Phase conjugation by degenerate four-wave mixing and temporal coherence

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The influence of the temporal-coherence properties of the pump and signal waves on the efficiency and the temporal fidelity of the phase-conjugation process associated with degenerate four-wave mixing is examined.

Analytical descriptions of optical phase conjugation, in both unbounded media[1] and optical fibers,[2] by means of degenerate four-wave mixing (DFWM) in Kerr-like media are usually obtained without taking into account the temporal fluctuations of the pump and signal waves associated with their finite coherence times T_p and T_s, respectively. Although this is justified by the tacit assumption that $L/V \ll T_p, T_s$ (L being the length over which nonlinear interaction takes place and V being the velocity of light in the medium), it is however possible to conceive situations in which, because of either the poor coherence properties of the sources employed or the length of the nonlinear medium, the above conditions are not both fulfilled. In this Letter, we treat these situations by working out the explicit expression of the reflected signal and by successively evaluating the significant statistical average, which permits us to show the influence of T_p, T_s and L/V on the efficiency and the temporal fidelity of the phase-conjugation process.

We write the electric field associated with the signal (+) and reflected (−) waves inside an optical fiber as

$$E_s{}^{\pm}(\mathbf{r}, t) = E_s(x, y)\exp[\mp i\beta_s(\omega_0)z + i\omega_0 t]\Phi_p{}^{\pm}(z, t), \tag{1}$$

where $E_s(x, y)$ is the transverse spatial configuration of a given propagation mode, β_s is the associated propagation constant, and $\Phi^{\pm}$ are slowly varying amplitudes. A similar expression is valid for the two counterpropagating pump waves, that is,

$$E_p{}^{\pm}(\mathbf{r}, t) = E_p(x, y)\exp[\mp i\beta_p(\omega_0)z + i\omega_0 t]\Phi_p{}^{\pm}(z, t). \tag{2}$$

Under the usual hypothesis $|\Phi_p| \gg |\Phi_s|$, the Φ's obey the two sets of equations[3,4]

$$\begin{aligned}[\partial/\partial z + (1/V_p)\partial/\partial t]\Phi_p{}^{+} &= iR_{pp}(|\Phi_p{}^{+}|^2 + 2|\Phi_p{}^{-}|^2)\Phi_p{}^{+},\\ [\partial/\partial z - (1/V_p)\partial/\partial t]\Phi_p{}^{-} &= iR_{pp}(|\Phi_p{}^{-}|^2 + 2|\Phi_p{}^{+}|^2)\Phi_p{}^{-}\end{aligned} \tag{3}$$

and

$$\begin{aligned}[\partial/\partial z + (1/V_s)\partial/\partial t]\Phi_s{}^{+} &= -2iR_{sp}(|\Phi_p{}^{+}|^2 + |\Phi_p{}^{-}|^2)\Phi_s{}^{+} - 2iR_{sp}\Phi_p{}^{+}\Phi_p{}^{-}\Phi_s{}^{-*},\\ [\partial/\partial z - (1/V_s)\partial/\partial t]\Phi_s{}^{-} &= 2iR_{sp}(|\Phi_p{}^{+}|^2 + |\Phi_p{}^{-}|^2)\Phi_s{}^{-} + 2iR_{sp}\Phi_p{}^{+}\Phi_p{}^{-}\Phi_s{}^{+*},\end{aligned} \tag{4}$$

where $V_s{}^{-1} = \mathrm{d}\beta_s/\mathrm{d}\omega$, $V_p{}^{-1} = \mathrm{d}\beta_p/\mathrm{d}\omega$ ($\omega = \omega_0$), and

$$R_{pp} = (\omega_0 n_2/c) \iint_{-\infty}^{+\infty} |E_p|^4 \mathrm{d}x\mathrm{d}y,$$

$$R_{sp} = (\omega_0 n_2/c) \iint_{-\infty}^{+\infty} |E_s|^2 |E_p|^2 \mathrm{d}x\mathrm{d}y, \tag{5}$$

n_2 being the nonlinear refractive index of the medium. A completely analogous set of equations describes propagation in unbounded media in the presence of nonuniform signal and pump waves.

The set of Eqs. (3) describing the pump evolution is uncoupled from the set of Eqs. (4) and can be preliminarily solved, the resulting expressions for $\Phi_p{}^{\pm}(z)$ then being inserted into the right-hand side of Eq. (4). By assuming that

$$\begin{aligned}\Phi_p{}^{+}(z = 0, t) &= \Phi_{\mathrm{op}}{}^{+}\exp[i\gamma(t)],\\ \Phi_p{}^{-}(z = L, t) &= \Phi_{\mathrm{op}}{}^{-}\exp[i\gamma(t)],\end{aligned} \tag{6}$$

L being the length of the interaction medium and $\gamma(t)$ being a stochastic phase accounting for the finite coherence time of the pump laser, Eqs. (4) can be rewritten, after the change of variables $\Phi_s{}^{+} = \exp(-iQz)\Phi_1$, $\Phi_s{}^{-*} = \exp(-iQz)\Phi_2$, with $Q = 4R_{sp}|\Phi_{op}|^2$ and $|\Phi_{op}{}^{+}| = |\Phi_{op}{}^{-}| \equiv |\Phi_{op}|$, is performed, in the form

$$[\partial/\partial z + (1/V_s)\partial/\partial t]\Phi_1 = -iTe^{i\theta}\Phi_2, \tag{7a}$$

$$[\partial/\partial z - (1/V_s)\partial/\partial t]\Phi_2 = -iT^*e^{-i\theta}\Phi_1, \tag{7b}$$

where $T = 2R_{sp}\Phi_{op}{}^{+}\Phi_{op}{}^{-}\exp(-3iR_{pp}|\Phi_{op}|^2L)$ and $\theta(z, t) = \gamma(t - z/V_p) + \gamma[t + (z - L)/V_p]$, which differs from the usual expression because of the presence of the factors $\exp(\pm i\theta)$ accounting for the statistical uncertainty associated with the pump waves. We can now

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obtain an integral equation for Φ_2 by formally integrating Eq. (7a), so that

$$\Phi_1(z,t) = -iT\int_0^z \exp\{i\theta[z', t + (z'-z)/V_s]\} \times \Phi_2[z', t+(z'-z)/V_s]dz' + \Phi_1(0, t - z/V_s), \quad (8)$$

by substituting the above expression into Eq. (7b) and by again formally integrating the resulting equation. This procedure yields

$$\begin{aligned}\Phi_2(z,t) = &-|T|^2\int_L^z dz' \int_0^{z'} dz'' \\ &\times \exp\{-i\theta[z', t + (z-z')/V_s]\} \\ &\times \exp\{i\theta[z'', t + (z-z')/V_s + (z''-z')/V_s]\} \\ &\times \Phi_2[z'', t + (z-z')/V_s + (z''-z')/V_s] - iT^* \\ &\times \int_L^z \exp\{-i\theta[z', t+(z-z')/V_s]\} \\ &\times \Phi_1[0, t + (z-z')/V_s - z'/V_s]dz', \end{aligned} \quad (9)$$

where we have set $\Phi_2(L,t) = 0$.

In the small-gain regime, $|T|L \ll 1$, we restrict ourselves in the following to consider only the second term on the right-hand side of Eq. (9). Furthermore, we set

$$\Phi_1(0,t) = S(t)e^{i\Psi(t)}, \quad (10)$$

where $S(t)$ represents the deterministic modulation (in both amplitude and phase) superimposed upon the amplitude-stabilized laser source, which excites the signal and exhibits a random phase fluctuation $\Psi(t)$ responsible for its finite bandwidth at $z = 0$. By limiting ourselves to situations in which the modulation time T_m of $S(t)$ is such that $T_m \gg L/V_s$ (this hypothesis corresponds to neglecting the filtering effects associated with the DFWM process[3,5,6]), we can finally write

$$\begin{aligned}\Phi_s{}^{-*}(0,t) &= iT^*S(t) \\ &\quad\times \int_0^L \exp[i\Psi(t - 2z'/V_s) \\ &\quad - i\theta(z', t - z'/V_s)]dz' \\ &= iT^*S(t)\exp[-i\gamma(t - L/V)] \\ &\quad\times\int_0^L \exp[-i\gamma(t - 2z'/V) \\ &\quad + i\Psi(t - 2z'/V)]dz' \equiv iT^*S(t)N(t), \end{aligned} \quad (11)$$

where we have assumed, for sake of simplicity, that $V_s \cong V_p \equiv V$.

According to Eq. (11), the backward signal wave is expressed by the product of a deterministic factor, which corresponds (in the small-gain limit) to perfect phase conjugation, times a noise factor $N(t)$, which accounts for the existence of a certain degree of randomness in the signal and pump waves. According to standard statistical procedures, in these cases one has to evaluate the autocorrelation function

$$H(\tau) = \langle N(t)N^*(t+\tau)\rangle, \quad (12)$$

where the angular brackets stand for an ensemble average (or time average over an interval long compared with the typical fluctuation times of the pump and signal waves). The value $H(0)$ at $\tau = 0$ and the correlation time τ_c of $H(\tau)$ give, respectively, estimates of the noise-induced reduction of the efficiency of the process and of the deterioration of its temporal fidelity. In order to perform the ensemble average appearing in Eq. (11), we assume that the pump and the signal are excited by two distinct laser sources, and we take advantage of the relation

$$\begin{aligned}&\langle\exp[i\gamma(t+\tau) - i\gamma(t)]\rangle \\ &= \exp\left[-\tfrac{1}{2}\iint_{00}^{\tau\tau}\langle\delta\omega_p(t')\delta\omega_p(t'')\rangle dt'dt''\right] \\ &= \exp(-\tfrac{1}{2}\langle\delta\omega_p{}^2\rangle\tau^2), \end{aligned} \quad (13)$$

which holds true provided that the instantaneous frequency fluctuations $\dot{\gamma} = \delta\omega_p$ are Gaussian distributed and that the correlation time of $\langle\delta\omega_p(t')\delta\omega_p(t'')\rangle$ is much longer than all the relevant times appearing in our equations. By using Eq. (13) and the analogous relation $\langle\exp[i\psi(t+\tau) - i\psi(t)]\rangle = \exp(-\tfrac{1}{2}\langle\delta\omega_s{}^2\rangle\tau^2)$ valid for the signal wave, we obtain

$$\begin{aligned}H(\tau) \equiv G_\xi(\eta) = &(\pi^{1/2}/2\xi)\exp(-\alpha\xi^2\eta^2) \\ &\times\{(1+\eta)\,\mathrm{erf}[\xi(1+\eta)]\,\mathrm{erf}[\xi(1-\eta)] - 2\eta\,\mathrm{erf}(\eta)\} \\ &+ (1/2\xi^2)\exp(-\alpha\xi^2\eta^2)\{\exp[-\xi^2(1-\eta^2)] \\ &+ \exp[-\xi^2(1+\eta^2)] - \exp(-\xi^2\eta^2)\}, \end{aligned} \quad (14)$$

where $\eta = \mu\tau/(L/V)$, with $\mu = (2\langle\delta\omega_p{}^2\rangle + \langle\delta\omega_s{}^2\rangle)/[2(\langle\delta\omega_p{}^2\rangle + \langle\delta\omega_s{}^2\rangle)]$, represents an adimensional time, $\alpha = \langle\delta\omega_p{}^2\rangle\langle\delta\omega_s{}^2\rangle/(2\langle\delta\omega_p{}^2\rangle + \langle\delta\omega_s{}^2\rangle)^2$, and $\xi = L/l$ is an adimensional parameter furnishing the ratio between the interaction length L and the longitudinal coherence length l defined by the relation

$$\begin{aligned} l &= V/(2\langle\delta\omega_p{}^2\rangle + 2\langle\delta\omega_s{}^2\rangle)^{1/2} \\ &= V/(2\pi\sqrt{2})(1/T_p{}^2 + 1/T_s{}^2)^{1/2}, \end{aligned} \quad (15)$$

with $T_{p,s} = 2\pi/\langle\delta\omega_{p,s}{}^2\rangle^{1/2}$.

As was mentioned above, $G_\xi(0)$ furnishes, as a function of ξ, the reduction in efficiency of the DFWM process associated with temporally limited coherence, whereas $G_\xi(\eta)$ gives, for any fixed ξ, an estimate of the

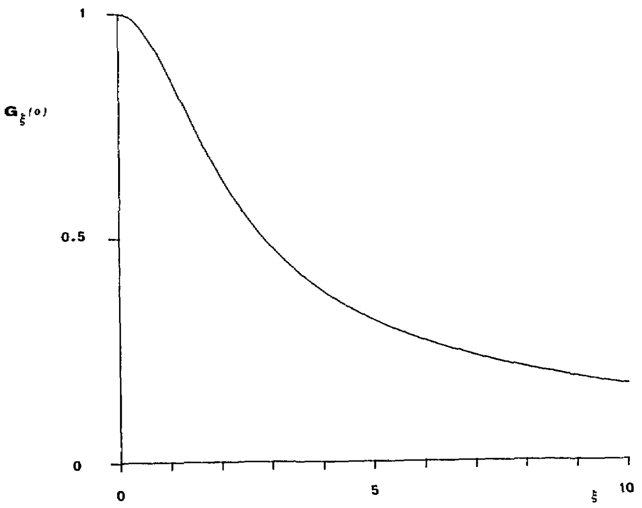

Fig. 1. $G_\xi(0)$ as a function of ξ.

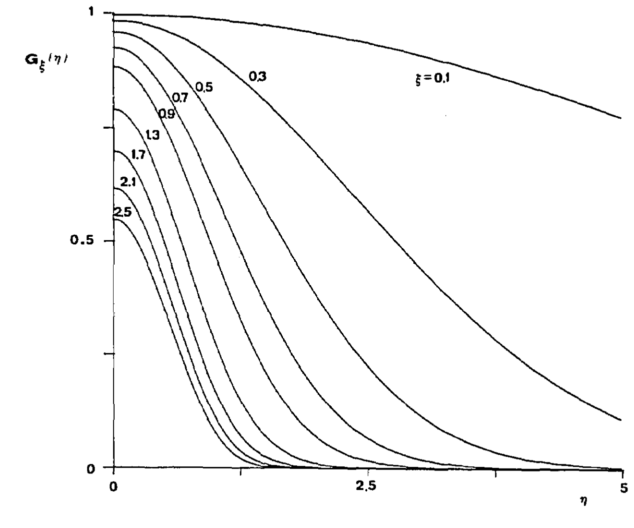

Fig. 2. $G_\xi(\eta)$ as a function of η for different values of the parameter ξ, with $\alpha = 0$.

fidelity of the process in reproducing a phase-conjugate replica of the input signal [in particular, $G_0(\eta) = 1$ for all η]. In Fig. 1, $G_\xi(0)$ is plotted as a function of ξ. Analysis of Fig. 1 shows that the efficiency is not significantly reduced until l becomes smaller than L ($\xi > 1$); for $\xi \gg 1$, $G_\xi(0) \cong \sqrt{\pi}/\xi$, and the efficiency drops as $1/\xi$. A faithful conjugate replica of the input modulation $S(t)$ is achieved whenever the characteristic time of the stochastic function $N(t)$ is much larger than that of $S(t)$ [see Eq. (11)]. Thus a sound criterion for temporal fidelity is furnished by the comparison between the correlation time τ_c and the modulation time T_m, good reproduction obviously being related to fulfillment of the condition that $\tau_c \gg T_m$. In Fig. 2, $G_\xi(\eta)$ is plotted for different values of ξ as a function of η. Although it refers to the case $\alpha = 0$ (corresponding to either pump or signal free from statistical uncertainty), it possesses a fairly general validity for $\xi \lesssim 1$ since $0 \leq \alpha \leq 1/8$ and the factor $\exp(-\alpha\xi^2\eta^2)$, through which α appears in the expression of $G_\xi(\eta)$, can in practice affect its behavior only in extremely incoherent situations ($\xi \gg 1$). In this last case and for α not too small (say, 1/10), $G_\xi(\eta)$ can be approximated by the expression

$$G_\xi(\eta) = (\sqrt{\pi}/\xi)\exp(-\alpha\xi^2\eta^2), \tag{16}$$

whose correlation time is $\tau_c = (1/\sqrt{\alpha})(l/V)$. In order to have a good temporal fidelity, one should then have $(1/\sqrt{\alpha})(l/V) \gg T_m$, which contradicts the assumption of $T_m \gg L/V$ underlying our analytical derivation. Accordingly, this extremely incoherent situation corresponds to a complete lack of temporal fidelity.

In conclusion, we have investigated the influence of the finite longitudinal coherence lengths of the signal and pump waves on the quality of the phase-conjugation process associated with DFWM in Kerr-like media. This has been achieved by explicitly evaluating the backward-traveling signal wave and by successively calculating its autocorrelation function. A different approach, based on a heuristic model for third-order susceptibility and on the assumption that the evaluation of the ensemble average of third-order polarizability is sufficient for estimating the efficiency of the phase-conjugation process, has been considered in Ref. 7. Although this last approach has the advantage of explicitly introducing the decay times of the phase gratings, thus also being suitable for the description of non-Kerr-like media, it does not seem to agree, both qualitatively and quantitatively, with the results of our rigorous method.

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