Influence of chromatic dispersion on degenerate four-wave mixing

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We investigate the way in which four-wave mixing in a waveguide is affected by chromatic dispersion. The frequency dependence of the associated phase-conjugate mirror reflectivity turns out not to be influenced by chromatic dispersion itself.

Degenerate four-wave mixing is investigated both in the stationary regime[1] (in which dispersion has no role) and in the nonstationary regime (in which the time behavior of the phase-conjugate signal is important, as when one is studying the frequency response of a phase-conjugate mirror[2]), without taking into account chromatic dispersion. Although this circumstance can be *a priori* justifiable over the short crystal samples in which four-wave mixing is usually performed, one is conversely led to think that chromatic dispersion can no longer be ignored when the process takes place in long glass fibers.

In this Letter, we study the time-dependent behavior of the phase-conjugate reflected signal by including in the equation describing its evolution the contribution that is due to chromatic dispersion, that is, a term containing $\beta'' = d^2\beta/d\omega^2$, β being the propagation constant of the incident signal inside the nonlinear medium [$\beta(\omega) = n(\omega)\omega/c$, where $n(\omega)$ is the material refractive index, if the medium is unbounded]. It turns out that this behavior (or, equivalently, the frequency response of the associated phase-conjugate mirror) is independent of β'', irrespective of the distance traveled in the medium.

Let us write the analytic signals of the electric fields associated with the incident and reflected waves in the form (see Fig. 1)

$$\hat{E}_i(\mathbf{r}, t) = E(x, y) \exp[-i\beta(\omega_0)z + i\omega_0 t]\Phi_i(z, t), \quad (1)$$

$$\hat{E}_r(\mathbf{r}, t) = E(x, y) \exp[i\beta(\omega_0)z + i\omega_0 t]\Phi_r(z, t), \quad (2)$$

where ω_0 is the central angular frequency, $\beta(\omega_0)$ is the associated propagation constant, and Φ_i and Φ_r are slowly varying amplitudes. The system of equations describing degenerate four-wave mixing, in the presence of a nondepleted continuous-wave pump, is[3]

$$\begin{aligned}[\partial/\partial z + (1/V)\partial/\partial t - (i/2A)\partial^2/\partial t^2]\Phi_i(z, t) &= -iS\Phi_i(z, t) - iP\Phi_r^*(z, t),\\ [\partial/\partial z - (1/V)\partial/\partial t + (i/2A)\partial^2/\partial t^2]\Phi_r(z, t) &= iS\Phi_r(z, t) + iP\Phi_i^*(z, t), \quad (3)\end{aligned}$$

where

$$V = (d\beta/d\omega)^{-1}_{\omega=\omega_0}, \qquad A = (d^2\beta/d\omega^2)^{-1}_{\omega=\omega_0} \quad (4)$$

represent, respectively, the group velocity and the group-velocity dispersion inside the medium, which generalizes the system of equations describing degenerate four-wave mixing in thin media[1] (where one can put $1/V = 1/A = 0$) and in relatively thick media[4,5] ($1/A = 0$). The coefficients S and P are given by[3]

$$S = 2\alpha\omega_0(n_2/c)(|\hat{E}_p{}^{(+)}|^2 + |\hat{E}_p{}^{(-)}|^2) \quad (5)$$

and

$$P = 2\alpha\omega_0(n_2/c)\hat{E}_p{}^{(+)}\hat{E}_p{}^{(-)}, \quad (6)$$

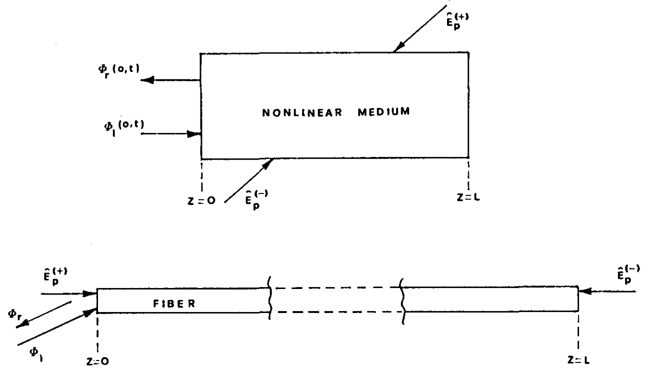

Fig. 1. Four-wave mixing geometry in a short medium and in a long optical fiber.

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where n_2 is the nonlinear refractive index of the medium and α is a suitable superposition integral between the normalized intensities of the pump and signal modes. If P does not depend on z [which can be shown[3] to hold true whenever $|\hat{E}_p{}^{(+)}(z = 0)|^2 = |\hat{E}_p{}^{(-)}(z = 0)|^2$], the system of Eqs. (3) can be solved exactly by employing the standard normal-mode expansion, that is, by looking for a solution of the kind

$$\Phi_i(z, t) = \tilde{\Phi}_i(\omega) \exp[ik(\omega)z - i\omega t],$$
$$\Phi_r{}^*(z, t) = \tilde{\Phi}_r{}^*(\omega) \exp[ik(\omega)z - i\omega t]. \quad (7)$$

By proceeding in this way, we obtain

$$\Phi_i(z, t) = e^{-iSz} \times \int_{-\infty}^{+\infty} d\omega G(\omega) \exp[-i(\omega^2 z/2A) - i\omega t] \times \frac{i(\omega/V) \sin[f(\omega)(z - L)] + f(\omega) \cos[f(\omega)(z - L)]}{f(\omega) \cos[f(\omega)L] - (i\omega/V) \sin[f(\omega)L]}, \quad (8)$$

$$\Phi_r{}^*(r, t) = -(i|P|^2/P^2)e^{-iSz} \times \int_{-\infty}^{+\infty} d\omega G(\omega) \exp[-i(\omega^2 z/2A) - i\omega t] \times \frac{\sin[f(\omega)(z - L)]}{f(\omega) \cos[f(\omega)L] - (i\omega/V) \sin[f(\omega)L]}, \quad (9)$$

where $f(\omega) = (|P|^2 + \omega^2/V^2)^{1/2}$ and

$$G(\omega) = (1/2\pi) \int_{-\infty}^{+\infty} \Phi_i(z = 0, t)e^{i\omega t}dt. \quad (10)$$

In particular, at $z = 0$ we have

$$\Phi_r{}^*(z = 0, t) = \int_{-\infty}^{+\infty} d\omega G(\omega)R(\omega)e^{-i\omega t}, \quad (11)$$

having introduced the complex frequency-dependent reflectivity

$$R(\omega) = -i \frac{|P|^2}{P^2} \frac{\tan[f(\omega)L]}{(i\omega/V) \tan[f(\omega)L] - f(\omega)}. \quad (12)$$

Remarkably, the factor $1/A$ accounting for chromatic dispersion does not appear in Eq. (12). As a matter of fact, Eq. (12) is equivalent to that worked out in Ref. 2, in which the authors consider a dispersionless medium.

Chromatic dispersion affects four-wave mixing only if one takes into account the presence of higher-order group dispersion, which is expressed by terms of the kind

$$B = (d^3\beta/d\omega^3)^{-1}_{\omega=\omega_0}, \qquad C = (d^4\beta/d\omega^4)^{-1}_{\omega=\omega_0}, \ldots. \quad (13)$$

In this case, the system of Eqs. (3) has to be modified by adding on the left-hand side the differential terms[3]

$$\mp \frac{1}{3!B} \frac{\partial}{\partial t^3} \pm \frac{i}{4!C} \frac{\partial}{\partial t^4} \mp \ldots \quad (14)$$

(the upper and the lower signs applying, respectively, to the first and the second equations). It turns out, however, that only the terms containing odd derivatives with respect to time are capable of affecting the complex reflectivity $R(\omega)$; in other words, if one could neglect all these terms [including $(1/V)\partial/\partial t$], degenerate four-wave mixing would give rise to an exact time-reversed replica of the incident signal.

On physical grounds, the lack of influence of even-order group dispersion on complex reflectivity can be associated with the property of four-wave mixing of providing a time-reversed replica of the signal. In fact, even-order dispersion corresponds, in the time domain, to terms that are invariant in the exchange $t \rightarrow -t$.

References

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