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IN DEFENSE OF UNANIMOUS JURY VERDICTS: MISTRIALS, COMMUNICATION, AND SINCERITY

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Abstract

It is a widely held belief among legal theorists that the requirement of unanimous jury verdicts in criminal trials reduces the likelihood of convicting an innocent defendant. This belief is, to a large extent, dependent upon the assumption that all jurors will vote sincerely based on their own impression of the trial evidence. Recent literature, however, has drawn this assumption into question, and simple models of jury procedure have been constructed in which, except under very strict conditions, it is never a Nash equilibrium for all jurors to vote sincerely. Moreover, Nash equilibrium behavior in these models leads to higher probabilities of both convicting an innocent defendant and acquitting a guilty defendant under unanimity rule than under a wide variety of alternative voting rules, including simple majority rule. The present paper extends these models by adding minimal enhancements that we argue bring the existing models closer to actual jury procedures. In particular, we separately analyze the implications of (1) incorporating the possibility of mistrial and (2) allowing limited communication among jurors. Under each of these enhancements, we identify general conditions under which sincere voting is, in fact, a Nash equilibrium. We further demonstrate that under such sincere voting equilibria, unanimous jury verdicts perform better than any alternative voting rule in terms of minimizing probability of trial error and maximizing expected utility.

In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Sincerity*

Peter J. Coughlan**

1 INTRODUCTION

It is a widely held belief among legal theorists that the requirement of unanimous jury verdicts in criminal trials reduces the likelihood of convicting an innocent defendant. This belief is, to a large extent, dependent upon the assumption that all jurors will vote sincerely -- that is, if the trial evidence leads a juror to believe that the defendant is innocent, the juror will vote to acquit, while if the evidence leads a juror to believe that the defendant is guilty, the juror will vote to convict. Recent literature, however, has suggested that the assumption of sincere voting by jurors may be inconsistent with Nash equilibrium behavior and has thus drawn into question the supposed benefits of unanimous jury verdicts.

The use of juries in criminal trials is based, at least in part, upon the belief that, when all individuals possess a common preference for selecting the "better" of two alternatives (in this case, conviction or acquittal), a group is more likely than any single individual to select the preferred option. This is the central argument behind the extensive literature that has developed based on Condorcet's Jury Theorem [Condorcet 1795/1976, Grofman and Feld 1988, Klevorick, Rothschild, and Winship 1984, Miller 1986, and Young 1988]. Analysis and extensions of this theorem have generally been statistical in nature, however, taking individual probabilities of correct decisions to be exogenously determined [Berg 1993; Ladha 1992, 1993, 1995]. An implicit element of this approach is the assumption that individuals behave in the same manner when they are acting as a dictator as when they are participating in a group decision process. In

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the framework of jury decision-making, this is equivalent to assuming that jurors vote sincerely based on their private information (and perhaps shared public information).

In a recent paper, however, Austen-Smith and Banks [1996] illustrate that sincere voting in such group decisions may be inconsistent with Nash equilibrium behavior under fairly general conditions. In response, McLennan [1996] and Wit [1996] have attempted to rehabilitate the central notion of Condorcet's Jury Theorem, by identifying reasonable conditions under which Nash equilibrium behavior, though it may be "insincere," still predicts that groups are more likely to make correct decisions than individuals.

Feddersen and Pesendorfer [1997] have adapted the general framework of Austen-Smith and Banks to the specific case of jury procedures in criminal trials and, in doing so, have derived some surprising results about unanimous jury verdicts. Feddersen and Pesendorfer construct a model of the jury process in which, except under very strict conditions, it is never a Nash equilibrium for all jurors to vote sincerely. Moreover, Nash equilibrium behavior in this model leads to higher probabilities of both convicting an innocent defendant and acquitting a guilty defendant under unanimity rule than under a wide variety of alternative voting rules, including simple majority rule. They conclude that, if their model is accurate, the societal objective of avoiding such jury errors may be better served by eliminating the requirement of unanimous verdicts in criminal cases.

The present paper extends the Feddersen and Pesendorfer model by adding certain minimal enhancements that we argue bring the model closer to actual jury procedures. In particular, we separately analyze the implications of (1) incorporating the possibility of mistrial and (2) allowing limited communication among jurors. Under each of these enhancements, we identify general conditions under which sincere voting is, in fact, a Nash equilibrium. We further demonstrate that under such sincere voting equilibria, the conclusion of the inferiority of unanimous jury verdicts does not persist. That is, if the possibility of either mistrial or limited communication is introduced, it is no longer the case that unanimous jury verdicts generally produce equilibrium probabilities of convicting an innocent defendant and acquitting a guilty defendant that are higher than under alternative voting rules. Moreover, within the sincere voting equilibria, unanimity rule maximizes ex ante expected utility for all jurors under these model enhancements.

2 THE BASIC MODEL

We first introduce the basic model of jury procedure which was analyzed by Feddersen and Pesendorfer and, more generally, by Austen-Smith and Banks. This model will serve as a point of departure and source of comparison for the new jury models introduced in this paper.

2.1 Basic Theoretical Framework

It is assumed that there are n jurors who will vote to determine the fate of a defendant. The set of jurors will be denoted by $N = \{1,2,...,n\}$ with an individual juror being represented by $j \in N$. There are two possible states of the world: the defendant is either guilty or innocent. We denote by G the state of the world in which the defendant is guilty and by I the state in which the defendant is innocent. Each state is assumed to occur with equal probability.

There are two possible outcomes of the jury vote: the defendant is convicted, denoted C, or the defendant is acquitted, denoted A. Each juror can either vote to convict (C) or acquit (A) the defendant. All votes are done by secret ballot and no abstentions are allowed. We will represent by $|C|$ the total number of votes for conviction and by $|A|$ the total number of votes for acquittal. In addition, $|C|_{-j}$ will denote the number of votes for conviction among all jurors other than j, or $N/\{j\}$, while $|A|_{-j}$ will denote the number of votes for acquittal among $N/\{j\}$.

A voting rule is described by a threshold $\hat{k}$, which is an integer between 0 and n. If $|C| \geq \hat{k}$ the defendant is convicted, and the defendant is acquitted otherwise. Unanimity rule is represented by the voting rule $\hat{k} = n$, while simple majority rule is represented by the voting rule with $\hat{k}$ equal to the smallest integer greater than $n/2$.

The impact of the trial evidence is represented by a private signal received by each juror. We will denote by s_j the signal received by juror j. There are two possible signals, g or i, and the signal is correlated with the true state of the world. In particular, for all j, $\text{Prob}(s_j=g \mid G) = \text{Prob}(s_j=i \mid I) = p \in (0.5,1.0)$. Thus, the parameter p is the probability that a juror receives the "correct" signal (g in state G and i in state I) and 1-p is the probability that the juror receives the "incorrect" signal (i in state G and g in state I). We will denote by $|g|$ the total number of g signals received and by $|i|$ the total number of i signals received. In addition, $|g|_{-j}$ will denote the number of g signals among $N/\{j\}$ while $|i|_{-j}$ will denote the number of i signals among $N/\{j\}$.

Note that, although juror signals are drawn independently given the true state of the world, they are correlated to each other in the sense that $\text{Prob}(s_j=g \mid s_i=g) = \text{Prob}(s_j=i \mid s_i=i) = p^2+(1-p)^2 > 1/2 > 2p(1-p) = \text{Prob}(s_j=g \mid s_i=i) = \text{Prob}(s_j=i \mid s_i=g)$. In other words, juror i's signal provides her information about juror j's signal and, in particular, she believes that juror j is more likely to have a signal that matches her own signal than one that does not.

Let $u_j(O,S)$ be juror j's utility given outcome O in state S. It is assumed that $u_j(C,G) = u_j(A,I) = 0$, $u_j(C,I) = -q_j$, and $u_j(A,G) = -(1-q_j)$ where $q_j \in (0,1)$. Note that, with this construction, any juror j will prefer conviction to acquittal whenever she believes the probability that the defendant is guilty is greater than q_j. In this sense, $1-q_j$ is a measure of what juror j considers to be "reasonable doubt." Also note that the analysis of the basic model presented by Feddersen and Pesendorfer uses common utilities (i.e. $q_i=q_j$ for all $i,j \in N$), although they state that this is purely for technical convenience. To assure the generality of our results, we will use individual utilities in all of our analysis.

We will denote by $\beta(k,n)$ the posterior probability that the defendant is guilty conditional on k of n guilty signals:

$$\beta(k,n) = \frac{p^k(1-p)^{n-k}}{p^k(1-p)^{n-k} + p^{n-k}(1-p)^k}$$

It is assumed that for all $j \in N$, there is a k^* with $n \geq k^* \geq 1$ such that $\beta(k^*-1,n) \leq q_j \leq \beta(k^*,n)$. This assumption rules out jurors who will always want to convict or always want to acquit the defendant regardless of the number of innocent and guilty signals. Thus, if it is known that $|g|=n$, all jurors will want to convict the defendant, while if it is known that $|i|=n$, all jurors will want to acquit the defendant.

The behavior of a given juror j is described by a strategy mapping, σ_j: $(0,1)\times\{g,i\} \rightarrow [0,1]$, with $\sigma_j(q_j,s_j)$ being the probability of voting to convict given utility parameter q_j and signal s_j. *Sincere voting* is defined as voting to convict whenever a guilty signal is received and voting to acquit whenever an innocent signal is received. In other words, the sincere voting strategy for juror j is given by:

$$\sigma_j(q_j,s_j) = \begin{cases} 1 \text{ if } s_j = g \\ 0 \text{ if } s_j = i \end{cases}$$

Note that sincere voting, defined in this manner, is a naive form of voting, since "voting one's signal" may be inconsistent with expected utility maximization for some jurors. For this reason, we also define *sophisticated sincere voting*. A strategy for juror j is considered sophisticated sincere voting when it consists of voting for the trial outcome which maximizes her expected payoff conditional on her signal and any other revealed signals. Thus, the general form of the sophisticated sincere voting strategy for juror j is given by:

$$\sigma_j(q_j, s_j) = \begin{cases} 1 \text{ if } q_j < \text{Prob}(G) \\ 0 \text{ if } q_j \geq \text{Prob}(G) \end{cases}$$

If juror j knows only signal s_j, her sophisticated sincere voting strategy becomes:

$$\sigma_j(q_j, s_j) = \begin{cases} 1 \text{ if } (s_j = g \ \& \ q_j < p) \text{ or } (s_j = i \ \& \ q_j < 1-p) \\ 0 \text{ if } (s_j = g \ \& \ q_j \geq p) \text{ or } (s_j = i \ \& \ q_j \geq 1-p) \end{cases}$$

If all n signals are revealed, the sophisticated sincere voting strategy for juror j is given by:

$$\sigma_j(q_j, s_j) = \begin{cases} 1 \text{ if } q_j < \beta(|g|, n) \\ 0 \text{ if } q_j \geq \beta(|g|, n) \end{cases}$$

2.2 Assumptions and Conclusions of the Basic Model

In analyzing this basic model, Feddersen and Pesendorfer make several assumptions to eliminate potential equilibria that do not satisfy certain normative criteria. In particular, they eliminate from consideration asymmetric equilibria and equilibria in which a juror's strategy is independent of the signal received. Certain restrictions are also placed upon the relationship between the parameters p and q_j. In particular, it is assumed for all $j \in N$ that:

$$1 - p \leq q_j \leq \beta(n-1, n) = \frac{p^{n-2}}{p^{n-2} + (1-p)^{n-2}}$$

The lower bound on q_j above is not particularly restrictive. To see this, note that violation of this bound means that $q_j < 1\text{-}p < 0.5$, and therefore that juror j prefers to convict even in cases where it is more likely that the defendant is innocent. Such juror preferences should rarely, if ever, exist, and thus requiring q_j to be greater than 1-p is a reasonable restriction. The upper bound on q_j is also relatively permissive. This bound says only that n-1 guilty signals (versus only one innocent signal) would be sufficient information for all jurors to prefer conviction.

With these assumptions placed on the basic model, Feddersen and Pesendorfer demonstrated that, for any voting rule $\hat{k}$, there does not exist a Nash equilibrium in which all jurors vote sincerely and that there is instead a unique mixed strategy equilibrium. Moreover, as the size of the jury increases towards infinity, equilibrium behavior under unanimity rule will lead to higher probabilities of both convicting an innocent defendant and acquitting a guilty defendant than under non-unanimous voting rules. Feddersen and Pesendorfer also show by example ($n=12$, $p=0.8$, and $q_j=0.9$ for all j) that this relationship can hold for smaller juries under fairly reasonable conditions (including $\hat{k} > n/2$).

The key to understanding these somewhat surprising results is to recognize that each juror will condition her strategy on the event that she is pivotal. In other words, each juror will behave as if she knew that her vote would determine the outcome of the trial. In the case of unanimity rule, this means that each juror will behave as if all other jurors are voting to convict the defendant. It is therefore not difficult to see that, regardless of one's own signal, being pivotal provides a strong incentive to vote for conviction since all other jurors are doing the same.

For non-unanimous rules, on the other hand, being pivotal may provide much less compelling information. For simple majority rule (with n odd), being pivotal means only that an equal number of the other jurors are voting in each direction. This information is not overwhelming for either guilt or innocence, and can therefore be expected to have much less influence on juror voting.

To explicitly demonstrate that sincere voting is not a Nash equilbrium under unanimity rule, suppose that all jurors do vote sincerely and consider the situation in which juror j receives an innocent signal ($s_j=i$). It is easy to see that juror j has a positive incentive to deviate from sincerity and vote to convict in this case. First note that since juror j will condition her vote on being pivotal, she will behave as if all other jurors are voting to convict. When jurors vote sincerely, this means that all other n-1 jurors received guilty signals and that juror j received the only innocent signal. Juror j's perceived probability of guilt is therefore $\beta(n-1,n)$ in this case. However, by assumption, $q_j \leq \beta(n-1,n)$ and thus juror j prefers conviction to acquittal. Hence, juror j has an incentive to deviate from sincerity, and therefore sincere voting is not a Nash equilibrium under unanimity rule in the basic model.

3 THE MISTRIAL MODEL

The first significant limitation of the basic model involves the delineation of trial outcomes. The basic model assumes that there are only two possible outcomes of the jury process: conviction or acquittal. Under unanimity rule, for example, a defendant is convicted if and only if all jurors vote for conviction, and the defendant is acquitted otherwise. In actual practice, however, almost all jurisdictions require unanimity to either convict or acquit a defendant in a criminal trial (see Schwartz and Schwartz 1992). If the jury vote results in neither a unanimous vote to convict nor a unanimous vote to acquit, then there is a "hung jury." If the hung jury situation persists through deliberations, a mistrial is declared and a new trial can be expected to take place. If the jury process is to be represented by a single vote, any non-unanimous vote would then immediately result in a mistrial.

3.1 Existence of Sincere Voting Equilibria with Exogenous Mistrial Utilities

Thus, consider an enhancement to the basic model in which there are three possible outcomes of the jury process: the defendant is convicted (C), the defendant is acquitted (A), or a mistrial is declared (M). A voting rule is still described by a threshold $\hat{k}$, however $\hat{k}$ must now be an integer between $^n/_2$ and n. If $|C| \geq \hat{k}$ the defendant is convicted, if $|A| \geq \hat{k}$ the defendant is acquitted, and a mistrial is declared otherwise. Note that $\hat{k}$ must be strictly greater than $^n/_2$, because if $\hat{k} = {}^n/_2$ then the trial outcome is indeterminate when $|C| = |A| = \hat{k}$.

Let $u_j(M,G) = -m_j^G$ $u_j(M,I) = -m_j^I$. We will make the natural assumption that the utility of a mistrial is strictly between the utilities of acquittal and conviction. That is, $0 < m_j^I < q_j$ and $0 < m_j^G < (1-q_j)$. In the next section, we will endogenize these mistrial utilities by equating them with the expected value of a new trial in a repeated trial process. For now, however, it is instructive to consider these mistrial utilities as exogenously determined.

Before proceeeding, it should be noted that Schwartz and Schwartz (1992) have also analyzed the impact of alternative voting rules within a model of jury procedure allowing for the possibility of mistrial. The Schwartz model, however, takes a very different approach, in which jurors have single-peaked preferences over a range of possible charges and the key choice variable is the prosecutorial decision about which charge (or charges) to prosecute.

The first result in the analysis of the current "mistrial model" presents the necessary and sufficient conditions for sincere voting to be a Nash equilibrium.

Proposition 1: Suppose that n is odd and that $\hat{k} = \frac{n+1}{2}$ (simple majority rule). In this case, sincere voting is a Nash equilibrium in the mistrial model if and only if $1\text{-}p \le q_j \le p$ for any juror $j \in N$. In all other cases, sincere voting is a Nash equilibrium if and only if, for any juror $j \in N$:

(a) $\left(p - q_j\right)\left(1-p\right)^{2\hat{k}-n-1} + \left(m_j^G p - m_j^I\left(1-p\right)\right)\left(p^{2\hat{k}-n-1} - \left(1-p\right)^{2\hat{k}-n-1}\right) \ge 0$ and

(b) $\left(p - \left(1 - q_j\right)\right)\left(1-p\right)^{2\hat{k}-n-1} + \left(m_j^I p - m_j^G\left(1-p\right)\right)\left(p^{2\hat{k}-n-1} - \left(1-p\right)^{2\hat{k}-n-1}\right) \ge 0.$

Proof: See Appendix.

The key element in the proof of Proposition 1 that distinguishes the predictions of the mistrial model from the predictions of the basic model is the understanding of what it means to be pivotal in the two different models. To illustrate the distinction, consider the case of unanimity rule. In the basic model under unanimity rule, a juror is pivotal only when all other jurors are voting to convict. This provides a strong incentive to vote for conviction, even for those jurors who receive an innocent signal. In the mistrial model, on the other hand, a juror is pivotal in two different cases: when all other jurors are voting to convict <u>and</u> when all other jurors are voting to acquit. Moreover, given an innocent signal in the mistrial model, a juror will believe that it is more likely that all other jurors are voting to acquit than that all other jurors are voting to convict. This provides such a juror a greater incentive to vote sincerely. The same is true for jurors who receive a guilty signal.

Although the conditions of Proposition 1 are fairly general, the structure of the inequalities in the proposition makes it difficult to immediately characterize all of the parameter values for which the proposition is satisfied. It is therefore helpful to examine more straightforward conditions that are simply sufficient (but not necessary) for sincere voting to be a Nash equilibrium in the mistrial model.

One set of such sufficient conditions is the following:

$$1-p \le q_j \le p \quad \text{and} \quad \frac{1-p}{p} \le \frac{m_j^G}{m_j^I} \le \frac{p}{1-p}.$$

These conditions indicate that sincere voting is a Nash equilibrium whenever: (a) the utility of the two "incorrect" trial outcomes (convicting the innocent and acquitting the guilty) are not significantly different, and (b) the utility of the two mistrial outcomes are not significantly

different. Depending upon the value of p, these conditions can be very general or rather restrictive, but they nonetheless illustrate that many non-trivial parameter values will satisfy Proposition 1.

It is important to note, however, that there are many parameter values that satisfy the conditions of Proposition 1 yet do not satisfy the easy-to-understand sufficient conditions specified above. For example, consider the Feddersen and Pesendorfer example in which n=12, p=0.8, and q_j=0.9 for all j (note that this example violates the above conditions). Under unanimity rule in this case, the conditions of Proposition 1 reduce approximately to:

$$\frac{1}{4} < \frac{m_j^G}{m_j^I} < 4.$$

This means that sincere voting will be a Nash equilibrium for this example so long as the utility (or disutility) of one mistrial outcome is not more than four times as large as the utility (or disutility) of the other mistrial outcome.

Further recognize that the existence of the sincere voting Nash equilibrium in this model once again does not require that juror utilities be common knowledge. Instead, it is sufficient for a given juror to know only her own utility function and that the other jurors vote sincerely (or, alternatively, that the other juror utilities satisfy conditions (a) and (b) of the proposition).

3.2 Existence of Sincere Voting Equilibria with Endogenous Mistrial Utilities

To further develop this mistrial model, we would like to endogenize the mistrial utilities, m_j^G and m_j^I, by specifying juror perceptions about the consequences of mistrial. These perceptions might incorporate many different factors, but it seems reasonable to model the utility of mistrial as simply the expected utility of an additional trial before a new jury.* In other words, we have:

$$m_j^G = (1 - q_j) \cdot \mathrm{Prob}_S(A \mid G) + m_j^G \cdot \mathrm{Prob}_S(M \mid G)$$
$$m_j^I = q_j \cdot \mathrm{Prob}_S(C \mid I) + m_j^I \cdot \mathrm{Prob}_S(M \mid I)$$

* We could also discount the expected utility of future trials or apply a fized cost/disutility to each new trial. Mistrial utilities incorporating these factors still allow us to calculate refined necessary and sufficient conditions for sincere voting, however analysis of such utility structures significantly increases the complexity of the presentation while providing minimal additional insight.

where $Prob_S(O|S)$ is the probability of outcome O in a single trial when the state of the world is S.

When the utility of mistrial is specified in this manner, the conditions for the existence of a sincere voting Nash equilibrium are simplified significantly:

Proposition 2: Suppose that the utility of mistrial is equal to the expected utility of an additional trial before a new jury. Sincere voting is then a Nash equilibrium in the mistrial model for any voting rule $\hat{k}$ if and only if $1\text{-}p \leq q_j \leq p$ for all $j \in N$.

Proof: See Appendix.

Note that Propositions 1 and 2 both suggest that the occurrence of sincere voting among jurors may increase as the "accuracy" of trials improves. As p increases, and thus trials become more truth revealing, all of the conditions of Propositions 1 and 2 become easier to satisfy, and thus sincere voting Nash equilibria will exist for more juries and more trials. This provides an additional argument for legal reforms that may be expected to improve the likelihood that the true state of the world, guilt or innocence, is revealed at trial.

It is helpful to discuss further the condition $1\text{-}p \leq q_j \leq p$ from Proposition 2, because this condition will appear again later in the paper. This constraint can be interpreted as the "one-man jury condition," because it is the same condition that would be required for a one-man jury to ever render a meaningful verdict. To see this, consider a jury consisting of a single juror j. If $q_j \leq 1\text{-}p$, then all defendants will be convicted, no matter which signal is received by juror j. Similarly, if $p \leq q_j$, then all defendants will be acquitted, no matter which signal is received by juror j. Thus, for this one-man jury to ever render a meaningful verdict (i.e., one that varies depending upon what happens at trial), it must be the case that $1\text{-}p \leq q_j \leq p$.

3.3 Comparison of Alternative Voting Rules

Once the existence of sincere voting Nash equilibria is established, it is important to compare the performance of alternative voting rules in terms equilibrium outcomes. One possible performance measure is the probability of a trial error, in other words, the probability of convicting an innocent defendant or acquitting a guilty defendant. Proposition 3 indicates that the probabilities of convicting an innocent defendant and acquitting a guilty defendant both decrease as $\hat{k}$, the number of votes required for a verdict, increases.

Proposition 3: Suppose that mistrial always results in a new trial and consider two voting rules, $\hat{k}_1$ and $\hat{k}_2$, with $\hat{k}_1 < \hat{k}_2$. If jurors vote sincerely, then:

(1) The probability of convicting an innocent defendant is lower under voting rule $\hat{k}_2$ than under voting rule $\hat{k}_1$.

(2) The probability of acquitting a guilty defendant is lower under voting rule $\hat{k}_2$ than under voting rule $\hat{k}_1$.

Proof: See Appendix.

Note that Proposition 3 implies that the probability of trial error is uniquely minimized by unanimity rule and uniquely maximized by simple majority rule. This result is in stark contrast to the conclusions from analysis of the basic model, in which Nash equilibrium behavior produced higher probabilities of both convicting an innocent defendant and acquitting a guilty defendant under unanimity rule than under any non-unanimous voting rule.

Another reasonable measure of the performance of alternative voting rules is in terms of expected utility. Our final result for the mistrial model, indicates that the expected utility for any juror increases as the number of votes required for a verdict increases.

Proposition 4: Suppose that the utility of mistrial is equal to the expected utility of an additional trial before a new jury and consider two voting rules, $\hat{k}_1$ and $\hat{k}_2$, with $\hat{k}_1 < \hat{k}_2$. If jurors vote sincerely, then the ex ante expected utility for an juror is higher under voting rule $\hat{k}_2$ than under voting rule $\hat{k}_1$.

Proof: See Appendix.

This proposition indicates that unanimity rule again performs uniquely best among all voting rules, this time in terms of maximizing expected utility. Moreover, Proposition 4 also implies that simple majority rule is again the uniquely worst voting rule under this performance measure.

While both Proposition 3 and Proposition 4 specifically apply to the version of the mistrial model in which mistrial utilities are determined endogenously, it is important to note that the basic results (i.e. unanimity rule minimizing error and maximizing utility) also hold true when mistrial utilities are specified exogenously. However, the analysis in the exogenous utility case is rather simple, and the appropriate interpretation of the results is less clear.

4 THE COMMUNICATION MODEL

Recall that the basic model effectively rules out any communication among jurors in that the entire jury process is assumed to be a single vote in which each juror has no information about the beliefs of other jurors. Let us therefore now consider a different enhancement to the basic model allowing for minimal communication among jurors.

In particular, suppose that the jury takes a single non-binding "straw vote" before taking the final binding vote for conviction or acquittal. All jurors must vote to either convict (C) or acquit (A) in both the preliminary and final vote, and the number of preliminary votes cast for each outcome are announced prior to conducting the final vote. It is assumed that no communication other than casting the anonymous preliminary vote takes place.

Note that this enhancement to the model is not meant to represent actual deliberation procedures, but is nonetheless intended to show the significance of including communication in any model of the jury process. The incorporation of a single non-binding straw-vote will demonstrate that the addition of even the most minimal communication can significantly change the conclusions of the model analysis.

4.1 Existence of Sincere Voting Equilibria

We start our analysis of this "communication model" by adapting our notion of sincerity to the distinctive voting framework of the model. The *sincere revelation strategy profile* for the communication model consists of each juror j voting according to the following guidelines:

(1) In the preliminary vote, juror j votes to convict iff $s_j=g$ (sincere voting);

(2) In the final vote, juror j votes to convict iff $\beta(k,n) \geq q_j$ where k is the number of votes to convict from the preliminary vote (sophisticated sincere voting);

Our first result for the communication model identifies the necessary and sufficient conditions for the sincere revelation strategy profile to constitute a subgame perfect Nash equilbrium.

Proposition 5: Let the jurors be numbered such that $q_1 \leq q_2 \leq \ldots \leq q_{n-1} \leq q_n$. Then the sincere revelation strategy profile is a subgame perfect Nash equilibrium for a given voting rule $\hat{k}$ if and only if one of the following conditions is true:

(a) $0 \leq q_{\hat{k}} \leq \beta(0,n)$;

(b) $\beta(n,n) < q_{\hat{k}} \leq 1$; or

(c) $\exists k^* \in \{1, \ldots, n\}$ such that $\beta(k^*-1,n) \leq q_j \leq \beta(k^*,n)$ for all $j \in N$.

Proof: See Appendix.

Proposition 5 says that sincerity is a Nash equilibrium in the communication model whenever juror utilities satisfy a certain "closeness" condition. The basic insight behind this proposition is that when juror utilities are similar enough for there to be a situation of "common interest," everyone can benefit from an honest sharing of information in the preliminary vote. Since jurors do not have competing interests, the sharing of information can only serve to enhance the probability of achieving the outcome that all jurors prefer. In fact, the basic results of Proposition 5 should hold for any game of incomplete information and common interest in which a choice must be made between two alternatives, such as between two candidates for office or two public projects. Also recognize that the existence of the sincere voting Nash equilibrium in this communication model does not require that juror utilities be common knowledge. Instead, it is sufficient for jurors to know only that the other jurors are following the sincere revelation strategy profile (or, alternatively, that one of the three conditions, (a), (b), or (c), is true).

One simple situation that meets the conditions of the proposition is the case of common utilities (i.e., when $q_i=q_j$ for all $i,j \in N$). Thus, in Feddersen and Pesendorfer's example where n=12, p=0.8, and $q_j=0.9$ for all j, sincere voting is a Nash equilibrium in the communication model under all possible voting rules. It is important to note, however, that it is possible for juror utilities to differ significantly and still satisfy the conditions of the proposition. Consider the case of a three-person jury (n=3), and suppose a correct signal is received 80% of the time (p=0.8). In this case, we have $\beta(0,3)=0.015$, $\beta(1,3)=0.200$, $\beta(2,3)=0.800$, and $\beta(3,3)=0.985$. Since $1-q_j$ is a measure of reasonable doubt, we would like to think that $q_j \geq 0.5$ for all j, and it is not unreasonable to suppose that $0.200 \leq q_j \leq 0.800$ or $0.800 \leq q_j \leq 0.985$ for all j. If all q_j values fall in either of these ranges, Proposition 5 says that sincere voting is a Nash equilibrium. While the second range is narrower than the first range, it may be more realistic, and still allows the reasonable doubt threshold to vary from 1-in-4 to about 1-in-70.

It may at first seem that as n increases (i.e., the size of the jury becomes larger), the difference between $\beta(k^*-1,n)$ and $\beta(k^*,n)$ will become smaller for all $k^* \in \{1, ..., n\}$, making the conditions of Proposition 1 increasingly difficult to satisfy. This is not entirely true, however. In fact, some of these differences remain constant (and potentially rather large) for all values of n. Our next proposition uses this fact to identify sufficient conditions for the existence of a sincere voting equilibrium that are independent of the size of the jury.

Proposition 6: For n odd, the sincere revelation strategy profile is a subgame perfect Nash equilibrium for any voting rule $\hat{k}$ if:

$$1\text{-}p \leq q_j \leq p, \ \forall j \in N.$$

For n even, the sincere revelation strategy profile is a subgame perfect Nash equilibrium for any voting rule $\hat{k}$ if:

$$\frac{(1-p)^2}{p^2+(1-p)^2} \leq q_j \leq 0.5, \ \forall j \in N \quad \text{or} \quad 0.5 \leq q_j \leq \frac{p^2}{p^2+(1-p)^2}, \ \forall j \in N.$$

For any value of n, odd or even, the sincere revelation strategy profile is a subgame perfect Nash equilibrium for any voting rule $\hat{k}$ if:

$$1\text{-}p \leq q_j \leq 0.5, \ \forall j \in N \quad \text{or} \quad 0.5 \leq q_j \leq p, \ \forall j \in N.$$

Proof: See Appendix.

To better understand the scope of the conditions in Proposition 6, consider the moderate case of p=0.8. Proposition 2 then says that, if there are an odd number of jurors (whether there be three jurors, 11 jurors, or 99 jurors), the sincere revelation strategy profile will be a Nash equilibrium for any voting rule whenever $0.2 \leq q_j \leq 0.8$ for all jurors $j \in N$. In addition, if there are an even number of jurors (whether there be 4 jurors, 12 jurors, or 100 jurors), such sincere voting will be a Nash equilibrium for any voting rule whenever all juror utilities satisfy either $0.06 \leq q_j \leq 0.5$ or $0.5 \leq q_j \leq 0.94$. Moreover, if $0.2 \leq q_j \leq 0.5$ for all jurors or $0.5 \leq q_j \leq 0.8$ for all jurors, sincere voting will be a Nash equilibrium for a jury of any size, odd or even. This example demonstrates that strategic jurors may vote sincerely in equilibrium under fairly general conditions for all juries and all voting rules.

Note that Propositions 6 suggests that the occurrence of sincere voting among jurors in the communication model may increase as the "accuracy" of trials improves. We observed the same

result in our analysis of the mistrial model. As p increases, and thus trials become more truth revealing, the conditions of Proposition 6 become easier to satisfy, and thus sincere voting Nash equilibria will exist for more juries and more trials. Also note that the "one-man jury condition," discussed previously, once again appears in Proposition 6.

Our next result for the communication model follows directly from Proposition 5.

Proposition 7: Suppose the juror utilities satisfy $0.5 \leq q_1 \leq q_2 \leq \ldots \leq q_{n-1} \leq q_n$. If condition (a), (b), or (c) from Proposition 5 is satisfied under voting rule $\hat{k}_1$, then the same condition is satisfied under any other voting rule $\hat{k}_2$ satisfying $\hat{k}_2 > \hat{k}_1$.

Proof: See Appendix.

This proposition indicates that, as long as $q_j \geq 0.5$ for all j (as we would expect), sincere voting is more likely to be a Nash equilibrium under unanimity rule than under any alternative voting rule.

4.2 Comparison of Alternative Voting Rules

We evaluate the performance of alternative voting rules in the communication model by once again examining the probability of trial error under different rules.

Proposition 8: Suppose that the sincere revelation strategy profile is a subgame perfect Nash equilibrium for two voting rules, $\hat{k}_1$ and $\hat{k}_2$. If jurors behave according to this Nash equilibrium, then:

(1) The probability of convicting an innocent defendant is the same under both voting rules.

(2) The probability of acquitting a guilty defendant is the same under both voting rules.

Proof: See Appendix.

Proposition 8 indicates that the sincere revelation Nash equilibrium results in the same probability of trial error under all voting rules. Thus, our conclusions once again contrast with the results from analysis of the basic model, in which unanimous jury verdicts were shown to be uniquely inferior under this performance measure.

Applying the alternative criterion of expected utility maximization, our results once again conflict with the negative assessment of unanimity rule from the analysis of the basic model. Instead, Proposition 9 indicates that the sincere revelation Nash equilibrium in the communication model produces the same expected utility under all voting rules.

Proposition 9: Suppose the subgame perfect Nash equilibrium described in Proposition 1 exists for any two voting rules, $\hat{k}_1$ and $\hat{k}_2$. If jurors behave according to this Nash equilibrium, then the expected utility for any juror is the same under both voting rules.

Proof: See Appendix.

5 CONCLUSIONS AND EXTENSIONS

Analysis of the basic model of jury procedure produces the somewhat surprising result that sincere voting can never be a Nash equilibrium for any voting rule, unless extreme restrictions are imposed. Instead, a mixed strategy equilibrium exists in which unanimous jury verdicts are uniquely inferior in terms of minimizing the probability of trial error.

The objective of the current paper was to evaluate the impact that certain extensions of this basic model have on the existence of sincere voting Nash equilibria. In particular, we examined the effects of introducing the possibility of mistrial and allowing limited communication upon the incentives for jurors to vote sincerely. In both cases, we find non-trivial conditions under which sincere voting is indeed a Nash equilibrium. In addition, we compare the outcomes of these sincere voting Nash equilibria under alternative voting rules and demonstrate that unanimity rule minimizes the probability of trial error and maximizes the ex ante expected utility of jurors.

An additional implication of the results of this paper is that the generality of sincere voting equilibria is strongly dependent upon the "accuracy" of trials. In particular, as the probability that the true state of the world is revealed at trial increases, the conditions for the existence of a sincere voting Nash equilibrium become more general in both the mistrial model and the communication model. This provides an additional argument in support of any legal reform that can be shown to produce more accurate impressions of guilt or innocence at trial.

While this paper was concerned only with the existence of pure strategy sincere voting Nash equilibria, the investigation of the impacts of mistrial and communication should be extended to

examine the existence and implications of mixed strategy and other non-sincere Nash equilibria. In particular, it is important to determine what happens when the conditions for existence of sincere voting Nash equilibria that are identified in this paper are violated. Do the equilibria that exist in such situations still produce outcomes that make unanimity rule superior in terms of minimizing error and maximizing utility? Or do the results of the basic model prevail, with unanimity rule being outperformed by other voting rules such as simple majority rule?

There are several other more basic extensions that should also be investigated to enhance the generality of these jury procedure models. For example, the prior probability of guilt or innocence is assumed to be 50% in all models, however empirical evidence, such as conviction rates, suggests that this probability may be closer to 80% or even 90%. Moreover, estimates of this prior probability may differ among jurors. It would be interesting to examine the impact of varying this parameter. Additionally, all of the models presented in this paper include the restriction that the utility of the two correct trial outcomes, convicting the guilty and acquitting the innocent, are equal. Any generalization of these models should examine the effect of loosening this restriction as well.

REFERENCES

Austen-Smith, David, and Jeffrey S. Banks. 1996. "Information Aggregation, Rationality, and the Condorcet Jury Theorem." *American Political Science Review* 90(1):34-45.

Berg, Sven. 1993. "Condorcet's Jury Theorem, Dependency Among Jurors." *Social Choice and Welfare* 10:87-95.

Condorcet, Marquis de. 1785/1976. "Essay on the Application of Mathematics to the Theory of Decision-Making." Reprinted in *Condorcet: Selected Writings*, Keith Michael Baker, ed., 33-70. Indianapolis: Bobbs-Marrill Co.

Fedderson, Timothy, and Wolfgang Pesendorfer. 1997. "Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts." *Discussion Paper*, Northwestern and Princeton Universities.

Grofman, Bernard, and Scott L. Feld. 1988. "Rousseau's General Will: A Condorcetian Perspective." *American Political Science Review* 82(2):567-576.

Klevorick, Alvin K., Michael Rothschild, and Christopher Winship. 1984. "Information Processing and Jury Decisionmaking." *Journal of Public Economics* 23:245-278.

Ladha, Krishna. 1992. "The Condorcet Jury Theorem, Free Speech, and Correlated Votes." *American Journal of Political Science* 36(3):617-634.

Ladha, Krishna. 1993. "Condorcet's Jury Theorem in Light of De Finnetti's Theorem." *Social Choice and Welfare* 10:69-85.

Ladha, Krishna K. 1995. "Information Pooling Through Majority-Rule Voting: Condorcet's Jury Theorem with Correlated Votes." *Journal of Economic Behavior & Organization* 26:353-372.

Ladha, Krishna, Gary Miller, and Joe Oppenheimer. 1996. "Information Aggregation by Majority Rule: Theory and Experiments." *Discussion Paper*, The John M. Olin School of Business, Washington University, St. Louis.

McLennan, Andrew. 1996. "Consequences of the Condorcet Jury Theorem for Beneficial Information Aggregation by Rational Agents." *Discussion Paper,* Department of Economics, University of Minnesota.

Miller, Nicolas R. 1986 "Information, Electorates, and Democracy: Some Extensions and Interpretations of the Condorcet Jury Theorem." *Information Pooling and Group Decision Making*, Bernard Grofman and Guillermo Owen, eds., 173-192. Greenwich: JAI Press Inc.

Schwartz, Edward P., and Warren F. Schwartz. 1992. "Decisionmaking by Juries Under Unanimity and Supermajority Voting Rules." *The Georgetown Law Journal* 80:775-807.

Wit, Jörgen. 1996. "Rational Choice and the Condorcet Jury Theorem." *Discussion Paper*, Division of Humanities and Social Sciences, California Institute of Technology.

Young, H.P. 1988. "Condorcet's Theory of Voting." *American Political Science Review* 82(4):1231-1244.

APPENDIX

Proposition 1: Suppose that n is odd and that $\hat{k} = \frac{n+1}{2}$ (simple majority rule). In this case, sincere voting is a Nash equilibrium in the mistrial model if and only if $1\text{-}p \le q_j \le p$ for any juror $j \in N$. In all other cases, sincere voting is a Nash equilibrium if and only if, for any juror $j \in N$:

(a) $\left(p - q_j\right)\left(1-p\right)^{2\hat{k}-n-1} + \left(m_j^G p - m_j^I\left(1-p\right)\right)\left(p^{2\hat{k}-n-1} - \left(1-p\right)^{2\hat{k}-n-1}\right) \ge 0$ and

(b) $\left(p - \left(1-q_j\right)\right)\left(1-p\right)^{2\hat{k}-n-1} + \left(m_j^I p - m_j^G\left(1-p\right)\right)\left(p^{2\hat{k}-n-1} - \left(1-p\right)^{2\hat{k}-n-1}\right) \ge 0.$

Proof: We will prove the second part of the proposition first.

<u>Case 1:</u> n is even and/or $\hat{k} \ne \frac{n+1}{2}$

Again recognize that a strategic voter will condition her strategy on the event that her vote is pivotal; that is, that her vote can change the trial outcome. For a given juror j, there are exactly four scenarios in which her vote is pivotal in this case:

(1) Defendant is guilty and $\hat{k}$-1 other jurors vote to convict $\left(G \cap |C|_{-j} = \hat{k} - 1\right)$

(2) Defendant is guilty and $\hat{k}$-1 other jurors vote to acquit $\left(G \cap |A|_{-j} = \hat{k} - 1\right)$

(3) Defendant is innocent and $\hat{k}$-1 other jurors vote to convict $\left(I \cap |C|_{-j} = \hat{k} - 1\right)$

(4) Defendant is innocent and $\hat{k}$-1 other jurors vote to acquit $\left(I \cap |A|_{-j} = \hat{k} - 1\right)$

Juror j's beliefs about the relative likelihood of each of these four scenarios will help determine her utility maximizing strategy. In particular, for any juror j, the expected utility of a vote to convict (ignoring the event in which the vote is not pivotal) is given by:

$$\begin{aligned} EU_j\left(C,s_j\right) &= \text{Prob}\left(G \cap |C|_{-j} = \hat{k} - 1\right)\cdot u_j(C,G) + \text{Prob}\left(G \cap |A|_{-j} = \hat{k} - 1\right)\cdot u_j(M,G) \\ &\quad + \text{Prob}\left(I \cap |C|_{-j} = \hat{k} - 1\right)\cdot u_j(C,I) + \text{Prob}\left(I \cap |A|_{-j} = \hat{k} - 1\right)\cdot u_j(M,I) \\ &= -q_j \cdot \text{Prob}\left(I \cap |C|_{-j} = \hat{k} - 1\right) - m_j^G \cdot \text{Prob}\left(G \cap |A|_{-j} = \hat{k} - 1\right) \\ &\quad - m_j^I \cdot \text{Prob}\left(I \cap |A|_{-j} = \hat{k} - 1\right) \end{aligned}$$

Similarly, the expected utility of a vote to acquit is given by:

$$
\begin{aligned}
EU_j(A,s_j) &= \text{Prob}\left(G \cap |C|_{-j} = \hat{k}-1\right)\cdot u_j(M,G) + \text{Prob}\left(G \cap |A|_{-j} = \hat{k}-1\right)\cdot u_j(A,G) \\
&\quad + \text{Prob}\left(I \cap |C|_{-j} = \hat{k}-1\right)\cdot u_j(M,I) + \text{Prob}\left(I \cap |A|_{-j} = \hat{k}-1\right)\cdot u_j(A,I) \\
&= -\left(1-q_j\right)\cdot \text{Prob}\left(G \cap |A|_{-j} = \hat{k}-1\right) - m_j^G \cdot \text{Prob}\left(G \cap |C|_{-j} = \hat{k}-1\right) \\
&\quad - m_j^I \cdot \text{Prob}\left(I \cap |C|_{-j} = \hat{k}-1\right)
\end{aligned}
$$

Now suppose all jurors vote sincerely. That is, $\sigma_j(g) = 1$ and $\sigma_j(i) = 0$ for all j, and thus $|C| = |g|$ and $|A| = |i|$. We must show that no juror can increase his or her utility by deviating from this strategy. More specifically, for all $j \in N$, we must show that:

(1) If $s_j = g$, then $EU_j(C,g) \geq EU_j(A,g)$

(2) If $s_j = i$, then $EU_j(A,i) \geq EU_j(C,i)$

<u>Subcase 1:</u> $s_j = g$

In this subcase, juror j's beliefs about the probability of the first scenario in which her vote is pivotal $\left(G \cap |C|_{-j} = \hat{k}-1\right)$ is given by:

$$
\begin{aligned}
\text{Prob}\left(G \cap |C|_{-j} = \hat{k}-1 \,\middle|\, s_j = g\right) &= \text{Prob}\left(G \cap |g| = \hat{k} \,\middle|\, s_j = g\right) \\
&= \frac{\text{Prob}\left(G \cap |g| = \hat{k} \cap s_j = g\right)}{\text{Prob}\left(s_j = g\right)} \\
&= \frac{\text{Prob}(G)\cdot \text{Prob}\left(|g| = \hat{k} \,\middle|\, G\right)\cdot \text{Prob}\left(s_j = g \,\middle|\, |g| = \hat{k}\right)}{\text{Prob}(G)\cdot \text{Prob}\left(s_j = g \,\middle|\, G\right) + \text{Prob}(I)\cdot \text{Prob}\left(s_j = g \,\middle|\, I\right)} \\
&= \frac{\frac{1}{2}\cdot \frac{n!}{\hat{k}!\cdot\left(n-\hat{k}\right)!}\cdot p^{\hat{k}}(1-p)^{n-\hat{k}}\cdot \frac{\hat{k}}{n}}{\frac{1}{2}\cdot p + \frac{1}{2}\cdot(1-p)} \\
&= \frac{(n-1)!}{\left(\hat{k}-1\right)!\cdot\left(n-\hat{k}\right)!}\cdot p^{\hat{k}}(1-p)^{n-\hat{k}} \\
&= \Psi\cdot p^{\hat{k}}(1-p)^{n-\hat{k}}
\end{aligned}
$$

where $\Psi = \dfrac{(n-1)!}{\left(\hat{k}-1\right)!\cdot\left(n-\hat{k}\right)!}$

In the same manner, we can show that:

$$\text{Prob}\left(G \cap |A|_{-j} = \hat{k}-1 \,\middle|\, s_j = g\right) = \Psi \cdot p^{n-\hat{k}+1}(1-p)^{\hat{k}-1}$$
$$\text{Prob}\left(I \cap |C|_{-j} = \hat{k}-1 \,\middle|\, s_j = g\right) = \Psi \cdot p^{n-\hat{k}}(1-p)^{\hat{k}}$$
$$\text{Prob}\left(I \cap |A|_{-j} = \hat{k}-1 \,\middle|\, s_j = g\right) = \Psi \cdot p^{\hat{k}-1}(1-p)^{n-\hat{k}+1}$$

Thus, the expected utility of a vote to convict is given by:

$$\begin{aligned} EU_j(C,g) &= -q_j\,\text{Prob}\left(I \cap |C|_{-j} = \hat{k}-1\right) - m_j^G\,\text{Prob}\left(G \cap |A|_{-j} = \hat{k}-1\right) \\ &\quad - m_j^I\,\text{Prob}\left(I \cap |A|_{-j} = \hat{k}-1\right) \\ &= -\Psi\left[q_j p^{n-\hat{k}}(1-p)^{\hat{k}} + m_j^G p^{n-\hat{k}+1}(1-p)^{\hat{k}-1} + m_j^I p^{\hat{k}-1}(1-p)^{n-\hat{k}+1}\right] \end{aligned}$$

Similarly, the expected utility of a vote to acquit is given by:

$$\begin{aligned} EU_j(A,g) &= -(1-q_j)\,\text{Prob}\left(G \cap |A|_{-j} = \hat{k}-1\right) - m_j^G\,\text{Prob}\left(G \cap |C|_{-j} = \hat{k}-1\right) \\ &\quad - m_j^I\,\text{Prob}\left(I \cap |C|_{-j} = \hat{k}-1\right) \\ &= -\Psi\left[(1-q_j)p^{n-\hat{k}+1}(1-p)^{\hat{k}-1} + m_j^G p^{\hat{k}}(1-p)^{n-\hat{k}} + m_j^I p^{n-\hat{k}}(1-p)^{\hat{k}}\right] \end{aligned}$$

$$\begin{aligned} \text{Condition (a)} &\Leftrightarrow (p-q_j)(1-p)^{2\hat{k}-n-1} + \left(m_j^G p - m_j^I(1-p)\right)\left(p^{2\hat{k}-n-1} - (1-p)^{2\hat{k}-n-1}\right) \geq 0 \\ &\Leftrightarrow (p-q_j p)(1-p)^{2\hat{k}-n-1} + m_j^G p\left(p^{2\hat{k}-n-1} - (1-p)^{2\hat{k}-n-1}\right) \\ &\geq (q_j - q_j p)(1-p)^{2\hat{k}-n-1} + m_j^I(1-p)\left(p^{2\hat{k}-n-1} - (1-p)^{2\hat{k}-n-1}\right) \\ &\Leftrightarrow (1-q_j)p(1-p)^{2\hat{k}-n-1} + m_j^G p^{2\hat{k}-n} + m_j^I(1-p)^{2\hat{k}-n} \\ &\geq q_j(1-p)^{2\hat{k}-n} + m_j^G p(1-p)^{2\hat{k}-n-1} + m_j^I p^{2\hat{k}-n-1}(1-p) \\ &\Leftrightarrow (1-q_j)p^{n-\hat{k}+1}(1-p)^{\hat{k}-1} + m_j^G p^{\hat{k}}(1-p)^{n-\hat{k}} + m_j^I p^{n-\hat{k}}(1-p)^{\hat{k}} \\ &\geq q_j p^{n-\hat{k}}(1-p)^{\hat{k}} + m_j^G p^{n-\hat{k}+1}(1-p)^{\hat{k}-1} + m_j^I p^{\hat{k}-1}(1-p)^{n-\hat{k}+1} \\ &\Leftrightarrow -\Psi\left[q_j p^{n-\hat{k}}(1-p)^{\hat{k}} + m_j^G p^{n-\hat{k}+1}(1-p)^{\hat{k}-1} + m_j^I p^{\hat{k}-1}(1-p)^{n-\hat{k}+1}\right] \\ &\geq -\Psi\left[(1-q_j)p^{n-\hat{k}+1}(1-p)^{\hat{k}-1} + m_j^G p^{\hat{k}}(1-p)^{n-\hat{k}} + m_j^I p^{n-\hat{k}}(1-p)^{\hat{k}}\right] \\ &\Leftrightarrow EU_j(C,g) \geq EU_j(A,g) \end{aligned}$$

Subcase 2: $s_j = i$

For this subcase, we can calculate juror j's beliefs about the relative probabilities of the four scenarios in which her vote is pivotal in the same manner as above. This gives us:

$$\text{Prob}\left(G \cap |C|_{-j} = \hat{k}-1 \,\middle|\, s_j = i\right) = \Psi \cdot p^{\hat{k}-1}(1-p)^{n-\hat{k}+1}$$
$$\text{Prob}\left(G \cap |A|_{-j} = \hat{k}-1 \,\middle|\, s_j = i\right) = \Psi \cdot p^{n-\hat{k}}(1-p)^{\hat{k}}$$
$$\text{Prob}\left(I \cap |C|_{-j} = \hat{k}-1 \,\middle|\, s_j = i\right) = \Psi \cdot p^{n-\hat{k}+1}(1-p)^{\hat{k}-1}$$
$$\text{Prob}\left(I \cap |A|_{-j} = \hat{k}-1 \,\middle|\, s_j = i\right) = \Psi \cdot p^{\hat{k}}(1-p)^{n-\hat{k}}$$
$$\text{where } \Psi = \frac{(n-1)!}{2\cdot\left(\hat{k}-1\right)!\cdot\left(n-\hat{k}\right)!}$$

Thus, the expected utility of a vote to convict is given by:

$$\begin{aligned} EU_j(C,i) &= -q_j\,\text{Prob}\left(I \cap |C|_{-j} = \hat{k}-1\right) - m_j^G\,\text{Prob}\left(G \cap |A|_{-j} = \hat{k}-1\right) \\ &\quad - m_j^I\,\text{Prob}\left(I \cap |A|_{-j} = \hat{k}-1\right) \\ &= -\Psi\left[q_j p^{n-\hat{k}+1}(1-p)^{\hat{k}-1} + m_j^G p^{n-\hat{k}}(1-p)^{\hat{k}} + m_j^I p^{\hat{k}}(1-p)^{n-\hat{k}}\right] \end{aligned}$$

Similarly, the expected utility of a vote to acquit is given by:

$$\begin{aligned} EU_j(A,i) &= -\left(1-q_j\right)\text{Prob}\left(G \cap |A|_{-j} = \hat{k}-1\right) - m_j^G\,\text{Prob}\left(G \cap |C|_{-j} = \hat{k}-1\right) \\ &\quad - m_j^I\,\text{Prob}\left(I \cap |C|_{-j} = \hat{k}-1\right) \\ &= -\Psi\left[\left(1-q_j\right)p^{n-\hat{k}}(1-p)^{\hat{k}} + m_j^G p^{\hat{k}-1}(1-p)^{n-\hat{k}+1} + m_j^I p^{n-\hat{k}+1}(1-p)^{\hat{k}-1}\right] \end{aligned}$$

$$\begin{aligned} \text{Condition (b)} &\Leftrightarrow \left(p-\left(1-q_j\right)\right)(1-p)^{2\hat{k}-n-1} + \left(m_j^I p - m_j^G(1-p)\right)\left(p^{2\hat{k}-n-1} - (1-p)^{2\hat{k}-n-1}\right) \geq 0 \\ &\Leftrightarrow q_j p(1-p)^{2\hat{k}-n-1} + m_j^I p\left(p^{2\hat{k}-n-1} - (1-p)^{2\hat{k}-n-1}\right) \\ &\geq \left(1-q_j-p+q_j p\right)(1-p)^{2\hat{k}-n-1} + m_j^G(1-p)\left(p^{2\hat{k}-n-1} - (1-p)^{2\hat{k}-n-1}\right) \\ &\Leftrightarrow q_j p(1-p)^{2\hat{k}-n-1} + m_j^G(1-p)^{2\hat{k}-n} + m_j^I p^{2\hat{k}-n} \\ &\geq \left(1-q_j\right)(1-p)^{2\hat{k}-n} + m_j^G p^{2\hat{k}-n-1}(1-p) + m_j^I p(1-p)^{2\hat{k}-n-1} \end{aligned}$$

$$\Leftrightarrow\ q_j p^{n-\hat{k}+1}(1-p)^{\hat{k}-1} + m_j^G p^{n-\hat{k}}(1-p)^{\hat{k}} + m_j^I p^{\hat{k}}(1-p)^{n-\hat{k}}$$
$$\geq\ (1-q_j)p^{n-\hat{k}}(1-p)^{\hat{k}} + m_j^G p^{\hat{k}-1}(1-p)^{n-\hat{k}+1} + m_j^I p^{n-\hat{k}+1}(1-p)^{\hat{k}-1}$$
$$\Leftrightarrow\ -\Psi\left[(1-q_j)p^{n-\hat{k}}(1-p)^{\hat{k}} + m_j^G p^{\hat{k}-1}(1-p)^{n-\hat{k}+1} + m_j^I p^{n-\hat{k}+1}(1-p)^{\hat{k}-1}\right]$$
$$\geq\ -\Psi\left[q_j p^{n-\hat{k}+1}(1-p)^{\hat{k}-1} + m_j^G p^{n-\hat{k}}(1-p)^{\hat{k}} + m_j^I p^{\hat{k}}(1-p)^{n-\hat{k}}\right]$$
$$\Leftrightarrow\ EU_j(A,i) \geq EU_j(C,i)$$

<u>Case 2:</u> n is odd and $\hat{k} = \frac{n+1}{2}$

Note that there can never be a mistrial in this case. If $|C| \geq \frac{n+1}{2}$, the defendant is convicted, while if $|C| < \frac{n+1}{2}$ (i.e., $|C| \leq \frac{n+1}{2} - 1 = n - \frac{n+1}{2}$), the defendant is acquitted. We also have that $|C|_{-j} = \hat{k}-1 \Leftrightarrow |A|_{-j} = \hat{k}-1$. Therefore, to be pivotal in this case is to decide between conviction and acquittal (as opposed to conviction and mistrial or acquittal and mistrial as above). Each juror is therefore concerned only with the relative likelihood of two possible scenarios: the defendant is guilty or the defendant is innocent.

Thus, for any juror j, the expected utility of a vote to convict is given by:

$$\begin{aligned} EU_j(C,s_j) &= Prob(G)\cdot u_j(C,G) + Prob(I)\cdot u_j(C,I) \\ &= -q_j\, Prob(I) \end{aligned}$$

Similarly, the expected utility of a vote to acquit is given by:

$$\begin{aligned} EU_j(A,s_j) &= Prob(G)\cdot u_j(A,G) + Prob(I)\cdot u_j(A,I) \\ &= -(1-q_j)\, Prob(G) \end{aligned}$$

Now suppose all jurors vote sincerely. That is, $\sigma_j(g) = 1$ and $\sigma_j(i) = 0$ for all j, and thus $|C| = |g|$ and $|A| = |i|$. We must show that no juror can increase his or her utility by deviating from this strategy. More specifically, for all $j \in N$, we must show that:

(1) If $s_j = g$, then $EU_j(C,g) \geq EU_j(A,g)$

(2) If $s_j = i$, then $EU_j(A,i) \geq EU_j(C,i)$

<u>Subcase 1:</u> $s_j = g$

In this subcase, juror j's beliefs about the probability of guilt or innocence are given by:

$$
\begin{aligned}
\mathrm{Prob}(G \mid s_j = g) &= \mathrm{Prob}(G \mid |g| = \hat{k}) \\
&= \beta(\hat{k}, n) \\
&= \frac{p^{\hat{k}}(1-p)^{n-\hat{k}}}{p^{\hat{k}}(1-p)^{n-\hat{k}} + p^{n-\hat{k}}(1-p)^{\hat{k}}} \\
&= \frac{p^{\hat{k}}(1-p)^{\hat{k}-1}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Prob}(I \mid s_j = g) &= \mathrm{Prob}(I \mid |g| = \hat{k}) \\
&= \beta(n-\hat{k}, n) \\
&= \frac{p^{n-\hat{k}}(1-p)^{\hat{k}}}{p^{\hat{k}}(1-p)^{n-\hat{k}} + p^{n-\hat{k}}(1-p)^{\hat{k}}} \\
&= \frac{p^{\hat{k}-1}(1-p)^{\hat{k}}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}}
\end{aligned}
$$

Thus, the expected utility of a vote to convict is given by:

$$
\begin{aligned}
EU_j(C,g) &= -q_j \,\mathrm{Prob}(I) \\
&= -q_j \cdot \frac{p^{\hat{k}-1}(1-p)^{\hat{k}}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}}
\end{aligned}
$$

Similarly, the expected utility of a vote to acquit is given by:

$$
\begin{aligned}
EU_j(A,g) &= -(1-q_j)\,\mathrm{Prob}(G) \\
&= -(1-q_j) \cdot \frac{p^{\hat{k}}(1-p)^{\hat{k}-1}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}}
\end{aligned}
$$

$$
\begin{aligned}
q_j \le p &\Leftrightarrow q_j - pq_j \le p - pq_j \\
&\Leftrightarrow q_j(1-p) \le (1-q_j)p \\
&\Leftrightarrow -q_j p^{\hat{k}-1}(1-p)^{\hat{k}} \ge -(1-q_j)p^{\hat{k}}(1-p)^{\hat{k}-1} \\
&\Leftrightarrow q_j \cdot \frac{p^{\hat{k}-1}(1-p)^{\hat{k}}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}} \ge (1-q_j) \cdot \frac{p^{\hat{k}}(1-p)^{\hat{k}-1}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}} \\
&\Leftrightarrow EU_j(C,g) \ge EU_j(A,g)
\end{aligned}
$$

Subcase 2: $s_j = i$

For this subcase, we can calculate juror j's beliefs about the probability of guilt or innocence in the same manner as above. This gives us:

$$\text{Prob}(G \mid s_j = i) = \frac{p^{\hat{k}-1}(1-p)^{\hat{k}}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}}$$

$$\text{Prob}(I \mid s_j = i) = \frac{p^{\hat{k}}(1-p)^{\hat{k}-1}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}}$$

Thus, the expected utility of a vote to convict is given by:

$$\begin{aligned} EU_j(C,g) &= -q_j \,\text{Prob}(I) \\ &= -q_j \cdot \frac{p^{\hat{k}}(1-p)^{\hat{k}-1}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}} \end{aligned}$$

Similarly, the expected utility of a vote to acquit is given by:

$$\begin{aligned} EU_j(A,g) &= -(1-q_j)\,\text{Prob}(G) \\ &= -(1-q_j) \cdot \frac{p^{\hat{k}-1}(1-p)^{\hat{k}}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}} \end{aligned}$$

$$\begin{aligned} 1-p \le q_j &\Leftrightarrow 1-q_j-p \le 0 \\ &\Leftrightarrow 1-q_j-p+pq_j \le pq_j \\ &\Leftrightarrow (1-q_j)(1-p) \le pq_j \\ &\Leftrightarrow -(1-q_j)p^{\hat{k}-1}(1-p)^{\hat{k}} \ge -q_j p^{\hat{k}}(1-p)^{\hat{k}-1} \\ &\Leftrightarrow -(1-q_j) \cdot \frac{p^{\hat{k}-1}(1-p)^{\hat{k}}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}} \ge -q_j \cdot \frac{p^{\hat{k}}(1-p)^{\hat{k}-1}}{p^{\hat{k}}(1-p)^{\hat{k}-1} + p^{\hat{k}-1}(1-p)^{\hat{k}}} \\ &\Leftrightarrow EU_j(A,i) \ge EU_j(C,i) \end{aligned}$$

Q.E.D.

Proposition 2: Suppose the utility of mistrial is equal to the expected utility of an additional trial before a new jury. Sincere voting is then a Nash equilibrium in the mistrial model for any voting rule $\hat{k}$ if and only if $1\text{-}p \le q_j \le p$ for all $j \in N$.

Proof: First note that, in a single trial, we have:

$$\text{Prob}_S(C \mid I) = \text{Prob}_S(A \mid G) = \sum_{x=\hat{k}}^{n} \binom{n}{x} p^{n-x}(1-p)^x$$

$$\text{Prob}_S(M \mid I) = \text{Prob}_S(M \mid G) = \sum_{x=n-\hat{k}+1}^{\hat{k}-1} \binom{n}{x} p^{n-x}(1-p)^x$$

This gives us:

$$m_j^G = (1-q_j)\cdot \text{Prob}_S(A \mid G) + m_j^G \cdot \text{Prob}_S(M \mid G)$$

$$m_j^G = (1-q_j)\sum_{x=\hat{k}}^{n} \binom{n}{x} p^{n-x}(1-p)^x + m_j^G \sum_{x=n-\hat{k}+1}^{\hat{k}-1} \binom{n}{x} p^{n-x}(1-p)^x$$

$$m_j^G = \frac{(1-q_j)\sum_{x=\hat{k}}^{n} \binom{n}{x} p^{n-x}(1-p)^x}{1-\sum_{x=n-\hat{k}+1}^{\hat{k}-1} \binom{n}{x} p^{n-x}(1-p)^x}$$

$$m_j^G = (1-q_j)\cdot \Omega$$

$$\text{where } \Omega = \frac{\sum_{x=\hat{k}}^{n} \binom{n}{x} p^{n-x}(1-p)^x}{1-\sum_{x=n-\hat{k}+1}^{\hat{k}-1} \binom{n}{x} p^{n-x}(1-p)^x} \ge 0$$

Similarly, we can show that:

$$m_j^I = q_j \cdot \Omega$$

Thus, condition (a) in Proposition 6 becomes:

$$(p-q_j)(1-p)^{2\hat{k}-n-1} + \left(m_j^G p - m_j^I(1-p)\right)\left(p^{2\hat{k}-n-1} - (1-p)^{2\hat{k}-n-1}\right) \ge 0$$

$$(p-q_j)(1-p)^{2\hat{k}-n-1} + \Omega\left((1-q_j)p - q_j(1-p)\right)\left(p^{2\hat{k}-n-1} - (1-p)^{2\hat{k}-n-1}\right) \ge 0$$

$$(p-q_j)(1-p)^{2\hat{k}-n-1} + \Omega(p-q_j)\left(p^{2\hat{k}-n-1} - (1-p)^{2\hat{k}-n-1}\right) \ge 0$$

$$\left(p - q_j\right)\left(\left(1-p\right)^{2\hat{k}-n-1} + \Omega\left(p^{2\hat{k}-n-1} - \left(1-p\right)^{2\hat{k}-n-1}\right)\right) \geq 0$$

$$p - q_j \geq 0$$

$$q_j \leq p$$

Similarly, we can show that condition (b) in Proposition 6 becomes:

$q_j \geq 1\text{-}p$

Thus, sincere voting is a Nash equilibrium for any voting rule $\hat{k}$ if and only if $1\text{-}p \leq q_j \leq p$ for all $j \in N$.

Recognize that when n is odd and $\hat{k} = \frac{n+1}{2}$, the conditions of this proposition are the same as the conditions of Proposition 1 dealing with exogenous mistrial utilities. Thus, endogenizing the mistrial utilities has no impact on the existence of the sincere voting Nash equilibrium in this case. This is not surprising, since we previously showed that a mistrial will never occur under these conditions.

Q.E.D.

Proposition 3: Suppose that mistrial always results in a new trial and consider two voting rules, $\hat{k}_1$ and $\hat{k}_2$, with $\hat{k}_1 < \hat{k}_2$. If jurors vote sincerely, then:

(1) The probability of convicting an innocent defendant is lower under voting rule $\hat{k}_2$ than under voting rule $\hat{k}_1$.

(2) The probability of acquitting a guilty defendant is lower under voting rule $\hat{k}_2$ than under voting rule $\hat{k}_1$.

Proof: First note that, due to the symmetry of the mistrial model, the probability of convicting an innocent defendant is equal to the probability of acquitting a guilty defendant. Therefore, it is sufficient to prove only part (1) of the proposition.

In addition, note that it is sufficient to prove only that the probability of convicting an innocent defendant is lower under voting rule $\hat{k}_1+1$ than under voting rule $\hat{k}_1$. It is then obvious by induction that, for any voting rule $\hat{k}_2$ with $\hat{k}_1 < \hat{k}_2$, the probability of convicting an innocent defendant is lower under $\hat{k}_2$ than $\hat{k}_1$.

The probability of convicting an innocent defendant in (possibly) repeated trials under voting rule $\hat{k}_1$ is given by:

$$\mathrm{Prob}_R^{\hat{k}_1}(C \mid I) = \sum_{x=\hat{k}_1}^{n} \binom{n}{x} p^{n-x}(1-p)^x + \mathrm{Prob}(C \mid I)_R^{\hat{k}_1} \cdot \sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1} \binom{n}{x} p^{n-x}(1-p)^x$$

$$\mathrm{Prob}_R^{\hat{k}_1}(C \mid I)\left(1 - \sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1} \binom{n}{x} p^{n-x}(1-p)^x\right) = \sum_{x=\hat{k}_1}^{n} \binom{n}{x} p^{n-x}(1-p)^x$$

$$\mathrm{Prob}_R^{\hat{k}_1}(C \mid I) = \frac{\sum_{x=\hat{k}_1}^{n} \binom{n}{x} p^{n-x}(1-p)^x}{1 - \sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1} \binom{n}{x} p^{n-x}(1-p)^x}$$

$$\mathrm{Prob}_R^{\hat{k}_1}(C \mid I) = \frac{\sum_{x=\hat{k}_1}^{n} \binom{n}{x} p^{n-x}(1-p)^x}{\sum_{x=\hat{k}_1}^{n} \binom{n}{x} p^{n-x}(1-p)^x + \sum_{x=\hat{k}_1}^{n} \binom{n}{x} p^{x}(1-p)^{n-x}}$$

$$\mathrm{Prob}_R^{\hat{k}_1}(C \mid I) = \frac{\sum_{x=\hat{k}_1}^{n} \binom{n}{x} p^{n-x}(1-p)^x}{\sum_{x=\hat{k}_1}^{n} \binom{n}{x} \left(p^{n-x}(1-p)^x + p^{x}(1-p)^{n-x}\right)}$$

Similarly, the probability of convicting an innocent defendant in possibly repeated trials under voting rule $\hat{k}_1+1$ is given by:

$$\mathrm{Prob}_R^{\hat{k}_1+1}(C \mid I) = \frac{\sum_{x=\hat{k}_1+1}^{n} \binom{n}{x} p^{n-x}(1-p)^x}{\sum_{x=\hat{k}_1+1}^{n} \binom{n}{x} \left(p^{n-x}(1-p)^x + p^{x}(1-p)^{n-x}\right)}$$

We now show that $\mathrm{Prob}_R^{\hat{k}_1}(C \mid I) > \mathrm{Prob}_R^{\hat{k}_1+1}(C \mid I)$:

$$(1-p)^{2(x-\hat{k}_1)} < p^{2(x-\hat{k}_1)} \quad \text{for any } x > \hat{k}_1$$

$$p^{\hat{k}_1-x}(1-p)^{x-\hat{k}_1} < p^{x-\hat{k}_1}(1-p)^{\hat{k}_1-x} \quad \text{for any } x > \hat{k}_1$$

$$\binom{n}{x} p^{n+\hat{k}_1-x}(1-p)^{n+x-\hat{k}_1} < \binom{n}{x} p^{n+x-\hat{k}_1}(1-p)^{n+\hat{k}_1-x} \quad \text{for any } x > \hat{k}_1$$

$$\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n+\hat{k}_1-x}(1-p)^{n+x-\hat{k}_1} < \sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n+x-\hat{k}_1}(1-p)^{n+\hat{k}_1-x}$$

$$p^{\hat{k}_1}(1-p)^{n-\hat{k}_1}\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n-x}(1-p)^{x} < p^{n-\hat{k}_1}(1-p)^{\hat{k}_1}\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{x}(1-p)^{n-x}$$

$$\left(p^{n-\hat{k}_1}(1-p)^{\hat{k}_1}+p^{\hat{k}_1}(1-p)^{n-\hat{k}_1}\right)\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n-x}(1-p)^{x}$$

$$< p^{n-\hat{k}_1}(1-p)^{\hat{k}_1}\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}\left(p^{n-x}(1-p)^{x}+p^{x}(1-p)^{n-x}\right)$$

$$\left(p^{n-\hat{k}_1}(1-p)^{\hat{k}_1}+p^{\hat{k}_1}(1-p)^{n-\hat{k}_1}+\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}\left(p^{n-x}(1-p)^{x}+p^{x}(1-p)^{n-x}\right)\right)\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n-x}(1-p)^{x}$$

$$< \left(p^{n-\hat{k}_1}(1-p)^{\hat{k}_1}+\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n-x}(1-p)^{x}\right)\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}\left(p^{n-x}(1-p)^{x}+p^{x}(1-p)^{n-x}\right)$$

$$\sum_{x=\hat{k}_1}^{n}\binom{n}{x}\left(p^{n-x}(1-p)^{x}+p^{x}(1-p)^{n-x}\right)\cdot\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n-x}(1-p)^{x}$$

$$< \sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^{x}\cdot\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}\left(p^{n-x}(1-p)^{x}+p^{x}(1-p)^{n-x}\right)$$

$$\frac{\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n-x}(1-p)^{x}}{\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}\left(p^{n-x}(1-p)^{x}+p^{x}(1-p)^{n-x}\right)} < \frac{\sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^{x}}{\sum_{x=\hat{k}_1}^{n}\binom{n}{x}\left(p^{n-x}(1-p)^{x}+p^{x}(1-p)^{n-x}\right)}$$

$$\mathrm{Prob}_R^{\hat{k}_1+1}(C\mid I) < \mathrm{Prob}_R^{\hat{k}_1}(C\mid I)$$

By induction, we have $\mathrm{Prob}_R^{\hat{k}_1}(C\mid I) > \mathrm{Prob}_R^{\hat{k}_2}(C\mid I)$, and since $\mathrm{Prob}_R(A\mid G) = \mathrm{Prob}_R(C\mid I)$, we also have that $\mathrm{Prob}_R^{\hat{k}_1}(A\mid G) > \mathrm{Prob}_R^{\hat{k}_2}(A\mid G)$.

Q.E.D.

Proposition 4: Suppose that the utility of mistrial is equal to the expected utility of an additional trial before a new jury and consider two voting rules, $\hat{k}_1$ and $\hat{k}_2$, with $\hat{k}_1 < \hat{k}_2$. If jurors vote sincerely, then the ex ante expected utility for an juror is higher under voting rule $\hat{k}_2$ than under voting rule $\hat{k}_1$.

Proof: Note that it is sufficient to prove only that the ex ante expected utility is higher under voting rule $\hat{k}_1+1$ than under voting rule $\hat{k}_1$. It is then obvious by induction that, for any voting rule $\hat{k}_2$ with $\hat{k}_1<\hat{k}_2$, the expected utility is higher under $\hat{k}_2$ than under $\hat{k}_1$.

If all jurors vote sincerely, the ex ante expected utility for juror j under voting rule $\hat{k}_1$ when the defendant is guilty is given by:

$$EU_j\left(\hat{k}_1 \middle| G\right) = -\left(1-q_j\right)\mathrm{Prob}_s^{\hat{k}_1}(A \mid G) + EU_j\left(\hat{k}_1 \middle| G\right)\mathrm{Prob}_s^{\hat{k}_1}(M \mid G)$$

$$EU_j\left(\hat{k}_1 \middle| G\right) = \frac{-\left(1-q_j\right)\mathrm{Prob}_s^{\hat{k}_1}(A \mid G)}{1-\mathrm{Prob}_s^{\hat{k}_1}(M \mid G)}$$

$$EU_j\left(\hat{k}_1 \middle| G\right) = \frac{-\left(1-q_j\right)\sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{1-\sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1}\binom{n}{x}p^{n-x}(1-p)^x}$$

Similarly, we can show that the ex ante expected utility for juror j under voting rule $\hat{k}_1$ when the defendant is innocent is given by:

$$EU_j\left(\hat{k}_1 \middle| I\right) = \frac{-q_j\sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{1-\sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1}\binom{n}{x}p^{n-x}(1-p)^x}$$

Thus, the overall ex ante expected utility for juror j under voting rule $\hat{k}_1$ is given by:

$$EU_j\left(\hat{k}_1\right) = \mathrm{Prob}(G)\cdot\frac{-\left(1-q_j\right)\sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{1-\sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1}\binom{n}{x}p^{n-x}(1-p)^x} + \mathrm{Prob}(I)\cdot\frac{-q_j\sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{1-\sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1}\binom{n}{x}p^{n-x}(1-p)^x}$$

$$= \frac{-\left(1-q_j\right)-q_j}{2}\cdot\frac{\sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{1-\sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1}\binom{n}{x}p^{n-x}(1-p)^x}$$

$$= \frac{-\sum_{x=\hat{k}_1}^{n} \binom{n}{x} p^{n-x}(1-p)^{x}}{2-2\sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1} \binom{n}{x} p^{n-x}(1-p)^{x}}$$

Similarly, the overall ex ante expected utility for juror j under voting rule $\hat{k}_1+1$ is given by:

$$EU_j\left(\hat{k}_1+1\right) = \frac{-\sum_{x=\hat{k}_1+1}^{n} \binom{n}{x} p^{n-x}(1-p)^{x}}{2-2\sum_{x=n-\hat{k}_1}^{\hat{k}_1} \binom{n}{x} p^{n-x}(1-p)^{x}}$$

We now show that $EU_j\left(\hat{k}_1+1\right) > EU_j\left(\hat{k}_1\right)$:

$$(1-p)^{2(x-\hat{k}_1)} < p^{2(x-\hat{k}_1)} \text{ for any } x > \hat{k}_1$$

$$p^{\hat{k}_1-x}(1-p)^{x-\hat{k}_1} < p^{x-\hat{k}_1}(1-p)^{\hat{k}_1-x} \text{ for any } x > \hat{k}_1$$

$$\binom{n}{x} p^{n+\hat{k}_1-x}(1-p)^{n+x-\hat{k}_1} < \binom{n}{x} p^{n+x-\hat{k}_1}(1-p)^{n+\hat{k}_1-x} \text{ for any } x > \hat{k}_1$$

$$\sum_{x=\hat{k}_1+1}^{n} \binom{n}{x} p^{n+\hat{k}_1-x}(1-p)^{n+x-\hat{k}_1} < \sum_{x=\hat{k}_1+1}^{n} \binom{n}{x} p^{n+x-\hat{k}_1}(1-p)^{n+\hat{k}_1-x}$$

$$\sum_{x=\hat{k}_1+1}^{n} \binom{n}{x}\left(p^{2n-\hat{k}_1-x}(1-p)^{\hat{k}_1+x} + p^{n+\hat{k}_1-x}(1-p)^{n+x-\hat{k}_1}\right)$$

$$< \sum_{x=\hat{k}_1+1}^{n} \binom{n}{x}\left(p^{2n-\hat{k}_1-x}(1-p)^{\hat{k}_1+x} + p^{n+x-\hat{k}_1}(1-p)^{n+\hat{k}_1-x}\right)$$

$$\binom{n}{\hat{k}_1}\left(p^{n-\hat{k}_1}(1-p)^{\hat{k}_1} + p^{\hat{k}_1}(1-p)^{n-\hat{k}_1}\right)\sum_{x=\hat{k}_1+1}^{n} \binom{n}{x} p^{n-x}(1-p)^{x}$$

$$< \binom{n}{\hat{k}_1} p^{n-\hat{k}_1}(1-p)^{\hat{k}_1} \sum_{x=\hat{k}_1+1}^{n} \binom{n}{x}\left(p^{n-x}(1-p)^{x} + p^{x}(1-p)^{n-x}\right)$$

$$\left(\sum_{x=\hat{k}_1}^{n} \binom{n}{x}\left(p^{n-x}(1-p)^{x} + p^{x}(1-p)^{n-x}\right)\right)\left(\sum_{x=\hat{k}_1+1}^{n} \binom{n}{x} p^{n-x}(1-p)^{x}\right)$$

$$< \left(\sum_{x=\hat{k}_1}^{n} \binom{n}{x} p^{n-x}(1-p)^{x}\right)\left(\sum_{x=\hat{k}_1+1}^{n} \binom{n}{x}\left(p^{n-x}(1-p)^{x} + p^{x}(1-p)^{n-x}\right)\right)$$

$$\frac{\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}\left(p^{n-x}(1-p)^x+p^x(1-p)^{n-x}\right)} < \frac{\sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{\sum_{x=\hat{k}_1}^{n}\binom{n}{x}\left(p^{n-x}(1-p)^x+p^x(1-p)^{n-x}\right)}$$

$$\frac{\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{1-\sum_{x=n-\hat{k}_1}^{\hat{k}_1}\binom{n}{x}p^{n-x}(1-p)^x} < \frac{\sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{1-\sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1}\binom{n}{x}p^{n-x}(1-p)^x}$$

$$\frac{-\sum_{x=\hat{k}_1+1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{2-2\sum_{x=n-\hat{k}_1}^{\hat{k}_1}\binom{n}{x}p^{n-x}(1-p)^x} > \frac{-\sum_{x=\hat{k}_1}^{n}\binom{n}{x}p^{n-x}(1-p)^x}{2-2\sum_{x=n-\hat{k}_1+1}^{\hat{k}_1-1}\binom{n}{x}p^{n-x}(1-p)^x}$$

$$EU_j\left(\hat{k}_1+1\right) > EU_j\left(\hat{k}_1\right)$$

By induction, we have $EU_j\left(\hat{k}_2\right) > EU_j\left(\hat{k}_1\right)$.

Q.E.D.

Proposition 5: Let the jurors be numbered such that $q_1 \le q_2 \le \ldots \le q_{n-1} \le q_n$. Then the sincere revelation strategy profile is a subgame perfect Nash equilibrium for a given voting rule $\hat{k}$ if and only if one of the following conditions is true:

(a) $0 \le q_{\hat{k}} \le \beta(0,n)$;

(b) $\beta(n,n) < q_{\hat{k}} \le 1$; or

(c) $\exists k^* \in \{1, \ldots, n\}$ such that $\beta(k^*-1,n) \le q_j \le \beta(k^*,n)$ for all $j \in N$.

Proof: First recognize that a strategic voter will condition her strategies in both the preliminary and final votes on the event that her vote is pivotal; that is, that her vote can change the trial outcome. In the event that her vote is <u>not</u> pivotal, her utility is unaffected by her vote and therefore such situations have no implications for strategic behavior.

We will evaluate strategy in the final vote first and then work backwards to examine the preliminary vote.

Final Vote Strategy:

Assume that in the preliminary vote, $\sigma_j(g)=1$ and $\sigma_j(i)=0$ for all jurors $j \in N$. Further assume that all jurors $j \in N$ vote to convict in the final vote iff $\beta(k,n) \geq q_j$, where k is the number of votes to convict from the preliminary vote. We must show that no juror has an incentive to deviate from this strategy in the final vote.

Note that, since all jurors vote sincerely in the preliminary vote, all jurors will know the total number of guilty (g) and innocent (i) signals before taking the final vote. Thus, all jurors will have the same estimate of the probability that the defendant is guilty, namely $\beta(k,n)$.

For any given juror, we need only consider the situation in which the juror's vote is pivotal. That is, if the given juror votes to convict, the defendant will be convicted, and if the given juror votes to acquit, the defendant will be acquitted. Thus, for any juror j, the expected utility of voting to acquit in this case is given by:

$$\begin{aligned} EU(A \mid |g| = k) &= -(1-q_j) \cdot Prob(G \mid |g| = k) \\ &= -(1-q_j) \cdot \beta(k,n) \end{aligned}$$

Similarly, the expected utility of voting to convict is given by:

$$\begin{aligned} EU(C \mid |g| = k) &= -q_j \cdot Prob(I \mid |g| = k) \\ &= -q_j \cdot (1-\beta(k,n)) \end{aligned}$$

Therefore, juror j will want to vote to convict iff:

$$\begin{aligned} EU(C \mid |g| = k) &\geq EU(A \mid |g| = k) \\ -q_j \cdot Prob(I \mid |g| = k) &\geq -(1-q_j) \cdot Prob(G \mid |g| = k) \\ -q_j \cdot (1-\beta(k,n)) &\geq -(1-q_j) \cdot \beta(k,n) \\ q_j \cdot (1-\beta(k,n)) &\leq (1-q_j) \cdot \beta(k,n) \\ q_j - q_j\beta(k,n) &\leq \beta(k,n) - q_j\beta(k,n) \\ q_j &\leq \beta(k,n) \end{aligned}$$

Therefore, sophisticated sincere voting in the final vote is a Nash equilibrium for this subgame. Recognize that this result is dependent only upon the assumption of sincere voting in the preliminary vote is independent of satisfaction or violation of conditions (a), (b), and (c).

Preliminary Vote Strategy:

Now assume that all jurors $j \in N$ vote to convict in the final vote iff $\beta(k,n) \geq q_j$, where k is the number of votes to convict from the preliminary vote. Further assume that $\sigma_j(g)=1$ and $\sigma_j(i)=0$ for all jurors $j \in N$ in the preliminary vote. We must show that, if one of the conditions, (a), (b), or (c), is satisfied, then no juror has an incentive to deviate from this sincere voting strategy in the preliminary vote. We must also show that, if all three conditions are violated, then at least one juror has an incentive to deviate from sincerity in the preliminary vote.

Case 1: Condition (a) is satisfied

In this case, we have that $0 \leq q_1 \leq \ldots \leq q_{\hat{k}} \leq \beta(0,n)$. This means that, in the final vote, at least $\hat{k}$ jurors will always vote to convict, and the defendant will thus always be convicted, regardless of the outcome of the preliminary vote. Therefore, no juror has a positive incentive to deviate from sincerity in the preliminary vote.

Case 2: Condition (b) is satisfied

In this case, we have that $\beta(n,n) \leq q_{\hat{k}} \leq \ldots \leq q_n \leq 1$ for all $j \in N$. This means that, in the final vote, at least $n-\hat{k}+1$ jurors will always vote to acquit, and the defendant will thus always be acquitted, regardless of the outcome of the preliminary vote. Therefore, no juror has a positive incentive to deviate from sincerity in the preliminary vote.

Case 3: Condition (c) is satisfied

In this case, we have that $\exists k^* \in \{1, \ldots, n\}$ such that $\beta(k^*-1,n) \leq q_j \leq \beta(k^*,n)$ for all $j \in N$. Thus, if juror j is pivotal in the preliminary vote, this means that $|g|_{-j}=k^*-1$. In other words, if juror j votes C in the preliminary vote, all other jurors will vote C in the final vote, and if juror j votes A in the preliminary vote, all other jurors will vote A in the final vote.

Note that this means that if juror j is pivotal in the preliminary vote, juror j can completely dictate the final trial outcome through her preliminary vote. Even under unanimity rule, juror j's

preliminary vote will determine the final vote of all other jurors, thus allowing juror j to choose the trial outcome with her final vote. Thus, we can say that a juror will prefer to vote C in the preliminary vote if and only if she prefers that the defendant be convicted in the final outcome $\left(\text{i.e., } EU\left(C|\,|g|_{-j} = k^* - 1\right) \geq EU\left(A|\,|g|_{-j} = k^* - 1\right)\right)$

Now suppose that s_j=i. In this case, we have that:

$$EU\left(C|\,|g|_{-j} = k^* - 1\right) = EU\left(C|\,|g| = k^* - 1\right) = -q_j \cdot \left(1 - \beta(k^* - 1, n)\right)$$

$$EU\left(A|\,|g|_{-j} = k^* - 1\right) = EU\left(A|\,|g| = k^* - 1\right) = -\left(1 - q_j\right) \cdot \beta(k^* - 1, n)$$

$$\begin{aligned}
\beta(k^* - 1, n) < q_j &\Rightarrow \beta(k^* - 1, n) - q_j \cdot \beta(k^* - 1, n) < q_j - q_j \cdot \beta(k^* - 1, n) \\
&\Rightarrow -\left(1 - q_j\right) \cdot \beta(k^* - 1, n) > -q_j \cdot \left(1 - \beta(k^* - 1, n)\right) \\
&\Rightarrow EU\left(A|\,|g|_{-j} = k^* - 1\right) > EU\left(C|\,|g|_{-j} = k^* - 1\right)
\end{aligned}$$

Now suppose that s_j=g. In this case, we have that:

$$EU\left(C|\,|g|_{-j} = k^* - 1\right) = EU\left(C|\,|g| = k^*\right) = -q_j \cdot \left(1 - \beta(k^*, n)\right)$$

$$EU\left(A|\,|g|_{-j} = k^* - 1\right) = EU\left(A|\,|g| = k^*\right) = -\left(1 - q_j\right) \cdot \beta(k^*, n)$$

$$\begin{aligned}
\beta(k^*, n) \geq q_j &\Rightarrow \beta(k^*, n) - q_j \cdot \beta(k^*, n) \geq q_j - q_j \cdot \beta(k^*, n) \\
&\Rightarrow \left(1 - q_j\right) \cdot \beta(k^*, n) \geq q_j \cdot \left(1 - \beta(k^*, n)\right) \\
&\Rightarrow -\left(1 - q_j\right) \cdot \beta(k^*, n) \leq -q_j \cdot \left(1 - \beta(k^*, n)\right) \\
&\Rightarrow EU\left(A|\,|g|_{-j} = k^* - 1\right) \leq EU\left(C|\,|g|_{-j} = k^* - 1\right)
\end{aligned}$$

Thus, a juror j will prefer to vote to convict in the preliminary vote if and only if s_j=g.

<u>Case 4:</u> Conditions (a), (b), and (c) are all violated

Violation of conditions (a) and (b) means that $\exists k^* \in \{1, ..., n\}$ such that $\beta(k^*-1,n) < q_{\hat{k}} \leq \beta(k^*,n)$. For a given juror j to be pivotal in the preliminary vote, it therefore means that $|g|_{-j}=k^*-1$. Violation of condition (c) means that $q_1 < \beta(k^*-1,n)$ and/or $\beta(k^*,n) < q_n$.

Suppose $q_1 < \beta(k^*-1,n)$ and consider the situation in which juror 1 is pivotal (i.e., $|g|_{-1}=k^*-1$) and s_1 = i. If juror 1 votes A in the preliminary vote (i.e., votes sincerely), the defendant will be

acquitted, since $\beta(k^*-1,n) < q_{\hat{k}}$. However, if juror 1 instead deviates and votes C, the defendant will be convicted, since $q_{\hat{k}} \leq \beta(k^*,n)$. Since $q_1 < \beta(k^*-1,n)$, juror 1 prefers that the defendant is convicted, and therefore juror 1 has a positive incentive to deviate and vote C.

Now suppose $\beta(k^*,n) < q_n$ and consider the situation in which juror n is pivotal (i.e., $|g|_{-n}=k^*-1$) and $s_n = g$. If juror n votes C in the preliminary vote (i.e., votes sincerely), the defendant will be convicted, since $q_{\hat{k}} \leq \beta(k^*,n)$. However, if juror n instead deviates and votes A, the defendant will acquitted, since $\beta(k^*-1,n) < q_{\hat{k}}$. Since $\beta(k^*,n) < q_n$, juror n prefers that the defendant is acquitted, and therefore juror n has a positive incentive to deviate and vote A.

Thus, if conditions (a), (b), and (c) are all violated, then sincere voting is not a Nash equilibrium in the preliminary vote.

Q.E.D.

Proposition 6: For n odd, the sincere revelation strategy profile is a subgame perfect Nash equilibrium for any voting rule $\hat{k}$ if:

$$1\text{-}p \leq q_j \leq p, \ \forall j \in N.$$

For n even, the sincere revelation strategy profile is a subgame perfect Nash equilibrium for any voting rule $\hat{k}$ if:

$$\frac{(1-p)^2}{p^2+(1-p)^2} \leq q_j \leq 0.5, \ \forall j \in N \quad \text{or} \quad 0.5 \leq q_j \leq \frac{p^2}{p^2+(1-p)^2}, \ \forall j \in N.$$

For any value of n, odd or even, the sincere revelation strategy profile is a subgame perfect Nash equilibrium for any voting rule $\hat{k}$ if:

$$1\text{-}p \leq q_j \leq 0.5, \ \forall j \in N \quad \text{or} \quad 0.5 \leq q_j \leq p, \ \forall j \in N.$$

Proof: First, suppose that n is odd. Proposition 5 says that the sincere revelation strategy profile is a subgame perfect Nash equilibrium for any voting rule $\hat{k}$ if:

$$\beta\left(\tfrac{n-1}{2}, n\right) \leq q_j \leq \beta\left(\tfrac{n+1}{2}, n\right), \ \forall j \in N.$$

This condition is equivalent to:

$$\frac{p^{\frac{n-1}{2}}(1-p)^{\frac{n+1}{2}}}{p^{\frac{n-1}{2}}(1-p)^{\frac{n+1}{2}}+p^{\frac{n+1}{2}}(1-p)^{\frac{n-1}{2}}} \le q_j \le \frac{p^{\frac{n+1}{2}}(1-p)^{\frac{n-1}{2}}}{p^{\frac{n-1}{2}}(1-p)^{\frac{n+1}{2}}+p^{\frac{n+1}{2}}(1-p)^{\frac{n-1}{2}}}$$

$$\frac{1-p}{1-p+p} \le q_j \le \frac{p}{1-p+p}$$

$$1-p \le q_j \le p$$

Now, suppose that n is even. Proposition 5 says that the sincere revelation strategy profile is a subgame perfect Nash equilibrium for any voting rule $\hat{k}$ if:

$$\beta\left(\tfrac{n}{2}-1,n\right) \le q_j \le \beta\left(\tfrac{n}{2},n\right),\ \forall j \in N \quad \text{or} \quad \beta\left(\tfrac{n}{2},n\right) \le q_j \le \beta\left(\tfrac{n}{2}+1,n\right),\ \forall j \in N.$$

The first of these two conditions is equivalent to:

$$\frac{p^{\frac{n}{2}-1}(1-p)^{\frac{n}{2}+1}}{p^{\frac{n}{2}-1}(1-p)^{\frac{n}{2}+1}+p^{\frac{n}{2}+1}(1-p)^{\frac{n}{2}-1}} \le q_j \le \frac{p^{\frac{n}{2}}(1-p)^{\frac{n}{2}}}{p^{\frac{n}{2}}(1-p)^{\frac{n}{2}}+p^{\frac{n}{2}}(1-p)^{\frac{n}{2}}}$$

$$\frac{(1-p)^2}{p^2+(1-p)^2} \le q_j \le 0.5$$

The second of these two conditions is equivalent to:

$$\frac{p^{\frac{n}{2}}(1-p)^{\frac{n}{2}}}{p^{\frac{n}{2}}(1-p)^{\frac{n}{2}}+p^{\frac{n}{2}}(1-p)^{\frac{n}{2}}} \le q_j \le \frac{p^{\frac{n}{2}+1}(1-p)^{\frac{n}{2}-1}}{p^{\frac{n}{2}-1}(1-p)^{\frac{n}{2}+1}+p^{\frac{n}{2}+1}(1-p)^{\frac{n}{2}-1}}$$

$$0.5 \le q_j \le \frac{p^2}{p^2+(1-p)^2}$$

Now recognize that for $p \in (0.5,1.0)$, we have that $1-2p < 0$ and $p-1 < 0$. This gives us:

$$\begin{aligned}
(p-1)(1-2p) > 0 &\Rightarrow 3p-2p^2-1 > 0 \\
&\Rightarrow p > p^2+(1-p)^2 \\
&\Rightarrow \frac{p}{p^2+(1-p)^2} > 1 \\
&\Rightarrow \frac{p^2}{p^2+(1-p)^2} > p
\end{aligned}$$

$$
\begin{aligned}
1-2p < 0 &\Rightarrow 1 < 2p \\
&\Rightarrow p < 2p^2 \\
&\Rightarrow p+1-p < 2p^2+1-p \\
&\Rightarrow 1-p < p^2+(1-p)^2 \\
&\Rightarrow \frac{1-p}{p^2+(1-p)^2} < 1 \\
&\Rightarrow \frac{(1-p)^2}{p^2+(1-p)^2} < 1-p
\end{aligned}
$$

Now we can combine the conditions for both odd and even n that are given above. Doing so, we see that, for any value of n, odd or even, the sincere revelation strategy profile is a subgame perfect Nash equilibrium for any voting rule $\hat{k}$ if:

$$1-p \le q_j \le 0.5,\ \forall j \in N \quad \text{or} \quad 0.5 \le q_j \le p,\ \forall j \in N.$$

Q.E.D.

Proposition 7: Suppose the juror utilities satisfy $0.5 \le q_1 \le q_2 \le \ldots \le q_{n-1} \le q_n$. If condition (a), (b), or (c) from Proposition 5 is satisfied under voting rule $\hat{k}_1$, then the same condition is satisfied under any other voting rule $\hat{k}_2$ satisfying $\hat{k}_2 > \hat{k}_1$.

Proof: Suppose condition (a) is satisfied for voting rule $\hat{k}_1$. This means that

$$0 \le q_{\hat{k}_1} \le \beta(0,n) = \frac{(1-p)^n}{p^n+(1-p)^n} < 0.5.$$

Since $q_j \ge 0.5$ for all j, condition (a) can not be satisfied for $\hat{k}_1$, and therefore the proposition is satisfied vacuously in this case.

Now suppose condition (b) is satisfied for voting rule $\hat{k}_1$. This means that $\beta(n,n) < q_{\hat{k}_1} \le 1$. Since $\hat{k}_1 < \hat{k}_2$, we have that $q_{\hat{k}_1} < q_{\hat{k}_2}$, and thus that $\beta(n,n) < q_{\hat{k}_2} \le 1$. Therefore, condition (b) is also satisfied for voting rule $\hat{k}_2$.

Finally, suppose that condition (c) is satisfied for voting rule $\hat{k}_1$. In this case, the condition is completely independent of the voting rule, thus condition (c) is also satisfied for voting rule $\hat{k}_2$.

Q.E.D.

Proposition 8: Suppose that the sincere revelation strategy profile is a subgame perfect Nash equilibrium for two voting rules, $\hat{k}_1$ and $\hat{k}_2$. If jurors behave according to this Nash equilibrium, then:

(1) The probability of convicting an innocent defendant is the same under both voting rules.

(2) The probability of acquitting a guilty defendant is the same under both voting rules.

Proof: Without loss of generality, assume $\hat{k}_1 \leq \hat{k}_2$. Existence of the sincere voting Nash equilibrium means that one of the three Proposition 5 conditions, (a), (b), or (c), is satisfied for each of the voting rules $\hat{k}_1$ and $\hat{k}_2$. It is also straightforward to show that both rules must satisfy the same condition (to see this, follow the same approach as used in the proof of Proposition 7).

Suppose both rules satisfy condition (a). In this case, $0 \leq q_j \leq \beta(0,n)$ for $j=1,2,\ldots,\hat{k}_1,\ldots,\hat{k}_2$. Thus, at least $\hat{k}_2$ jurors will always vote to convict in the final vote regardless of the outcome of the preliminary vote and regardless of the voting rule. Therefore, all defendants are convicted under both voting rules, and the probability of trial error under both voting rules is simply 0.5 (the prior probability that the defendant is innocent).

Now suppose both rules satisfy condition (b). In this case, $\beta(n,n) \leq q_j \leq 1$ for $j=\hat{k}_1,\ldots,\hat{k}_2,\ldots,n$. Thus, no more than $\hat{k}_1$ jurors will ever vote to convict in the final vote regardless of the outcome of the preliminary vote and regardless of the voting rule. Therefore, all defendants are acquitted under both voting rules, and the probability of trial error under both voting rules is simply 0.5 (the prior probability that the defendant is guilty).

Finally suppose both rules satisfy condition (c). In this case, $\exists k^* \in \{1, \ldots, n\}$ such that $\beta(k^*-1,n) \leq q_j \leq \beta(k^*,n)$ for all $j \in N$. Recall that the number of votes to convict in the preliminary vote will be equal to $|g|$ in equilibrium. Thus, if $|g| \geq k^*$, all jurors will vote to convict in the final vote, and if $|g| < k^*$, all jurors will vote to acquit in the final vote. Since all final votes are unanimous, if a defendant is convicted under one voting rule, she would also be convicted under the other voting rule. Therefore, the probability of convicting an innocent defendant must be the same under both

voting rules. Similarly, if a defendant is acquitted under one voting rule, she would also be acquitted under the other voting rule. Therefore, the probability of acquitting a guilty defendant must also be the same under both voting rules.

Q.E.D.

Proposition 9: Suppose the subgame perfect Nash equilibrium described in Proposition 1 exists for any two voting rules, $\hat{k}_1$ and $\hat{k}_2$. If jurors behave according to this Nash equilibrium, then the expected utility for any juror is the same under both voting rules.

Proof: In the proof of Proposition 8, we showed that the trial outcome will always be the same under both voting rules. Therefore, the expected utility (and, in fact, the final realized utility) must be the same under both voting rules, also.

Q.E.D.