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THE REAPPORTIONMENT REVOLUTION AND BIAS IN U.S. CONGRESSIONAL ELECTIONS

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The Reapportionment Revolution and Bias in U.S. Congressional Elections*

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Abstract

We develop a simple formal model of the redistricting process that highlights the importance of two factors: first, partisan or bipartisan control of the redistricting process; second, the nature of the reversionary outcome, should the state legislature and governor fail to agree on a new districting plan. Using this model, we derive various predictions about the levels of partisan bias and responsiveness that should be observed under districting plans adopted under various constellations of partisan control of state government and reversionary outcomes, testing our predictions on postwar (1946–70) U.S. House electoral data. We find strong evidence that both partisan control and reversionary outcomes systematically affect the nature of a redistricting plan and the subsequent elections held under it. Further, we show that the well-known disappearance circa 1966 of what had been a long-time pro-Republican bias of about 6% in nonsouthern congressional elections can be explained completely by the changing composition of northern districting plans.

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The Supreme Court's landmark decision in *Baker v. Carr* (369 U.S. 186, 1962) – holding that state legislative apportionments were justiciable – unleashed a flood of litigation that engulfed all but 14 states within the year. Using several of these cases as vehicles to clarify its substantive position, the Court affirmed that the votes of all citizens must count equally, a position that implied equality of population between legislative districts. In *Wesberry v. Sanders* (376 U.S. 1, 1964), the Court extended its "one man, one vote" principle to cover congressional elections.

The practical result of this flurry of Court decisions and litigation was a massive redistricting effort throughout the country, as state legislators redrew both their own districts and their states' congressional districts in order to eradicate the substantial levels of malapportionment that then existed. Five states redrew their congressional districts between the time the *Wesberry* decision was handed down in February and the 89th Congress was elected in November 1964. The main attack on malapportionment, however, was launched after the 1964 election – with twenty-four states redrawing their congressional districts before the 90th, seventeen before the 91st, and six before the 92nd Congress.

In this paper, we develop a general model of the redistricting process in the American states and apply it to the study of the reapportionment revolution. We focus in particular on how redistricting in the 1960s affected the partisan outcome of nonsouthern congressional elections.[1] The conventional wisdom is that redistricting (whether in the 1960s or not) is unlikely to produce net partisan gains but we show that nonsouthern Democrats were substantial net beneficiaries of redistricting in the 1960s. Our results explain a long-standing puzzle in the literature on congressional elections: the sudden disappearance of pro-Republican bias in the translation of votes into seats in nonsouthern elections circa 1966.

The rest of the paper proceeds as follows. We first review previous work on "the case of the disappearing bias." We then present a model of the redistricting process, deriving hypotheses about levels of partisan bias and responsiveness under different constellations of partisan control of the redistricting process and reversionary outcome. After reviewing the literature on whether 1960s redistricting plans were partisan, incumbent-protecting, or neither, we suggest why these plans should have benefited northern Democrats. In section 4, we apply our model to congressional redistricting action from 1946 to 1970. Section 5 tests our theoretical expectations against the empirical record. Unlike previous studies, we find that a model in which parties are the key actors explains the data well and that control of redistricting produced substantial gains for the redistricting party throughout this period. Moreover, our results explain the disappearance of pro-Republican bias in nonsouthern elections in the mid-1960s. Section 6 concludes.

[1] We define as nonsouthern all states not in the former Confederacy. We shall also refer to these states as "northern," although they of course include western, mountain and border states in addition to the states that traditionally fall under the rubric of "northern".

1 The case of the disappearing bias

A series of studies (e.g., Erikson 1972; Jacobson 1990; Brady and Grofman 1991; King and Gelman 1991) show the disappearance (circa 1966) of a substantial pro-Republican bias in nonsouthern House elections. Erikson (1972:1234), for example, estimated partisan bias by regressing the Democratic share of nonsouthern seats on the aggregate Democratic share of the nonsouthern vote. Over the period 1952–64 he found that, "had the aggregate vote division been a 50–50 partisan split, the Democrats could have expected to win only about 44.6 per cent of the seats." In contrast, his study found no pro-Republican bias in the translation of congressional votes into seats outside the south after 1966. Although well-known and replicated with longer time series of data (by, e.g., Jacobson 1990; Brady and Grofman 1991; King and Gelman 1991), this abrupt disappearance of partisan bias in nonsouthern House elections has not been adequately explained in the literature.

Some have suggested that the disappearing bias is a simple reflection of the nature of malapportionment in the north. In particular, Republicans were strongest in rural areas, which were the most heavily overrepresented — thus, eradicating malapportionment must necessarily have hurt them. There is a well-known problem with this story, however. As Erikson (1972:1236) reports, the correlation between Democratic vote share and population in nonsouthern districts was negative prior to the mid-1960s, not positive. The Republicans were well aware of this, having commissioned an internal study to forecast the consequences of eradicating malapportionment in 1964 — and concluding that they would benefit (cf. Prendergast 1965).

The most prominent explanation of the disappearing bias is Erikson's (1972) and King and Gelman's (1991) suggestion that growth in the incumbency advantage explains the pro-Democratic trend in partisan bias. King and Gelman's figures on northern bias (1991:128) show no linear trend over the period 1946–64 (with the year-by-year estimates averaging about -10%), a jump from -10% in 1964 to +2% in 1966 (by far the biggest shift between consecutive years), followed by no consistent linear trend in the period 1966–86 (with an average of about +0.5%). Their figures on the incumbency advantage can be read as showing a shallow rate of growth from 1946 to 1964, followed by a large (10%) increase between 1964 and 1966, with no linear trend thereafter. King and Gelman (1991:131) thus suggest that "the explanation of the trend in bias seems to be that the incumbency advantage happened to begin its dramatic increase at a time (the 1940s) when Democrats held a majority of House seats."

The main problem with this story is that the Gelman-King measure of the incumbency advantage is vote-denominated, not seat-denominated. If incumbency confers more votes on average but does not increase the probability of victory (as in Jacobson 1987), then the efficiency with which Democratic votes were translated into Democratic seats would not be improved; it would be worsened. The Erikson, King and Gelman story line requires that the seat-denominated incumbency advantage increased but there is no evidence that this happened over the relevant time period (Jacobson 1987). Thus, the disappearance of pro-Republican bias outside the south remains a puzzle.

In our view, the explanation for the disappearing bias lies in the wave of redistricting sparked by the Supreme Court's reapportionment decisions. Erikson (1972) rejected this possibility after dividing the sample of congressional districts into redrawn and unchanged categories, plotting the distribution of Democratic vote shares in the two sub-samples side-by-side, and finding no difference.

Visual inspections of graphs, however, are not always reliable. In order to test the claim that redistricting did not affect congressional vote shares more sytematically, we ran a standard regression model predicting the Democratic vote share (based on Cox and Katz 1996 and Gelman and King 1990). The regression controls for national partisan swings, the lagged Democratic vote share, lagged and current incumbency status, and lagged and current challenger quality. (The lagged variables are present in the model to capture the "normal vote" — that is the vote a typical Democratic candidate would get in the district.) We also included a variable indicating if the district was redrawn prior to the election, to capture any systematic differences between redrawn and unchanged districts. Estimating the model using data from all incumbent-contested, nonsouthern congressional districts from 1966 to 1970, we found the effect of redistricting to be a loss of 1.58% points for the Democratic candidate (with a standard error of 0.44). Thus, contrary to Erikson's (1972) speculation, redistricting *did* affect the distribution of the congressional vote.

Moreover, running similar analysis (now as a probit) to model the probability of a Democartic victory, we found that the coefficient on the redistricting variable switched signs but was again statistically significant. That is, Democratic incumbents whose districts were redrawn were more likely to win, while Republican incumbents whose districts were redrawn were less likely to win. If one asks how Democratic incumbents in redrawn districts could get smaller vote shares on average while at the same time winning more often, the most readily intelligible answer is in terms of redistricting. Pro-Republican gerrymanders give most Democratic incumbents inefficiently large margins of victory, while perhaps isolating a few for defeat. Pro-Democratic gerrymanders similarly give most Republican incumbents inefficiently large margins of victory, while targeting a few for defeat. Thus, if the 1960s saw a change from mostly pro-Republican districting plans to mostly bipartisan and Democratic plans (as we shall show that it did), one expects a lower average Democratic vote share but a better average chance at victory — precisely what we found. The case of the disappearing bias should be reopened.

2 A model of the redistricting process

The literature on redistricting categorizes gerrymanders according to the varying goals that those who redraw district lines pursue. For present purposes, the most important categories are pro-incumbent gerrymanders (when a bipartisan alliance draws the lines to preserve the current incumbents' chances of victory) and partisan gerrymanders (when a single party draws the lines to maximize its seat share). In this section, we develop a model in which these two types of gerrymander — along with a third "mixed" type —

emerge endogenously as a function of partisan control of the redistricting process and the nature of the reversionary outcome.

Preliminaries. Since 1913, the U.S. House has had a fixed number (435) of seats. Since 1929, these seats have been automatically apportioned among the states according to population after each decennial census. It has then been up to each state to define the boundaries of its allotted number of districts. Typically, district boundaries have been established by the passage of a state law, with state legislatures and governors bargaining within the confines of the ordinary statutory processes of their states.

The literal output of the redistricting process is a description of the boundary lines of each congressional district in the state. But parties care about who wins the various seats at stake, not about lines per se, and so districting plans are usually described in terms of their political rather than literal characteristics. One approach (see, e.g., Owen and Grofman 1988; McDonald N.d.) focuses on how boundary lines affect the resultant partisan make-up of each district (measured, for example, by the number of registered Republicans and Democrats).

In our model, a particular set of district lines affects two broad features of the translation of votes into seats, what we shall call partisan bias and responsiveness. Partisan bias is a standard concept in the literature, usually defined as the difference between the average seat share that the Democrats would get with an average vote share of 0.5 and their "fair share" of 0.5 (half the seats for half the votes). For example, if the Democrats on average win 55% of New Jersey's state assembly seats when their candidates average 50% of the vote, then the translation of votes into seats in New Jersey legislative elections exhibits a 5% pro-Democratic bias. If a single party controls the redistricting process, there is a standard recipe it can follow in increasing partisan bias in its favor: pack as many of the other party's supporters in as few districts as possible (creating inefficiently safe districts), while spreading its own supporters across as many districts as possible (creating winnable but not inefficiently safe districts).

Increasing partisan bias is one way by which a redistricting party might seek to maximize its expected seat share, but it is not the only way. If a party is quite sure that it will get a majority of votes cast in its favor over the life of a districting plan, another way to maximize its expected seat share is to make every district a microcosm of the state. As compared to the packing strategy, this creates more marginal districts, and will thus increase what is typically called the responsiveness of the system. Another way to think of the microcosm strategy is that it makes the system more like a winner-take-all election: whoever gets the most votes statewide typically gets the most votes in all the districts (which are microcosms of the state by hypothesis) and therefore wins all the seats. Given sufficient certainty about its electoral strength, a redistricting party might rationally seek to maximize its expected seat share by maximizing responsiveness, thus producing a system that favors big parties (where "big" is defined in terms of vote share), rather than engineering a bias specifically in its own favor. This strategy runs the risk of giving the "out" party all the seats, should it manage an unexpectedly good electoral showing; but it does not concede any districts at all to the opposition, as the packing strategy does.

Thus, although what is literally chosen in the redistricting process are the lines that define the various districts into which a state is partitioned, we assume that each set of lines induces a particular level of partisan bias (in favor of one or the other party) and responsiveness (in favor of whichever party gets more votes). Parties, when they choose districting plans, can then be thought of as choosing levels of partisan bias and responsiveness, in light of the consequences that these choices will have for their success in winning seats.

Votes into Seats. How do parties think levels of bias and responsiveness affect the translation of votes into seats? Here, we assume that their understanding of the matter is similar to that embodied in the standard seats-votes curve. Consider a series of elections to a given legislature (all districts being single-member and only two parties competing). Let s denote the share of legislative seats that the Democrats win in some state; and v denote the mean share of the vote that the Democrats garner in that election. The standard seats-votes curve (see, e.g., King and Browning 1987; King 1989; Campagna and Grofman 1990), a generalization of the classic cube law equation (Kendall and Stuart 1950), is:

$$\frac{s}{1-s} = e^{\lambda}\left(\frac{v}{1-v}\right)^{\rho} \tag{1}$$

Obviously, parties need not carry this equation around in their collective heads. What is important is that this equation capture the abstract characteristics of districting plans identified above (partisan bias and responsiveness); and that these terms in fact affect the translation of votes into seats in qualitatively the way described above. These requirements are met by Equation (1).

First, the parameter λ relates directly to pro-Democratic bias. Recall that bias is defined as the share of seats the Democrats are predicted to win with a mean vote share of 0.5, less 0.5. If $v = 0.5$, then $(\frac{v}{1-v})^{\rho} = 1$ for any value of ρ. Hence, from Equation (1), the value of $\frac{s}{1-s}$ is e^{λ}, the value of s is $\frac{e^{\lambda}}{e^{\lambda}+1}$, and the partisan bias in the system can accordingly be expressed as:

$$\textsc{Partisan Bias} = \frac{e^{\lambda}}{e^{\lambda}+1} - 0.5. \tag{2}$$

Negative values of partisan bias indicate a Republican advantage in the efficiency of votes-to-seats translations, while positive values indicate a Democratic advantage.

Second, the parameter ρ reflects responsiveness: how much a party's seat share can be expected to increase with a given increment in its vote share. Figure 1 illustrates the shape of the seats-votes curve for three different values of ρ, from which it is evident that an even more direct interpretation of this parameter is in terms of how large the seat bonus to the larger party is. If $\rho < 1$, the party winning the smaller vote share is overrepresented, while the party winning the larger vote share is underrepresented. In the extreme, as ρ approaches negative infinity, one has a "loser-take-all" situation, with the smaller party (in terms of vote share) winning all the seats at stake.[2] If $\rho = 1$, both parties' seat shares equal their vote shares; neither small parties nor large parties are favored. If $\rho > 1$, the party winning the smaller vote share is underrepresented, while the party winning the larger vote share is overrepresented. In the extreme, as ρ approaches infinity, one has a "winner-take-all" situation, with the larger party winning all the seats at stake.[3] We will continue to refer to ρ as the districting plan's responsiveness but remember that it also relates to the over-representation of large and small parties.

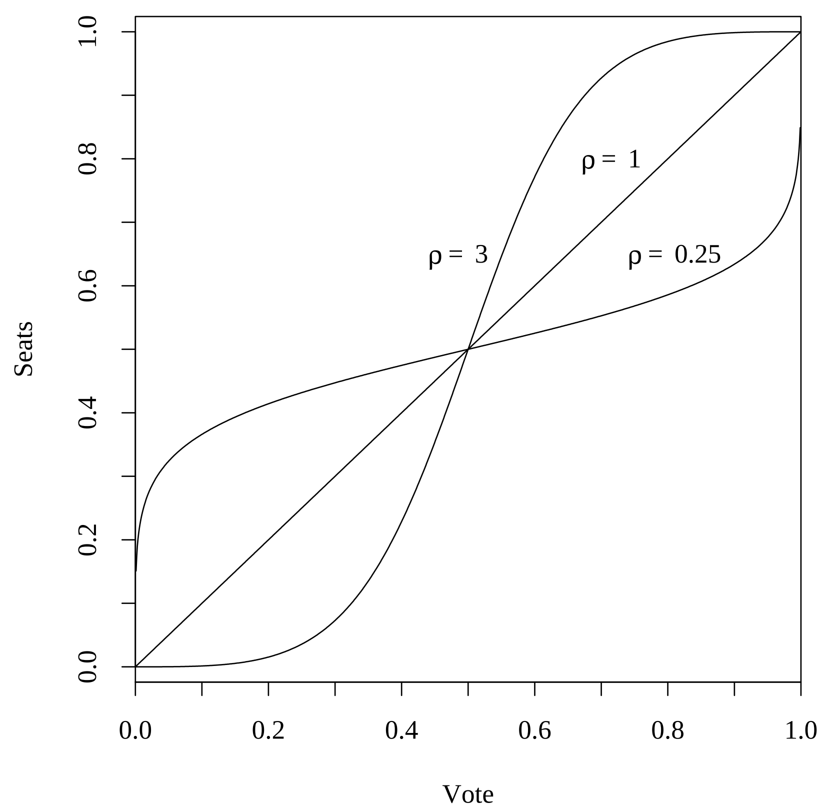

Figure 1: *Responsiveness*: This figure shows how the seats-vote curve given in Equation (1) changes for three values of ρ: 0.25, 1, and 3.

A party's maximization problem. We now embed Equation (1) in an explicit max-

[2]It should be remembered that responsiveness is separate from partisan bias. Severe malapportionment in favor of one party, for example, might produce a system in which the smaller party won all the seats, but this result would be due to heavy partisan bias (a big λ), not to low responsiveness ($\rho < 1$).

[3]Note that an infinite ρ necessarily ensues in a "legislature" with only one seat: for then whichever of the two parties wins more votes in the election necessarily wins the (only) seat at stake.

imization problem. We propose a simple partisan model, in which two (unitary) state parties are the redistricters, both seeking to maximize a concave function of their expected share of seats, subject to various constraints that limit the range of available plans. By focusing on state parties as the primary redistricting agents, we ignore strife between party factions within the state.[4] By assuming that parties maximize a concave function of their expected seat share, we allow the parties to be risk-averse and in particular to value holding on to the seats that they already have more than gaining some new seats. State parties might be risk averse because congressional incumbents (who naturally value the seats the party already holds more than new ones it might win) are able to influence them.[5] State parties might also be risk averse, independently of any influence by congressional incumbents, because congressional seniority underpins the state's ability to get a larger share of federal largesse. The more valuable congressional seniority is, the more valuable it is to keep one's own incumbents and the less valuable it is to knock off the other party's incumbents.

Finally, by assuming that redistricting parties face constraints, we recognize a variety of real-world limitations on redistricters' power. It is theoretically possible to define a district that has 100% registered Republicans, but this would require a lot of information and a tremendously cumbersome statute. Even before the reapportionment decisions, state provisions requiring that plans respect subunit lines — e.g., cities and counties — made such a district impossible to draw. We deal with these constraints simply by positing an exogenously given set $\mathcal{A}$ of attainable redistricting plans, defined as a subset of $\rho - \lambda$ space.[6]

Given any choice of ρ and λ, a party's expected seat share can be derived from Equation (1) as

$$\mathrm{E}(s|\rho,\lambda) = \left[1 + \exp\left(-\lambda - \rho \ln\left(\frac{v}{1-v}\right)\right)\right]^{-1}.$$

Letting $x = \ln(\frac{v}{1-v})$, f be the probability density function for x, and u give the party's utility from a given expected seat share, a party's maximization problem — $\max E[u(s|\rho,\lambda)]$

[4]Factional strife — e.g., rural partisans doing in their urban co-partisans — appears to have been frequent in the south (which is excluded from our analysis) and to a lesser extent in the border states. Factionally motivated plans should have produced bias against one majority-party faction or another, not necessarily bias against the minority party. The more common such factional plans were in the period we study (1946-70), the less our predictions – based on the assumption that interparty competition was the primary engine driving events – will pan out. Our purpose in assuming unitary parties is not to deny the existence of strains within the parties (or the possible redistricting consequences of such strains) but to assert the general primacy of partisan motivations.

[5]Such influence might not be automatic, if state legislators as a class coveted higher office, and thus favored plans that would oust as many incumbents as possible. This sort of anti-congressional-incumbent sentiment is mentioned from time to time in case histories but does not appear to have been a dominant consideration in any redistricting in our dataset.

[6]We believe that maximizing λ (e.g., by packing opposition supporters in a few districts) entails lowering ρ (the redistricting party, even if it wins a handsome statewide majority, is unlikely to win the packed districts; hence, the winner will not take all and an upper bound on ρ is implied), and vice versa. This sort of trade-off can be modeled by assuming a particular shape for $\mathcal{A}$ in $\rho - \lambda$ space, although we shall not pursue that line of argument here.

— can be expressed (with a few steps of algebra) as

$$\max_{(\rho,\lambda)\in\mathcal{A}} \int_{-\infty}^{\infty} u\left(\frac{e^{\rho x}}{e^{-\lambda}+e^{\rho x}}\right) f(x)dx \tag{3}$$

Optimal Choices. Since we assume that u is a concave increasing function of expected seats ($u' > 0$ and $u'' < 0$), it is immediately evident from Equation (3) that each party prefers the largest possible bias in its favor. The optimal choice of λ, denoted λ^*, would be $+\infty$ if the set of attainable plans permitted such a choice (which we assume it does not!).

The optimal level of ρ, denoted ρ^*, is more complicated. It depends on the expected (as opposed to the desired) level of partisan bias and also on how risk-averse the parties are. The basic intuition, however, is straightforward: the surer a party is that it will end up with more votes than its opponent, the more it wishes to bet on a highly responsive system — one which gives a larger seat bonus to the larger party. For a formal proof of this assertion, see Appendix A.

The model as developed thus far deals essentially with a single party's preferences — that is, with the ideal levels of responsiveness and partisan bias that a party would establish, if it controlled the redistricting process. Another part of our model, which focuses on the interaction of the two parties, will be developed next.

Reversionary plans. In addition to the constraint of choosing from a given set of attainable plans, parties must also take account of the reversionary outcome to the redistricting process – that is, the districting plan that would be used, should the state legislature and governor fail to agree on a legally acceptable plan.[7] We distinguish two types of reversionary plans: conservative and radical. A conservative reversionary plan is simply the current plan: if no new statute is passed, the old lines are retained. A radical reversionary plan is any reversionary plan that is not conservative. For example, in some cases of postwar congressional redistricting, the reversionary plan entailed an election in which all candidates ran at-large in the state; in other cases, the reversionary plan was one that a court had devised.

Consider what the values of ρ and λ will be under three different constellations of partisan control and reversionary outcome: (i) unified partisan control (i.e., one party controls all three branches of state government or sufficient majorities in the legislature to override any gubernatorial veto); (ii) divided control with a conservative reversion; and (iii) divided control with a radical reversion. In the first case, the nature of the reversionary plan — whether conservative or radical — should not matter. Whichever party controlled the state should simply impose a plan in its own interests, hence we refer to these as *partisan plans*. The Republicans, for example, would choose a plan associated with the largest possible pro-Republican bias. Moreover, since they had recently done

[7]The emphasis on reversion points in this section is akin to that in setter models, on which see Rosenthal 1990 and Cox N.d

well enough to win unified control of the state, presumably they expect a vote share in excess of 0.5, which is to say that $\mu = \mathrm{E}[x] = \mathrm{E}[ln(\frac{v}{1-v})]$ exceeds zero. In this case, we expect (see Proposition 1 in the appendix) that $\rho^* > 0$, with ρ^* being larger the more optimistic the party is about its prospects of outpolling its rival.[8]

Now consider the case of divided control with a conservative reversion — in a state, let us say, in which the current plan is Republican. The current plan will, by the argument above, exhibit a substantial pro-Republican bias and possibly a fairly large responsiveness as well. Since the current plan is the reversion, and the Republicans and Democrats have diametrically opposed interests when it comes to bias, any new plan will largely maintain the pro-Republican bias of the old plan.[9] But if there is divided government, then typically the expected vote share for both parties will be closer to 0.5 than in the partisan case considered above; that is, the mean of x will be closer to zero. Thus, the optimal responsiveness for the Republicans will be nearer to zero than it was when they last redistricted, while the optimal responsiveness for the Democrats will still be small. As the reversionary level of responsiveness is the "high" level established earlier, when the Republicans thought their vote support was more substantial, both parties will agree on reducing responsiveness. We will refer to these as *mixed plans* since bias is high (as in a partisan plan) but responsiveness is low (as in a bipartisan plan; see below).

Consider finally the case of divided control with a radical reversion — say, an all at-large election. All at-large elections should have produced very high values of responsiveness (higher as voters are more partisan) and low values of partisan bias (lower as voters are more partisan). Just as in the conservative reversion cases, neither party will wish to change partisan bias. Thus, the partisan bias under any new plan is again expected to be near the reversionary level — but in this case that level is low, not high. Also just as in the conservative reversion case, both parties will prefer to reduce responsiveness below the level observed under partisan plans. Thus, the responsiveness under any new plan is expected to be low too. We will refer to these as *bipartian plans.*

All told, our expectations are as summarized in Table 1. Under unified control, we expect partisan gerrymanders, with high levels of both partisan bias and responsiveness. Under divided control with a radical reversion, we expect incumbent-protecting gerrymanders, with low levels of both partisan bias and responsiveness. Finally, under divided control with a conservative reversion, we expect high levels of bias but low levels of responsiveness – a mixed case that has features of both partisan and incumbent-protecting gerrymanders.

[8] Note that as a party approaches risk neutrality, it is willing to set responsiveness to infinite levels, when $\mu > 0$, since this gives it an advantage in terms of expected seat share. So, depending on how risk-averse parties are, the optimal level of responsiveness could be quite large. Note also that a party would have to be quite risk-averse to keep ρ below unity: given that $\mu > 0$, such a low responsiveness level would give an advantage in expected seat share to the other party, an advantage that could be overcome only in terms of avoiding the additional risk that a larger value of ρ would entail

[9] In principle, if the parties' degrees of risk aversion are common knowledge, one party might be able to extract concessions from the other on the partisan bias dimension, in return for lowering responsiveness. This, however, is a second-order consideration.

Table 1: Predictions for Bias and Responsiveness by Government Control and Reversion

	Conservative Reversion	Radical Reversion
Unified Control	Partisan Plan: High Bias & High Resp.	Partisan Plan: High Bias & High Resp.
Divided Control	Mixed Plan: High Bias & Low Resp.	Bipartisan Plan: Low Bias & Low Resp.

In Section 5, we shall test more specific forms of these hypotheses for the early postwar period (1946-70). First, however, we set the stage for this analysis by discussing the literature concerning the reapportionment revolution.

3 Gerrymandering in the 1960s

The most prominent early hypothesis about the wave of redistrictings in the 1960s, due to Tufte (1973; 1974), was that they were exercises in incumbent protection. The subsequent literature, however, has dismissed this notion. Fiorina (1977:17–19), for example, raises three objections. First, why would state legislators and executives have become suddenly more solicitous of congressional incumbents in the 1960s? Second, if district lines were being redrawn to create safer Republican districts for Republican incumbents and safer Democratic districts for Democratic incumbents, then why did the distribution of the presidential vote in congressional districts not become markedly more bimodal? Third, if redistricting affected the incumbency advantage, then how does one explain the evidence marshalled by Cover (1977) and Ferejohn (1977) — which shows that incumbent vote margins were growing just as fast in states that did not redistrict as in those that did?

If the 1960s redistrictings were not pro-incumbent gerrymanders, were they partisan? Most studies of partisan gerrymanders have focused on redistrictings occurring after each federal census and have found mixed or relatively moderate partisan effects state by state, which cumulate into even smaller net national effects (e.g., Abramowitz 1983; Bullock 1975; Cain 1985; Campagna and Grofman 1990; Glazer, Grofman and Robbins 1987; Niemi and Winsky 1989). The view now prevailing in the literature is that redistricting is unlikely to produce any net partisan gains at the national level because (1) partisan gerrymanders occur only when one party controls both the legislative and executive branches in a state, making them relatively rare given the high incidence of divided government in the states;[10] (2) partisan gerrymanders sometimes fail; and (3)

[10]Even when one party controls the policy-making apparatus, partisan gerrymanders are likely to occur only if the in party has some reason to fear the out party (which they might not in one-party areas of the country).

pro-Republican gerrymanders in some states balance pro-Democratic gerrymanders in other states (Butler and Cain 1992:8–9).

Why the 1960s may have been different. There are theoretical reasons to suspect that the 1960s redistrictings differed from their post-census cousins, however. First, traditional Republican dominance of the north meant that most redistricting actions in the 1960s overturned districting plans that were favorable to Republicans. Table 2 classifies the last pre-revolutionary and first post-revolutionary districting plans in each of the thirty-three nonsouthern states that had more than one district and did not use multi-member elections exclusively in the pre-revolutionary period.[11] As can be seen, fourteen of the thirty-three plans in place when the reapportionment revolution arrived were Republican. Another nine were bipartisan modifications of older Republican plans (four of which had a Republican cast to them due to the conservative nature of the reversionary outcome). Less than a third of the pre-revolutionary plans were Democratic.

Even Table 2's accounting — by which over two-thirds of the pre-revolutionary plans potentially favored Republicans — understates the degree of Republican dominance. First, California and Washington operated under Republican plans until 1961 and 1959, respectively, so that 75% of the total number of pre-revolutionary state elections were held under plans that potentially favored Republicans. Second, the Republicans dominated most of the larger states, so that over 80% of district elections were contested under partisan Republican, mixed Republican or bipartisan Republican plans in the pre-revolutionary period.

Interacting with traditional Republican dominance in the north was LBJ's landslide victory over Barry Goldwater in 1964 — which meant that the Republicans found themselves at a local minimum in terms of their state legislative power, just as the wave of court-mandated congressional redistricting peaked. The consequence of this electoral disaster was that the 1960s saw the replacement of mostly Republican plans with mostly bipartisan and partisan Democratic plans.

A second reason to expect that the 1960s redistrictings were different is that they were conducted under the threat of court action, should the state legislature and governor not agree on a bill. Below we argue that much of the time expectations about the court's reversionary plan advantaged the Democrats.

4 How the Reapportionment Revolution Affected Reversionary Outcomes

We believe that the puzzle of the disappearing pro-Republican bias can be solved by considering how the entry of the courts into the "political thicket" of redistricting altered

[11]Throughout the paper, we exclude the five single-district states — Alaska, Delaware, Nevada, Vermont and Wyoming — from our analysis. In addition, we exclude Hawaii because of its exclusive use of multi-member elections prior to 1970

Table 2: Last Pre-Revolutionary and first Post-Revolutionary Plans in 33 Non-southern States

	Post-Revolutionary Plans				
Pre-Revolutionary Plans	Partisan: Republican	Bipartisan: Republican	Bipartisan: Democratic	Partisan: Democratic	No Plan until 1972
Partisian: Republican	CO (4) KS (5) NH (2) SD (2)	MT (2) NY (41) OR (4) CT (6)	—	UT (2) IN (11)	IA (7) ME (2) ND (2) RI (2)
Mixed: Republican	OH (24)	MI (19) WI (10)	—	NJ (15)	—
Bipartisan: Republican	—	MA (12) NE (3) IL (24) PA (27)	—	—	MN (8)
Bipartisan: Democratic	—	—	—	—	—
Mixed: Democratic	—	—	—	—	—
Partians: Democratic	ID (2)	—	AZ (3) CA (38) OK (6) MD (8) NM (2)	WV (3) KY (7) MO (10) WA (7)	—

Note: The number of districts in each state is indicated in parentheses.

the strategic situation facing the parties. In particular, the courts' entry affected the nature of the reversionary outcomes to the redistricting process.

Reversionary plans — pre-revolutionary. Prior to the reapportionment revolution, a state's current plan was usually the reversionary outcome. The only exceptions arose when a state gained or lost seats pursuant to the decennial federal census. If a state had gained seats, the exception was minor: absent agreement on a new set of districts, a state could simply elect the new members at-large, preserving all the old districts. This was a relatively painless solution that incumbent members of congress often found preferable to the disruption of their current districts. If a state had lost seats in the reapportionment, however, all members had to be elected at-large, absent a new districting plan. This was a much less palatable reversionary plan and its unpleasantness explains why the bulk of pre-revolutionary redistricting action occurred in states that had lost representation in Congress. In terms of the model developed in Section 2, prior to the reapportionment revolution all states not losing seats had conservative reversionary outcomes, while all states losing seats had radical reversionary outcomes.[12]

Reversionary plans — post-revolutionary. Now consider how the entry of the judiciary altered the strategic situation. The role of the courts in the 1960s varied from case to case, but often things played out as follows. First, a federal or state court would declare a state's current districting plan null and void. The court would then give the state a more or less clear and more or less constraining deadline; if the legislature and governor could not agree on a plan by the deadline, the court would impose a plan. Sometimes the plan that the court planned to impose was clear ex ante, sometimes not. But the court's plan was never the pre-existing plan. Thus, all redistricting action in the 1960s took place under threat of a radical reversionary outcome of one kind or another.

Reversionary plans — qualitative evidence. Our model predicts that mixed redistricting acts in the pre-revolutionary era should have left the level of partisan bias established by the next-previous partisan plan unchanged, when the reversionary outcome was conservative. In a state with an existing Democratic districting plan, for example, the Democrats would have had no reason to accept a plan that was any worse for them than the reversionary plan. Missouri provides a case in point. As Short (1931:637) reports, "The Democratic party had a majority in both houses of the General Assembly in 1911; but, inasmuch as the number of Missouri's representatives in Congress was not changed by the reapportionment act of that year, and the existing districts were distinctly advantageous to that party, no serious attempt was made to formulate a redistricting measure which would meet with the approval of the Republican chief executive."[13]

On the other hand, our model predicts that bipartisan plans concluded in the face of a radical reversionary outcome would produce lower levels of partisan bias. Whether a party needed to accept substantial reductions in the level of partisan bias in its favor, however, depended on the two parties' expectations about how each would fare under the reversionary plan and their skill in bargaining. Consider the example of Missouri again. After Missouri lost a seat in the reapportionment of 1930, the Democratic legislature in 1931 put a highly partisan redistricting bill before the Republican governor, then

[12]Technically, at-large elections were illegal from 1842 until 1929, then again after 1968. The legal basis for the reversions noted in the text appears to go back to some court decisions in 1932 (see Apportionment Act. 1982. *U.S. Code*. Vol. 4, sec. 2a, p. 64). For present purposes, it is sufficient to note that the 1941 Apportionment Act repeated the relevant provisions (and that they were not overturned until the 1960s). See Apportionment Act. 1941. *Statutes At Large*. Vol. 55, sec. 1, p. 762

[13]The Democrats were presumably confident that they would continue to poll a majority of votes in legislative elections, hence uninterested in reducing the level of responsiveness. Missouri in 1911 thus provides an example in which vote expectations under divided government were not substantially different than under unified government — as we assume they were, at least on average, in the postwar era

refused to compromise. In the 1932 election this intransigence meant that all representatives were elected at-large. But the Democrats regained unified control of the state and promptly passed a new districting bill to their liking (see Short 1931). Thus, in this case, the preexisting Democratic bias in Missouri's districts was preserved despite the state's episode of divided government. In states that were not traditionally dominated by one party, as Missouri was, at-large elections were less likely to present a clear advantage for either party over the other; in these states, especially if both parties' congressional incumbents were influential with the state legislature, one expects a rather different outcome: an incumbent-protecting gerrymander in which responsiveness is substantially reduced. Given the inherent uncertainties involved in forecasting election results, and incumbents' perennial interest in their own reelection, this was probably the more common outcome when states faced radical reversions under divided government.

Court deadlines and reversionary plans could have a substantial impact in states where partisan control was less firm than in Missouri. In Michigan, for example, the Republicans controlled both houses of the state legislature and the governorship when a three-judge federal court declared, on 27 March 1964, that the state's 1963 redistricting plan was unconstitutional in light of *Wesberry*. The state legislature was preoccupied with its own redistricting problem and, by the time it got around to congressional redistricting in early May, the strategic balance had tilted in favor of the Democrats. The court had clearly warned that all 19 House seats in Michigan would be filled at-large if the legislature did not act before the 1964 election. This was a reversionary outcome that the Republicans found extremely distasteful, in light of their estimates of the relative statewide strengths of the two parties. Moreover, state law required a 90 day waiting period before any new law became effective, a requirement that could only be waived by a two-thirds vote. Consequently, because the redistricting plan had to be in place before the primary election season began (in less than 90 days) and because the Republican advantage in the state House was only 58–52 (less than two-thirds), the federal court's firm deadline and clear reversionary plan meant that concessions had to be made to the Democrats (Congressional Quarterly 1966:2066–67).

These few anecdotes suggest that the nature of the reversionary plan could sometimes have a substantial impact under divided control. In the next section we look more systematically at whether the model's various expectations are borne out in the data.

5 How the Reapportionment Revolution Affected Bias and Responsiveness

In this section, we look at how bias and responsiveness changed with the wave of court-ordered redistrictings in the 1960s. Our strategy is to estimate the bias and responsiveness parameters in Equation 1 via maximum likelihood for each of the kinds of plan identified above (see Appendix B for estimation details). Thus the relevant seat and vote shares in our analysis are $s_{j,t}$, the Democratic seat share in state j at election t, and $v_{j,t}$, the mean Democratic share of the vote in state j at election t.[14] As the whole point of the analysis is to study the effect of redistricting, and redistricting happens within individual states, we do not pool votes and seats across the entire nonsouthern portion of the U.S., as have most previous studies. Instead, we take as our unit of analysis the state-year: a particular election of U.S. House members from a particular state.[15]

Our data cover the years from 1946 to 1970 and include all nonsouthern House elections, except contests held in single-seat states (for which the responsiveness parameter is necessarily infinite and the bias parameter necessarily zero) and those held under multi-member rules in four two-seat states (Hawaii 1958–70; Arizona 1946; New Mexico 1946–58; North Dakota 1946–60). We stop the analysis in 1970 so that the immediate effects of the reapportionment revolution are not confounded with the effects of later redistrictings.

Naturally, correct identification of the nature of partisan control in each state at the time of each redistricting is crucial to our analysis, as is the correct assignment of each election in each state to a particular plan. Fortunately, Martis (1982) lists the date of passage of each redistricting plan in all fifty states. We have used various sources to identify the partisan composition of each house of the state legislature and the partisanship of the governor at the time of each redistricting, taking account when necessary of the fact that some states allow overrides of gubernatorial vetoes with less than a 2/3 vote by the legislature.[16]

Responsiveness. Our results are displayed in Tables 3 and 4. Consider the responsiveness results — displayed in Table 3 — first. We expect significantly lower responsiveness under any plans adopted under divided government — i.e., bipartisan and mixed plans. Consistent with this expectation, the four lowest responsiveness values in the table are observed for such plan types. As it turns out, one cannot reject the null hypothesis

[14] Using the statewide Democratic share of the vote, instead of the mean Democratic share affects the level of bias because Democrats tend to win seats in lower turnout races. But it does not much affect estimates of how bias responded to 1960s redistricting. This gibes with previous results in the literature using the overall vote (e.g., Tufte 1973; Jacobson 1990), which show a change in bias in the pro-Democratic direction in 1966–70 similar to that found in studies using the mean district vote.

[15] In fact, if one believes that optimal gerrymandering is occurring, constructing a sensible model at the district level of the Democratic candidate's share of the two-party vote, as is usually done, is a daunting task. Consider just the case of a state with a partisan Democratic plan. As noted before, the plan that maximizes partisan bias is constructed using two types of districts: "marginal" Democratic districts with expected vote share just large enough to ensure victory and a small number of ultra-safe Republican districts where the expected Democratic vote share will be as near zero as feasible. Thus, with just one type of plan we observe data that are mixtures of two types of districts. With more plan types we only increase the number of constituent elements to this mixture. We would not have a problem if we could identify the type of each district *ex ante* , as we could then use standard regression tools to model district vote shares and estimate quantities of interest, such as partisan bias and responsiveness. However, absent this identifying information, estimation becomes problematic. The advantage of the seats-vote approach we are taking is that we do not need this information at all to consistently estimate partisan bias and responsiveness.

[16] Our data, including our identification of the plan types for each state can be found at `http://jkatz.caltech.edu`.

that the responsiveness values in all the divided control plans are the same, nor the null hypothesis that the responsiveness values in all the partisan plans are the same — but one can reject the null hypothesis that these two responsiveness levels are the same at conventional levels of significance.

Table 3: Responsiveness under Eight Different Districting Plans in 33 Nonsouthern States, 1946–1970.

Plan Type	Pre-Revolutionary	Post-Revolutionary
Partisan: Republican	3.53	4.31
	(0.30)	(0.58)
Mixed: Republican	2.16	–
	(0.80)	
Bipartisan: Republican	2.06	2.32
	(0.28)	(0.52)
Bipartisan: Democratic	–	1.15
		(0.65)
Mixed: Democratic	–	–
Partisan: Democratic	3.43	5.32
	(0.35)	(0.76)

Notes: Standard errors are in parentheses.
Full estimation notes are in Appendix B.

If one examines the enactment of plans under divided government, it is not always obvious that incumbent protection has carried the day. But in some cases, it is abundantly clear. Consider, for example, the 1962 redistricting in Massachusetts. Initially, the Democratic state legislature had considered a partisan gerrymander, despite threats of a veto from the Republican governor. Ultimately, however, a bipartisan group of Congressional incumbents (led by Tip O'Neill) proposed a plan to protect as many incumbents as possible from the consequences of the state's loss of two seats. The incumbents met with state legislators and the governor, even enlisting the White House to lobby a few powerful legislative dissidents rumored to have an eye on running for Congress themselves (and therefore favoring the less predictable reversionary outcome). One dissident " … attacked the state's U.S. House delegation for 'dictating a freeze-in for themselves" and as "vultures sweeping down over the State House to preserve their own jobs" (Congressional Quarterly 1962, 1643). But in the end the deed was done.

Partisan Bias. Consider next the results on partisan bias, displayed in Table 4. We have two main expectations about bias that we shall consider in turn.

Table 4: Partisan Bias under Eight Different Districting Plans in 33 Nonsouthern States, 1946–1970.

Plan Type	Pre-Revolutionary	Post-Revolutionary
Partisan: Republican	−8.26	−0.92
	(1.08)	(2.61)
Mixed: Republican	−8.35	–
	(1.98)	
Bipartisan: Republican	−5.32	−3.13
	(1.24)	(2.37)
Bipartisan: Democratic	–	1.69
		(2.89)
Mixed: Democratic	–	–
Partisan: Democratic	4.76	8.70
	(2.00)	(3.73)

Notes: Standard errors are in parentheses.
Full estimation notes are in Appendix B.

First, we expect partisan plans, whether before or after the reapportionment revolution, to produce significant levels of bias in favor of the redistricting party. This expectation is borne out in three of the four cases in which it is at risk. Pre-revolutionary partisan Republican plans produced a pro-Republican bias of 8.26%, pre-revolutionary partisan Democratic plans produced a pro-Democratic bias of 4.76%, and post-revolutionary Democratic plans produced a pro-Democratic bias of 8.70%. The only exception is that post-revolutionary partisan Republican plans produced a tiny and insignificant bias.[17]

Second, we expect that mixed plans — plans enacted under divided government when the reversionary outcome was conservative — should have preserved any pre-existing partisan bias. As all post-revolutionary reversions were radical, we can only test this expectation by comparing pre-revolutionary plans. Partisan Republican plans and mixed

[17]Why does the level of bias attained by Democrats increase from pre- to post-revolutionary plans, while that attained by Republicans decreases? Part of the reason may be that pre-revolutionary Democrats were often as concerned with intra-party fights as with inter-party fights, so that partisan bias was not always specifically pro-Democratic or anti-Republican. During the 1960s, in contrast, Democrats became more concerned with Republicans and targeted their districting plans more specifically against them. Another part of the reason may be that Democrats could find plenty of voters in the urban and suburban areas that were gaining representation due to the removal of malapportionment, while this was harder for Republicans. That is, the ratio of Republican support in rural areas to nonrural areas may have been higher in traditionally Republican states than the corresponding ratio was for Democrats in traditionally Democratic states.

Republican plans are indeed statistically indistinguishable in terms of partisan bias. (As it turns out, there were no mixed Democratic plans in our dataset and so we cannot test the hypothesis that such plans produce high levels of pro-Democratic bias.)

The net partisan effect of 1960s redistrictings. Overall, how much did the reapportionment revolution hurt the Republicans? Some light is shed on this question in Figure 2, which displays the overall partisan bias in the 322 nonsouthern districts under study here for each year from 1946 to 1970. The overall bias is calculated by taking the proportion of the 322 districts that fell into each plan type in each year, multiplying by the estimated average partisan bias for that plan type (using the estimates in Table 4), and summing across all plan types. As can be seen, bias in the north was, by our estimates, running between 6.0% and 6.4% pro-Republican in every year from 1946 to 1960 inclusive (worth about 20 seats). The stability of these figures (which largely conform to previous estimates) reflected a stable distribution of districts across plan types: there was relatively little consequential redistricting in this period. The pro-Republican nature of bias in the early postwar period simply reflected the dominance of Republican-favoring plan types. The Republican advantage in the north begins to erode noticeably in 1962, when California, Massachusetts and Pennsylvania replaced Republican plans with Democratic or bipartisan/radical plans.[18] With the entry of the courts into the redistricting process, the number of Democratic and bipartisan/radical plans further increased and, by 1966, the Republican advantage was largely gone.

Thus, we have a proximal answer to the puzzle of why a long-standing pro-Republican bias in nonsouthern congressional elections disappeared circa 1966: redistricting in the 1960s — mostly sparked by the Supreme Court's reapportionment decisions — removed this bias. As neither the 1970s redistrictings (Glazer, Grofman and Robbins 1987) nor the 1980s redistrictings (Campagna and Grofman 1990) much affected the net partisan balance, Democratic gains in 1964-70 were preserved for some time.

6 Conclusion

The Supreme Court's landmark apportionment decisions, beginning with *Baker v. Carr*, led to what on the face of it would appear to be the most important single change in the conduct of elections in the U.S. since the 19th amendment gave women the vote. Yet, although legal scholars were quick to recognize the enormous jurisprudential importance of the decisions, hailing them as nothing less than a "reapportionment revolution," political scientists have not found that the decisions produced sweeping consequences —

[18] To confirm that the upward shift in 1962-64 was due to a compositional change rather than a change in plan effects, we examined whether any plan effect differed significantly in 1962-64 from previous years. The liklihood ratio test statistic was 1.69 with two degrees of freedom, far from any critical values. That is, we could not reject the null hypothesis that the plan effects were the same in 1962-64 as they had been previously. We might also note that the 1962 redistricting in California was widely considered to be a Democratic gerrymander and produced a dramatically different distribution of vote shares from the preceding Republican gerrymander.

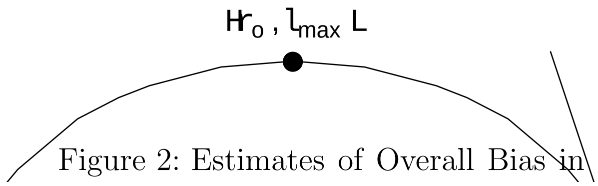

Figure 2: Estimates of Overall Bias in 33 Nonsouthern States, 1946–1970

other than the immediate consequence of redistricting itself.

In this paper, we have developed a model of the redistricting process that highlights the importance of two factors: first, partisan or bipartisan control of the redistricting process (a factor that many others have recognized); second, the nature of the reversionary outcome, should the state legislature and governor fail to agree on a new districting plan (a factor that has been almost entirely neglected). Using this model, we derive various predictions about the levels of partisan bias and responsiveness that should be observed under districting plans adopted under various constellations of partisan control of state government and reversionary outcomes, testing our predictions on postwar (1946–70) U.S. House electoral data.

We find strong evidence that both partisan control and reversionary outcomes systematically affect the nature of a redistricting plan and the subsequent elections held under it. That partisan control matters may not strike some as particularly surprising, but it runs counter to much of the literature, which finds only modest differences between partisan and bipartisan plans. That reversionary outcomes matter has been largely ignored in the previous literature.

In addition to properly identifying the effects of various kinds of plan, our work provides clear evidence that the reapportionment revolution substantially affected two important macro-features of postwar congressional elections. First, we show that the well-known disappearance circa 1966 of what had been a long-time pro-Republican bias of about 6% in nonsouthern congressional elections can be explained completely by the changing composition of northern districting plans. Prior to the 1960s, the vast bulk of nonsouthern districting plans were partisan Republican plans (the chief exceptions being in the border states). By the mid-1960s, the vast bulk of nonsouthern districting plans were bipartisan or partisan Democratic plans.

Second, our results also shed light on an even more famous puzzle: the case of the vanishing marginals. Prior to the 1960s, almost all districting plans were partisan. By the mid-1960s the bulk of plans were bipartisan. Our model clearly predicts that responsiveness should be lower under bipartisan than partisan plans, and that is what we find. Thus the large compositional change of districting plans in the north offers a partial explanation for the abruptly increasing margins of incumbents documented by Mayhew (1974), along neo-Tuftean lines (Tufte 1972; 1973). In future work, we intend to explore further how redistricting affected the growth of incumbents' margins, along with the closely related issue of growth in the incumbency advantage.

Appendix A A Party's Optimization Problem

In this appendix, we formalize the claim in the text that "the surer a party is that it will end up with more votes than its opponent, the more it wishes to bet on a highly responsive system." It is difficult to show analytically how ρ^* varies for a completely general density f and utility function u. We shall henceforth assume, consistent with our estimation procedure below, that f is the normal density with mean μ and standard deviation σ. Given this assumption, ρ^* turns out to be a function of μ and σ. In particular, a party prefers a more responsive system the larger is its expected vote (i.e., the larger is μ). Moreover, if $\mu > 0$, then a party prefers a more responsive system the smaller is σ. More formally:

Proposition 1 *Let (ρ^*, λ^*) be a solution to the maximization problem in Equation (3) and let $x(=\ln(\frac{v}{1-v}))$ be distributed as Normal with mean, μ, and standard deviation, σ. If a party displays sufficently high absolute risk aversion, such that*

$$\frac{-u''}{u'} > e^{-2\lambda} - 1$$

then:

1. *$\frac{\partial \rho^*}{\partial \mu} > 0$; and*
2. *if $\mu > 0$ then $\frac{\partial \rho^*}{\partial \sigma} < 0$.*

In the rest of this appendix we prove Proposition 1 for the case in which $\mathrm{E}[u(S)] = \mathrm{E}[\ln(S)]$.

Recall that the general problem is of the form

$$\max_{(\rho,\lambda)\in\mathcal{A}} \int_{-\infty}^{\infty} u\left(\frac{e^{\rho x}}{e^{-\lambda} + e^{\rho x}}\right) f(x)dx$$

where $u' > 0$, $u'' < 0$ and f is a probability density function. Differentiating Eu using Leibniz's rule, one gets

$$\frac{\partial Eu}{\partial \rho} = e^{-\lambda} \int_{-\infty}^{\infty} u' \frac{x e^{\rho x}}{(e^{-\lambda} + e^{\rho x})^2} f(x)dx$$

and

$$\frac{\partial^2 Eu}{\partial \rho^2} = e^{-\lambda} \int_{-\infty}^{\infty} \frac{x^{-\lambda} e^{2\rho x}}{(e^{-\lambda} + e^{\rho x})^4} \left\{u'' + u'(e^{-\lambda} + e^{\rho x})(e^{-\rho x - \lambda} - 1)\right\} f(x)dx$$

One special case concerns a pdf f that is symmetric with mean zero. Note that with such an f, $\rho = 0$ implies:

$$\frac{\partial Eu}{\partial \rho} = \frac{e^{-\lambda} u'}{(e^{-\lambda} + 1)} \int_{-\infty}^{\infty} x f(x)dx$$

which equals zero since the mean $\int_{-\infty}^{\infty} xf(x)dx = 0$. A sufficient condition for $\partial^2 Eu/\partial\rho^2 < 0$ (hence for $\rho^* = 0$) is that the term in curly brackets be negative for $\rho = 0$. A little algebra shows that this is equivalent to

$$\frac{-u''}{u'} > e^{-2\lambda} - 1$$

This inequality is satisfied by risk-neutral u's if $\lambda > 0$. For $\lambda \leq 0$, u must exhibit enough absolute risk aversion ($-u''/u'$ must be large enough) in order for the inequality to be satisfied.

Now consider the case in which f is the normal density, which we denote ϕ, with mean μ and standard deviation σ and u is the natural logarithm. In this case, a party's maximization problem is

$$\max_{(\rho,\lambda)\in\mathcal{A}} \int_{-\infty}^{\infty} \ln\left(\frac{e^{\rho x}}{e^{-\lambda} + e^{\rho x}}\right)\phi(x)dx \longleftrightarrow$$
$$\max_{(\rho,\lambda)\in\mathcal{A}} \int_{-\infty}^{\infty} (\rho x - \ln(e^{-\lambda} + e^{\rho x}))\phi(x)dx \longleftrightarrow$$
$$\max_{(\rho,\lambda)\in\mathcal{A}} \rho\mu - \int_{-\infty}^{\infty} \ln(e^{-\lambda} + e^{\rho x})\phi(x)dx.$$

Differentiating

$$\frac{\partial Eu}{\partial\rho} = \mu - \int_{-\infty}^{\infty} \frac{e^{\rho x}x}{e^{-\lambda} + e^{\rho x}}\phi(x)dx$$

Since $\frac{\partial Eu}{\partial\rho}(\rho^*(\mu,\sigma),\mu,\sigma) \equiv 0$ (ignoring corner solutions), one can take the total derivative to show that

$$\frac{\partial\rho^*}{\partial\mu} = \frac{-\frac{\partial^2 Eu}{\partial\rho\partial\mu}}{\frac{\partial^2 Eu}{\partial\rho^2}}$$

But

$$\frac{\partial^2 Eu}{\partial\rho^2} = -\int_{-\infty}^{\infty} \frac{x^2 e^{\rho x-\lambda}}{(e^{-\lambda} + e^{\rho x})^2}\phi(x)dx < 0$$

and

$$\begin{aligned}\frac{\partial^2 Eu}{\partial\rho\partial\mu} &= 1 - \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} \frac{e^{\rho x}x}{e^{-\lambda} + e^{\rho x}} e^{-(x-\mu)^2/2\sigma^2} - \frac{1}{2\sigma^2}2(x-\mu)(-1)dx \\ &= 1 - \frac{1}{\sigma^2}\int_{-\infty}^{\infty}\left(\frac{e^{\rho x}}{e^{-\lambda} + e^{\rho x}}\right)x(x-\mu)\phi(x)dx\end{aligned}$$

Since the term in parentheses is strictly less than 1 (and greater than zero), the integral is less than σ^2 and $\frac{\partial^2 Eu}{\partial\rho\partial\mu} > 0$. Thus, $\frac{\partial\rho^*}{\partial\mu} > 0$.

The second part of Proposition 1 can be proven using similar techniques. QED.

Appendix B Estimation

In this appendix we consider estimation of Equation (1). As written the equation is deterministic and can not directly be used to estimate the parameters of interest from observed data. However, if we assume a stochastic model — following King and Browning (1987; see also King 1990) — then Equation (1) defines the expected portion of seats in a state i in election t going to the Democrats as

$$\begin{aligned} \mathrm{E}[s_{i,t}] &= \left[1 + e^{\lambda}\left(\frac{v_{i,t}}{1-v_{i,t}}\right)^{\rho}\right]^{-1} \\ &= \left[1 + \exp\left(-\lambda - \rho\ln\left(\frac{v_{i,t}}{1-v_{i,t}}\right)\right)\right]^{-1}. \end{aligned} \tag{4}$$

The second expression for the expected seat proportion is same as the mean function for the standard logit model for grouped data with a constant, λ, and a single independent variable, $ln(\frac{v_{i,t}}{1-v_{i,t}})$. If we were to further assume that the probability of the Democrats winning a district were independently and identically distributed, we could model the process with a binomial distribution. The binomial assumption and Equation (4) then set up a standard grouped logit model that we could estimate either via maximum likelihood (as in King and Browning 1987) or two-step minimum Chi-Square methods (see Greene 1993:653–657 or Maddala 1983:28–34).

However, we suspect that there is still some unmodeled heterogeneity — beyond that being picked up by the logistic of the vote shares — and possibly some correlation in the probabilities across districts. In fact, an optimal partisan gerrymander would require such heterogeneity across districts. Assuming that there were not enough partisan voters for the dominant party to win every district, there would be two types of districts in the state: a handful that the minority party wins overwhelmingly and the remaining districts in which the dominant party wins but not by huge margins. In order to handle this we assume that the seat shares follow an extended beta-binomial, instead of a standard binomial distribution. The extended beta-binomial is generated by assuming that the probability (from a binomial model) that a district is won by the democrats varies according to a beta distribution.[19]. Let $S_{i,t}$ be the number of seats the Democrats win in state i in election t and N_i the total number of districts in state i. The extended beta-binomial can then be written as

$$f(S_{i,t}|\pi_i,\gamma) = \frac{N_i!}{S_{i,t}!(N_i - S_{i,t})!}\frac{\prod_{j=0}^{S_{i,t}-1}(\pi_i + \gamma j)\prod_{j=0}^{N_i-S_{i,t}-1}(1-\pi_i+\gamma j)}{\prod_{j=0}^{N_i-1}(1+\gamma j),}$$

where we assume the convention that if any of the constituent products are negative, then the term is set to 1. Note that since we are explicitly conditioning on N_i, the model incorporates the heteroskedastity caused by the varying number of districts across states in our sample.

[19]See King 1989:45–48 for a complete derivation of the extended beta-binomial distribution.

The parameter π_i is the average probability that a given district in state i is won by the Democrats. Thus,

$$\pi_i = \frac{\mathrm{E}[S_{i,t}]}{N_i} = \mathrm{E}[s_{i,t}].$$

So we can use Equation (4) to model the systematic variation in the underlying probability. The parameter γ captures the amount that π_i varies over the districts or the correlation across districts. If γ is zero, then the extended-binomial is just the binomial and districts are identically and independently distributed. If $\gamma > 0$, there is positive correlation between districts and when $\gamma < 0$ there is negative correlation between districts.

The log likelihood is straight forward to derive assuming independence across states. The contribution of each state i, ignoring terms that do not depend on the parameters, is

$$\mathcal{L}_i(\pi_i, \gamma | S_{i,t}, N_i) \propto \sum_{j=0}^{S_{i,t}-1} (\pi_i + \gamma j) + \sum_{j=0}^{N_i - S_{i,t}-1} (1 - \pi_i + \gamma j) - \sum_{j=0}^{N_i - 1} (1 + \gamma j).$$

We then substitute Equation (4) for π_i to get $\mathcal{L}_i(\lambda, \rho, \gamma | S_{i,t}, N_i, v_{i,t})$. The likelihood for the entire sample is found by summing the $\mathcal{L}_i$ across the states. In our actual estimation we are going to allow λ and ρ to vary over plan types, but we will assume a common γ to ensure a comparable scale of the coefficients.

The raw results can be found in Table 5. These serve as the basis of Tables 4 and 3. Responsiveness can be read directly off the estimation, however to calculate the partisan bias we need to plug the estimated λ into Equation (2). This is a consistent estimate of partisan bias, since ML is invariant to reparamaterization. However, in order to calculate the standard errors for the estimate we need to use the Delta method (Greene 1993:297). If $\hat{\theta}$ is a ML estimate, then the standard error of $f(\hat{\theta})$ is

$$\mathrm{var}[f(\hat{\theta})] = \left(\frac{df(\hat{\theta})}{d\hat{\theta}} \right)^2 \mathrm{var}[\hat{\theta}].$$

The square root of this variance is reported in Table 4 as the estimate of the standard error of the partisan bias estimate.

As can be seen from Table 5, the estimate of γ is negative and significantly different from zero, as expected given an optimal gerrymander. Unfortunately, the maximization for the extended-beta binomial likelihood is not straight forward since γ has a lower bound that depends on the true, but unknown, π. Therefore to confirm our results we estimated our model using a binomial model — i.e., fixing $\gamma = 0$ — both via maximum likelihood methods and via a two-stage minimum Chi-Square method with the seat logits smoothed as suggested by Cox (1970:33). Although both techniques assume independent and identically distributed data, they are still consistent — but inefficient – if this assumption fails (Gourieroux, Monfort and Trongon 1984). The results are similar from all

Table 5: Untransformed Estimates of Partisan Bias and Responsiveness by Plan Type in 33 Nonsouthern States, 1946–1970

Plan Type	λ	ρ
	Pre-Revolutionary Plans	
Partisan: Republican	−0.33	3.53
	(0.04)	(0.30)
Mixed: Republican	−0.34	2.16
	(0.08)	(0.80)
Bipartisan: Republican	−0.21	2.06
	(0.05)	(0.28)
Partisan: Democratic	0.19	3.42
	(0.08)	(0.35)
	Prost-Revolutionary Plans	
Partisan: Republican	−0.04	4.31
	(0.10)	(0.58)
Bipartisan: Republican	−0.13	2.32
	(0.10)	(0.52)
Bipartisan: Democratic	0.07	1.15
	(0.12)	(0.65)
Partisan: Democratic	0.35	5.32
	(0.15)	(0.76)
γ	−0.012	
	(0.002)	
N	413	
Log-likelihood	−2517.00	

three estimations boosting our confidence in the findings. As further test of robustness of our findings we ran our model allowing year effects and then allowing responsiveness to vary by number of districts in the state. While these changed the exact numeric results, the underlying pattern of results remained the same.

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