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COLLUSION IN MULTIPLE OBJECT SIMULTANEOUS AUCTIONS: THEORY AND EXPERIMENTS

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SOCIAL SCIENCE WORKING PAPER 1010

March 1998

Revised November 1998

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Abstract

The choice of strategies by bidders who are allowed to communicate in auctions is studied. Using the tools of mechanism design, the possible outcomes of communication between bidders participating in a series of simultaneous first-price auctions are investigated. A variety of mechanisms are incentive compatible when side payments are not allowed. When attention is restricted to mechanisms that rely only on bidders' ordinal ranking of markets, incentive compatibility is characterized and the ranking mechanism of Pesendorfer (1996) is interim incentive efficient. Laboratory experiments were completed to investigate the existence, stability, and effect on bidder and seller surplus of cooperative agreements in multiple object simultaneous first-price auctions. Collusive agreements stable in the laboratory. The choices of the experimental subjects often closely match the choices predicted by the ranking and serial dictator mechanisms presented earlier. However, a few notable exceptions raise interesting prospects for the theoretical development of models of cooperative behavior.

1 Introduction

Collusion by bidders is thought to be a prominent feature of auctions for antiques, fish, wool, timber, school milk, and oil drainage leases (Cassady (1967), Pesendorfer (1996), Hendricks and Porter (1988)). In fact, from 1979 to 1988, 81% all of Sherman Act violations filed by the U.S. Department of Justice involved auctions (Froeb (1988)). Bidders have incentives to coordinate their behavior to increase their surplus by eliminating competition amongst each other. If they can find an equitable technique for dividing the spoils from such collusive behavior, bidder *rings* can be quite successful.

In auctions, bidders are asymmetrically informed; they know their own values for the objects but not those of the other agents. In order to limit the amount of surplus

*Thank you to John Ledyard, Simon Wilkie, Dave Porter, and Julie Scott for their comments and patience. Any errors are the author's. Please send comments or correspondence to akwas@hss.caltech.edu.

that the auctioneer accumulates, the bidders would like to reach a preauction bidding agreement. However, any agreement may reveal the bidders' private information, causing their decisions to change. All bidders face a temptation to increase their one period profits by defecting from the collusive bidding agreement. Three primary questions which need to be addressed in the auction setting are:

1. Do bidders form cooperative agreements in simultaneous first-price auctions?
2. If they do, what sort of strategies do they utilize?
3. How do these strategies effect market efficiency, bidder surplus, and seller surplus?

The objective of this paper is to begin grappling with these questions by providing a theoretical and experimental examination of cooperative agreements in first-price sealed bid auctions. While others have already examined collusion in single object auctions (Graham and Marshall (1987), McAfee and McMillan (1992), Güth and Peleg (1996)), other than in Pesendorfer (1996), the properties of collusive behavior in the multiple object auction environment remains undiscovered. However, multiple object simultaneous sealed bid auctions are not completely unfamiliar. For example, auctions for school milk contracts are held under this procedure (Pesendorfer (1996)). Milgrom (1996) has recently suggested that simultaneous sealed bid auctions be used for determining the Carrier of Last Resort (COLR) privileges by the FCC.

Collusion is modeled as the choice of a *collusive mechanism* by the bidders. Given that they face a game defined by a series of simultaneous first-price auctions, bidders select a mechanism that maps from their valuations for each object to a set of bids in the auction. While noncooperative (Bayes Nash equilibrium) bidding is one possible outcome, there are potentially many other, more profitable, mechanisms. When side payments are allowed between bidders, an interim incentive efficient mechanism which dominates the noncooperative outcome is identified. A class of reasonable mechanisms may be eliminated on its face. I then examine collusive mechanisms under the restriction that no side payments may be made between bidders. In the multiple object setting, the number of potential incentive compatible mechanisms increases significantly. Three mechanisms that, in general, dominate Bayes Nash bidding are presented. On a restricted domain, the ranking mechanism of Pesendorfer (1996) is interim incentive efficient. These findings suggest that, if given the opportunity, bidders should be able to find a mechanism which they prefer to noncooperative behavior (cooperative agreements will be formed) and that there are some intuitively simple mechanisms that can be predicted as possible stable outcomes. Laboratory experiments are then conducted that often support these theoretical predictions. However, in a few experiments, bidders appear to deviate from theoretical predictions. They choose mechanisms that are not consistent with individual incentives yet lead to higher profits. These deviations suggest an avenue for future research.

In Section 2, the general framework of this institution is developed. The tools of mechanism design are used to develop a model of cooperative behavior in simultaneous first-price sealed bid auctions in Section 3. The experimental design is presented in Section 4. Section 5 is a general discussion of the findings of these experiments. Proofs and relevant lemmas are provided in Appendix A.

2 The Model

There are n bidders bidding on m objects in m simultaneous first-price auctions. Bidder i's valuation for object j is drawn from a continuous distribution F_{ij}. It is assumed that for all i and for all j, F_{ij} has a common support given by $[\underline{v}, \overline{v}]$, and the density, f_{ij}, is defined and strictly positive. Assume that bidders' valuations in each market are symmetric, or $F_{ij} = F_{kj}$ for all i, j, k.[1] Let $v = (v_1, v_2, \ldots, v_n)$ be the vector of individual valuations where $v_i = (v_{i1}, v_{i2}, \ldots, v_{im})$ is the vector of valuations in each market for individual i. Let b be a vector of bids similarly defined.

The simultaneous first-price auctions determine an allocation $x \in \{0,1\}^{m \cdot n}$ and prices based on the bids placed, where $x_{ij} = 1$ indicates that bidder i has been allocated object j. Feasibility requires that $\sum_{i=1}^{n} x_{ij} = 1$ for all j. The function $g : [\underline{v}, \overline{v}]^{m \cdot n} \to [0,1]^{m \cdot n}$ determines the probability that each bidder is allocated each object:

$$g_{ij}(b) = \begin{cases} \frac{1}{k} & b_{ij} \geq b_{\ell j} \text{ for all } \ell \\ 0 & \text{otherwise} \end{cases}$$

where $k = \#\{b_{ij} | b_{ij} \geq b_{\ell j}\ \forall \ell\}$ is the number of high bidders. Thus, each object is allocated to the highest bidder with ties broken randomly. The price paid by each bidder is given by $p : X \to \mathbb{R}^{m \cdot n}$ which is defined as

$$p_{ij}(x) = \begin{cases} b_{ij} & x_{ij} = 1 \\ 0 & \text{otherwise.} \end{cases}$$

If a bidder wins an item, then he pays his bid. Let $G_{ij}(b_{ij})$ be the expected probability that a bid of b_{ij} by bidder i is highest in market j. Let $P_{ij}(b_{ij})$ be the expected price paid by bidder i for object j when he has placed a bid of b_{ij}. Since first-price auctions are being modeled, the expected price can be simplified to $P_{ij}(b_{ij}) = b_{ij} G_{ij}(b_{ij})$. Assume that bidders are risk neutral. The expected utility for individual i is given by,

$$U_i(b_i, v_i) = \sum_{j=1}^{m} G_{ij}(b_{ij})(v_{ij} - b_{ij}).$$

The auctioneer may want to set a reserve price $c > \underline{v}$ to maximize revenue. For simplicity, assume that the auctioneer is passive and sets $c = \underline{v}$. Also assume that the bidders cannot resell the objects; the allocation decision of the auctioneer is binding.

[1] Many of the results presented here are also true when values are drawn from different distributions, but symmetry is maintained for simplicity.

The outcome of noncooperative behavior in this environment has been extensively studied. The optimal bidding strategy of each player is given by the Bayes Nash equilibrium of a game with asymmetric information. Maskin and Riley (1996) provide the most general sufficient conditions for the existence and uniqueness of a Bayes Nash equilibrium bid function. Given the assumption that F_{ij} and f_{ij} are strictly positive and bidders are risk neutral, a unique, monotonic Bayes Nash equilibrium exists. In the case of symmetric distributions, the symmetric bid function for each bidder is given by the simple bid function

$$b_{ij}(v_{ij}) = v_{ij} - \int_{\underline{v}}^{v_{ij}} \left(\frac{F_j(y)}{F_j(v_{ij})} \right)^{n-1} dy \text{ for all } i, j \tag{1}$$

where $F_j(v) = F_{ij}(v)$ for all i.

3 Cooperative Equilibria

If all bidders act noncooperatively, their attempts to outbid each other will give most of the surplus to the auctioneer. When all bidders' values are drawn from the uniform distribution, bidders will obtain only $1/n$ of the surplus. If the bidders can find an agreement in which they place very low bids in the auction, they can expropriate most of the surplus from the seller. However, finding such an agreement is not necessarily an easy task. In single unit first-price auctions, collusion is considered to be difficult to sustain. Robinson (1985) shows that, with commonly known values, collusive agreements are not stable. However, in an independent private values framework, McAfee and McMillan (1992) show that collusion is possible. However, Güth and Peleg (1996) note that, by using repeated play to support their collusive equilibrium, McAfee and McMillan (1992) diminish the problem of enforcement in their analysis. Güth and Peleg (1996) show that no collusive mechanism satisfies both no envy and their weaker form of incentive compatibility when the item is being sold at the first-price. However, under more general conditions, Güth and Peleg (1996) describe equilibrium strategies. They find that when the object is being sold in a first-price sealed bid auction a ring leads to the same profits for both the buyer and seller as in the competitive case. In their view, the inability of collusion in first-price auctions to lead to profitable agreements may explain the general predominance of first-price sealed bid auctions. In the multiple object setting, however, the opportunities for collusive equilibria increase.

In order to collude in this auction environment, bidders must come to a voluntary agreement about what bids are to be placed at the auction (which, in turn, determines who will be the winner of each item) as well as what sort of side payments are to be made between members. Assume that bidders can communicate, and that they coordinate their bidding in each market in some sort of group decision process.

How is this group decision process modeled? Assume that bidders formulate a *collusive mechanism.* A collusive mechanism is a game played by the bidders, the outcome of which is a set of bids in the auction. As in Laffont and Martimort (1998), assume that the objective of the mechanism is to maximize the expected utility of each bidder.[2]

Attempting to characterize the collusive mechanisms that may arise as the outcome of all possible cooperative games between bidders is a daunting task. Fortunately, by assuming that any collusive agreement must be compatible with individual incentives, that search can be drastically limited. The Revelation Principle says that any outcome which can be attained as the Bayes Nash equilibrium of some mechanism can also be attained as the Bayes Nash equilibrium of a direct revelation mechanism (Gibbard (1973), Dasgupta et al. (1979)). A direct revelation mechanism is a direct mechanism which satisfies individual incentive compatibility (IC).[3] Thus, the outcome of communication between bidders can be thought of as a mechanism, (β, s), which determines the bids to be placed and the payments to be made between members. In other words, $\beta : [\underline{v}, \overline{v}]^{m \cdot n} \to [\underline{v}, \overline{v}]^{m \cdot n}$ is a function such that $\beta_{ij}(r)$ specifies a bid by i in market j. The function $s : [\underline{v}, \overline{v}]^{m \cdot n} \to \mathbb{R}^{m \cdot n}$ specifies the payment (possibly negative) that each bidder pays in addition to his bid price if he is the winning bidder. Hence, $s_{ij}(r)$ is the payment bidder i pays in market j.

Assume that bidders decide upon such a mechanism after they have seen their own values in each market. However, they remain uncertain as to the actual valuations of the other bidders. No bidder has the ability to coerce another bidder to reveal his valuation. Thus, all information about individual preferences for markets must come from the mechanism itself. At the *interim* stage, each bidder's expected utility is given by

$$U_i(r_i, v_i) = \sum_{j=1}^{m} G_{ij}(B_{ij}(r_i))(v_{ij} - B_{ij}(r_i)) - S_{ij}(r_i) \tag{2}$$

where

$$B_{ij}(r_i) = \int_{r_{-i} \in V_{-i}} \beta_{ij}(r, r_{-i}) dF_{-i}(r_{-i})$$

and

$$S_{ij}(r_i) = \int_{r_{-i} \in V_{-i}} s_{ij}(r, r_{-i}) dF_{-i}(r_{-i})$$

are the *reduced form* equations which represent the bidder's expected bid and expected payment. G_{ij} is given by the rules of the auction; it is the probability that i's bid is greater than the $n-1$ other bids placed. r_i is the vector of reported values of agent i and v_i is i's vector of actual valuations for the j objects being auctioned.

$V_i((\beta, s)|v_i)$ is agent i's indirect utility function over mechanisms determined by the

[2]Laffont and Martimort (1998) examine collusive mechanisms in public goods environments. They propose that the collusive mechanism is designed by a benevolent planner (or centre). The perspective taken here is similar, but I aspire to allow the bidders to select the mechanism themselves.

[3]A mechanism is direct if the strategy space is equivalent to the type space. In this case, agents report a vector of valuations r_i.

expected utility attained from maximization of Equation 2:

$$V_i((\beta, s)|v_i) = \max_{r_i \in [\underline{v}, \overline{v}]^m} \sum_{j=1}^{m} G_{ij}(B_{ij}(r_i))(v_{ij} - B_{ij}(r_i)) - S_{ij}(r_i)$$

A mechanism satisfies IC if it is in the best interest of every individual to report his true valuation for the objects for all possible values that the other bidders might have (or $r_{ij} = v_{ij}$ for all i, j).

3.1 Definition (Incentive Compatible) *A mechanism (β, s) is Incentive Compatible iff $U_i(v_i, v_i) \geq U_i(r_i, v_i)$ for all r_i, and for all v_i.*

Assume that bidders do not deviate from the collusive mechanism. While bidders are able to misrepresent their values *within* the mechanism, once bids are determined by the mechanism the bids are perfectly enforced in the auction. This approach may be justified by repeated play. If bids are placed that are inconsistent with the mechanism, bidders will use a trigger strategy which punishes deviant bidders. McAfee and McMillan (1992) and Pesendorfer (1996) use this approach to find profitable collusive mechanisms. Otherwise, bidders' incentives to increase their bids cannot be avoided. The negative results of Güth and Peleg (1996) are largely due to the fact that they assume that bidders may place any bid in the auction. The repeated game approach appears to be consistent with previous experimental evidence on cooperative agreements (see Section 4). Also, assume that bidders' values are not ex post observable. After an auction, bidders cannot observe values in order to determine whether bids were truthful. Therefore, collusive mechanisms must be independent of actions in previous auctions.

A restriction which makes analysis of the various mechanisms substantially easier is *anonymity*, which requires that bidders with the same valuations are treated the same under the mechanism.

3.2 Definition (Anonymity) *A mechanism (β, s) satisfies anonymity iff for all permutations $\sigma : N \rightarrow N$, $B_i(v_i) = B_{\sigma(i)}(v_{\sigma(i)})$ and $S_i(v_i) = S_{\sigma(i)}(v_{\sigma(i)})$ for all v_i, and for all i.*

As in Ledyard and Palfrey (1994), when examining situations in which agents' valuations are drawn from identical distributions, it is assumed that mechanisms are anonymous (or symmetric).[4] Lemmas A.1 and A.2 in the appendix justify this approach; by restricting attention to anonymous mechanisms, when bidders are symmetric, non-anonymous mechanisms that are socially preferred to anonymous mechanisms are not excluded.

An IC collusive mechanism that is always feasible is the *noncooperative mechanism*: bids are placed that are consistent with the symmetric Bayes Nash equilibrium.

[4]In situations in which values are drawn from different distributions for the same market, the mechanism should be allowed to vary with different distributions as well as with different values.

3.3 Example (Noncooperative Mechanism)

$$\begin{aligned}\beta_{ij}(r) &= v_{ij} - \int_{\underline{v}}^{v_{ij}} \left(\frac{F_j(y)}{F_j(v_{ij})}\right)^{n-1} dy \\ s_{ij}(r) &= 0\end{aligned}$$

△

If bidders cannot find a collusive mechanism that is preferred to this strategy, there is little hope for successful collusion. Laffont and Martimort (1998) examine, in a public goods setting, whether some mechanism dominates the noncooperative mechanism. The objective here is to go a step further by describing the possible mechanisms.

A first step in determining what mechanisms might be expected is to propose a reasonable mechanism and investigate its characteristics. A *reduced bidding* mechanism is one possibility. Under this mechanism, each bidder agrees to bid some fraction (α_j) of his value in each market.

3.4 Example (Reduced Bidding Mechanism)

$$\begin{aligned}\beta_{ij}(r) &= \alpha_j r_{ij} \\ s_{ij}(r) &= 0\end{aligned}$$

△

The reduced bidding mechanism represents limited competition between bidders. By choosing such a mechanism, if the bidders truthfully report their valuations, the objects will be won by the bidders with the highest valuations, and, if the α's are small, the cartel will capture most of the surplus. The following lemma characterizes IC reduced bidding mechanisms.

3.5 Lemma *For each market, j, there exists an $\alpha_j \in (0,1]$ such that the Reduced Bidding mechanism is Bayesian Incentive Compatible iff $\frac{d\Gamma_{ij}(r_{ij})}{dr_{ij}} = c_j$ for all i where $c_j \in \mathbb{R}_+$. Furthermore, α_j is given by $\frac{d\Gamma_{ij}(r_{ij})}{dr_{ij}} = \frac{1-\alpha_j}{\alpha_j}$ where $\Gamma_{ij}(r_{ij}) = \frac{G_{ij}(r_{ij})}{g_{ij}(r_{ij})}$.*

The implication of Lemma 3.5 is that the only IC reduced bidding mechanisms are those that yield bidder profits identical to noncooperative bidding.

3.6 Theorem *If an IC α_j exists for all markets then the resulting bid function is equivalent to the noncooperative mechanism.*

If $\alpha_j v_{ij}$ is not equal to the noncooperative bid strategy, bidders have an incentive to increase their reported values to increase their probability of winning in the auction. Since the cartel members cannot directly observe each other's values, all agents will partake in this destructive behavior as long as they are bidding below the Bayes Nash equilibrium. Therefore, the only IC reduced bidding mechanisms is the Bayes Nash equilibrium. This result is similar to Güth and Peleg (1996)'s; any mechanism which allows for positive bidding must be equivalent to the Bayes Nash equilibrium.

Given that mechanisms of this sort don't seem very realistic, what types of mechanisms might one expect to see bidders select? Holmström and Myerson (1983) suggest that a reasonable class of mechanisms to eliminate are those that are *interim dominated* by another mechanism.

3.7 Definition (Interim Dominated) *A mechanism* (β, s) *is interim dominated by* (β', s') *iff* $V_i((\beta', s')|v_i) \geq V_i((\beta, s)|v_i)$ *for all* i *and for all* v_i *with at least one strict inequality.*

If the bidders select a mechanism that is interim dominated, even before they learn their values, bidders would unanimously agree to switch to a mechanism which dominates it. Interim incentive efficient (Holmström and Myerson (1983)) mechanisms are those which are not dominated.

3.8 Definition (Interim Incentive Efficient) (β, s) *is interim incentive efficient iff there does not exists another IC, feasible mechanism* (β', s') *such that* (β, s) *is interim dominated by* (β', s').

Holmström and Myerson (1983) show that, if a mechanism is interim incentive efficient, it can *never* be common knowledge that another IC mechanism interim dominates it. Thus, interim efficiency is a minimal standard for what is expected as the outcome of a cooperative process.[5]

In order to limit the incentives to misrepresent their valuations as evidenced in the reduced bidding mechanism, bidders might select one bidder as the sole bidder in each market. Define β° as follows.

$$\beta^\circ_{ij}(v) = \begin{cases} \underline{v} & \text{with probability } q_{ij}(v) \\ \emptyset & \text{with probability } (1 - q_{ij}(v)) \end{cases}$$

where

$$\sum_{i=1}^{n} q_{ij} = 1 \text{ for all } j$$

[5]It is likely that the cooperative process would lead bidders to actually select a set of mechanisms which is a subset of the interim incentive efficient mechanisms. For example, Holmström and Myerson (1983) argue that the appropriate restriction in the face of communication between agents is the concept of *durability* (the bidders would never unanimously approve a change from one mechanism to another). For the sake of this paper, I take the set of interim incentive efficient mechanisms to be a good first approximation.

and $\emptyset$ indicates that the bidder does not participate in the auction.[6] The function which determines the side payments as a function of bidders' valuations is now given by t. Let the class of mechanisms of this form be indicated by $\mathcal{B}^\circ$. Thus, (q, t) is now a new direct revelation mechanism that defines a bidder's ex post expected utility as

$$U_i(r, v) = \sum_{j=1}^{m} q_{ij}(r)(v_{ij} - \underline{v}) - t_{ij}(r),$$

and interim expected utility as

$$U_i(r_i, v_i) = \sum_{j=1}^{m} Q_{ij}(r_i)(v_{ij} - \underline{v}) - T_{ij}(r_i)$$

where Q_{ij} is the reduced form probability of being selected as the sole bidder and T_{ij} tax is:

$$\begin{aligned} Q_{ij}(r_i) &= \int q_{ij}(r_i, r_{-i}) dF_{-i} \\ T_{ij}(r_i) &= \int t_{ij}(r_i, r_{-i}) dF_{-i}. \end{aligned}$$

The following theorem establishes that attention can be restricted to this particular class of mechanisms.

3.9 Theorem *If (β, s) is an incentive compatible direct mechanism such that $(\beta, s) \notin \mathcal{B}^\circ$ then there exists an incentive compatible, direct mechanism $(\beta', s') \in \mathcal{B}^\circ$ which interim dominates (β, s).*

The set of interim incentive efficient mechanisms lie within $\mathcal{B}^\circ$. Bidding leads to profits for the auctioneer which necessarily implies losses to the cartel. To achieve the greatests possible surplus bidders will allocate the object to the bidder who would have won the same object under the mechanism not in $mathcalB^\circ$. Then, the bidders can divide up the gains from not bidding in a manner that does not affect incentives. This is the same approach taken by Graham and Marshall (1987) when modeling collusion in second-price auctions. Noncooperative bidding specifies strictly positive bids for all bidders. The noncooperative mechanism is not in $\mathcal{B}^\circ$, implying that there exist collusive mechanisms which dominate it.

3.10 Corollary *The noncooperative mechanism is dominated when side payments are allowed.*

McAfee and McMillan (1992) provide an insight into possible mechanisms that might arise in this setting.

[6]A nearly equivalent version would allow one bidder to bid $\underline{v} + \epsilon$ and all others to bid $\underline{v}$.

3.11 Definition (Ex post Efficient) (q, t) *is ex post efficient iff there does not exist another mechanism* (q', t') *such that* $V_i((q',t')|v) \geq V_i((q,t)|v)$ *for all* i*, and for all* v *with strict inequality somewhere.*[7]

A mechanism is said to be ex post efficient if it always assigns bidding rights in each market to the bidder with the highest valuation. Thus, the bidder with the highest valuation is chosen as the winning bidder with probability one.

3.12 Remark In order for a mechanism to be ex post efficient it must be that for all j, $q_{ij}(v) = 1$ iff $v_{ij} = \max\{v_{1j}, v_{2j}, \ldots, v_{nj}\}$.

In the single object setting, McAfee and McMillan (1992) show that there exists an ex post efficient mechanism which can easily be extended to the generic multiple object environment developed here.

3.13 Example (Efficient Strong Cartel Mechanism)

$$
\begin{aligned}
q_{ij}(v) &= \begin{cases} 1 & v_{ij} = \max\{v_{1j}, v_{2j}, \ldots, v_{nj}\} \\ 0 & \text{otherwise} \end{cases} \\
t_{ij}(v) &= (F_j(v_{ij})^{-n}) \int_{\underline{v}}^{v_{ij}} (v_{ij} - \underline{v})(n-1)F_j(s)^{n-1} f_j(s)ds + \underline{v}
\end{aligned}
$$

if $v_{ij} = \max\{v_{1j}, v_{2j}, \ldots, v_{nj}\}$ and otherwise

$$
t_{ij}(v) = -\frac{[t_{ik}(v) - \underline{v}]}{(n-1)} \tag{3}
$$

△

Under this mechanism, the bidder with the highest valuation in each market is selected and splits between each of the $n-1$ other bidders the gain in surplus from limiting competition. Since this mechanism is dependent only upon valuation reports for each particular market it can be extended to the multiple object setting.

3.14 Theorem (McAfee and McMillan (1992)) *The efficient strong cartel mechanism is both incentive compatible and ex post efficient.*

When side payments are allowed, there exists a collusive mechanism that allows the bidders to capture all available surplus. Since ex post efficiency uniquely characterizes q (Remark 3.12), it must be that an interim efficient mechanism also satisfies that restriction. Therefore, the strong cartel mechanism must be interim incentive efficient.

3.15 Corollary *The efficient strong cartel mechanism is interim incentive efficient when side payments are allowed.*

[7] V_i is now an agent's ex post utility where all other agents' valuations are revealed.

3.1 Weak Cartels

McAfee and McMillan (1992) also examine collusive agreements in single object first-price auctions that prohibit side payments. A likely explanation for this restriction is that antitrust laws and the threat of detection make actual side payments extremely risky, if not impossible. It is hard to imagine a large firm actually transferring funds to another firm. Thus, the only method for collusion is the division of bidding rights in various markets. Assume side payments are not possible.[8] Under these *weak cartel* agreements, McAfee and McMillan (1992) show that the best mechanism for a ring of bidders is one in which they all place identical bids, or they commit to a rotation scheme that randomly chooses an exclusive bidder. Such rotation schemes are often called *phases of the moon* agreements (Bane (1973)). In the mechanism design model just developed, such a restriction can be implemented by requiring that no transfers are made.

3.16 Assumption (No side payments) $t_{ij}(v) = 0$ *for all* i *and for all* j

Let $\mathcal{B}^s$ be the subset of $\mathcal{B}^\circ$ such that Assumption 3.16 is satisfied. A mechanism with no side payments cannot be ex post efficient since the condition given by Remark 3.12 violates IC. The following result is a generalization of the result of Dudek et al. (1995).

3.17 Theorem *Let* $G_{ij}(r) = \prod_{k \neq i} F_{kj}(r)$. *If there is some* i *such that* $G_{ij}(\hat{v}_{ij}) \neq G_{ij}(v_{ij})$ *for some positive* $\hat{v}_{ij}, v_{ij} \in [\underline{v}, \overline{v}]$, *then there does not exist an IC* (β, s) *in* $\mathcal{B}^s$ *such that it is ex post efficient.*

Without the extra lever of side payments, the cartel cannot ensure that the bidder with the highest valuation is chosen. Every bidder (even the lowest types) must be given some positive probability of being chosen as the sole bidder. Therefore, there will always be mechanisms in $\mathcal{B}^\circ$ that (ex post) dominate mechanisms without side payments. If, instead of comparing mechanisms in $\mathcal{B}^s$ to *all* other mechanisms, attention is restricted to mechanisms *only* in $\mathcal{B}^s$, then any mechanism is ex post efficient. Raising any bidder's probability that he is the sole bidder in some market (given that all type information is revealed) necessarily requires lowering other bidders' probabilities of being the sole bidder. The decreased probability of winning cannot be offset by side payments as it is in the strong cartel situation.

Unfortunately, examining all possible mechanisms in $\mathcal{B}^s$ is still a very difficult task due to the multi-dimensionality of each bidder's type (each bidder's type is an m-tuple of valuations). While Rochet (1987) provides necessary and sufficient conditions for a mechanism to be IC in very general multi-dimensional settings, finding the interim incentive efficient mechanism in this class is still not trivial. I proceed by proposing a potential collusive mechanism in $\mathcal{B}^s$. In the single object setting, McAfee and McMillan (1992) show that random assignment of a winning bidder is the only IC collusive mechanism without side payments other than the noncooperative outcome. The random assignment mechanism generalizes their result to the multiple object setting.

[8] Assume that side payments are illegal and that this prohibition is perfectly enforced.

3.18 Example (Random Assignment Mechanism)

$$q_{ij}(v) \quad = \quad \frac{1}{n} \tag{4}$$

△

The random assignment mechanism is IC since it does not depend upon *any* individual information. Three procedures describe how bidders might arrive at the probabilities specified by this mechanism. First, in each market, the group could randomly select a sole bidder. Second, if the auctioneer randomizes amongst tie bids, all bidders could simply agree to place identical bids of $\underline{v}$ in each market. Finally, when $F_{ij} = F_{kj}$ for all i, j, and k and the auction is repeated many times, the assignment mechanism could also be approximated by each bidder bidding in only one market for all periods.

Is it the case that even this simple mechanism interim dominates noncooperative bidding? For some distributions there will always be values for which bidders prefer the noncooperative mechanism to the random assignment mechanism. Consider an example where $v_{ij} \in [0, 1]$ and $F_{ij}(v) = v^{1/3}$ for all i. The Bayes Nash equilibrium bid function is given by

$$b_{ij}(v_{ij}) = \frac{n-1}{n+2} v_{ij}.$$

For any v_{ij}, a bidder's expected utility from the random assignment mechanism is $\frac{1}{n} v_{ij}$. A bidder will prefer the noncooperative outcome in market j if

$$v_{ij} > \left(\frac{n+2}{3n} \right)^{\frac{3}{n-1}}. \tag{5}$$

For $n > 1$, there are feasible values which satisfy this condition. As n increases, the right hand term of Equation 5 approaches 1. In general, as the set of bidders grows, the set of values under which the noncooperative mechanism is preferred to the random assignment mechanism shrinks.

3.19 Theorem *As $n \to \infty$, the set of v such that all bidders strictly prefer the noncooperative mechanism to the random assignment mechanism shrinks to a set of measure zero.*

Only bidders with high valuations will prefer the noncooperative outcome. However, as n increases, it is more likely that there are other bidders with high values. This makes the Bayes Nash equilibrium strategy less profitable. If F_j is convex for all markets, the random assignment mechanism will dominate the Bayes Nash equilibrium mechanism (i.e. random assignment will be preferred for all values). A convex F_j implies that higher value draws are more likely, which encourages bidders to bid closer to their values in the Bayes Nash equilibrium. An implication of Theorem 3.19 is that for the single object case the random assignment mechanism is interim incentive efficient. For all symmetric distributions, there is a non-zero cut-off between preference for the random assignment mechanism and the noncooperative mechanism. However, the noncooperative mechanism is the only other incentive compatible mechanism. Thus, random assignment cannot be dominated.[9]

3.20 Proposition *When $m = 1$, the random assignment mechanism is interim incentive efficient.*

The fact that bidders are more likely to prefer the random assignment mechanism to noncooperative bidding when n is large is opposed to conventional wisdom on collusive behavior. Both experimentally and empirically, cartelization is thought to be much easier in small groups. However, as the group size increases, the benefits from noncooperative behavior shrink significantly.

While random assignment mechanisms are the only IC collusive mechanisms in the single object environment, other IC mechanisms are available in the multiple object environment. Bidders may use more sophisticated *rotation schemes* which utilize the increased dimensionality of the type space to increase efficiency. Bidders are willing to trade-off probability of winning a lower valued object for increased probability of winning a higher valued object. These mechanisms are characterized by the strategic choice of sole bidders for each market based upon their reported values. The serial dictator mechanism (Satterthwaite and Sonnenschein (1981), Olson (1991)) is an example of a rotation scheme.

3.21 Example (Serial Dictator Mechanism) For each random permutation of bidders: $(n_1, n_2, \ldots, n_m)$, where $n_k = i$ indicates that bidder i selects in spot k,

$$q_{(n_k)j} = \begin{cases} 1 & j = R(n_k) \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$\tag{7}$$

where $R(n_k)$ is defined iteratively as follows. Let $R(n_0) = \emptyset$ and for $k \geq 1$

$$R(n_k) = \arg \max_{j \in \{1,\ldots,m\} \setminus \bigcup_{i=1}^{k-1} R(n_i)} \{v_{(n_k)j}\}.$$

△

The serial dictator mechanism selects the order in which each bidder is allowed to select the market in which he is the sole bidder. Each bidder is a *dictator* over the outcomes at a single point in time. Assume that the choice of which bidder is selected as a dictator at any point is random.[10] If there are m objects and n bidders, the probability that any bidder is selected to be the dictator for market i is $1/m$. In the example described by Figure 1, there are five bidders and five objects. The numbers indicate each bidder's relative ranking of

[9] Any randomization will be IC, but only the random assignment mechanism satisfies anonymity.
[10] This is necessary to maintain anonymity.

		Bidder				
		1	**2**	**3**	**4**	**5**
	A	1	3	3	5	5
	B	2	5	2	3	2
Market	**C**	3	2	1	1	1
	D	4	1	4	2	3
	E	5	4	5	4	4

Figure 1: An example

his values. If the random draw of dictators yields the order $(1, 2, 3, 4, 5)$, then bidder 1 would select first and choose market A, 2 would select market D, 3 would select C, 4 would select B, and 5 would have no choice but to select market E. When the number of objects is less than or equal to the number of bidders, the serial dictator mechanism requires that each bidder be selected at most one time. The number of possible allocations predicted by the serial dictator mechanism can be large. In principle, each different permutation of the dictator order could lead to a different outcome.[11] Thus, the number of possible orderings, $n!$, acts as an upper bound on the number of possible outcomes.

The serial dictator mechanism is IC because stating one's true valuations maximizes the probability that higher valued objects are chosen first. The serial dictator mechanism highlights the increased richness of the set of possible mechanisms when examining multiple object auctions. More importantly, the multiple object environment makes the random assignment mechanism an inferior choice.

3.22 Proposition *The serial dictator mechanism interim dominates the random assignment mechanism.*

In an assignment model, Olson (1991) shows that the serial dictator mechanism is ex post efficient when considering implementation in Nash equilibria. However, I am examining implementation of mechanisms in Bayes Nash equilibria. Therefore, it is possible that other mechanisms may exist which interim dominate the serial dictator mechanism.

Ideally, I would continue examining generic weak cartel mechanisms. However, the serial dictator mechanism suggests a class of mechanisms that seem particularly reasonable as a first guess at the expected choice of mechanism in this setting and are easier to analyze. They are *ordinal mechanisms* which rely only on each individual's ranking of his markets.

3.23 Definition (Ordinal Mechanism) *Let* $M = \{1, 2, \ldots, m\}$. *Let* $f : [\underline{v}, \overline{v}]^{m \cdot n} \to M^{m \cdot n}$ *be a function defined as*

$$f(v_{ij}) = k \iff \#\{v_{il} : v_{il} \geq v_{ij}\} = k - 1.$$

[11] Although that is not necessarily true. It is easy to imagine circumstances in which only one solution is possible. For example, suppose each bidder's maximal valuation is in a different market. Then, for any combination, each bidder will select the market he ranks highest.

q is an ordinal mechanism iff for all v and v′

$$f(v) = f(v') \Rightarrow q(v) = q(v').$$

When a mechanism is ordinal, both q and the reduced form probabilities Q can be expressed as a function of each agent's ranks of the markets. Since any incentive compatible collusive mechanism will not be ex post efficient (Theorem 3.17), it must be that the mechanism makes limited use of the bidders' information. Also, when $m = 1$, in order to satisfy IC, the mechanism must not depend on any private information. Ordinal mechanisms are one class of mechanisms that satisfy these constraints. There may be other incentive compatible mechanisms that use more information than ordinal mechanisms. However, as a first cut, the possible mechanisms given this restriction are examined. When considering ordinal mechanisms, IC is characterized by the following proposition.

3.24 Proposition *Any ordinal mechanism q is Bayesian Incentive Compatible iff for all i,*

1. *Q_{ij} is decreasing in the ranks (i.e. $Q_{ij}(m_i|m_{ij} = 1) \geq Q_{ij}(m_i|m_{ij} = 2) \ldots \geq Q_{ij}(m_i|m_{ij} = m)$), and*
2. *$(Q_{ij}(m_i|m_{ij} = l) - Q_{ij}(m_i|m_{ij} = p)) = (Q_{ik}(m_i|m_{ik} = l) - Q_{ik}(m_i|m_{ik} = p))$ for all j, k and for all l, p.*

The first condition is a standard IC constraint that says a bidder will be willing to place each market in its proper rank only if doing so results in an increase in his probability of winning that object. The second condition constrains how the mechanism may vary across markets. The relative differences in Q between each rank must be the same in each market. Otherwise, there may be values for which the bidder would prefer to change his reported ranks.

The random assignment and serial dictator mechanisms are IC ordinal mechanisms. Pesendorfer (1996) suggests another ordinal mechanism that satisfies Bayesian incentive compatibility: the ranking mechanism. Bidders submit reports of their ranks.[12] Then, the bidder with the highest rank is selected in each market as the sole bidder in that market. If more than one bidder happens to report the same rank, then the sole bidder is chosen at random from those bidders. The example in Figure 1 is an illustration of such a mechanism. The ranking mechanism would select bidder 1 as the sole bidder in market A, either 1, 3, or 5 in market B, either 3, 4, or 5 in market C, 2 in market D, and 2, 4, or 5 in market E. Three features of the ranking mechanism are apparent. First, bidders can be selected as the sole bidder in more than one market. In this example, bidder 5 could potentially be selected as the bidder in three markets. It is possible that bidder 3 not be selected at all. Second, the sole bidder's rank can be very low. For example, in

[12] If one wishes to stick to the strict definition of a direct mechanism, imagine bidders submitting their valuations and some cartel centre ranking their values. Bidders are indifferent between reporting their true valuations and reporting any other order-preserving set of valuations.

market E, the potential winning bidders' ranks are all 4, indicating that their valuations are likely to be quite low.

3.25 Example (The Ranking Mechanism)

$$q_{ij}(m) = \begin{cases} \frac{1}{k} & m_{ij} \leq m_{\ell j} \text{ for all } \ell \\ 0 & \text{otherwise} \end{cases}$$

where $k = \#\{\ell | m_{\ell j} = m_{ij}\}$ is the number of bidders who ranked market j the same as i. △

The reduced form probabilities for each bidder and each market are given by

$$Q_{ij}(m_{ij}) = \sum_{k=1}^{n} \left(\frac{1}{k}\right) \left(\frac{(n-1)!}{(k-1)!(n-k)!}\right) \left(\frac{1}{m}\right)^{k-1} \left(\frac{m - m_{ij}}{m}\right)^{n-k}. \tag{8}$$

Under the ranking mechanism, each agent's probability of being selected as the sole bidder in a particular market is independent of his ranks for the other markets. Obviously, the ranking mechanism satisfies incentive compatibility since the interim probability of being selected as the sole bidder is decreasing in the ranking. The probability that an individual is selected as the bidder in any particular market is simply the probability that no one ranked that market higher than he did, which is clearly decreasing in his ranking (for higher ranks (m_{ij}) each term in Equation 8 is smaller).

For a fixed number of bidders, Pesendorfer (1996) shows that expected efficiency converges to 100% as the number of markets increases.[13] The ranking mechanism will always select a bidder who ranked a particular market the highest as opposed to the serial dictator mechanism which may, due to the order of draws, select a bidder who does not have a high rank. Thus, in expectation, bidders' valuations should be higher. In fact, the ranking mechanism is an interim incentive efficient ordinal mechanism.

3.26 Theorem *If $\overline{v} > (n-1)\underline{v}$, the ranking mechanism is interim incentive efficient in the class of all anonymous ordinal mechanisms without side payments.*

If $\underline{v} = 0$, then the condition on the support of the distribution is satisfied for all n. This suggests that, if a group of bidders are deciding on how to collude, they may very well want to pick the ranking mechanism since no other mechanism can do better for all possible values.[14]

[13] Bidder surplus as a percentage of the maximum possible surplus can be readily substituted for efficiency in these situations since bidders are essentially bidding zero which implies no seller's surplus.

[14] A similar result likely holds for asymmetric distributions and a slightly redefined ranking mechanism where the probability a bidder is assigned a market when there is a tie is dependent on his distribution of values in that market. However, the current version of the proof relies heavily upon the anonymity of the mechanism.

There may exist other (non-ordinal) mechanisms which dominate the ranking mechanism. Since the noncooperative mechanism is not an ordinal mechanism, it is even possible that it may dominate the ranking mechanism. However, since the ranking mechanism dominates random assignment, Theorem 3.19 can be applied to the ranking mechanism: the ranking mechanism is not dominated by noncooperative bidding.

3.27 Corollary *As $n \to \infty$, the set of v such that all bidders strictly prefer the noncooperative mechanism to the ranking mechanism shrinks to a set of measure zero.*

Holmström and Myerson (1983) suggest a stronger standard which may be more logical when a decision rule is chosen in the interim stage. A mechanism is *durable* if the agents would never (for all possible draws of values) unanimously approve a change from that mechanism to another mechanism. Interim incentive efficiency guarantees that, from an ex ante perspective, the mechanism will not be blocked. However, once agents have observed their values, one agent (not *knowing* that the others will prefer a new mechanism) may propose a change which is unanimously accepted. Corollary 3.27 suggests that the ranking mechanism may not be durable. There are always distributions such that for some values all agents prefer the Bayes Nash equilibrium strategy to the ranking mechanism.

3.28 Proposition *For all n and m, there exist distributions such that for a set of positive measure the Bayes Nash equilibrium is unanimously preferred to the ranking mechanism.*

This, however, does not mean that the ranking mechanism is not durable. Holmström and Myerson (1983) model durability by a specific voting game. It is necessary to consider what the bidders would learn if they unanimously approved a change to another mechanism. For example, if all bidders agreed to move from the ranking mechanism to the noncooperative mechanism, then each bidder could infer that everybody had high valuations. This updated information, however, would cause them to bid higher in the Bayes Nash equilibrium, making it a less attractive agreement. Consider an example where $m = 1$ and $n = 2$. Let $v_i \in [0, 1]$ and $F_i(v) = v^{1/2}$. Both bidders will prefer the Bash Nash bidding only if $v_i > 9/16$. However, if the bidders condition their bids on the fact that their opponent has a value above 9/16, they will bid so high that they will no longer prefer noncooperative behavior. Unfortunately, the ranking mechanism does not satisfy the sufficient conditions given by Holmström and Myerson (1983) for a mechanism to be durable. Thus, the question of whether there are any durable collusive mechanisms without side payments remains open.

Clearly, collusive agreements will most likely involve selecting a sole bidder to bid in each market. When side payments are allowed, an ex post efficient mechanism exists. However, with no side payments, ex post efficiency cannot be achieved. The fact that bidders are bidding on multiple objects allows them to choose a collusive agreement that yields higher expected surplus (and efficiency) than the best IC mechanism in the single object case (random assignment). The serial dictator and ranking mechanisms are two ordinal mechanisms which interim dominate the random assignment mechanism. This is only a partial analysis of the outcomes of collusive behavior in the multiple object setting. There remain many unanswered questions. For example, what is the full characterization of interim incentive efficient mechanisms? Also, what is the impact of communication and repeated play on the choice of strategies?

While the ranking mechanism is an interim incentive efficient mechanism (in the class of ordinal mechanisms), the serial dictator mechanism may have an advantage due to its simplicity. The structure of the serial dictator mechanism is similar to a typical description of a bidder *ring* in which each bidder takes a turn (in a ring) picking what he wants to bid on (Cassady (1967)). However, this intuitive simplicity is at the cost of expected efficiency. Both these rotation schemes interim dominate the random assignment mechanism. On the other hand, the random assignment mechanism would be extremely simple for a group of bidders to utilize and monitor. An experimental examination of this mechanism design problem will give some initial insight into this trade-off between efficiency and simplicity.

4 Experimental Design

In the previous section, it was shown that different forms of collusive strategies could be used in multiple object simultaneous first-price auctions. A few strategies highlighted as possible choices by bidders are:

- Competitive bidding,
- Reduced bidding,
- Random assignment, and
- Rotation schemes (serial dictator or ranking).

The theory suggests that some of these mechanisms will most likely be preferred to others. For example, both the particular rotation schemes examined, the serial dictator and ranking mechanism, interim dominate the random assignment mechanism. Reduced bidding agreements are generally only IC if they yield the same profitability as competitive bidding.

In the analysis of Section 3, some possible collusive mechanisms are discussed given the assumption that bidders have agreed to cooperate. Will bidders actually decide to form cooperative agreements? In this vein, the experimental literature on cooperative behavior provides some initial insights. As is the case in prisoners' dilemma or public goods experiments, there are incentives for participants to coordinate their behavior to increase their overall payoffs. However, each participant also has an incentive to defect from any cooperative agreement. While only Isaac and Walker (1985) examine collusive behavior in sealed bid auctions,[15] numerous other experimental studies have highlighted three factors that appear to affect the ability of groups to cooperate:

1. Communication,
2. Repeated play, and
3. Institutional structure.

In general, participants cannot form successful cooperative agreements unless they are given an opportunity to communicate and coordinate their strategies. Isaac et al. (1985) found that allowing communication in a public goods experiment led to a small but stable increase in the amount contributed to the public good. Daughety and Forsythe (1987) found that, with written communication, experimental subjects made choices closer to the collusive optimum. In addition, the method by which communication is allowed appears to be important. For example, Palfrey and Rosenthal (1991) found that in a public goods experiment, where binary signals were the only form of communication allowed, the resultant behavior was no more efficient, despite the fact that participants conditioned their behavior heavily on the signals. This suggests that the more extensive the communication that is allowed, the more likely it is that stable, cooperative outcomes will be observed. The psychology literature has focused on the ability of group discussion to change individual choices (Pruitt (1971)). Numerous psychological factors can play important roles in the ability of a group discussion to lead to outcomes that are preferred by the group but may be contrary to individual incentives (i.e. providing a public good or participating in a cartel).

Repeated interaction appears to be a significant factor in the effectiveness of cooperation. If participants meet only one time, there is little incentive to choose a cooperative outcome. However, cooperative choices can be supported in repeated settings through the use of trigger strategies or Tit-for-Tat type behavior. Selten and Stoecker (1986) report a significant end-game effect in which participants tend to defect from cooperative agreements when they know the end of the experimental session is near. Palfrey and Rosenthal (1994) compare games in which participants in a public goods experiment are repeatedly matched with different individuals to games in which participants repeatedly interact with the same individual. They find that contributions increase slightly under the repeated treatment.

The institutional structure of the environment can drastically affect the level of cooperation observed. The best example of such a contrast is the difference in the effectiveness of collusion in double auction, posted-offer, and sealed bid auction institutions. Isaac et al. (1984) and Clauser and Plott (1992) report that collusive agreements are more successful when sellers can place posted offers. In the double auction environment, in which each participant can change the current offer at any time, collusive efforts almost always break down. However, Isaac and Walker (1985) show that collusive agreements

[15] Kagel and Roth (1995) describe a series of in-class experiments that Kagel conducted with common values.

are relatively stable in sealed bid first-price auctions. In 7 out of 10 experiments, stable collusive agreements developed. One explanation for the contrast in the success of collusion under these various institutions is that in the double auction there is a continuous incentive to defect from the cooperative agreement. However, in both the posted-offer and sealed bid auctions, participants only make a single, binding decision; if they do not deviate when making that decision, it is impossible for them to deviate until the next period. In addition to communication and repeated play, the overall susceptibility of the underlying economic environment to cooperation should also be considered when determining the likelihood of cooperative results.

The multiple unit simultaneous sealed bid auction combines all of the above factors to create a situation that is conducive to cooperative outcomes. First, bidders are allowed to verbally communicate. Second, bidders repeatedly interact with the same individuals and, in most cases, do not know when the experiment will end.[16] Finally, the institution is an extension of the sealed bid auctions studied by Isaac and Walker (1985), which are susceptible to collusion.

With this previous experimental work in mind, stable and successful cooperative agreements are expected to form. However, participants can choose among many different cooperative strategies that vary significantly in their relative sophistication. Palfrey and Rosenthal (1994) note that, despite the increase in contributions from repeated play, participants fail to use sophisticated and more profitable strategies. However, when bidders are allowed to communicate, Isaac and Walker (1985) find that some groups attempt to use more sophisticated strategies where the bidder with the highest value is picked. The primary objective of this experimental study is to determine what types of strategies bidders are actually using in this environment. Also, as shown in Section 3, when side payments are not allowed these strategies are not expected to lead to ex post efficient auctions. The choice of collusive mechanism will affect the final efficiency of the auction as well as the surplus of both the bidders and the seller. In order to provide a better understanding of collusive agreements in first-price auctions, a series of laboratory experiments was designed that allowed for observation of bidders' choice of collusive mechanism.

Subjects for these experiments were students at the California Institute of Technology. All subjects participated in only one experiment. Experiment instructions can be found in Appendix B. The simultaneous first-price auctions were implemented on auction software designed by Wes Boudeville and Dave Porter.

In each experiment, five bidders participated in 5 simultaneous single unit first-price auctions. In the first 5 periods of each experimental session, no communication was allowed. In the next 13-18 periods, subjects were allowed to communicate between each period.

[16]Two experiments were conducted in which the final period was announced in order to test the end-game effect.

Bidders were required to place a bid of at least one experimental dollar (franc) in each market.[17] This restriction ensured that subjects were unable to monitor adherence to collusive agreements via the sound of computer keys being hit for the submission of bids. Also, this allowed the experimenter to easily determine when all the bids had been placed. If ties occurred in the highest bids, the computer software randomized between the high bidders to determine the winner.

4.1 Communication

After the fifth period it was announced that communication would be allowed between bidders. The following statement was handed out and read to subjects, who were then allowed to ask questions.

> **Communication with Other Participants**
>
> Sometimes in previous experiments, participants have found it useful when the opportunity arose, to communicate with one another. You are going to be allowed this opportunity while the computers are reset between periods.
>
> There will be some restrictions.
>
> You are free to discuss any aspects of the experiment (or the market) that you wish, except that:
>
> - You may not discuss any quantitative aspects of the private information on your value sheets.
> - You are not allowed to discuss side payments or to use physical threats.
>
> Since there are still some restrictions on your communications with one another, an experimenter will monitor your discussion between periods. To make this easier, all discussions will be at this site.
>
> Remember, after the computers have been reset between periods (and the next period has begun) there will be no discussion until after the end of the next period.
>
> We will allow a maximum of 4 minutes in any one discussion session.

Subjects were also told that the number of rounds had been fixed. This announcement was intended to make bidders aware that there was no trade-off between conversation and number of periods (and thus profits). In most experiments, subjects had no problem understanding the limitations of their communication and only occasional reminders (or clarifications about the form of acceptable information) were required.

4.2 Information Conditions

The *limited information* environment was the most restrictive information condition utilized by Isaac and Walker (1985). The only information available to participants was

[17] The conversion rate of francs to dollars was either 250 or 500. Thus, a minimum bid of 1 franc was generally a trivial amount.

the identity of the winning bidders and the prices they paid. A second, more limited, information condition not used by Isaac and Walker (1985) was the *zero information* condition which reported only the winning bids to the bidders. The identity of the winning bidder in each auction was unknown to everyone except the winner. Under the zero information condition, the participants could only determine who had placed winning bids through voluntary discussion. The increased difficulty in identifying and punishing deviant bidders was expected to make the zero information condition less conducive to cooperative behavior.

4.3 Symmetry

In the symmetric environment, valuations for all five markets and bidders were drawn from the same distribution. Integer values between 1 and 1000 were drawn using the discrete uniform distribution. Under the assumption that bidders are risk neutral, the unique, symmetric Bayes Nash equilibrium bid function is:

$$b_{ij}(v_{ij}) = .8v_{ij} \text{ for all } i, j.$$

Since the bid functions are symmetric and strictly monotonic, under competitive bidding the auction is expected to be ex post efficient.[18]

4.4 Asymmetry

In the asymmetric environment, valuations for four of the markets for each bidder were drawn from the same discrete uniform distribution with values between 1 and 1000. In the fifth market, valuations were drawn from a first-order stochastic dominant distribution, $F(v) = \frac{v^2}{1000^2}$, taking values between 1 and 1000 as well.[19] In each market, one bidder had a valuation drawn from this preferred distribution. The identity of that bidder was announced to all participants.

When bidders are behaving noncooperatively, the Bayes Nash equilibrium bid function can be estimated numerically. Figure 2 is a plot of the estimated bid functions for each market when bidders have values drawn from the above distributions.[20] If bidder 1 has values drawn from the stochastically dominant distribution, $b_1(v) \leq b_i(v)$ for all other i and for all valuations. Thus, competitive bidding will not necessarily lead to full efficiency. However, in this case, the expected efficiency of competitive bidding is extremely close to 100% (at 99.983%).

[18]An auction is ex post efficient if the winning bidder has the highest valuation for the object.
[19]The discrete analog to this distribution was actually used.
[20]BIDCOMP2, a program developed by John Riley, was used to estimate these bid functions.

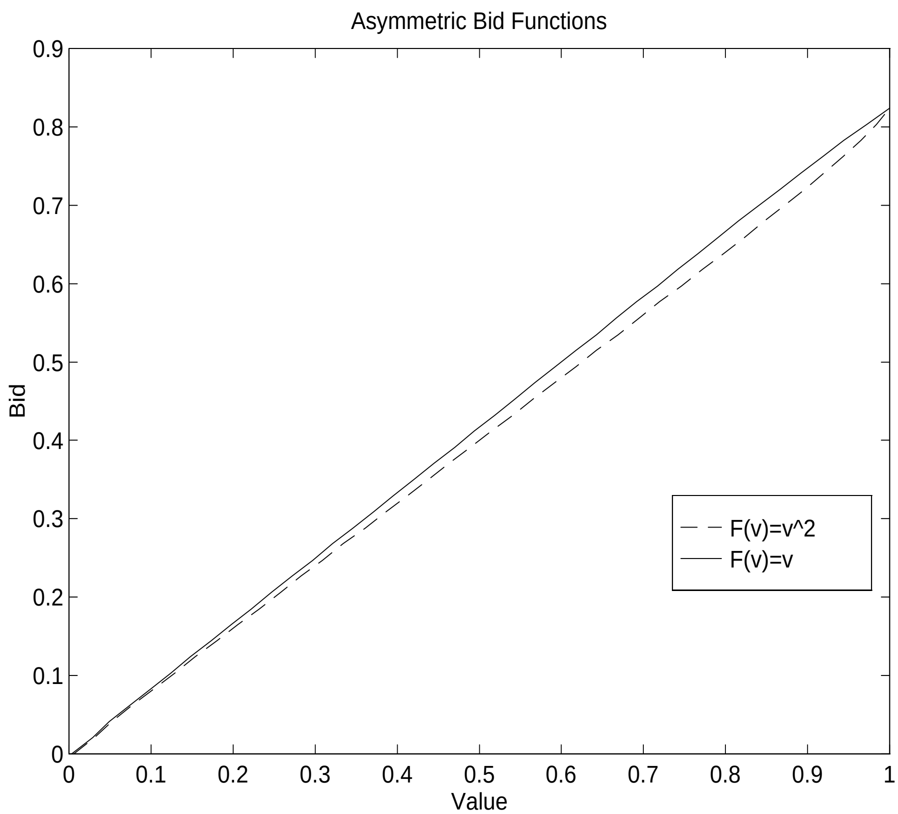


Figure 2: Asymmetric Bid Functions

4.5 End of Experiment Changes

In order to determine whether communication or repeated play were important factors in the success of collusive agreements, two changes at the end of 5 of the 10 experiments were implemented. The first change was intended to determine the value of repeated play in this environment. Since it was not practical to conduct experiments in which cartel members did not repeatedly interact as in Palfrey and Rosenthal (1994), the end-game effect (EG) was studied (Selten and Stoecker (1986)). At the end of experiments six and seven, it was announced that one more period would be conducted. In this final period, communication was allowed but otherwise complete anonymity was induced. Bidders drew their values randomly from a set of five envelopes. The identity of the winning bidders and their exact earnings were unknown to the experimenter and the other subjects.[21]

The second treatment was designed to demonstrate the importance of communication. In experiments 8 through 10, subjects were told at the beginning of their discussion for period 18 that it would be the last period of discussion (the experiment lasted for five periods beyond that).[22] Isaac and Walker (1988) and Daughety and Forsythe (1987) report that, while cooperation is greater with prior communication (PC) than with no communication, once communication ends the level of cooperation tends to gradually erode.[23]

Both of these changes were made near the end of the experimental session and subjects did not have any a priori knowledge of these treatments. Thus, observations of cooperative agreements in earlier periods should not be affected by either the EG or PC treatment.

5 Experiment Results

Ten experiments were completed with six experiments utilizing the symmetric environment and four using asymmetric valuation draws. Six experiments were conducted under the limited information setting; four experiments used the more limited zero information condition. A general summary of the experiments can be found in Table 1.

Subject earnings averaged $33.75 across all experiments. No experimental session lasted longer than two hours, with the average length closer to one hour and thirty min-

[21]A third party not involved with the experiments paid the subjects for that period by placing their earnings in envelopes marked with an ID known only to the bidder.

[22]In experiment 10, discussion was ended after period 17 and 6 periods without communication were completed.

[23]Actually Isaac and Walker (1988) found that in 3 out of 4 experiments in their first experimental series no participants defected from the collusive agreement after communication was ended. However, in their second set of experiments, contributions declined in 11 of 17 experiments.

Exp.	Number of Periods	Environment	Information Condition
1	20	Symmetric	Limited
2	22	Symmetric	Limited
3	22	Symmetric	Limited
4	22	Symmetric	Limited
5	20	Asymmetric	Limited
6	20	Asymmetric	Limited
7	20	Asymmetric	Zero
8	18	Asymmetric	Zero
9	18	Symmetric	Zero
10	17	Symmetric	Zero

Table 1: Experimental Design

utes. There was no significant variance of subject profits between and within periods.[24]

The behavior of the bidders in the first five periods of the auction, when communication was not allowed, was roughly similar to previously observed results. In previous experiments, bidders bid somewhat above the bid predicted by the risk neutral prediction (Cox et al. (1988)). However, for extremely low valuations where bidders have little chance of winning, they typically place extremely low bids (often 0). The estimation of a simple linear regression on the bids placed in the auctions with symmetric valuations demonstrates the similarity of the initial five periods with these results. Estimating the linear regression of $b = \beta_1 + \beta_2 v + \epsilon$ yields estimates of $\hat{\beta}_1 = -12.75$ and $\hat{\beta}_2 = .815$, which are both statistically significant. Thus, after accounting for unusually low bids near zero (the negative intercept), bidding appears to be slightly above the prediction of the risk neutral Nash equilibrium ($\beta_2 = .8$).[25]

5.1 Do bidders form cooperative agreements?

The results of Isaac and Walker (1985) suggest that successful cooperative is expected here. A significant drop in bidding prices is one indicator of collusive behavior. The average bid in periods with communication drops to near zero. While the average bid in no communication periods was 428 francs, it was only 9.6 francs in communication periods. However, a reduction in bid levels is not necessarily an indicator of profitable collusive behavior; Isaac et al. (1984) found that while prices increased when communication was allowed in posted-offer markets, profits did not necessarily increase. Isaac and Walker (1985) use an index of monopoly effectiveness (M), which is the proportion of maximum

[24]However, there was significant variation of profits across experiments due to the choice of cooperative strategy.

[25]The standard errors for the two coefficients are 5.205 and .0087 respectively. The null hypothesis that $\beta_2 \leq .8$ can be rejected at a 95% level of significance.

total possible surplus captured by the bidders:

$$M = \frac{\sum_{j=1}^{5} v_j^* - b_j^*}{\sum_{j=1}^{5} \max_i v_{ij}},$$

where v_j^* is the valuation of the winning bidder in market j and b_j^* is his bid. In these experiments, M increases from an average of .265 in no communication periods to .912 when communication is allowed. Bidders capture a significantly large proportion of the total surplus available. Perhaps the strongest evidence of successful cooperative behavior is that, despite a change in the conversion rate from 250 francs per dollar to 500 francs per dollar, average bidder per period profits rose from \$ .93 to \$ 1.51.

1 Conclusion *When communication is allowed, under both environments and information conditions, collusive agreements are formed and are stable.*

Few deviations from collusive agreements were evident in the ten experiments. In early periods, bidders occasionally placed bids that were not in line with the collusive agreement. Excluding the first two periods of communication, there were only three out of 129 periods in which bidders made notable deviations from the cooperative agreements. In contrast to Isaac and Walker (1985), where collusion occasionally broke down, there is no evidence of sustained deviations in these experiments.[26]

Given the apparent strength of collusive agreements, the two changes mentioned in Section 4.5 were made to try to gain an insight into the source of the strength of these ties. Under the EG treatment, 9 out of 10 subjects did not deviate from the collusive agreement; only one bidder in experiment seven deviated.[27] This seems to indicate that even in one shot environments such collusive agreements are fairly stable. Thus, repeated play is not a particularly important factor in the success of cooperation in this setting.

However, a second change indicated weakness in collusive agreements. Three experiments were conducted with the PC treatment. As expected, in all three experiments, the bidders formulated an agreement on how to collude when discussion was not allowed. However, bidders were quick to deviate from their ex ante agreements. In the first period of no communication, one bidder deviated in every experiment (see Figure 3). The number of deviations typically increased and most bidders began to bid more aggressively. In one experiment, by the last period four of the five bidders placed bids roughly in line with competitive bidding. However, in the other two experiments, a few bidders were typically able to take advantage of the optimistic behavior of the other bidders. All in all, 12 of 15 bidders placed bids that were significantly different than the ex ante agreement reached by the group. Bidder surplus as a percentage of maximum total surplus dropped from 87.88% in the communication periods to 80.64% in the no communication periods.[28]

[26]The graphs of bidders' surplus in Appendix C demonstrate the consistency of the cooperative agreements.

[27]That bidder placed a bid out of line with the collusive agreement in only one market.

[28]The null hypothesis that the mean surplus from the communication periods is less than or equal to the mean surplus with communication can be rejected at a 90% level of confidence by a rank sum test

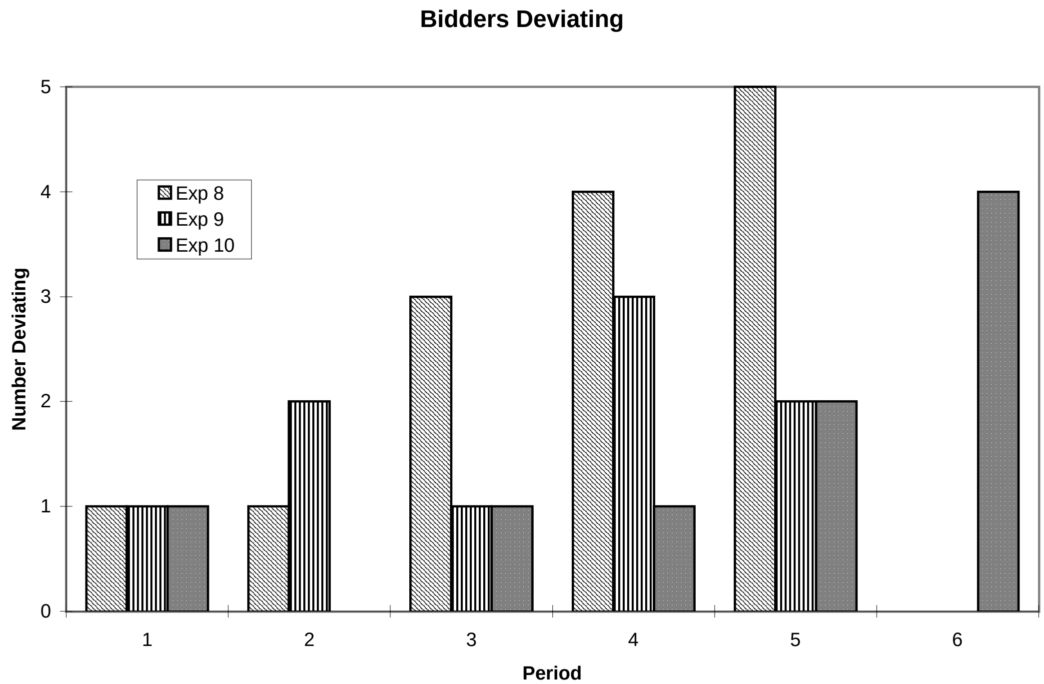


Figure 3: Bidders Deviating when No Communication was allowed

2 Conclusion *Communication is more important than repeated play in fostering successful collusive agreements.*

These results indicate that one of the most important features of such collusive agreements is the ability to discuss the outcomes and make after plans every period. A possible explanation is the need for the cartel to coordinate punishment strategies at the end of each period.

5.2 What types of strategies do bidders utilize?

Closer examination of the periods in which communication was allowed reveals heterogeniety in the choice of cooperative strategies between some experimental sessions. Two distinct strategies can be discerned from the data and observation of preplay communication. The first, and most common strategy, was the utilization of bid rotation. Bid rotation strategies can be characterized by the selection of one bidder as the sole bidder

($z = 1.317$).

in each auction. This bidder placed a low bid greater than 1 franc (typically 2–5 francs) while all other bidders bid 1 franc in the auction. A second strategy observed in the data was a reduced bidding agreement. This strategy entails the agreement by all bidders to place bids which are linear transformations of their actual valuations. Since bidders are required to submit whole franc bids of at least 1 franc, reduced bidding will, in general, lead to higher average bids than bid rotation.[29]

3 Conclusion *In 7 out of 10 experiments, bidders used a bid rotation strategy. In experiments where bid rotation was not used, bidders used a reduced bidding strategy.*

The easiest method for discerning these two different strategies was observation of preplay communication. In the 7 experiments in which bid rotation was used, bidders attempted to reach some resolution of who would bid in each market. However, in the 3 bid reduction schemes, bidders determined a level of bidding. The difference between these experiments can also be seen in the level of bids placed. In the 7 rotation experiments, the average bid placed was 2.8 francs. In the reduced bidding experiments, the average bid was 23 francs.

5.2.1 Reduced Bidding

In two of the reduced bidding experiments, the cartel agreed to place bids that were 1% of redemption values.[30] In the other, bidders agreed to place bids that were 10% of valuations. Such agreements violate individual incentive compatibility (Section 3). Only an agreement to bid 80% of valuations is incentive compatible. Since their values are not ex post verifiable, bidders can increase their bids beyond either the 1% or 10% level without detection, and increase their probability of winning the object. Therefore, bidders would be expected to bid higher than their particular reduced bidding agreement dictates. Figure 4 shows the deviations from the agreed upon strategy. A deviation of zero indicates that the bidder placed his bid at the whole number nearest either 1% (for experiments 1 and 4) or 10% (for experiment 3) of his value. In all three experiments the null hypothesis that the mean deviation is equal to zero can be rejected at the 95% confidence level. Surprisingly, however, in two of the experiments, mean deviations are significantly below zero implying bidders were actually bidding below the agreement. Only in one experiment were deviations significantly above zero (See Table 2).

The fact that these reduced bidding agreements are replicated and appear to be relatively stable creates problems for the theory. Why did bidders not shade their bids up in two experiments? Bidders seem to ignore individual incentives, despite the fact that detection of placing higher bids is very difficult.

[29]Bid rotation strategies lead to average bids that are close to 1 franc since all bidders except one bid 1 franc.

[30]Bidders in experiment 4 quickly switched from a 10% rule to a 1% rule after two periods.

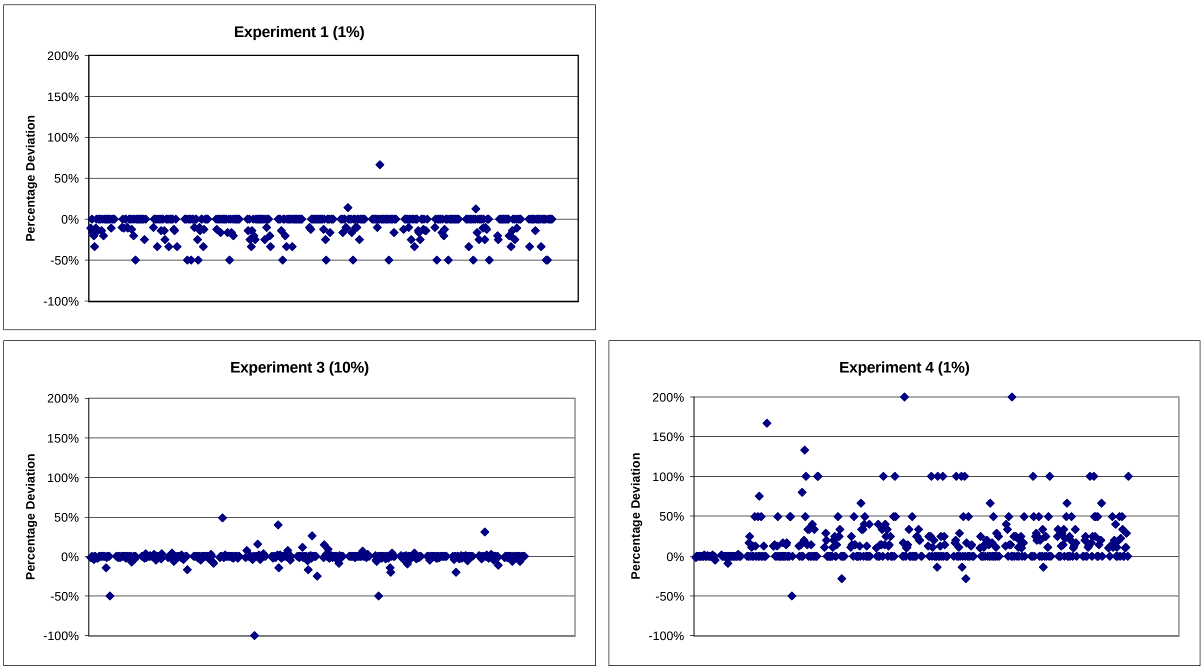


Figure 4: Deviations from Reduced Bidding Agreements

	Experiment		
	1	3	4
Mean Deviation	-0.06481	-0.00774	0.167219
Std. Error	0.131955	0.076929	0.286164
Observations	375	425	425

Table 2: Mean Deviations from Reduced Bidding Agreement

5.2.2 Bid Rotation

The majority of the experiment sessions (7 out of 10) lead to bidding strategies that were classified as bid rotation agreements. There are many different mechanisms which are incentive compatible and look like bid rotation outcomes. The choice of mechanism by the group has significant implications for efficiency and thus the percentage of maximum total surplus captured by the bidders. Four behavioral strategies which can lead to outcomes similar to those observed in these seven experiments are:

1. Ranking mechanism (R),
2. Serial dictator mechanism (SD),
3. Random assignment mechanism (A), and
4. Perfect information (P).

The ranking (R), serial dictator (SD), and random assignment (A) mechanisms were discussed in Section 3. The perfect information (P) strategy describes the possibility that bidders may perfectly collude by somehow determining the bidder with the highest valuation in each market.[31] The objective is to determine which of these possible mechanisms was most likely utilized in each of these experiments. Three techniques that shed light on the choice of a strategy by bidders are:

1. Observation of preplay discussion,
2. Comparison of expected efficiencies with observed efficiencies, and
3. Comparison of predicted market division with observed choices.

- **Discussion**

While observing bidder discussion is not a rigorous test for the predominance of one model over the other, simply listening to the conversations of the bidders can provide a great deal of insight into the intentions of the bidders. Bidder discussion was typically closer to the ranking mechanism than to the serial dictator mechanism. In most cases, bidders would begin their discussion by naming what they wanted first (their highest rank). If there was no conflict, discussion ended. If there was disagreement, those who had chosen conflicting markets would attempt to reach a compromise by naming their next best market. It is easy to see that such an iterative procedure leads to outcomes predicted by the ranking mechanism under the restriction that no bidder be chosen more than once. If the group discussion was consistent with the serial dictator mechanism, once a bidder had named a market in which he wished to bid, no other bidder could pick that market. Typically, conversation between bidders did not take this form.[32]

[31] This is a highly unexpected outcome given the limitations on bidder communications. However, it is still possible that this may be the *best* predictor of group behavior.

[32] However, it is possible that there may have been some first-mover advantage; the bidder who made his announcement of preferred markets first got his favored market more often. Since the order of discussion was not recorded, this factor cannot be analyzed for these experiments.

Behavioral Strategy	**Predicted Efficiency** Symmetry	Asymmetry
R	90.60%	92.12%
SD	85.20%	86.63%
A	60.00%	80.00%
P	100.0%	100.0%

Table 3: Predicted Efficiencies

	Experiment 2	5	6	7	8	9	10
Mean Efficiency	90.52%	92.70%	90.48%	92.77%	87.94%	89.94%	90.69%
Std. Error	0.0466	0.0545	0.0985	0.0755	0.1100	0.0729	0.0841
Observations	17	15	15	15	13	13	12

Table 4: Mean Efficiencies – Rotation

- **Efficiencies**

An auction is efficient if the winner of each object is the bidder with the highest valuation. Efficiency is denoted by

$$\text{Efficiency} = \frac{\sum_{j=1}^{5} v_j^*}{\sum_{j=1}^{5} \max_i v_{ij}}.$$

The predicted efficiencies for each of the behavioral strategies in this particular setting are given in Table 3. If a group is utilizing a particular mechanism, the average of the observed efficiencies should converge to the above efficiencies. The null hypothesis that the mean efficiency for each experiment was different than 90.60%, for the symmetric environment, and 92.12%, for the asymmetric environment, predicted by the ranking mechanism cannot be rejected at a 95% level of confidence in any of the seven experiments (See Table 4). However, in five of the seven experiments, the null hypothesis that the mean efficiency was equal to that predicted by the serial dictator mechanism (85.20% and 86.63%) can be rejected at a 95% level of confidence.[33] The observed efficiencies are also significantly different from the 60% and 80% predicted by an assignment mechanism. The perfect information model can also be rejected under this test in all seven experiments. A simple comparison of observed results seems to strongly favor the ranking mechanism as the best determinant of behavior in each of the seven experiments.

- **Comparing the Choices**

An analysis of bidder discussion and efficiencies provides some support for the ranking mechanism. However, analysis of discussion is purely ad hoc and relies upon the judgement of the experimenter who observed the experimental session. Comparison of mean observations utilizes outcomes rather than choices.

[33]Comparison of the mean bidder surplus yields similar results since bids placed are close to zero.

A more rigorous test involves comparing the choices of the bidders to the choices predicted by each model. Initial examination of choices in each particular experiment indicates that the ranking mechanism is a good predictor of choices; 87% of all observed choices are consistent with the ranking mechanism. However, other mechanisms also correlate well with the observed choices. The likelihood-based classification procedure of El-Gamal and Grether (1995) provides a more rigorous statistical comparison of all the proposed models. Let $C_t = \{(c_1, c_2, c_3, c_4, c_5) |\, c_i \in \mathbb{Z}, 1 \leq c_i \leq 5,\; i = 1, 2, 3, 4, 5\}$ be the class of behavior rules for each period such that each bidder is selected as the sole bidder in a particular market. For example, $c_1 = 2$ indicates that bidder 2 was selected as the sole bidder in market A. Each model predicts a subset $B_t \subset C_t$ and $B = B_1 \times B_2 \times \cdots \times B_{p_s}$ where p_s is the number of periods completed in an experiment. Each experimental session is treated as a single subject, s, and it is assumed that each s chooses exactly one behavioral strategy. The error probability, ϵ, is assumed to be the same for all individuals, experimental sessions and choices. The choice by individual i in period t for a particular experimental session is denoted by a_{ti}. Then, for all B, let

$$x^s_{B,ti} = \begin{cases} 1 & a_{ti} \in B_t \\ 0 & \text{otherwise.} \end{cases}$$

Let

$$X^s_B = \sum_{i=1}^{n} \sum_{t=1}^{p_s} x^s_{B,ti}$$

be the total number of choices predicted correctly for a particular session. The likelihood can be found to be

$$f^{B,\epsilon}(x^s) = \left(1 - \frac{\epsilon}{3}\right)^{X^s_B} \left(\frac{2\epsilon}{3}\right)^{p_s n - X^s_B}$$

for each behavioral strategy.[34] Under the assumption that participants in all S experiments are using the same mechanism, the maximum likelihood estimate is given by

$$(\hat{B}, \hat{\epsilon}) = \arg\max_{B,\epsilon} \prod_{s=1}^{S} f^{B,\epsilon}(x^s).$$

The algorithm suggested by El-Gamal and Grether (1995) is used to obtain the maximum likelihood estimate for any set of k behavioral strategies. Then, using a penalty function given by

$$g(k) = k \ln(4) + k \ln(3) + S \ln(k),$$

k is chosen to maximize the information criterion,

$$IC(k) = \ln \left(\prod_{s=1}^{S} \max_{h \in \{1,\ldots,k\}} f^{\hat{B},\hat{\epsilon}}(x^s) \right) - g(k).$$

[34] It is assumed here that, if a bidder made an error, he chose the correct strategy with probability one third and another strategy with probability two thirds. In reality, a bidder could have a choice of between 5 (if he happens to be choosing first or if there is little conflict) to 1 (if he is choosing last or there is a great deal of conflict) markets. Since, on average, he will have a choice of three markets, $(\frac{1}{3}, \frac{2}{3})$ is selected as an approximation.

	Experiment						
	2	**5**	**6**	**7**	**8**	**9**	**10**
R	85.88%	90.67%	85.33%	93.33%	78.46%	89.23%	85.00%
SD	84.71%	65.33%	62.67%	64.00%	86.15%	86.15%	86.67%
A	29.41%	49.33%	40.00%	42.67%	33.85%	23.08%	30.00%
P	56.47%	58.67%	61.33%	60.00%	50.77%	53.85%	60.00%

Table 5: Percentage of Choices Explained by Models – Individual Experiments

Using this technique, I can test the ability of the four possible mechanisms to explain the observed choices by each experimental session. The choices of the ranking mechanism (R) are easily characterized by saying that an error was made in a particular market if the bidder chosen was not the individual with the highest rank in that market. Unfortunately, the serial dictator mechanism (SD) cannot be characterized as easily. Each possible permutation of the five bidders can potentially lead to a different choice of market assignment predicted by the mechanism. Almost any observed choice can be predicted by the mechanism. For any particular experiment the number of possible combinations of choices across periods is 120^{p_s} (which is 8.92×10^{24} in the experiment with the fewest periods). The choices predicted by the serial dictator mechanism are limited to a smaller set. It is assumed that each experimental group agrees to rotate the order of selection in each period. Thus, if the order of choosing was 1,2,3,4,5 in period t then it would be 2,3,4,5,1 in period $t+1$. This limits the number of combinations predicted by the serial dictator mechanism to a more manageable 120 combinations. While limiting the serial dictator mechanism in this manner makes it less likely that it will be classified as the best fitting model, it is reasonable to assume that no individual bidder would approve of any combination that did not evenly spread out the right to pick early since early picking leads to higher individual surplus. The assignment mechanism (A) assumes that each bidder is selected as the sole bidder in his favored market when distributions are not symmetric. Thus, bidder 1 is assumed to always be the sole bidder in market A, bidder 2 in B, bidder 3 in C, bidder 4 in D, and bidder 5 in E. Finally, the perfect information model (P) represents the choices that would be made if the bidders were able to actually aggregate their information perfectly. The bidder with the highest value is picked in each market.

Table 5 presents the data for each experiment. In all experiments, the ranking and serial dictator mechanisms better explain the data than either assignment or perfect information. Table 6 reports the results of the maximization of the information criterion to determine the optimal number of rules to choose. Using two rules best explains the choices observed in the seven experiments. In experiments 2, 5, 6, 7 and 9, the ranking mechanism is the behavioral strategy that best fits the experimental data. However, the serial dictator mechanism significantly adds to the explanatory power of the model in experiments 8 and 10. Using this classification procedure, it is possible to rule out the random assignment model of collusive behavior. Also, bidders were apparently unable to perfectly aggregate information. However, the serial dictator mechanism can not be eliminated.

No. of Models	Rule(s) Chosen	No. Classified	$\hat{\epsilon}$	$g(k)$	IC
1	R	435	0.39	2.485	-203.101
2	**R,SD**	**333,108**	**0.354**	**9.822**	**-197.893**
3	R,SD,*	333,108,0	0.354	15.145	-203.216
4	R,SD,*,*	333,108,0,0	0.354	19.644	-207.715

Table 6: Estimated Models

		Information	
		Limited	**Zero**
Values	**Symmetric**	3 Reduced 1 Rotation	2 Rotation
	Asymmetric	2 Rotation	2 Rotation

Table 7: The Effect of Treatments

The combination of these three methods of determining which bid rotation scheme was used gives strong evidence in favor of the ranking mechanism. The serial dictator mechanism, however, still appears to be a strategy which is used occasionally by groups in this setting, especially in experiment 8, in which both the observed efficiency and the choices of markets correlate well with the serial dictator mechanism.

4 Conclusion *Reduced bidding mechanisms are only observed under the limited information and symmetric environments.*

All three instances of utilization of reduced bidding strategies were in experiments in which bidders had uniform valuation draws in all five markets and were informed of the identity of the winning bidders (Table 7). While Isaac and Walker (1985) found no significant patterns between collusive agreements and their two information conditions of full information and limited information,[35] this result demonstrates that information matters. While it may not be significant in determining whether bidders collude, it does alter their choice of strategy. Bidders seem to be less willing to select a strategy which violates incentive constraints when they have less ex post information. Second, the switch to a less cooperative strategy in the asymmetric environment has some precedence. Isaac and Walker (1988) found that asymmetries in public goods experiments tended to decrease the level of voluntary contributions. While a complete breakdown of cooperation is never evident here, this result suggests that bidders' choice of strategies is affected by the environment.

[35] The full information condition was a less restrictive environment which reported all the bids placed in the auction.

5.3 What effect do different strategies have on the outcome of the auction?

The choice of cooperative strategies can drastically effect the results of the auction. The differences between mechanisms can best be seen by examining the efficiency of the auction and the amount of surplus accruing to the bidders.

5.3.1 Efficiency

Despite the apparent problems with enforceability, reduced bidding agreements have advantages from a social welfare standpoint. In the three experiments which exhibited these collusive agreements, average efficiencies were 99.26%, 99.38%, and 98.36%. A rank sum test shows that the mean efficiency for these experiments is significantly different than the mean efficiency of experiments in which bidders used rotation schemes.[36]

5 Conclusion *Reduced bidding yields higher average efficiency than bid rotation.*

This result is due to the ability of reduced bidding to select the highest bidder (assuming people do not deviate from the agreement). Figure 5 shows the efficiencies for the experiments in which reduced bidding was observed and the efficiencies for the experiments in which bid rotation was observed. Efficiency is also fairly stable under the reduced bidding agreements. When bidders are using rotation schemes, efficiency varies significantly due to the imprecision of the ranks. However, the reduced bidding agreement consistently yields efficiencies near 100%. The variance of the observed efficiencies for the seven non-reduced bidding experiments was always higher than the variance for the three reduced bidding experiments.

5.3.2 Bidder Surplus

The overall level of profitability for the bidders is best described by the index of monopoly effectiveness which reports the proportion of total possible surplus captured by the bidders. In experiments in which bidders used rotation schemes or the 10% reduced bidding agreement, the average M was 0.898, whereas the two 1% reduced bidding experiments yielded an average effectiveness of 0.970.

6 Conclusion *The index of monopoly effectiveness is highest for the bidders under the 1% reduced bidding rule.*

The 1% reduced bidding agreement was the most successful (profitable) collusive agreement. This result highlights the apparent trade-offs between these strategies. If bidders

[36] The Wilcoxon-Mann-Whitney test with correction for ties yielded $z = 8.267$, which is greater than any reasonable critical value of the standard normal distribution.

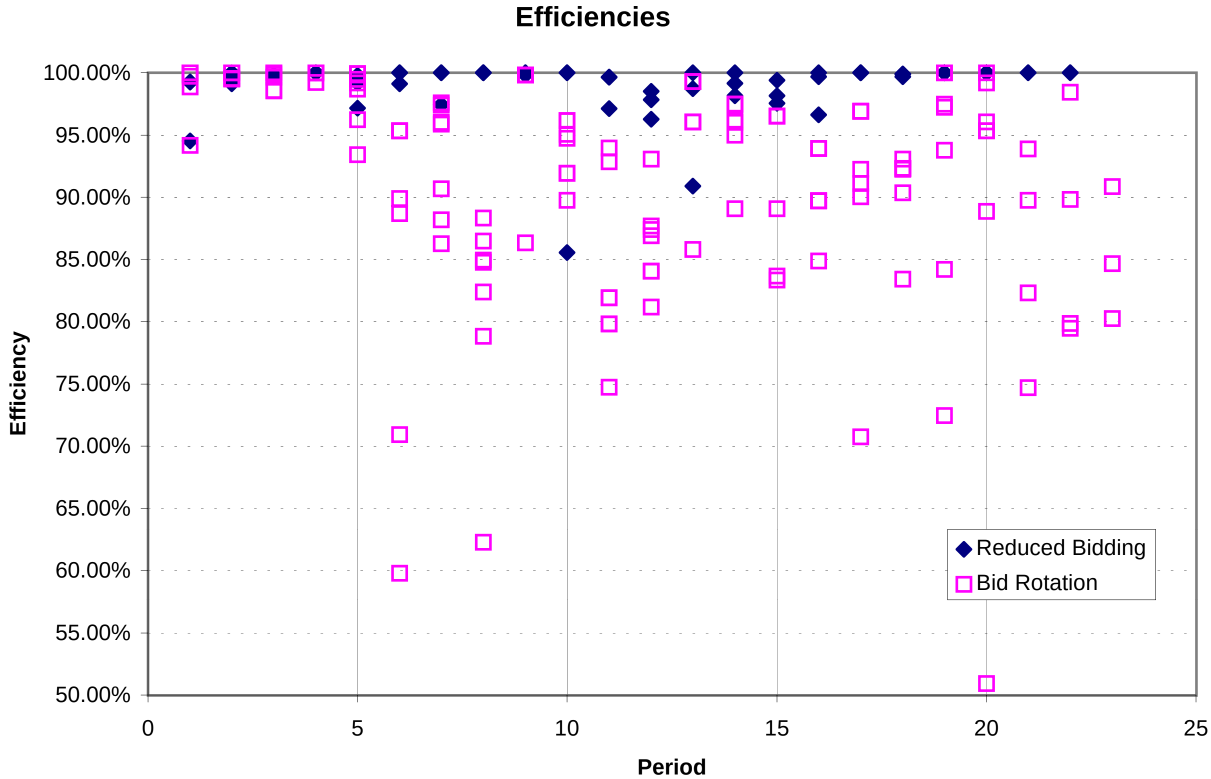


Figure 5: Reduced Bidding v. Rotation Efficiencies

do not lie about their values, reduced bidding yields a much higher efficiency than rotation schemes. This increase in the size of the available surplus more than accounts for the increased level of bids required by a 1% agreement. The 10% agreement, on the other hand, entails too high a level of bidding to actually increase profitability over rotation schemes.

These two conclusions are the best argument in favor of a reduced bidding mechanism. Bidders select reduced bidding because it is more profitable than rotation mechanisms, despite the fact that it is not consistent with individual incentives. It appears that something in the nature of communication in the group decision making process allowed the bidders to ignore this problem.

6 Conclusion

The two primary contributions of this paper are:

1. A description of possible collusive mechanisms when the number of objects is greater than 1,
2. An analysis of experimental data to identify the strategies chosen by bidders.

While incentive compatibility constraints severely limit the set of possible cooperative strategies in single unit auctions, there are many more sophisticated and profitable possible mechanisms when multiple objects are being auctioned simultaneously. Rotation schemes take advantage of bidders' willingness to trade-off probability of winning in lesser valued markets in return for an increased probability in higher valued markets. Of all rotation schemes that use only ordinal information, the ranking mechanism is interim incentive efficient and provides outcomes strictly preferred to those that are possible when only one item is being auctioned (random assignment).

Previous experimental investigations of cooperation in a wide variety of settings (auctions, markets, public goods, prisoners' dilemma) have almost solely focused on the formation of cooperative agreements. It has been well established that, in environments similar to the auction environment discussed here, experimental subjects will agree to cooperate. In this paper, I examine the choice of cooperative strategies. Bidders can choose from a variety of strategies (including noncooperative behavior) that vary significantly in their complexity, profitability, and adherence to incentive constraints. Subjects exhibit behavior which is often consistent with strategies predicted by theory.

However, deviations in three of the experimental sessions from the choices predicted by theory suggest that a better theory of the cooperative choice of a decision rule needs to be formulated. In these experiments, bidders used a strategy that is not incentive compatible, but leads to higher profits when bidders do not lier about their values. The theory developed here assumes that bidders do not voluntarily communicate and that any information that is used must be consistent with their incentives. However, if, a priori, bidders could agree to credibly reveal their information, then reduced bidding agreements become possible. A complete theory of the choice of strategies when bidders are asymmetrically informed will treat the level of communication as an additional choice variable. Wilson (1978) proposes versions of interim efficiency that assume different levels of information sharing (coarse and fine). Interim efficiency is a very weak standard on the strategies chosen. Potentially, there are many interim efficient mechanisms. Since the behavior being modeled is explicitly cooperative, a more cooperative solution concept is in order. In many domains that concept is the core. However, finding core allocations in this setting is more difficult. For example, the feasible set of strategies for each coalition depends upon the actions of those outside the coalition, and the question of information sharing within coalitions becomes relevant. Myerson (1984) provides some initial insights by defining threat points as minimal levels of expected utility that each coalition must receive.

The inclusion of durability may also lead to a more satisfying theory. If a mechanism is not durable, then there will be instances in which bidders will reject it in favor of another mechanism. This behavior might be observable experimentally. Is there evidence of a move away from one mechanism based upon the values drawn? In terms of collusion, durability might even predict when collusion breaks down. Unfortunately, in this experimental design, the random assignment, serial dictator, and ranking mechanisms dominate the noncooperative mechanism (due to the choice of distributions).

Finally, the auctioneer was assumed to be completely passive. In reality, the auctioneer can take steps to combat collusive behavior. Graham and Marshall (1987) highlight some techniques that the auctioneer may use in an English auction. In sealed bid auctions, the use of a reserve price becomes even more important for the auctioneer to earn revenue. Collusive strategies are also easily identifiable by a lack of bidding. If the auctioneer can punish collusive behavior, then bidders may need to formulate agreements that are less obvious. A full understanding of collusion in auctions requires an analysis of the steps an auctioneer can take to combat collusion.

Appendix A Lemmas and Proofs

A.1 Lemma *Let* $(Q,T) = \{(Q_i,T_i)\}_{i=1}^n$ *and* $(Q',T') = \{(Q'_i,T'_i)\}_{i=1}^n$ *be two generic, feasible, IC mechanisms. Then for all* $\alpha \in [0,1]$, $(\alpha Q + (1-\alpha)Q', \alpha T + (1-\alpha)T')$ *is also a feasible, IC mechanism.*

Proof: Since (Q,T) and (Q',T') are IC

$$\sum_{j=1}^m Q_{ij}(v_i)v_{ij} - T_{ij}(v_i) \geq \sum_{j=1}^m Q_{ij}(\overline{v}_i)v_{ij} - T_{ij}(\overline{v}_i)\ \forall \overline{v}\ \forall i \tag{9}$$

$$\sum_{j=1}^m Q'_{ij}(v_i)v_{ij} - T'_{ij}(v_i) \geq \sum_{j=1}^m Q'_{ij}(\overline{v}_i)v_{ij} - T'_{ij}(\overline{v}_i)\ \forall \overline{v}\ \forall i \tag{10}$$

which implies that

$$\begin{aligned}\alpha\left(\sum_{j=1}^m Q_{ij}(v_i)v_{ij} - T_{ij}(v_i)\right) + (1-\alpha)\left(\sum_{j=1}^m Q'_{ij}(v_i)v_{ij} - T'_{ij}(v_i)\right) \geq \\ \alpha\left(\sum_{j=1}^m Q_{ij}(\overline{v}_i)\overline{v}_{ij} - T_{ij}(\overline{v}_i)\right) + (1-\alpha)\left(\sum_{j=1}^m Q'_{ij}(\overline{v}_i)\overline{v}_{ij} - T'_{ij}(\overline{v}_i)\right)\end{aligned} \tag{11}$$

Rearranging and bringing the α inside the sum yields the desired result

$$\begin{aligned}\sum_{j=1}^m(\alpha Q_{ij}(v_i) + (1-\alpha)Q'_{ij}(v_i))v_{ij} - (\alpha T_{ij}(v_i) + (1-\alpha)T'_{ij}(v_i)) \geq \\ \sum_{j=1}^m(\alpha Q_{ij}(\overline{v}_i) + (1-\alpha)Q'_{ij}(\overline{v}_i))\overline{v}_{ij}) - (\alpha T_{ij}(\overline{v}_i) + (1-\alpha)T'_{ij}(\overline{v}_i))\end{aligned} \tag{12}$$

Feasibility follows by simply allowing each agent to report their types v_i and using a public randomization device to choose (Q,T) with probability α and (Q',T') with probability $(1-\alpha)$. ∎

A.2 Lemma *If* $F_{ij} = F_{kj}$ *for all* i,j,k *and* $(Q,T) = \{(Q_i,T_i)\}_{i=1}^n$ *is feasible and*

$$\omega = \sum_{i=1}^n \frac{1}{n}\int(\lambda(v_i)(Q_i(v_i)v_i - T_i(v_i)))dF_i(v_i)$$

where λ *is a social welfare weight on types,* Q_i *and* T_i *are* $j \times 1$ *vectors and* F_i *is the joint distribution of the* j *values of each agent. Then* $\exists(\hat{Q},\hat{T})$ *such that* $\hat{Q}_i = \hat{Q}_k$ *and* $\hat{T}_i = \hat{T}_k$ *for all* i,k *and*

$$\int(\lambda(v)\hat{Q}(v)v - \hat{T}(v))dF(v) = \omega$$

Also, if (Q,T) *is IC then* $(\hat{Q},\hat{T})$ *is IC as well.*

Proof: Since values for all individuals are drawn from identical distributions and utilities are of an identical form, if (Q,T) is feasible and IC then for all $\sigma : \{1,\ldots,n\} \to \{1,\ldots,n\}$ one-to-one (permutations) $\{Q_{\sigma(i)}, T_{\sigma(i)}\}_{i=1}^n$ is also feasible and IC. By Lemma A.1, every mechanism in the convex hull of all permutations of (Q,T) is feasible and IC. Let $(\hat{Q},\hat{T})$ be the mechanism created by the convex combination of all $n!$ permutations of (Q,T) equally weighted by $\frac{1}{n!}$. Thus, since each (Q_i,T_i) appears exactly $(n-1)!$ times, $\hat{Q} = \frac{1}{n}\sum_{i=1}^n Q_i$ and $\hat{T} = \frac{1}{n}\sum_{i=1}^n T_i$. Thus, given λ, we have that

$$\begin{aligned}
\int (\lambda(v)(\hat{Q}(v)v - \hat{T}(v)))dF(v) &= \int (\lambda(v)(\frac{1}{n}\sum_{i=1}^n Q_i(v)v - \frac{1}{n}\sum_{i=1}^n T_i(v)))dF(v) \\
&= \sum_{i=1}^n \frac{1}{n}\int (\lambda(v)(Q_i(v)v - T_i(v)))dF_i(v) \\
&= \omega \qquad (13)
\end{aligned}$$

Thus, $(\hat{Q},\hat{T})$ is symmetric, feasible and IC and leads to the same ex ante social value. ∎

proof of Lemma 3.5: Let the Reduced Bidding mechanism be IC. Then given the first order conditions for maximization of each agent's expected utility, it must be that

$$g_{ij}(v_{ij})v_{ij}(1-\alpha_j) - G_{ij}(v_{ij})\alpha_j = 0 \qquad (14)$$

$$\Gamma_{ij}(v_{ij}) = v_{ij}\frac{(1-\alpha_j)}{\alpha_j} \qquad (15)$$

Since this must be true for all $v_{ij} \in [\underline{v}, \overline{v}]$, differentiating with respect to v_{ij} yields

$$\frac{d\Gamma_{ij}(v_{ij})}{dv_{ij}} = \frac{1-\alpha_j}{\alpha_j} = c_j$$

Obviously, the other direction can be trivially shown to hold by setting $\alpha_j = \frac{1}{\frac{d\Gamma_{ij}(v_{ij})}{dv_{ij}}+1}$. ∎

Proof of Theorem 3.6: Let (β, s) be a reduced bidding mechanism satisfying incentive compatibility (and thus the conditions of Lemma 3.5). Suppose that $b_{ij}(v_{ij}) = \alpha_j v_{ij}$ is not a Bayes Nash equilibrium. In the noncooperative setting the first order conditions for maximization are given by Equation **??**. If $b_{ij}(v_{ij}) = \alpha_j v_{ij}$ is not an equilibrium then it must be that

$$\frac{1}{v_{ij} - \alpha_j v_{ij}} \neq \sum_{k \neq i} H_{kj}(v_{ij})\left(\frac{1}{\alpha_j}\right)$$

for some i and j where

$$H_{kj}(v) = \frac{f_{kj}(v)}{F_{kj}(v)}$$

which implies that

$$\alpha_j \neq v_{ij}(1-\alpha_j)\sum_{k \neq i} H_{kj}(v_{ij})$$

Note that

$$g_{ij}(v_{ij}) = G_{ij}(v_{ij}) \sum_{k \neq i} H_{kj}(v_{ij})$$

Thus, multiplying by $G_{ij}(v_{ij}) > 0$ leads to

$$G_{ij}(v_{ij})\alpha_j \neq v_{ij}(1 - \alpha_j)g_{ij}(v_{ij})$$

which is a contradiction with the first order conditions for IC given by Equation 14. Thus, if the reduced bidding mechanism is IC it is also the outcome of competitive behavior. ∎

Proof of Theorem 3.9: Let (β, s) be an incentive compatible mechanism. Define

$$Q_{ij}(v_i) = G_{ij}(B_{ij}(v_i))$$

and

$$T_{ij}(v_i) = G_{ij}(B_{ij}(v_i))(B_{ij}(v_i) - \underline{v}) + S_{ij}(v_i).$$

Then,

$$\sum_{j=1}^{m} Q_{ij}(v_i)(v_{ij} - \underline{v}) - T_{ij}(v_i) \equiv \sum_{j=1}^{m} Q_{ij}(B_{ij}(v_i))(v_{ij} - B_{ij}(v_i)) - S_{ij}(v_i) \; \forall v_i \; \forall i.$$

Since, (β, s) satisfies that necessary and sufficient conditions for IC then so to must (q, t).

To show the second part assume that $S(v) = \sum_{i=1}^{n} \sum_{j=1}^{m} S_{ij}(v_i)$ and note that $T(v) = \sum_{i=1}^{n} \sum_{j=1}^{m} G_{ij}(B_{ij}(v_i))(B_{ij}(v_i) - \underline{v}) + S_{ij}(v_i)$. Thus, since $G_{ij}(B_{ij}(v_i))$ is a probability and $(B_{ij}(v_i) - \underline{v}) \geq 0$ ($\underline{v}$ is the lower bound of the range of $B_{ij}(v_i)$), it must be that $T(v) \geq S(v)$ for all v. Thus, we can define a function $c_{ij}(v_{-i})$ such that $c_{ij}(v_{-i}) \geq 0$ and $\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}(v_{-i}) = T(v) - S(v)$. Let

$$c_{ij}(v_{-i}) = \frac{\sum_{k \neq i}^{m} G_{kj}(B_{kj}(v_k))(B_{kj}(v_k) - \underline{v})}{n - 1}$$

Then, let

$$\hat{T}_{ij}(v_i) = T_{ij}(v_{ij}) - \int_{\underline{v}}^{\overline{v}} c_{ij}(v_{-i}) dF_{-i}$$

be the new expected tax then $(q, \hat{t})$ is a new mechanism that is still incentive compatible (since the new term is just a constant for any agent) but yields higher expected utility due to the lower expected taxes. ∎

Proof of Theorem 3.17: Suppose q is an ex post efficient Bayesian Mechanism without transfers. Let $v_i = (v_{i1}, v_{i2}, \ldots, v_{im})$ and $\hat{v}_i = (v_{ij}, \hat{v}_{i,-j})$ and agent i is as given above. Incentive Compatibility requires that

$$U_i(\hat{v}_i, \hat{v}_i) \geq U_i(v_i, \hat{v}_i) \text{ and } U_i(v_i, v_i) \geq U_i(\hat{v}_i, v_i).$$

Thus,

$$\sum_{j=1}^{m}\int \hat{v}_{ij}q_{ij}(\hat{v}_i,v_{-i})dF_{-i}(v_{-i}) \geq \sum_{j=1}^{m}\int \hat{v}_{ij}q_{ij}(v_i,v_{-i})dF_{-i}(v_{-i})$$

and

$$\sum_{j=1}^{m}\int v_{ij}q_{ij}(v_i,v_{-i})dF_{-i}(v_{-i}) \geq \sum_{j=1}^{m}\int v_{ij}q_{ij}(\hat{v}_i,v_{-i})dF_{-i}(v_{-i}).$$

Since $q_{ij}(v) = 1$ only if $v_{ij} > v_{kj}$ for all $k \neq i$ and $q_{ij}(v) = 0$ otherwise, from ex post efficiency, I may simplify, so that $\int q_{ij}(v_i,v_{-i})dF_{-i}(v_{-i}) = G_{ij}(v_{ij})$. Thus,

$$\sum_{j=1}^{m}\hat{v}_{ij}G_{ij}(\hat{v}_{ij}) \geq \sum_{j=1}^{m}\hat{v}_{ij}G_{ij}(v_{ij})$$

and

$$\sum_{j=1}^{m}v_{ij}G_{ij}(v_{ij}) \geq \sum_{j=1}^{m}v_{ij}G_{ij}(\hat{v}_{ij})$$

But for all $k \neq j$ $v_{ik} = \hat{v}_{ik}$ implying that $G_{ik}(v_{ik}) = G_{ik}(\hat{v}_{ik})$. This allows me to simplify the expression to

$$\hat{v}_{ij}G_{ij}(\hat{v}_{ij}) \geq \hat{v}_{ij}G_{ij}(v_{ij})$$

and

$$v_{ij}G_{ij}(v_{ij}) \geq v_{ij}G_{ij}(\hat{v}_{ij}).$$

Rearranging terms yields

$$\hat{v}_{ij}[G_{ij}(\hat{v}_{ij}) - G_{ij}(v_{ij})] \geq 0$$

and

$$v_{ij}[G_{ij}(v_{ij}) - G_{ij}(\hat{v}_{ij})] \geq 0.$$

Since both $\hat{v}_{ij}, v_{ij} > 0$, it must be that $G_{ij}(\hat{v}_{ij}) = G_{ij}(v_{ij})$, which is a contradiction. ∎

Proof of Theorem 3.19: Since the probabilities defined by the random assignment mechanism and the optimal bid function are independent for each market. If suffices to show that the result holds for one j. Let $v_{ij} < \overline{v}$ be a value for bidder i in market j. If the noncooperative outcome is preferred then it must be that

$$(v_{ij} - b_{ij}(v_{ij}))F_j(v_{ij})^{n-1} > \frac{1}{n}v_{ij} \tag{16}$$

which implies that

$$v_{ij}F_j(v_{ij})^{n-1} > \frac{1}{n}v_{ij}$$

and

$$nF_j(v_{ij})^{n-1} > 1.$$

Since $F_j(v_{ij}) < 1$, then $\lim_{n\to\infty} nF_j(v_{ij})^{n-1} = 0$. Thus, for some n, random assignment must be preferred at $v_{ij} < \overline{v}$. As $n \to \infty$, the set of v_{ij} such that (16) holds converges to $\overline{v}$ a set of measure zero. ∎

In order to prove Proposition 3.22, we need the following lemma.

A.3 Lemma *Let* α, $\alpha' \in \mathbb{R}^n_+$ *and* $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n \alpha'_i = 1$. *If there exists a* k *such that for all* $j \leq k$, $\alpha_j \geq \alpha'_j$ *and for all* $j > k$, $\alpha_j \leq \alpha'_j$, *then for all* $x \in \mathbb{R}^n$ *such that* $x_1 \leq x_2 \leq \cdots \leq x_n$, $\alpha \cdot x \geq \alpha' \cdot x$.

Proof: Assume $\exists x \ni \alpha \cdot x < \alpha' \cdot x$. Then it must be that

$$(\alpha_1 - \alpha'_1)x_1 + (\alpha_2 - \alpha'_2)x_2 + \cdots + (\alpha_n - \alpha'_n)x_n < 0$$

Since for all $j \leq k$, $(\alpha_j - \alpha'_j) \geq 0$ and for all $j > k$, $(\alpha_j - \alpha'_j) \leq 0$

$$\left(\sum_{j \leq k} \alpha_j - \sum_{j \leq k} \alpha'_j\right) x_k + \left(\sum_{j > k} \alpha_j - \sum_{j > k} \alpha'_j\right) x_{k+1} < 0$$

Since α and α' both sum to 1, we can simplify to get

$$\left(\sum_{j \leq k} \alpha_j - \sum_{j \leq k} \alpha'_j\right) x_k < \left(\sum_{j \leq k} \alpha_j - \sum_{j \leq k} \alpha'_j\right) x_{k+1}$$

which implies the contradiction that $x_k < x_{k+1}$. ∎

Proof of Proposition 3.22: Let Q^{SD}_{ij} be the reduced form probabilities given by the serial dictator mechanism, and let $Q^A_{ij} = \frac{1}{n}$. Note that $\sum_{j=1}^m Q^{SD}_{ij} = \sum_{j=1}^m Q^A_{ij} = \frac{m}{n}$. Also, note that Q^{SD}_{ij} is a decreasing function of each markets ordinal ranking. Thus, w.l.o.g. let v_i be such that $v_{i1} \geq v_{i2} \cdots \geq v_{im}$. Note that $Q^{SD}_{i1} > Q^A_{i1} = \frac{1}{n}$ since with probability $\frac{1}{n}$, i gets to choose first, however, there is also a positive probability that i chooses at some other point but market 1 is still available. This is enough to apply Lemma A.3 (multiply the whole equation by $\frac{n}{m}$ in order to get a linear combination). Thus, it must be that for all v_i, $v_i \cdot Q^{SD}_{ij} \geq v_i \cdot Q^A_{ij}$. All that remains to be shown is that $\exists v_i \ni \ v_i \cdot Q^{SD}_{ij} > v_i \cdot Q^A_{ij}$ Let $v_i = (\overline{v}_{i1}, \ldots, \overline{v}_{ik}, \underline{v}_{i(k+1)}, \ldots, \underline{v}_{im})$. This yields our result. ∎

Proof of Proposition 3.24: ($\Rightarrow$) Let an ordinal mechanism be IC and assume that 1 or 2 don't hold. Suppose $\exists l < p$ and a j such that $Q_{ij}(m_{ij} = p) > Q_{ij}(m_{ij} = l)$. Let v_i be such that $v_{i1} > v_{i2} > \cdots > v_{im}$ such that $\#\{v_{ik} | v_{ik} > v_{ij}\} = l - 1$. IC implies that

$$Q_{ij}(m_{i1} = 1)v_{i1} + \cdots Q_{ij}(m_{ij} = l)v_{ij} + \cdots + Q_{ik}(m_{ik} = p)v_{ik} + \cdots + Q_{im}(m_{im} = m)v_{im} \geq \tag{17}$$
$$Q_{ij}(m_{i1} = 1)v_{i1} + \cdots + Q_{ij}(m_{ij} = p)v_{ij} + \cdots + Q_{ik}(m_{ik} = l)v_{ik} + \cdots + Q_{im}(m_{im} = m)v_{im} \tag{18}$$

which implies that

$$Q_{ij}(m_{ij} = l)v_{ij} + Q_{ik}(m_{ik} = p)v_{ik} \geq Q_{ij}(m_{ij} = p)v_{ij} + Q_{ik}(m_{ik} = l)v_{ik} \tag{19}$$
$$v_{ij}(Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p)) \geq v_{ik}(Q_{ik}(m_{ik} = l) - Q_{ik}(m_{ik} = l)) \tag{20}$$

given our assumptions, this implies that $v_{ij} \leq v_{ik}$ which is a contradiction.

Suppose $\exists Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p) \neq Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p)$. W.l.o.g. assume $Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p) > Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p)$. Let $c = \frac{Q_{ij}(m_{ij}=l) - Q_{ij}(m_{ij}=p)}{Q_{ij}(m_{ij}=l) - Q_{ij}(m_{ij}=p)} \geq 1$. Then chose v_i such that $\#\{v_{il} | v_{il} > v_{ij}\} = l - 1$ and $\#\{v_{il} | v_{il} > v_{ik}\} = p - 1$ and $v_{ik} < cv_{ij}$. Then, using the same argument as above, IC implies that

$$\frac{Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p)}{Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p)} \geq \frac{v_{ik}}{v_{ij}} \tag{21}$$

$$> \frac{cv_{ij}}{v_{ij}} \tag{22}$$

$$= c \tag{23}$$

which is a contradiction.

($\Leftarrow$) Suppose 1 and 2 hold but the ordinal mechanism is not IC. Then $\exists j, k \ni v_{ij} > v_{ik}$ and

$$Q_{ij}(m_{i1} = 1)v_{i1} + \cdots Q_{ij}(m_{ij} = l)v_{ij} + \cdots + Q_{ik}(m_{ik} = p)v_{ik} + \cdots + Q_{im}(m_{im} = m)v_{im} < \tag{24}$$

$$Q_{ij}(m_{i1} = 1)v_{i1} + \cdots + Q_{ij}(m_{ij} = p)v_{ij} + \cdots + Q_{ik}(m_{ik} = l)v_{ik} + \cdots + Q_{im}(m_{im} = m)v_{im} \tag{25}$$

which implies that

$$v_{ij}(Q_{ij}(m_{ij} = l) - Q_{ij}(m_{ij} = p)) < v_{ik}(Q_{ik}(m_{ik} = l) - Q_{ik}(m_{ik} = p))$$

given that 1,2 hold it must be that $v_{ij} < v_{ik}$ which is a contradiction. ∎

Before showing the proof that the ranking mechanism is interim incentive efficient, some additional notation is in order. Since the ranking mechanism is assumed to be anonymous, it must be that $q_{ij}(m)$ is a function only of the number of individuals who have ranked each particular market in each spot. Thus, for simplicity let $E_j = \{(n_{1j}, n_{2j}, \ldots, n_{mj}) | \sum_{i=1}^{m} n_{ij} = n\}$ be the set of possible total ranks for a market where n_{ij} indicates that n_{ij} bidders ranked market j in their ith spot. Thus, $E = E_1 \times E_2 \cdots E_m$ is the set of possible events over which q_{ij} may vary. Let $\pi(e)$ be the probability that $e \in E$ occurs.

Proof of Theorem 3.26: Suppose that there exists another ordinal mechanism such that $\sum_{j=1}^{m} Q'_{ij}(v_i)v_{ij} \geq \sum_{j=1}^{m} Q_{ij}(v_i)v_{ij}$ for all i and for all v_i Let v_i be such that $v_{i1} \geq v_{i2} \cdots \geq v_{im}$. Then it must be that

$$Q'_{i1}(1)v_{i1} + Q'_{i2}(2)v_{i2} \ldots + Q'_{im}(m)v_{im} \geq Q(1)v_{i1} + Q(2)v_{i2} \ldots + Q(m)v_{im}$$

which implies that

$$(Q'_{i1}(1) - Q(1))v_{i1} + (Q'_{i2}(2) - Q(2))v_{i2} \ldots + (Q'_{im}(m) - Q(m))v_{im} \geq 0$$

This inequality implies that $\exists k \ni Q'_{ik}(k) \geq Q(k)$. If for all $k \leq m$, $Q'_{ik}(k) = Q(k)$ then for all i the outcome of Q' is identical to Q and they are equivalent mechanisms. On the other hand, let j be the first market such that $Q'_{ij}(j) > Q(j)$ Notice that,

$$Q'_{ij}(j) = \sum_{e \in E(j)} q'_{ij}(e)\pi(e) \tag{26}$$

$$Q(j) = \sum_{e \in E(j)} q_{ij}(e)\pi(e) \tag{27}$$

where $E(j) = \{e \in E | n_{jj} \geq 1\}$. Let $E(j)$ be partitioned into two sets: $E(j)_1 = \{e \in E(j) | q_{ij}(e) = 0\}$ and $E(j)_2 = \{e \in E(j) | q_{ij}(e) > 0\}$. In order for $Q'_{ij}(j) > Q(j)$ it must be that $\exists e \in E \ni q'_{ij}(e) > q_{ij}$. Now, show, by cases, that an increase in q_{ij} for any event in either $E(j)_1$ or $E(j)_2$ will lead to a contradiction.

Case 1: Suppose $\exists e \in E(j)_1 \ni q'_{ij}(e) > q_{ij}(e)$.

Let $c = q'_{ij}(e)$ Then, since $q_{ij}(e) = 0$ it must be that $\exists k < j \ni n_{kj} > 0$ or some other individual ranks the events lower than you. Let $k^* = \min\{k | n_k > 0\}$. Under the ranking mechanism, it must be that $q_{ik^*}(e) = \frac{1}{n_{k^*}}$ and $q_{il}(e) = 0$ otherwise. Thus, since $q_{ij}(e) \geq 0$ for all j and $n_i q_{ik} = 1$ it must be that any change increase in $q'_{ij}(e)$ must come at a reduction in q_{ik}. Thus it must be that $q'_{ik} = \frac{1 - n_j c}{n_k}$. Thus given the choice of j, it must be that

$$\left[\frac{1 - n_j c}{n_k} - \frac{1}{n_k}\right] \pi(e) v_{ik} + [c\pi(e)] v_{ij} \geq 0$$

which is true only if

$$v_{ij} \geq \frac{n_j}{n_k} v_{ik}$$

Notice that both n_j and n_k are greater than zero. This is only true for all $e \in E(j)_1$ if

$$v_{ij} \geq \frac{1}{n-1} v_{ik}$$

Let v_i be such that $v_{i1} = v_{i2} = \ldots = v_{i(j-1)} = \overline{v}$ and $v_{ij} = v_{i(j+1)} = \ldots = v_{im} = \underline{v}$. If q' is preferred to q it must be that $\underline{v} \geq \frac{1}{n-1}\overline{v}$. Since $\overline{v} > (n-1)\underline{v}$, it must be that $\underline{v} > \underline{v}$ which is a contradiction. Thus, Case 1 cannot hold.

Case 2: Suppose $\exists e \in E(j)_2 \ni q'_{ij}(e) > q_{ij}(e)$. If $e \in E(j)_2$, then it must be that $q_{ij}(e) = \frac{1}{n_j}$. Thus, it must be that $q'_{ij}(e) > \frac{1}{n_j}$ which violates feasibility of anonymous mechanisms (since this implies $q'_{kj}(e) \neq q'_{ij}(e)$ for some other individual who ranks the market in spot j. Thus, Case 2 cannot hold.

Thus, there cannot exist a q' such that it improves each agent's interim expected utility for all values. ∎

Appendix B Experiment Instructions

Experiment Instructions

Introduction

You are about to participate in an experiment in the economics of market decision making in which you will earn money based on the decisions you make. All earnings you make are yours to keep and will be paid to you at the end of the experiment. In this experiment you are going to participate in a market in which you will be buying units in a sequence of independent market days or trading periods. You will each receive a sequence of numbers, five for each period, which describe the value to you of any decisions you might make. These numbers may differ among individuals. *You are not to reveal this information to anyone. It is your own private information.* From this point forward, you will be referred to by your bidder number. You are bidder number __ in this experiment. In each trading period you will be able to place bids to purchase a single unit in all of five markets (labeled A-E).

Redemption Values and Earnings

During each market period you are free to purchase a unit in any of the five markets if you want. If you purchase a unit in that market, you will receive the redemption value indicated on your redemption value sheet for that period and that market. Your earnings from a unit purchase, which are yours to keep, are the difference between your redemption value for that unit and the price you paid for the unit. That is:

$$\text{Your earnings} = (\text{redemption value}) - (\text{purchase price})$$

Suppose for example that you buy a unit in market A and that your redemption value is 200 in market A. If you pay 150 for the unit then your earnings are

$$\text{Earnings from unit} = 200 - 150 = 50$$

You can calculate your earnings on your accounting sheet at the end of each period. The currency used in the markets is francs. The conversion rate of francs to dollars will be listed on your redemption value sheets. Your total earnings in any period are given by the sum of your earnings in each market. For example, if you purchased a unit in market A for earnings of 50 and a unit in market B for earnings of 80, then your total earnings that period would be 130 francs. Remember, if you purchase a unit in a particular market, you must use the redemption value from that market.

Market Organization

In each period five markets will be open. There will be 5 participants in each market. In the markets, buyers may submit bids by entering bids into the computer. The bids will be arranged from the highest bid to the lowest. The highest bid in each market will be announced by the computer as the buyer in that market. The identity of the highest bidder will not be announced. The buyer will pay a price equal to the bid and as a result will earn the difference between his/her redemption value for the unit and the highest bid placed. The bids of all other bidders are nullified. They receive no redemption value and pay nothing and so have earnings of zero for that market. If more than one bidder submits an identical high bid in a market, the buyer will be determined randomly (each tied bidder has an equal chance) and the price paid will be equal to their high bid in that market.

Submitting Bids

On your screen you will see a window titled, *Make A Bid*. In this window you select the market you want to bid in by clicking the square beneath an item's letter. When you click on a market the button will appear to be depressed in order to indicate that the market has been selected. Once you mark the desired market, you can enter the amount (in francs) you are willing to bid in the box with a dollar sign. Bids should be in whole francs only. After your order is specified, you can send it to the market by selecting *save*. Each bid you make must have only one market selected. You must place a bid of at least 1 franc in every market. However, you may bid as much as you choose in any period and any market. You will have approximately two minutes in order to submit your bids. The period will end when all bidders have place a bid in each market. You may view your bids by clicking on the *Bids* button in your main window. Once all bidders have submitted their bids, the period will be closed and the results calculated. When the results are available, you may view the bids by clicking on the *Results* button in your main window. Selecting *Show* will display the results.

Determination of Redemption Values

For each buyer the redemption value for each market and each period will be between 1 and 1000. In four of the five markets, each number from 1 to 1000 has equal chance of appearing. It is as if each number between 1 and 1000 is stamped on a single ball and placed in an urn. A draw from the urn determines the redemption value for an individual. The ball is replaced and a second draw determines the redemption value for another player. The redemption values each period are determined the same way. The following is a table in which the probability of getting a value in a certain range is listed: (It is for your reference)

Range of Redemption value	Probability of a value in this range
1-100	10%
1-200	20%
1-300	30%
1-400	40%
1-500	50%
1-600	60%
1-700	70%
1-800	80%
1-900	90%
1-1000	100%

In the fifth market, redemption values are drawn in a different manner. Redemption values close to 1000 have a higher chance of appearing than do those close to 1. It is as if the number 1 is stamped on a single ball, 2 is stamped on 3 balls, 3 is stamped on 5 balls, and so on. For any value n between 1and 1000, the number of balls equals 2n-1. All the balls are placed in an urn.

A draw from the urn determines the redemption value for an individual. The ball is replaced and a second draw determines the redemption value for another player. The redemption values each period are determined the same way. The following is a table in which the probability of getting a value in a certain range is listed: (It is for your reference)

Range of Redemption value	Probability of a value in this range
1-100	1%
1-200	4%
1-300	9%
1-400	16%
1-500	25%
1-600	36%
1-700	49%
1-800	64%
1-900	81%
1-1000	100%

There will be one bidder whose values are drawn from this set of draws in each market. Bidder 1 will receive redemption values drawn in this manner in market A. Likewise, 2 in B, 3 in C, 4 in D, and 5 in E. For each bidder, the redemption values in the four other markets will be given by draws determined as previously described.

Your redemption value sheet may look something like this:

A 520
B 128
C 200
D 750
E 776

This indicates that you would receive a redemption value of 520 in market A if you place the highest bid in that market. Likewise, your value in market B would be 128 and so on. The first period will be practice. You will receive no earnings for this period. If you have a question, please raise your hand and a monitor will come by to answer your question.

To be read after round 5

Communication with Other Participants

Sometimes in previous experiments, participants have found it useful when the opportunity arose, to communicate with one another. You are going to be allowed this opportunity while the computers are reset between periods. There will be some restrictions. You are free to discuss any aspect of the experiment (or the market) that you wish, except that:

- You may not discuss any quantitative aspects of the private information on your value sheets.

- You are not allowed to discuss side payments or to use physical threats.

Since there are still some restrictions on your communications with one another, an experimenter will monitor your discussion between periods. To make this easier, all discussions will be at this site. Remember, after the computers have been reset between periods (and the next period has begun) there will be no discussion until after the end of the next period. We allow a maximum of 4 minutes in any one discussion session.

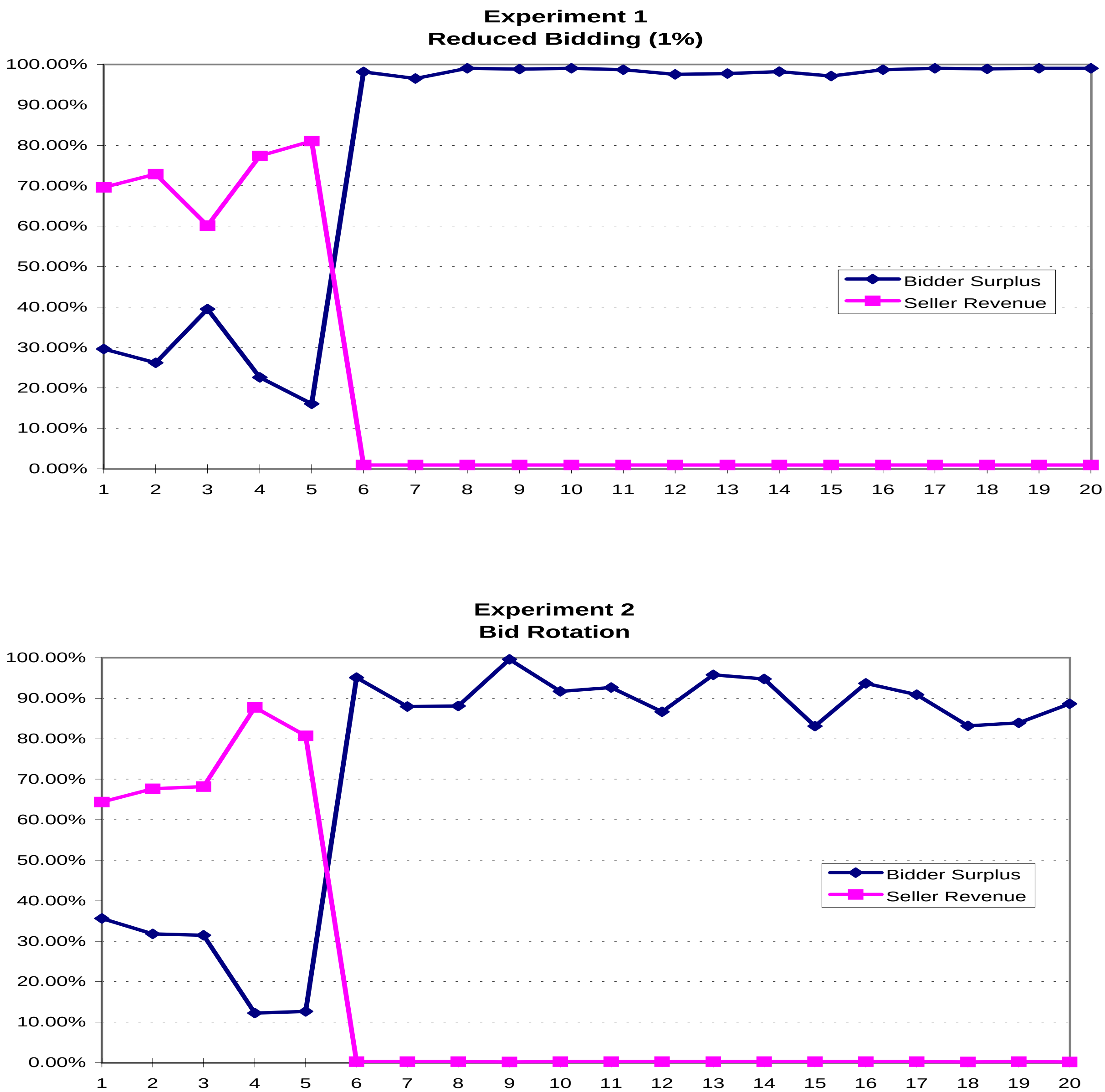
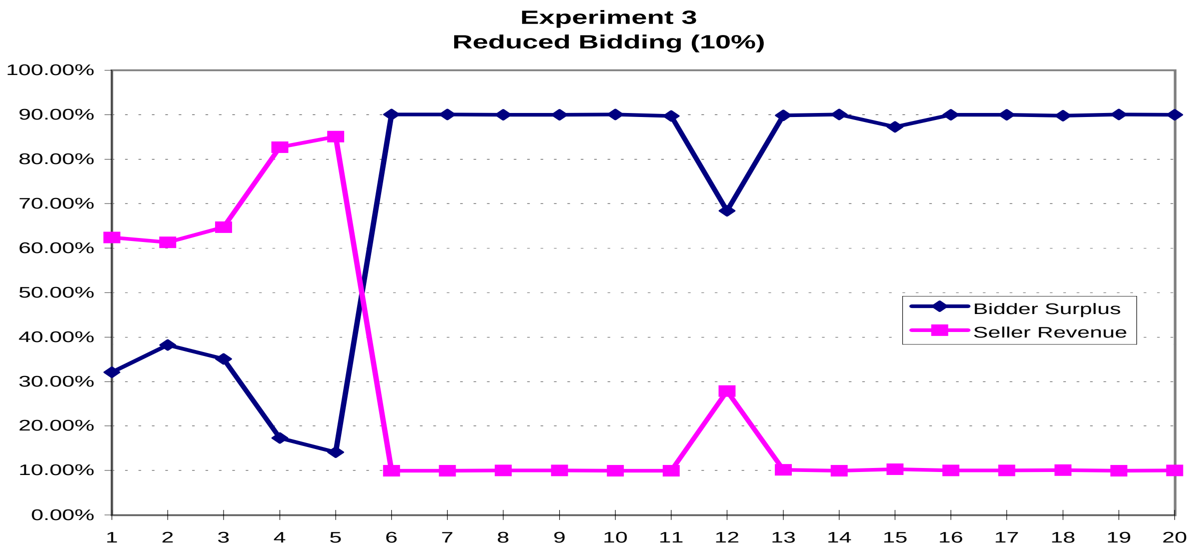


Figure 7: Experiment 2



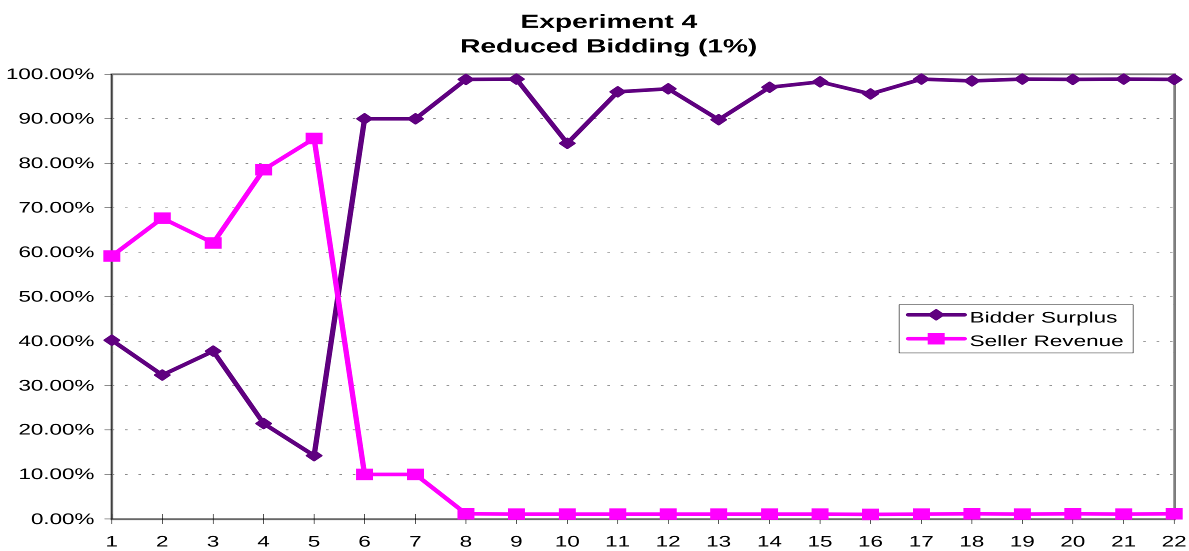
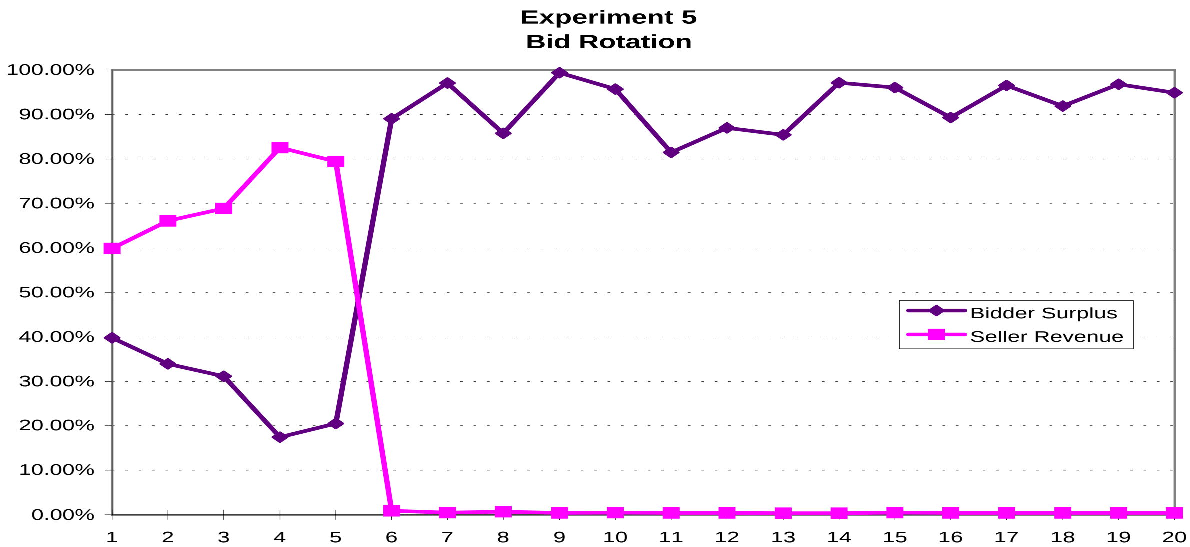


Figure 9: Experiment 4



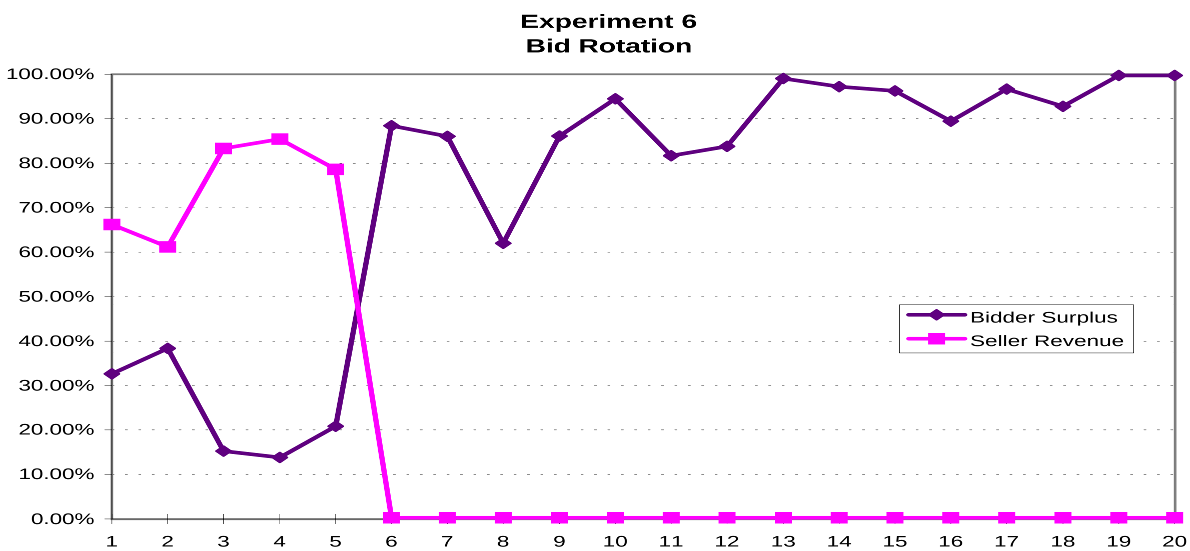
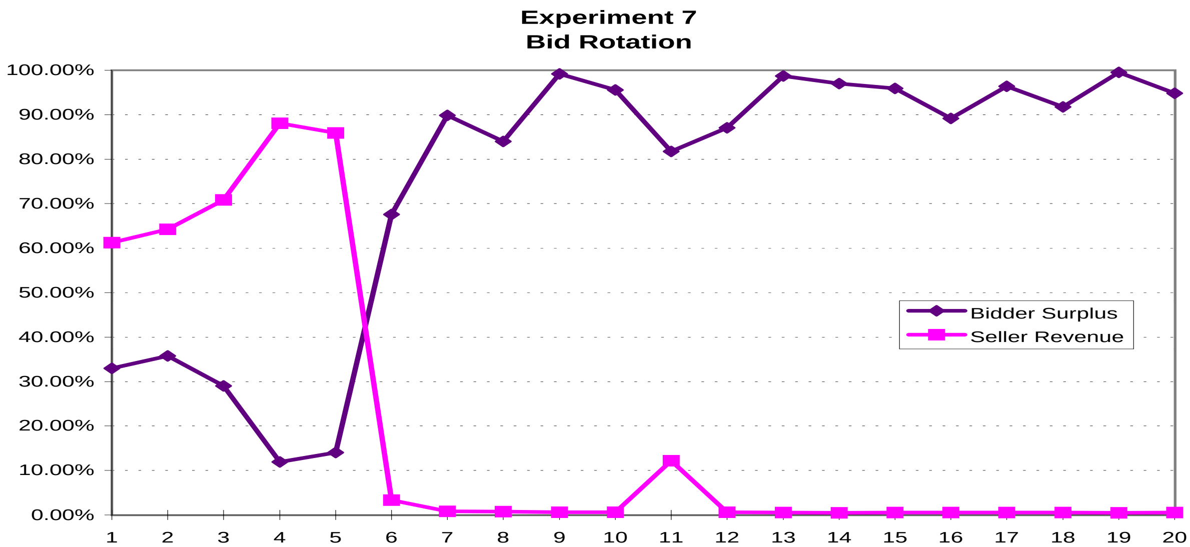


Figure 11: Experiment 6



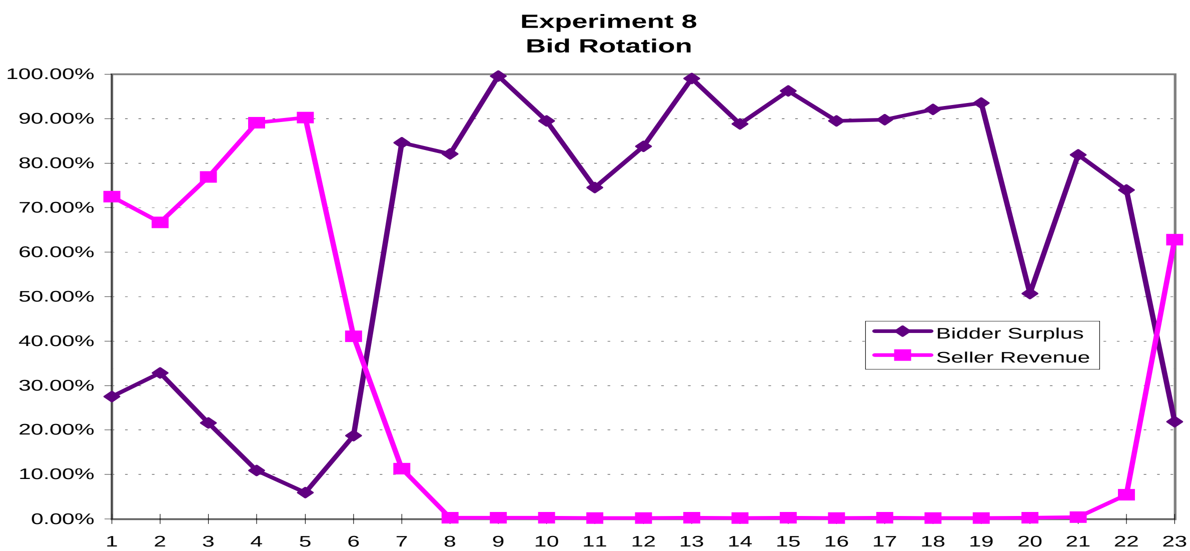
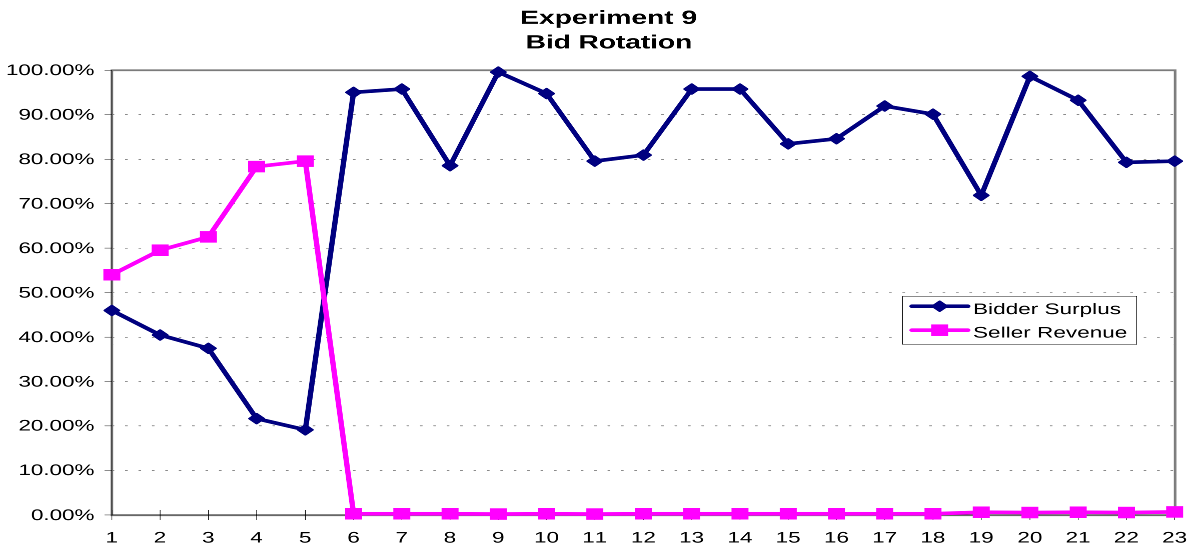


Figure 13: Experiment 8



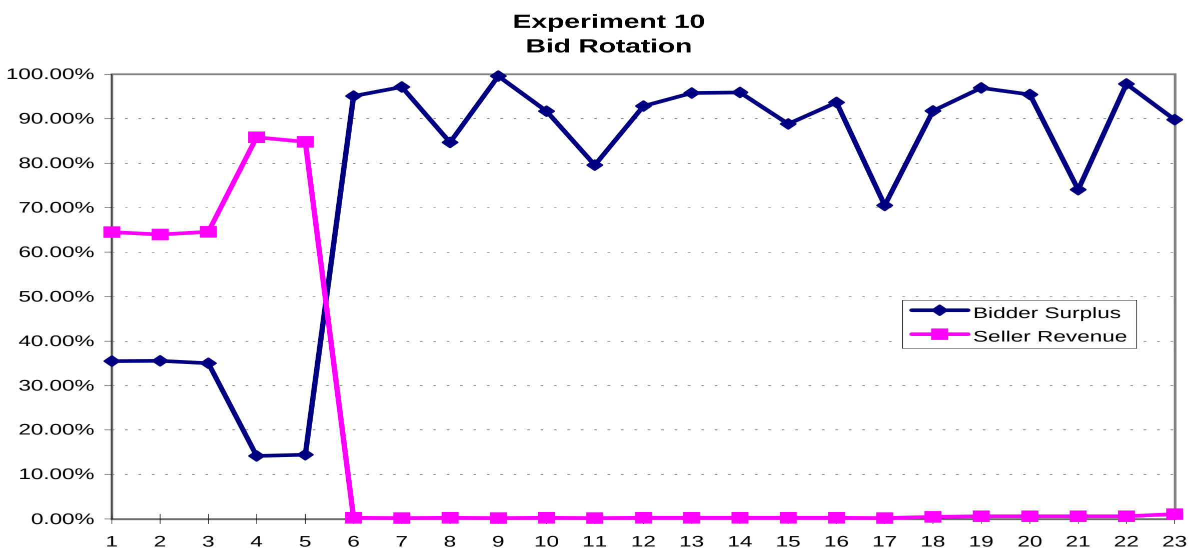


Figure 15: Experiment 10

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