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ON THE EMERGENCE OF CITIES

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Abstract

This paper considers the formation of cities in a simple model in which the preferences of agents depend on at most two characteristics of a location: its population and its average distance to the other agents. In such a simple model it is possible to recreate phenomena such as path dependency and centrally located cities which have been generated in more sophisticated models. Moreover, an example is provided in which cities emerge in the sense that the micro level preferences of agents do not appear to favor locating near or with other agents. When nonlinear effects are included then it is possible to show that even if efficient equilibria exist, they are not likely to occur and that there may exist extreme sensitivity to initial conditions. The model suggests that the mapping from individual preferences to population distributions merits further study.

ON THE EMERGENCE OF CITIES

Scott E. Page*

1 Introduction

This paper examines population aggregation in a model in which individual agents make location decisions on a two dimensional lattice. The analysis greatly simplifies earlier models of city formation by isolating the mapping from individual level relocation decisions to macro-level population patterns. Individuals, who may be thought of as either firms or people, have simple preferences, and as a result their behavior can be characterized by rules. The accumulation of these rules forms a complex adaptive system in which either expected or emergent phenomena may occur. This paper does not claim to provide definitive answers as to how and why cities form but instead, to demonstrate two facts: first, that skeletal models can often provide the same insights as more sophisticated approaches and second, that macro phenomena need not bear a strong resemblance to their micro foundations. The question of why cities form clarifies the latter point. Many models which generate cities include explicit gains from aggregation at the micro level. To presume micro level preferences which mimic macro level phenomena, though an obvious modelling choice, may be an incorrect one. That cities form when all agents want to reside near one another would not surprise anyone. That they can form when agents do not appear to have incentives to agglomerate, for example when agents want to live as far away from other agents as possible, does offer a hint into the complex map between agent level preferences and equilibrium population distributions.

The main results of this paper are as follows: Cities are shown to form under a variety of settings. In some scenarios their location can be sensitive to initial conditions and/or path dependent : identical micro–level preferences can lead to a single city in the center or four cities in the corners of the lattice. In other scenarios, the size and spatial distribution of cities is inevitable. In either case, the equilibrium distribution of cities need not be utility maximizing. Finally, restrictions on mobility can change the set of equilibrium population distributions in nontrivial ways.

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In the formal model, agents relocate in response to the distribution of agents on the lattice. As mentioned, these agents may be thought of as either individuals or firms. Preference characteristics of the agents often make one of the these assumptions more likely than the other. In the spirit of Schelling (1978), the model is not burdened with many moving parts. Agent behavior is a function of only two characteristics of the population distribution: *separation* and *population*. Separation refers to the average distance to other agents, and population equals the number of agents at a particular location. These characteristics may enter agents' utilities positively, negatively, or both positively and negatively in the case of higher order effects. The mapping from preferences to equilibrium distributions is then analyzed under two assumptions about mobility: global and local relocation ability. Under global relocations, agents can move anywhere on the lattice. Under local relocations, they are restricted to neighboring cells. The analysis focuses on three aspects of the equilibrium population distributions: their spatial characteristics, their sensitivity to initial conditions, and their aggregate utility.[1] Owing to the dynamic nature of the phenomena of city formation, a static theorem proof approach permits only a limited view. Additional knowledge can be gained from watching cities form in computational experiments under various assumptions about micro–level behavior. Accordingly, the analysis to follow combines mathematical and computational theory (Judd 1995).

Many earlier models of city formation include either an explicit, or a not so deeply buried assumption of increasing returns from agglomeration Krugman 1996). Such assumptions are motivated by references to complementarities between industries, savings owing to transportation costs, or sharing of the fixed costs of operating a market. The assumption that the benefits to agglomeration increase with city size guarantees the formation of cities. To refer to the phenomena of city formation in such environments as *emergent* may be misleading. The term emergent applies to epi-phenomena which are unexpected (Forrest 1990), or at a minimum not obvious as is the case in many city formation models, and as mentioned, there are environments in which cities emerge in this stricter sense – without an explicit assumption of positive returns from agglomeration.

Some comments are in order as to this model's intended contribution to the growing literature on city formation. Recently, economists have reconsidered the role of cities in the economy (Arthur 1991, Krugman 1993, Lucas 1988, Berliant and Konishi 1994). This renewed interest in city formation stems partially from the engaging historical accounts of Jacobs (1969, 1984) and Cronon (1991) as well as from increased recognition

[1] A equal weight utilitarian social welfare function measures the total utility of a population distribution.

of increasing returns (Arthur 1991). More generally, an awareness of human capital's contribution to growth has directed economists towards the role of cities. In his review of the literature on economic development, Lucas (1988) concludes that in order for cities to exist there must be some external benefits from agglomeration. If cities play a central role in the growth of the economy, i.e. if as in Jacobs' theory they are the nucleus of the atom, then the study of their formation, size, and location should be central to the study of the macroeconomy.

Models of city development range from general equilibrium models which show the existence of pareto efficient equilibria to examples assuming particular functional forms (Krugman 1993). As an example of the former, Berliant and Konishi (1994) assume that cities allow agents to exploit gains from trade and share transportation and marketplace setup costs. They show that provided certain technical assumptions are satisfied that there exists a pareto efficient equilibrium. While important, this approach to modelling does not address the relationship between micro–level preferences and equilibrium distributions much less issues such as the amount of sensitivity to initial conditions.

In contrast, the approaches taken by Krugman (1996) and Arthur (1991) permit the study of the sensitivity of city location to initial conditions. Silicon Valley and Route 128 are often trotted out as evidence that such sensitivity may actually exist. Neither region possessed a natural advantage for high technology industries, yet both attracted them in large numbers. In Arthur's model, firms possess complementary technologies depending upon location – firms benefit from locating near one another. These locational complementaries can create extreme sensitivity to initial conditions. In contrast, Krugman's model considers an economy with both agricultural and manufacturing sectors. His model combines a Dixit–Stiglitz monopolistic competition model with Samuelson's spoilage or "iceberg" transportation costs model, in which a fraction of the goods are lost during transport. Transportation costs and the benefits from agglomeration lead to city formation. However, in his model, the city forms at or near the midpoint of the linear world. Thus, he finds a moderate level of sensitivity to initial conditions but nowhere near the levels found in Arthur's model.[2]

Krugman and Arthur offer stylized models of city formation. As a result, care must be taken not to overinterpret either model. Krugman's modelling decisions, made with an eye for tractability, are not benign. His emphasis on transportation costs drives the limited sensitivity to initial conditions. Similarly, Arthur's assumption of positive externalities between firms creates the extreme sensitivity to initial conditions which

[2]Krugman (1996) has also constructed a model on a ring in which he does generate similar sensitivity to initial conditions as found in Arthur.

he finds. As shall be demonstrated in the paper, this variation in sensitivity to initial conditions depends in an understandable way upon the micro-level incentives of the agents. If agents care primarily about their distance to other agents, in the language of this paper their *separation*, then sensitivity to initial conditions is limited. If agents care about population, then extreme sensitivity to initial conditions may exist.

Urban economists (Mills 1980) have long recognized both negative and positive effects from agglomeration. [3] Negative effects of agglomeration, or congestion effects, mitigate the value of moving to a single large city for citizens. In the case of firms, negative complementarities, such as Pigou's famous example of laundries and industrial smoke may induce firms to choose spatially disparate locations. In the case of individuals, crowding can bring crime, traffic, and pollution. The inclusion of negative complementarities often results in environments in which the equilibria distributions of agents are not Pareto efficient.

The remainder of this paper is organized as follows. Section 2 contains the basic model which is defined on an N by N lattice. Section 3 presents the mathematical and computational theory for the eight cases with linear preferences, and section 4 considers nonlinear preferences. The reliance on computation stems from the number of cases considered and the fact that the intent here is to exhibit the subtleties of the micro to macro transition. The discussion at the end of the paper addresses possible extensions of the model, many of which have been explored computationally.

2 A Model of City Formation

A skeletal model is designed to capture spatial population accumulation brought about by agents' relocation decisions. In the model, there are assumed to be a finite number of agents who reside on an N by N lattice. These agents may be thought of as either individuals or firms. In several of the scenarios considered, firms may be a more appropriate interpretation. The assumption of a square lattice is not benign. Several findings depend upon it. This shortcoming appears unavoidable and suggests the need for future work with irregular lattices and lattices which approximate actual geography.

[3] Krugman's and Arthur both mention the potential for negative effects from agglomeration. but neither makes them a central feature of his model. To criticize Arthur, Krugman, and others for downplaying negative agglomeration effects would be unfair. Krugman refers to his work as a "pilot–study" and admits that the particular functional forms were chosen because they were 'tractable." These early models have been attempts to focus attention on heretofore neglected issues: in Krugman's case the importance of regional economics and in Arthur's the prevalence of increasing returns in economics. Both models have been successful in this respect. This paper builds from their contributions in an obvious direction.

Def'n: *The set of* **agents** $M = \{1, 2, 3, ...m\}$

The set of all possible locations is an N by N lattice.

Def'n: *The set of* **locations** $N \times N$, *where* $N = \{1, 2, 3, ...n\}$

An agent's utility level, and, perforce, her relocation decision depend upon characteristics of the distribution of agents on the lattice. For example, agents may prefer to reside at highly populated locations, or they may prefer to minimize their distance from other agents. Let F denote a distribution of agents on the set of locations. F can be formalized as a map from the set of locations into M so that F_{ij} denotes the number of agents residing in the ith row and jth column of the lattice. A constraint must be included so that total number of agents equals m.

Def'n: *The set of* **distributions** *of agents*

$$\Psi = \{F : F : N \times N \to M \cup \{0\}, \text{ s.t. } \sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} = m\}$$

Agents have identical preferences over the distributions of agents. An agent's utility depends upon her own location, and on the entire distribution.

Def'n: *The* **utility function** $u : N \times N \times F \to \Re$

Since all agents are identical, a distribution F is *utility maximizing* if it maximizes a utilitarian social welfare function.

Def'n: *The distribution* $F \in \Psi$ **utility maximizing** *if and only if*

$$\sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} u(i, j, F) \geq \sum_{i=1}^{n} \sum_{j=1}^{n} F'_{ij} u(i, j, F') \quad \textit{for all } F' \in \Psi$$

An agent's utility may depend upon the population at her home location. The population may enter positively; agents want to live near other agents. As discussed in the introduction, there are several motivations for such an assumption, technological

externalities, the formation of marketplaces, and the creation of infrastructure to name just a few. Alternatively, the population may enter negatively; agents might prefer to have fewer agents living at their home location. This assumption is applicable when agents are involved in agricultural production, where crowding occurs, or if agents prefer to live in isolation than in a crowded city.

An agent's utility may also depend upon her average distance from other agents. If agents face significant transportation costs and if they trade with a significant percentage of the other agents, then they may wish to minimize their average distance to other agents. Average distance could also enter into utility positively; agents may wish to be as far from other agents as possible. If the agents represent firms and firms create negative externalities and, moreover, if these externalities depend upon separation, then this assumption is not unreasonable. Pigou's example of smokestacks and laundries would be an example of this sort of preferences.[4]

When computing average distance, either of two measures can be used: A city block distance measure, denoted by $d^c(i, j, F)$, which counts the number of lattice points horizontally and vertically which separate two locations or the standard Euclidean distance measure, denoted by $d^e(i, j, F)$. The dynamics and end states depend in predictable and unimportant ways on the choice of distance measure. The analysis which follows includes only the city block distance measure as it lends itself more readily to formal analysis.

Def'n: *The* **city block distance** *from* (i^*, j^*) *given* F,

$$d^c(i^*, j^*, F) = \frac{1}{m} \sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} \cdot (\mid i^* - i \mid + \mid j^* - j \mid)$$

2.1 Equilibria

Assumptions about the ability of agents to relocate are necessary before the equilibrium distributions or the dynamics can be characterized. Two rules: *global relocations* and *local relocations* are used in this analysis. Under global relocations, each agent chooses the location on the lattice generating the highest utility. Agents take into account the effect of their own movement on population.[5] In the event of a tie, an agent chooses the first location evaluated from among those generating the highest utility.

[4]This would not be an appropriate model of isolationists, who would wish to minimize the number of people living within a specified distance.

[5]This is in contrast to many Tiebout models which assume that agents do not consider the effect of their own relocations.

Def'n: *The distribution F is an* **equilibrium with respect to global relocations** *if and only if $F_{ij} > 0$ implies $u(i,j,F) \geq u(i',j',F - \delta_{ij} + \delta_{i'j'})$ for all i',j', where $\delta_{ij} : N \times N \rightarrow \{0,1\}$ where $\delta_{ij}(i,j) = 1$ and $\delta_{ij}(\hat{i},\hat{j}) = 0$ for $(\hat{i},\hat{j}) \neq (i,j)$.*

Under local relocations, agents do not search the entire lattice for the best location. They are restricted in how far they can move in any one iteration. They are also myopic. They move to the best possible location within a fixed neighborhood as opposed to moving in the direction of the best location on the entire lattice. Agents are assumed to be capable of computing their utility from each location in their local neighborhood.[6]

Def'n: *The distribution F is an* **equilibrium with respect to local relocations of distance** *d if and only if $F_{ij} > 0$ implies $u(i,j,F) \geq u(i',j',F - \delta_{ij} + \delta_{i'j'})$ for all i',j' such that $(\mid i - i' \mid + \mid j - j' \mid) \leq d$* where $\delta_{ij}(i,j) = 1$ and $\delta_{ij}(\hat{i},\hat{j}) = 0$ for $(\hat{i},\hat{j}) \neq (i,j)$.

In the examples described below the size of a neighborhood equals one ($d = 1$). Given the city block distance measure, agents may only move to one of the four horizontal and vertical locations on the lattice. An agent residing at a corner location has only two alternative locations in which to reside.

Two methodological issues remain: how agents time their relocation decisions and how the initial population distributions are determined. The timing of updating, whether synchronous or asynchronous, often qualitatively effects both dynamics and end states (Huberman and Glance 1992). Given that relocation decisions might be made at any time, in this model agents do not move simultaneously but instead relocate at different times. The order of the asynchronous updating is random.[7] Agents are identified by numbers ranging from 1 to m and placed in a que. In each period, the agents sequentially choose to reside in the location offering the highest utility given the relocations of all agents ahead of them in the que. The initial distribution of agents is random according to a uniform distribution: agents have equal probability of being assigned to each location. An assumption that agents were involved in agriculture prior to the formation of cities makes this not an unreasonable assumption.

[6] This assumption is a bit troublesome but is made for convenience. For some of the preferences considered, agents need information about populations at all locations on the lattice in order to compute the utility they obtain from the locations in their neighborhood.

[7] An alternative approach is to use incentive based asynchronous updating. Under incentive based asynchronous updating, the order in which agents relocate is determined by their utility from updating. Those agents with the most to gain from relocating are the first to relocate. Page (1995) shows that incentive based asynchronous updating alters both dynamics and the distribution over end states for several classes of cellular automata.

3 Linear Preferences

The first class of models to be analyzed assumes linear preferences. Recall that an agent's utility at a location may depend upon either average distance from other agents, *separation*, or the location's population. In the models considered initially, these components enter into the utility functions linearly. Preferences that depend on location population alone are first considered, then preferences that depend only on separation, and, finally, linear combinations of location population and separation.

3.1 Population Preferences

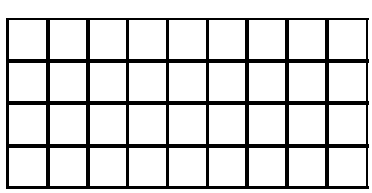
Agents' utilities may either increase or decrease with the population at their home location. Each possibility is considered in turn with both global and local relocations. Population does not create a smooth utility gradient, so local relocations can result in suboptimal equilibrium distributions.

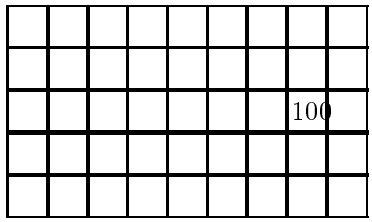
3.1.1 Agglomeration

In the first scenario, an agent's utility equals the population at her home location: an agent residing at location (i, j) obtains a utility equal to F_{ij}. The larger the local population, the more utility accruing to the agent. In the global relocations scenario, the first agent to relocate chooses from among those locations with the largest population. The next agent to relocate necessarily chooses the same location as the first agent, as that location now contains strictly more agents than any other. In turn all remaining agents choose the same location, so that after one round of relocations, all agents reside in a single large city. These dynamics are not particularly interesting other than that they exhibit sensitivity to initial conditions: whichever location begins with the most agents becomes the large city. The graph below shows the percentage of the population living at each site on a nine by nine lattice. This and other examples are drawn from sample computational experiments using one thousand agents. The nine by nine lattice makes the presentation clearer than if much larger lattices, say one hundred by one hundred, are used.

Global Relocations

$$u(i, j, F) = F_{ij}$$





The next two claims state that a distribution F is an equilibrium with respect to global relocations if and only if all agents reside at a single location and that these equilibrium allocations are utility maximizing.

Claim 3.1 *If $u(i,j,F) = F_{ij}$ then F is an equilibrium with respect to sequential global relocations if and only if there exists an (i,j) such that $F_{ij} = m$*

pf: If $F_{ij} = m$ then the utility from remaining at (i,j) equals m and the utility from any other location equals 1. Thus, F is an equilibrium.

To prove the other direction, let $K = \max_{(i,j)}\{ F_{ij}\}$ where $K < m$ and show that this leads to a contradiction. There are two cases to consider. First, suppose there is a unique (i^*, j^*) such that $F_{ij} = K$. For all $(i,j) \neq (i^*, j^*)$, $u(i,j,F) < u(i^*, j^*, F)$, which implies that all agents must be located at (i^*, j^*), a contradiction. Second, suppose $F_{ij} = F_{i^*j^*} = K$ and that $(i,j) \neq (i^*, j^*)$. Consider an agent located at (i,j). Her utility equals K. Her utility from (i^*, j^*) equals $K + 1$, a contradiction.

Claim 3.2 *If $u(i,j,F) = F_{ij}$ then F is utility maximizing if and only if there exists an (i,j) such that $F_{ij} = m$*

pf: Maximizing the utilitarian social welfare function is equivalent to the following constrained maximization problem.

$$\max \sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} \cdot F_{ij}$$

subject to

$$\sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} = m \text{ and } F_{ij} \geq 0 \text{ for all } (i,j)$$

The convexity of the objective function implies that optimum must occur at a corner. All corners obtain an identical value of m^2, which completes the proof.

In the local relocations scenario, the dynamics becomes more complicated. Each agent in turn chooses a location in her neighborhood with maximal population. There are two reasons why, unlike in the global relocation scenario, all agents do not choose

to reside at the same location in the first round of relocations. First, the neighborhoods of two agents need not intersect, in which case it is impossible for the second agent to choose to reside at the same location as the first agent. Second, among those agents whose neighborhoods do intersect, the locations within their neighborhoods with maximal population may differ. After a few rounds of relocations, the population pattern consists of a congeries of small villages. For a distribution to be an equilibrium given these dynamics, any two locations with strictly positive populations may not be adjacent, such as in the diagram below. By claim 3.2 we know that the distribution is not utility maximizing.

Local Relocations ($d = 1$)

$u(i, j, F) = F_{ij}$

								8
	7							
				12			7	
								7
	6				11			
		8						
					9		12	
	7							
						6		

Under local relocations, the equilibrium population distributions are more sensitive to initial conditions then they are under global relocations. Under global relocations, the equilibrium distribution changes only if the initial location with the largest population changes. Under local relocations, the distribution of the villages can be changed more easily.

3.1.2 Isolation

Making the opposite assumption and assuming that agents want to live in lightly populated areas results in a very different equilibrium distribution. If $u(i, j, F) = -F_{ij}$, then under global relocations, the agents spread themselves uniformly over the lattice as is stated in Claim 3.3.

Claim 3.3 *Assume $m = \alpha \cdot n^2$, where α is an integer. If $u(i, j, F) = -F_{ij}$, then F is an equilibrium with respect to global relocations if and only if $F_{ij} = \alpha$ for all (i, j)*

pf: First, $F_{ij} = \alpha$ for all (i, j) is shown to be an equilibrium. All agents obtain a utility equal to $-\alpha$. The utility to an agent currently located at (i, j) from an alternative location equals $-(\alpha + 1)$, thus F is an equilibrium.
The uniqueness of the equilibrium is shown by contradiction. Suppose that there exists an (i, j) such that $F_{ij} \geq (\alpha+1)$. It follows that there exists an (i', j') such that $F_{i'j'} \leq (\alpha-1)$. Any agent located at (i, j) would obtain strictly greater utility by moving to (i', j').

As the next claim states, this distribution is utility maximizing.

Claim 3.4 *Assume $m = \alpha \cdot n^2$, where α is an integer. If $u(i, j, F) = F_{ij}$ then F is utility maximizing if and only if $F_{ij} = \alpha$ for all (i, j).*

pf: As before, set up the Lagrangian

$$\max_{F,\lambda} \sum_{i=1}^{n} \sum_{j=1}^{n} -F_{ij} \cdot F_{ij} \; + \; \lambda \cdot (\sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} \; - m)$$

The first order necessary conditions:

$$2 \cdot F_{ij} = \lambda \quad \text{for all} \quad (i, j)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} = m$$

are sufficient because the function is strictly concave. Therefore, the interior critical point describes a unique maximum in which all locations have identical populations.

Under local relocations, the equilibrium distributions which result from computational experiments are often nearly uniform. If the initial distribution is not approximately uniform, then the final distribution may differ substantially as shown in the next figure.

Local Relocations ($d = 1$)

$$u(i, j, F) = -F_{ij}$$

				1				
		1	1	2	1	1		
	1	1	2	3	2	1	1	
1	1	2	3	4	3	2	1	1
1	2	3	4	5	4	3	2	1
1	1	2	3	4	3	2	1	1
	1	1	2	3	2	1	1	
		1	1	2	1	1		
				1				

3.2 Separation Preferences

Separation preferences assume that agents care about their average distance to other agents. They may wish to be as close to other agents as possible on average, which might be the case if agents trade extensively, or alternatively, agents may wish to distance themselves from other agents. This latter assumption is defensible if agents do not want other agents to trade or visit and if the probability that an agent visits another agent is linear in distance.

An implication of separation preferences is that they create smooth utility gradients. In other words, if preferences are monotonic in separation, then the equilibria for global and local relocations are identical. This is formally stated in the next claim.

Claim 3.5 *If $u(i, j, F) = h(d^c(i, j, F))$ where h is strictly monotonic real valued function then F is an equilibrium with respect to global relocations if and only if it is an equilibrium with respect to local relocations.*

pf: The only if direction holds by definition. Therefore, it suffices to show that if a distribution F is an equilibrium with respect to local relocations, then it is also an equilibrium with respect to global relocations when $u(i, j, F) = h(d^c(i, j, F))$ and h is

monotonic. Without loss of generality, assume that h is monotonically increasing. It suffices to show that the result holds for the case where h is the identity function.

The proof proceeds by contradiction. Suppose that an agent residing at (i, j) would benefit by moving to (i', j'), where $i' > i$ and $j' \geq j'$. The utility increase in moving from (i, j) to (i', j') is the sum of the increase of moving from (i, j) to (i', j) and the increase in moving from (i, j) to (i, j'). Therefore, let Δ_i equal the change in utility if the agent moves from (i, j) to (i', j). It is sufficient to prove that if this is strictly positive then Δ_1, the change in utility if the agent moves from (i, j) to $(i + 1, j)$ is also strictly positive. Let $s(i) = \sum_{j=1}^{n} F_{ij}$. It is straightforward to show that

$$\Delta_1 = 1 - \sum_{k=i+1}^{n} s(k) + \sum_{k=1}^{i} s(k)$$

If $i' - i = 1$ then $\Delta_i = \Delta_1$, which completes the proof. Assume $i' - i \geq 2$, it follows that

$$\Delta_i = (i' - i) + \sum_{k=i'}^{n} (i' - i) \cdot s(k) + \sum_{k=1}^{i} (i' - i) \cdot s(k) + \sum_{k=i+1}^{i'-1} ((i' - i) - 2(k - i)) \cdot s(k)$$

Multiplying Δ_1 by $(i' - i)$ and subtracting Δ_i yields

$$(i' - 1) \cdot \Delta_1 \; - \; \Delta_i \; = \; \sum_{k=i+1}^{i'-1} 2(k - i) s(k)$$

Since all of the $s(k)$'s are greater than or equal to zero, it follows that $\Delta_1 > 0$ which completes the proof.

Note that even though the set of equilibria are the same, the dynamics may differ substantially and as a result, identical starting points may lead to distinct equilibria, and they typically do under global and local relocations.

3.2.1 Attraction

The more reasonable assumption is that agents prefer to be close to other agents so that they might trade. Therefore, $u(i, j, F) = -d^c(i, j, F)$. In the global relocation scenario, the first agent to relocate chooses the location which has minimal average distance to all other agents. Given the assumption of a uniform initial distribution of agent locations, this agent locates near the center of the lattice. The location chosen by the next agent and all subsequent agents will be either be the same, or differ by a small distance.[8]

[8] If for example, all agents are located in the center with one agent residing far to the north, then the first agent to relocate, provided she is not the northerner, may move out of the city. If the northerner locates next, then she will chose to live in the city.

After only a few rounds of adaptation, typically two or three, all agents reside in a single location. Both the dynamics and the end state differ from the earlier case in which agents cared only about population at their home location. The dynamics are more interesting because convergence is not immediate. More importantly, the location of the end state is not especially sensitive to initial conditions: the resulting single city is located at or near the center of the lattice, just as in Krugman's more sophisticated model.

Under local relocations, in each time period agents march towards the center of the lattice. Eventually, all agents reside at a single location at or near the center of the lattice. The next two claims state that a single city is an equilibrium distribution for both local and global relocations and that it is utility maximizing.

Claim 3.6 *If $u(i,j,F) = -d^c(i,j,F)$ then F is an equilibrium with respect to global and local relocations if and only if there exists an (i,j) such that $F_{ij} = m$*

pf: First suppose that there exists an (i,j) such that $F_{ij} = m$. It follows that $u(i,j,F) = 0$. If any agent relocates, her utility would be strictly negative. Therefore, F is an equilibrium. The other direction is proven by contradiction. Suppose that $F_{ij} < m$ for all (i,j). It suffices to show that an agent can benefit by relocating. Choose (i,j) and (i',j') such that $F_{ij} > 0$, $F_{i'j'} > 0$, and $i < i'$. Let $s(i) = \sum_{j=1}^{n} F_{ij}$. If an agent moves from (i,j) to $(i+1,j)$, her change in utility Δ_i given by

$$\Delta_i = 1 - \sum_{k=1}^{i} s(k) + \sum_{k=i+1}^{n} s(k)$$

The one is because the agent does not move one unit away from herself when she relocates. If an agent moves from (i',j') to $(i'-1,j')$, her change in utility $\Delta_{i'}$ given by

$$\Delta_{i'} = 1 - \sum_{k=i'}^{n} s(k) + \sum_{k=1}^{i'-1} s(k)$$

There are two cases to consider. $i' = i+1$ and $i' > i+1$. If $i' = i+1$ then

$$\Delta_i + \Delta_{i'} = 2$$

If $i' > i+1$, then

$$\Delta_i + \Delta_{i'} = 2 + 2 \cdot \sum_{k=i+1}^{i'-1} s(k)$$

In either case, $\Delta_i + \Delta_{i'} > 0$ implying that one of the terms exceeds zero and completing the proof.

Claim 3.7 *If $u(i,j,F) = -d^c(i,j,F)$ then F is utility maximizing if and only if there exists an (i,j) such that $F_{ij} = m$*

pf: $u(i,j,F) \leq 0$ for all F. Therefore, it suffices to show that there exists an (i,j) such that $F_{ij} = m$ if and only if

$$U(f) = \sum_{i=1}^{n} \sum_{j=1}^{n} u(i,j,F) F_{ij} = 0$$

Suppose $F_{ij} = m$, then $F_{i'j'} = 0$ for all $(i',j') \neq (i,j)$. A straightforward calculation shows that $U(F) = 0$. To prove the other direction, suppose that $F_{ij} < m$ for all (i,j). Choose (i,j) and (i',j') so that $F_{ij} > 0$ and $F_{i'j'} > 0$. It follows that $d^c(i,j) > 0$ and $d^c(i',j') > 0$. which implies that the $U(F) > 0$.

Interestingly, the set of equilibrium and utility maximizing distributions are not biased towards the center. The city can lie anywhere on the lattice. In computational experiments, the city always lies near the center because during the formation process agents want to be close to other agents as well. Note the difference between this and Krugman's model in which farmers' remain in the surrounding areas and the city is near the center to stay close to the markets. In the model presented here, the equilibrium distribution contains no population outside of the city, yet the city is still centrally located. The city's location is an artifact of the initial population distribution and does not fulfill any purpose in equilibrium.

3.2.2 Repulsion

If the alternative assumption is made so that agents wish to *maximize* their average distance to other agents, then the dynamics and the distribution of end states changes dramatically. In the global relocation scenario, the first agent to relocate chooses a corner, as do all other agents in turn.

Global and Local Relocations

$u(i,j,F) = d^c(i,j,F)$

18								32

32								18

If the populations in opposite corners are not equal, then agents continue to relocate. The sensitivity to initial conditions is not extreme. The populations in the corners may change, but not by much. In the local relocation scenario, agents crawl towards the corners of the space, and within a few rounds of relocation, all agents reside along the edges of the lattice. Eventually, the agents locate an equilibrium in which the populations at opposite corners are equal. Again in this case, local and global relocation lead to similar end states, although the dynamics differ. This is a case in which a macro phenomenon emerges in the true sense. The aggregation of population in the corners runs counter to the microlevel incentives which are to separate.

This intuition can be formalized without too much difficulty. Claim 3.8 below states that all stable allocations consist of all agents in the four corners with equal populations in opposite corners.

Claim 3.8 *Assume m is even. If $u(i,j,F) = d^c(i,j,F)$, then F is an equilibrium with respect to global relocations if and only if F satisfies the following equalities:*

$$\begin{array}{rcll} F_{11} & = & F_{nn} & \\ F_{1n} & = & F_{n1} & \\ F_{ij} & = & 0 & \text{if } \{i,j\} \not\subseteq \{1,n \end{array}$$

pf: The proof proceeds in two parts. First, if $\{i,j\} \not\subseteq \{1,n\}$ then it is shown that $F_{ij} = 0$. Then, the equality of the populations in opposite corners is shown.

Part 1: Suppose that an agent resides at (i,j) and $\{i,j\} \not\subseteq \{1,n\}$. Without loss of generality assume that $i \not in \{1,n\}$. Let $s(i) = \sum_{j=1}^{n} F_{ij}$. Let Δ_+ equal the change in utility if the agent moves to $(i+1,j)$ and Δ_- equal the change in utility if the agent moves to $(i-1,j)$. It is straightforward to show that

$$\Delta_+ = 1 - \sum_{k=i+1}^{n} s(k) + \sum_{k=1}^{i} s(k)$$

and that

$$\Delta_- = 1 + \sum_{k=i}^{n} s(k) - \sum_{k=1}^{i-1} s(k)$$

adding the two terms obtains

$$\Delta_+ + \Delta_- = 2 + 2 \cdot s(i)$$

Since the sum of the two terms is strictly positive, one of the two terms must be positive, which completes the first part of the proof.

Part 2: A straightforward calculation shows that if $F_{11} = F_{nn}$ and $F_{1n} = F_{n1}$ then no agent increases her utility by relocating. By symmetry it suffices to show that if $F_{ij} = 0$ for all $\{i,j\} \not\subseteq \{1,n\}$ and if $F_{11} < F_{nn}$ then an agent would relocate from (n,n) to $(1,1)$. Given these conditions, it follows that

$$u(1,1,F) - u(n,n,F) = 2(n-1) \cdot [F_{nn} - F_{11}] > 0$$

which completes the proof.

Claim 3.9 states that this distribution is utility maximizing.

Claim 3.9 *Assume m is even. If $u(i,j,F) = d^c(i,j,F)$, then F is utility maximizing if and only if F satisfies the following equalities:*

$$\begin{array}{lcll} F_{11} & = & F_{nn} & \\ F_{1n} & = & F_{n1} & \\ F_{ij} & = & 0 & \text{if } \{i,j\} \not\subseteq \{1,n\} \end{array}$$

pf: The Lagrangian for this problem is as follows:

$$\max_{F,\lambda} 2 \cdot \sum_{i=1}^{n-1} \sum_{i'>i} \sum_{j=1}^{n} \sum_{j'=1}^{n} (i'-i) \cdot F_{ij} \cdot F_{i'j'} + 2 \cdot \sum_{j=1}^{n-1} \sum_{j'>j} \sum_{i=1}^{n} \sum_{i'=1}^{n} (j'-j) \cdot F_{ij} \cdot F_{i'j'} + \lambda \cdot (\sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} - m)$$

Let $s(i) = \sum_{j=1}^{n} F_{ij}$ and $r(j) = \sum_{i=1}^{n} F_{ij}$. The first order necessary conditions can be written as follows:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} = m$$

$$\sum_{i'=1}^{n} \mid i' - i \mid s(i') \; + \sum_{j'=1^n} \mid j' - j \mid r(j') \; = \; \lambda \text{ for all } (i,j)$$

The crux of the proof is that $s(i) = 0$ for $i \notin \{1,n\}$ and that $r(j) = 0$ for $j \notin \{0,1\}$. Holding j fixed and subtracting the first order necessary condition for $(n-1,j)$ from the first order necessary condition for (n,j) obtains:

$$-s(1) + \sum_{i=1}^{n} s(i) = 0$$

Similarly, subtracting the first order necessary condition for $(2,j)$ from the first order necessary condition for $(1,j)$ obtains:

$$-s(n) + \sum_{i=1}^{n-1} s(i) = 0$$

Summing these two equalities yields

$$\sum_{i=2}^{n-1} s(i) = 0$$

which implies that $s(i) = 0$ for $i \in \{2, .., n-1\}$. Substituting into the previous equation, gives that $s(1) = s(n)$. A similar argument shows that $r(i) = 0$ for $i \in \{2, ., n-1\}$ and that $r(1) = r(n)$. A straightforward calculation shows that if $s(1) = s(n) = \frac{m}{2} = r(1) = r(n)$ then all of the first order conditions are satisfied. It also follows from the definitions of s and r that $F_{ij} = 0$ if $\{i, j\} \not\subseteq \{1, n\}$. Therefore, $s(1) = s(n)$ can be rewritten as $F_{11}+F_{1n} = F_{n1}+F_{nn}$ and $r(1) = r(n)$ as $F_{11}+F_{n1} = F_{1n}+Fnn$. Adding the first equation to the second yields $F_{11} = F_{nn}$. Plugging this into either equation yields $F_{1n} = F_{n1}$. A straightforward calculation shows that all distributions which satisfy $F_{11} = F_{nn}$ and $F_{1n} = F_{n1}$ have identical values under the utilitarian social welfare function.

It remains to show that the first order necessary conditions are sufficient. As a first step in showing sufficiency, it is proven that any distribution F with $F(i, j) > 0$ for some $i \notin \{1, n\}$, has a lower value under the utilitarian social welfare function than a distribution, in which which an agent at location (i, j) is moved to either $(i-1, j)$ or $(i+1, j)$. Let Δ_+ equal the change in the sum of the agents' utilities if the agent moves to $(i+1, j)$ and Δ_- equal the change in the sum of the agents' utilities if the agent moves to $(i-1, j)$. It is straightforward to show that

$$\Delta_+ = 2 \cdot [1 - \sum_{k=i+1}^{n} s(k) + \sum_{k=1}^{i} s(k)]$$

and that

$$\Delta_- = 2 \cdot [1 + \sum_{k=i}^{n} s(k) - \sum_{k=1}^{i-1} s(k)]$$

adding the two terms gives

$$\Delta_+ + \Delta_- = 4 + 4 \cdot s(i) > 0$$

By symmetry, at the global optimum all agents must be located in the four corners.

To complete the proof, the population in opposite corners must be shown to be equal. The proof is by contradiction. Suppose that $F_{11} < F_{nn}$. There are two cases to consider.

Case 1: $F_{nn} \geq F_{11} + 2$: If an agent at location $(1, 1)$ moves to location (n, n) then the change in aggregate utility, Δ is given by:

$$\Delta = 4n[F_{nn} - 1 - F_{11}] > 0$$

Case 2: $F_{nn} - F_{11} = 1$: By assumption m, the total number of agents is even; therefore, $F_{1n} \neq F_{n1}$. Without loss of generality assume that $F_{1n} > F_{n1}$. If an agent at location (n, n) moves to location $(n, 1)$, then the change in aggregate utility, Δ, is given by:

$$\Delta = 2n[F_{nn} - 1 - F_{11} + F_{1n} - F_{n1}] > 0$$

which completes the proof.

3.2.3 Summary

The table below summarizes the four cases with pure separation and agglomeration. The entries in *italics* are those environments for which equilibrium distributions are sensitive to the initial distribution.

Summary of Single Component Models			
$u(i, j, F)$	*Global Relocations*	*Local Relocations*	*Utility Max*
$F_i j$	*one city anywhere*	*isolated villages*	one city anywhere
$-F_i j$	uniformly spread	uniformly spread[9]	uniformly spread
$d_i j$	four corners	four corners	four corners
$-d_i j$	one city near center	one city near center	one city anywhere

3.3 Separation and Agglomeration

The previous scenarios consider either population or separation but not both simultaneously, which is the next step in the analysis. For the moment, only linear combinations of separation and agglomeration effects are considered. In the ensuing section, nonlinear terms are included. For expediency, a genetic algorithm is used to approximate the optimal population patterns (Holland 1975, Goldberg 1989) is applied to the environment. The genetic algorithm uses tournament selection and uniform crossover to search for utility maximizing population distributions. Moreover, formal claims that population distributions are equilibria are replaced by simulation findings.

The coefficients of population and distance may be either positive or negative, so there are four cases to consider. In each case, the relative weights on the two components can be varied.

3.3.1 Isolated Attraction

If both coefficients are negative, then agents prefer lightly populated areas which are close to the other agents spatially. The variable $\alpha > 0$ measures the relative importance of the distance component. With either global or local relocations, the equilibria resemble piles of sand near the center of the lattice. The weight of the simulation evidence suggests that the pile of sand is invariant to the initial distribution up to translations and rotations.

Agents are balancing off their desire to be nearer other agents with their preference for a less crowded home location. Thus, agents at the very center are nearer to other agents, but must endure having a larger home location.

Global and Local Relocations

$$u(i,j,F) = -F_{ij} - \alpha d^c(i,j,F)$$

				3	3			
			4	8	8	4		
		1	7	11	11	7	1	
			4	8	8	4		
				3	3			

Changes in the relative weight of the two components changes the equilibrium distributions in understandable directions. As α is increased (decreased) the piles of sand grows taller (shorter) and encompasses a smaller (larger) area. Larger α imply that the agents want to be closer together, which increases the population at the center. The sandpile formation appears to vary smoothly with changes in α.

The sandpile formation also appears to be optimal. A genetic algorithm searching the space of distributions discovered the sandpile formation and, in many applications, was unable to find another distribution generating higher utility.

3.3.2 Agglomerated Attraction

Switching the sign on the coefficient of F_{ij} creates agents who prefer to live in highly populated cities and close to other agents. With global relocation, the equilibrium distribution is obvious. Agents locate in a single city near the center of the lattice. Given the separation component the location of the city is less sensitive to initial conditions than in the case where utility depended only on population. The proof that this distribution utility maximizing follows from Claim 3.2 and Claim 3.7. With local relocations, agents could become stuck in moderately sized cities which are spatially separated. The findings vary depending upon the relative weights on population and separation. If the population term predominates, then the equilibrium distributions may be similar to those shown below, while if the coefficient of the separation term predominates, then the agents move to a single city in the center.

Local Relocations

$$u(i,j,F) = +F_{ij} - d^c(i,j,F)$$

			16					
						41		
				27				
						16		

Notice also in the local relocation scenario, that the city sizes can vary. In the single component models, the cities took on at most two distinct values, here they may take on may more.

3.3.3 Agglomerated Repulsion

If the coefficients of both the population and the separation term are positive, then the end states vary significantly under local and global relocation. Under global relocation, the agents move to two opposing corners in the first iteration. Typically, one of these corners has a larger population than the other. In the second generation, all agents move to the corner with the larger population. The outcome is a single city located in a corner. This occurs because the population term begins to predominate. The single city in the corner is also utility maximizing for small α. Under local relocation, the agents move towards the corners. Unless the population coefficient is especially large, within a few generations, the population is spread unevenly over the four corner locations. When the population term predominates, then in addition to the four corner location, there may also be locations just off center with positive population as shown below.

Local Relocations

$$u(i,j,F) = F_{ij} + d^c(i,j,F)$$

19								23
	7							
	6							
21								24

3.3.4 Isolated Repulsion

The last linear case to consider is where the coefficient on the population is negative and the coefficient on separation is positive. Less formally, agents wish to live at a location with small population and want to be as far as possible from the other agents. Under global relocation, the agents reside along the edges of the lattice, with a concentration of agents near the corners. The end state appears to occur regardless of the initial distribution. Under local relocation, the agents move towards the edges and often locates the same equilibrium distribution as under global relocation. Sometimes the end state differs slightly as asymmetries cannot be overcome by local movements. Computations using a genetic algorithm suggest that these configurations are also utility maximizing.

Local Global Relocations

$$u(i,j,F) = -F_{ij} + d^c(i,j,F)$$

6.2	3.7	2.5	1.8	1.6	1.8	2.5	3.7	6.2
3.7	1.2						1.2	3.7
2.5								2.5
1.8								1.8
1.6								1.6
1.8								1.8
2.5								2.5
3.7	1.2						1.2	3.7
6.2	3.7	2.5	1.8	1.6	1.8	2.5	3.7	6.2

The following table summarizes the four linear cases. Again, those entries in *italics* represent the outcomes that are sensitive to initial conditions.

Summary of Linear Models			
Preferences	*Global*	*Local*	*Utility Maximizing*[10]
$F_ij + d_ij$	*one city in corner*	*cities in four corners*	one city in corner
$F_ij - d_ij$	one city in center	one city in center	one city in center
$-F_ij + d_ij$	edges with corner peaks	edges with corner peaks	edges with corner peaks
$-F_ij - d_ij$	central sand pile	central sand pile	central sand pile

Note that there is substantial additivity in the equilibrium configurations. Attraction leads to a city in the center, and isolation leads to a uniform distribution. Combining the two (isolated attraction) yields a centrally located sand pile. Checking other pairs of effects reveals a common pattern. Therefore, there seems to be no meaningful externalities between the two effects.

4 Nonlinear Preferences

The model is now extended to include nonlinear effects. Two cases are considered: one with a nonlinear population term which shows that equilibria need not be efficient, and one with a nonlinear separation term which exhibits extreme sensitivity to initial conditions. In the first scenario, a negative second order effect on population is assumed. Standard explanations for negative external effect include congestion, pollution, crime, or inefficiencies in public good provision. The utility to an agent from being in location (i, j) can be written as

$$u(i, j, F) = F_{ij} - \frac{1}{k} \cdot (F_{ij})^2$$

where α is a positive constant. This change in preferences has a substantial impact on the dynamics of city formation. In the global relocation scenario, the dynamics begin similar to the linear case described above. The first agent chooses from among the locations with the largest population, and the second agent chooses the same location. However, at some point the "city" becomes overcrowded and an agent chooses the location with the second largest population. The result at the end of one round of relocations will be several cities, the exact number of which varies directly with α. All but the last of these cities to form will have identical populations. In the second round of relocations, agents move from the larger cities to the one smaller city until all cities have equal populations.

Global Relocations

$$u(i, j, F) = F_{ij} - \frac{1}{22} \cdot (F_{ij})^2$$

					20			
	20							
					20			
							20	
				20				

The size of these cities is not utility maximizing. The first city to form stops increasing in population only when agents prefer to live in a location with a small population. Therefore, the cities that form will be too large. The following claim states that beginning from a uniform distribution, the resulting cities are exactly twice as large as is optimal provided that the lattice is large relative to the population. A corollary to the claim states that the equilibrium distribution offers no higher utility than the initial distribution.

Claim 4.1 *Let $N^2 = m$, $u(i,j,F) = F_{ij} - \frac{1}{k} \cdot (F_{ij})^2$, where $k \geq 4$. Suppose initially that $F_{ij} = 1$ for all (i,j). Under global relocations the equilibria satisfy $F_{ij} \leq k-1$ for all (i,j).*

pf: By assumption, all locations have identical populations equal to one initially. An agent is selected at random and chooses from among the locations generating maximal utility. The agent obtains utility $1 - \frac{1}{k} \leq 1$ from her home location and $2 - \frac{4}{k} \geq 1$ from any other location. The second inequality follows from the assumption that $k \geq 4$. Let (i', j') denote the location that the first agent chooses. In the future, agents will choose this location until the utility from beginning another city exceeds the utility of joining this city. Let a denote the smallest number such that an agent would prefer to choose to remain alone at her location over a city with a agents, i.e.

$$a = \min\{b : 1 - \frac{1}{k} \geq b - \frac{b^2}{k}\}$$

The function $f(b) = b - \frac{b^2}{k}$ is single peaked with a negative second derivative. A simple calculation shows that $f(k-1) = 1 - \frac{1}{k}$. Therefore, no cities can form which have populations larger than $k-1$. Typically, cities no larger than $k-2$ will form as agents can pair with other isolated agents to form cities of size two rather than join the larger city.

Remember that agents relocate sequentially. The first agent will relocate in the location with the largest population. Subsequent agents choose the same location until

that location's population equals $k-2$. This is nearly double the optimal size which can be shown to be $\frac{k}{2}$. This intuition is formalized in the next claim.

Claim 4.2 *Suppose that* $u(i,j,F) = F_{ij} - \frac{1}{k} \cdot (F_{ij})^2$ *and that* $m = ck$, *where* c *is an integer. The utility maximizing distributions satisfy* $F_{ij} \in \{0, \frac{k}{2}\}$ *for all* (i,j).

pf: For $k > 0$, the function $G(x) = x - \frac{1}{k}x^2$ has a unique optimum at $x = \frac{k}{2}$. The proof follows immediately.

Although these claims do not require sophisticated mathematics, they are interesting nonetheless. There exist utility maximizing population distributions which are equilibria. In fact, every utility maximizing distribution is an equilibrium. Moreover, they are locally stable in the sense that if you randomly relocate a few agents, relocations will return you to these equilibria. Unfortunately, these locally stable equilibria are not likely to be realized if agents begin uniformly distributed across the space. In fact, the more likely equilibrium distributions have utility which is not much better than the uniform distribution.

In the local relocation scenario, the dynamics do not change significantly from when utility monotonically increases with population. Agents choose the location in their neighborhood with the largest population. Only in the case where k is quite small, so that congestion appears quickly, does behavior change.

If utility is not monotonic in average distance to other agents, then the equilibrium distribution changes in predictable ways, though it appears to be highly path dependent. Under global relocation, the agents tend toward the center initially and then separate. After the first round of relocations, the agents are spread nearly uniformly near the center of the lattice. In a series of trials, agents tended to concentrate in a few cities whose distance from the center depends on the size of the nonlinear terms. The system does not always stabilize. The next two figures show the enormous path dependence. Each represents an equilibrium distribution for the case $\alpha = -1.25$:

Local Relocations

$$u(i,j,F) = d^c(i,j,F) - 1.25(d^c(i,j,F))^2$$

	11						40	

	1							
	9	3					38	

		9.5			15.5			
	3.3							
9.3							7.4	
7.4							9.3	
	5					3.3		
		15.5			9.5			

Each of these distributions has approximately the same aggregate utility. A genetic algorithm attempting to find a utility maximizing distribution located these as utility maximizing distributions.

5 Discussion

This paper has examined the formation and emergence of cities, their spatial distributions, and the optimality of equilibrium distributions by employing a simple model. Two effects, separation and population, can create much of the interesting behavior seen in more sophisticated models. Path dependency results from preferences for population. Central placement is the residue of preferences to minimize distances. More interestingly, the transition from micro level preferences to macro level population distributions was shown to be far from intuitive. Including higher order effects for even simple preferences obtains equilibrium population distributions which would not have been predicted. Such findings suggest that a deeper analysis of this mapping would be worth undertaking before settling on a standard model or class of models for city formation.

This paper also addressed the notion of the emergence of cities. Emergence can be characterized as unpredicted order or structure which results from the interaction of simple local rules. Many earlier models, such as those in which agglomeration follows immediately from assumptions of increasing returns, fail to satisfy this definition of emergence. This paper has shown that cities can emerge in the strong sense – when agents wish to maximize their average distance to other agents, they accumulate into cities.

Finally, the skeletal model in this paper can be extended in many directions.

For example, natural advantage can be included by giving agents higher utility from residing at or near certain locations. The introduction of such features can alter findings substantially. Natural advantage tends to reduce the sensitivity to initial conditions in the pure agglomeration model. Other features such as population growth and heterogeneous preferences can also be included which will enable researchers to get a better handle on how the relocation incentives for agents contribute to the formation of cities.

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