10^{15} sec^{-1}, $\mathfrak{N} = 10^{23}$ cm^{-3}) gives $\delta = 1.2 \times 10^{-6}$ cm stat volt^{-1}, in good agreement with Miller's experimental value.

5A-2 Theory of the Optical Parametric Oscillator, A. Yariv, *California Institute of Technology, Pasadena, Calif.*, and W. H. Louisell, *Bell Telephone Laboratories, Inc., Murray Hill, N. J.*

We have considered the problem of parametric oscillation in the optical region. The mathematical approach consists of solving Maxwell equations in the normal-mode formulation[1] for the amplitudes of the pump (ω_p) signal (ω_s) and idler (ω_i) modes in the presence of a nonlinear optical medium characterized by an SHG-like tensor χ_{ijk} and with $\omega_p = \omega_i + \omega_s$. The derivation of the equations of motion for the normal mode amplitudes is similar to those of previous resonator analyses.[1,2] The original contribution of the paper, however, is the extension of the analysis into the nonlinear *above threshold* region. This requires that the pump energy density inside the optical resonator and the pump power input be considered as two independent physical quantities. The analysis shows that below threshold the two are proportional to each other, while above threshold the pump energy density saturates, in a manner analogous to the single-pass gain of a laser oscillator, at a constant value. The analysis is also used to obtain explicit expressions for the power outputs at ω_i and ω_s as a function of the pumping power. The conditions of optimum coupling are derived and the validity of the Manley-Rowe relations in the presence of losses is established.

[1] W. H. Louisell, A. Yariv, and A. E. Siegman, *Phys. Rev.*, vol. 124, p. 1646, 1961.
[2] R. H. Kingston and A. L. McWhorter, *Proc. IEEE*, vol. 53, pp. 4–12, January 1965.

5A-3 Measurement of Optical Second-Harmonic Generation in Centrosymmetric and Noncentrosymmetric Crystals Using Focused Gaussian Laser Beams,[1] J. E. Bjorkholm, G. E. Francois, and A. E. Siegman, *Microwave Laboratory, Stanford University, Stanford, Calif.*

The optical second-harmonic generation (SHG) caused by a focused, lowest order Gaussian laser beam in index-matched crystals of ADP and calcite has been analyzed in detail, and measured experimentally with CW 6328 Å laser beams, leading to the following results.

1) For a collimated laser beam, the dependence of SHG upon crystal orientation near index matching differs from the simple $(\sin\theta)^2/\theta^2$ dependence in small but significant details. These small differences are fully explained by the Gaussian plane-wave spectrum associated with the finite Gaussian transverse variation of the laser beam.

2) Using a collimated laser beam, the nonlinearity in ADP is found to be d_{36}(ADP, 6328 Å) $= 1.36 \times 10^{-9}$ esu $\pm$ 12 percent. The estimated error is significantly smaller then previous measurements. The enhancement factor $(2n-1)/n$ due to n randomly phased longitudinal modes is experimentally checked.

3) A general analytical expression is developed for the case of a lowest order Gaussian beam focused to a focal spot of size w_0 in a nonlinear crystal of length l, at or near index-matching. The amount of SHG depends upon the ratios of l to a "Rayleigh length" $z_R = \pi w_0^2/\lambda$ characteristic of the focal region and a double-refraction length $l_\alpha = w_0/\alpha$, where α is the angle of double refraction in the index-matched crystal. Going in the direction of decreasing focal spot size w_0, there are in general four asymptotic regions, in which the amount of SHG varies as w_0^{-2}, w_0^{-1}, w_0, and w_0^2. Between the middle two regions, there is an optimum degree of focusing, for maximum SHG, occurring when $z_R = l/\pi$; the amount of SHG at optimum focusing varies as $l^{\frac{1}{2}}$. (For very short crystals or for crystals in which α is very small not all of the four regions exist, and the optimum condition is modified.)

These analytical results are verified in detail using a wide range of degrees of focusing in ADP crystals of three different lengths. The absolute value of d_{36} in ADP when measured with a focused beam also agrees very closely with the value measured with a collimated beam, thus further verifying the analysis.

4) Quadrupolar SHG, dc electric-field-induced SHG (ESHG), and the classical Kerr effect have been measured in the centrosymmetric crystal calcite using a focused CW 6328 Å laser beam. The observed quadrupolar effect in calcite, compared to d_{36} in ADP, has a magnitude $\chi_{eff}'/d_{36} = 0.85 \times 10^{-3} \pm 14$ percent, or somewhat smaller than the value of a similar (but not identical) quantity measured with pulsed ruby lasers by Terhune et al. The observed coefficient for ESHG, relative to ADP, was $\chi_{51}/d_{36} = 2.1 \times 10^{-5} \pm$ 24 percent. The measured effective Kerr coefficient (not corrected for the combined effects of photoelasticity and stresses proportional to the square of the electric field) was $\chi_\beta = 2.0 \times 10^{-14}$ esu $\pm$ 16 percent. The center of the quadratic variation of ESHG with dc electric field was observed to be displaced from zero dc field, as previously observed by Terhune et al. One previously proposed explanation of this displacement as due to inhomogeneous dc electric fields is ruled out by these experiments. Alternative explanations remaining are surface harmonic generation (previously proposed) and the effects of a quadrupolar correction to the Poyning theorem.

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5A-4 Theoretical and Experimental Studies of Optical Second-Harmonic Generation in Birefringent Crystals, D. D. Bhawalkar, R. C. Smith, and L. S. Watkins, *Department of Electronics, Southampton University, Southampton, England.*

A theory has been developed to describe second-harmonic generation (SHG) from a laser beam for the various modes of a resonator. The scalar diffraction integral is solved for the propagation of an extraordinary harmonic wave close to index matching. The approximations involved are identical to those used in deriving the modes of a resonator. The theory developed is completely general and can be applied for crystals of any length, 1. Kleinman et al.[1,2] have treated two special cases where $1 \gg z_0$ and $1 \ll z_0$ ($2z_0$ is the confocal parameter of the beam). Our theory can be used for these regions but the experimental and theoretical results to be presented apply to the region where $1 \simeq z_0$ which is most important for practical devices.

A detailed solution of the diffraction integral in a birefringent medium shows that the effect of anisotropy can be described completely by moving the source by $z \tan\alpha$ (z is the distance traveled in the medium and α is the angle between the Poynting vector and the wave normal). This result is applied to the propagation of waves radiating from the second harmonic polarization generated by a Gaussian beam. The expression for the SH polarization is obtained by squaring the expression for the field in the Gaussian beam. The diffraction integral for the harmonic wave has to be integrated over the x and y directions which are normal to the propagation direction z, and also along the z direction to include SHG along the entire length of the crystal. The first two integrations can be performed exactly but the final integral has to be carried out numerically. The effects of absorption and scatter at both the fundamental and harmonic frequencies are included.

Two types of calculation are made, one of the form of the harmonic beam at the exit face of the crystal which we define as the near-field pattern, and the other of the form of the beam at large distances defined as the far-field pattern. Both the intensity and phase distributions are computed for these two cases. A number of computations have been made of both the near-field and far-field patterns of harmonic from a TEM_{00q} beam with $2z_0 = 31.4$ mm for ADP crystals of thicknesses between 3 and 40 mm. An approximation for the near field has been found which avoids numerical integration but gives results in good agreement with the exact theory. Far-field patterns have been computed for different positions of the crystal in the Gaussian beam and the SHG efficiency obtained. Calculations have also been performed to obtain the efficiency for various crystal thicknesses.

The field patterns of second harmonic generated by TEM_{0q} have also been investigated. No exact calculations have been carried out for this mode since an approximate solution has been found adequate to describe the different patterns. In this description the harmonic wave can be expressed as two orthogonal modes, TEM_{00q} and TEM_{01q} modes, with a phase difference between them which is determined by the position of the crystal in the beam. As a result of this phase difference the far-field pattern consists of three maximas when the

[1] Boyd et al., *Phys. Rev.*, vol. 137, p. A1305, February 1965.
[2] Kleinman et al., private communication.