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The Core of an Economy With Multilateral Environmental Externalities

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Abstract

When environmental externalities are international–i..e. transfrontier–they most often are multilateral and embody public good characteristics. Improving upon inefficient laissez-faire equilibria requires voluntary cooperation for which the game-theoretic core concept provides optimal outcomes that have interesting properties against free riding.

To define the core, however, the characteristic function of the game associated with the economy (which specifies the payoff achievable by each possible coalition of players–here, the countries) must also specify in each case the behavior of the players which are not members of the coalition. This has been for a long time a major unsolved problem in the theory of the core of economies with many producers of a public good.

Among the several assumptions that can be made in this respect, a plausible one is defined in this paper, for which it is then shown that the core is nonempty. The proof is constructive in the sense that it exhibits a solution (i.e., an explicit coordinated abatement policy) that has the desired property of nondomination by any proper coalition of countries, given the assumed behavior of the other countries.

The Core of an Economy With Multilateral Environmental Externalities*

Parkash Chander Henry Tulkens

1 Introduction

We deal in this paper with an economy with several agents, whose productive activities generate "multilateral" externalities, i.e., externalities that each one of them can both generate and be a recipient of. We have in mind externalities that are detrimental for the recipients. We also call these externalities "environmental" because we assume that they exhibit public goods (actually, public "bads") characteristics in the sense that when they are generated, they affect all agents in the economy. We should also call them "additive" because we further assume that what is received by any recipient is simply the sum of what is emitted by the various generators.

Being motivated by an interest of long standing[1] for the sources of cooperation between countries on issues of transfrontier pollution (an excellent example of multilateral environmental externalities), we call upon the cooperative game theoretic concept of core for identifying joint actions that would improve upon inefficient laissez-faire equilibria by means of voluntary cooperation, and achieve more than mere optimality. In this interpretation, our model below is to be seen as that of an international economy, with the

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[1]A first version of the international environmental model alluded to here appeared in TULKENS 1979, and was elaborated upon from a game theoretic point of view in CHANDER and TULKENS 1991 and 1992a and b.

countries being also the players in the games associated with it.[2] Other interpretations are conceivable, however.

To define the core, the characteristic function of the game–which specifies the payoff achievable by each possible coalition of players–must also specify, in the case of games with externalities, the behavior of the players which are *not* members of the coalition. This is a disputed issue, and alternative assumptions in this respect lead to alternative core concepts such as, *e.g.* the α- and β -cores, which are contrasted to each other in an externalities context by SCARF 1971, STARRETT 1972 and LAFFONT 1977 (chapter V).[3] In discussing his externalities model applied to acid rains, MÄLER 1989 claims that the core concept (actually a particular version of the α-core, as we shall show below) is useless because it results in too large a set of outcomes.

Among the several assumptions that can be made on the behavior of players outside each coalition, we propose one in this paper which we think is a plausible one in the context of environmental externalities, and for which we show that the core is nonempty. The proof is constructive in the sense that it exhibits a solution (*i.e.*, an explicit coordinated abatement policy, in our pollution interpretation) that has the desired property of nondomination by any proper coalition of players, given the assumed behavior of the other players.

2 The Model of an Economy with Multilateral Environmental Externalities

The agents of the economy (countries in our international interpretation) are indexed by i and denoted by the set $N = \{i \mid i = 1, 2, \ldots, n\}$. Three categories of commodities are considered: (i) a standard private good, whose quantities are denoted by $x \geq 0$ if they are consumed, and by $y \geq 0$ if they are produced; (ii) pollutant discharges, the quantities of which are denoted by $p \geq 0$; and (iii) ambient pollutant quantities, denoted by $z \leq 0$.

Each agent i's preferences are represented by a utility function $u_i(x_i, z)$ satisfying:

Assumption 1: $u_i(x_i, z) = x_i + v_i(z)$, quasi-linearity; and

Assumption 2: $v_i(z)$ concave, differentiable and such that $\frac{dv_i}{dz} \equiv \pi_i(z) > 0$ for all $z \geq 0$.

With each agent i there is furthermore associated a technology, described by the production function $y_i = g_i(p_i)$, satisfying:

Assumption 3: $g_i(p_i)$ strictly concave and differentiable over an interval; and

[2]The "acid rain game" of MÄLER 1989 is a similar international environmental model, to which we intend to apply, in CHANDER and TULKENS 1994, the concepts and results developed here.

[3]The issue also arises in games associated with economies with public goods, as in FOLEY 1970, CHAMPSAUR 1975, MOULIN 1987 and CHANDER 1993.

Assumption 4: There exists a $p_i^o > 0$ such that

$$\frac{dy_i}{dp_i} \equiv \gamma_i(p_i) \begin{cases} > 0 \text{ if } \; p_i < p_i^o & \text{(i)} \\ = 0 \text{ if } \; p_i \geq p_i^o & \text{(ii)} \\ = \infty \text{ if } \; p_i = 0. & \text{(iii)} \end{cases}$$

Inputs, which are not explicitly mentioned in the production functions, are subsumed in the functional symbols g_i. Although this amounts to treat them as fixed in the analysis that follows, our results will not rest in an essential way on that assumption, which is made here mostly for expositional convenience.

Finally, all possible behaviors within the economy, in terms of the consumption, production and pollutant discharge decisions taken by its agents, as well as in terms of the resulting values of ambient pollutant are formally described by feasible states:

Definition 1: **Feasible states** of the economy (or "allocations") are vectors

$$(x, p, z) \equiv (x_1, \ldots, x_n; p_1, \ldots, p_n; z)$$

such that

$$\sum_{i \in N} x_i \leq \sum_{i \in N} g_i(p_i) \tag{1}$$

and

$$z = -\sum_{i \in N} p_i. \tag{2}$$

3 Efficient and Equilibrium States of the Economy

To provide background to our search for an appropriate core concept, which is of a cooperative nature, we first define in this section efficient states of which core allocations, when they exist, are a subset, and then define various equilibrium concepts intended to describe alternative states of the economy under either absence of cooperation or only partial cooperation.

Definition 2: **A Pareto efficient state** of the economy is a feasible state (x, p, z) such that there exists no other feasible state (x', p', z') for which $u_i(x_i', z') \geq u_i(x_i, z)$ for all $i \in N$ with strict inequality for at least one i.

To characterize efficient states, the usual first order conditions take in this case the form of the following system of equalities:

$$\sum_{j \in N} \pi_j(z) = \gamma_i(p_i), i = 1, 2, \ldots, n. \tag{3}$$

Henceforth, we shall always write π_N for $\sum_{j \in N} \pi_j$. Notice that, due to Assumption 4(iii), one has $p_i > 0$ for all i in any efficient state. It is easily seen that there exists a Pareto efficient state.

Proposition 1: Assumptions 1 and 3 imply that in all Pareto efficient states, the vector of emission levels $(p_1, \ldots, p_n)$ is the same.

Proof: Suppose not, and let $(\bar{x}, \bar{p}, \bar{z})$ and $(\tilde{x}, \tilde{p}, \tilde{z})$ be two states, both Pareto efficient, with $\bar{p}_i \neq \tilde{p}_i$ for some i. On the one hand, if $\tilde{z} = \bar{z}$, then $\pi_N(\tilde{z}) = \pi_N(\bar{z})$ implies by (3) that $\gamma_i(\bar{p}_i) = \gamma_i(\tilde{p}_i) \ \ \forall i \in N$; but this, by strict concavity of all functions $g_i(p_i)$, is only possible if $\tilde{p}_i = \bar{p}_i \ \ \forall i$. If, on the other hand, $\tilde{z} \neq \bar{z}$, say $\tilde{z} < \bar{z}$ without loss of generality, the fact that $\pi_N(\tilde{z}) \geq \pi_N(\bar{z})$ now implies by (3) that $\gamma_i(\tilde{p}_i) \geq \gamma_i(\bar{p}_i) \ \ \forall i$, which in turn implies by concavity that $\tilde{p}_i \leq \bar{p}_i \ \ \forall i$. However, since $\tilde{z} = -\sum_{i \in N} \tilde{p}_i$ and $\bar{z} = -\sum_{i \in N} \bar{p}_i$, one would have $\tilde{z} \geq \bar{z}$, which is a contradiction.

Q.E.D. ∎

Let $x^o = \sum_{i \in N} g_i(p_i^o)$. Then x^o is the maximum amount of total consumption.

We now consider each agent i of our economy as a player in an n-person noncooperative game. To this effect, let

$$T_i = \{(x_i, p_i) | 0 \leq p_i \leq p_i^o; 0 \leq x_i \leq x^o\}, i \in N$$

be the strategy set of player i, and $T = T_1 \times \ldots \times T_n$ be the space of joint strategies of all players. Any joint strategy choice $[(x_1, p_1), \ldots, (x_n, p_n)] \in T$ of the players induces a feasible state (x, p, z) of the economy if (x, p, z) satisfies (1) and (2).

Then, if for each $i = 1, \ldots, n$ and any $[(x_1, p_1), \ldots, (x_n, p_n)] \in T$ we choose $u_i(x_i, z) = x_i + v_i(z)$ with $z = -\sum_{j \in N} p_j$ as the payoff for player i, and write $u = (u_1, \ldots, u_n)$, we have defined a noncooperative game $[N, T, u]$, associated with the economy.

Definition 3: For the noncooperative game $[N, T, u]$, the joint strategy choice $[(\bar{x}_1, \bar{p}_1), \dots, (\bar{x}_n, \bar{p}_n)]$ is a **Nash equilibrium** if for all $i \in N, (\bar{x}_i, \bar{p}_i, \bar{z})$ maximizes $x_i + v_i(z)$ subject to $x_i \leq g_i(p_i)$ and $p_i + z = -\sum_{j \neq i} \bar{p}_j$.

Definition 4: For the economy, a **disagreement equilibrium** is the state $(\bar{x}_1, \dots, \bar{x}_n; \bar{p}_1, \dots, \bar{p}_n; \bar{z})$ induced by the Nash equilibrium $[(\bar{x}_1, \bar{p}_1), \dots, (\bar{x}_n, \bar{p}_n)]$ of the game $[N, T, u]$.

To characterize a Nash equilibrium or a disagreement equilibrium, the first order conditions of the maximization problem in Definition 3 yield the well known system of equalities:

$$\pi_i(\bar{z}) = \gamma_i(\bar{p}_i), i = 1, \dots, n. \tag{4}$$

From the fact that the system (4) differs from (3), the standard statement is derived that a disagreement equilibrium is not an efficient state of the economy. Furthermore, it will be useful to establish the following two properties:

Proposition 2: For the game $[N, T, u]$ there exists a Nash equilibrium which is unique.

Proof: The existence of a Nash equilibrium follows from standard theorems (see, e.g., FRIEDMAN 1990), whose conditions require that each player's strategy set be compact and convex, and each player's payoff function be concave, continuous, and bounded. All these conditions are obviously met in our model.

To prove uniqueness of the equilibrium $[(\bar{x}_1, \bar{p}_1), \dots, (\bar{x}_n, \bar{p}_n)]$, suppose contrary to the assertion that there exists another Nash equilibrium

$$[(\hat{x}_1, \hat{p}_1), \dots, (\hat{x}_n, \hat{p}_n)] \neq [(\bar{x}_1, \bar{p}_1), \dots, (\bar{x}_n, \bar{p}_n)].$$

Without loss of generality assume that $\sum \hat{p}_i \leq \sum \bar{p}_i$, entailing $\hat{z} = -\sum \hat{p}_i \geq \bar{z} = -\sum \bar{p}_i$. From the characterization of the disagreement equilibrium and the concavity of the functions $v_i(z)$, we would then have $\pi_i(\hat{z}) \leq \pi_i(\bar{z})$ as well as $\gamma_i(\hat{p}_i) \leq \gamma_i(\bar{p}_i)$ for each $i \in N$. But given the concavity of the production functions, this last inequality would imply that $\hat{p}_i \geq \bar{p}_i$ for each $i \in N$, which contradicts the assumption that $\sum \hat{p}_i \leq \sum \bar{p}_i$, and $[(\hat{x}_1, \hat{p}_1), \dots, (\hat{x}_n, \hat{p}_n)] \neq [(\bar{x}_1, \bar{p}_1), \dots, (\bar{x}_n, \bar{p}_n)]$.

Q.E.D.
∎

Turning to the economy, the existence and uniqueness of a disagreement equilibrium state is also established by Proposition 2.

Definition 5: For the noncooperative game $[N, T, u]$, the joint strategy choice $[(\approx{x}_1, \approx{p}_1), \dots, (\approx{x}_n, \approx{p}_n)]$ is a **strong Nash equilibrium** if for all $S \subset N$,

$$[(\approx{x}_i)_{i\in S}, \approx{z}] \text{ maximizes } \sum_{i\in S} [x_i + v_i(z)] \tag{5.a}$$

$$\text{subject to } \sum_{i\in S} x_i \leq \sum_{i\in S} g_i(p_i) \tag{5.b}$$

$$\text{and } \sum_{i\in S} p_i + z = \begin{cases} 0 \text{ if } S = N \\ -\sum_{j\in N\setminus S} \approx{p}_j \text{ if } S \neq N. \end{cases} \tag{5.c}$$

Note that, unlike the Nash equilibrium, a **strong Nash equilibrium**, if one exists, induces a Pareto efficient state of the economy.

Proposition 3: For the game $[N, T, u]$, there does not exist a strong Nash equilibrium.

A proof of Proposition 3 follows from Proposition 2, which shows that the Nash equilibrium is unique, and the observation made earlier that the Nash equilibrium does not induce a Pareto efficient state of the economy. Since a strong Nash equilibrium is also a Nash equilibrium and induces a Pareto efficient state of the economy, it follows that there exists no strong Nash equilibrium for the game $[N, T, u]$.[4]

BERNHEIM, PELEG, and WHINSTON (1987) note that strong Nash equilibria almost never exist or the strong Nash concept is "too strong". Therefore, they propose an alternative equilibrium concept: namely; coalition-proof Nash equilibrium. However, as in the case of strong Nash equilibrium, the concept of coalition-proof Nash equilibrium involves the assumption that when a coalition deviates, it takes as given, the strategies of the complement.[5]

Turning to the cooperative part of our analysis, let us now associate, with every coalition S, the number $w(S)$, called the "worth" of the coalition S and defined as the highest aggregate payoff $\sum_{i\in S} u_i$ that the members of the coalition can achieve using some strategy. Thus, the pair $[N, w(.)]$, consisting of the players set N and the characteristic function $w(.)$, defines a cooperative game (with transferable utilities) associated with our economy.

Stated in full, the characteristic function reads

$$w(S) = \sum_{i\in S} [x_i + v_i(z)]. \tag{6}$$

For the association of this function with our economy to be meaningful–i.e. to correspond to feasible states, the variable z should satisfy condition (2). However, this equality involves strategic choices made by players who are *not* members of S. Thus, the

[4]MÄLER (1989) shows that there exists no strong Nash equilibrium for the "acid rain game".

[5]Moreover, it can be shown that for the game $[N, T, u]$ no coalition-proof Nash equilibrium is Pareto efficient, if there exists one at all.

worth of a coalition in our game is not only a function of actions taken by its members, but also of actions of players outside the coalition.

This typical feature of cooperative games associated with economies with externalities and/or public goods requires the characteristic function to specify explicitly what the actions are of both the members of S and of the other players. A familiar way to get around this problem has been to assume that the players outside the coalition adopt those strategies that are least favorable to the coalition. That is, the characteristic function is defined as

$$w^{\alpha}(S) = \max_{\{x_i, i\in S, z\}} \sum_{i\in S}[x_i + v_i(z)] \tag{7.a}$$

$$\text{subject to } \sum_{i\in S} x_i \leq \sum_{i\in S} g_i(p_i) \tag{7.b}$$

$$\sum_{i\in S} p_i + z \begin{cases} = & -\sum_{j\in N\setminus S} p_j^o \text{ if } S \neq N \\ = & 0 \text{ if } S = N. \end{cases} \tag{7.c}$$

Games with characteristic function of this type are used in many studies of economies with public goods (see, e.g., FOLEY 1970, CHAMPSAUR 1975, MOULIN 1987 and CHANDER 1993). This is also the case with the game used by SCARF 1971 in his study of economies with externalities: his "α-characteristic" function is similar to the one defined above. Pushing the "pessimistic" view to the extreme, MÄLER 1989 assumes in his model that there is no upper bound such as our p_i^o for the individual pollutant discharges; thus, coalitions can be hurt by up to infinite amounts of pollutants emitted by players outside the coalition. Such strategies may, however, not maximize the individual payoffs of these players.

On the one hand, the notions of strong Nash equilibrium and coalition-proof Nash equilibrium assume that when a coalition deviates it takes as given the strategies of its complement. On the other hand, the notion of α-core presumes that when a coalition deviates, its payoff is what it would get when members of the complementary coalition act to minimax this coalition payoff. There is no reason why the complimentary coalition should behave in this war-like fashion, and there is no reason for the deviating coalition to necessarily expect or fear such behavior.[6]

We therefore introduce a concept in which when a coalition deviates it does not take as given the strategies of its complement, nor does it fear the worst. It instead looks ahead to a resulting equilibrium that its actions induce. More specifically, we assume that when S deviates, the members of $N\setminus S$ break-up into singletons and the resulting equilibrium is the Nash equilibrium between S and the remaining players with members of S playing best response to the strategies of others.[7]

[6]A similar conceptual criticism can be made of the β-core.

[7]Our analysis might be extendable to the case when $N\setminus S$ may continue as a coalition, but we do not pursue this possibility in the present paper.

Let $S \subset N$ be some coalition and, associated with it, the set T^S defined as

$$T^S = \{(x_i, p_i)_{i \in S} | 0 \leq p_i \leq p_i^o, x_i \geq 0, \ \ \forall i \in S; \sum_{i \in S} x_i \leq \sum_{i \in S} g_i(p_i)\}, \tag{8}$$

that is the set of strategies accessible to the members of S as a coalition. As the last inequality in this definition allows for some members of S possibly to consume more, or less, than what they produce, such a strategy set includes the possibility of transfers of private goods among the members of the coalition. The inequality also implies that the algebraic sum of these transfers be zero within the coalition.

We now introduce the following concept:

Definition 6: For the economy, given a coalition S, a **partial agreement equilibrium with respect to** S is the joint strategy choice $[(\tilde{x}_1, \tilde{p}_1), \ldots, (\tilde{x}_n, \tilde{p}_n)] \in T$ where

$$\text{(i)} \ \ [(\tilde{x}_i)_{i \in S}, \tilde{z}] \ \text{ maximizes } \ \sum_{i \in S} [x_i + v_i(z)] \tag{9.a}$$

$$\text{subject to } \ \sum_{i \in S} x_i \leq \sum_{i \in S} g_i(p_i) \tag{9.b}$$

$$\text{and } \ \sum_{i \in S} p_i + z = - \sum_{j \in N \setminus S} \tilde{p}_j, \tag{9.c}$$

$$\text{(ii)} \ \ \forall j \in N \setminus S, (\tilde{x}_j, \tilde{z}) \ \text{ maximizes } \ x_j + v_j(z) \tag{9.d}$$

$$\text{subject to } \ x_j \leq g_j(p_j) \tag{9.e}$$

$$\text{and } \ p_j + z = - \sum_{i \neq j} \tilde{p}_i, \ \text{ and} \tag{9.f}$$

$$\text{(iii)} \qquad \tilde{z} = - \sum_{j \in N} \tilde{p}_j. \tag{9.g}$$

The joint strategy choice that induces this state of the economy may be seen as a Nash equilibrium$[(\tilde{x}_1, \tilde{p}_1), \ldots, (\tilde{x}_n, \tilde{p}_n)]$ between the coalition S acting as one individual player and the other players acting alone.

In terms of first order conditions, a partial agreement equilibrium $[(\tilde{x}_1, \tilde{p}_1), \ldots, (\tilde{x}_n, \tilde{p}_n)]$ with respect to a coalition S is characterized by the system of n equalities:

$$\sum_{j\in S}\pi_j(\tilde{z}) = \gamma_i(\tilde{p}_i), i \in S$$

and

$$\pi_j(\tilde{z}) = \gamma_j(\tilde{p}_j), j \in N\backslash S,$$

where

$$\tilde{z} = -\sum_{i\in N}\tilde{p}_i.$$

Proposition 4: For any proper coalition $S \subset N$,

(i) there exists a partial agreement equilibrium with respect to S;

(ii) the individual emission levels corresponding to such equilibrium are unique;

(iii) the individual emission levels of the players outside S are not lower than those at the disagreement equilibrium,

(iv) although the total emissions are not higher.

Proof: The existence proof follows from similar arguments as in Proposition 2. The uniqueness of individual emission levels follows from similar arguments as in Proposition 1. We therefore only formally prove here the remaining two parts of the proposition.

Let $\tilde{z} = -\sum\tilde{p}_i$ and $\bar{z} = -\sum\bar{p}_i$, where $(\tilde{p}_1,\ldots,\tilde{p}_n)$ and $(\bar{p}_1,\ldots,\bar{p}_n)$ are the emission levels corresponding to a partial agreement equilibrium with respect to S and to the disagreement equilibrium, respectively. We first show that $\tilde{z} \geq \bar{z}$.

Suppose contrary to the assertion that $\tilde{z} < \bar{z}$. We must then have $\pi_i(\tilde{z}) \geq \pi_i(\bar{z})$ for each $i \in N$. From the characterizations of a partial agreement equilibrium and of the disagreement equilibrium it follows that

$$\sum_{j\in S}\pi_j(\tilde{z}) = \gamma_i(\tilde{p}_i) \geq \gamma_i(\bar{p}_i) = \pi_i(\bar{z}), \forall i \in S$$

and

$$\pi_j(\tilde{z}) = \gamma_j(\tilde{p}_j) \geq \gamma_j(\bar{p}_j) = \pi_j(\bar{z}), \forall j \in N \backslash S.$$

From the strict concavity of each function g_i, it follows that $\tilde{p}_i \leq \bar{p}_i$ for each $i \in N$. But this contradicts our supposition that $\tilde{z} < \bar{z}$. Hence, we must have $\tilde{z} \geq \bar{z}$.

Finally, since $\tilde{z} \geq \bar{z}, \pi_j(\tilde{z}) \leq \pi_j(\bar{z})$ for each $j \in N \backslash S$. The inequalities above and strict concavity of g_j imply that $\tilde{p}_j \geq \bar{p}_j$ for each $j \in N \backslash S$.

Q.E.D.
∎

Note that in view of Assumption 2, the total emissions corresponding to a partial agreement equilibrium with respect to a coalition of two or more players are strictly lower than those corresponding to the disagreement equilibrium. This means that as compared to the disagreement equilibrium, the players outside a coalition of two or more players are strictly better-off at a partial agreement equilibrium with respect to that coalition–which is actually a form of free riding on the part of those players.

We introduce now an alternative characteristic function–hence an alternative game; namely, the game $[N, w^\gamma]$ defined by the "partial agreement" characteristic function:

$$w^\gamma(S) = \max_{\{x_i, i \in S, z\}} \sum_{i \in S} [x_i + v_i(z)] \tag{10.a}$$

$$\text{subject to } \sum_{i \in S} x_i \leq \sum_{i \in S} g_i(p_i) \tag{10.b}$$

$$\text{and } \sum_{i \in S} p_i + z = -\sum_{j \in N \backslash S} p_j, \tag{10.c}$$

$$\text{where } \forall j \in N \backslash S, (x_j, z) \text{maximizes } x_j + v_j(z) \tag{10.d}$$

$$\text{subject to } x_j \leq g_j(p_j) \tag{10.e}$$

$$\text{and } p_j + z = -\sum_{i \neq j} p_i. \tag{10.f}$$

Here it is assumed that when coalition S forms, the players outside the coalition choose their strategies according to Nash behavior, given the strategies of others, so as to maximize their own individual payoffs. And S knows this and similarly chooses its strategy (acting as one individual player) so as to maximize its own payoff. The worth of coalition S is therefore equivalent to its payoff that corresponds to a partial agreement equilibrium with respect to S. It is thus not assumed that the players outside the coalition S do the worst; nor is it assumed, as in the concepts of strong and coalition-proof Nash equilibria, that they do not react to the actions of S. Instead they are assumed to act noncooperatively so as to reach an equilibrium in their own best individual interest.

It may be noted that for each S, the value assigned by the "partial agreement" characteristic function w^γ is at least as much as that assigned by the "α-characteristic" function w^α, i.e., $w^\gamma(S) \geq w^\alpha(S)$ for each $S \subset N$. In fact, examples can be constructed such that $w^\gamma(S) > w^\alpha(S)$ for all $S \subset N$. This means that the core of the game $[N, w^\gamma]$, i.e, the "γ-core" is, if nonempty, contained in the "α-core"; and possibly smaller.

If we assume that when a coalition S forms, the complementary coalition $N \backslash S$ also forms and chooses the best response strategy for its members, given what S does, then the core of the so-defined game, if nonempty, might be smaller than even that of the game $[N, w^\gamma]$.[8]

4 A Strategy in the Core of Games with Linear Payoff Functions

Thanks to the specification of a characteristic function, every strategy that a coalition may consider can be evaluated in terms of its aggregate payoff, and compared with the payoff yielded by strategies that other coalitions might contemplate. In particular, considering the coalition N of all players, we have:

Definition 7: A strategy of the coalition N is said to belong to the **core** of the cooperative game $[N, w]$ if the payoff it yields for each coalition is larger than the payoff $w(S)$ that any coalition $S \subset N$ can achieve.

If it is impossible to find a strategy for N with that property, the core of the game is empty.

Emptiness or nonemptiness of the core typically depends upon the form of the characteristic function, and more specifically upon the power that the game assigns to each coalition. In the presence of externalities, this power is crucially affected by the assumed behavior of the players outside the coalition. Thus, with the α-characteristic function,

[8]CARRARO and SINISCALCO 1993 in a model with identical agents assume that when S forms and achieves the aggregate payoff $w(S)$, if some $i \in S$ leaves S, the coalition $S \backslash \{i\}$ remains formed. They show that then it may be better for i to leaves S; as this advantage grows with the size of coalition, they conclude that only small coalitions can prevail, and N will never form.

coalitions are weakened by the presumed minimax behavior, letting hope that the corresponding α-core be nonempty, as is the case in SCARF 1971, in the version given by LAFFONT 1977 of the SHAPLEY and SHUBIK 1969 "garbage game",[9] as well as in MÄLER's 1989 "acid rain game".

The former two results, however, do not bear on an economy with externalities of the environmental type we are dealing with here, and Mäler's argument is only an informal one. Furthermore, no results are available, to our knowledge, for the type of characteristic function we have introduced above. We therefore shall tackle directly, i.e. from scratch, our central issue of whether any core property can be established for the cooperative game defined in the previous section.

To find out whether or not the core of any cooperative game is nonempty various approaches can be used. Two qualitative ones are offered by SCARF 1967 and SHAPLEY 1971 who respectively show nonemptiness if the game is balanced or convex. None of these proved successful in our case. We therefore turn to another, constructive, approach, which is to exhibit a strategy and show that it satisfies Definition 7.

We follow here this latter approach. Specifically, let $(x^*, p^*, z^*) = (x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$ be the Pareto efficient state defined as follows

$$x_i^* = g_i(\bar{p}_i) - \frac{\pi_i^*}{\pi_N^*}\Big(\sum_{i\in N} g_i(\bar{p}_i) - \sum_{i\in N} g_i(p_i^*)\Big), i \in N, \tag{11}$$
$$z^* = -\sum p_i^*,$$

where $\pi_i^* = \pi_i(z^*)$ for each $i \in N$ and $(p_1^*, \ldots, p_n^*)$ and $(\bar{p}_1, \ldots, \bar{p}_n)$ are the (unique) individual emission levels corresponding to the Pareto efficient states and to the disagreement equilibrium, respectively.[10] By Pareto efficiency $\pi_N^* = \gamma_i(p_i^*)$ for each $i \in N$.

As can be easily seen, the above implies that for each $i \in N$,

$$(x_i^*, p_i^*, z^*) \text{ maximizes } x_i + v_i(z)$$

subject to

$$x_i \leq g_i(\bar{p}_i) - \frac{\pi_i^*}{\pi_N^*}\Big(\sum_{j\in N} g_j(\bar{p}_j) - \sum_{j\neq i} g_j(p_j^*) - g_i(p_i)\Big),$$

[9] This is also the case in the many studies of economies with public goods alluded to above of FOLEY 1970, CHAMPSAUR 1975, MOULIN 1987 and CHANDER 1993, where the unfavorable behavior consists in producing no public good at all; these have typically large cores.

[10] CHANDER 1993 analyzes an instantaneous analog of this cost sharing rule in a public good context.

$$p_i + z = -\sum_{j \neq i} p_j^*$$

and

$$0 \leq p_i \leq p_i^o.$$

This means that the state $(x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$ is an equilibrium concept. As it can be given an interpretation analogous to that of the ratio equilibrium (KANEKO 1977; MAS-COLELL and SILVESTRE 1989), we shall refer to it as **the ratio equilibrium with respect to the disagreement equilibrium**.[11]

We prove the nonemptiness of the core by showing that the ratio equilibrium with respect to the disagreement equilibrium belongs to the core of the game $[N, w^\gamma]$.

We first consider a special case of our general model, namely the one where it is assumed that the payoff functions are linear, i.e.:

Assumption 1$'$: $u_i(x_i, z) = x_i + \bar{\pi}_i z, \bar{\pi}_i > 0$.

This is actually the case for which MÄLER 1989 proves the nonexistence of a strong Nash equilibrium. Note also that in SHAPLEY and SHUBIK's 1969 garbage game, the payoff functions are linear (unlike here, however, their externalities are directional and involve no diseconomies of scale).

As can be easily seen from the characterizations of the Nash equilibrium and of a partial agreement equilibrium, an important consequence of the linearity assumption is that in a partial agreement equilibrium with respect to a coalition the emission levels of the players outside the coalition are the same as in the Nash equilibrium. In fact, under the linearity assumption the Nash equilibrium is a dominant strategy equilibrium.

Theorem 1 Under the linearity Assumption 1$'$, the strategy for the grand coalition N that induces the ratio equilibrium (x^*, p^*, z^*) of the economy belongs to the core of the game $[N, w^\gamma]$.

Proof: Suppose contrary to the assertion, that the strategy inducing $(x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$ is not in the core of the game $[N, w^\gamma]$. Then there exists a coalition $S \subset N$ and a strategy for S inducing the feasible state $(\tilde{x}_1, \ldots, \tilde{x}_n; \tilde{p}_1, \ldots, \tilde{p}_n; \tilde{z})$ such that $(\tilde{x}_1, \ldots, \tilde{x}_n; \tilde{p}_1, \ldots, \tilde{p}_n; \tilde{z})$ is a partial agreement equilibrium with respect to S, and $\tilde{x}_i + \bar{\pi}_i \tilde{z} > x_i^* + \bar{\pi}_i z^*$ for all $i \in S$. From the characterization of a partial agreement equilibrium with respect to a

[11]Note that one can also consider the Pareto efficient state defined as: $x_i^* = g_i(p_i^o) - \frac{\pi_i^*}{\pi_N^*}(\sum_{i \in N} g_i(p_i^o) - \sum_{i \in N} g_i(p_i^*)), i \in N$. But we are unable to establish the same properties for this Pareto efficient state. It seems that the reference point matters.

coalition, it follows that $\tilde{p}_i = \bar{p}_i$ for all $i \in N \backslash S$, that $\tilde{p}_i \geq p_i^*$ for all $i \in S$, and that $\sum_{i \in S} \tilde{x}_i = \sum_{i \in S} g_i(\tilde{p}_i)$.

Consider now the alternative efficient state $(\hat{x}_1, \ldots, \hat{x}_n; p_1^*, \ldots, p_n^*; z^*)$, defined as:

$$\hat{x}_i = g_i(\tilde{p}_i) - \frac{\bar{\pi}_i}{\bar{\pi}_N}(\sum_{i \in N} g_i(\tilde{p}_i) - \sum_{i \in N} g_i(p_i^*)), i \in N, \tag{12}$$
$$z^* = -\sum_{i \in N} p_i^*.$$

We show below that, as far as the members of S are concerned, one has

$$\sum_{i \in S} \hat{x}_i + \sum_{i \in S} \bar{\pi}_i z^* > \sum_{i \in S} \tilde{x}_i + \sum_{i \in S} \bar{\pi}_i \tilde{z}, \tag{13}$$

which implies

$$\sum_{i \in S} \hat{x}_i + \sum_{i \in S} \bar{\pi}_i z^* > \sum_{i \in S} x_i^* + \sum_{i \in S} \bar{\pi}_i z^* \tag{14}$$

if S is able to achieve $\tilde{x}_i + \bar{\pi}_i \tilde{z} > x_i^* + \bar{\pi}_i z^*$ for all $i \in S$ as it is supposed to do.

We further show that as far as the other players are concerned,

$$\hat{x}_i + \bar{\pi}_i z^* \geq x_i^* + \bar{\pi}_i z^* \text{ for all } i \in N \backslash S. \tag{15}$$

As together the inequalities (14) and (15) imply that the state $(\hat{x}_1, \ldots, \hat{x}_n; p_1^*, \ldots, p_n^*; z^*)$ is Pareto superior to the Pareto efficient allocation $(x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$, we get an impossibility. Proving (13) and (15) will thus establish the theorem.

To show (13), the definition (12) allows one to write

$$\sum_{i \in S} \hat{x}_i + \sum_{i \in S} \bar{\pi}_i z^* = \sum_{i \in S} g_i(\tilde{p}_i) - \frac{\sum_{i \in S} \bar{\pi}_i}{\bar{\pi}_N}(\sum_{i \in N} g_i(\tilde{p}_i) - \sum_{i \in N} g_i(p_i^*)) + \sum_{i \in S} \bar{\pi}_i z^*$$

$$= \sum_{i \in S} \tilde{x}_i - \frac{\sum_{i \in S} \bar{\pi}_i}{\bar{\pi}_N}(\sum_{i \in N} g_i(\tilde{p}_i) - \sum_{i \in N} g_i(p_i^*)) + \sum_{i \in S} \bar{\pi}_i z^* - \sum_{i \in S} \bar{\pi}_i \tilde{z} + \sum_{i \in S} \bar{\pi}_i \tilde{z}$$

$$= \sum_{i \in S} \tilde{x}_i + \frac{\sum_{i \in S} \bar{\pi}_i}{\bar{\pi}_N} \left[\bar{\pi}_N (z^* - \tilde{z}) - (\sum_{i \in N} g_i(\tilde{p}_i) - \sum_{i \in N} g_i(p_i^*)) \right] + \sum_{i \in S} \bar{\pi}_i \tilde{z}. \tag{16}$$

From the respective characterizations of a Pareto efficient state, and of a partial agreement equilibrium, we have for all $i \in N, \bar{\pi}_N = \gamma_i(p_i^*)$ and $\tilde{p}_i \geq p_i^*$. Hence, the strict concavity of each function g_i implies

$$\bar{\pi}_N \geq \frac{g_i(\tilde{p}_i) - g_i(p_i^*)}{\tilde{p}_i - p_i^*}, \quad \text{for all} \quad i \in N,$$

which in turn implies that

$$\bar{\pi}_N(z^* - \tilde{z}) > \sum_{i \in N} g_i(\tilde{p}_i) - \sum_{i \in N} g_i(p_i^*).$$

Then (13) follows from (16).

On the other hand, from the respective characterizations of the disagreement equilibrium and of a partial agreement equilibrium with respect to a coalition, we have $\tilde{p}_i \leq \bar{p}_i$, for all $i \in N, \tilde{p}_i = \bar{p}_i$ for all $i \in N \backslash S$, and thus $\sum_{i \in N} g_i(\tilde{p}_i) \leq \sum_{i \in N} g_i(\bar{p}_i)$. Therefore,

$$\hat{x}_i = g_i(\tilde{p}_i) - \frac{\bar{\pi}_i}{\bar{\pi}_N}(\sum_{i \in N} g_i(\tilde{p}_i) - \sum_{i \in N} g_i(p_i^*))$$

$$\geq g_i(\bar{p}_i) - \frac{\bar{\pi}_i}{\bar{\pi}_N}(\sum_{i \in N} g_i(\bar{p}_i) - \sum_{i \in N} g_i(p_i^*)) = x_i^* \quad \text{for all} \quad i \in N \backslash S$$

These inequalities establish (15).

Q.E.D.
∎

It was noted earlier that the core of the game $[N, w^\gamma]$ is contained in the α-core. Theorem 1 thus simultaneously establishes the existence of the α-core.

Finally, note that the linearity of the payoff functions can be substituted by a slightly weaker assumption, namely:

Assumption 1$''$: For all $S \subset N, S \neq N, |S| \geq 2, \sum_{i \in S} \pi_i(z^*) \geq \pi_j(\bar{z}), j \in S$, where $\bar{z}$ and z^* correspond to the disagreement equilibrium and a Pareto efficient state.

Clearly, the condition is satisfied when the payoff functions are linear. It is easily seen that our results carry over to this case.

5 Nonemptiness of the Core Under Nonlinear Payoff Functions

Doing away with the linearity assumption of course makes the interaction between the strategic variables more complex. To see this, note that the Nash equilibrium is no longer a dominant strategy equilibrium. We thus impose an additional condition on our general model.

Assumption 3 ′: For all $i, j \in N, g_i = g_j$, i.e., the players have identical production functions.

Note that this may appear to be restrictive, but such an assumption is very often made in the public goods literature.[12] In the environmental externalities model of CARRARO and SINISCALCO 1993, as well as of BARRETT 1990, games are studied in which all the players are identical on both the production and the consumption sides. Similarly, MÄLER 1989 analyzes a game in which all players have identical characteristics.

Henceforth, the production function of each player will thus be denoted simply by g and $dg(p_i)/dp_i = \gamma(p_i), i \in N$. We now note an important consequence of Assumption 3 ′:

Proposition 5: Under Assumption 3 ′, the emission level of each player in the coalition of a partial agreement equilibrium must not be higher than that corresponding to the Nash equilibrium.

Proof: Let $S \subset N$ be some coalition, and let $((\tilde{x}_1, \tilde{p}_1), \ldots, (\tilde{x}_n, \tilde{p}_n)) \in T$ be a partial agreement equilibrium with respect to S. From Proposition 4, we have $\sum_{i \in S} \tilde{p}_i \leq \sum_{i \in S} \bar{p}_i$, where $((\bar{x}_1, \bar{p}_1), \ldots, (\bar{x}_n, \bar{p}_n))$ is the Nash equilibrium.

Since the players have identical production functions, it follows from the characterization of a partial agreement equilibrium with respect to S that $\gamma(\tilde{p}_i) = \gamma(\tilde{p}_j)$ for all $i, j \in S$. Thus $\tilde{p}_i = \tilde{p}_j$. Since $\sum_{i \in S} \tilde{p}_i \leq \sum_{i \in S} \bar{p}_i$, we have $\tilde{p}_i \leq \bar{p}_i$ for each $i \in S$.

Q.E.D.
∎

Theorem 2 Under Assumption 3 ′, the core of the game $[N, w^\gamma]$ is nonempty.

Proof: We show again that the ratio equilibrium $(x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$ is in the core of the game $[N, w^\gamma]$. That is, the allocation defined as

[12] In fact, this brings our model closer to the standard one of pure public goods and thus our results can be extended to such a model.

$$x_i^* = g(\bar{p}_i) - \frac{\pi_i^*}{\pi_N^*}(\sum_{i\in N} g(\bar{p}_i) - \sum_{i\in N} g(p_i^*)), i \in N,$$

where $((\bar{x}_1, \bar{p}_1), \ldots, (\bar{x}_n, \bar{p}_n))$ is the Nash equilibrium, $\pi_i^* = dv_i(z^*)/dz^*$, and $z^* = -\sum p_i^*$. From Pareto efficiency we must also have $\pi_N^* = \gamma(p_i^*)$ for all $i \in N$.

Suppose contrary to the assertion that the above allocation is not in the core of the economy. Then there exists a coalition $S \subset N, S \neq N$, and a partial agreement equilibrium with respect to $S, [(\tilde{x}_1, \tilde{p}_1), \ldots, (\tilde{x}_n, \tilde{p}_n)]$, such that

$$\tilde{x}_i + v_i(\tilde{z}) > x_i^* + v_i(z^*) \quad \text{for all} \quad i \in S, \tag{17}$$

where $\tilde{z} = -\sum_{i\in N} \tilde{p}_i$.

Define a new Pareto efficient allocation $(\hat{x}_1, \ldots, \hat{x}_n; p_1^*, \ldots, p_n^*; z^*)$ as follows:

$$\hat{x}_i = g(\tilde{p}_i) - \frac{\pi_i^*}{\pi_N^*}(\sum_{i\in N} g(\tilde{p}_i) - \sum_{i\in N} g(p_i^*)).$$

We claim that inequality (17) implies that

$$\sum_{i\in S} \hat{x}_i + \sum_{i\in S} v_i(z^*) > \sum_{i\in S} \tilde{x}_i + \sum_{i\in S} v_i(\tilde{z}) \tag{18}$$

and

$$\sum_{i\in N\setminus S} \hat{x}_i > \sum_{i\in N\setminus S} x_i^*. \tag{19}$$

As these two inequalities together with (17) clearly contradict the Pareto efficiency of $(x_1^*, \ldots, x_n^*; p_1^*, \ldots, p_n^*; z^*)$, our theorem is proved if we establish them.

We first prove (18). By the definition of the new allocation,

$$\begin{aligned}\sum_{i\in S} \hat{x}_i &= \sum_{i\in S} g(\tilde{p}_i) - (\sum_{i\in S} \pi_i^*/\pi_N^*)(\sum_{i\in N} g(\tilde{p}_i) - \sum_{i\in N} g(p_i^*)) \\ &\geq \sum_{i\in S} \tilde{x}_i - (\sum_{i\in S} \pi_i^*/\pi_N^*)(\pi_N^*(\sum_{i\in N} \tilde{p}_i - \sum_{i\in N} p_i^*)),\end{aligned}$$

using the concavity of the function g, as well as the Pareto efficiency condition (3); but the last expression is equal to:

$$\sum_{i\in S}\tilde{x}_i - (\sum_{i\in S}\pi_i^*/\pi_N^*)\pi_N^*(z^* - \tilde{z}).$$

Thus,

$$\sum_{i\in S}\hat{x}_i + \sum_{i\in S}\pi_i^* z^* \geq \sum_{i\in S}\tilde{x}_i + \sum_{i\in S}\pi_i^*\tilde{z},$$

that is

$$\begin{aligned}\sum_{i\in S}\hat{x}_i + \sum_{i\in S}v_i(z^*) &\geq \sum_{i\in S}\tilde{x}_i + \sum_{i\in S}v_i(\tilde{z}) + (\sum_{i\in S}v_i(z^*) - \sum_{i\in S}v_i(\tilde{z}) - \sum_{i\in S}\pi_i^*(z^* - \tilde{z}))\\ &\geq \sum_{i\in S}\tilde{x}_i + \sum_{i\in S}v_i(\tilde{z}),\end{aligned}$$

(i.e. (18)), since from concavity $v_i(z^*) - v_i(\tilde{z}) \geq \pi_i^*(z^* - \tilde{z})$ for all i.

Next we prove inequality (19). By definition,

$$\begin{aligned}\sum_{i\in N\setminus S}\hat{x}_i &= \sum_{i\in N\setminus S}g(\tilde{p}_i) - (\sum_{i\in N\setminus S}\pi_i^*/\pi_N^*)(\sum_{j\in N}g(\tilde{p}_j) - \sum_{j\in N}g(p_j^*))\\ &= \sum_{i\in N\setminus S}g(\bar{p}_i) - (\sum_{i\in N\setminus S}\pi_i^*/\pi_N^*)(\sum_{j\in N}g(\bar{p}_j) - \sum_{j\in N}g(p_j^*))\\ &\quad +(\sum_{i\in N\setminus S}g(\tilde{p}_i) - \sum_{i\in N\setminus S}g(\bar{p}_i))\\ &\quad +(\sum_{i\in N\setminus S}\pi_i^*/\pi_N^*)(\sum_{j\in N}g(\bar{p}_j) - \sum_{j\in N}g(\tilde{p}_j))\\ &= \sum_{i\in N\setminus S}x_i^* + (\sum_{i\in N\setminus S}g(\tilde{p}_i) - \sum_{i\in N\setminus S}g(\bar{p}_i))\\ &\quad +(\sum_{i\in N\setminus S}\pi_i^*/\pi_N^*)(\sum_{j\in N}g(\bar{p}_j) - \sum_{j\in N}g(\tilde{p}_j))\\ &= \sum_{i\in N\setminus S}x_i^* + [(\sum_{i\in N\setminus S}g(\tilde{p}_i) - \sum_{i\in N\setminus S}g(\bar{p}_i)\\ &\quad -(\sum_{i\in N\setminus S}\pi_i^*/\pi_N^*)(\sum_{i\in N\setminus S}g(\tilde{p}_i) - \sum_{i\in N\setminus S}g(\bar{p}_i))]\\ &\quad -(\sum_{i\in N\setminus S}\pi_i^*/\pi_N^*)(\sum_{i\in S}g(\tilde{p}_i) - \sum_{i\in S}g(\bar{p}_i)).\end{aligned}$$

As Propositions 4 and 5 imply that $\tilde{p}_i \geq \bar{p}_i$ for all $i \in N \backslash S$ and $\tilde{p}_i \leq \bar{p}_i$ for all $i \in S$, we have $\sum_{i \in N \backslash S} \hat{x}_i > \sum_{i \in N \backslash S} x_i^*$, i.e. (19)

Q.E.D.

6 Concluding Remarks

We have been able to show that it is logically possible to find a state of the economy such that no coalition S can do better for its members, when one assumes that the other agents would play their best response strategies if S were to form.[13] This is true even in the context of games for which no strong Nash equilibrium or Pareto efficient coalition-proof Nash equilibrium exists. We take this as an argument supporting the view that full cooperation (coalition N) can prevail and a Pareto efficient state can be achieved. Although the issue of how to find that state is not thereby resolved, the constructive result obtained in this paper invites further research in that direction.

It seems that our analysis can be generalized to the case of a more general class of utility functions. However, we postpone this task to a later paper. Presently, our purpose has been to introduce the concepts and illustrate them in a simple context. The assumption of quasi-linearity of the utility functions leads to a more transparent analysis.

We assumed that when a coalition S deviates the members of $N \backslash S$ break-up into singletons. Although a generalization of our concepts to the case when $N \backslash S$ may continue as a coalition is straightforward, the existence of a nonempty core becomes a problem.[14]

[13] As noted earlier, this result implies that this is also true when one assumes that the other agents do the worst they can.

[14] It is only under such hypothesis that CARRARO and SINISCALCO 1993 arrive at the result that N will never form.

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