Supplementary Information:
Navigability of Random Geometric Graphs in the Universe and Other Spacetimes

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Supplementary Figures

Figure S1: Convergence of success ratio and stretch. The boxplots summarize the distributions of the success ratio (a) and stretch (b) as functions of the number $N_p$ of random source-destination node pairs sampled in 10 random geometric graphs ($N_p$ pair samples in each graph) in the Einstein-de Sitter (dust) manifold with $\tau_0 = 4.64$, $\kappa = 10$, and $N = 2^{20}$. The orange boxes range from the first to third quartiles, while the bars are minima and maxima. The distributions stabilize at $N_p \ll N$. 
Figure S2: **Fraction of geodesically disconnected node pairs** in the graphs in Figs. 2-4 in the main text. Panels (a,b) correspond to the graphs in the de Sitter (dark energy) and mixed manifolds with $q = 60$, $\rho_0 = 6$ and $N = 2^{20}, \bar{k} = 10$, respectively. The graphs in the Einstein-de Sitter (dust) manifold have trivially no geodesically disconnected node pairs since the manifold is geodesically connected.

Figure S3: **Clustering in Lorentzian RGGs.** The figure shows the average clustering $\bar{c}(k)$ of nodes of degree $k$ in random geometric graphs with $q = 60, \rho_0 = 6, \tau_0 = 0.84$ in the three studied manifolds. The mean clustering excluding nodes with $k = \{0, 1\}$ in the de Sitter, Einstein-de Sitter, and mixed manifolds are $\bar{c}_E = 0.145, \bar{c}_D = 0.164$, and $\bar{c}_M = 0.166$, respectively.

Figure S4: **Degree distribution in Lorentzian RGGs.** Panels (a) and (b) show the degree distribution in the random geometric graphs in the three considered manifolds in the constant-$q$ and constant-$N, \bar{k}$ experiments, respectively, at the largest considered cut-off times $\tau_0$. Specifically, in panel (a) $q = 60, \bar{k} = 130, N = 2518528, \tau_0 = 2.11$, and $\rho_0 = 6$, while in panel (b) $q = 0.564, \bar{k} = 10, N = 2^{20}, \tau_0 = 4.64$, and $\rho_0 = 1.68$. 
Figure S5: **Hub density in Lorentzian and hyperbolic random graphs.** The hub density is defined as the number of links among the \( N_H \) nodes with largest degrees, divided by the maximum possible number \( \binom{N_H}{2} \) of such links. Panels (a,b) compare the hub density in two random graphs of the same size \( N = 2^{20} \) and average degree \( \bar{k} = 10 \). Panel (a) shows the data for the mixed-content (M) Lorentzian manifold graph with \( \rho_0 = 1.68 \) and \( \tau_0 = 4.64 \), while panel (b) shows the same data for the hyperbolic graph generated using [http://named-data.github.io/Hyperbolic-Graph-Generator/](http://named-data.github.io/Hyperbolic-Graph-Generator/) with parameters \( N = 2^{20} \), \( \bar{k} = 10 \), \( \gamma = 2 \), and \( T = 0 \) (the resulting radial cutoff is \( \rho_0 = 32.36 \)). There are exactly zero links between 25 largest-degree nodes in the Lorentzian graph, while the subgraph induced by the first 103 highest-degree nodes in the hyperbolic graph is the complete graph.

Figure S6: **A typical navigation path in a Lorentzian RGG.** The figure shows the greedy geometric routing navigation path from the spacelike-separated green source and red destination in the same graph as in Figure 1 in the main text. The greedy path, which is also the shortest (stretch-1) path in the graph, alternates between hubs and peripheral nodes. Any timelike-separated pairs of nodes are directly linked, resulting in trivial one-hop stretch-1 paths.