SESSION V: Linear Application

5.3: Optimum Noise Performance of Transistor Input Circuits

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A SINGLE COMMON-EMITTER transistor amplifier fed from a resistive source exhibits a minimum noise figure F_m when the source resistance has an optimum value R_{gm}. Figure 1 shows a simplified representation of such an amplifier, in which the transistor internal small-signal resistances are neglected and the noise sources are lumped into an emitter noise voltage v_{ne} and a collector noise current i_{nc}, each defined in a bandwith of 1 cps and assumed constant at all frequencies; i.e., $1/f$ noise is neglected.

In Figure 2 is shown a generalized amplifier fed from a complex source impedance and containing a single common-emitter transistor in the first stage. It is assumed that noise in succeeding stages is negligible. Various feedback paths to either the base or emitter of the first transistor are shown, and an effective shunt input resistor with thermal noise (R_1, v_{n1}) and an effective emitter degeneration resistor with thermal noise (R_2, v_{n2}) are also shown.

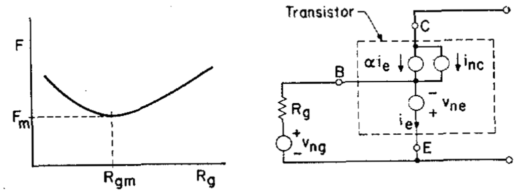
If the signal source is purely resistive, analysis of the circuit of Figure 2 shows that a minimum noise figure F_m' is obtainable with an optimum source resistance R_{gm}'. Expressions for F_m' and R_{gm}' may be given in terms of F_m and R_{gm} (as defined in Figure 1), and are shown in Figure 3. These expressions are all independent of any feedback. It is seen that F_m' is always greater than F_m in the presence of R_1 or R_2, and that the optimum source resistance R_{gm}' may be either less than or greater than R_{gm}, depending on the relative magnitudes of R_1 and R_2. In most practical cases, the thermal noise introduced by R_1 and R_2 may be neglected. Figures 4 and 5 show comparison between predicted and measured curves of noise figure versus source resistance for various values of R_1 and R_2.

If the source is a complex impedance, the quantity of interest is the signal-to-noise ratio S_O at the output of the amplifier. This quantity will in general be dependent on the signal frequency chosen, and on the gain characteristics of the entire amplifier. An expression for S_O in the circuit of Figure 2 is given in Figure 6. Again the result is independent of any feedback, except insofar as the feedback affects the gain characteristic.

If the source is a pure inductance, the expression for S_O reduces to that given in Figure 7. In practice, a typical source of this type would be a magnetic tape reproduce head or a magnetic phonograph pickup. To a good approximation, for a tape head the quantity E^2/f^2L is independent of frequency and of the number of turns, and hence of its inductance, if the tape recording characteristic is constant flux amplitude at all frequencies. Similarly, for a phonograph pickup, the same remarks apply if the disk recording characteristic is constant amplitude at all frequencies. Thus, a *Figure of Merit* M of the source may be specified, which is independent of signal frequency and of the source inductance; see Figure 7.

Under the above condition, the S_O of the amplifier exhibits a maximum value S_{Om} if the source inductance has an optimum value L_{gm}; see Figure 8. It may be observed that these quantities are expressed in terms of the source *Figure of Merit*, the first transistor noise properties, amplifier gain parameters, and circuit parameters containing only R_1 and R_2. Some typical figures for a magnetic tape head and suitably-compensated amplifier are given in Figure 9, from which may be determined that a maximum output signal-to-noise ratio of 72 db may be obtained if the number of turns on the head is increased to make its inductance 260 mh, or if a step-up input transformer of ratio $(260/3)^{1/2}=9.3$ is used at the input.

It is to be emphasized that the criteria for best noise performance are in no way connected with the criteria for maximum power transfer from the source.



Minimum noise figure F_m with optimum source resistance R_{gm}:

$$F_m = 1 + \frac{2v_{ne}^2}{4kTR_{gm}} \qquad R_{gm} = \frac{\alpha v_{ne}}{i_{nc}}$$

Define "emitter effective noise resistance" R_{ne} by

$$v_{ne}^2 = 4kTR_{ne}$$

Hence $\quad R_{ne} = R_{gm}(F_m - 1)/2$

Figure 1—Basic common-emitter amplifier fed from resistive source. For a given transistor operating point, the noise figure of the circuit shows a minimum for an optimum value of source resistance.

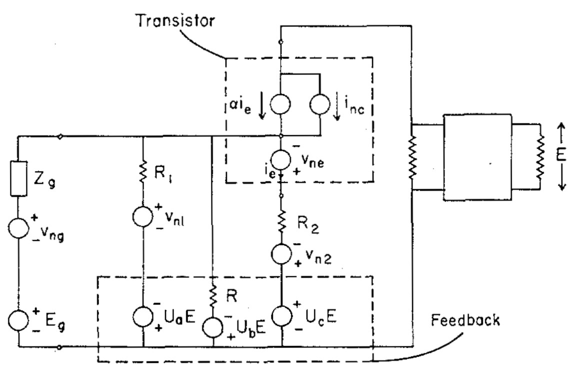


Figure 2—Generalized transistor amplifier fed from reactive source, including input (base) shunt resistance and emitter degeneration resistance with thermal noise, also feedback generators which are functions of amplifier output voltage.

(1) $R_2 = 0$

$$R_{gm}' = R_{gm} \Big/ \sqrt{1 + \frac{R_{gm}^2}{R_1^2}\left(1 + \frac{R_1}{R_{ne}}\right)} < R_{gm}$$

$$\approx R_{gm} \Big/ \sqrt{1 + \frac{R_{gm}^2}{R_1^2}} \quad \text{if } R_{ne} \gg \frac{R_{gm}}{2}, \text{ i.e. } F_m \gg 2$$

$$\frac{F_m' - 1}{F_m - 1} = R_{gm}\left(\frac{1}{R_1} + \frac{1}{R_{gm}'}\right) > 1$$

(2) $R_1 = \infty$

$$R_{gm}' = R_{gm}\sqrt{1 + \frac{R_2^2}{R_{gm}^2} + \frac{R_2}{R_{ne}}} > R_{gm}$$

$$\approx R_{gm}\sqrt{1 + \frac{R_2^2}{R_{gm}^2}} \quad \text{if } R_{ne} \gg \frac{R_{gm}}{2}, \text{ i.e. } F_m \gg 2$$

$$\frac{F_m' - 1}{F_m - 1} = \frac{1}{R_{gm}}\left(R_2 + R_{gm}'\right) > 1$$

Figure 3—Equations for optimum source resistance and minimum noise figure for generalized amplifier of Figure 2 if source is purely resistive. Thermal noise generated in R_1 and R_2 can usually be neglected.

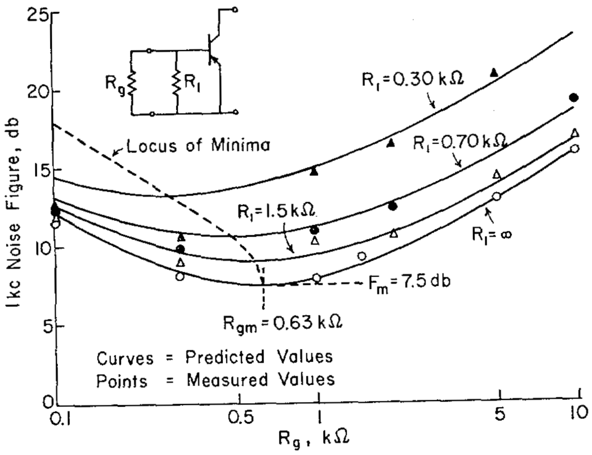


Figure 4—Comparison between predicted and measured noise figure curves as functions of source resistance, for resistive source and $R_2 = 0$. Thermal noise generated in R_1 was neglected in computing the predicted curves.

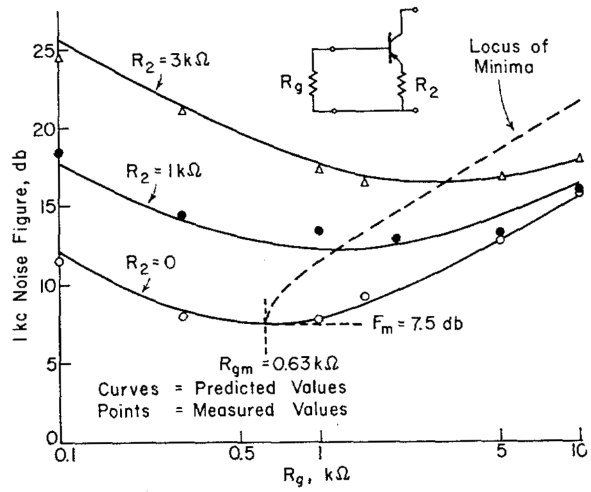


Figure 5—Comparison between predicted and measured noise figure curves as functions of source resistance, for resistive source and $R_1 = \infty$. Thermal noise generated in R_2 was neglected in computing the predicted curves.

$$S_o = \frac{\left(\dfrac{2E_{go}^2}{4kT\,R_{gm}\,(F_m-1)}\right)}{\displaystyle\int_0^\infty \left[\frac{R_g}{R_{ne}} + \left|1+\frac{Z_g}{R_1}\right|^2 + \frac{1}{R_{gm}^2}\left|Z_g + R_2\left(1+\frac{Z_g}{R_1}\right)\right|^2\right]\left|\frac{G(f)}{G(f_o)}\right|^2 df}$$

where $G(f) = E/E_g$ = gain

S_o = Signal-to-noise ratio at output at frequency f_o, when input signal is E_{go} and gain is $G(f_o)$

Figure 6—Expression for signal-to-noise ratio at frequency f_o at output of generalized amplifier of Figure 2 when source is a complex impedance. Thermal noise in R_1 and R_2 is neglected.

If $Z_g \approx j2\pi f L_g$

$$S_o = \frac{\left(\dfrac{2M}{F_m-1}\right) 2\pi L_g R_{gm} f_o^2}{f_o R_{gm}^2\left(1+\dfrac{R_2^2}{R_{gm}^2}\right)\displaystyle\int_0^\infty\left|\frac{G(f)}{G(f_o)}\right|^2 df + (2\pi L_g)^2\left[\left(1+\frac{R_2}{R_1}\right)^2+\frac{R_{gm}^2}{R_1^2}\right]\int_0^\infty\left|\frac{G(f)}{G(f_o)}\right|^2 f^2 df}$$

where $M \equiv \dfrac{E_{go}^2}{8\pi kT f_o^2 L_g}$ = Figure of Merit of source

For signal flux $\phi = \Phi \sin 2\pi ft$ and N turns,

$E_g \sim N\dfrac{d\phi}{dt} \sim Nf$ Hence $\dfrac{E_g^2}{f^2 L_g} = \dfrac{E_{go}^2}{f_o^2 L_g}$ = const.

$L_g \sim N^2$

Figure 7—Expression of Figure 6 restricted to case where source is purely inductive. A *Figure of Merit* of the source can be defined which is independent of its inductance.

S_o has a maximum value S_{om} when source inductance L_g has an optimum value L_{gm}:

$$L_{gm} = \frac{R_{gm}}{2\pi}\frac{A_1 C_1}{A_2 C_2} \qquad S_{om} = \frac{M}{(F_m-1)A_1 A_2 C_1 C_2}$$

where $M = \dfrac{E_g^2}{8\pi kT f^2 L_g} = \dfrac{2(\text{max. available signal energy in 1 cycle})}{\text{thermal energy in 1 cycle}}$

$$A_1 = \frac{1}{f_o}\sqrt{\int_0^\infty\left|\frac{G(f)}{G(f_o)}\right|^2 df} \qquad A_2 = \frac{1}{f_o}\sqrt{\int_0^\infty\left|\frac{G(f)}{G(f_o)}\right|^2 f^2 df}$$

Gain parameters

$$C_1 = \sqrt{1+\frac{R_2^2}{R_{gm}^2}} \qquad C_2 = \sqrt{\left(1+\frac{R_2}{R_1}\right)^2+\frac{R_{gm}^2}{R_1^2}}$$

Circuit parameters

Figure 8—In the circuit of Figure 2, L_{gm} and S_{om} may be expressed in terms of source *Figure of Merit,* input transistor noise parameters, circuit parameters and gain parameters.

TYPICAL FIGURES

Magnetic tape head
E_g = 0.6 mv at f = 1kc, L_g = 3 mh
Hence $M = 1.2 \times 10^9$
or $M_{db} \equiv 10 \log_{10} M$ = 91db

First stage transistor
F_m = 6db R_{gm} = 1kΩ

Circuit parameters
Equivalent input shunt resistance R_1 = 5kΩ
Equivalent emitter degeneration resistance R_2 = 1kΩ
Hence C_1 = 1.41 C_2 = 1.22

Gain parameters
Response from f_1 = 50cps decreasing at 6db/octave to f_2 = 10kc
Hence $A_1 = 1/\sqrt{f_1}$ $A_2 = \sqrt{f_2}$

Results
L_{gm} = 260mh, S_{om} = 72db, independent of signal frequency in this case

Figure 9—Typical figures for circuit of Figure 2 for a magnetic tape head source energized by constant flux amplitude tape. A compensating amplifier gain characteristic is therefore required, and S_o is the same for all signal frequencies.