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EQUILIBRIA WITH UNRESTRICTED ENTRY IN MULTI MEMBER DISTRICT PLURALITY (SNTV) ELECTIONS
Part I: Theory

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Abstract

Extending Duverger's Law to electoral districts of arbitrary district magnitude would imply just one "extra" candidate running in each race. In this paper we analyze equilibrium properties (possible equilibrium configuration and then existence) of a plurality electoral system returning more than one legislator per district. We look at sincere Downsian voters and strategically behaving candidates (who can change their policy platforms at no cost, while new candidates can enter the race). In Part II we find empirical evidence in favor of the implications of this analysis in the performance of actual SNTV electoral systems, such as the one in Japan and Taiwan.

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Part I: Theory

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Political engineering in the form of designing democratic institutions is a growth industry, even among states that have a reasonably long history of democratic government (e.g., Italy, Israel). However, newly introduced forces of democratic competition can exacerbate the internal problems of some states, as when elections give rise to ethnic parties that find it in their interest to raise the salience of divisive issues. But even those who disagree about specifics agree that institutional rules matter and that some rules are less problematical than others, depending on circumstances (see, for example, Lijphart 1977 and 1984, Linz 1990, Riker 1982, Horowitz 1991, Shugart and Carey 1992). And as long as the designing process occurs, we must strive to find those institutional arrangements that have the best chance of preventing conflict from wholly destabilizing a political system.

As we and others argue elsewhere (Shvetsova and Filippov 1993, Horowitz 1985) the period prior to the formation of a coherent political party structure is the most dangerous stage of democratic development, although once formed, the character and function of political parties becomes relatively predictable and stable. But whether a stable democratic equilibrium is achieve at all depends on many things and that chief among them is the state's election laws and ethnic composition (Ordeshook and Shvetsova 1993, Horowitz 1991, Mainwaring 1993). Thus, although we cannot model this dynamic process of party formation, the design of an electoral system requires that we understand better how election laws help generate a political party system of a specific type. In particular, of interest is the question of the principal ability of a specific electoral procedure to yield stable and predictable outcomes - in other words, to yield an equilibrium.

To this end, this essay seeks to understand the nature of election competition fostered by a specific type of election system - the single non-transferable vote (SNTV) - that is used today in parliamentary elections in Japan and Taiwan. Our interest in this system is motivated by an evident empirical regularity that warrants theoretical explanation. Specifically, Reed's (1990) analysis of Japanese elections to the lower House reveals the eventual emergence of k+1 "serious" candidates in each of its k-member electoral districts.

Cox (1993) identifies Reed's findings as the "extension" of Duverger's Law (1954) to plurality systems with multi-candidate electoral districts. By looking at the district-level electoral results, Reed concludes that

- in equilibrium, only k+1 candidates compete in the Japanese SNTV system;
- over time the vote shares of these k+1 candidates tend towards uniformity;
- although there is some evidence of strategic voter behavior, the process whereby this equilibrium is achieve appears to depend primarily on the ability of candidate (parties) to coordinate their actions; and
- convergence to this equilibrium is slow, at least in Japan.

The approach we take with respect to Reed's findings are both theoretical and empirical. The theoretical part of our analysis is devoted to studying the equilibrium properties of the SNTV system, when only the voters preferences are predetermined, but positions and the number of competing candidates are endogenous. It shows that in addition to k+1 equilibria, there may also exist k-equilibria, depending on the specific form of the distribution of preferences. And for the special case of a uniform distribution of preferences, equilibria exist for any number of candidates greater than or equal to k. Also, we see that, except for the special case of a uniform preference distribution, in equilibrium successful candidates should obtain uneven shares of the total vote. The reanalysis of Reed's data as well as data from Taiwan that we provide in the companion manuscript to this essay, moreover, shows that candidates achieve an equilibrium configuration relatively quickly - at least more quickly than Reed suggests - and that if the support of candidates becomes more uniform over time, this is due largely to the actions of voters rather than to the positioning of the candidates on the issues.

Ours is not the first theoretical analysis of SNTV.[1] Cox (1993), motivated specifically by Reed's data, focuses on the analysis of the properties of the equilibria which SNTV yields. He offers a model of SNTV in which any number of candidates greater than k can compete and he approaches the problem from the standpoint of strategic voters rather

[1]Denzau, Katz and Slutsky (1985) analyze multi-candidate elections under various rules and candidate objectives that admit of SNTV as a special case. But, in addition to assuming that preferences are uniformly distributed and that all voters are sincere, they are precluded from addressing Reed's empirical findings by the assumption that the number of candidates is determined exogenously.

than strategic candidates. Admittedly, his treatment has the advantage of not imposing any specific assumptions about the distribution of voter preferences (aside from assuming that there is at least one voter of any type) and of not restricting the issue space to any specific number of dimensions. On the other hand, a treatment that does not impose some geometry on the set of alternatives such as a spatial representation of issues cannot allow the candidates any strategic role. Thus, Cox assumes simply that there is a given number of candidates (some number greater than k) with fixed policy positions. Assuming that voters begin with a common knowledge assessment of the electoral prospects of each candidate, he then establishes that the rational expectations equilibrium has the first k candidates receiving equal vote shares and the remaining candidates receiving declining vote shares.

In contrast to Cox, our approach is to take voters as sincere, with single-peaked preferences over a one-dimensional policy space, and to take candidates as strategic players who not only can move along the policy dimension, but can also enter the competition in unlimited quantities if doing so yields each entrant a positive probability of winning a seat. *Equilibrium* is defined as a combination of policy positions and number of candidates such that

- all candidates have a positive probability of winning a seat,
- no candidate, by altering his position, can increase his probability of winning a seat, and
- no new candidate can enter and by doing so receive a positive probability of winning a seat.

When allowing an unlimited number of candidates to enter the race, we make the assumption that no candidate *enters* the race unless he obtains a positive probability of getting a seat immediately by doing so. Such an assumption rules out the entrants who, like Japan's Communist Party, often use the race as an opportunity for political propaganda, rather than to get into the Parliament. When assuming individual electoral gains for the entrant, we also prohibit anyone from entering the race merely to hurt some otherwise successful candidate. Based on the assumptions made, we show first that if more than one candidate can occupy the same policy position, then regardless of district magnitude, k, and the distribution of preferences (as long as there are no mass points), a k-equilibrium never exists, and that when $f(x)$ is strictly quasi-concave there does not exist any equilibrium number of competing candidates.

We then modify this structure by supposing that candidates must be spatially separated by some minimal distance. It is in this context that we establish the existence of the k and k+1 equilibria. More specifically, for a broad class of electoral distributions we describe some of the necessary conditions for the existence of an equilibrium when entry is allowed, whereas for unimodal distributions we identify the form of that equilibrium. Briefly,

- for unimodal (quasi-concave) distributions of voter ideal points, if an equilibrium exists, it can only be a k or a k+1 equilibrium;
- a k+1 equilibrium necessarily exists for all symmetric unimodal distributions;
- a k equilibrium exists for all symmetric and convex preference distributions and values of k of 1, 2, and 3;
- uniform distributions yield equilibria with any number of candidates if not less than k; and
- unless voters preferences are distributed uniformly, the vote shares of the k leading candidates do not have to be equal or similar, but the difference must be narrow for the k^{th} and the $k+1^{st}$ candidates.

The structure of this paper is as follows. In Section 1 we offer some basic notation and outline the model's essential structure. In Section 2 we analyze the properties that an equilibrium must satisfy when candidates are allowed to occupy the same spatial position, and we use these properties to show that no such equilibrium is possible. In Section 3 we assume that candidates cannot get spatially closer than some minimal distance δ, whereupon we establish the existence of various equilibria, depending on the distribution of preferences. Finally, in Section 4 we offer some testable propositions about competition in SNTV systems that we test through a reanalysis of Reed's data and data from Taiwan in Part II of this essay.

1. The Basic Model

Among the systems of proportional representation, SNTV is one that allows a relatively unambiguous definition of candidate objectives. The goal of a candidate is to win a seat, and for that he must score above the $k+1^{st}$ best competitor. Aside from this, objectives such as maximization of vote share or rank are irrelevant considerations. And since under SNTV there are but three possibilities for a candidate - (1) he receives a seat with certainty; (2) he receives a seat with some positive probability less than one; and (3) his probability of being elected is zero - we assume that candidates maximize the probability of winning a seat.

That we focus on individual candidates rather than political parties is not merely a matter of convenience. In theory, the treatment of parties as strategic players should be different from the treatment of candidates. Some studies show (Balinski and Young 1982) that when parties are many, no voting system guarantees that the increase in the party's share of vote would not lead to the decrease of its share of legislative seats. Thus, a candidate's probability of winning a seat may not always be monotone with his party's vote share. One can still suppose that when elections are conducted in a large (such as national) district, there is no conflict in the objectives of party leaders and a party's candidates once the party list has been compiled, and it is reasonable to assume that both maximize vote share. However, within a local electoral district it is often the case, that the number of seats subject to allocation is comparable to the number of parties, represented in the race. Therefore, it might as well be party strategy to promote few individual candidates within the district, even if the system is not SNTV. It may be rational, hence, for parties maximizing the sum of seats won across all districts, to assume the objective function of its candidates running within the districts. Of course, as long as the ballot structure requires voters to choose between parties, not candidates, we can at best look for similarities between the parties' and candidates' incentives within the district, but cannot substitute candidates for parties as active players in the formulation of the model.

Things are different under SNTV. We clearly cannot suppose that a party can maximize its share of seats by merely maximizing the summed vote share of its candidates (Cox and Niou 1993). A party's candidates can split the vote in a district in ways that preclude any of them from winning a seat. Whereas if they coordinate their support or if fewer of them run, several of them might secure seats even if their smaller number decreases the party's overall share of the vote. Under SNTV, then, a natural harmony of interests between party leaders and potential candidates puts the interests of political candidates first.

What strengthens the role of individual candidates even more, is that SNTV is an election system that operates exclusively at the local level - without the at-large district or allocation by remainders. There is, then, a strong local component to legislative elections in both Taiwan and Japan in which candidates are the key players and parties are merely the non-binding coordinating entities that negotiate on behalf of the candidates before elections and facilitate legislative structure afterwards. Indeed, in

Taiwan, for example, it is not unusual to find members of the Kuomintang running in an election against other KMT members without official party sanction. Thus, our analysis assumes that under SNTV the key strategic players are candidates rather than parties. We emphasize, however, that this focus does not preclude us from discussing the role of parties in SNTV systems. In particular, considerable coordination may be required to preclude entry or to otherwise prevent an unintended supply of candidates. In fact, the model that we suggest here rationalizes the evolution of the extensive *cooperation between the "parties"* at the district level, that is so typical of the Japanese political system, even if it takes place under the label of "factional politics".

Turning now to more formal matters, we begin by assuming that candidates locate themselves somewhere on a one-dimensional policy space, $\mathbf{R}$, and that voters have single-peaked preferences in this space corresponding to the usual Euclidean distance model (Enelow and Hinich 1984) so that the distribution of voters ideal points, $f(x)$, is continuous and contains no mass points. The electoral rule is plurality in a k-member districts (SNTV). Thus, the k candidates with the largest vote shares each win a seat and ties are broken by coin tosses. Insofar as candidate motives are concerned, we assume that each candidate, actual or potential, maximizes his or her probability of winning a seat. In contrast to Cox (1993), we assume that voters vote sincerely for the candidate closest to their ideal (tossing coins if indifferent), but that candidates are strategic in their selection of policy platforms. Entry is allowed, but a new candidate enters only if he can secure a non-zero probability of winning a seat. Existing candidates must choose their positions under the threat that new opponents might enter the contest.

Developing this structure further requires some additional notation. Briefly, we let

C	a finite set of candidates, where $i, j, ..., v \in C$;
x_i	the position of candidate i in the policy space;
$\mathbf{x}$	the vector of candidate positions, $(x_1, x_2, ..., x_v)$
n_i	the number of candidates at the policy position of candidate i;
$l_{x_i} = l_i = \frac{\lvert x_i + x_{i-1} \rvert}{2}$	the left-most ideal point of voters who most prefer candidate i;
$r_{x_i} = r_i = \frac{\lvert x_i + x_{i+1} \rvert}{2}$	the right-most ideal point of voters who most prefer candidate i;

$s_{x_i} = \int_{l_i}^{r_i} f(x)\,dx = F(r_i) - F(l_i)$ the proportion of voters who most prefer x_i. Also, let $s_{x_i}^L$ be the proportion of voters to the left of x_i who most prefer x_i and $s_{x_i}^R = s_{x_i} - s_{x_i}^L$ (since $f(x)$ has no mass points, we need not concern ourselves with anyone who might most prefer x_i).

$S_i = \frac{1}{n_i} s_{x_i}$ candidate i's share of the vote;

$s_i = s_{x_i} = [\,l_{x_i};\ r_{x_i}\,]$ the interval of policy positions that are closer to x_i than any other candidate position;

$P_i(\mathrm{x})$ candidate i's probability of winning a seat, where $P_i = P_j$ if $x_i = x_j$. Note that $P_i > P_j$ only if $S_i > S_j$.

Finally, an equilibrium (x,C) to the election game with k seats at stake is a ν-element vector x and a set C of ν candidates such that

a. no new candidate can enter and, ceteris paribus, secure a non-zero probability of winning a seat; and
b. no candidate $i \in C$ can unilaterally alter his position and increase P_i; and
c. $P_i >$ for all candidates in C.

2. The Model When Two or More Candidates Can Occupy the Same Policy Platform

We can now prove several lemmata that characterize the properties - in terms of number and location of candidates - that any equilibrium (x,C) must possess. We emphasize at the outset, however, that we offer these lemmata to establish that, unless we impose some additional restrictions on the candidate's positions (specifically, unless we preclude the possibility of two or more candidates adopting the same policy platform), no equilibrium exists. Admittedly, there may be easier routes to prove non-existence, but the method we offer here paves the way to establishing existence when candidates are constrained to adopt distinct positions. First, then,

Lemma 1: If (x,C) is an equilibrium, then,

1. P_i equals either 1, or 0, or α $(0 < \alpha < 1)$ for all i in C (if both P_i and P_j are not equal to 1, but greater than 0, then both are equal to α, where α is some positive number less than one).

2. if $n_i \geq 2$ and $P_i > 0$, then $P_i < 1$. (a candidate can receive a seat with certainty only if he or she stands alone at a policy position).

3. if $P_i > 0$ then $n_i \leq 2$ (not more than two candidates will occupy the same policy position and still have a chance to win).

4. if $n_i = 2$ and $P_i > 0$, then $s^L_{x_i} = s^R_{x_i}$

5. $n_i = 2$ only if for all σ such that $\hat{x}_i = x_i + \sigma \in [x_{i-1}; x_{i+1}]$, and $\sigma > 0$

$$\int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx < 2\int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\,dx,$$

and for $\sigma < 0$

$$\int_{\hat{l}_{x_i}}^{l_{x_i}} f(x)\,dx < 2\int_{x_i+\frac{\sigma}{2}}^{x_i} f(x)\,dx.$$

6. For all x_i and x_j, if $n_i = n_j = 2$ and P_i, $P_j > 0$, then $s_{x_i} = s_{x_j}$ (paired candidates everywhere have support of the same size).

7. if $n_i = 1$ and $P_i < 1$, then for all σ such that $\hat{x}_i = x_i + \sigma \in [x_{i-1}; x_{i+1}]$, and $\sigma > 0$

$$\int_{l_{x_i}}^{l_{\hat{x}_i}} f(x)\,dx \geq \int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx,$$

and for $\sigma < 0$

$$\int_{l_{\hat{x}_i}}^{l_{x_i}} f(x)\,dx \leq \int_{r_{\hat{x}_i}}^{r_{x_i}} f(x)\,dx.$$

8. if $n_i = 1$ and $P_i = 1$, and if there exists a candidate j in C such that $P_j = \alpha$, then

$$s_j < s_{x_i} < 2s_j \text{ and } s^L_{x_i} < s_j;\ s^R_{x_i} < s_j.$$

9. no two adjacent policy positions x_i and x_j in x can be located so that there exists an interval $[a, b]$ in the interval $[x_i, x_j]$ such that

$$|a - b| \leq |x_i - x_j|/2$$

and

$$\int_a^b f(x)\,d\hat{x} \geq S_{\min},$$

where $S_{\min}$ is the vote share of a candidate with the lowest positive probability of winning a seat.

The proof of Lemma 1 is in the Appendix.

At this point it is perhaps useful to say something about ties. Unlike models in which voters are strategic (Cox 1993, Palfrey 1989, Myerson and Weber 1989), a tie here is not a "knife-edged" case involving a single voter. We are not concerned with ties as an electoral outcome but with the candidate's estimates of their electoral prospects. Even if candidates possess perfect knowledge about preferences and candidate positions, a candidate's electoral support remains a random variable subject to determination by such things as variations in turnout (which we do not model) and vote counting errors. In this event a tie in a candidate's calculations becomes a robust possibility.[2]

Our next lemma establishes a restriction on the location of the paired candidates in the case, when more than one seat is allocated through the tie (i.e. for example five candidates are tied for three seats), and Lemma 3 says that if the distribution of preferences is single-peaked, the tie can occur for not more than just one seat out of k.

Lemma 2: If (x,C) is an equilibrium, if $|P_i = 1| < k - 1$ (if a tie occurs for more than one seat), and if there is an x_i in x such that $n_i = 2$ (say candidates i and j), then for all σ such that $\hat{x}_i = x_i + \sigma \in [x_{i-1}; x_{i+1}]$, , if $\sigma > 0$

$$\int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx \le \int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\,dx,$$

and for $\sigma < 0$

$$\int_{l_{\hat{x}_i}}^{l_{x_i}} f(x)\,dx \le \int_{x_i+\frac{\sigma}{2}}^{x_i} f(x)\,dx.$$

Proof: If more than one seat is allocated by breaking ties, then at least three candidates must be tied. Since, from Lemma 1.2, candidates i and j win a seat with probability less than 1 and since, from Lemma 1.1, all tied candidates win a seat with the same probability, α, then there exists a third candidate, say t, such that $P_i = P_j = P_t$. So the candidate who increases his share of the vote wins a seat for certain. So, by deviating from x_i, candidate i secures the vote share

[2] And derived later from the model, this possibility should also be consciously reinforced by the actions of the strategic candidates themselves. Namely, in order to prevent further entry, "safe" candidates are interested in maintaining competition somewhere along the policy space. Because they do not care so much about keeping their excess votes, "safe" candidates can afford to maintain proper distances from "weak" candidates, in order to preserve or generate a tie between them for the sake of a mutually beneficial no-entry balance.

$$S_{\hat{x}_i} = S_{x_i} - \int_{x_i}^{x_i+\frac{a}{2}} f(x)\,dx + \int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx$$

Hence, if the conditions of the lemma are not satisfied, a deviation increases i's vote share. Although j's share may also increase, l's cannot increase, so i wins a seat for certain, and (x,C) cannot be an equilibrium. QED

Lemma 3: If the distribution of preferences, $f(x)$, is strictly quasi-concave, then in equilibrium, at most one seat can be allocated through a tie, i.e.,

$$|P_i = 1| \geq k - 1.$$

Proof: Suppose to the contrary that $|P_i = 1| < k - 1$. Then,

a. from Lemma 2, for all x_i in x such that $n_i = 2$, the mode of $f(x)$, m(f), is in s_{x_i}. Hence, there does not exist an x_j such that $x_j \neq x_i,\ n_j = n_i = 2$.. That is, there can be only one set of paired candidates.

b. for all x_i in x such that $n_i = 1$ and $0 < P_i < 1$, it follows from the single-peakedness of $f(x)$ and Lemma 1.7 that $m(f) \in s_{x_i}$. Hence, there does not exist an x_j in x such that $x_j \neq x_i,\ n_j = n_i = 1$,. That is, there can be at most one candidate in C who does not receive a seat with certainty.

c. From (a) and (b) it follows that

$$\{\exists x_i \in x \ s.t.\ n_i = 1,\ 0 < P_i < 1\} \rightarrow$$
$$\{\nexists x_j \in \mathbf{x} \ s.t.\ x_j \neq x_i,\ n_j \leq 2,\ 0 < P_j < 1\}$$

Hence,

1. there does not exist an $x_i \in$ x such that $n_i = 1$, $0 < P_i < 1$,
2. if there is an $i \in C$ such that $0 < P_i < 1$, then there must be two such candidates who are paired at x_i,
3. (1) and (2) together imply that $|P_i = 1| \geq k - 1$, as at most one seat can be allocated through a tie. QED

It follows immediately from this lemma that,

Corollary 1: If (x,C) is an equilibrium, f(x) is strictly quasi-concave, then $|C| \leq k+1$, as by Lemma 3, at most one seat can be allocated through the tie, and at most two candidates can be tied for it.

Lemmata 1 through 3 now allow us to establish a non-existence result when no restrictions are placed on the candidates spatial positions.

Proposition 1: Under the assumptions of the model, for any distribution of voters' ideal points and any k, $|P_i=1| < k$, *i.e., no k-equilibrium exists.*

Proof: Suppose to the contrary that $|P_i=1| = k$. But then for any established candidate $i \in C$, $P_j = \frac{1}{2} > 0$, where $j \notin C$, $x_i = x_j$. Hence entry will occur, and *(x,C)* cannot be an equilibrium. QED

Proposition 1, though, does not rule out the possibility of equilibria with more than k candidates. To that end, suppose $f(x)$ is quasi-concave (unimodal). Then,

Proposition 2: If the distribution of voters' ideal points is quasi-concave, then under the assumptions of the model no equilibrium (x,C) exists.

Proof: By Proposition 1, $|P_i=1| < k$, $\rightarrow \exists i \in C$, s.t. $0 < P_i < 1$. By Lemma 3 - for $f(x)$ quasi-concave, $|0 < P_i < 1| = 2$, while both tied candidates are located in the same position. Hence, in equilibrium there cannot be other than $k+1$ candidates, $k-1$ of which receive seats with certainty, and two others (suppose, candidates i and j) are tied for a single seat. By Lemma 1.4. $S^L_{x_i} = S^R_{x_i}$. Hence

$$\forall \delta > 0,\ \exists \sigma \geq \delta,\ s.t.\ S_{x_i+\sigma} \geq S^R_{x_i} - \int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\, d(x) > \frac{1}{2}\left(S^L_{x_i} + \int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\, dx\right).$$

In other words, by hurting at once both candidates i and j, the entrant receives more than what is left to each of the tied candidates. So such an entrant receives a seat with certainty, which implies that entry occurs and *(x,C)* is not an equilibrium.QED

3. The Model when Candidates Must Maintain Some Minimal Separation

The nonexistence result for the unimodal case presented above is driven by the assumption that two or more candidates can run on indistinguishable platforms. Instead, we may want to introduce a minimal distance δ, that must separate any two candidates in the policy space.[3] Lemma 5, therefore, is similar to Lemma 1, in its listing of the conditions that must hold in an equilibrium, if it exists.

Lemma 5: If no two candidates can adopt spatial positions that are closer than δ, and if *(x,C)* is an equilibrium for any small δ, then,

1. Lemma 1.1 holds.
2. If $|P_i=1| = k$, i.e. if all candidates receive seats with certainty, then

   $$\forall i, \quad S^L_{x_i} + [F(x_i+\frac{\delta}{2}) - F(x_i)] > S^R_{x_i} - [F(x_i+\frac{\delta}{2}) - F(x_i)]$$

   and

   $$S^R_{x_i} + [F(x_i) - F(x_i-\frac{\delta}{2})] > S^L_{x_i} - [F(x_i) - F(x_i-\frac{\delta}{2})]$$

   That is, no candidate i can be located further from the median of his support m_i, than $\frac{\delta}{2}$. (Suppose not. Then new candidate j can either win a seat, or tie with candidate i for it, if j enters at $x_i+\delta$.)
3. Lemma 1.7 holds, when modified as follows: If there exists candidate **i**, such that $P_i < 1$, then for

$\forall \sigma \;\; s.t. \;\; \hat{x}_i = x_i + \sigma \in [x_{i-1}+\delta ; x_{i+1}-\delta]$, either

$$\int_{l_{x_i}}^{l_{\hat{x}_i}} f(x)\,dx \geq \int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx, \qquad \text{or} \qquad \int_{l_{\hat{x}_i}}^{l_{x_i}} f(x)\,dx \leq \int_{r_{\hat{x}_i}}^{r_{x_i}} f(x)\,dx.$$

4. Lemma 1.8 holds.
5. Lemma 1.9 holds.

Note, that Lemma 5.2 hints at the possibility of a k-equilibrium. We want to explore this possibility further here. No general result with respect to a k-equilibrium's existence or non-existence has been established so far. Its existence depends not only on the form of the distribution of voters' ideal points and the size of the district (k), but in certain cases - on the size of δ as well (equilibrium may disappear with the decrease of the δ parameter, while existing for its greater values). As the rest of our discussion here

[3] Provided, that nowhere in the distribution is $s=2\delta$ associated with positive probability of winning, which for some δ is always true by the continuity of the distribution and finiteness of an integral.

is aimed to establishing the results for δ arbitrarily small, we present only one particular case of k-equilibrium existence, which fits this requirement:

Proposition 3: For any concave and symmetric distribution of voters preferences when k=1, 2, or 3, k-equilibria exist for any small δ.

Proof: In order to prove the result for any small δ, we need to show that all k candidates can be positioned exactly at their respective means, and that none of them gets a twice smaller share of the vote than another (by L.5.2, and L.1.9). Because the cases of k=1 and k=2 are trivial, consider k=3 and locate the middle candidate x_2 at the mode of the distribution. Choose l_2, such that

$$\int_{x_1}^{l_2} f(x)\,dx = \int_0^{x_1} f(x)\,dx, \quad |l_2 - x_1| = |x_2 - l_2|$$

Note, that such an l_2 always exists. Now we want to show that $s_1^R > s_2^L$ in order to assure, that the middle candidate has less than twice more votes than the candidate on the left. If we show this for the linear slope, it must hold for any other slope of a concave distribution. The vote share of the first candidate on his left by construction is equal to that on his right, i.e.

$$a*b = \frac{1}{2}(a+\Delta_a)*(b+\Delta_b) \ \ i.e. \ \ a*b = a*\Delta_a + b*\Delta_b + \Delta_a*\Delta_b,$$

where

$$a = x_1, \ b = f(x_1)$$
$$a+\Delta_a = l_2, \ b+\Delta_b = f(l_2).$$

Thus, as we have to show that $s_1^L < 2s_2^L$, and we know, that

$$s_2^L = (a+2\Delta_a)*(b+2\Delta_b) - 2a*b$$

Hence,

$$s_2^L = a*b + 2\Delta_a*\Delta_b < 2a*b$$

QED

The following proposition shows that for the special case of a uniform distribution of voter ideal points, for all values of k equilibria exist for any number of candidates greater than or equal to k.

Proposition 4: If $f(x) \sim U[0,1]$, *then* $\forall k\ \exists$ *equilibria* (x,C) *such that* $|C| \geq k$.

Proof: We prove the existence simply by presenting the examples of equilibria for all cases describing $|C| \geq k$. Specifically,

1) for $|C| > 2k$ odd, let $x_i = \frac{2i-1}{2|C|}$ $\forall i \in C,\ x_i \in x$;

2) for $|C| \geq 2k$ even, either let $x_i = \frac{2i-1}{2|C|}$,

or let $x_{\{i \mid odd\}} = \frac{2i-1}{2|C|} - \frac{\delta}{2}$ and $x_{\{i \mid even\}} = \frac{i}{|C|} + \frac{\delta}{2}$;

3) for $2k > |C| > k$ any combination of certain and tied candidates is possible, including the arrangement described in (1).

Lemma 6: *If (x,C) is an equilibrium*, if f(x) is strictly quasi-concave, and if there exists a candidate $i \in C$, such that $0 < P_i = \alpha < 1$, then $\exists j\ s.t.\ |x_i - x_j| = \delta, P_j = P_i = \alpha$ and $\arg\max f(x) \in \{s_i \cup s_j\}$.

Proof: By Lemma 5.3, if such a candidate *i* exists, it must either be located so that $\arg\max f(x) \in s_{x_i}$, or be blocked from above by another candidate , say *i+1*, such that $|x_{i+1} - x_i| = \delta$. As at least two candidates must have a probability of winning a seat less than one, than at least one such candidate must be "blocked" from above.

If a candidate *i* is blocked from above, he must be blocked by *i+1* such that $0 < P_{i+1} = P_i < 1$. Suppose otherwise. Then $P_{i+1} = 1$. But by definition, $S_{\underline{i+1}} - S_{\underline{i}} = M$, where M is some number strictly greater than 0. Then for any eligible distribution $f(x)$ there exists δ^*, such that

$$\forall \delta < \delta^*,\ \int_{x_i+\frac{\delta}{2}}^{x_{i+1}+\frac{\delta}{2}} f(x)\,dx < M$$

Note, that $|x_{i+1} - x_i| = \delta$. Hence, an entrant at $x_{i+1}+\delta$ receives a seat with certainty, and (x,C) is not an equilibrium.

But then $P_{i+1} < 1$, and by Lemma 5.3 it must be that $\arg\max f(x) \in s_{i+1}$, which implies that $|0 < P_i < 1| = 2$, $|x_{i+1} - x_i| = \delta$, and

$$F(x_i + \frac{\delta}{2}) - F(x_i + \frac{\delta - \sigma}{2}) \geq F(l_{x_i}) - F(l_{x_i} - \frac{\sigma}{2}),$$

and

$$F(x_{i+1}+\frac{\sigma-\delta}{2})-F(x_{i+1}-\frac{\delta}{2}) \geq F(r_{x_{i+1}}+\frac{\sigma}{2})-F(r_{x_{i+1}}).$$

Hence, as $f(x)$ is strictly quasi-concave, l_{x_i} and $r_{x_{i+1}}$ straddle the mode and $argmax\, f(x) \in \{s_i \cup s_{i+1}\}$. QED

Two corollaries follow immediately from Lemma 6,

Corollary 2: If f(x) is quasi-concave, and if (x,C) is an equilibrium, then $|C| \leq k+1$, and $|P_i=1| = k-1$.

Corollary 3: If f(x) is quasi-concave, and if (x,C) is an equilibrium, and if there exists **i**, such that $0<P_i<1$, then $P_i = 1/2$.

We are now positioned to prove the central result of this section of the paper -

Proposition 5: I. For any strictly quasi-concave distribution of ideal points, if there are equilibria other than k-equilibrium existing for all small $\delta>0$, they must be of the following form:

- $|C|=k+1$;

- two candidates are located maximally close to each other, the union of their constituencies includes the mode of the distribution, and these two candidates are tied for a seat;

- the remaining k-1 candidates each receive a seat with certainty and are located so, that Lemma 5 holds.

II. For all symmetric single-peaked distributions and for all odd k's (and some even), such equilibria exist for any small $\delta>0$.

Proof: Part I of the proposition follows directly from Lemma 6 and Corollaries 2 and 3 - for a quasi-concave distributions no other equilibria with $|C|>k$ may exist. Part II is proved by the construction of the corresponding equilibrium.

Since the case of $k=1$ is trivial, we start with $k=3$. For δ small enough, set

$$x_i = argmax\, f(x) - \frac{\delta}{2}, \quad x_j = argmax\, f(x) + \frac{\delta}{2}$$

Now on $(\infty; x_i]$ we need to choose l_i to separate the constituencies of candidates *i* and *i-1* so that

$$|l_i - x_{i-1}| = |x_i - l_i| \geq \frac{\delta}{2}$$

Notice, that the choice of l_i uniquely determines x_i. By the strict monotonicity of $f(x)$ on $(\infty; x_i]$,

$$F(x_i) - F(l_i) > F(l_i) - F(x_{i-1})$$

If in particular we choose l_i such that

$$F(x_i) - F(l_i) = F(x_{i-1} = x_i - 2|x_i - l_i|)$$

the conditions of Lemma 5 are satisfied. But by the strict monotonicity of $f(x)$ on $(\infty; x_i]$ such an l always exists: $F(x_i) - F(l_i)$ is continuous, monotonically decreasing from $0.5 - \xi(\delta)$ to 0 as l_i goes from 0 to $x_i - \frac{\delta}{2}$, while $F(x_{i-1} = x_i - 2|x_i - l|)$ is continuous, monotonically increasing from 0 to $0.5 - \xi(\delta)$. Therefore, there exists l_i^* such that $F(x_i) - F(l_i^*) = F(x_{i-1})$. Finally, by symmetry, locate x_{j+1} at $F^{-1}(1 - F(x_{i-1}))$, so that $((x_{i-1}, x_i, x_j, x_{j+1}), \{1, 2, 3, 4\})$ constitutes a {k+1}-equilibrium for $k=3$.

To construct an equilibrium for $k=5$, choose x_i and x_j as before. From our previous argument we know that the choice of l_{i-1}^* separating the constituencies of candidates $i-1$ and $i-2$ uniquely determines the choice of l_i^* and, hence, x_{i-1} and x_{i-2} (see Figure 1). And again, for the same reason as in the case of $k=3$, we want the choice of l_{i-1}^* to satisfy

$$F(x_{i-1}) - F(l_{i-1}^*) = F(x_{i-2} = x_{i-1} - 2|x_{i-1} - l_{i-1}^*|)$$

The existence of such l_{i-1}^* is again asserted by a fixed-point argument. The theorem holds because this method of construction can be extended to any odd k[4]. QED

4. Implications of the Model that Admit Empirical Testing

That Reed's (1990) work documents the stability of the $k+1$ pattern in Japan is an encouraging fact. Indeed, although our analysis asserts the existence of a $k+1$-equilibrium only for symmetric and unimodal distributions of voters' preferences, there clearly exists in each particular case a large set of equilibrium spatial configurations with the same number of candidates. For this reason we can speculate that symmetry is not a strict

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[4] The reason we do not claim the uniqueness of an equilibrium for any particular value of k, is the arbitrariness of the choice of x_i and x_j, for which the only restrictions are $\operatorname{argmax} f(x) \in \{s_i \cup s_j\}$ and that Lemma 5.3 holds.

requirement. For small variations in a symmetric distribution there must still exist a subset of configurations that remain equilibria. At the same time, if we relax the restriction that the number of candidates to the right of the mode equal the number to the left, we suspect that for any unimodal distribution (no matter how asymmetric) there exists a district size k for which a $k+1$-equilibrium exists. Thus, the preconditions for testing our analysis are not necessarily as strong as the assumptions vis-a-vis symmetry that we employ in proving our formal results.

The next precondition to empirical testing is to establish that the assumption of a unimodal preference distribution provides a reasonable characterization of individual districts. Fortunately, extremely high ethnic and linguistic homogeneity in both Taiwan and Japan allows us to suppose that constituencies are unlikely to be polarized along these lines. Japan's districts, moreover, are quite small (3-5), as are Taiwan's (with but a few notable exceptions), thereby giving us some confidence in the validity of the unimodality assumption. That those exceptions (i.e., Taipei and Kaoshung) are also likely to be the most heterogeneous districts provides a basis for seeing whether the $k+1$ rule holds better in homogeneous than heterogeneous election districts. Of course, even taking all this into account, we still cannot claim that the empirical analysis that follows in Part II of this essay is a rigorous test of our model - rather it is merely a piece of empirical evidence in support of it.

Insofar as specific hypotheses are concerned, both Reed's empirical and Cox's theoretical analysis agree that the vote shares of the candidates should be approximately the same. Reed asserts this hypothesis for all $k+1$ candidates. Cox, by approaching the problem from the standpoint of voters' rationality, derives that only the top k candidates should all receive identical vote shares, and the $k+1^{\underline{st}}$ and lower ranked candidates get arbitrarily lower shares of the vote. In contrast, our analysis predicts that candidates in equilibrium do not necessarily secure equal electoral support. Although over time their support may become even as Reed reports, this trend should be the result of strategic voting rather than of any strategic action by the candidates.

An empirical assessment of this hypothesis, though, requires a reconsideration of how we count candidates - the number of "serious" candidates competing in each district. Reed's (1990) approach is not satisfactory for our needs. Reed equates the number of candidates to the number of "effective" candidates, i.e. the inverse of the sum of the candidates' squared shares of the vote. Although the motivation for the use of "effective" number of candidates is to avoid separating "serious" candidates from non-serious ones by some *ad hoc* criterion, the use of this measure involves a number of unsatisfactory assumptions. In particular, testing the hypothesis of $k+1$ candidates running in a district by looking at "effective" number of candidates implies that all such candidates earn identical support from voters in equilibrium. If there are indeed only $k+1$ candidates, but with uneven shares of the vote, the "effective" number will be below $k+1$. Conversely, for "effective" number to be $k+1$ when vote shares differ, more than k+1 candidates must compete. Indeed, Reed does assert that in equilibrium all candidates should be equally successful at the polls. Looking merely at the "effective" number of candidates makes it impossible to test this part of his hypothesis and confounds the influence of several hypotheses.

Unlike Reed, we will not compute "effective" numbers of candidates, but instead will draw a line rather arbitrarily, cutting off those unsuccessful candidates whose vote shares are "significantly" lower than the candidates' immediately above them. To avoid obvious criticism, the cut-off will be set at a 20% vote decrease, 33%, 50%, and 100 percent. In other words, we count the number of candidates in the district as k plus all those candidates, who gathered not less than 80 (67, 50, or 0) percent of the vote of the competitor immediately above them. For example, if the $k^{\underline{th}}$ candidate receives 100,000 votes, and

$k+1^{\underline{st}}$ receives 79,000,
$k+2^{\underline{nd}}$ receives 52,000,
$k+3^{\underline{rd}}$ receives 25,000,

and the last one, the $k+4^{\underline{th}}$, only 10 votes, the number of "serious" candidates in the district will be k, $k+1$, $k+2$, and $k+4$ correspondingly for the 20, 33, 50 and 100 percent cut-offs.

Insofar as attempting to see whether a candidate-oriented analysis (our) provides a better explanation of the data than does a voter-oriented explanation (Cox, and Reed), we must look as some additional things. Because Reed predicts that there should be $k+1$ "serious" candidates per district and since we predict either k or $k+1$ such candidates, we cannot use simple counts to discriminate between Reed's analysis and our own. Moreover, Cox does not infer any predictions about the number of candidates. He states only that all victorious candidates should obtain identical shares of the vote. We can, though, get a handle of discriminating between the two alternative approaches by looking at the ratios of the candidates' vote shares within districts. Specifically,

> Cox's hypothesis can be restated as predicting that the ratio of the *vote shares of the* $k-1^{\underline{st}}$ candidate and the $1^{\underline{st}}$ candidate being close to 1.

- Our analysis predicts that the corresponding ratio of the $k+1^{\underline{st}}$ to the $k^{\underline{th}}$ being close to 1.
- Reed's hypothesis predicts that both ratios should be 1.

In Part II of this essay we analyze the complete set of Taiwanese elections and reexamine the electoral data from Japan assembled by Reed with these hypotheses in mind. However in the Addendum to this paper we present a partial set of Taiwanese elections, to provide evidence immediately that our hypothesis is worth consideration.

Finally, we want to offer several comments on the how our analysis might be used to shed light on the potential rational grounds for the party formation under SNTV. One widely held opinion is that for party membership to be individually rational for the candidates, it must be significantly reducing campaigning costs, or otherwise strengthening the candidates positions with the voters (Holler, 1987). Alternatively, Downs's political party does not yield benefits to members on an individual basis, but instead collects them "lump-sum," in the form of the control over the government, which serves as a kind of public good for party members. A political party, then, is but "a team seeking to control the governing apparatus by gaining office in a duly constituted election", "whose members agree on all their goals instead of just part of them" (Downs, 1957 p.25).

In contrast, our analysis suggests the concept of a party as a coordinating agent among candidates. That is, the particular difficulty candidates confront in an SNTV system is being certain that only the "correct" number of them compete, and that they compete at the "correct" positions on the issue space. Notice now that this perspective admits of the existence of factions within parties and the free use of faction labels in a campaign. The phenomenon of factions, of course, is not foreign to either the Japanese or the Taiwan political systems.

Specific aspects of intra- and inter-party coordination include, for example, Curtis' (1972) and Reed's (1990) observation in the Japanese system of (for example) "the young entrepreneur... [who] runs as an independent to demonstrate his campaigning ability to the [major party] in the hope that they will reward him with nomination" (Reed 1990, p. 355). Japanese parties also tend to grant their "joint" nomination to independent candidates - something that hints of the attention given by parties to defining the "spatial" location of a candidate. Cases of collusion between the parties in terms of vote transfers are also common.

Notice moreover that our treatment of parties in fact corresponds to Schumpeter's famous definition of party and machine politicians as "simply the response to the fact that the electoral mass is incapable of actions other than a stampede, and they constitute the attempt to regulate political competition exactly similar to the corresponding practice of trade associations" (Schumpeter, 1947 p.283). Owing to the complexity of the equilibrium our model describes - a complexity that requires the right number of candidates at the right spatial locations - it seems only reasonable to view parties in SNTV systems as both agents of coordination as well as of enforcement.

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ADDENDUM:

The table below shows the dynamics of candidate participation in three consecutive Taiwanese national elections - 1980, 1983, and 1986 - prior to the drastic change of electoral laws that occurred in 1989 (26 electoral districts were formed in place of 8, with average district magnitude reduced from 6.9 to 4.6). Columns 4, 5, and 6 of the table show the number of "serious" candidates in excess of k competing in the race within districts for the three cut-off levels specified above. Column 8 gives the ratio of the support for the k-$1^{\underline{st}}$ leading candidate to that of the first-ranked candidate - statistics that should converge to 1 by both Reed's and Cox's hypotheses. Column 9 reports the ratio of the vote for the strongest loser (the k+$1^{\underline{st}}$ candidate) to that of a last victor (the $k^{\underline{th}}$ candidate), which should converge to 1 by both Reed's hypothesis and ours (except for the k-equilibrium cases). The data on this set of elections are interesting in particular because, with opposition to the Kuomintang illegal during this period, there has been no process of party formation during the time covered by the data in this table. This data, then, derives from a period of minimal coordination during a campaign (when, for example, the total number of candidates exceeded the number of seats in the district 2 to 4 times).

Looking at the numbers this table reports, we see that, as predicted by our model (except for the clearly expressed cases of k-candidate competition - the Third district in 1983; the Sixth district in 1980, 1983, and 1986; and the Eighth district in 1980 and 1986), the ratio of the vote for the first loser and the last winner stays close to 1, beginning with the first elections. The average for the year 1980 is 0.89, for the year 1983, 0.94, and for the year 1986, 0.93. At the same time and contrary to Reed and Cox's analyses, the numbers in column 8 stay far from 1 (averaging 0.55 in 1980, 0.76 in 1983, and 0.65 in 1986). Of course, we cannot say whether this ratio would converge to 1 over a longer period (as Reed claims based on his analysis of Japan). But these data do encourage us to examine the Japanese case more closely for support of the hypothesis that the "k+1 rule" is driven more by candidate and party strategies than by individually motivated voters voting strategically.

DISTRICT LEVEL ELECTORAL DYNAMICS IN TAIWANESE GENERAL ELECTIONS, 1980-1986

District	Year	District magnitude k	Number of "serious" candidates in excess of *k* when cutoff is when the vote reduces by			Total number of candidates	Ratio of the vote	
			20%	33.3%	50%		k-1st/1-st	k+1st/k-th
1	2	3	4	5	6	7	8	9
1	1980	8	2	3	3	21	0.6158	0.9736
	1983	9	2	2	2	22	0.5451	0.9776
	1986	9	1	3	3	17	0.5826	0.9976
2	1980	6	0	5	12	18	0.4657	0.7792
	1983	6	2	4	6	16	0.7747	0.9027
	1986	6	2	2	2	11	0.6335	0.9859
3	1980	9	1	1	5	22	0.5559	0.9357
	1983	9	0	0	0	17	0.6829	0.4749
	1986	10	1	1	2	17	0.4668	0.9486
4	1980	8	2	5	14	23	0.4169	0.9172
	1983	8	2	2	2	20	0.6863	0.9998
	1986	9	2	4	4	20	0.6055	0.8387
5	1980	5	1	1	3	11	0.7576	0.856
	1983	5	1	1	2	10	0.6254	0.9904
	1986	5	2	2	2	8	0.8556	0.8094
6	1980	2	0	0	2	6	0.7103	0.5677
	1983	2	0	0	0	5	0.9156	0.4735
	1986	2	0	0	0	2	0.8571	0
7	1980	8	2	2	25	33	0.3371	0.932
	1983	8	2	2	8	25	0.5305	0.9484
	1986	8	2	2	2	16	0.574	0.9885
8	1980	5	0	0	7	16	0.5622	0.5435
	1983	5	2	3	5	17	0.742	0.8473
	1986	6	0	0	0	11	0.5968	0.3742

DISTRICT LEVEL ELECTORAL COMPETITION IN TAIWANESE GENERAL ELECTIONS, 1989

District	District magnitude k	Number of "serious" candidates in excess of k when cutoff is when the vote reduces by			Total number of candidates	Ratio of the vote	
		20%	33%	50%		k-1st/1-st	k+1st/k-th
1	11	5	7	7	29	0.566	0.914
2	2	0	1	1	4	0.513	0.768
3	1	0	0	0	2	1.000	0.346
4	5	1	1	1	8	0.601	0.910
5	1	0	2	2	3	1.000	0.793
6	1	1	1	1	6	1.000	0.959
7	2	2	2	2	4	0.574	0.846
8	4	2	2	2	12	0.974	0.982
9	3	3	3	3	12	0.956	0.980
10	4	2	2	2	13	0.523	0.949
11	2	0	2	2	7	0.984	0.710
12	3	1	1	1	7	0.371	0.959
13	2	1	1	1	6	0.663	0.959
14	1	0	0	0	3	1.000	0.418
15	4	0	0	0	10	0.851	0.271
16	2	0	3	3	9	0.997	0.684
17	4	1	2	2	8	0.566	0.894
18	3	0	0	0	6	0.521	0.423
19	1	0	0	0	2	1.000	0.318
20	1	1	1	1	4	1.000	0.803
21	1	0	0	0	6	1.000	0.317
22	6	2	3	3	20	0.316	0.963
23	6	0	2	2	17	0.639	0.695
24	4	1	1	1	13	0.643	0.988
25	4	2	2	2	11	0.643	0.939
26	1	0	0	1	4	1:000	0.602

District	District magnitude k	Number of "serious" candidates in excess of k when cutoff is when the vote reduces by			Total number of candidates	Ratio of the vote	
		20%	33%	50%		k-1st/1-st	k+1st/k-th
1	16	9	9	48	48	0.157	0.965
2	2	1	1	1	5	0.756	0.834
3	7	4	4	4	13	0.489	0.868
4	2	1	1	1	7	0.809	0.748
5	3	1	1	1	10	0.960	0.816
6	7	0	3	3	16	0.753	0.734
7	7	1	2	2	16	0.664	0.870
8	3	1	1	1	7	0.588	0.897
9	4	2	2	2	8	0.835	0.975
10	3	2	2	2	6	0.899	0.997
11	5	3	4	4	11	0.603	0.912
12	6	2	2	2	14	0.554	0.908
13	5	1	1	2	11	0.697	0.927
14	1	1	1	1	7	1.000	0.597
15	2	2	2	2	8	0.573	0.998
16	1	1	1	1	3	1.000	0.926
17	2	1	1	2	5	0.770	0.960
18	2	2	2	2	10	0.673	0.890
19	4	1	1	1	11	0.755	0.825
20	1	1	1	1	2	1.000	0.996
21	4	1	1	3	13	0.694	0.912
22	1	1	1	1	5	1.000	0.828
23	1	0	1	1	2	1.000	0.703
24	9	3	4	13	28	0.185	0.935
25	9	7	7	15	40	0.386	0.939
26	6	1	1	1	14	0.439	0.999
27	6	2	2	2	13	0.569	0.997