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An Experimental Examination of the Walrasian Tatonnement Mechanism

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SOCIAL SCIENCE WORKING PAPER 824

December 1992

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1.0 Introduction

Joyce (1984) reports results of experiments of a Walrasian tatonnement auction that show that the mechanism is stable, exhibits strong convergence properties and generates efficiency averaging better than 97%.[1] He also found that when subjects could see part of the order flow (excess demand), price tended to be lower (favorable to buyers). His experiments consisted of a stationary environment where subjects were provided with single-unit supply and demand functions. This paper assesses the robustness of his results in a more complex setting and systematically investigates the effect of various order flow information and message restriction rules on the performance of the Walrasian mechanism. In particular, our subjects were provided with multi-unit demands and supplies where equilibrium price and subject values or costs were changed each trading period.

Part of the motivation for our experiments is to assess the extent of the inefficiencies predicted by theory as follows:

1. When there are both buyers and sellers in the market, each of which has one unit to buy or sell, the only Nash equilibria of the Walrasian tatonnement mechanism are those that support the competitive equilibrium outcome. Furthermore, a Walrasian tatonnement process can be designed that has a dominant strategy equilibrium where each participant reveals value or cost (see McAffe (1990)). The design imposes constraints on participant messages; specifically, at the announced price at t, if excess demand is positive (negative), any seller (buyer) not registering a sell (buy) order at t cannot register an order at time t+1. Without this improvement rule the dominant strategy equilibrium outcome no longer exists. However, even with this improvement rule, the dominant strategy revelation property does not hold when demands and supplies are multi-unit, since a participant may influence price without being entirely out of the market. When suppliers and demanders have multiple units to trade, theory provides very little guidance to the market designer on the appropriate price discovery rules by which to organize a Walrasian tatonnement.

[1] *Efficiency* is defined as the percent of maximum producer and consumer surplus realized by a trading mechanism.

2. The results reported in Noussair (1991) also raises questions about the robustness of Joyce's results in an environment with multiple unit supply and demand. He shows that the isomorphism between the English auction (which gives full order flow information) and a uniform price sealed-bid auction (which gives no order flow information) does not hold when demands are multi-unit and multiple units are to be allocated. It seems reasonable to assess the impact of multi-unit supply and demand as well as the role of information on demand and supply conditions on the performance of the tatonnement auction.

3. With prices sensitive to revealed supply and demand, the price adjustment process[2] in a tatonnement auction typically results in a Nash equilibrium in which participants underreveal demands and supplies (see Hurwicz (1972) and Otani and Sicilian (1990)). The strategic underrevelation can lead to outcomes that are different than the competitive equilibrium outcome. Without a consistency restriction placed on traders' messages in a Walrasian tatonnement, the revealed willingness to buy or to sell at a particular price are nonbinding (a form of cheap talk). Thus, we assess the impact of imposing a consistency requirement on subjects' bids and offers on the performance of the Walrasian mechanism.

Not only is there a theoretical interest on the performance of the Walrasian mechanism, but there are field implementations of such a process where questions have been raised about its efficiency; the opening of the New York Stock Exchange (NYSE) and special opening and closing procedures on the Commodity Exchange Inc. (COMEX). At the opening on the NYSE, the specialist in each security listed on the exchange calls out prices until he or she finds the price that maximizes the volume of matched buy and sell order quantities submitted over the electronic system and those held by floor traders actively participating in the auction at the specialists' post. If there are more buys (sells) than sells (buys) and the specialist is not prepared to absorb the imbalance, the price is adjusted upward (downward). The efficiency of this process has been questioned by Amihud and Mendelson (1987) and Stoll and Whaley (1990) because of the inflated volatility of opening prices. Amihud and Mendelson (1991) compare day to day price volatility and demonstrate that there is less volatility of the call auction at the start of the afternoon session of the Tokyo Stock Exchange (TSE)

[2] Price is adjusted in the direction of excess demand (downward if there are more sells than buys, upward if there are more buys than sells)

relative to the one at the start of the trading day, and suggest that less current information about supply and demand conditions may underlie the observed volatility at the opening. Bronfman and Schwartz (1992) provide theoretical support for the hypothesis that price discovery is particularly complex after a non-trading period. The COMEX has proposed a five minute period prior to the opening and close of every trading day during which brokers holding orders to buy and/or sell can attempt to match quantities in an open outcry (at a price to be determined during the opening or closing minute, respectively). The price at which these matched orders will be deemed to have traded is the unweighted "average" of the prices at which contracts traded during the opening or closing minute; that is, it gives equal weight to all execution prices and does not take into account the volume of trading at each price. Notice that this price will not have been determined when the matching of buy and sell quantities take place. Since rules matter in the performance of a mechanism, it is important to investigate the impact of different implementations of the Walrasian mechanism on market efficiency.

Control over supply and demand conditions and the information provided to traders is impossible in the field, and therefore minimal conditions for studying the role of auction rules on price discovery are not possible. The use of laboratory methods in economics allows key parameters in the market to be under the control of the experimenter. In particular, using monetary incentives, underlying demand and supply conditions can be induced so that equilibrium price and quantity exists and is known to the experimenter. Given the induced supply and demand environment, performance of a pricing institution (auction rules) in terms of the efficiency of price discovery and in the allocation of asset holdings can be assessed. In addition, the strategic behavior of subjects can be compared relative to fully revealing strategies. Our experiments are fully computerized allowing for greater control in differentiating private and public information and enforcing different message restrictions on subjects.

In our experiments, the underlying supply and demand conditions allow for strategic price manipulation by buyers and sellers and the conditions are not stationary for each market period. This experimental environment provides a difficult test of the efficiency of price discovery and allocations for a Walrasian tatonnement mechanism. We also provide evidence on the performance of the Walrasian process under different levels of "transparency" (market information).[3] For the Walrasian mechanism this translate to providing information on the current buy and sell orders in the market and potential price movement. It is unclear whether such information can assist or hinder the market. In addition to the transparency issue, we also investigate the properties of bid-offer restriction rules that facilitate orderly price discovery.

The basic results show that all versions of the computerized multiple unit Walrasian auction produces prices consistent the with the competitive equilibrium prediction. Unfortunately, the allocations fall short of the competitive prediction and result in efficiencies lower than those provided by a continuous double auction. Among all the Walrasian auction designs we tested, the treatment in which full order flow information is provide and there are no bid-offer restrictions performs best. As theory would predict, there is a strong correlation between per unit profit and the amount of underrevelation by players. Finally, unlike the result found by Joyce, we find no significant strategic behavior differences between buyers and sellers.

2.0 Experimental Environment and Walrasian Auction Design

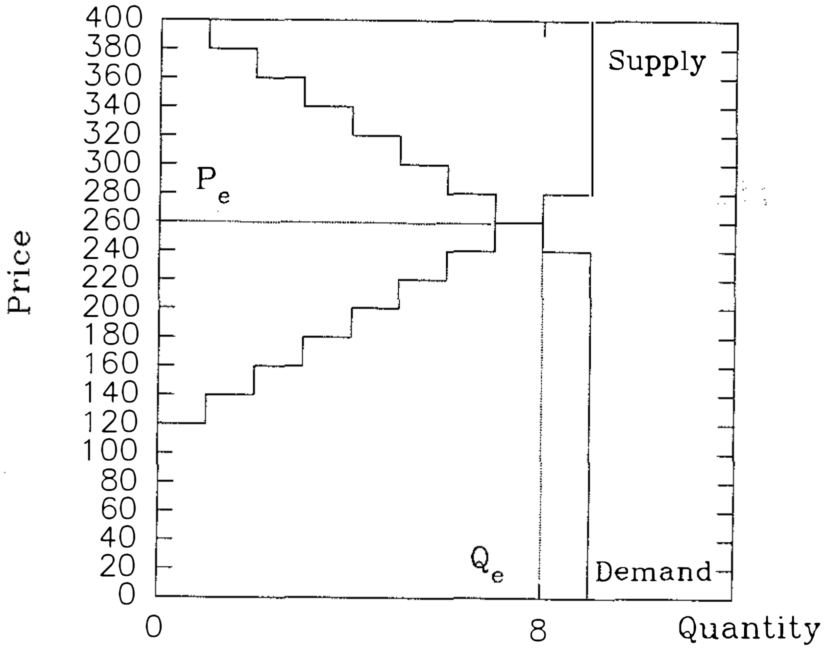
We first consider a simple experimental environment developed by Joyce (1984) in his examination of the Walrasian auction. We then describe our multi-unit nonstationary supply and demand environment and computerized implementation of the Walrasian auction process.

2.1 Baseline

Consider the following environment faced by n buyers and n sellers of a single commodity. Each buyer has a value for a discrete single unit. Each seller has the capacity to supply only one discrete unit to the market for a fixed cost. Given the values and costs of the potential market participants, a supply and demand array can be constructed as in Figure 1, which we will call environment E1.

[3] Transparency is defined as the amount of real time information on quotes, transaction prices and volume that is disclosed.

Figure 1
The E1 Environment



In the simple environment described in Figure 1, Joyce implements a specific Walrasian Auction as follows:

1. An initial price $P_o > 0$ is selected by an auctioneer.
2. Either:

a. With all participants present in the same room, each buyer and seller indicates to the auctioneer whether they wanted to buy or sell a single unit at the announced price by raising their hand. However, only the auctioneer knew if a particular subject was a buyer or seller, that is, each trader's identity (buyer or seller) was his own private information; or

b. Buyers and sellers were in separate rooms. Thus, buyers could observe the number of buy orders and sellers could observe the number of sell orders during each iteration and infer the opposite side's demand and supply at the end of each iteration.

3. If the number of buyers demanding a unit at that price equals the number of sellers supplying a unit at that price, the process stops and a market was made.

4. If there was an imbalance of supply and demand at that price, i.e. excess demand E(P) was nonzero, the auctioneer updated the price using the following formula:

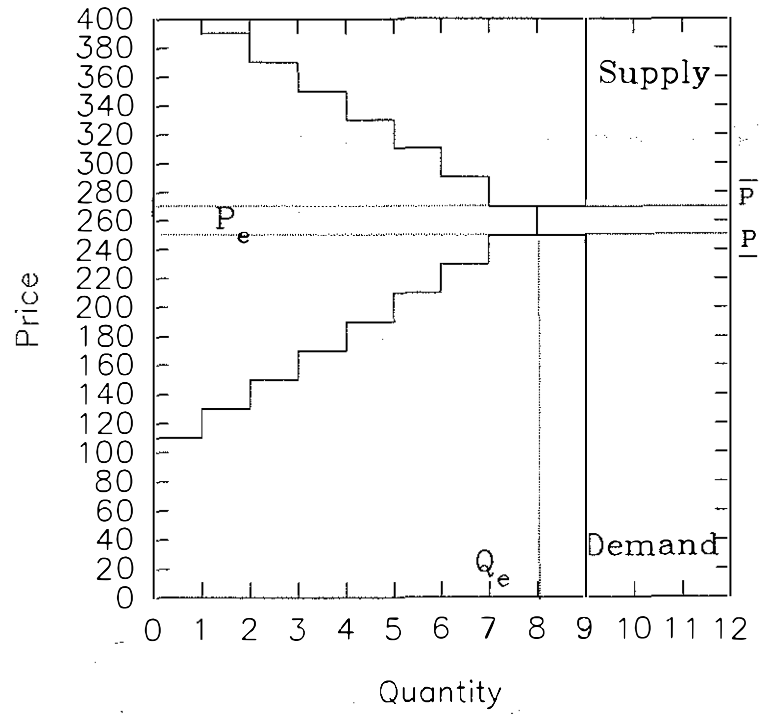
$$\Delta P = \begin{cases} \$.05E(P) & \text{if } |E(P)| > 1 \\ \$\text{“Z”}E(P) & \text{if } |E(P)| = 1, \text{ where “Z” } < \$.05 \end{cases}$$

Under treatment 2a, participants were not told the exact imbalance of supply and demand. In all cases, current buy or sell messages were not constrained by past messages (there was no improvement rule), and the price adjustment rule was linear. However, there was a strong restriction placed on each participants' message; they could only register demands and supplies for one unit.

2.2 Nash Equilibrium Strategies with Single Unit Demands and Supplies

When individuals have demands or supplies for one unit nonrevelation is a risky strategy since, should the market clear, the individual will fail to make a profitable transaction. If there is complete knowledge concerning values and costs, then in a nonrepeated process of this type, any pure strategy Nash equilibrium must be at (Q_e,P_e). This follows because at any outcome different than (Q_e,P_e) those individuals using a nonrevealing strategy, that are not part of the allocation, would do better by revealing. In particular, with the arrays given in Figure 1, revelation is a Nash equilibrium. However, suppose that the market environment is as pictured in Figure 2. Then, if $P_o > \bar{P}$, a Nash equilibrium is where one (or several) demanders with value greater than $\underline{P}$ do not reveal until price is at $\underline{P}$ and all sellers reveal. A similar result can be found for the case when $P_o < \underline{P}$; sellers with cost less than $\underline{P}$ do not reveal until price is at $\bar{P}$. With full information on the order flow, the actual price determined in the auction period will be affected by the relative bargaining power of buyers and sellers, and to some extent, the initial price P_o will affect each sides' ability to manipulate the outcome. However, with single unit demands and supplies, the only Nash equilibria are where the outcome is in the equilibrium "price tunnel" and all valuable units are traded.

Figure 2
Competitive Price Tunnel Design



If there is incomplete information about supply and demand, then individual participants must balance the probability of failing to make a profitable trade with the profits from affecting the final price by underrevealing. In this case, the competitive outcome need not be a Nash equilibrium and thus ex post inefficiencies can occur in equilibrium. In Joyce's study, his Walrasian auction had average efficiencies over 97%. Under the treatment 2b, prices were significantly lower than treatment 2a. This suggested that information on the actual amount of excess demand is important.

From the simple example above, we can see that the Walrasian mechanism allows strategic manipulation and that information concerning the composition of excess demand is important in determining an outcome. The questions that motivate our research are:

a. Which rules result in greater efficiencies in a Walrasian Tatonnement Process?

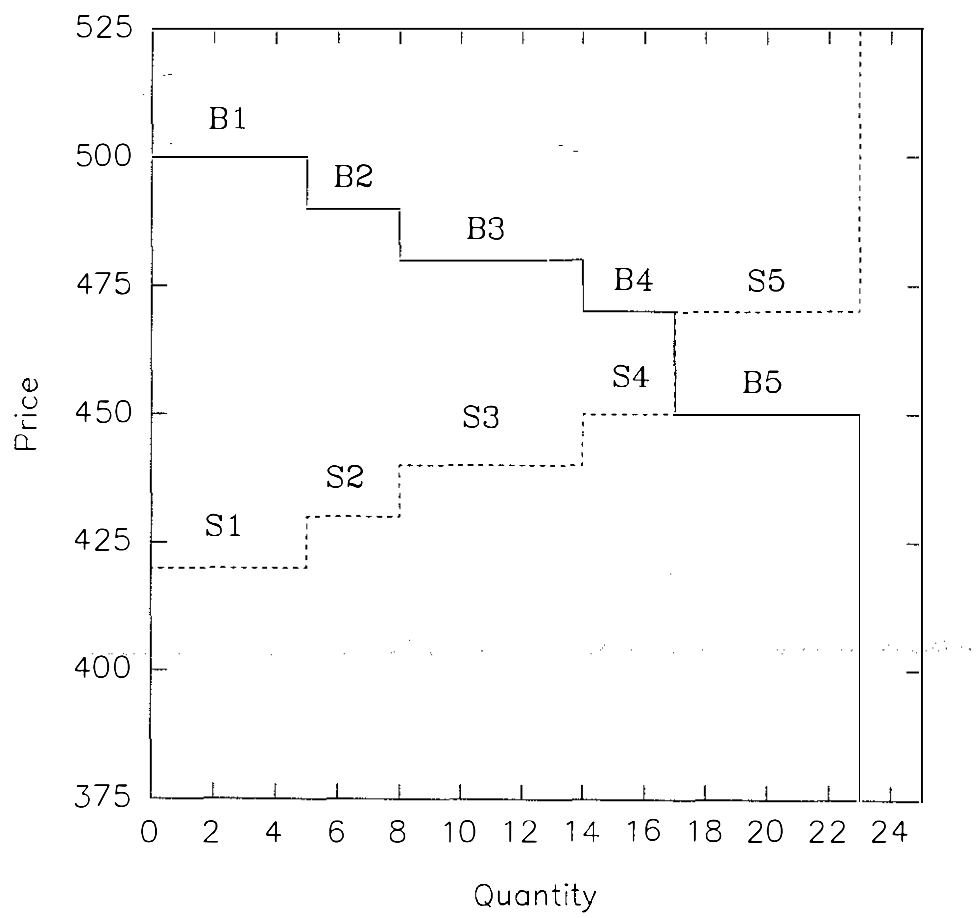
b. How well does the Walrasian mechanism perform when there are significant opportunities to manipulate prices?

c. How well does the mechanism perform in a non-stationary environment?

2.3 Multi-Unit Non-stationary Supply and Demand Environment

Our experiments utilized five buyers and five sellers. The basic demand and supply configuration is provided in Figure 3. The aggregate supply and demand arrays are step functions where each step identifies a particular individual's value or cost. Only one trader is assigned to a step on these functions. In addition, each participant has multiple units to bid or to offer all on the same step. As shown in Figure 3, there are three buyers (B1,B3,B5) and three sellers (S1,S2,S3) endowed with six units and two buyers (B2,B4) and two sellers (S2,S3) endowed with three units. Thus, there are twenty-four buy and sell units in the market; of these, eighteen are potentially tradable within the equilibrium price tunnel [450,470].

Figure 3
Period 4 Supply and Demand Arrays



The parameter values described in Figure 3 were used in period 4 of our experiments. During an experiment, buyers remained buyers and sellers remained sellers, period to period, although two important changes occurred each period:

1. The equilibrium prices was changed by parallel and equal shifts in the aggregate demand and supply arrays. In particular, from period to period, a random constant from the interval [100,490] is added to (or subtracted from) each step on the aggregate demand and supply functions. In period 4, the steps on the demand function correspond to the prices 500, 490, 480, 470, 450, 440, 430, and 420; the midpoint of the equilibrium price tunnel was 460. If the price at the midpoint of the equilibrium price tunnel in the next period was 250, the steps on the demand function would be 290, 280, 270, 260, 240, 220, 200, and 180. This change is a result of the demand and supply curves being shifted downward by 210, with their basic shape and point of intersection unchanged.

2. Buyers and sellers relative competitiveness changes. This takes place as follows: within each period, buyers are assigned (by a random rotation procedure) to one of the demand steps (B1-B5), and sellers are assigned to one the the supply steps (S1-S5). Each period they are assigned to a new step. For example, buyer 1 who in period 4 was endowed with the right to resell up to 6 units at a price of 500, could find himself or herself on a step with 6 units, but with no tradable units within the equilibrium price tunnel, as is buyer 5 in Figure 3. Alternatively, she could be on a step with only three units and where she can influence the price as the marginal buyer (as is buyer 4).

Thus, there are always 18 units to be traded at the competitive equilibrium price, always five steps on the demand function with buyers distributed one to a step, and always five steps on the sell function with sellers distributed one to a step. This experimental environment, which we will identify as E2, allows us to assess the performance of the Walrasian auction in an environment where participants have multiple units and where relative competitiveness is variable period to period. This environment has been used in previous experimental studies (see Campbell et al. (1991) and McCabe et al. (1991)) and has been shown to provide a difficult test for price discovery. The environment, from the participants perspective, seems to be changing each period and thus relying on past market data can hinder price discovery.

2.4 Walrasian Auction Design and Computerized Implementation

A Walrasian tatonnement must specify the following rules in order to implement the auction:

i. The process must determine a starting or *initial price* P_o.

For each experiment a set of initial prices was selected from three possible vectors. The vectors were determined as follows: Let $P^e = (P^e_1, ..., P^e_n)$ be the vector of equilibrium prices for periods 1 to n. Let θ_i be a random variable drawn from the interval [-50,50] for each i=1,...,n. Let ν_i be a random variable drawn from the interval [-25,25]. The first vector of initial prices was $P^1_o = (P^e_1 - \theta_1, \ldots, P^e_n - \theta_n)$; the second initial price vector was given by $P^2_o = -P^1_o$; the third vector was given by $P^3_o = (P^e_1 - \nu_1, \ldots, P^e_n - \nu_n)$. These initial price vectors were used to investigate the affect of the opening price on price discovery.

ii. The *price adjustment function* ϵ,[4]

$$P_t = P_{t-1} + \epsilon(D_{t-1}, S_{t-1})$$

The price adjustment rule we use in our experiments is piece-wise linear. Specifically, after 4 iterations the adjustment factor ϵ used in the previous four iterations was halved.[5] Notice that this piecewise adjustment rule reduces the benefits from nonrevelation because as the number of iterations increases, it takes a larger imbalance to significantly adjust the price. In our experiments, the following form of this piecewise rule was used:

$$P_t = P_{t-1} + 4\left[\frac{1}{2(1+[\![\frac{t}{4}]\!])}\right][D(P_{t-1}) - S(P_{t-1})]$$

where $[\![y]\!]$ denotes the greatest integer less than or equal to y and t is the current iteration in the period. For example, at iteration 6 with an announced price of 200 and reported excess demanded of 10, the price next period will be 210. Unlike the experiments conducted by Joyce, our experiments are computerized and thus there is no human auctioneer judgment on "appropriate" price changes.

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[4] We define $D_t = (D_o(P_o), D_1(P_1), \ldots, D_t(P_t))$, $S_t = (S_o(P_o), S_1(P_1), \ldots S_t(P_t))$ as the aggregate supply and demand responses for each price iteration up to P_t and D^i_t, S^j_t the individual supply and demand responses for each price iteration up to P_t. We will represent the true demands and supplies with the lower case letters, d_t, s_t, d^i_t, s^j_t.

[5] We define an *iteration* as the time between two successive price changes and a *period* as the time between two successive allocations.

iii. The *order flow information* available to participants, i.e. the information available to participant i <u>during</u> iteration t,

$$I^i(t) \subseteq \{D^1{}_o(P_o),..,D^n{}_t(P_t),S^1{}_o(P_o),..,S^m{}_t(P_t),E_o(P_o),..,E_t(P_t),\mu(D^i{}_t,S^i{}_t),..\}$$

Notice that this construction can include information on the current iteration buy and sell orders so that real-time information updates can be included. It also allows for basic market statistics (μ) such as the number of active participants, etc.

As has been noticed by Joyce (1986) and Noussair (1991), information supplied to players concerning the composition of excess demand can have a pronounced effect on the outcome of a tatonnement process. In our experiments, we consider two alternative information structures: *minimum information* and *complete order flow information.* Under the minimum information treatment, subjects are told (on their computer screens) the current trial price, the adjustment factor for the current trial, the number of seconds remaining for the current trial, and a full history of past trial prices and past order flow imbalances. However, they are given no information on any imbalance at the current price.

Our second treatment provides subjects with order flow information as well. In addition to the information in the minimum information treatment, subjects are provided at each trial with the real-time updated buy and sell orders as they arrived during the current price iteration, and what the next iteration price would be, based on the current imbalance information. Allowing participants to see the exact real time composition of buy and sell imbalances allows for individuals to update their strategies which can either hinder or assist in the price discovery process. The usefulness (efficiency promoting) of order flow information will likely depend on the underlying environment under study. In our experimental design the environment is not stationary. Therefore, this continual shifting equilibrium price also allows us to assess the effects of order flow information on the speed of price discovery.

iv. A *message restriction* specification limits the messages that can be sent, i.e.

$$(D^i{}_t(P_t),S^i{}_t(P_t)) \in M^i(D^i{}_t,S^i{}_t)$$

This rule restricts the potential buy and sell orders that can be placed during iteration t as a function of past responses. In all replications we placed the following restrictions on the messages participants could send at each iteration:

1. Individuals could not sell short or buy on margin. Thus, individuals

were not permitted to offer more units than their maximum capacity or demand more units than they had positive values.

2. Once an order was sent to the market it could not be canceled.

In several replications we put additional restrictions on subject messages. We imposed a *restriction* rule that requires a buyer who was willing to purchase m units at a price Y to be willing to purchase AT LEAST m units at prices lower than Y. Similarly, a seller who was willing to sell n units at at price Z must be willing to sell AT LEAST n units at prices above Z. For example, a buyer who is willing to buy 2 units at a price of 325 may state he or she is willing to purchase only 1 unit at a price of 290. This kind of inconsistency makes it difficult for participants to update their priors on the shape of the underlying demand and supply conditions. The motivation for this rule was to restrict inconsistency to allow for more consistent conjectures by subjects.

This rule does not prevent withholding, but it does minimize inconsistency. Thus a subject who was willing to buy 2 units at 325 is not precluded from revealing later in the period that in fact he or she is willing to buy 3 units at 325. A seller who was willing to sell only 1 unit at 340 can in the end agree to sell 4 units at 340. The rule is flexible enough to permit subjects to explore their relative competitiveness; however, it does limit "reneging".

v. Finally, a *stopping rule* defines the amount of time (ω) allowed for making buy and sell order decision for an iteration (t) and the rule (t^*) to end the period (ψ) and determine a final allocation X^* and price P^*. These are defined by:

$$\omega_t = O(D^i{}_t, S^i{}_t)$$
$$t^*{}_\psi = Q(D^\psi{}_t, S^\psi{}_t)$$
$$(P^*, X^*) = H(D_{t^*}, S_{t^*})$$

The stopping rule used in our experiments has two dimensions. First during an iteration, the time remaining to submit an order was endogenous. A clock was set at 15 seconds when the iteration price was posted. Any new order quantity submitted at the price reinitialized the clock to 15 seconds. This rule provided an implementation of a "soft close" procedure. A soft close enforces a unanimity requirement in that no one can guarantee himself or herself the last say. The second dimension dealt with the exact close of the market period. We closed the market period, at trial t^*, when $P_{t^*}=P_{t^*-1}$ or $E(P_{t^*})=0$. Notice that given our price adjustment rule, this stopping rule does not imply that $E(P) = 0$. Thus, if at t^*, $E(P_{t^*}) \neq 0$, we ration by a time priority.

A strategy S in this game is a mapping from past order flow information given the price adjustment, restriction and stopping rules, i.e., S^i_t: (P_t, I(t); $M^i(\cdot)$, $\epsilon(\cdot)$, $O(\cdot)$, $Q(\cdot)$, $H(\cdot)$,d^i, s^i) $\rightarrow$ (D^i_t,S^i_t). The final allocation will depend on the interaction of all these rule specifications on the strategies that participants select.

3.0 Experimental Design

Figure 4 shows the computer screens that subjects were viewing. Order flow information is provided in the lower middle box. Under the minimum information treatment, the only information provided in this box is the current iteration price (this is referred to as Potential Price on the subject screen). In all treatments, past trials and imbalances are recorded in the lower left-hand box, and the subjects current submitted quantity (and the number of profitable units at the current price) are recorded in the lower right-hand box.

Table 1 provides an overview of the experimental treatments and the number of experiments conducted per cell in our design. The design consists of two factors (improvement rule and order flow information) which are either present or not in each experiment. Appendix A contains an abbreviated set of instructions used in the experiments.

Table 1

Experimental Treatments*

(Number of experiments conducted is listed in cells)

Message Restriction	Information: Minimal	Information: Order Flow
No	4	5
Yes	3	3

* All of our treatments were conducted by using the piece-wise linear price adjustment rule described in this section.

Figure 4

Computer Display

Period						
Unit		10	11	12	13	
1	Value	400				
	Price					
	Profit					
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					
4	Value	100				

Iteration	D-S	P (BUY)
2	-2	385(1)
1	5	360(2)

Potential Price

375

Adjustment Factor:

2

Buys	Sells
17	16

Your current buy order:

1 units

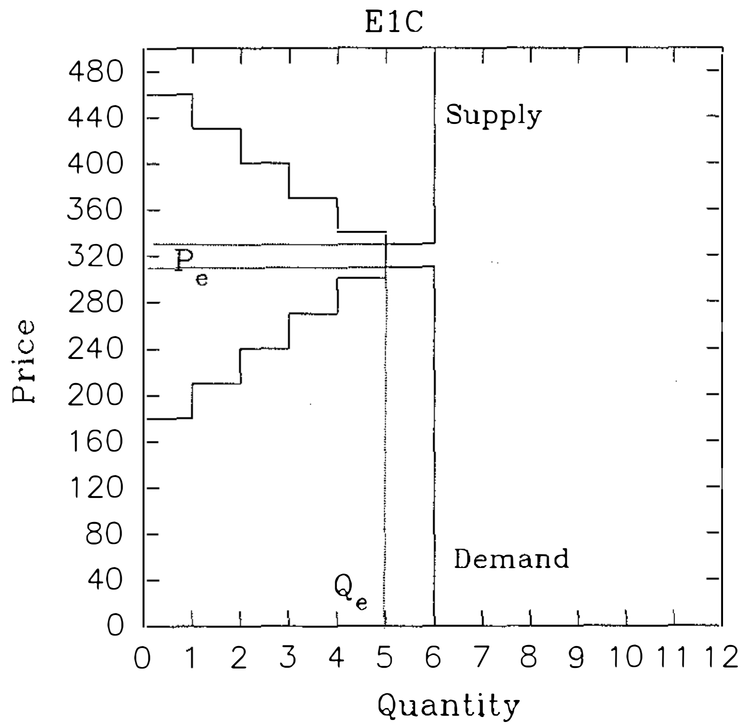
Profitable Units: 1

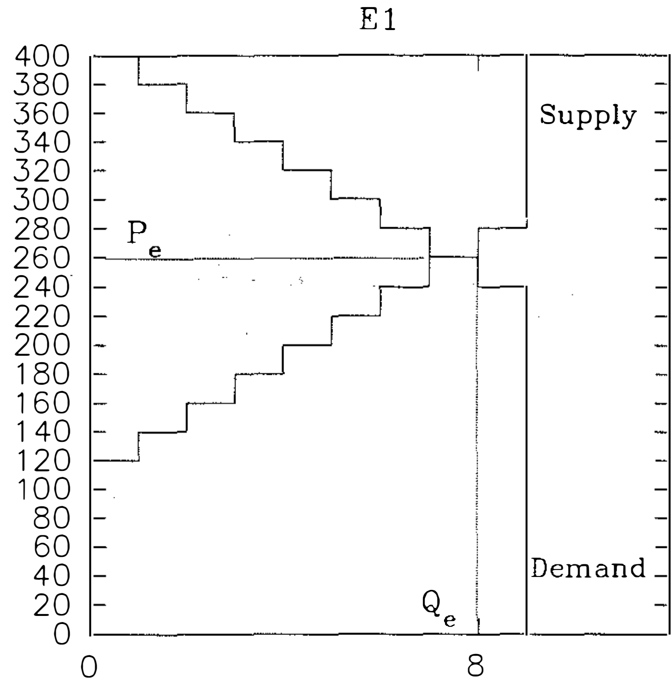
4.0 Experimental Results

4.1 The Simple Environment

In order to test the computer implementation of the Walrasian Tatonnement mechanism, we conducted two experiments using the environment described in Figure 5 which we call E1C. This environment is similar to the E1 environment used by Joyce. These two replications allow us to check our procedures and compare the results of our computerized tatonnement with the oral auction version reported in Joyce.

Figure 5
Simple Environments





In both Joyce and our experiments, subjects were endowed with single unit supplies and demands, and there was a 20 cent range defining the competitive equilibrium price. Price above the midpoint of this tunnel give greater surplus to sellers, and price below to the buyers. In our experiments, the tunnel was defined by subject valuations and costs; in Joyce's design, it was created by providing subjects with a 10 cent commission for each trade. In both experiments, subjects were paid the difference between their limit price and the market clearing price (plus the commission in Joyce).

Table 2 shows the mean efficiency (percent of the maximum producer plus consumer surplus generated) for periods 1-6 and periods 7+ for both the E1 and E1C environments. The efficiencies are high in later periods. However, in the computerized implementation of the mechanism there is a significant increase in efficiency in later periods of an experiment.

Result 1: A comparison of the efficiencies in periods 1-6 with that for periods beyond period 6 shows that efficiencies significantly increase with E1C and there is no

significant change with E1.
Support: The t-statistic is 2.25 for E1C and -1.64 for E1.

Table 2
Mean Efficiency By Periods for E1C and E1

	E1C	E1
Periods 1-6	85.3	98.9
	(1.8)	(2.6)
Periods 7+	97.7	96.3
	(5.6)	(6.1)

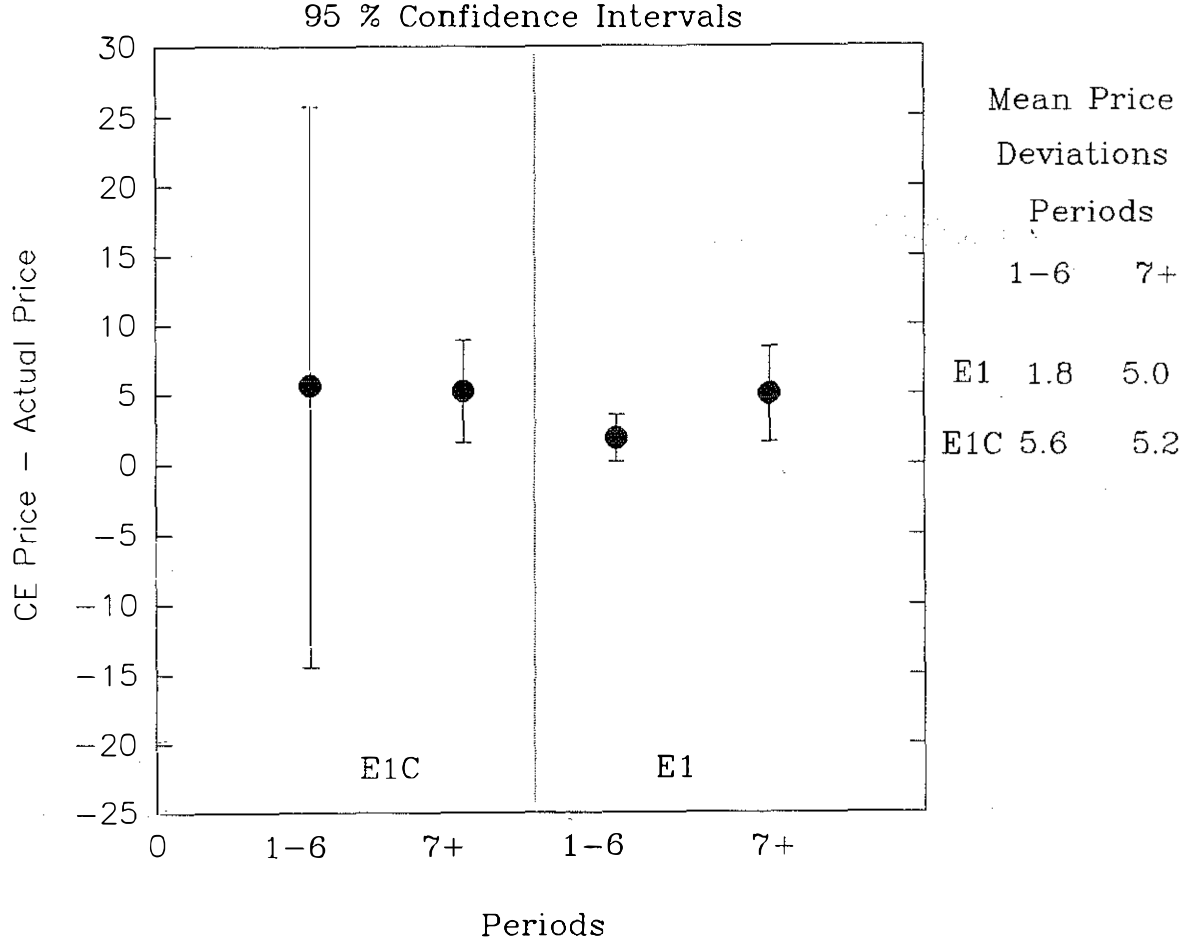
We also tested for differences in efficiency between the oral auctions conducted by Joyce and our computerized auction and found that they are not different for periods 7+. The strong period effect in the computerized treatment relative to the oral implementation is consistent with the results found in Williams (1980) where convergence to the competitive equilibrium price was slower with a computerized versus an oral implementation of a continuous bid-offer trading system.

Result 2: There is no significant difference in efficiency between the E1 and E1C cases.
Support: The t-statistic for periods 7+ is .605.

In terms of price formation, there is no difference between the computerized implementation and the oral implementation. Prices lie in the equilibrium price tunnel, near the midpoint (see Figure 6). This suggests that the Nash equilibrium prediction for single unit demand and supply is supported by this data, i.e., the Competitive Equilibrium outcome.

Result 3: There is no significant difference in the distribution of prices between the E1 and E1C cases.
Support: For periods 7+, the t-statistic is .106 (p-value = .90).

Figure 6
Price Dispersion Relative to CE Midpoint Price



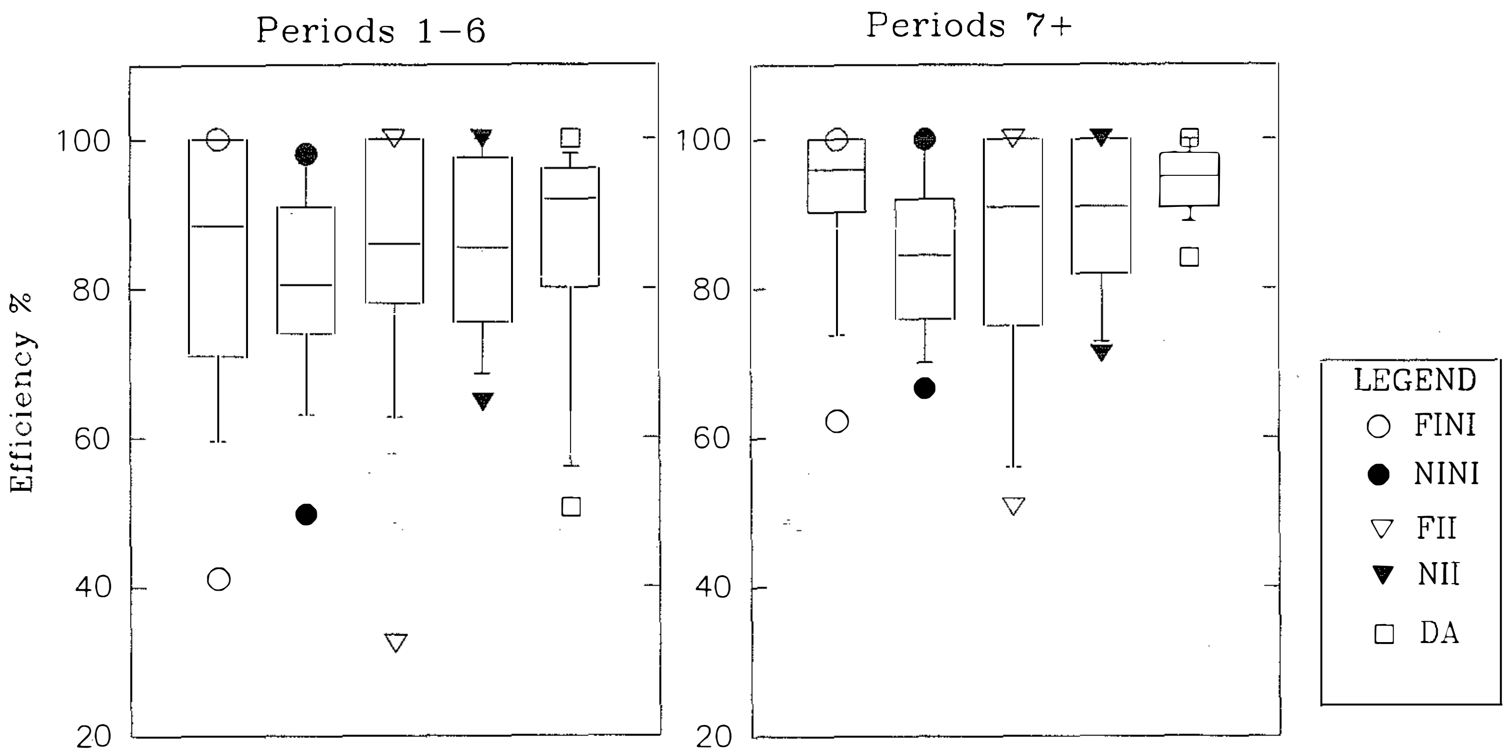
4.2 Baseline and Treatment Effects

For the remainder of this paper, we will use the following abbreviations for the treatments in our design: FINI = Full Information with No bid-offer restriction; NINI = No Information with No bid-offer restriction; FII = Full Information with bid-offer restriction; NII = No Information with bid-offer restriction .

Figure 7 shows the efficiency distribution (boxplots) for each of the four treatments in our implementation of the Walrasian auction in the E2 environment. The boxplots show the median, interquartiles, the 10th and 90th percentile caps. The 5th and 95th percentiles are shown as symbols below and above the caps. In addition to the Walrasian treatments, we report the results of 6 baseline double auction (DA) experiments using the E2 environment. A double auction is a real-time continuous process in which traders submit bids and offers with the bid-offer spread determined by a standard bid-ask improvement rule. The DA has been used extensively in experimental studies of markets and has the robust capacity to implement the Competitive Equilibrium outcome. The time series of efficiency for each experimental treatment can be found in Appendix B.

Figure 7

Distribution of Efficiency of Walrasian Auction Treatments and DA Baseline



Notice that the DA has a very tight distribution, while the Walrasian treatments have a large dispersion in efficiency.

Result 4: The DA outperforms each of the Walrasian auction designs we tested. FINI performs best among the Walrasian auction treatments.

Support: An ANOVA was undertaken based on the following dummy variable regression:

Efficiency = α_1DA*Periods1-6 + β_1FINI*Periods1-6 + ... + α_2DA*Periods7+ + β_5FINI*Periods7+ + ... + ϵ

Table 3 supplies the outcome of this regression and the associated statistics.

Table 3

ANOVA Estimates on Efficiency

Independent Variable	Estimated Coefficient	Standard Error
DA*Periods1-6	85.344	1.663
FINI*Periods1-6	83.000	1.358
NINI*Periods1-6	80.720	3.036
FII*Periods1-6	83.667	3.030
NII*Periods1-6	85.125	2.629
DA*Periods7+	94.619	1.358
FINI*Periods7+	91.718	2.062
NINI*Periods7+	84.031	2.526
FII*Periods7+	85.082	2.630
NII*Periods7+	88.147	2.209

Result 5: Each treatment yields an increase in efficiency in later periods.
Support: Additional support is provided by the results reported in Table 3.

Result 6: The following efficiency rankings, for periods 7+, show that only the full information without improvement (FINI) treatment approaches the efficiency of the double auction.: DA = FINI $\geq$ NII = FII = NINI.

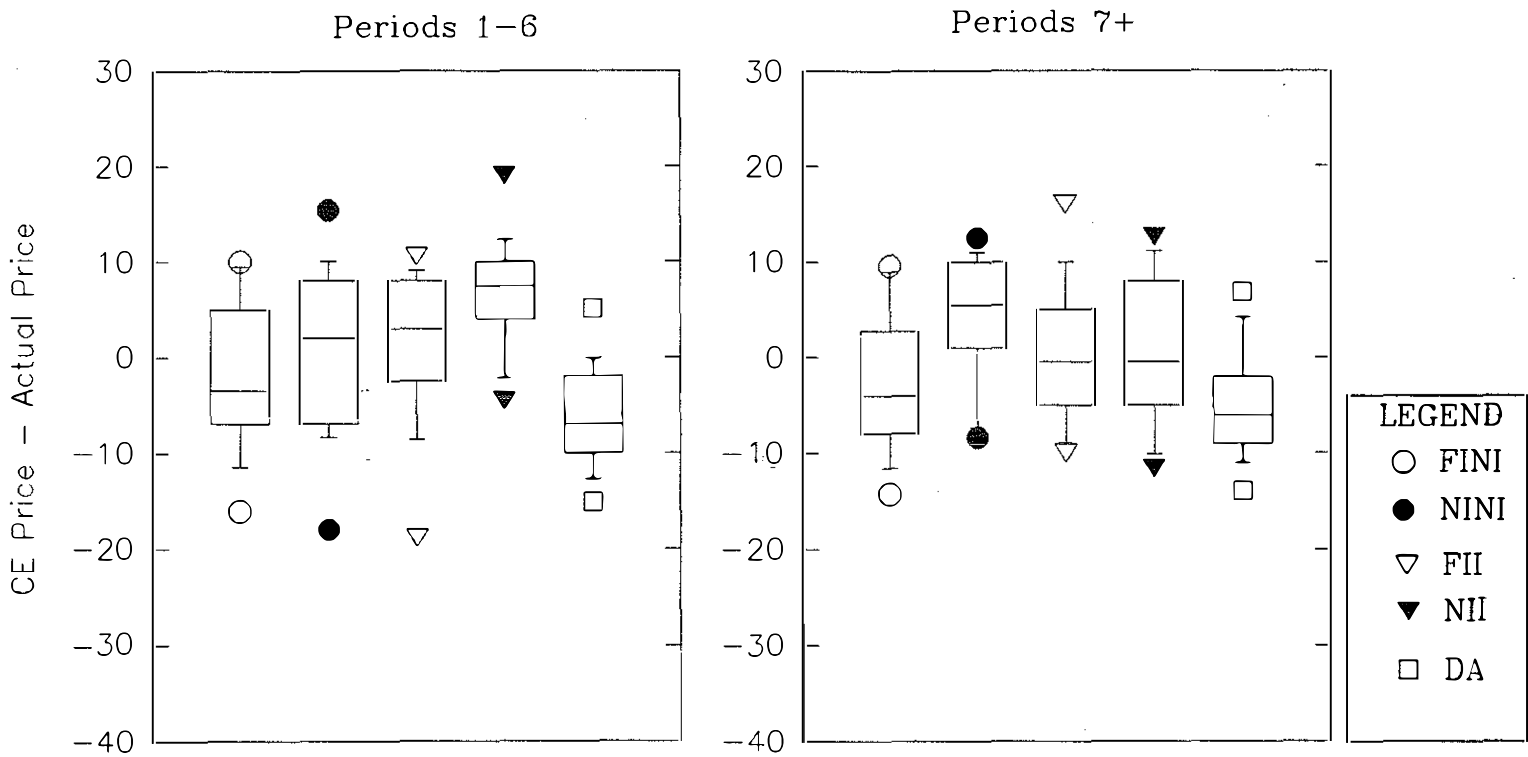
We can ascertain the following comparative static results.

i. Conditional on having no bid-offer restriction rule, full-information helps in obtaining more efficient allocations.

ii. Conditional on only minimal information being provided, the restriction rule helps in obtaining more efficient allocations. However, the level of efficiency does not approach that of FINI or DA.

With respect to price formation, Figure 8 shows the price dispersion relative to the competitive equilibrium price tunnel. From the boxplots it is easy to see that each treatment results in prices that lie within the tunnel. However, the low efficiencies reported in Table 3 show that the supply and demand match is not correct, and suggests the presence of significant underrevelation on both sides of the market: if one side underreveals to gain an advantage, the other side underreveals to neutralize that advantage.

Figure 8
Price Dispersion Around the CE Price Midpoint



To see if the final price is dependent on the initial price P_0 we estimate the following equation for each treatment for periods 7+:[6]

(Revealed Price - Actual Price) = $\alpha + \beta$(Revealed Price - Actual Price) + ϵ

Result 7: In no treatment does the initial price have an effect on the final price obtained in the market.

Support:

The estimates are:

Treatment	α	Standard Error	β	Standard Error
FINI	-2.65	1.45	-.0256	.0578
NINI	3.69	1.61	-.0290	.0486
FII	-0.84	1.93	-.0560	.0860
NII	0.26	1.28	.0330	.0520

Before we investigate individual behavior, we consider the relationship between the number of iterations and the efficiency of each treatment. Under price taking behavior that the process should stop within 2 iterations. This rarely happened in any of our treatments so that significant misrepresentation was occurring. Since subjects are trying to discover price and determine the terms of trade, an increase in the number of iterations may reflect more strategic withholding resulting in lower efficiencies.

Result 8: The number of iterations required to match supply and demand has an insignificant effect on efficiency for all of the Walrasian treatments. Significantly more price iterations are required to clear the market with the improvement rule.

Support: The following regression (for periods 7+) was estimated:

$$\text{Efficiency} = \alpha + \beta(\text{iterations}) + \epsilon$$

The estimates are:

Treatment	α	Standard Error	β	Standard Error	Average Per Period Number of Iterations
FINI	96.9	3.97	-.83	.54	6.30
NINI	89.3	4.27	-.89	.52	6.77
FII	90.4	5.72	-.56	.50	8.57
NII	77.7	4.65	1.18	.52	9.75

[6] Revealed price is defined as the price that would have occurred if every participant acted as a price taker in the market.

Result 8 reflects the difficult strategic problem faced by traders in a Walrasian auction. When there is no order flow information and trading strategies are constrained in subsequent iterations by the improvement rule (NII), slow revelation of demand is not risky in so far that it allows subjects to assess their relative competitiveness. However, underrevelation can result in an inefficient market clearing.

4.3 Individual Behavior

Three types of individual behavior can be identified in our experiments:

(1) *Overrevelation*: A buy or sell response that can result in a marginal loss in profit if the process stops, i.e., for each iteration t and participant i at the price P_t[7]

$$D^i{}_t(P_t) > d^i{}_t(P_t)$$
$$S^j{}_t(P_t) > s^j{}_t(P_t)$$

(2) *Underrevelation*: A buy or sell response that is less than the number units that are profitable at the current price

$$D^i{}_t(P_t) < d^i{}_t(P_t)$$
$$S^j{}_t(P_t) < s^j{}_t(P_t)$$

(3) *Revelation*: A buy or sell response that contains all profitable units and no unprofitable units at the current price

$$D^i{}_t(P_t) = d^i{}_t(P_t)$$
$$S^j{}_t(P_t) = s^j{}_t(P_t)$$

For each treatment, less than 3 percent of responses are consistent with underrevelation. For periods 7+, the number of responses that are consistent with overrevelation is less than 1 percent. Thus, overrevelation is a very rare occurrence in the Walrasian market treatments. If we only focus on cases in which positive profits can be made at the current price, there can be no cases of overrevelation since we have imposed short-selling restrictions. Thus, under the condition of positive profits, the table below shows the percent of responses consistent with underrevelation.

Table 4 shows that both buyers and sellers underreveal nearly one-third of the time. In FII, over 65% of the buyer responses are consistent with underrevelation.

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[7]Recall that uppercase letters represent actual aggregate supply and demand responses while lowercase letter represent true demand and supply functions.

Notice that under our improvement rule, in later periods, once a buyer (seller) has revealed a willingness to purchase (sell) x units at a particular price, he or she is required to purchase (sell) that many units at a lower (higher) price; therefore, underrevealing at the beginning of a period is the only way to obtain strategic bargaining room for later in a period.

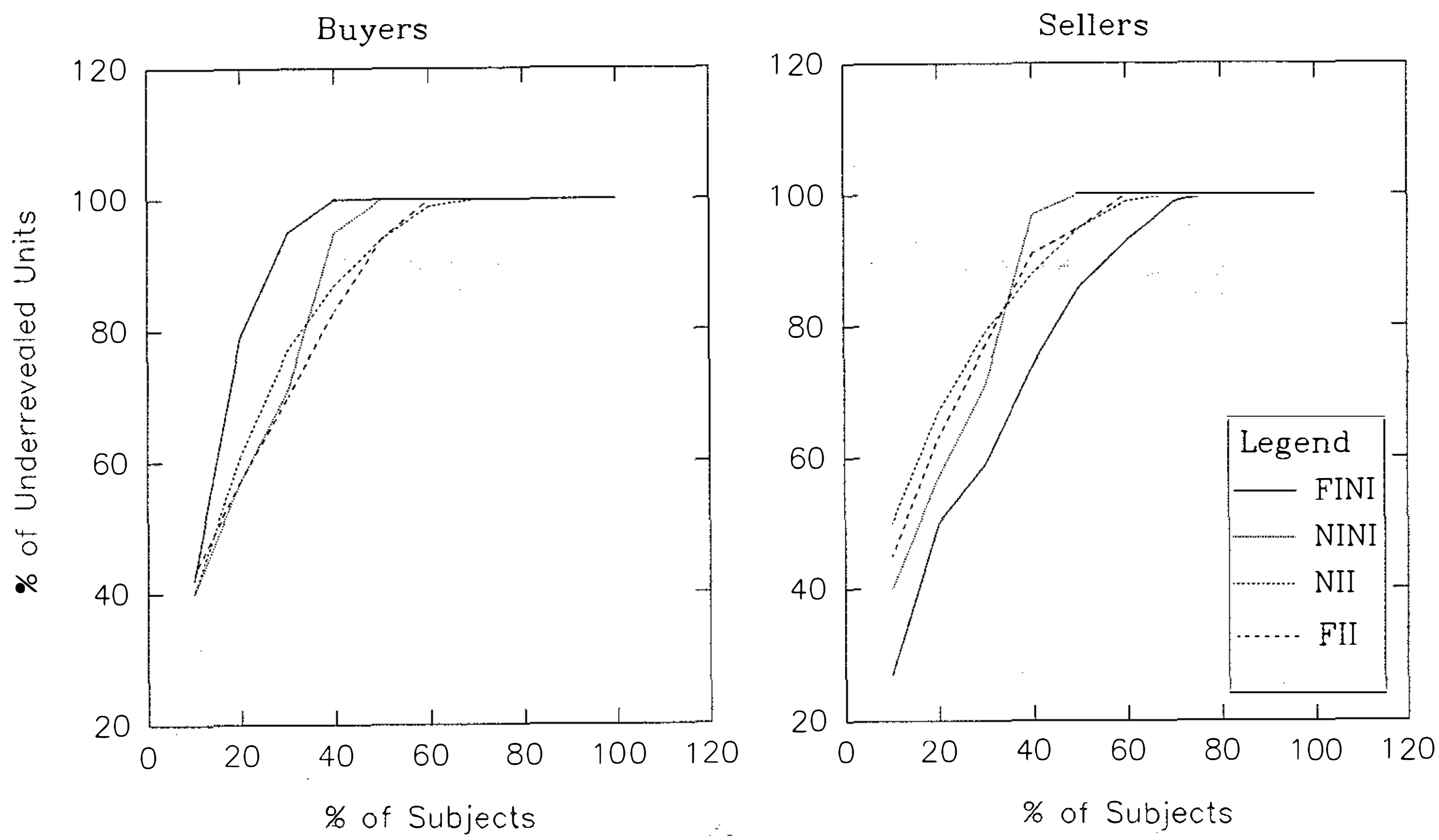
Table 4

Percent Underrevelation Responses by Type, Treatment and Periods

Treatment	Periods 1-6		Periods 7+	
	Buyers	Sellers	Buyers	Sellers
FINI	33	38	31	35
NINI	33	34	37	26
FII	37	33	66	38
NII	39	31	40	37

Table 4 suggests that underrevelation is common. Indeed, there are no cases in which a subject always reveals in every iteration and period of an experiment. However, if we only consider the final outcomes of a period in an experiment, we can investigate the distribution of underevelation by subjects. In Figure 9, for periods 7+, the sum of units underrevealed in the last iteration of each period was determined for each treatment, and then the sum of the units underrevealed in the last iteration of a period by each subject in a treatment was determined; the percent of underrevealed units by subjects is provided in the figure. For example, in the case of buyers, in all treatments, approximately one-half of the buyers account for all the underrevealed units. Alternatively, approximately half of the buyers were revealing by the end of a period.

Figure 9
Distribution of Underrevelation Responses



Result 9: By the end of a period, nearly one-half of all buyers reveal and one-third of all sellers reveal. In general, underrevelation is concentrated among a few of the participants.

If we focus on the underrevelation cases and determine what influences the <u>amount</u> underrevealed, theory shows that an underrevealing strategy is undertaken because the foregone profit on the unrevealed units is more than compensated for on the low/higher price paid/received on the accepted units. Thus, there should be an effect on the amount of underrevelation on the part of a subject based on per unit profit. The following equation was estimated for each treatment (for period 7+):

units underrevealed = $\alpha + \beta$ (per unit profit) + γ (buyer/seller dummy) + ϵ

We would predict that $\alpha < 0$, $\beta > 0$ and γ=0. Table 5 presents the estimates of this equation.

Table 5
Underrevelation Estimates*

Treatment	α	β	γ	R^2
FINI	-4.75	.057	.27	.34
	(.15)	(.0049)	(.21)	
NINI	-4.29	.0520	.30	.28
	(.16)	(.0057)	(.23)	
NII	-4.14	.0301	.05	.18
	(.17)	(.004)	(.24)	
FII	-4.36	.056	-.42	.40
	(.16)	(.006)	(.23)	

* Standard error are listed in parentheses under each estimate.

Result 10: For all treatments, the amount of underrevelation is significantly effected by per unit profits. The level of underevelation is not significantly different between buyers and sellers.

5.0 Summary

Walras's knowledge of the operations of the Paris Bourse and his need for a price adjustment mechanism that, in principle, could coordinate general equilibrium price adjustments led him to introduce into the economics literature the mechanism that bears his name. Its theoretical appeal was to define a virtual, or 'fictitious play' process which allowed the dispersed information of agents to be aggregated before binding contracts could occur. This characteristic allowed one to finesse the complexity of path dependent processes that result if contracts occur out of equilibrium. This feature no doubt accounts for the extensive theoretical study of its dynamic and stability properties. While the tatonnement has been found to be unstable in multiple markets, this research has left open its potential for application to single market price calls in securities and other markets.

Joyce was the first to examine this mechanism empirically (some 100 odd years after Walras's work), and found that it performed well in single unit per person environments using a human auctioneer. We find that all versions of the computerized multiple unit Walrasian auction perform less efficiently than the continuous double auction. The full information version, however, with no restrictions on bid-offer behavior, performs best. Since the Paris Bourse used a Walrasian process until recently, and it has a similar long application to the London Bullion price 'fixing' (Jarecki, 1976), why, given its relatively poor efficiency properties, has it been so durable? A likely possibility is that it works in field applications because it uses a live auctioneer, better informed and more flexible than a computer algorithm, and able to avoid backtracking or to minimize its effects. An alternative possibility is that mechanisms survive in the field for historical and other reasons unrelated to efficiency.

With the decline of interest in general equilibrium theory and the concomitant ascendancy of work in game theory, other auction mechanisms, popular throughout the world of commodity and financial markets, have been exposed to theoretical and empirical examination. A comparison of alternative call market mechanisms shows clearly that backtracking mechanisms, such as the Vickrey (1976) version of the multiple unit English auction perform badly relative to the one-sided convergence characterized by the English clock auction (McCabe, Rassenti and Smith, 1991). This result is robustly corroborated in two-sided auctions using the 'Dutch English' (DE) Clock mechanism (McCabe, Rassenti and Smith, 1992a). In the latter a price clock is started high; buyers report their demand quantities (say Q_D), and sellers report their supply quantity (say $Q_S>Q_D$). As the clock price ticks down, buyers enter and Q_D increases, sellers exit and Q_S decreases, until $Q_D=Q_S$ and Q_S units are sold to the active buyers. In the DE procedure, buyers who enter must commit; sellers who exit cannot reenter. If there is an overshoot, the penultimate trial becomes binding and the long side is rationed. Consequently, DE is like a Walrasian adjustment process but with tighter controls on exit/entry -- a Walrasian auction with a heavy-handed auctioneer, if you like. An obvious disadvantage of DE is that if new disrupting information arrives during a call, the committed traders cannot escape.

The principles here seem clear. Efficiency and strategy proofness can be enhanced by restricting the message space of traders. Prices are called exogenously, responses are restricted to exit/entry commitments that are binding, and backtracking is ruled out. An alternative to requiring commitment is to levy a charge for pulling your bid or offer. This provides an incentive to commit, but still escape. This could lead to a 'premature' stop with rationing or a 'failed' market. The latter of course may be desirable since it leads to a restart of the auction, when you want to restart expost new information.

The charge approach to the incentive problem is used effectively by the Arizona Stock Exchange (AZX). A commission is charged by AZX for a trade, but you pay it if you pull your bid (offer), and pay it again if you reenter. AZX is a uniform price double auction call market with open display of all bids and offers in real time. Currently the call is for one hour after the New York exchanges close. This mechanism was found to be as efficient as the continuous double auction in a stochastic environment examined by McCabe, Rassenti and Smith (1992b).

The results of our examination of the Walrasian mechanism show that it lacks robustness in environments in which multi-unit demands and supplies are present and there is little depth at the margin, so that underrevelation has a direct influence on price. Traders act strategically by underrevealing and in the process there is an incorrect match between supply and demand. The outcome results in approximately the correct price signal, but in order to support this price, both sides strategically under reveal.

Appendix A

Abbreviated Set of Instructions

Buyer 1 Tatonnement Auction

Unit		Period 1	Period 2	Period 3	Period 4	Period 5
1	Value	400				
	Price					
	Profit					
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					

Above you will find a record sheet. The numbers in your personal record sheet may be different and may change between periods in the actual experiment. The rows labeled with the word 'Value' represent the values to you (in cents) of purchasing each of the indicated units.

Each value may be thought of as the price the experimentor will pay you for that unit if you can purchase it and then resell it to him. All values will be in cents (.01 dollar).

You are BUYER 1 for the whole experiment.
Press -NEXT- to continue.

Buyer 1 Tatonnement Auction

Unit		Period 1	2	3	4	5
1	Value	400				
	Price					
	Profit					
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					

New values will be given to you prior to the beginning of each period. They may or may not be the same as those for the previous period.

Press -NEXT- to continue or -BACK- to review.

Buyer 1 Tatonnement Auction

Unit		Period 1	2	3	4	5
1	Value	400				
	Price					
	Profit					
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					

The main thing to remember is that your cash profits will depend upon your ability to buy a unit at a price below its value. Any loss (profit < 0) from buying a unit above its value will be deducted from your total profits.

Profits are accumulated over the whole experiment, with your total profits at the end of the experiment being the summation of your profits over all periods.

Press -NEXT- to continue or -BACK- to review.

Instructions without order flow information

Buyer 1 — Tatonnement Auction

		Period				
Unit		1	2	3	4	5
1	Value	400				
	Price					
	Profit					
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					

When the number of buy orders does not equal the the number of sell orders, the trial price will be changed. The adjustment in trial price is given by:

Adjustment factor x (BUY orders - SELL orders)

The value of the adjustment factor used to determine the next trial price is always shown above the center box.

Press <NEXT> to continue or <BACK> to review

Adjustment factor: 5.00

Trial	B-S	P (BUY)	Trial 1 Price	Your current buy order:
1	5	360 (2)	360	2 units
				Profitable units = 1

Buyer 1 Tatonnement Auction

Unit		Period 1	2	3	4	5
1	Value	400				
	Price					
	Profit					
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					

In our example, we see that the price for the next trial will be increased by 25 cents = [5.00 x (20 - 15)]. So the price for trial #2 will be 385 cents (360+25).

In the upcoming experiment the adjustment factor will start at 2 and will be halved every 4 trials.

Press <NEXT> to continue or <BACK> to review

Adjustment factor: 5.00

Trial	B-S	P(BUY)	Trial 1 Price	Your current buy order:
1	5	360 (2)	360	2 units
				Profitable units = 1

Buyer 1 Tatonnement Auction

Unit		Period 1	2	3	4	5
1	Value	4ØØ				
	Price					
	Profit					
2	Value	35Ø				
	Price					
	Profit					
3	Value	2ØØ				
	Price					
	Profit					

If at some later trial the adjustment factor is only .5Ø and there are 2 more sells than buys, then the next trial price would decrease by -1 cents (.5Ø×(-2)). But if the factor were only .125, then the price ajustment, .125×(-2)=-.25, would be less than half a cent and the price would not change at all. At that point the period would end, the current trial price would become the market price, and the last 2 sell orders would not be executed.

Press <NEXT> to continue or <BACK> to review

Adjustment factor: 5.ØØ

Trial	B-S	P(BUY)	Trial 1 Price	Your current buy order:
1	5	36Ø(2)	36Ø	2 units
				Profitable units = 1

Instructions with the bid-offer improvement rule

Buyer 1 Tatonnement Auction

Unit		Period 1	2	3	4	5
1	Value					
	Price	375				
	Profit	25				
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					

There are restrictions that will be placed on the orders you make in this market. As a buyer you can only submit buy orders. In addition, your buy order quantity must improve upon any previous order quantity at a less favorable trial price. The improvement rule works as follows.

Press ←NEXT→ to continue or ←BACK→ to review
Adjustment factor: 5.00

Trial	B-S	P (BUY)
2	-2	385 (1)
1	5	360 (2)

Trial 3 Price: 375

Your current buy order: 1 units

Profitable units = 1

Buyer 1 Tatonnement Auction

Unit		Period 1	2	3	4	5
1	Value					
	Price	375				
	Profit	25				
2	Value	35Ø				
	Price					
	Profit					
3	Value	2ØØ				
	Price					
	Profit					

Your orders must be consistent. If you are willing to buy 2 units at 355, you must be willing to buy at least 2 units at a lower price, say, 35Ø. The computer remembers your order quantity at every trial price. If in the current period there has been one or several prices higher than the current trial price, your order box will display the largest order quantity that you previously submitted. If you try to submit a lesser quantity the computer will ignore your order.

Press <NEXT> to continue or <BACK> to review

Adjustment factor: 5.ØØ

Trial	B-S	P(BUY)
2	-2	385 (1)
1	5	36Ø (2)

Trial 3 Price: 375

Your current buy order: 1 units

Profitable units = 1

Buyer 1		Tatonnement Auction				
		Period				
Unit		1	2	3	4	5
1	Value		45Ø			
	Price	375				
	Profit	25				
2	Value	35Ø	41Ø			
	Price					
	Profit					
3	Value	2ØØ	4ØØ			
	Price					
	Profit					

Notice that the computer simply ignores new orders for less than 3 units. Your default order is 2 units at the current trial price of 39Ø. Now submit an improved order for 3 units and observe the market results.

Adjustment factor: 5.ØØ

Trial	B-S	P (BUY)	Trial 3 Price	Your current buy order:
2	-8	43Ø (Ø)	39Ø	2 units
1	6	4ØØ (2)		Type a new order (>2) ≫
				Profitable units = 3

Remember that the adjustment factor for the upcoming experiment will start at 2 and be halved every 4 trials. This limits the number of possible prices which will be tried. The period ends and the current trial price becomes the market price when either:

1. The imbalance(B-S) at the current trial price is zero.

or

2. The adjustment factor is small enough that given the current imbalance no price change of at least one cent can be tried.

If condition 2 holds when the clock strikes Ø, then the latest orders submitted on the side of the market which has excess orders will not be executed.

Press -NEXT- to continue or -BACK- to review.

Quick Review

1. Each period you will be given a set of values for up to six units.

2. During a period there will be several trials to find a market-clearing price; you will be notified on your terminal of each trial price. You will then enter how many units you wish to buy at that price and press DATA to confirm your order.

3. After all market participants have submitted their buy and sell orders, there will be 15 seconds which you will have to improve your order quantity. The total number of buy and sell orders submitted will be posted on your screen.

4. If during a 15 second interval there are no further order quantities entered, the final imbalance will be calculated. The trial price will be increased if buy orders outnumber sell orders and decreased if sell orders outnumber buy orders. You will be asked to submit your order quantities at the new price.

Press ←NEXT→ to continue or ←BACK→ to review

Quick Review

5. The market will close when the number of buy orders equals the number of sell orders, or the required price change would be less than .5 cents. The current trial price will become the market clearing price. Your profit for every unit you bought at that price will be calculated for you. When there is an imbalance, the latest submitted orders on the side of the market with the excess orders will not execute.

Press ←NEXT→ to continue or ←BACK→ to review

Quick Review

5. The market will close when the number of buy orders equals the number of sell orders, or the required price change would be less than .5 cents. The current trial price will become the market clearing price. Your profit for every unit you bought at that price will be calculated for you. When there is an imbalance, the latest submitted orders on the side of the market with the excess orders will not execute.

6. Whenever no next trial price is showing, it means that if nobody submits any further orders before the clock strikes Ø then the current trial price will become the market clearing price for the current period.

Press ←NEXT→ to continue or ←BACK→ to review

Buyer 1 Tatonnement Auction

Unit		Period 1	2	3	4	5
1	Value	400				
	Price					
	Profit					
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					

Suppose at the end of trial #1 there are 20 buy and 15 sell orders. You tried to buy 2 units at 360 which gets recorded in the Price (BUY) column of the left hand box. There is an excess of buys (5) which gets recorded in the B-S (Buys minus Sells) column of the left hand box. Note that the number of buys and sells at the current price are shown in the center box.

Press <NEXT> to continue or <BACK> to review

Adjustment factor: 5.00

Next trial Price: 385

Trial	B-S	P (BUY)
1	5	360 (2)

Trial 1 Price

360

Buys 20 Sells 15

Your current buy order:

2 units

Profitable units = 1

Buyer 1 Tatonnement Auction

		Period				
Unit		1	2	3	4	5
1	Value	400				
	Price					
	Profit					
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					

Above the center box you are also shown what the next trial price would become (based on the present imbalance) if the current price does not clear the market. If the imbalance (B-S) is 0, or the adjustment is too small to force a new price based on the current imbalance, there will be <u>no next trial price</u> posted.

Press <NEXT> to continue or <BACK> to review

Adjustment factor: 5.00
Next trial Price: 385

Trial	B-S	P (BUY)
1	5	360 (2)

Trial 1 Price

360

Buys 20 Sells 15

Your current buy order:

2 units

Profitable units = 1

Buyer 1 Tatonnement Auction

Unit		Period 1	2	3	4	5
1	Value					
	Price	375				
	Profit	25				
2	Value	350				
	Price					
	Profit					
3	Value	200				
	Price					
	Profit					

There are some restrictions that will be placed on orders you make in this market. As a buyer, you can only submit buy orders. In addition, once you have submitted an order at a given trial price, you can not cancel or reduce the size of the order. You can however increase the number of units. If at some trial price you enter an order to buy 3 units, then you may increase the order to 4 units, but not decrease it to 0,1 or 2 units.

Press ←NEXT→ to continue or ←BACK→ to review

Adjustment factor: 5.00

Next trial Price:

Trial	B-S	P(BUY)
2	-2	385 (1)
1	5	360 (2)

Trial 3 Price: 375

Buys: [ ] Sells: [ ]

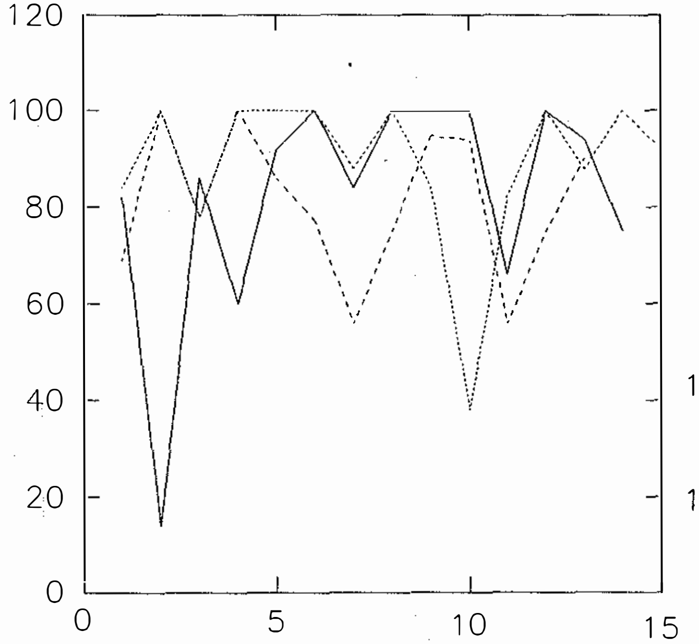
Your current buy order: 1 units

Profitable units = 1

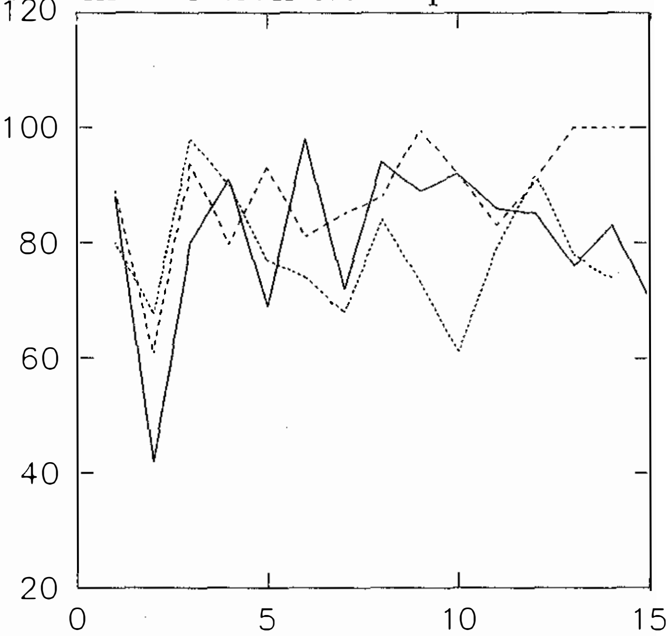
Appendix B

Time Series of Efficiency
By Treatment and Experiment
E2 Environment

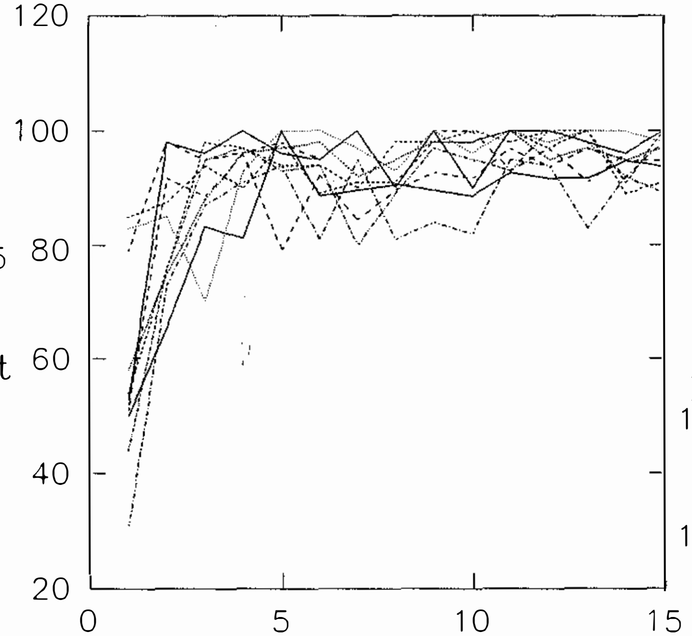
Full Information with Improvement



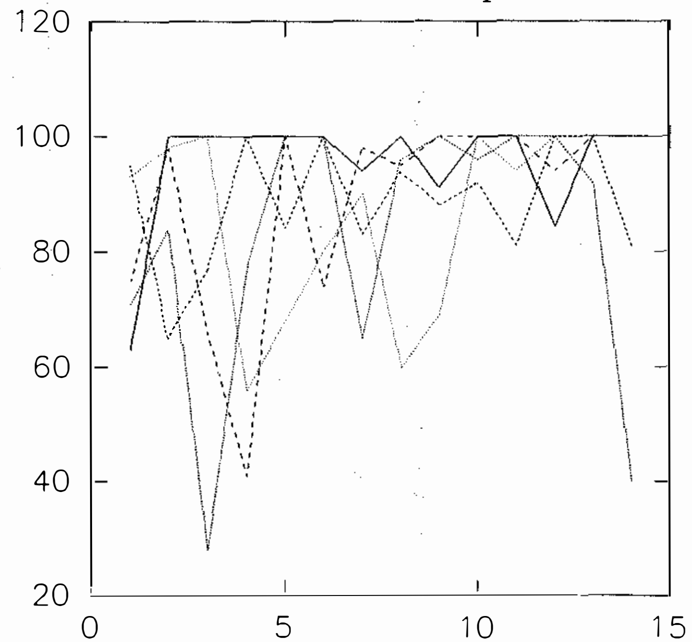
No Information No Improvement



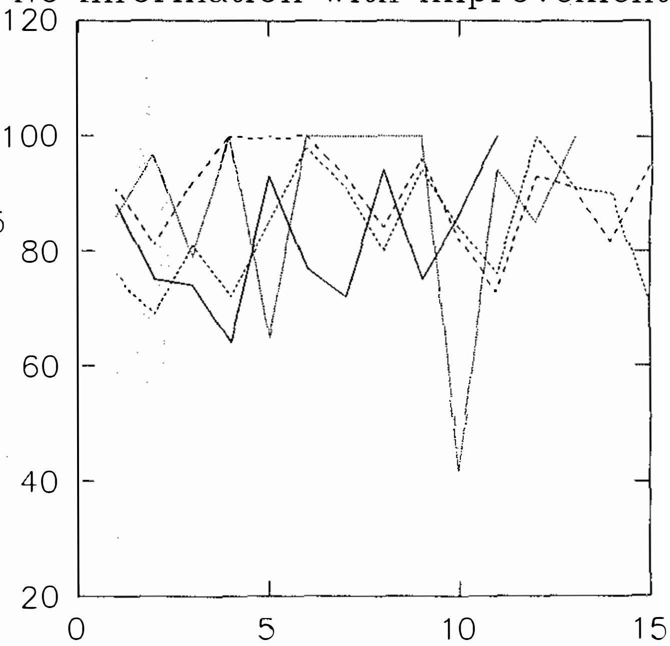
Double Auction Baseline



Full Information No Improvement



No Information with Improvement



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