

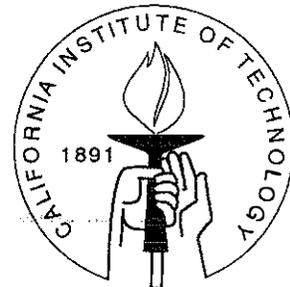
DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

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THE OPTIMAL CHOICE OF PRIVATIZING STATE-OWNED ENTERPRISES: A POLITICAL  
ECONOMIC MODEL

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## Abstract

We study the choice of a maximizing Bureaucrat over privatization policies and their effects on consumer welfare in a transition economy. We study a Bureaucrat whose objective function is maximizing a surplus budget subject to the constraint of staying in office, and a Bureaucrat who maximizes popularity/consumer welfare subject to the constraint of a balanced budget. Other things being equal, both types of Bureaucrat will privatize the sector (firms) with the least market power and the most subsidy first. This is the “cheapest” way to privatize state-owned enterprises. Also, it is shown that it is relatively easier and faster to privatize in a less democratic society.

# THE OPTIMAL CHOICE OF PRIVATIZING STATE-OWNED ENTERPRISES: A POLITICAL ECONOMIC MODEL

Yan Chen\*

## 1 Introduction

Because of the universally recognized deficiencies of state-owned enterprises compared to private enterprises, there is almost no controversy over the necessity to transform central-planned economies into market economies. The controversy lies in how to perform the transition. In this paper, we study the impact of privatization policies on consumer welfare, the consumers' preferences over different privatization bundles, and hence, what policies will be chosen by politicians with different objectives and under different political institutions. In particular, we study which kind of enterprises they would choose to privatize first.

In the past few years, socialist countries have tried different routes of transition from central-planned to market economies. There have been two types of transition policies: a "big-bang" policy of rapid and thorough privatization through a comprehensive plan, as practiced in most of the Eastern European countries and Russia; and an evolutionary policy, chiefly employed in China, where economic reform has been introduced step-by-step over the past decade<sup>1</sup>. Both types of policies have had rocky transitions. The transition period is usually marked by substantial price increases, imperfect competition and a fluctuation in people's living standards. Most of the time, these characteristics are the result of certain features of the old planned economy. Studying the mixed effects of the old and new economic and political institutions during this special period reveals the influence of different policies, especially the sequencing of policies, which can have a substantial impact on the reform process as well as on people's living standards.

Debates over privatization policies among economists of socialist countries usually focus on the sequencing problem, i.e., how to find the optimal sequence of privatization so as to minimize the problems characterizing the transition period. The sequence suggested and practiced in these countries roughly follows the size of different sectors<sup>2</sup>: rapid privatization of small

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<sup>1</sup>See John McMillan and Barry Naughton (1992) for a detailed comparison.

<sup>2</sup>See Yining Li (1989); Hans Blommesteine and Michael Marrese [ed.] (1991), p.15 and p.23

businesses first; establishment of a social safety net; demonopolization; privatization of medium state-owned enterprises; and last, privatization of large state-owned enterprises.

Despite the heated discussions and various experiments over sequencing, there is virtually no theoretical work on sequencing. Most of the privatization literature concentrates on the sale mechanisms<sup>3</sup>, ownership and incentives<sup>4</sup>, the regulation of privatized firms<sup>5</sup>, and the problems of the transition period<sup>6</sup>. In particular, there is no formal model in the privatization literature that incorporates the influence of politics on privatization policies. We need to remember that politicians are the ones that choose the privatization policies. Therefore, it is important to see what kind of policies a maximizing politician would choose under different political institutions.

In this paper, we set up a model to test the rationale of the sequences suggested above by economists of central planned economies and to study what kind of sequence a Bureaucrat with different objectives would choose. Section 2 introduces the setup and basic assumptions of a highly simplified transition economy and a Bureaucrat, a simplified representation of the political institution. In Section 3, we analyse the problem of the consumer, the firm and the Bureaucrat respectively. We consider two types of Bureaucrats: one who maximizes the surplus budget subject to the constraint of staying in office and one who maximizes popularity/consumer welfare subject to the constraint of a balanced budget<sup>7</sup>. In Section 4, we present the main results of the model: other things being equal, both types of Bureaucrat will first privatize the sector(s) which have the least market power<sup>8</sup> and which receive the largest subsidy. Also, we show that it can be relatively easier and faster to privatize in a less democratic society. Finally, in Section 5, we discuss the limitations and possible extensions of the model and conclude the paper.

## 2 Setup and Basic Assumptions

In this section we present a simple two period model of a transition economy that consists of  $I$  consumers,  $N$  sectors of firms, and a Bureaucrat.

Consumers have different utility functions and incomes, which are exogenously given. At time  $t$ , consumer  $i$  is rationed to a fixed amount of products from the public sectors at fixed prices. Because of the low prices and minimum amounts supplied, we assume that he buys all the quantities that are rationed to him. This assumption closely approximates the actual situations in many central planned economies. He uses the rest of his income to choose consumption bundles from the products of the private sectors to maximize his utility.

**Assumption 1** *Each consumer has a quasilinear utility function,  $u_i(q_1, \dots, q_N) + m_i$ , where  $q_1, \dots, q_N$  are the amounts of products 1, ...,  $N$  he consumes, and  $m$  is a numeraire good.*

*$u : R_+^{N+1} \rightarrow R_+^1$  is monotonically increasing, twice continuously differentiable, strictly quasi-*

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<sup>3</sup>See Patrick Bolton and Gerard Roland (1992) Appendix 2; and Fernando Branco and Antonio S. Mello (1991).

<sup>4</sup>See Jean Tirole (1991)

<sup>5</sup>See Jean Tirole (1991)

<sup>6</sup>See David Lipton and Jeffrey Sachs (1990) for a model of repressed inflation and the corresponding strategies of transition. Murphy, Shleifer and Vishny (1991) has an interesting model of supply diversion.

<sup>7</sup>We will see later that this is equivalent to maximizing budget.

<sup>8</sup>This concept is introduced later. It basically captures the competitiveness of the sector and the elasticity of the product. The higher the market power of a sector, the more it can raise the price above marginal cost.

concave, and satisfies the following Inada conditions:

$$\lim_{q_j \downarrow 0} \frac{\partial}{\partial q_j} u_i(q_1, \dots, q_N) = +\infty, \quad \lim_{q_j \uparrow \infty} \frac{\partial}{\partial q_j} u_i(q_1, \dots, q_N) = 0.$$

In the model, firms produce  $N$  distinct products and are hence divided into  $N$  sectors. Each sector  $j$  consists of  $L_j$  identical firms, where  $L_j \in [1, +\infty)$ . When  $L_j = 1$ , sector  $j$  is a monopoly; when  $L_j$  is large enough (approaching infinity), sector  $j$  is competitive; when  $L_j$  is between 1 and infinity, it can have different degrees of competitiveness<sup>9</sup>. At time  $t - 1$ ,  $k$  sectors are public (denoted by sector 1, ...  $k$ ), and  $n - k$  sectors are private (denoted by sectors  $k + 1, k + 2, \dots, N$ ). In each period the Bureaucrat can choose to privatize one or more sectors. At time  $t$ , the Bureaucrat decides whether to continue privatizing, and which sector(s) to privatize. Without loss of generality, assume sector  $k$  is picked. We can then study the characteristics of  $k$  and the influence of its privatization on the changes in consumer welfare.

Assume that a firm  $lj$  in public sector  $j$  fulfills quota  $\bar{Q}_{lj}$  imposed by the Bureaucrat, and sells its output at a fixed price  $\bar{P}_j$ , which is below the market-clearing price. This assumption reflects a basic feature of the central planned economies, where prices are fixed for historical reasons and reflect neither cost nor market demand. Therefore, the total output of sector  $j$  is  $\bar{Q}_j = \bar{Q}_{lj} * L_j$ . For simplicity, assume the population in the economy is fixed. Therefore, we can assume that  $\bar{Q}_j$  and  $\bar{Q}_{lj}$  are fixed as long as sector  $j$  remains public, since the quota is decided by rationing over the total population. Let the revenue of (or subsidy to) sector  $j$  be  $B_j = \bar{P}_j * \bar{Q}_j - C_{lj}(\bar{Q}_j) * L_j$ , where  $C_{lj}(\cdot)$  is the cost function of firm  $lj$ .

After sector  $j$  is privatized, firm  $lj$ 's objective becomes profit maximization. It chooses the optimal output  $Q_{lj}$  and sells it at the market-clearing price  $P_j$ . Let  $\pi_{lj}$  be the firm's profit function, and  $C_{lj}$  its cost function which satisfies the following assumptions:

**Assumption 2**  $C: R_+^1 \rightarrow R_+^1$  is differentiable and monotonic.

Let  $P_j(Q_1, \dots, Q_N)$  be sector  $j$ 's inverse demand function. To ensure the existence of a Cournot equilibrium<sup>10</sup>, we need some assumptions on  $P_j(\bar{Q})$ .

**Assumption 3**  $P_j(\bar{Q})$  is twice continuously differentiable, monotonic, and satisfies  $\frac{\partial P_j(\bar{Q})}{\partial Q_j} + Q_j \frac{\partial^2 P_j(\bar{Q})}{\partial Q_j^2} \leq 0$ , which requires the inverse demand function to be concave.

Note that the cost function of firm  $lj$  does not change before or after privatization. Here we implicitly assume that technology does not change. What is changed is the production quantity and price, which is adjusted for the purpose of profit maximization. This implies that the objective functions of the firms change after privatization, but any efficiency gain occurs after the transition period.

In order to simplify the structure in the later part of the model, we assume that none of the products of the  $N$  sectors are substitutes for each other. They can be either independent or complements. Another way to think about this assumption is to group all the substitutes in the economy in the same sector and simplify them into one product by using marginal rates of substitution.

**Assumption 4** The cross elasticity of any two products  $\varepsilon_{ij} = -\frac{\partial q_i}{\partial P_j} \frac{P_j}{q_i} \geq 0, \forall i \neq j, i, j = 1, 2, \dots, n$ .

<sup>9</sup>See Jean Tirole (1988), p.218 - p.221, p.226 - p.228.

<sup>10</sup>See, e.g., Richard Schmalensee and Robert D. Willig [ed.] (1988), p.334 - p.335.

This assumption is equivalent to  $\frac{\partial Q_i}{\partial P_j} \leq 0, \forall i \neq j$ , which implies that the consumers' utility functions need to satisfy the following condition:  $\frac{\partial^2 u_i(\vec{Q})}{\partial q_i \partial q_l} \leq 0, \forall l = 1, \dots, N$ . (See Appendix for proof.)

The Bureaucrat is a simplified representation of the government. He is assumed to maximize the surplus budget<sup>11</sup>. For comparison we also consider what happens if he maximizes consumer welfare, which reflects the Bureaucrat's popularity and job security. He knows the distribution of consumers' utility functions<sup>12</sup> and the distributions of shares in the private sectors. At any given time  $t$ , he makes three decisions – whether to continue privatizing, which sector to privatize and how to compensate the consumers. In order to concentrate on the characteristics of the transition period, we neglect some other important functions of the government, such as public good provision, and assume that the Bureaucrat's only functions are privatization and compensation. We use a parameter  $\alpha$  to characterize the political institutions, where  $\alpha$  is the percentage of consumers he needs to satisfy in order to stay in office.

### 3 Analysis of the Model

#### 3.1 The Firm's Problem

Public firm  $lj$  in sector  $j$  is given the quota  $\bar{Q}_{lj}$ . Suppose it can fulfill the quota and sell its output at the fixed price  $\bar{P}_j$ . Then it will provide revenue (or demand subsidy) of the amount  $B_{lj}$ , where  $B_{lj} = \bar{P}_j \bar{Q}_{lj} - C(\bar{Q}_{lj})$ . So the total revenue (or subsidy) of sector  $j$  is

$$B_j = \bar{P}_j \bar{Q}_j - C(\bar{Q}_j).$$

After the firm is privatized, it becomes a profit maximizer. It chooses its optimal output  $Q_{lj}$  to maximize its profit. The price of product  $j$  is determined by the total output of the sector, which depends on the decisions of the other identical firms in the same sector and the total output of other sectors. Note that by Assumption 1, consumers all have quasilinear utility functions, so the inverse demand functions exist. We use Cournot equilibrium analysis for the private firms' decisions.

Firm  $lj$  chooses the optimal output  $Q_{lj}$  in order to

$$\max_{Q_{lj}} P_j(Q_1, \dots, Q_N) Q_{lj} - C(Q_{lj}).$$

From Assumption 2, we know that the second order condition for the above maximization problem is satisfied, so we only need to look at the first order condition, which is,

$$P_j + Q_{lj} \left[ \frac{\partial P_j}{\partial Q_j} + \sum_{h \neq j} \frac{\partial P_j(\vec{Q})}{\partial Q_h} \frac{\partial Q_h}{\partial Q_j} \right] - MC_j = 0.$$

Here we use  $MC_j$  to denote the marginal cost of firms in sector  $j$ . Rearranging terms we get

$$\frac{P_j(\vec{Q}) - MC_j}{P_j(\vec{Q})} = \frac{1}{L_j} \left[ \frac{1}{\varepsilon_{jj}} + \sum_{h \neq j} \frac{1}{\varepsilon_{jh}} \right] \equiv \alpha_j,$$

<sup>11</sup>To be defined later.

<sup>12</sup>But he does not know the exact utility function of each consumer.

where  $\varepsilon_{jj}$  is the own elasticity of demand at  $Q_j$ , and  $\varepsilon_{jh}$  is the cross elasticity of demand between product  $j$  and product  $h$ . Since all firms of the same sector are identical, namely, they all have the same cost functions, the market share of firm  $lj$  equals the inverse of the number of firms in sector  $j$ , i.e.,  $\frac{Q_{lj}}{Q_j} = \frac{1}{L_j}$ . Note that the above formula is very similar to the Cournot oligopoly pricing formula, where  $\alpha_j = \frac{P_j(\bar{Q}) - MC_j}{P_j(\bar{Q})} = \frac{Q_{lj}}{Q_j} \frac{1}{\varepsilon_{jj}}$ .<sup>13</sup> We call  $\alpha_j$  firm  $lj$ 's *market power index*, which also characterizes sector  $j$ 's market power. Alternatively, we can write the above equation as  $P_j = \frac{MC_j}{1 - \alpha_j}$ , which will be used later.

Note that the market power index is quite general with regard to the degree of competitiveness in a sector. When  $L_j = 1$ , the above formula becomes the monopoly pricing formula. On the other hand, if  $L_j \rightarrow \infty$ , the equilibrium converges to the competitive equilibrium<sup>14</sup>. Therefore, the market power index shows how much a sector can raise the price of its product above its marginal cost. It is inversely related to the number of firms in the sector and the elasticities of demand of the product.

### 3.2 The Consumer's Problem

We assumed above that in a socialist economy consumer  $i$  buys all the rationed products first, and then uses the rest of his income to choose a consumption bundle from the private sector. In order to study the effects of the privatization of a certain sector, say  $k$ , on the change of his utility, we study his maximization problem in two arbitrarily chosen contiguous time periods,  $t-1$  and  $t$ .

At time  $t-1$ , sectors  $1, \dots, k$  are in the public sector, fulfilling quotas; sectors  $k+1, \dots, N$  are in the private sector, maximizing profits. Consumer  $i$ 's rationed quantities of products  $1$  through  $k$  are  $\bar{q}_1, \dots, \bar{q}_k$ , which are allocated equally to everybody in the economy. In reality, the allocations vary from person to person according to age, sex and other personal characteristics. Here, for simplicity of analysis, and also because we can not distinguish among individual consumers, we assume an equal allocation. He then chooses a consumption bundle of products from the private sector and the amount of numeraire good to maximize his utility subject to his budget constraint:

$$\begin{aligned} & \max_{\{q_j^{t-1}\}_{j=k+1}^N, m_i^{t-1}} [u_i(\bar{q}_1, \dots, \bar{q}_k, q_{k+1}^{t-1}, \dots, q_{iN}^{t-1}) + m_i^{t-1}] \\ \text{s.t. } & \sum_{j=k+1}^N P_j^{t-1} q_{ij}^{t-1} + m_i^{t-1} = y_i + \sum_{j=k+1}^N \theta_{ij}(1 - \tau_j)\pi_j^{t-1} - \sum_{j=1}^k \bar{P}_j \bar{q}_j = y_i^{t-1}. \end{aligned}$$

Here  $\tau_j$  is the tax rate of sector  $j$ , and  $\theta_{ij}$  is consumer  $i$ 's proportion of shares in sector  $j$ . From the budget constraint, we see that consumer  $i$ 's income comes from two sources: his exogenously given income  $y_i$ , which can be interpreted as wage and other personal endowments, and his share of the after tax profit from the private sector,  $\sum_{j=k+1}^N \theta_{ij}(1 - \tau_j)\pi_j^{t-1}$ . His *effective income*,  $y_i^{t-1}$ , with which he can choose his consumption bundle among products produced in the private sector, is total income less the expenditure on rationed products.

<sup>13</sup>See, e.g., Richard Schmalensee and Robert D. Willig [ed.] (1989), Ch.6. We rewrite it using the notations of this paper.

<sup>14</sup>See Jean Tirole (1988), p.226 - p.228.

Solving this maximization problem, we can get the demand function of consumer  $i$  for good  $j$

$$q_{ij}^{t-1} = q_{ij}^{t-1*}(P_{k+1}^{t-1}, \dots, P_N^{t-1})$$

and the aggregate demand function for product  $j$

$$Q_j^{t-1} = \sum_{i=1}^I q_{ij}^{t-1*}(P_{k+1}^{t-1}, \dots, P_N^{t-1}).$$

Plugging the demand function into consumer  $i$ 's utility function gives us his indirect utility function

$$v_i(\bar{P}_1, \dots, \bar{P}_k, P_{k+1}^{t-1}, \dots, P_N^{t-1}, y_i^{t-1}) = \phi_i(\bar{P}_1, \dots, \bar{P}_k, P_{k+1}^{t-1}, \dots, P_N^{t-1}) + y_i^{t-1},$$

where  $\phi_i(\bar{P}^{t-1}) = u_i(\bar{q}_1, \dots, \bar{q}_k, q_{i,k+1}^{t-1*}(\bar{P}^{t-1}), \dots, q_{iN}^{t-1*}(\bar{P}^{t-1})) - \sum_{j=k+1}^N P_j^{t-1} q_{ij}^{t-1*}(\bar{P}^{t-1})$ . Since consumer  $i$  has quasi-linear utility function, his indirect utility function can be written in two parts, with effective income separate from  $\phi(\bar{P}^{t-1})$ .

At time  $t$ , if another sector, say, sector  $k$ , is privatized, consumer  $i$  is given compensation  $T$  for the price increase in product  $k$  and the price changes in the other private sectors<sup>15</sup>. At the same time, he can buy shares in the newly privatized sector and reap his corresponding profit in this sector. We set up his optimization problem in a similar way to that at time  $t-1$ , but there are some differences. Now that product  $k$  is no longer rationed, he has one more choice variable; also, because of the shares he buys in the newly privatized sector and the change in the profits in the old private sector, his effective income is changed. Now he chooses  $q_{ik}^t, \dots, q_{iN}^t, m_i^t$  in order to

$$\begin{aligned} & \max_{\{q_j^t\}_{j=k}^N, m_i^t} [u_i(\bar{q}_1, \dots, \bar{q}_{k-1}, q_{ik}^t, \dots, q_{iN}^t) + m_i^t] \\ \text{s.t. } & \sum_{j=k}^N P_j^t q_{ij}^t + m_i^t = y_i + \sum_{j=k}^N \theta_{ij}(1 - \tau_j)\pi_j^t - \theta_{ik}S_k - \sum_{j=1}^{k-1} \bar{P}_j \bar{q}_j + T \equiv y_i^t, \end{aligned}$$

where  $S_k$  is the total revenue from the sale of sector  $k$ . Solving this maximization problem, we can get the demand function of consumer  $i$  for good  $j$

$$q_{ij}^t = q_{ij}^{t*}(P_k^t, \dots, P_N^t),$$

and the aggregate demand function for product  $j$

$$Q_j^t = \sum_{i=1}^I q_{ij}^{t*}(P_k^t, \dots, P_N^t).$$

Plugging the demand function into consumer  $i$ 's utility function gives us his indirect utility function

$$v_i(\bar{P}_1, \dots, \bar{P}_{k-1}, P_k^t, \dots, P_N^t, y_i^t) = \phi_i(\bar{P}_1, \dots, \bar{P}_{k-1}, P_k^t, \dots, P_N^t) + y_i^t,$$

where  $\phi_i(\bar{P}^t) = u_i(\bar{q}_1, \dots, \bar{q}_{k-1}, q_{ik}^{t*}(\bar{P}^t), \dots, q_{iN}^{t*}(\bar{P}^t)) - \sum_{j=k}^N P_j^t q_{ij}^{t*}(\bar{P}^t)$ .

<sup>15</sup>Compensation is a frequently used scheme during the transition period in socialist countries.

We would like to know the minimal amount of compensation for consumer  $i$  necessary to keep him on the same indifference curve, after we privatize sector  $k$ , as he was before. This can be done by equating his indirect utility function at time  $t$  to that at  $t-1$ , i.e.,

$$\phi_i(\bar{P}_1, \dots, \bar{P}_{k-1}, P_k^t, \dots, P_N^t) + y_i^t = \phi_i(\bar{P}_1, \dots, \bar{P}_k, P_{k+1}^{t-1}, \dots, P_N^{t-1}) + y_i^{t-1},$$

which enables us to solve the *individual consumer's minimal compensation*,

$$T_i^* = \phi_i(\bar{P}^{t-1}) - \phi_i(\bar{P}^t) + \sum_{j=k+1}^N \theta_{ij}(1 - \tau_j)(\pi_j^{t-1} - \pi_j^t) + \theta_{ik}[S_k - (1 - \tau_k)\pi_k^t] - \bar{P}_k \bar{q}_k.$$

The first two terms are the change of his indirect utility due to price changes, which we label the *price effect*; the next two terms show the consumer's income changes due to the changes in his after tax profit shares, which we label the *profit effect*. This is an important expression in the later analysis of the Bureaucrat's problem.

Figure 1 shows the consumer's consumption before and after privatization in a simple two-good economy. At time  $t-1$ , sector 2 is private, while sector 1 is public. Since  $\bar{q}_1$  is the rationed amount, the consumer's consumption bundle  $(\bar{q}_1, \bar{q}_2)$  usually is not the tangency point. At time  $t$ , sector 1 is privatized. The price of product 1 goes up to the market clearing price  $P_1$ , and the price of product 2 also changes. With the new price ratio and effective income, the consumer maximizes his utility subject to his budget constraint. For some consumers, the new consumption bundle can lie on a higher indifference curve; for others, it can lie on a lower indifference curve. The minimal compensation,  $T^*$ , shows the amount of transfer needed to get the consumer to the tangency point consumption bundle,  $(q_1, q_2)$ , on the previous indifference curve. Note that it could be positive, zero or negative.

### 3.3 The Bureaucrat's Problem

The Bureaucrat is a highly simplified representation of the government. At any given time  $t$ , he decides whether to continue privatizing, and, if yes, what sector(s) to privatize and how to compensate the consumers. Assume at time  $t$ , his budget comes from three sources:

- (1) Revenue and subsidies from the public sectors,  $\sum_{j=1}^{k-1} B_j$ ;
- (2) Revenue from the sale of the public sector  $k$ ,  $S_k$ ;
- (3) Taxes from the private sectors,  $\sum_{j=k}^N \tau_j \pi_j^t$ .

It would be interesting to understand the details of the sale process. But since it depends on the bargaining power of the seller and the buyers, the future profitability of the firms and a number of other political considerations, we do not study these in this paper. We assume that the sale mechanism is efficient enough such that the sale revenue approximates the after tax profit, i.e.,  $S_k = (1 + \epsilon_k)(1 - \tau_k)\pi_k^t$ , where  $\epsilon_k \in (-1, 1)$  represents the percentage difference between the sale amount and the actual after tax profit due to the bargaining power of the buyers and seller, political considerations or other factors.

We give the Bureaucrat the objective function of maximizing the surplus budget, i.e., total budget less total consumer compensation, subject to the constraint of staying in office. The surplus can be used to build up the Bureaucracy, or on personal gratification, if he is a corrupt bureaucrat. This objective function can be justified under a range of circumstances<sup>16</sup>. In a

<sup>16</sup>See William A. Niskanen (1971), Chapter 3 and 4.

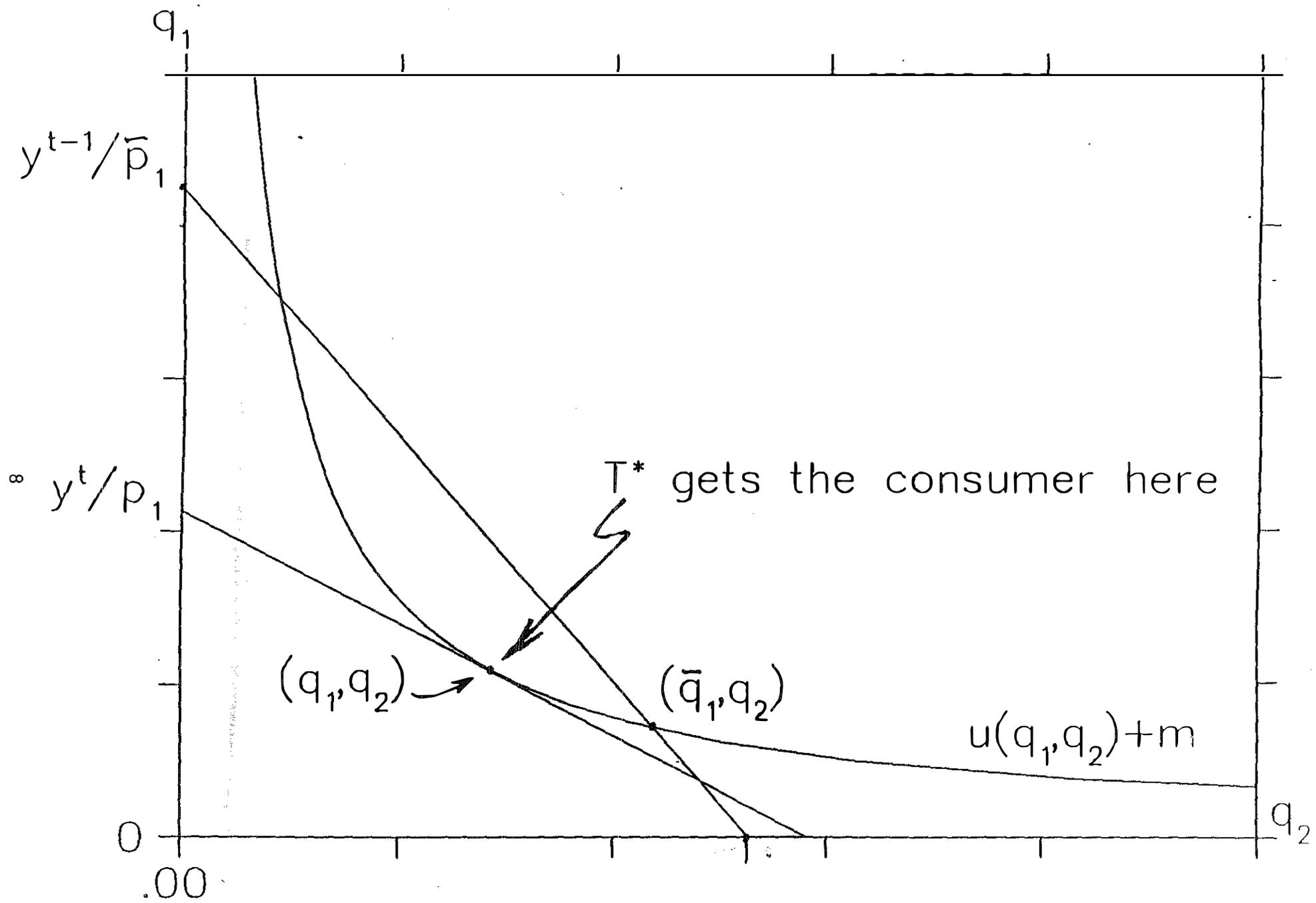


Figure 1: Consumption Before And After Privatization of Sector 1

society with elections, suppose that consumers/voters use a retrospective voting rule, i.e., they will vote for the Bureaucrat if they occupy the same or a higher utility curve in this period as in the last period, and vote against him if they are on a lower utility curve. We denote the proportion of votes he has to get to stay in office as  $d$ . In a society without elections, the Bureaucrat also needs to satisfy a certain percentage of consumers to be able to stay in office, though this  $d$  could be much lower than the one in a democratic society. For example, suppose that consumers in a society without elections judge the Bureaucrat's policy in a similar retrospective way as those in a democratic society, and they can throw the Bureaucrat out of the office by revolt or other means, if the percentage of dissatisfied consumers exceeds  $1 - d$ . Therefore, the Bureaucrat's constraint is to satisfy at least  $d$  percent of the consumers to stay in office.

We assume that the Bureaucrat knows the distribution of the consumers' utility functions, but does not know the utility functions of individual consumers. In each period, therefore, he compensates everybody the same amount<sup>17</sup>. Depending on their utility functions, some consumers will be better off and some will be worse off after the privatization and the compensation than they were before.

To formalize the problem, we let the Bureaucrat choose the sector and the level of consumer compensation to

$$\begin{aligned} \max \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] - IT \\ \text{s.t. } \frac{1}{J} |i : T \geq T_i^*| \geq d, \end{aligned}$$

where the constraint means that at least  $d$  of the voters are content with the level of compensation offered by the government.

For comparison with the behavior of the Bureaucrat who maximizes surplus budget, we model another kind of Bureaucrat whose objective function is to maximize popularity or consumer welfare, subject to a balanced budget, i.e.,

$$\begin{aligned} \max |i : T \geq T_i^*| \\ \text{s.t. } \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] = IT. \end{aligned}$$

In the next section, we will analyse the decisions of both types of Bureaucrat and compare their optimal behavior.

## 4 Main Results and Discussions

### 4.1 Individual Consumer's Minimal Compensation

In order to study the Bureaucrat's decision, we need to know how an individual consumer's minimal compensation changes with the characteristics of sector  $k$ ,  $\alpha_k$ , and how the number of

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<sup>17</sup>This is a feasible and practical compensation scheme. It is used in China after each successive "price liberation" reform and in Russia.

shares consumer  $i$  purchases in sector  $k$  affects the magnitude of his minimal compensation. To do this, we need a lemma that shows how  $\phi_i(\vec{P}^t)$  changes with the change in  $P_k^t$ .

**Lemma 1**  $\frac{\partial \phi_i(\vec{P}^t)}{\partial P_k^t} \leq 0$ .

*Proof of Lemma 1:*

We know that  $\phi_i(\vec{P}^t) = u_i(q_1^*, \dots, q_{k-1}^*, q_k^*(\vec{P}^t), \dots, q_N^*(\vec{P}^t)) - \sum_{j=k}^N P_j^t q_{ij}^*(\vec{P}^t)$ .

Differentiating  $\phi_i(\vec{P}^t)$  with respect to  $P_k^t$ , we get

$$\frac{\partial \phi_i(\vec{P}^t)}{\partial P_k^t} = \sum_{j=k}^N \frac{\partial u_i(q_j^*(\vec{P}^t))}{\partial q_j^*} \frac{\partial q_j^*(\vec{P}^t)}{\partial P_k^t} - \sum_{j=k}^N P_j^t \frac{\partial q_{ij}^*(\vec{P}^t)}{\partial P_k^t} - q_k^*(\vec{P}^t).$$

Since consumer  $i$  has quasilinear utility function, we have  $\frac{\partial u_i(q_j^*(\vec{P}^t))}{\partial q_j^*} = P_j^t$ . Therefore,

$$\frac{\partial \phi_i(\vec{P}^t)}{\partial P_k^t} = -q_k^*(\vec{P}^t) \leq 0.$$

**Q.E.D.**

So  $\phi_i(\vec{P}^t)$  decreases with the increase of  $P_k^t$ , the price of product  $k$ .

**Proposition 1** *When a sector  $k$  is privatized, and consumer  $i$ 's share in sector  $k$  is  $\theta_{ik} \leq A_i$ , then other things being constant, the minimal individual compensation increases with an increase in  $P_k^t$ , and with an increase in the market power of sector  $k$ . I.e.,*

$$\frac{\partial T_i^*}{\partial P_k^t}, \frac{\partial T_i^*}{\partial \alpha_k} \begin{cases} \geq 0 & \text{if } \epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} < 0 \text{ and } \theta_{ik} \leq A_i, \text{ or } \epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} \geq 0; \\ < 0 & \text{if } \epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} < 0 \text{ and } \theta_{ik} > A_i; \end{cases}$$

$$\text{where } A_i = \frac{\frac{\partial \phi_i(\vec{P}^t)}{\partial P_k^t} + \sum_{j=k+1}^N \theta_{ij}(1-\tau_j) \frac{\partial \pi_j^t}{\partial P_k^t}}{\epsilon_k(1-\tau_k) \frac{\partial \pi_k^t}{\partial P_k^t}}.$$

*Proof of Proposition 1:*

We know that the minimal compensation for consumer  $i$  is

$$T_i^* = \phi_i(\vec{P}^{t-1}) - \phi_i(\vec{P}^t) + \sum_{j=k+1}^N \theta_{ij}(1-\tau_j)(\pi_j^{t-1} - \pi_j^t) + \theta_{ik}[S_k - (1-\tau_k)\pi_k^t] - \bar{P}_k \bar{q}_k,$$

and that the sale amount of sector  $k$  is  $S_k = (1 + \epsilon_k)(1 - \tau_k)\pi_k$ , where  $\epsilon_k \in (-1, 1)$ . Plugging this into  $T_i^*$ , we get

$$T_i^* = \phi_i(\vec{P}^{t-1}) - \phi_i(\vec{P}^t) + \sum_{j=k+1}^N \theta_{ij}(1-\tau_j)(\pi_j^{t-1} - \pi_j^t) + \theta_{ik}\epsilon_k(1-\tau_k)\pi_k^t - \bar{P}_k \bar{q}_k.$$

Differentiating  $T_i^*$  with respect to  $P_k^t$ , we get

$$\frac{\partial T_i^*}{\partial P_k^t} = -\frac{\partial \phi_i(\vec{P}^t)}{\partial P_k^t} - \sum_{j=k+1}^N \theta_{ij}(1-\tau_j) \frac{\partial \pi_j^t}{\partial P_k^t} + \theta_{ik}\epsilon_k(1-\tau_k) \frac{\partial \pi_k^t}{\partial P_k^t}.$$

From Lemma 1, we know that  $\frac{\partial \phi_i(\bar{P}^t)}{\partial \bar{P}_k^t} \leq 0$ . So the first term is positive.

Next, we want to sign the second term. The profit function of private firm  $lj$  can be written as

$$\pi_{lj} = P_j Q_{lj}(\bar{P}) - C_j(Q_{lj}(\bar{P})).$$

Differentiating the above expression with respect to  $P_k$ , we have

$$\frac{\partial \pi_{lj}}{\partial P_k} = \frac{\partial Q_{lj}(\bar{P})}{\partial \bar{P}_k^t} [P_j - MC_j(Q_{lj})] \leq 0,$$

since we assume that the products are not substitutes. Therefore,  $\frac{\partial \pi_{lj}^t}{\partial \bar{P}_k^t} \leq 0$ . So the second term is also positive.

Since the profit in sector  $k$  can increase or decrease with an increase in the price of product  $k$ , the sign of  $\frac{\partial \pi_k^t}{\partial \bar{P}_k^t}$  is ambiguous.

(1) If  $\epsilon_k \frac{\partial \pi_k^t}{\partial \bar{P}_k^t} \geq 0$ , then we have  $\frac{\partial T_i^*}{\partial \bar{P}_k^t} \geq 0$ .

(2) If  $\epsilon_k \frac{\partial \pi_k^t}{\partial \bar{P}_k^t} < 0$ , however, the exact change in the amount of compensation caused by the change in the price of product  $k$  depends on the proportion of shares he holds in sector  $k$ .

When  $0 \leq \theta_{ik} \leq A_i$ , we have  $\frac{\partial T_i^*}{\partial \bar{P}_k^t} \geq 0$ . On the other hand, if  $\theta_{ik} > A_i$ , we have  $\frac{\partial T_i^*}{\partial \bar{P}_k^t} < 0$ .

Since  $P_j = \frac{MC_j}{1-\alpha_j}$ , it follows that  $\frac{\partial P_k^t}{\partial \alpha_k} > 0$ . We know that  $\frac{\partial T_i^*}{\partial \alpha_k} = \frac{\partial T_i^*}{\partial \bar{P}_k^t} \frac{\partial \bar{P}_k^t}{\partial \alpha_k}$ . Therefore, the sign of  $\frac{\partial T_i^*}{\partial \alpha_k}$  is the same as the sign of  $\frac{\partial T_i^*}{\partial \bar{P}_k^t}$ . **Q.E.D.**

Proposition 1 tells us that when consumer  $i$ 's share in the newly privatized sector is below a certain level, his minimal compensation goes up when the price of product  $k$  increases; when his share is above a certain level, his minimal compensation actually decreases with an increase in the price of product  $k$ . Intuitively, as a small shareholder or somebody who does not hold any shares in the newly privatized sector, the price effect dominates the profit effect – he mainly suffers from the price increase as a consumer; if he is a large shareholder, however, he shares in the profits of privatization, and the profit effect dominates the price effect.

Figure 2 illustrates Proposition 1 with a simple computer simulation. The economy consists of 100 consumers and two goods. Consumer  $i$ 's utility function takes the form of  $u_i = \alpha_i \sqrt{q_1} + (1 - \alpha_i) \sqrt{q_2} + m_i$ , where the indices  $\alpha_i \in [0, 1]$  are generated randomly by the computer. At time  $t-1$ , both sectors are public. We normalize  $\bar{P}_1 = \bar{P}_2 = \bar{q}_2 = 1$ . At time  $t$ , sector 2, is privatized, but sector 1 is still public. Let the tax rate be 0.3, and  $\epsilon_2$  be 0. Let sector 2 have a cubic cost function,  $C_2 = .04q_2^3 - .9q_2^2 + 10q_2 + 5$ . For a randomly picked consumer  $i$ , we give him different proportion of shares in sector  $k$ , and plot out how his minimal compensation,  $T_i^*$ , changes with the change in  $P_k$ , when his proportion of shares,  $\theta = 0, 0.1, 0.3, 0.5$ . We can see that when  $\theta = 0, 0.1$ , his minimal compensation increases with an increase in  $P_k$ ; when  $\theta = 0.3$ , the cutpoint, the graph goes to the other direction from  $P_k = 2$ ; when  $\theta = 0.5$ , this large shareholder's minimal compensation decreases with an increase in  $P_k$ . Note that this is only a 100-consumer economy. In a large economy, the cutpoint should be much smaller.

Apart from the above, we would like to know the change in the minimal compensation for consumer  $i$  as the proportion of shares he holds changes. The result is summed up in Proposition 2 as follows:

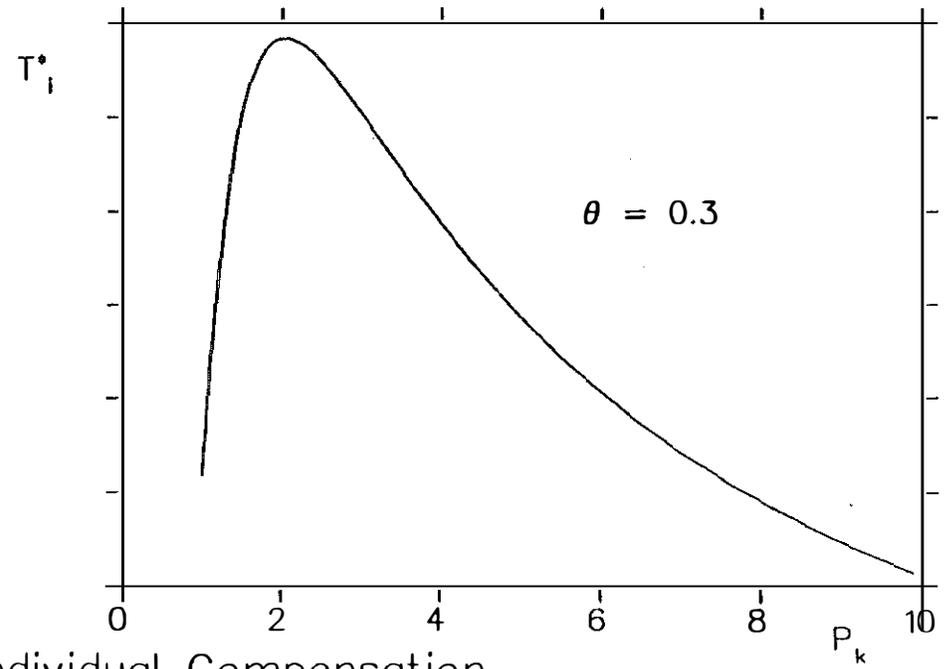
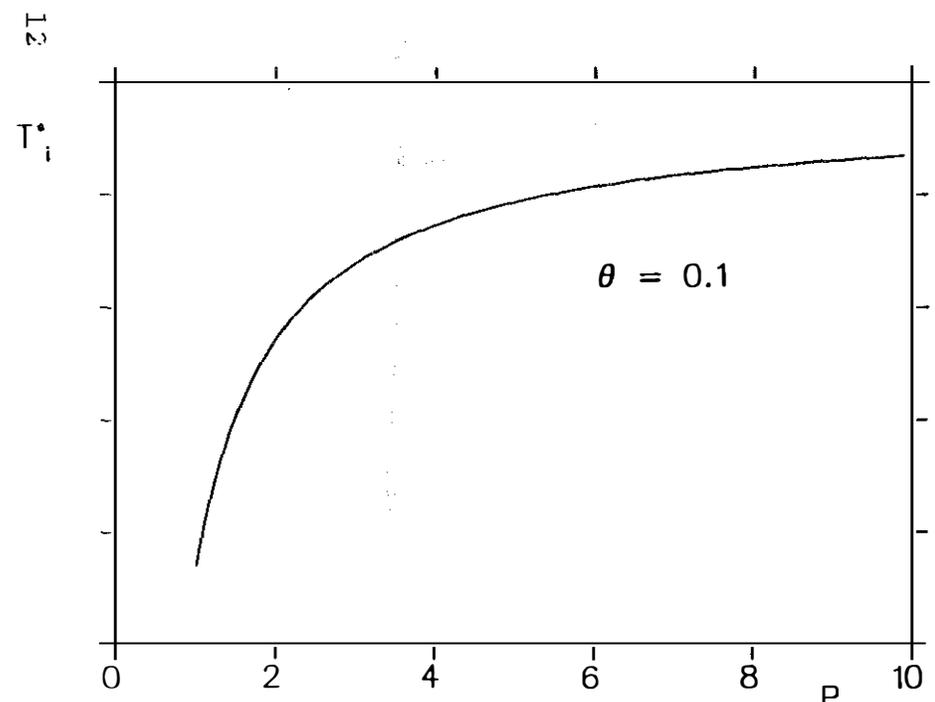
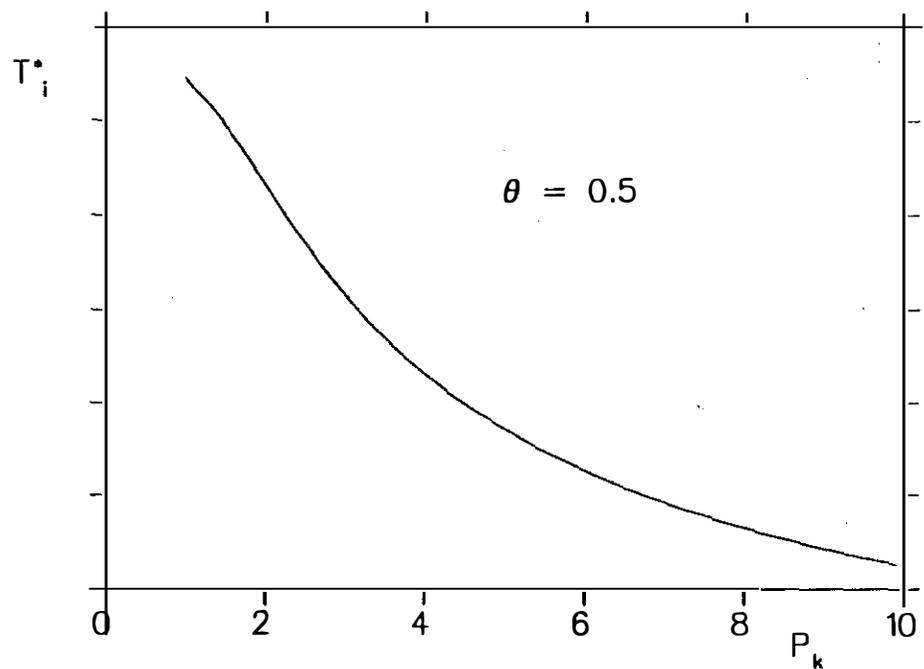
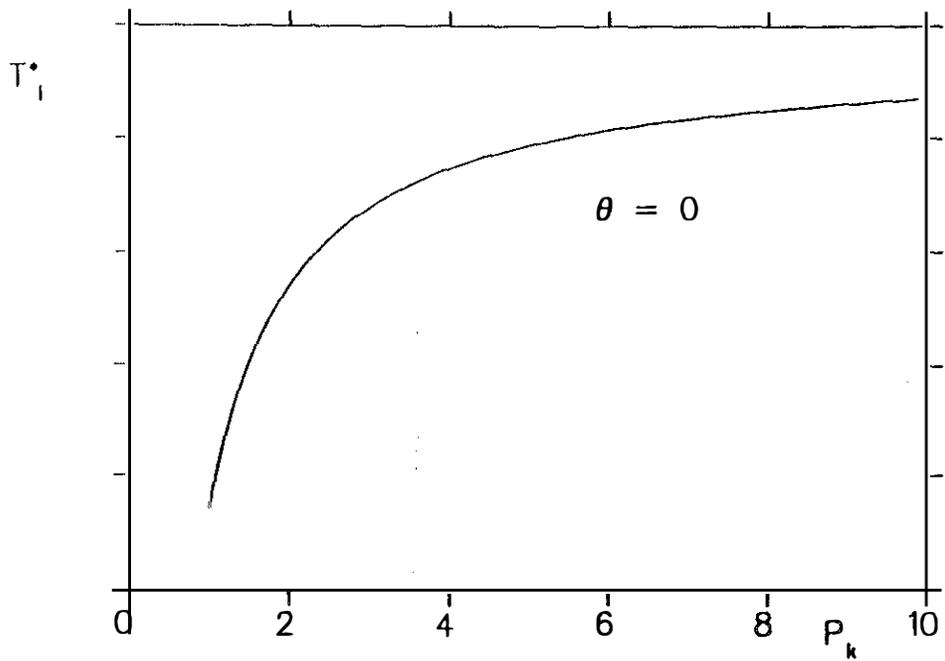


Figure 2: Minimal Individual Compensation

$$\text{Proposition 2 } \frac{\partial T_i^*}{\partial \theta_{ik}} = \epsilon_k(1 - \tau_k)\pi_k^t \begin{cases} > 0 & \text{if } \epsilon_k > 0 \\ = 0 & \text{if } \epsilon_k = 0 \\ < 0 & \text{if } \epsilon_k < 0 \end{cases}$$

*Proof of Proposition 2:*

Differentiating  $T_i^*$  with respect to  $\theta_{ik}$ , we can get Proposition 2 directly. **Q.E.D.**

This proposition tells us that when the sale is underpriced,  $\epsilon_k < 0$ , we need to compensate the shareholders less, the more shares they have. If the sale price is exactly the same as the after tax profit,  $\epsilon_k = 0$ , the change in the proportion of shares of a consumer does not affect the minimal compensation for him. When the sale is overpriced, however, we need to compensate the shareholders more, the more shares they have<sup>18</sup>.

## 4.2 Minimal Aggregate Compensation

In order to solve the Bureaucrat's problem, we need to know the distribution of an individual consumer's minimal compensation, since the Bureaucrat does not know each individual's utility functions and shares. He can only make his decision based on the aggregate behavior of the individual's minimal compensation. Recall the form of the minimal individual compensation,  $T_i^*$ ,

$$T_i^* = \phi_i(\bar{P}^{t-1}) - \phi_i(\bar{P}^t) + \sum_{j=k+1}^N \theta_{ij}(1 - \tau_j)(\pi_j^{t-1} - \pi_j^t) + \theta_{ik}\epsilon_k(1 - \tau_k)\pi_k^t - \bar{P}_k\bar{q}_k.$$

Note that there are two kinds of distributions in the above expression, the indirect utility function  $\phi(\cdot)$  and the consumer's proportion of shares in a private sector,  $\theta_j$ . So in order to know the distribution of  $T_i^*$ , we need to know the distributions of  $\phi_i(\bar{P}^{t-1}) - \phi_i(\bar{P}^t)$  and  $\theta_j$ .

Different individuals usually have different utility functions. Let  $\mathcal{F}$  be all possible functional forms of  $\phi(\cdot)$ , and let  $\Phi$  be the admissible set of  $\mathcal{F}$ , i.e.,  $\Phi \subset \mathcal{F} : \mathcal{R}^N \rightarrow \mathcal{R}$ . We can label the indirect utility functions in  $\Phi$  by  $\omega$ . Let the index set  $\Omega$  be a subset of the real line, i.e.,  $\omega \in \Omega \subset R$ . We assume that  $\phi(\cdot, \omega)$  depends continuously on index  $\omega$ , and that  $\frac{\partial \phi(\cdot, \omega)}{\partial \omega} > 0$ .  $\omega$  has cumulative distribution function  $M(\omega)$ .

Let  $\theta_j \sim F_j, j = k, k+1, \dots, N$ . Let the joint distribution of  $\theta_k, \dots, \theta_N$  be  $F(\theta)^{n-k+1}$ , where  $\theta \in \Theta$ , and  $\Theta$  is the admissible set of  $\theta_j$ . We employ the following notation:  $\vec{b} = (b_k, b_{k+1}, \dots, b_N)$ , where the  $b_j$ 's are the coefficients of the  $\theta_j$ 's;  $c \equiv \bar{P}_k\bar{q}_k$ , which is a constant because it is the expenditure on the rationed allotment of product k. For simplicity of calculation, we assume that  $\omega$  and  $\theta_j$  are independent of each other. Suppose  $T_i^* \sim G(\cdot, \vec{\beta})$ , then we get the following lemma for the distribution of  $T_i^*$ :

**Lemma 2** *For any given level of compensation  $T$ , the cumulative distribution function of  $T_i^*$ , i.e., the percentage of consumers for which  $T_i^* \leq T$ , is*

$$G(T, \vec{\beta}) = \int \cdots \int_{\Theta} \left[ \int_{\{\omega \in \Omega : \phi(\bar{P}^{t-1}, \omega) - \phi(\bar{P}^t, \omega) + \vec{\beta}\vec{\theta} - c \leq T\}} dM(\omega) \right] dF(\theta)^{n-k+1}.$$

*Proof of Lemma 2:*

<sup>18</sup>Perhaps we do not need to worry about this situation because nobody will buy any shares of sector  $k$  if the expected after tax profit share is less than the amount of money they pay for the shares.

Let  $\phi(\vec{P}^{t-1}, \omega) - \phi(\vec{P}^t, \omega) \equiv z$ , and let  $z$  follow the distribution  $H(\cdot, \vec{P}^{t-1}, \vec{P}^t)$ . Then

$$H(\cdot, \vec{P}^{t-1}, \vec{P}^t) = \text{Prob}(z \leq a) = \int_{\{\omega \in \Omega: \phi(\vec{P}^{t-1}, \omega) - \phi(\vec{P}^t, \omega) \leq a\}} dM(\omega).$$

This equation gives us the cumulative distribution function of  $z$ , expressed in terms of  $\omega$ . Using simplifying notations, we can rewrite the minimal individual compensation as  $T_i^* = z + \vec{\beta}\vec{\theta} - c$ . Suppose  $T_i^* \sim G(\cdot, \vec{\beta})$ , then for any given level of compensation  $T$ , we have the cumulative distribution function

$$\begin{aligned} G(T, \vec{\beta}) &= \int \cdots \int_{\Theta} \left[ \int_{\{z: z + \vec{\beta}\vec{\theta} - c \leq T\}} dH(b) \right] dF(\theta)^{n-k+1} \\ &= \int \cdots \int_{\Theta} H(T + c - \vec{\beta}\vec{\theta}) dF(\theta)^{n-k+1} \\ &= \int \cdots \int_{\Theta} \left[ \int_{\{\omega \in \Omega: \phi(\vec{P}^{t-1}, \omega) - \phi(\vec{P}^t, \omega) + \vec{\beta}\vec{\theta} - c \leq T\}} dM(\omega) \right] dF(\theta)^{n-k+1}. \end{aligned}$$

**Q.E.D.**

Therefore, we can express the cumulative distribution of  $T_i^*$  in terms of the distribution of  $\omega$  and  $\vec{\theta}$ . And this facilitates our method of solving the Bureaucrat's problem.

Consider the surplus budget maximizing Bureaucrat's problem:

$$\begin{aligned} \max \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] - IT \\ \text{s.t. } \frac{1}{I} |i : T \geq T_i^*| \geq d. \end{aligned}$$

The constraint is equivalent to  $G(T, \vec{\beta}) \geq d$ . We can convert his constrained maximization problem into one of unconstrained maximization by finding the minimal  $T$ ,  $T_{min}$ , to keep him in office. Therefore, he chooses the public sector  $k$  to

$$\max \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] - IT_{min},$$

where  $T_{min}$  is the solution to  $G(T_{min}, \vec{\beta}) = d$ . We call  $T_{min}$  the *minimal aggregate compensation* in a transition economy.

Figure 3 illustrates the concept of the minimal aggregate compensation by using the same economy as in Figure 2. We calculate the minimal individual compensation for all 100 consumers and plot out the cumulative distribution function. Then for any given level of  $d$ , the proportion of consumers to be left not worse off by the privatization of sector  $k$  and the amount of compensation, we can find a corresponding  $T_{min}$ , the minimal amount of compensation paid to every consumer in the economy so that at least  $d$  percent of the consumers are better off.

We need to find out the properties of  $T_{min}$  — how it changes with the changes in the underlying parameters.

**Proposition 3** *In a large population, the minimal aggregate compensation,  $T_{min}$ , increases with an increase in the market power of sector  $k$ ,  $\alpha_k$ , i.e.,*

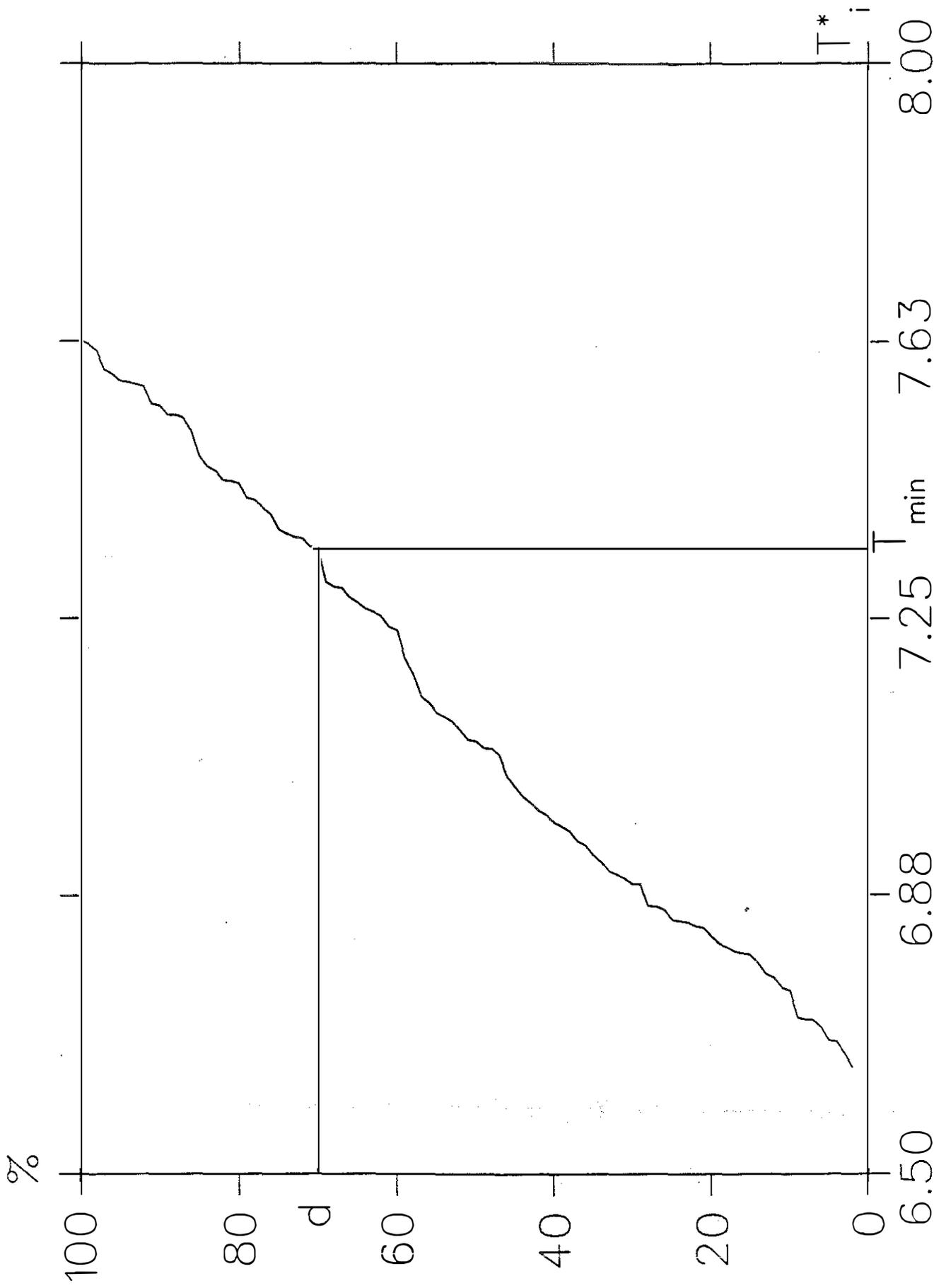


Figure 3: Minimal Aggregate Compensation

$$\frac{\partial T_{min}}{\partial \alpha_k} \geq 0,$$

(1) if  $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} \geq 0$ , or

(2) if  $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} < 0$  and  $Prob\{\theta_k \leq \underline{A}\} = 1$ , which holds in a large population;

$$\text{where } \underline{A} = \frac{\frac{\partial \phi(\vec{P}^t, \omega)}{\partial P_k^t}}{\epsilon_k (1 - \tau_k) \frac{\partial \pi_k^t}{\partial P_k^t}}.$$

*Proof of Proposition 3:* We know that

$$G(T_{min}, \vec{\beta}) = \int \cdots \int_{\Theta} \left[ \int_{\{\omega \in \Omega: \phi(\vec{P}^{t-1}, \omega) - \phi(\vec{P}^t, \omega) + \vec{\beta} \vec{\theta} - c \leq T_{min}\}} dM(\omega) \right] dF(\theta)^{n-k+1} = d.$$

Let  $T \equiv \phi(\vec{P}^{t-1}, \omega) - \phi(\vec{P}^t, \omega) + \vec{\beta} \vec{\theta} - c$ . First, we want to show that  $\frac{\partial T}{\partial P_k^t} \geq 0$ ,  $\forall \omega, \vec{\theta}$ , if  $Prob\{\theta_k \leq \underline{A}\} = 1$ .

Differentiating  $T$  with respect to  $P_k^t$ , we get

$$\frac{\partial T}{\partial P_k^t} = -\frac{\partial \phi(\vec{P}^t, \omega)}{\partial P_k^t} - \sum_{j=k+1}^N \theta_j (1 - \tau_j) \frac{\partial \pi_j^t}{\partial P_k^t} + \theta_k \epsilon_k (1 - \tau_k) \frac{\partial \pi_k^t}{\partial P_k^t}.$$

From the proof of Proposition 1, we know that the first two terms are both positive, while the sign of the third term is ambiguous.

(1) If  $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} \geq 0$ , then we have  $\frac{\partial T}{\partial P_k^t} \geq 0$ .

(2) If  $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} < 0$ , we need more conditions to decide the sign of  $\frac{\partial T}{\partial P_k^t}$ . Let  $A = \frac{\frac{\partial \phi(\vec{P}^t, \omega)}{\partial P_k^t} + \sum_{j=k+1}^N \theta_j (1 - \tau_j) \frac{\partial \pi_j^t}{\partial P_k^t}}{\epsilon_k (1 - \tau_k) \frac{\partial \pi_k^t}{\partial P_k^t}}$ .

Note that there are two kinds of distributions in  $A$ : the index of utility functions,  $\omega$ , and the proportion of shares in the existing private sectors,  $\theta_j$ ,  $j = k + 1, \dots, N$ . Since  $\phi(\vec{P}^t, \omega)$  is decreasing in  $P$ , and strictly increasing in  $\omega$ , we use  $\underline{\omega}$  to denote the highest absolute value of  $\frac{\partial \phi(\vec{P}^t, \omega)}{\partial P_k^t}$ , and  $\bar{\omega}$  to denote its lowest absolute value. And we know that the proportion of shares in each private sector,  $\theta_j \in [0, 1]$ . Then it follows that the lower bound of  $A$  is

$$\underline{A} = \frac{\frac{\partial \phi(\vec{P}^t, \bar{\omega})}{\partial P_k^t}}{\epsilon_k (1 - \tau_k) \frac{\partial \pi_k^t}{\partial P_k^t}}.$$

Therefore, if  $Prob\{\theta_k \leq \underline{A}\} = 1$ , we have  $\frac{\partial T}{\partial P_k^t} \geq 0$ ,  $\forall \omega, \vec{\theta}$ .

Next, we want to show that  $T_{min}$  has a similar property. It follows from the first part of the proof that for any  $P_k^{t'} \geq P_k^t$ , if  $Prob\{\theta_k \leq \underline{A}\} = 1$ , we have  $T' \geq T$ ,  $\forall \omega, \vec{\theta}$ . We want to show that  $T'_{min} \geq T_{min}$ , where  $T'_{min}$  satisfies

$$G'(T'_{min}, \vec{\beta}') = \int \cdots \int_{\Theta} \left[ \int_{\{\omega \in \Omega: \phi(\vec{P}^{t-1}, \omega) - \phi(\vec{P}^{t'}, \omega) + \vec{\beta}' \vec{\theta} - c \leq T'_{min}\}} dM(\omega) \right] dF(\theta)^{n-k+1} = d.$$

Suppose not, then  $T'_{min} < T_{min}$ . Let  $A = \{\omega \in \Omega : T' \leq T_{min}\}$ , and  $B = \{\omega \in \Omega : T \leq T_{min}\}$ . Since  $T' \geq T$ , it follows that  $A \subseteq B$ . We know that

$$\int \cdots \int_{\Theta} \left[ \int_B dM(\omega) \right] dF(\theta)^{n-k+1} = d.$$

It follows that

$$\int \cdots \int_{\Theta} \left[ \int_A dM(\omega) \right] dF(\theta)^{n-k+1} \leq d.$$

Let  $C = \{\omega \in \Omega : T' \leq T'_{min}\}$ . Since  $T_{min} > T'_{min}$ , we have  $C \subset A$ . Therefore,

$$\int \cdots \int_{\Theta} \left[ \int_C dM(\omega) \right] dF(\theta)^{n-k+1} < d,$$

but this contradicts the definition of  $T'_{min}$ . So  $T'_{min} \geq T_{min}$ . Then we have  $\frac{\partial T_{min}}{\partial P_k^t} \geq 0$ , and equivalently,  $\frac{\partial T_{min}}{\partial \alpha_k} \geq 0$ .

Finally, we want to show that  $Prob\{\theta_k \leq \underline{A}\} = 1$  holds in a large population.

$$Prob\{\theta_k \leq \underline{A}\} = 1 - Prob\{\theta_k > \underline{A}\} = 1 - \frac{|i : \theta_{ik} > \underline{A}|}{I},$$

Since  $\sum_{i=1}^I \theta_{ik} = 1$ ,  $|i : \theta_{ik} > \underline{A}| < \min \text{int}[\frac{1}{\underline{A}}]$ , which is bounded and independent of  $I$ . Therefore, as  $I \rightarrow +\infty$ ,  $\frac{|i : \theta_{ik} > \underline{A}|}{I} \rightarrow 0$ , we have

$$Prob\{\theta_k \leq \underline{A}\} = 1 - \frac{|i : \theta_{ik} > \underline{A}|}{I} = 1.$$

**Q.E.D.**

Therefore, when the population is large, the probability that  $\theta_k \leq \underline{A}$  approaches 1. The intuition behind this result is quite clear. Figure 4 shows the upper bound of all possible density functions of  $\theta_k$ . We can see that there are at most 100 people who can own 1% or more of the shares of sector  $k$  at the same time, which is negligible in a large population. Most people will own a very small percentage of the total shares, a percentage that approximates zero. If the shares are distributed evenly in the population,  $\theta_k$  is approximately the inverse of the population, which is very small,  $10^{-6}$ , even if there are only one million people in the economy. If there are several large shareholders, their mere existence makes the shares/person in the rest of the economy approach zero. And because the small number of large shareholders, they have measure zero in the whole distribution. So both scenarios lead to the same conclusion: the probability that  $\theta_k \leq \underline{A}$  approaches one in a large population.

So Proposition 3 shows that in a large population the minimal aggregate compensation,  $T_{min}$ , increases with an increase in the market power of sector  $k$ ,  $\alpha_k$ . Apart from this, we would also like to know how  $T_{min}$  changes when the other underlying characteristics shift, such as the sale price of sector  $k$ ,  $S_k$ , and the minimal percentage of consumers who are better off as a result of the privatization policy,  $d$ .

**Proposition 4** *Holding other things the same, the minimal compensation,  $T_{min}$ , increases with an increase in the sale price of sector  $k$ ,  $S_k$ , and the threshold for the Bureaucrat to stay in office,  $d$ . That is,*

$$\frac{\partial T_{min}}{\partial S_k} \geq 0, \text{ and } \frac{\partial T_{min}}{\partial d} > 0.$$

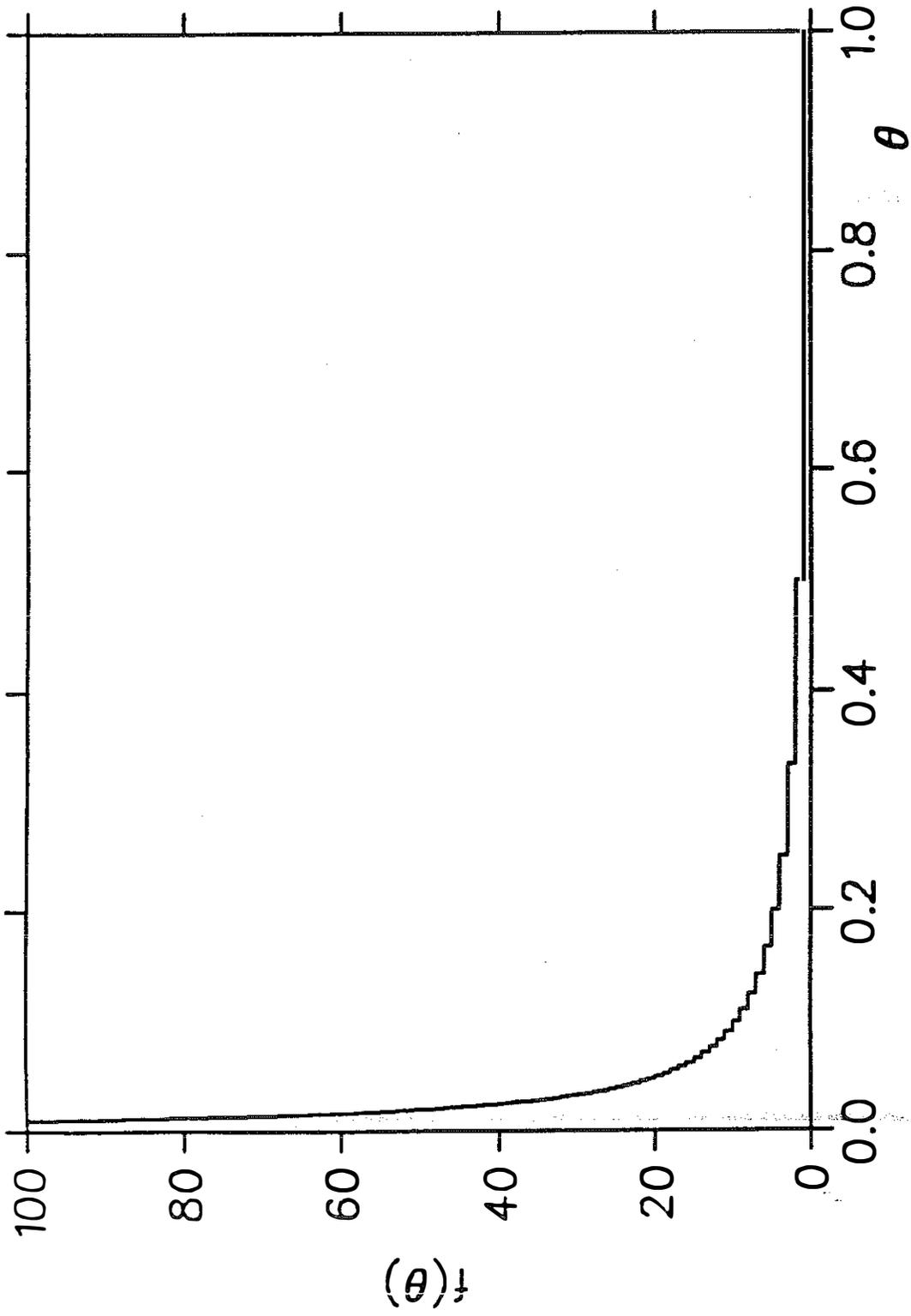


Figure 4: The Upper Bound of All Possible Density functions

*Proof of Proposition 4:*

Differentiating  $T$  with respect to  $S_k$ , we get  $\frac{\partial T}{\partial S_k} = \frac{\partial \vec{\beta}}{\partial S_k} \cdot \vec{\theta} = \theta_k \geq 0$ . From this, we can infer that  $\frac{\partial T_{min}}{\partial S_k} \geq 0$ .

The other result,  $\frac{\partial T_{min}}{\partial d} \geq 0$ , is obvious from the property of cumulative distribution functions. **Q.E.D.**

The first part of Proposition 4 tells us that the minimal aggregate compensation increases with an increase in the sale price of sector  $k$ . This is because the shareholders of sector  $k$  will get less net profit as a result of the sale price increase of sector  $k$  and need more compensation to keep them on the same or higher indifference curves.

The second part of Proposition 4 is obvious from Figure 3. If the proportion of consumers required to be better off increases, the minimal aggregate compensation need to increase.

### 4.3 The Bureaucrat's Optimal Behavior – Surplus Budget Maximizing Bureaucrat

We simplify the Bureaucrat's constrained maximization problem by introducing the concept of minimal aggregate compensation. He will

$$\max \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] - IT_{min}.$$

Since the Bureaucrat maximizes surplus budget, his decision as to whether to privatize another sector at time  $t$  depends on whether the privatization of that sector would increase his surplus budget. If not, he might as well maintain the status quo. For the analysis of his problem, we define his time  $t-1$  surplus budget as

$$SB^{t-1} = \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^{t-1},$$

and his time  $t$  surplus budget as (if he chooses to privatize another sector):

$$SB^t = \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] - IT_{min}.$$

We define the *incremental budget* between these two periods as

$$IB \equiv SB(t) - SB(t-1) = \sum_{j=k+1}^N \tau_j (\pi_j^t - \pi_j^{t-1}) + [-B_k + S_k + \tau_k \pi_k^t] - IT_{min}.$$

Therefore, he will privatize another sector  $k$  if and only if there exists a sector such that  $IB \geq 0$ .

When  $IB \geq 0$  is satisfied, the Bureaucrat will choose the public sector that gives him the highest surplus budget. We define the maximal budget at time  $t$  as

$$SB^* = \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] - IT_{min}.$$

We can see that maximizing  $SB^*$  is equivalent to maximizing  $IB$ . We would like to know the characteristics of sector  $k$  that give the Bureaucrat his maximal surplus budget. That is, what kind of sector would he like to choose?

**Proposition 5** *For a Bureaucrat whose objective function is maximizing the surplus budget subject to the constraint of staying in office, in a large population, his maximal budget increases with a decrease in the market power of sector  $k$ ; the incremental budget increases with an increase in the amount of subsidy sector  $k$  receives from the government, i.e.,*

$$\frac{\partial SB^*}{\partial \alpha_k} = \frac{\partial IB}{\partial \alpha_k} \leq 0, \text{ if } Prob\{\theta_k \leq \underline{A}\} = 1; \text{ and } \frac{\partial IB}{\partial B_k} = -1.$$

*Proof of Proposition 5:*

Differentiating  $SB^*$  and  $IB$  with respect to  $P_k^t$ , we get

$$\begin{aligned} \frac{\partial SB^*}{\partial P_k^t} &= \frac{\partial IB}{\partial P_k^t} = \sum_{j=k+1}^N \tau_j \frac{\partial \pi_j^t}{\partial P_k^t} + \tau_k \frac{\partial \pi_k^t}{\partial P_k^t} - I \frac{\partial T_{min}}{\partial P_k^t} \\ &= I \left[ \frac{1}{I} \left( \sum_{j=k+1}^N \tau_j \frac{\partial \pi_j^t}{\partial P_k^t} + \tau_k \frac{\partial \pi_k^t}{\partial P_k^t} \right) - \frac{\partial T_{min}}{\partial P_k^t} \right] \end{aligned}$$

When  $I \rightarrow +\infty$ , we have  $\frac{1}{I} \left( \sum_{j=k+1}^N \tau_j \frac{\partial \pi_j^t}{\partial P_k^t} + \tau_k \frac{\partial \pi_k^t}{\partial P_k^t} \right) \rightarrow 0$ ; also, from Proposition 3, it follows that when  $I \rightarrow +\infty$ , we have  $Prob\{\theta_k \leq \underline{A}\} = 1$ , and thus  $\frac{\partial T_{min}}{\partial P_k^t} \geq 0$ . Therefore, it follows that  $\frac{\partial SB^*}{\partial P_k^t} = \frac{\partial IB}{\partial P_k^t} \leq 0$ .

To express the result in terms of the market power index, we have  $\frac{\partial SB^*}{\partial \alpha_k} = \frac{\partial IB}{\partial \alpha_k} \leq 0$ .

Now we differentiate  $IB$  with respect to  $B_k$ , and easily get  $\frac{\partial IB}{\partial B_k} = -1$ . **Q.E.D.**

Proposition 5 is our main result. It tells us that in a large population, the Bureaucrat who maximizes surplus budget will gain most by privatizing the public sector with the least market power and the most subsidy first, if all other characteristics of the public sectors are the same.

Now let us turn to the influence of the characteristics of political institutions on the Bureaucrat's behavior. We get the following proposition:

**Proposition 6** *The maximal surplus budget increases with a decrease in the threshold of the satisfaction level,  $d$ , i.e.,  $\frac{\partial SB^*}{\partial d} < 0$ .*

*Proof of Proposition 6:*

Differentiating  $SB^*$  with respect to  $d$ , and apply Proposition 4, we get the result. **Q.E.D.**

This proposition shows that in an economy with a smaller  $d$ , i.e., a less democratic society, the Bureaucrat actually benefits more from the privatization process. If the surplus budget becomes his personal property, he becomes richer consequently. If it is used to ease the operation of the Bureaucracy or privatization process, it could be relatively easier and faster to privatize in a less democratic society.

#### 4.4 The Bureaucrat's Optimal Behavior — Popularity/Consumer Welfare Maximizing Bureaucrat

As a comparison to the surplus budget maximizing Bureaucrat, we sketch a popularity or consumer welfare maximizing Bureaucrat. We give him the objective function of maximizing

the number of consumers better off by privatization, subject to the constraint of a balanced budget, i.e.,

$$\begin{aligned} & \max |i : T \geq T_i^*| \\ \text{s.t. } & \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] = IT. \end{aligned}$$

Note that the constraint gives us the maximal compensation to the consumers,  $T_{max}$ , which is the average of the total budget over the whole population. From the maximand, we can see that the higher  $T_{max}$  is, the larger the number of consumers better off by the privatization policy. Therefore, the consumer welfare or popularity maximizing problem can be turned into a budget maximizing problem as follows,

$$\max \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t].$$

We denote the maximal budget from the above maximization problem by  $B^*$ ,

$$B^* = \sum_{j=1}^k B_j + \sum_{j=k+1}^N \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t].$$

In a similar way as the previous section, we define the incremental budget between  $t$  and  $t-1$  by

$$IB = \sum_{j=k+1}^N \tau_j (\pi_j^t - \pi_j^{t-1}) + [-B_k + S_k + \tau_k \pi_k^t].$$

We would like to know the characteristics of sector  $k$  that gives the Bureaucrat the maximal budget. In other words, what kind of sector is he more likely to pick to maximize his objective function?

**Proposition 7** *For a Bureaucrat who maximizes popularity/consumer welfare/budget, his maximal budget increases with a decrease in the market power of sector  $k$  when there is a big enough private sector. I.e.,*

$$\begin{aligned} \frac{\partial B^*}{\partial \alpha_k} = \frac{\partial IB}{\partial \alpha_k} \leq 0 & \quad \text{if } \frac{\partial \pi_k^t}{\partial P_k^t} \leq 0, \text{ or} \\ & \quad \text{if } \frac{\partial \pi_k^t}{\partial P_k^t} > 0, \text{ and } \sum_{j=k+1}^N \tau_j \frac{\partial \pi_j^t}{\partial P_k^t} \geq -\tau_k \frac{\partial \pi_k^t}{\partial P_k^t}; \end{aligned}$$

*while his incremental budget increases with an increase in subsidy sector  $k$  receives when it belongs to the public sector. I.e.,  $\frac{\partial IB}{\partial B_k} = -1$ .*

*Proof of Proposition 7:*

Differentiating  $B^*$  and  $IB$  with respect to  $P_k^t$ , we get

$$\frac{\partial B^*}{\partial P_k^t} = \frac{\partial IB}{\partial P_k^t} = \sum_{j=k+1}^N \tau_j \frac{\partial \pi_j^t}{\partial P_k^t} + \tau_k \frac{\partial \pi_k^t}{\partial P_k^t}.$$

We know that the first term is negative and the second term is ambiguous.

(1) If  $\frac{\partial \pi_k^t}{\partial P_k^t} \leq 0$ , we get  $\frac{\partial B^*}{\partial P_k^t} = \frac{\partial IB}{\partial P_k^t} \leq 0$ . Equivalently,  $\frac{\partial B^*}{\partial \alpha_k} = \frac{\partial IB}{\partial \alpha_k} \leq 0$ .

(2) If  $\frac{\partial \pi_k^t}{\partial P_k^t} > 0$ , then if the private sector is big enough so the decrease in tax revenue from all the private sectors outweighs the increase in tax revenue from the newly privatized sector, i.e.,  $\sum_{j=k+1}^N \tau_j \frac{\partial \pi_j^t}{\partial P_k^t} \geq -\tau_k \frac{\partial \pi_k^t}{\partial P_k^t}$ , we get the same result as that in case (1).

Similarly, differentiating  $IB$  with respect to  $B_k$ , we get

$$\frac{\partial IB}{\partial B_k} = -1.$$

#### Q.E.D.

Proposition 7 shows that holding the other characteristics of the firms in the public sector the same, a Bureaucrat who maximizes popularity/consumer welfare/budget will privatize the public sector with the most subsidy first; and if the private sector is big enough, he will privatize the sector with the least market power first.

## 5 Summary and Extensions

From the analysis of the strategies of Bureaucrats with different objective functions, we can see that the comparative statics results are very similar under ordinary situations. Among the public sectors with all other characteristics the same, each will choose to privatize the sector with the least market power and the most subsidy from the state. Intuitively speaking, this is the “cheapest” way to privatize from the Bureaucrat’s point of view.

This is a two-period static model. We assumed that the number of firms remained the same. If we add entry into the model, we can see that entry drives down the market power of any sector,  $\alpha_j = \frac{1}{L_j} \left[ \frac{1}{\varepsilon_{jj}} + \sum_{h \neq j} \frac{1}{\varepsilon_{jh}} \right]$ . Encouragement of entry will increase the number of firms in a sector and thus drive down  $\alpha_j$ . Therefore, from the Bureaucrat’s point of view, he should encourage measures that can drive down the market power of a sector, such as entry and demonopolization.

If we go back to the sequencing policy discussed in the introduction, we can see that the size of a sector is not the only factor that should be taken into consideration in the Bureaucrat’s optimal policy. Other important factors, such as the subsidy a sector gets, the elasticity of demand of the product, and the competitiveness of a sector (the latter two are included in the concept of the market power index) should all be taken into consideration.

In our model, we assume that all goods are non-substitutes to each other. We consider substitutes to be in the same sector. A more realistic way would be to solve the problem without this assumption. It would be more complicated and we are not sure how the result would change. The wage income of the consumers and the cost functions of firms are taken as exogenously given. Also the mechanism for the sale of firms is not considered. Future work should be done to make these factors endogenous within the economy, so as to make the whole economy a closed system. Some crude thinking suggests that the privatization of a sector in the economy would lead to a total change in the supply and demand of labor. This will form the bases for the change in wage income. Therefore, for the consumers in a transition economy, the compensation from the government and the change in their wage income will be the decisive factor in coping with price increases.

## Appendix

We assume that  $\varepsilon_{ij} = -\frac{\partial q_i}{\partial P_j} \frac{P_j}{q_i} \geq 0, \forall i \neq j, i, j = 1, 2, \dots, N$ . This assumption implies that the consumer's utility functions need to satisfy the following condition:  $\frac{\partial^2 u_i(\vec{q})}{\partial q_i \partial q_i} \leq 0$ .

*Proof:*

Let the consumer solve the following maximization problem:

$$\max_{\{q\}, m} u(\vec{q}) + m$$

$$\text{s.t. } \vec{P} \cdot \vec{q} + m = y.$$

This is equivalent to the unconstrained maximization problem:

$$\max_{\{q\}} u(\vec{q}) + y - \vec{P} \cdot \vec{q}.$$

Differentiating the maximand with respect to  $q_i$ , we get the first order condition:

$$\frac{\partial u(\vec{q})}{\partial q_i} = P_i.$$

Differentiating the first order condition with respect to  $P_j$ , where  $i \neq j$ , we get

$$\sum_l \frac{\partial^2 u_i(\vec{q})}{\partial q_i \partial q_l} \frac{\partial q_l}{\partial P_j} = \sum_l \frac{\partial P_i(\vec{Q})}{\partial Q_l} \frac{\partial Q_l}{\partial P_j}.$$

Since  $\varepsilon_{ij} \leq 0, \forall i \neq j$ , the righthand side of the above expression is greater or equal to zero, also  $\frac{\partial q_l}{\partial P_j}$ , we get the necessary condition of Assumption 4:

$$\frac{\partial^2 u_i(\vec{q})}{\partial q_i \partial q_i} \leq 0.$$

**Q.E.D.**

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