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LOWER BOUNDS ON ASSET RETURN COMOVEMENT

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Abstract

Under standard assumptions from dynamic asset pricing theory (value additivity, complete markets, rational expectations, and strict stationarity and ergodicity) and absence of arbitrage, lower bounds on the conditional and unconditional cross-moments of the returns on two assets are derived. They are expressed in terms of the second moment of a linear combination of option premia. The restrictions are probed with data from the foreign exchange market covering the period 1983-1991. Assuming that the value of the economy's benchmark payoff never exceeds one, and substituting linear projection for conditional expectation, several violations of the conditional lower bounds are discovered. The violations are attributed to unit roots in the data.

Lower Bounds on Asset Return Comovement

Peter Bossaerts*

1 Introduction

How much comovement should we expect between the returns on two assets? Returns equal next period's prices plus dividends, divided by this period's prices, and prices are determined in equilibrium by agents who implement their optimal intertemporal investment and consumption plans. More specifically, prices of assets reflect the value that an additional unit would add to the consumer's utility, and, hence, are set with reference to the consumer's optimal portfolio. Because prices, and, hence, returns, are determined in equilibrium with respect a common benchmark (the consumer's optimal portfolio), one would expect a minimum level of comovement between asset prices, and, hence, asset returns. This paper investigates how much comovement would be required in equilibrium. The theoretical restrictions are investigated empirically on a dataset of foreign currency prices.

The issue is important. The fraction of the volatility of prices that can be attributed to changes in expectations about future dividend flows is very small, as the variance bounds literature amply illustrates (e.g., Grossman and Shiller [1981], West [1988]). Hence, to reconcile asset price volatility with standard asset pricing theory, one should point to changes in the price of risk, i.e., changes in the value of the benchmark portfolio, as the main source. But, if the benchmark price generates most of the volatility of asset prices, one would expect to see substantial comovement between asset prices.

General equilibrium models confirm this, at least if they feature restricted supply or non–constant stochastic–returns–to–scale, otherwise asset prices, and, hence, returns, are

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not endogenous. As a matter of fact, when derived in a general equilibrium framework, popular asset pricing models such as the Capital Asset Pricing Model (CAPM) restrict comovements unrealistically. It is well-known that the CAPM leads to the nonsensical result that returns must be perfectly correlated (Rosenberg and Ohlson [1976]). It has proven to be frustrating to avoid perfect correlation without destroying the main feature of the CAPM, namely the relationship between mean returns and risk measures that are defined in terms of correlation with a common benchmark. Assuming dividend processes satisfy a K-factor specification, Connor and Korajczyk [1988] obtained a model where prices lie in a K-dimensional plane. In Bossaerts and Green [1989], who built on earlier work by Brennan [1973], asset prices were less restricted, but still conditionally colinear. Bossaerts [1988] derived a much more general relationship between asset prices (in particular, co-integration), at the cost of destroying the linear relationship between mean returns and risk found in other models. Nevertheless, the assumption about the nature of dividend processes remains highly specific in all these papers. This contrasts with consumption-based asset pricing models, which provide much more generality. Empirical verification of the latter, however, depends on arguably unreliable aggregate consumption data. In consumption-based asset pricing models, the correlation between returns is directly restricted through the first-order conditions ("stochastic Euler equations") of a representative consumer. The correlation structure will depend on the particular parametrization of this consumer's preferences (such as Epstein and Zin's [1991] non-expected-utility preferences).

Rather than starting from a particular set of assumptions about the stochastic nature of dividends or about the utility function of the representative consumer, this paper investigates restrictions on return comovement that follow from a set of assumptions that is common to the above models. This means that, if violations are discovered, there will not be the ambiguity that the rejections might be due to the *ad hoc* assumptions about dividend processes (as in, e.g., Bossaerts and Green [1989]) or preferences (as in, e.g., Epstein and Zin [1991]). The set of assumptions that are retained are: value additivity (no taxes or transactions costs), complete markets, ergodicity and strict stationarity of returns, rational expectations, and absence of arbitrage opportunities. The analysis builds on recent work by Hansen and Richard [1987], who investigated certain testable restrictions on returns that can be derived under the same set of assumptions, and that require a minimal use of conditioning information. Related empirical work includes Hansen and Jagannathan [1991] and Gallant, Hansen and Tauchen [1990].

At such a level of generality, unfortunately, it is not possible to obtain lower bounds on correlations or covariances directly. Only noncentral cross-moments can be restricted without having to observe the complete information set that investors use when making decisions. This negative result is analogous to Hansen and Richard [1987]'s finding that CAPM-type restrictions on central moments do not survive when information is conditioned out. For instance, Hansen and Richard showed that the only restriction on the return of the benchmark portfolio that can be expressed in terms of unconditional moments is that its noncentral second moment is the minimum possible in the economy. Nevertheless, the lower bounds on return cross–moments of this paper do have implications for correlations: if the former are violated, the latter will be too low. The converse is obviously not true.

Empirical verification of the cross–moment bounds does require a restriction on the price of the economy's benchmark asset. In particular, it must be assumed that this price never exceeds one. In terms of Lucas' [1978] endowment economy, this means that the economy should never move into a state where it expects, with high probability, to crash. If this were ever the case, the expected squared marginal rate of substitution of tomorrow's consumption for today's consumption, i.e., the price of the benchmark asset, would exceed one. This paper also reports results that assume risk neutrality, in which case the price of the benchmark asset equals the squared price of a one–period riskfree zero–coupon bond.

The tests are performed on a set of foreign exchange data. There are three reasons for focusing on the foreign exchange market. First, the lower bounds involve prices of exchange options, and an option to buy, say, deutsche mark out of U.S. dollar is an exchange option if the numéraire currency is a third currency such as the Japanese yen. One could reexpress the cross–moment bounds of this paper in terms of standard options, and apply them to, for instance, stock price data, at the cost of a substantial weakening. Second, foreign exchange markets are highly integrated, involve very few transaction costs, and, hence, a violation of the bounds should be attributed to incomplete markets, nonstationarity, nonergodicity, and/or lack of rational expectations. Third, there is substantial evidence that prices in foreign exchange markets do not conform to a model of rational, risk–neutral traders. This evidence is most obvious in the correlation between the prediction error of the forward foreign exchange premium and its level (for a summary, see Hodrick [1987]). Consequently, there is a yardstick to measure the power of the cross–moment bounds tests: they should reject when the price of the benchmark asset is set equal to its value under risk neutrality.

The remainder of the paper is organized as follows. The next section presents the cross–moment bounds. Section 2 expresses them directly in terms of foreign exchange data. Section 3 presents the results from the empirical investigation. Section 4 interprets the violations of the cross–moment bounds. Section 5 concludes.

2 Derivation of the Cross–Moment Bounds

This section will discuss the assumptions and several steps needed to establish the main result.[1] We shall work in a standard economy with strictly stationary and ergodic payoffs. This means that we automatically exclude a large class of economies, most notably, those of interest to diffusion theorists, or, for instance, a Lucas–type economy where consumption growth is stationary but preferences are negative exponential. Nevertheless, the stationary and ergodic economy has been the focus of most empirical work.

We also assume that investors have rational expectations, which means that they know the probabilistic structure (possible states of nature, information flow and corresponding probability measure). We postulate absence of transaction costs (and taxes), i.e., value additivity. This does not seem to be a strong assumption for the foreign exchange market, where we shall verify the lower bounds, and, hence, rejections are unlikely to be caused by violations of this assumption.

Information flows to the economy in the form of strictly stationary, ergodic signals. Investors use this information to assess the likelihood of future returns on the various assets. This does impose a restriction: there must not be any seasonalities in returns (such as January effects), for otherwise, a nonstationary variable (time–dependent) will be part of the investors' information set.

Finally, we assume complete markets. In other words, given the information flow (and we assume that all investors receive the same messages, otherwise the definition of complete markets is ambiguous), investors can insure all possible risk. The assumption of complete markets can be relaxed, at a cost of slightly weakening the cross–moment bounds. In particular, if markets are complete, the economy's benchmark payoff will always be positive. If markets are incomplete, this payoff could become negative, and the derivation of the cross–moment bounds has to be altered accordingly.

If this were a static economy, absence of arbitrage opportunities would immediately imply that prices can be written as the expectation of the payoff on a benchmark portfolio times the payoff on the asset itself. This is a straightforward implication from the Riesz representation theorem for Hilbert spaces (see, e.g., Harrison and Kreps [1979]). In a dynamic economy, however, prices are random variables, and, hence, cannot be associated with bounded, continuous linear functionals, which map payoffs into the real line rather than a space of random variables (functions). Hansen and Richard [1987], however, introduced the concept of a "conditional functional", and extended the Riesz representation theorem. Consequently, in the absence of arbitrage, prices can be written as the *conditional* expectation of next period's payoff times the payoff on a benchmark

[1]The complete, formal derivation of the results in Hansen and Richard's [1987] framework can be obtained from the author.

portfolio. Letting P_t denote the price at time t on an asset that promises a (random) payoff of X_{t+1} the next period, and letting I_t denote the investors' information set, Hansen and Richard [1987] showed that, in the absence of arbitrage opportunities,

$$P_t = E[\hat{X}_{t+1} X_{t+1} | I_t] \quad \text{(almost surely)}, \tag{1}$$

for some unique payoff $\hat{X}_{t+1}$ that can be obtained as a portfolio of the available assets in the economy. The portfolio weights may change over time, and generally will, but must be in the investors' information set. Moreover, this payoff is positive with unit probability. (In the sequel, we shall drop the qualifier "almost surely"; it will not lead to ambiguities.)

We are interested in applying this result to obtain a lower bound on the conditional cross–moments of the payoffs on two assets, where the conditioning is done with respect to a subset of the information set, say, an instrument $Y_t \in I_t$. Let X_{1t+1} and X_{2t+1} denote the (strictly positive) promised payoffs on two assets, labelled 1 and 2, respectively. First, rearrange Schwartz' inequality for conditional expectations:

$$E[X_{1t+1} X_{2t+1} | I_t] \geq \frac{(E[|\hat{X}_{t+1} (X_{1t+1} X_{2t+1})^{\frac{1}{2}} || I_t])^2}{E[|\hat{X}_{t+1}|^2 | I_t]}. \tag{2}$$

Notice that $E[|\hat{X}_{t+1} (X_{1t+1} X_{2t+1})^{\frac{1}{2}} || I_t]$ can be written as $E[\hat{X}_{t+1} (\frac{X_{1t+1}}{X_{2t+1}})^{\frac{1}{2}} X_{2t+1} | I_t]$. Approximate the square–root of $\frac{X_{1t+1}}{X_{2t+1}}$ times X_{2t+1} from below by a linear combination of payoff on exchange (call) options:

$$(\frac{X_{1t+1}}{X_{2t+1}})^{\frac{1}{2}} X_{2t+1} \geq \sum_{k=0}^{M_t} A_{kt} \max(\frac{X_{1t+1}}{X_{2t+1}} - B_{kt}, 0) X_{2t+1}. \tag{3}$$

We allow the weights A_{kt} and exercise prices B_{kt} to change over time, but they must be in the investors' information set. Similarly, M_t, the number of options used to approximate a square–root payoff, can be time–variable as well. Obviously, the higher M_t, the better the approximation. By (1), the time–t value of this portfolio of options is given by the conditional expectation of the right–hand side in (3) times $\hat{X}_{t+1}$. Let P_{3t} denote this value. The conditional expectation of the left–hand side in (3) times $\hat{X}_{t+1}$ is always larger than or equal to P_{3t}, because $\hat{X}_{t+1}$ is never negative. But this conditional expectation appears in the numerator of the right-hand side of (2). Finally, let $\hat{P}_t$ denote the value of the benchmark portfolio, i.e., $\hat{P}_t = E[\hat{X}^2_{t+1} | I_t]$, the expression in the denominator of the right–hand side of (2). Hence, combining (2) and (3):

$$E[X_{1t+1} X_{2t+1} | I_t] \geq \frac{(P_{3t})^2}{\hat{P}_t}.$$

Applying the law of iterated expectations (this is possible because we are dealing with noncentral moments), it follows that, for $Y_t \in I_t$:

$$E[X_{1t+1} X_{2t+1} | Y_t] \geq E[\frac{(P_{3t})^2}{\hat{P}_t} | Y_t]. \tag{4}$$

Equation (4) binds the conditional cross–moment of the payoff on two assets in terms of the squared value of a portfolio of options and the price of the economy's benchmark portfolio. The conditioning can be done with respect to any variable in the investors' information set, even a constant (which would give the bound in terms of unconditional moments). It is not necessary to observe the complete information set.

By interpreting X_{1t+1} and X_{2t+1} as returns (tomorrow's prices plus dividends divided by today's prices), (4) becomes a bound on the cross–moment of two returns. At this moment, we shall translate the bounds directly in terms of foreign exchange data, in order to make the empirical verification transparent.

3 An Application to the Foreign Exchange Market

Assume currency i is the numéraire currency. i will be taken to be the Deutsche mark ($i = DM$), the Japanese yen ($i = Y$), the British pound ($i = \$$), Swiss frank ($i = SF$), the French franc ($i = FF$) or the Canadian dollar ($i = C\$$). Let s^{i*}_{jt+1} denote the time $t+1$ value, in units of currency i, of one unit of currency j, where j can be any of the aforementioned currencies or \$, the U.S. dollar, but $j \neq i$. Let r_{it} and r_{jt} denote the rate over the period $t, t+1$ on a deposit in currency i or j, respectively. Thus, the payoff, in terms of the numéraire currency (currency i) of a deposit in currency j, equals $s^{i*}_{jt+1}(1 + r_{jt})$. Finally, let $s^{\$*}_{jt+1}$ denote the time $t+1$ value of currency j in U.S. dollar. The bound in (4) becomes:

$$E[\frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}|Y_t] \geq E\left[\frac{(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2}{\hat{P}_t}|Y_t\right], \tag{5}$$

where $s^{i*}_{j\$t}$ is the time t value of a portfolio that promises to pay

$$\sum_{k=0}^{M_t}[A_{jkt}(1+r_{jt})] \max(s^{\$*}_{jt+1} - B_{jkt}\frac{1+r_{\$t}}{1+r_{jt}}s^{\$*}_{jt}, 0)s^{i*}_{\$t+1}. \tag{6}$$

In order to interpret the portfolio with payoff (6), we should be more specific about the choice of the A_{jkt}s and B_{jkt}s. To a certain extent, this choice is arbitrary. In particular, we can set $B_{j0t} = 0$, such that the payoff in (6) becomes the payoff (in currency i) on a portfolio of (i) a deposit in currency j, (ii) U.S. dollar call options written on currency j. The ratio of the value of this portfolio (in terms of currency i), $s^{i*}_{j\$t}$, to the value of currency j, s^{i*}_{jt}, can be rewritten:

$$\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}} = A_{j0t} + \sum_{k=1}^{M_t}[A_{jkt}(1+r_{jt})]\frac{C^{i*}_{\$jkt}}{s^{i*}_{jt}},$$

where $c^{i*}_{\$jkt}$ denotes the time t currency i price of a U.S. dollar call written on currency j with exercise price $B_{jkt}\ \frac{1+r_{\$t}}{1+r_{jt}}\ s^{\$*}_{jt}$.

The A_{jkt}s and B_{jkt}s should be chosen such that (i) they are in the investors' information set at t, (ii) they form a lower approximation to the square–root function: $\sum_{k=0}^{M_t} A_{jkt}\max(y-B_{jkt},0)\leq\sqrt{y}$, for all $y>0$, (iii) the exercise prices that they generate $(B_{jkt}\ \frac{1+r_{\$t}}{1+r_{jt}}\ s^{\$*}_{jt})$ correspond to U.S. dollar currency options that are traded at t. A simple way to satisfy these requirements would be to pick the A_{jkt}s and B_{jkt}s as follows. First, set $B_{j0t}=0$. Second, assume that M_t currency options are traded at time t. Let $e^{\$*}_{jkt}$ be, at time t, the exercise price of the kth option to buy currency j $(e^{\$*}_{j1t}<e^{\$*}_{j2t}<\ldots<e^{\$*}_{jM_tt})$. Set:

$$B_{jkt}=\frac{e^{\$*}_{jkt}}{s^{\$*}_{jt}}\frac{1+r_{jt}}{1+r_{\$t}}. \tag{7}$$

Third, determine the A_{jkt}s $(k=0,1,2,\ldots,M_t-1)$ recursively, as follows.

$$\begin{aligned} A_{j0t} &= B_{j1t}^{-\frac{1}{2}}, \\ A_{j1t} &= \frac{\sqrt{B_{j2t}}-\sqrt{B_{j1t}}}{B_{j2t}-B_{j1t}}-A_{j0t} \\ A_{j2t} &= \frac{\sqrt{B_{j3t}}-\sqrt{B_{j2t}}}{B_{j3t}-B_{j2t}}-(A_{j0t}+A_{j1t}) \\ &\ldots \end{aligned} \tag{8}$$

$$\begin{aligned} &A_{jM_t-1t} \\ &= \frac{\sqrt{B_{jM_tt}}-\sqrt{B_{jM_t-1t}}}{B_{jM_tt}-B_{jM_t-1t}}-(A_{j0t}+A_{j1t}+\ldots+A_{jM_t-2t}). \end{aligned}$$

Finally, set $A_{jM_tt}=-\sum_{k=0}^{M_t-1}A_{jkt}$. This way, the A_{jkt}s and B_{jkt}s will statisfy the above restrictions. In particular, they will define a lower, piecewise linear approximation to the square–root function (restriction (ii)).

The role of currencies j and the dollar can be exchanged, because the payoff on a call option to purchase dollar with currency j is related to that of a put option to purchase currency j with dollar:

$$\max(s^{j*}_{\$t+1}-e,0)s^{i*}_{jt+1}=e\max(\frac{1}{e}-s^{\$*}_{jt+1},0)s^{i*}_{\$t+1},$$

where e is a positive scalar. From this, we can obtain a cross–moment bound in terms of put options:

$$E[\frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}}\frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}|Y_t]\geq E\left[\frac{(\frac{s^{i*}_{\$jt}}{s^{i*}_{\$t}})^2}{\hat{P}_t}|Y_t\right], \tag{9}$$

where $s^{i*}_{\$jt}$ is the time t value of a portfolio that promises to pay

$$\sum_{k=0}^{M_t} \frac{A_{\$kt}B_{\$kt}(1+r_{jt})}{s^{\$*}_{jt}} \max\left(\frac{1}{B_{\$kt}}\frac{1+r_{\$t}}{1+r_{jt}}s^{\$*}_{jt} - s^{\$*}_{jt+1}, 0\right)s^{i*}_{\$t+1}. \tag{10}$$

Again, there is a certain freedom in setting the $A_{\$kt}$s and $B_{\$kt}$s. We shall choose values that match put options that have traded at time t. Order the corresponding exercise prices, $e^{\$*}_{jkt}$ $(k \geq 1)$, now from high to low: $e^{\$*}_{j1t} > e^{\$*}_{j2t} > \ldots > e^{\$*}_{jM_t t}$. Add: $e^{\$*}_{j0t} = \infty$. Set:

$$B_{\$kt} = \frac{s^{\$*}_{jt}}{e^{\$*}_{jkt}}\frac{1+r_{\$t}}{1+r_{jt}}. \tag{11}$$

Consequently, $B_{\$0t} = 0 < B_{\$1t} < \ldots < B_{\$M_t t}$. Compute the $A_{\$kt}$s as in (8). With $B_{\$\bullet t} = 0$, $s^{i*}_{\$jt}$ is the currency i value of a portfolio consisting of (i) a dollar deposit, (ii) currency put options denominated in dollars. Moreover,

$$\frac{s^{i*}_{\$jt}}{s^{i*}_{\$t}} = A_{\$0t} + \sum_{k=1}^{M_t}[A_{\$kt}B_{\$kt}(1+r_{jt})]\frac{\P^{i*}_{\$jkt}}{s^{i*}_{jt}},$$

where $\P^{i*}_{\$jkt}$ denotes the time t currency i price of a U.S. dollar put written on currency j with exercise price $\frac{1}{B_{\$kt}}\frac{1+r_{\$t}}{1+r_{jt}}s^{\$*}_{jt}$.

Several comments about the cross-moment bounds can now be made. First, a family of bounds can be obtained from Hölder's inequality, as a way to generalize the bounds (which derive from Schwartz' inequality). Second, sharper bounds result if the tails of the payoff distribution are restricted. If deep out-of-the-money options have low probability of paying out anything, the piecewise linear, lower approximation to the square root-function can be allowed to cross the square root function without violating the bound. In particular, $A_{jM_t t} > -\sum_{k=1}^{M_t-1} A_{jkt}$.

Third, notice that the weights on the currency options in the bounds are negative. Since the noncentral second moment equals the sum of the variance and the squared average, this implies that, the cheaper the options are on average, the more likely the bounds will be violated. This is intuitive: options to exchange an asset for another one with highly correlated payoffs will carry a low premium. Also, the more volatile the option premia are, the more the returns on the underlying assets should move together. This is a consequence of the assumption of absence of arbitrage opportunities. If asset prices in general, and option premia in particular, are very volatile, then the probability of one day finding an arbitrage opportunity (ignoring the risk for a moment) is high. Option market traders like volatility for this reason. If prices move together rather than being uncorrelated, it is hard, however, to exploit the apparent arbitrage opportunities: even a portfolio consisting of a large number of assets will still carry substantial risk. The risk cannot be diversified away. The Law of Large Numbers does not work because of the correlation, and, hence, it is impossible to find the "well diversified" portfolios that are needed to exploit the arbitrage opportunity.

Fourth, by choosing $Y_t = 1$, the bounds can be related to the work of Hansen and Jagannathan [1991]. By conditioning on $Y_t = 1$ before applying Schwartz' inequality in the derivation of the lower bounds, Equation (5) becomes:

$$E[\frac{s_{jt+1}^{i*}(1+r_{jt})}{s_{jt}^{i*}}\frac{s_{\$t+1}^{i*}(1+r_{\$t})}{s_{\$t}^{i*}}] \geq \frac{(E[\frac{s_{j\$t}^{i*}}{s_{jt}^{i*}}])^2}{E[(\hat{X}_{t+1})^2]}. \tag{12}$$

Equation 12 generates a lower bound on the unconditional second moment of the benchmark payoff (marginal rate of substitution in Hansen and Jagannathan's terminology). Hansen and Jagannathan derive a lower bound on the marginal rate of substitution from the average prices and second moments of a set of assets. Our bound derives from cross–moments only; there will be one bound per pair of assets. In this sense, the bound provides a value for the minimum unconditional second moment of the benchmark payoff that is consistent with the pattern of cross–moments in the data. Several candidates of the benchmark payoff (such as consumption–data–based marginal rates of substitutions) can be checked against this lower bound.

Notice, however, that the cross–moment bounds depend on $\hat{P}_t$, the price of the economy's benchmark portfolio. We would prefer to test the bounds without having to observe this price, and this is possible if one is willing to assume that it never exceeds one. In that case, the bounds would still hold with 1 substituted for $\hat{P}_t$. As mentioned in the Introduction, the assumption that the benchmark value never exceeds one corresponds, in a Lucas–type economy, to the assumption that the representative consumer never expects the economy's output to worsen to the extent that her expected marginal rate of substitution of consumption squared is above one. Alternatively, let P_{Rt} denote the price of a one–period riskfree pure–discount bond. In the foreign currency example, $P_{Rt} = \frac{1}{1+r_{it}}$, provided i is the numéraire currency. If assets are priced under risk neutrality, $\hat{X}_{t+1} = P_{Rt}$, and we can substitute $(P_{Rt})^2$ for $\hat{P}_t$ without violating the bound.

4 Empirical Tests

In order to test the cross-moment lower bounds, a series of spot foreign currency quotes and one-month eurocurrency quotes were obtained from DRI. These daily quotes cover the period February 1, 1983–June 30, 1991. The spot quotes are expressed as number of U.S. dollars per unit of foreign currency ("American terms"). For each trading day, a bid and ask spot quote is available, reflecting the New York open (when Europe is still active, i.e., when the foreign currency market is at its deepest). Bid and ask quotes across non-U.S. dollar currencies are obtained using triangular arbitrage. This way, cross rates are obtained for the U.S. dollar, the Deutsche mark, the Japanese yen, the British pound, the Swiss franc, the French franc and the Canadian dollar. The eurocurrency quotes are bid and ask quotes reflecting London's mid-morning situation. They were obtained for the U.S. dollar, the Deutsche mark, the Japanese yen, the British pound, the Swiss franc, the French franc, and the Canadian dollar.

Simultaneously, prices of opening transactions in foreign currency options on the Philadelphia Stock Exchange were obtained. They cover the period February 28, 1983 to December 31, 1990. Transaction prices were obtained for calls and puts written on the Deutsche mark, the Japanese yen, the British pound, the Swiss franc, the French franc, and the Canadian dollar. The options are American, i.e., could be exercised before maturity. The cross-moment bounds in the previous section involve, however, European options (that cannot be exercised before the end of the period). This does not pose any problem for call options written on currencies that pay an interest substantially below the U.S. dollar interest rate (the option contracts are denominated in U.S. dollar), because the early exercise feature would be close to worthless anyway. It does cause problems, however, for call options written on high-interest currencies, and put options. An investigation of the cross-moment bounds reveals, however, that the American feature merely biases the bounds downward, as the value of each option is multiplied by a negative number. In other words, the fact that American options usually sell at a premium relative to their European counterparts biases the tests of the cross-moment bounds towards acceptance. Notice that the Philadelphia Stock Exchange actually recently introduced European foreign currency options. No use was made of the transaction prices of these European calls and puts, because they trade only rarely.

The construction of the linear combination of option premia and the calculation of the return to foreign currency deposits in the estimation of the cross–moment bounds deserve more discussion. For each trading day, all trading records for put and call options with the shortest maturity were collected. Erroneous records were discarded (unless it was evident what the mistake was, e.g., the exercise price was recorded in 10ths of U.S. dollar rather than U.S. cents). With these option transactions, linear combinations were formed as detailed in the previous section. The spot quotes from the Philadelphia Stock Exchange were not used; DRI quotes were used instead. For each trading day, returns to foreign currency deposits were computed, provided there were option, spot and eurocurrency quotes. The interest on the foreign currency deposit was taken to be the one–month eurocurrency rate. The number of days that the deposit was held was matched with the maturity of the option(s). The DRI database was searched for a spot rate quote closest to the liquidation date of the deposit. There is obviously a certain level of approximation in this calculation: one–month eurocurrency quotes are good for *exactly* one month, and not anything shorter or longer than one month.

The number of options that traded on any given day varied substantially. Moreover, there appeared to be periods when predominantly in–the–money call options and out–

of–the–money put options changed hands, whereas the opposite type of options were traded in other periods. There were quite a bit of missing observations (trading days without an option trading record) for the French franc and the Canadian dollar. Some of these reflected genuine thin trading in options written on those currencies. Other trading days without observations on French franc or Canadian dollar options reflected omissions by the Philadelphia Stock Exchange's recording–keeping service. Moreover, the option premia for the French franc and Canadian dollar were higher, and, as will be documented shortly, the bounds were violated correspondingly less often. For brief periods, certain currencies had options with different contract sizes. When this happened, data for only one type of contract were extracted (always the newly introduced one).

The results from the estimation of the lower bounds are provided in Table 1. To save space, the results for the French franc and Canadian dollar are not reported.[2] As mentioned before, there were many missing observations for French franc and Canadian dollar options. Nevertheless, the results were similar to those for the other currencies. Two versions of the lower bounds were tested. The first one requires the assumption that $P\{\hat{P}_t > 1\} = 0$ so that $\hat{P}_t$ does not need to be observed. The results are reported in the rows labeled "$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$". The second one assumes risk neutrality, i.e., it sets $\hat{P}_t = P^2_{Rt}$. The results are reported in the rows labeled "$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1 + r_{it})$". The bounds were computed for the following instruments: $Y_t = 1$ (this produces the bounds with unconditional moments), $Y_t = \frac{1+r_{it}}{1+r_{jt}}$, $Y_t = \frac{1+r_{it}}{1+r_{\$t}}$, $Y_t = \frac{1+r_{jt}}{1+r_{\$t}}$. Ordinary Least Squares (linear projection) provided estimates of the conditional moments. The percentage variance that is explained by the projection (R^2) is displayed as well in Table 1. For each bound, the range of Y_t for which it is violated, if at all, is indicated.

Ordinary Least Squares is not the most efficient estimation procedure, because of the presence of heteroscedasticity and autocorrelation. The latter follows because of a substantial overlap in time (the data are daily, yet the maturities of the foreign currency deposits are matched with the option maturities, which easily extend beyond the subsequent month). Asymptotically, heteroscedasticity– and autocorrelation–adjusted estimation is superior, yet substantial problems in its finite–sample implementation remain. In particular, different outcomes are generated depending on the implementation (in well–defined cases, certain criteria can be developed to compare the performance using simulation –see, e.g., Andrews [1991]). Consequently, Ordinary Least Squares estimation was preferred. Ordinary Least Squares is numerically well–behaved and provides consistent estimates.

Ordinary Least Squares, however, does not provide the right confidence bands. Unfortunately, heteroscedasticity– and autocorrelation–adjusted confidence bands suffer the same finite–sample sensitivity as the corresponding point estimates. On the other hand,

[2]They can be obtained from the author.

the adjustment of confidence intervals for heteroscedasticity or autocorrelation would not alter any rejections at the (consistent) Ordinary Least Squares estimates. Incidentally, there is an issue whether one can "reject" anything at all in the present context. Under the hypothesis that the lower bounds hold, there is a range of possible distances between the bound and the value of the variable that is bounded. Classical hypothesis testing requires one to pre–specify a particular distance and declare it the null hypothesis. Confidence bands are then designed (heteroscedasticity– and autocorrelation–adjusted, if necessary) to "reject" this null hypothesis. Yet, in the present case, the model does not naturally pre–specify a value for this distance. The absence of a clear null hypothesis deprives confidence bands of their meaning.

Ordinary Least Squares implements linear projection. This means that conditional expectations are assumed to be linear in the conditioning variable. Linearity will follow from conditional normality. In other words, if the product of the returns on two foreign currency deposits is normally distributed, conditional on the ratio of the interest rates (Y_t above), or if such conditional normality holds for linear combinations of option premia, then linear projection can substitute conditional expectation. Alternatively, one could use nonparametric regression, in order to capture possible nonlinearities.

The violations that are reported in Table 1 are, to a certain extent, systematic. First, if the bounds are violated under the assumption that the value of the benchmark payoff never exceeds one ($Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$), they are as well under the assumption of risk neutrality ($Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1 + r_{it})$). There are some exceptions. Second, when violations occur for call options, they usually do as well for put options. Again there are exceptions, mainly due to differences in exercise prices and maturities of calls *vs.* puts. Third, excluding the Swiss franc, violations usually occur when the numéraire's interest level is at an historical high relative to that of currency j. Conversely, violations most often emerge when the U.S. dollar interest rates are low relative to those of the numéraire currency or currency j. Fourth, the coefficient of determination (R^2) is often surprisingly high. In the case the regressand equals $(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, for instance, this must mean that interest rate differentials appear to be capable of predicting not only the option premia themselves, but also the type (exercise price and maturity) of the options that were traded. The R^2s are generally lower when the regressand equals $(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$. Most remarkable, however, is the ability of the interest rate differential to predict the product of the returns on two foreign currency deposits $(\frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}})$. Table 1a shows, for instance, that the Deutsche mark / U.S. dollar interest rate differential explains 41% of the variation in the product of the Deutsche mark return on a yen eurocurrency deposit times that on a eurodollar deposit.

The violations that are reported in Table 1 against risk neutrality confirm the rejections of the unbiasedness hypothesis that have been reported in studies of the spot and forward foreign exchange markets only (see Hodrick [1987] for a survey). (Whereas most of the literature has used the U.S. dollar as the numéraire currency, rejections have been discovered also in terms of different currencies; Bossaerts and Hillion [1991], for instance, provide evidence –albeit less pronounced– against the unbiasedness hypothesis with the French franc as numéraire.)

Despite the fact that Table 1 displays a substantial number of violations, most of them cover only a small part of the range of the conditioning variable. Bounds involving the British pound are violated more severely, however. In only one case has the unconditional bound been violated, namely when risk neutrality is assumed, the numéraire is the Japanese yen, currency j is the Canadian dollar, and put option premia are used in the computation of the bound.

5 Discussion

Why are there so many rejections of the cross–moment bounds? One of the assumptions behind the theory, or a combination, must be violated. As argued before, the foreign exchange market is well–integrated, very deep, and, hence, transaction costs or taxes (violations of value additivity) are the least likely source of the rejections. Markets may be incomplete, but this affects the cross–moment bounds only in so far as it would lead the benchmark payoff to be negative a substantial proportion of time. Arbitrage opportunities (possibilities to construct riskfree zero–investment portfolios with positive payoff) are unlikely as well. Investors may not have rational expectations, but absent a clear alternative model, this would be hard to check.

There is substantial evidence of nonstationarity in the data. Stationarity is not only important from a theoretical point of view: together with ergodicity, it permits consistent estimation of the moments in the bounds. If several random variables behave like unit root processes, as they appear to do, estimators will not be consistent, and inferences invalid. The estimates of the slope coefficients in the linear projections, for instance, may as well be spurious, and, hence, violations of the conditional cross–moment bounds a statistical artefact.

The presence of unit roots was not immediately clear from an investigation of the autocorrelation function: the first–order autocorrelation coefficient was often low. This is documented in Table 2. Nevertheless, the autocorrelation function decreased only slowly as a function of the lag. This may be an indication of fractional integration (see, e.g., Sowell [1990]). It may also indicate that the variables are the sum of a random walk plus noise, where the noise is of a level such that the presence of the unit root will be detected in the first–order autocorrelation coefficient only when samples are sufficiently large. For reasonable values of order of integration, however, the fractionally integrated process is stationary and ergodic. Consequently, fractional integration cannot explain the violations of the lower bound. Therefore, it seemed reasonable to focus on the "random walk plus noise" hypothesis. A very intuitive check was performed. Using Yule's [1926] results on spurious statistics when regressing one random walk on another, the t–statistic of the slope coefficient in a regression of the relevant random variables onto simulated standard normal random walks were calculated. This approach was preferred over sophisticated unit root tests because of its simplicity and intuitive appeal. The results are reported in Table 2. It is clear that too many t–statistics are high (in absolute value) relative to what is to be expected by chance.

Unit roots in interest rates have been documented elsewhere (for international evidence, see, e.g., Kugler [1990]). Hence, unit roots in interest spreads should not surprise us. I did not find, however, a reference where the nonstationarity of interest spreads was analyzed explicitly. Unit roots in the product of the returns on two foreign–currency deposits are more surprising, but may be related to unit roots in the interest rate processes. Perhaps most puzzling are the unit roots in the linear combinations of option premia. Option premia are usually thought of as stationary (they will be, for instance, in the Black–Scholes option pricing model, provided exercise prices are regularly reset relative to the price of the underlying asset). The nonstationarities may be related to the exercise prices in reported option transactions that were used to compute the parameters A_t and B_t. The results in Bossaerts and Hillion [1992] confirm this.

The presence of unit roots in both the regressors and regressands in Table 1 explains why some of the R^2s could be as high as 53%. As explained in Granger and Newbold [1986] (p. 207), this is to be expected when regressing one random walk on another. In this sense, the ability of the Deutsche mark / U.S. dollar interest rate differential to explain 41% of the variation in the product of the Deutsche mark return on a yen eurocurrency deposit times that on a eurodollar deposit should not be interpreted as an opportunity to make money.

6 Conclusion

This paper discovered several violations of conditional lower bounds on asset return cross–moments and attributes them to unit roots in the data. Does this mean that dynamic asset pricing theory has to be rewritten, in order to reflect these nonstationarities? It is well known that tests of unit roots have low power (Schwert [1989]). Moreover, the focus on unit roots may be unwarranted in view of the number of plausible alternatives (e.g., Sims and Uhlig [1991]). In fact, Den Haan [1991] argued that the unit roots in the interest rates are to be expected even in a strictly stationary and ergodic rational expectations production economy. Only in large samples will it become possible to confirm that the equilibrium interest rates are indeed stationary and ergodic.

Despite the size of the samples (although they do not cover an extended period of time - only 7 years), this paper still discovered the spurious correlation with unrelated random walks that is characteristic of unit root processes. This affects the tests of the cross-moment bounds. Only if one is convinced that unit roots in the interest rates, returns on foreign currency deposits and linear combinations of option premia are implausible *a priori*, can one attribute violations of the conditional cross-moment bounds to lack of comovement in asset prices.

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Table 1a

Violations of the cross–moment bounds

i = *Deutsche mark*

	j = *Japanese yen* Calls	Puts	j = *British pound* Calls	Puts	j = *Swiss franc* Calls	Puts	j = *French franc* Calls	Puts	j = *Canadian dollar* Calls	Puts
Number of Observations:	1584	1755	1751	1719	1759	1578	738	363	1598	885
Estimation of $E[Z]$*:*										
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$,	1.0242	1.0226	1.0214	1.0209	1.0154	1.0166	1.0123	1.0166	1.0204	1.0197
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$,	0.9842	0.9891	0.9841	0.9824	0.9864	0.9801	0.9538	0.9473	0.9889	0.9882
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$,	0.9944	0.9989	0.9941	0.9925	0.9961	0.9901	0.9732	0.9673	1.0032	1.0041
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{it}}{1+r_{jt}}$*:*										
Min(Y):	0.9898	0.9853	0.9637	0.9597	0.9930	0.9932	0.9298	0.9247	0.9490	0.9598
Max(Y):	1.0115	1.0115	0.9996	0.9995	1.0083	1.0138	0.9999	0.9998	0.9997	0.9996
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.215	.306	.110	.138	.074	.094	$<$.001	.069	.093	.125
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.328	.253	.336	.296	.028	.125	.525	.434	.336	.255
Violation for $Y \in$	(1.0008, 1.0115]	(1.0006, 1.0115]			[0.9930, 0.9982)	[0.9932, 0.9983)			(0.9995, 0.9997]	(0.9975, 0.9996]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.080	.018	$<$.001	$<$.001	.020	.003	.102	.122	.442	.382
Violation for $Y \in$	(1.0006, 1.0115]	(1.0004, 1.0115]			[0.9930, 0.9985)	[0.9932, 0.9985)			(0.9992, 0.9997]	(0.9962, 0.9996]
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{it}}{1+r_{\$t}}$*:*										
Min(Y):	0.9647	0.9681	0.9685	0.9653	0.9698	0.9673	0.9631	0.9669	0.9654	0.9586
Max(Y):	1.0018	1.0011	1.0011	1.0011	1.0011	1.0022	1.0043	1.0033	1.0032	1.0032
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.413	.444	.254	.331	.377	.390	.035	.176	.217	.241
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.463	.284	.182	.261	.227	.355	.403	.464	.166	.175
Violation for $Y \in$	(0.9997, 1.0018]	(0.9995, 1.0011]	(1.0009, 1.0011]	(1.0007, 1.0011]	(1.0000, 1.0011]	(1.0002, 1.0022]		(1.0028, 1.0033]	(0.9993, 1.0032]	(0.9982, 1.0032]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.021	.003	.021	$<$.001	.003	.026	.046	.156	.326	.332
Violation for $Y \in$	(0.9994, 1.0018]	(0.9991, 1.0011]	(1.0005, 1.0011]	(1.0004, 1.0011]	(0.9995, 1.0011]	(0.9999, 1.0022]		(1.0021, 1.0033]	(0.9984, 1.0032]	(0.9970, 1.0032]
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{jt}}{1+r_{\$t}}$*:*										
Min(Y):	0.9696	0.9788	0.9920	0.9863	0.9648	0.9541	0.9852	0.9983	0.9950	0.9950
Max(Y):	1.0003	1.0003	1.0343	1.0266	1.0048	1.0079	1.0538	1.0594	1.0402	1.0162
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.372	.376	.030	.073	.328	.343	.039	$<$.001	.054	.128
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.375	.214	.062	.003	.188	.331	.301	.142	.197	.041
Violation for $Y \in$	(0.9999, 1.0003]	(0.9996, 1.0003]	(1.0185, 1.0343]	(1.0100, 1.0266]	(1.0004, 1.0048]	(1.0007, 1.0079]			(1.0067, 1.0402]	(1.0035, 1.0162]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.004	.016	.027	.003	.006	.021	.103	.025	.103	.008
Violation for $Y \in$	(0.9996, 1.0003]	(0.9993, 1.0003]	(1.0141, 1.0343]	(1.0079, 1.0266]	(0.9997, 1.0048]	(1.0002, 1.0079]	(1.0471, 1.0538]		(1.00358, 1.0402]	(1.0025, 1.0162]

Table 1b

Violations of the cross-moment bounds
i = *Japanese yen*

	j = *Deutsche mark* Calls	Puts	j = *British pound* Calls	Puts	j = *Swiss franc* Calls	Puts	j = *French franc* Calls	Puts	j = *Canadian dollar* Calls	Puts
Number of Observations:	1768	1602	1751	1719	1759	1578	738	363	1599	886
Estimation of $E[Z]$:										
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$,	1.0038	1.0024	1.0111	1.0109	1.0047	1.0033	1.0096	0.9968	1.0113	1.0007
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$,	0.9904	0.9846	0.9841	0.9824	0.9864	0.9801	0.9538	0.9473	0.9889	0.9882
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$,	1.0009	0.9955	0.9950	0.9934	0.9970	0.9912	0.9750	0.9701	1.0043	1.0063
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{it}}{1+r_{jt}}$:										
Min(Y):	0.9980	0.9980	0.9570	0.9689	0.9886	0.9888	0.9397	0.9328	0.9359	0.9642
Max(Y):	1.0092	1.0075	0.9997	0.9997	1.0154	1.0173	1.0000	1.0000	0.9999	0.9999
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.042	.053	.074	.054	.004	.002	.002	<.001	.087	.046
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.361	.336	.316	.232	.141	.288	.486	.397	.320	.221
Violation for $Y \in$	(1.0090, 1.0092]		(0.9996, 0.9997]			[.9888, 0.9935)			(0.9988, 0.9999]	(0.9971, 0.9999]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.004	.010	.002	.007	.141	.001	.023	.038	.390	.430
Violation for $Y \in$	(1.0008, 1.0092]	(1.0016, 1.0075]	(0.9992, 0.9997]						(0.9997, 0.9999]	(0.9920, 0.9999]
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{it}}{1+r_{\$t}}$:										
Min(Y):	0.9809	0.9845	0.9773	0.9705	0.9797	0.9706	0.9504	0.9699	0.9521	0.9629
Max(Y):	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0028	1.0011	1.0005	1.0008
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.017	.018	.041	.060	.029	.047	.079	.040	.131	.121
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.228	.271	.175	.217	.174	.284	.242	.366	.182	.152
Violation for $Y \in$									(0.9993, 1.0005]	(0.9979, 1.0008]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.075	<.001	.061	.029	.057	.001	.001	.058	.437	.467
Violation for $Y \in$	(1.0001, 1.0003]						(1.0026, 1.0028]		(0.9979, 1.0005]	(0.9951, 1.0008]
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{jt}}{1+r_{\$t}}$:										
Min(Y):	0.9719	0.9779	0.9920	0.9863	0.9648	0.9541	0.9852	0.9983	0.9950	0.9950
Max(Y):	1.0011	1.0011	1.0343	1.0266	1.0048	1.0079	1.0538	1.0594	1.0402	1.0162
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.001	.001	.016	<.001	.006	.014	.125	.048	<.001	.066
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.322	.352	.062	.003	.188	.331	.301	.142	.197	.041
Violation for $Y \in$			[0.9920, 0.9939)			(1.0039, 1.0079]	(1.0209, 1.0538]	(1.0482, 1.0594]		(1.0030, 1.0162]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.054	.001	.038	.008	.058	<.001	.041	.004	.035	<.001
Violation for $Y \in$	[0.9719, 0.9841)		([.9920, 0.9967)			(1.0073, 1.0079]	(1.0096, 1.0538]	(1.0134, 1.0594]	(1.0118, 1.0402]	(1.0011, 1.0162]

Table 1c

Violations of the cross–moment bounds
i = British pound

	j = Deutsche mark Calls	j = Deutsche mark Puts	j = Japanese yen Calls	j = Japanese yen Puts	j = Swiss franc Calls	j = Swiss franc Puts	j = French franc Calls	j = French franc Puts	j = Canadian dollar Calls	j = Canadian dollar Puts
Number of Observations:	1765	1599	1752	1581	1756	1575	738	363	1595	883
Estimation of $E[Z]$:										
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$,	1.0142	1.0176	1.0222	1.0258	1.0142	1.0175	1.0245	1.0215	1.0222	1.0244
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$,	0.9904	0.9846	0.9892	0.9842	0.9864	0.9800	0.9538	0.9473	0.9889	0.9882
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$,	1.0098	1.0045	1.0089	1.0048	1.0060	1.0005	0.9976	0.9927	1.0181	1.0208
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{it}}{1+r_{jt}}$:										
Min(Y):	1.0004	1.0004	1.0003	1.0003	1.0006	1.0006	0.9750	0.9773	0.9878	0.9832
Max(Y):	1.0372	1.0280	1.0446	1.0446	1.0424	1.0504	1.0417	1.0226	1.0272	1.0275
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.050	.070	.025	.027	.036	.122	.005	.009	.240	.317
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.333	.298	.232	.345	.322	.507	.040	.022	.129	.053
Violation for $Y \in$									(1.0040, 1.0272]	(1.0040, 1.0275]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.406	.187	.423	.389	.380	.236	.407	.089	.262	.168
Violation for $Y \in$							(1.0158, 1.0417]	(1.0117, 1.0226]	(1.0017, 1.0272]	(1.0019, 1.0275]
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{it}}{1+r_{\$t}}$:										
Min(Y):	0.9923	0.9923	0.9923	0.9923	0.9923	0.9865	0.9963	0.9975	0.9884	0.9819
Max(Y):	1.0131	1.0131	1.0225	1.0237	1.0399	1.0587	1.0486	1.0354	1.0594	1.0441
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.132	.149	.163	.168	.101	.045	<.001	.006	.176	.310
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.005	<.001	.012	.007	.057	.053	.304	.203	.196	.064
Violation for $Y \in$	(1.0054, 1.0131]	(1.0061, 1.0131]	(1.0056, 1.0225]	(1.0065, 1.0237]	(1.0085, 1.0399]	(1.0182, 1.0587]			(1.0083, 1.0594]	(1.0067, 1.0441]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.067	.070	.073	.118	.087	.094	.423	.304	.345	.176
Violation for $Y \in$	(1.0024, 1.0131]	(1.0032, 1.0131]	(1.0031, 1.0225]	(1.0038, 1.0237]	(1.0032, 1.0399]	(1.0056, 1.0587]	(1.0172, 1.0486]	(1.0154, 1.0354]	(1.0036, 1.0594]	(1.0036, 1.0441]
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{jt}}{1+r_{\$t}}$:										
Min(Y):	0.9719	0.9779	0.9788	0.9696	0.9648	0.9541	0.9852	0.9983	0.9950	0.9950
Max(Y):	1.0011	1.0011	1.0003	1.0003	1.0048	1.0079	1.0538	1.0594	1.0402	1.0162
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.248	.298	.329	.331	.174	.256	.005	<.001	.063	.126
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.323	.348	.214	.376	.188	.332	.301	.142	.096	.041
Violation for $Y \in$	(0.9997, 1.0011]	(1.0001, 1.0011]	(0.9997, 1.0003]	(1.0000, 1.0003]	(1.0011, 1.0048]	(1.0010, 1.0079]			(1.0064, 1.0402]	(1.0040, 1.0162]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.249	.074	.256	.169	.201	.085	.040	.128	.300	.083
Violation for $Y \in$	(0.9979, 1.0011]	(0.9988, 1.0011]	(0.9987, 1.0003]	(0.9992, 1.0003]	(0.9990, 1.0048]	(0.9996, 1.0079]	(1.0148, 1.0538]	(1.0221, 1.0594]	(1.0020, 1.0402]	(1.0018, 1.0162]

Table 1d

Violations of the cross–moment bounds
i = *Swiss franc*

	j = *Deutsche mark*		j = *Japanese yen*		j = *British pound*		j = *French franc*		j = *Canadian dollar*	
	Calls	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*
Number of Observations:	1768	1602	1755	1584	1751	1719	738	363	1598	885
Estimation of $E[Z]$:										
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$,	1.0123	1.0142	1.0197	1.0227	1.0190	1.0194	1.0162	1.0203	1.0201	1.0178
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$,	0.9904	0.9846	0.9891	0.9842	0.9841	0.9824	0.9538	0.9473	0.9889	0.9882
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$,	0.9988	0.9930	0.9977	0.9929	0.9929	0.9912	0.9717	0.9655	1.0015	1.0018
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{it}}{1+r_{jt}}$:										
Min(Y):	0.9907	0.9889	0.9800	0.9844	0.9630	0.9530	0.9270	0.9208	0.9474	0.9512
Max(Y):	1.0019	1.0018	1.0031	1.0027	0.9994	0.9994	0.9998	0.9997	0.9997	0.9995
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.200	.203	.324	.314	.112	.178	.004	.092	.126	.153
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.051	.097	.216	.385	.333	.315	.510	.428	.263	.214
Violation for $Y \in$	(1.0007, 1.0019]	(1.0009, 1.0018]	(1.0006, 1.0031]	(1.0007, 1.0027]					(0.9991, 0.9997]	(0.9969, 0.9995]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.002	.032	.028	.133	.005	.019	.156	.180	.271	.300
Violation for $Y \in$	(1.0003, 1.0019]	(1.0006, 1.0018]	(1.0003, 1.0031]	(1.0004, 1.0027]					(0.9982, 0.9997]	(0.9955, 0.9995]
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{it}}{1+r_{\$t}}$:										
Min(Y):	0.9707	0.9712	0.9637	0.9599	0.9669	0.9533	0.9525	0.9518	0.9610	0.9500
Max(Y):	1.0016	1.0014	1.0016	1.0011	1.0018	1.0041	1.0033	1.0035	1.0068	1.0024
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.306	.311	.404	.417	.225	.314	.066	.242	.229	.252
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.275	.326	.250	.439	.163	.239	.392	.445	.120	.151
Violation for $Y \in$	(0.9999, 1.0016]	(1.0003, 1.0014]	(0.9997, 1.0016]	(0.9999, 1.0011]	(1.0015, 1.0018]	(1.0014, 1.0041]		(1.0011, 1.0035]	(0.9995, 1.0068]	(0.9977, 1.0024]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.009	.070	.008	.100	.001	.026	.098	.218	.121	.240
Violation for $Y \in$	(0.9991, 1.0016]	(0.9998, 1.0014]	(0.9991, 1.0016]	(0.9994, 1.0011]	(1.0009, 1.0018]	(1.0008, 1.0041]		(1.0000, 1.0035]	(0.9982, 1.0068]	(0.9964, 1.0024]
Estimation of $E[Z\|Y]$, $Y = \frac{1+r_{jt}}{1+r_{\$t}}$:										
Min(Y):	0.9719	0.9779	0.9788	0.9696	0.9920	0.9863	0.9852	0.9983	0.9950	0.9950
Max(Y):	1.0011	1.0011	1.0003	1.0003	1.0343	1.0266	1.0538	1.0594	1.0402	1.0162
$Z = \frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$, R^2:	.280	.282	.364	.394	.050	.092	.051	.005	.072	.135
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$, R^2:	.322	.352	.214	.375	.062	.003	.301	.142	.197	.041
Violation for $Y \in$	(0.9999, 1.0011]	(1.0003, 1.0011]	(0.9997, 1.0003]	(0.9998, 1.0003]	(1.0137, 1.0343]	(1.0091, 1.0266]			(1.0059, 1.0402]	(1.0035, 1.0162]
$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$, R^2:	.010	.069	<.001	.159	.004	.004	.122	.039	.214	.023
Violation for $Y \in$	(0.9993, 1.0011]	(0.9999, 1.0011]	(0.9993, 1.0003]	(0.9996, 1.0003]	(1.0089, 1.0343]	(1.0069, 1.0266]	(1.0266, 1.0538]		(1.0034, 1.0402]	(1.0026, 1.0162]

Table 2a

Evidence of Nonstationarities
i = *Deutsche mark*

	j = *Japanese yen*		j = *British pound*		j = *Swiss franc*		j = *French franc*		j = *Canadian dollar*	
	Calls	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*
Number of Observations:	1584	1755	1751	1719	1759	1578	738	363	1598	885
$\frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$	-19.6	-13.0	13.4	-15.3	-15.9	5.2	-1.6	0.5	3.8	-13.6
	(.88)	(.82)	(.89)	(.83)	(.88)	(.79)	(.52)	(.56)	(.70)	(.77)
$(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$	-11.9	12.5	-9.2	10.2	5.9	-8.1	7.8	-3.5	3.9	0.7
	(.47)	(.55)	(.29)	(.43)	(.46)	(.47)	(.30)	(.18)	(.19)	(.31)
$(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$	-1.9	3.5	2.6	-3.1	-5.4	-8.7	3.4	-1.9	2.1	-9.9
	(.29)	(.42)	(.19)	(.29)	(.38)	(.34)	(.33)	(.21)	(.31)	(.29)
$\frac{1+r_{it}}{1+r_{jt}}$	28.7	5.1	-8.8	14.5	-26.7	8.4	6.7	-3.3	-1.4	10.8
	(.84)	(.74)	(.67)	(.63)	(.87)	(.72)	(.28)	(.32)	(.32)	(.44)
$\frac{1+r_{it}}{1+r_{\$t}}$	33.8	13.9	-27.6	22.3	31.8	-14.4	7.0	-2.2	-4.0	15.6
	(.83)	(.70)	(.81)	(.71)	(.84)	(.75)	(.35)	(.39)	(.58)	(.59)
$\frac{1+r_{jt}}{1+r_{\$t}}$	28.1	15.2	-18.0	8.0	34.7	-13.9	-3.6	3.3	-4.4	10.0
	(.81)	(.69)	(.77)	(.79)	(.85)	(.74)	(.40)	(.44)	(.32)	(.61)

Table 2b

Evidence of Nonstationarities
i = *Japanese yen*

	j = *Deutsche mark*		j = *British pound*		j = *Swiss franc*		j = *French franc*		j = *Canadian dollar*	
	Calls	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*
Number of Observations:	1768	1602	1751	1719	1759	1578	738	363	1599	886
$\frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$	-5.0	-0.4	0.7	-2.6	-8.7	-1.8	8.4	-3.0	-0.3	7.5
	(.92)	(.90)	(.84)	(.86)	(.90)	(.82)	(.49)	(.62)	(.67)	(.71)
$(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$	0.1	-10.0	-11.3	-5.7	1.3	16.5	5.1	-1.0	8.5	-4.2
	(.58)	(.60)	(.29)	(.43)	(.46)	(.48)	(.30)	(.18)	(.19)	(.31)
$(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$	5.8	-5.8	8.6	5.2	-3.3	0.6	7.4	-0.8	-18.3	9.3
	(.43)	(.32)	(.23)	(.26)	(.37)	(.28)	(.24)	(.12)	(.40)	(.32)
$\frac{1+r_{it}}{1+r_{jt}}$	16.3	8.3	-10.9	-6.4	3.8	-26.9	3.4	-0.4	8.7	-10.5
	(.86)	(.86)	(.58)	(.59)	(.84)	(.73)	(.24)	(.29)	(.22)	(.42)
$\frac{1+r_{it}}{1+r_{\$t}}$	-1.9	-1.0	-23.5	-6.0	11.4	18.6	-2.1	4.1	19.7	-18.3
	(.84)	(.86)	(.78)	(.69)	(.85)	(.73)	(.35)	(.41)	(.48)	(.60)
$\frac{1+r_{jt}}{1+r_{\$t}}$	-6.6	-3.5	-7.9	0.9	5.2	24.1	-7.5	4.7	12.0	-11.6
	(.83)	(.85)	(.77)	(.79)	(.85)	(.72)	(.40)	(.44)	(.32)	(.61)

Table 2c

Evidence of Nonstationarities
i = British pound

	j = Deutsche mark Calls	Puts	*j = Japanese yen* Calls	Puts	*j = Swiss franc* Calls	Puts	*j = French franc* Calls	Puts	*j = Canadian dollar* Calls	Puts
Number of Observations:	1765	1599	1752	1581	1756	1575	738	363	1595	883
$\frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$	-8.4	10.4	11.8	-6.6	13.3	-6.4	3.6	-0.5	-3.7	10.3
	(.89)	(.89)	(.87)	(.80)	(.89)	(.77)	(.60)	(.64)	(.70)	(.76)
$(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$	19.7	-17.4	-6.5	19.3	-12.5	13.1	5.6	2.2	5.2	-4.3
	(.58)	(.59)	(.47)	(.55)	(.46)	(.48)	(.30)	(.18)	(.19)	(.30)
$(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$	-4.2	0.9	-3.9	0.4	7.8	-1.1	2.1	1.6	-6.4	7.9
	(.56)	(.37)	(.45)	(.40)	(.48)	(.31)	(.14)	(.06)	(.25)	(.27)
$\frac{1+r_{it}}{1+r_{jt}}$	-17.1	13.5	-0.2	-8.8	18.3	-15.2	0.8	2.9	-0.8	-8.9
	(.78)	(.81)	(.72)	(.58)	(.70)	(.60)	(.38)	(.48)	(.60)	(.67)
$\frac{1+r_{it}}{1+r_{\$t}}$	13.5	-13.2	-12.8	9.4	-14.7	3.4	-4.0	-0.7	0.3	-12.2
	(.93)	(.93)	(.88)	(.81)	(.74)	(.64)	(.27)	(.27)	(.42)	(.61)
$\frac{1+r_{jt}}{1+r_{\$t}}$	33.3	-26.8	-12.1	21.2	-35.7	18.8	-6.7	-3.5	1.5	-13.1
	(.83)	(.85)	(.81)	(.69)	(.85)	(.72)	(.40)	(.44)	(.32)	(.61)

Table 2d

Evidence of Nonstationarities
i = Swiss franc

	j = Deutsche mark Calls	Puts	*j = Japanese yen* Calls	Puts	*j = British pound* Calls	Puts	*j = French franc* Calls	Puts	*j = Canadian dollar* Calls	Puts
Number of Observations:	1768	1602	1755	1584	1751	1719	738	363	1598	885
$\frac{s^{i*}_{jt+1}(1+r_{jt})}{s^{i*}_{jt}} \frac{s^{i*}_{\$t+1}(1+r_{\$t})}{s^{i*}_{\$t}}$	-11.0	-13.4	6.6	-1.5	-17.8	12.8	-3.4	5.4	-8.3	0.3
	(.91)	(.89)	(.88)	(.79)	(.89)	(.82)	(.54)	(.57)	(.67)	(.71)
$(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$	9.9	18.6	-12.2	-6.3	10.2	-7.5	-7.0	3.0	6.1	5.0
	(.58)	(.60)	(.47)	(.55)	(.29)	(.43)	(.30)	(.18)	(.19)	(.31)
$(\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1+r_{it})$	-2.2	12.2	-6.3	-7.1	-1.7	-2.3	-11.4	5.4	-2.1	-1.4
	(.45)	(.42)	(.29)	(.45)	(.18)	(.32)	(.35)	(.23)	(.23)	(.22)
$\frac{1+r_{it}}{1+r_{jt}}$	20.4	24.2	-0.5	-8.0	25.0	-18.1	-7.5	4.5	18.7	7.6
	(.86)	(.87)	(.88)	(.75)	(.67)	(.61)	(.28)	(.32)	(.41)	(.44)
$\frac{1+r_{it}}{1+r_{\$t}}$	22.2	32.8	-4.6	-2.9	31.6	-26.0	-5.1	2.7	26.6	10.6
	(.84)	(.85)	(.84)	(.70)	(.81)	(.70)	(.34)	(.38)	(.61)	(.58)
$\frac{1+r_{jt}}{1+r_{\$t}}$	19.9	30.1	-7.0	1.0	6.4	-12.3	7.4	-5.0	11.5	8.9
	(.83)	(.85)	(.81)	(.69)	(.77)	(.79)	(.40)	(.44)	(.32)	(.61)

Remarks about Table 1. Violations of the bounds in Corollaries 2 and 3 are displayed, for different conditioning variables Y, and assuming $P\{\hat{P}_t > 1\} = 0$ (rows labelled "$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2$") or risk neutrality (rows labelled "$Z = (\frac{s^{i*}_{j\$t}}{s^{i*}_{jt}})^2(1 + r_{it})$"). s^{i*}_{jt} is currency j's spot rate, in units of currency i, on the trading day corresponding to the tth observation. s^{i*}_{jt+1} is currency j's spot rate, in units of currency i, at the maturity date of the options from the tth observation. $s^{i*}_{\$t}$ is the spot rate of the U.S. dollar, in units of currency i, on the trading day corresponding to the tth observation. $s^{i*}_{\$t+1}$ is the spot rate of the U.S. dollar, in units of currency i, at the maturity date of the options from the tth observation. $s^{i*}_{j\$t}$ is the value of a linear combination of call or put option premia on the trading day corresponding to the tth observation. Call option premia were used to compute $s^{i*}_{j\$t}$ in the columns marked "Calls". Put option premia were used in the columns marked "Puts". Both the option premia and the weights change over time depending on the value of the options, the spot rates, the interest rates, and the quantity and characteristics (exercise price and time-to-maturity) of the component options. r_{it}, r_{jt} and $r_{\$t}$ are, respectively, currency i's, currency j's and the U.S. dollar's interest rates on the trading day corresponding to the tth observation (the one-month eurocurrency rates multiplied by the time-to-maturity of that observation's options). The entries in the rows indicated by "*Violation for* $Y \in$" are the subranges of the conditioning variable, Y, where, according to the estimates of the conditional expectations (i.e., regression estimates), the bound was violated. The regression estimates were obtained by Ordinary Least Squares. R^2 is the regression coefficient of determination. The data were obtained from DRI (spot and eurocurrency rates) and the Philadelphia Stock Exchange (option transactions). They cover the period February 1, 1983 to June 30, 1991.

Remarks about Table 2. t-statistics are reported for the slope coefficient in a regression of the variables indicated at the top of each column onto a simulated $N(0,1)$ random walk. The simulations differ across columns. A high t-statistic (in absolute value) is evidence of nonstationarity. The numbers in brackets are the first-order autocorrelation coefficient of the variables indicated at the top of each column. The variables and dataset are described in the footnote to Table 1.