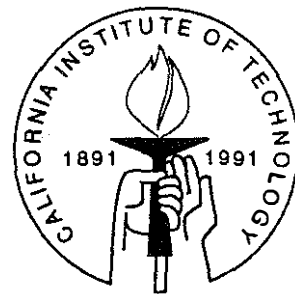DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

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Noisy Signalling in Financial Markets

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Abstract

Separating signalling equilibria of financial markets with anonymous insiders are investigated. Definitions of separating signalling equilibria are extended to allow for the noise that provides anonymity. The role of noise in equilibrium existence results is clarified. In particular, the result of Glosten and Madhavan, that noise is necessary for dealer markets to remain open, is qualified. The separating signalling equilibrium is written as the solution to a central planner's problem. Besides facilitating computation. this formulation highlights: (i) the critical nature of incentive compatibility constraints. (ii) the welfare aspects. The former causes many equilibrium price-quantity schedules to be non-linear and non-differentiable. An analysis of the latter leads to the conclusion that Pareto-efficient outcomes can be approximated by a repeated version of an insider game.

Keywords: *noisy signalling, dealership market, market microstructure, asymmetric information.*

Journal of Economic Literature Classification: 026, 311, 313.

Noisy Signalling in Financial Markets

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1   Introduction

Consider the following market situation. An asset with a future random payoff will be exchanged between two individuals. One has better information about the likely payoff. He is called the insider. The other is one among many market makers, who are prepared to take the opposite side of the trade for an appropriate price. The insider may be risk-averse. Market makers are risk-neutral. We assume that there are a sufficient number of market makers for them to behave competitively. The exchange operates in the following way. First, the insider receives a signal about the likely payoff of the risky asset. Next, he offers a quantity to the market makers, who react by quoting prices. Finally, exchange takes place between the insider and the market maker who offers the most favorable price.

This situation is reminiscent of the signalling literature (see Spence [1973], Milgrom and Roberts [1982], Cho and Kreps [1987], and others). The signalling literature, however, investigates non-anonymous exchange, in the sense that everything beyond the signal about the payoff of the risky asset is common knowledge. In contrast, the market microstructure literature has been interested in models with a greater degree of anonymity. The usual methods of assuring more anonymity are either lumping the insider's trades together with orders from uninformed "noise" traders (Kyle [1985], Admati and Pfleiderer [1988], Foster and Viswanathan [1990], Bernhardt and Hughson [1990], Subrahmanyam [1990], etc.) or by assuming the market makers do not observe the insider's endowment (Glosten [1989], Madhavan [1987]).

Unfortunately, the models of the market microstructure literature are highly specialized. When anonimity is introduced by aggregating informed trades with orders from noise traders, agents are assumed to be risk-neutral. In addition, orders from noise traders, signals about security payoffs and payoffs themselves are taken to be normally distributed.[1] Little guidance is provided as to how the models can be generalized, e.g., to the case where agents are risk-averse or payoffs are non-normal. The purpose of this paper is to investigate such generalizations.

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[1]Subrahmanyam [1990] has risk-averse insiders, but sets their endowments of the risky asset to zero. Consequently, insiders trade only for speculative reasons. This avoids the incentive compatibility conditions which are at the heart of signaling games with risk-averse agents.

The market microstructure literature is complicated by the fact that part of it actually considers *screening* models, whereby the market makers move first. In Glosten and Madhavan's framework, the market makers quote price schedules and insiders follow by choosing optimal quantities along the most favorable price schedules. This is reminiscent of the insurance models of Rothschild and Stiglitz [1975] and Riley [1979]. Glosten and Madhavan have risk-averse insiders (they have negative exponential preferences). but signals, payoffs and endowments continue to be normally distributed. They obtain nonexistence results for low levels of noise. Our analysis will determine whether these results are general in the sense that they will reappear in the signalling version of their model or when distributional assumptions are relaxed.[2]

We start with the signalling model without anonimity. This is a straightforward application of the existing literature on signalling models. We provide definitions for a Bayes-Nash as well as a *Bayes-Stackelberg* separating equilibrium. The reason is that it is possible to reinterpret Glosten and Madhavan's screening equilibrium as a signalling equilibrium (where the insider, rather than the market makers, moves first) with market makers behaving as Stackelberg leaders.

Although our equilibrium concepts are taken from the game-theoretic literature. our market microstructure model is not a fully-specified game, since the market makers' strategies are derived from a zero-profit constraint rather than payoff optimization. The zero-profit constraint is justified as the outcome of a Bertrand competition that we do not model explicitly. Consequently, when we state that the market makers "behave as Stackelberg leaders", we mean that their zero-profit constraint is solved taking into account the insider's reaction to the solution.

We provide three equivalent formulations of a Bayes-Nash separating equilibrium. The first is standard. The second is taken from Mailath [1987], who shows conditions for existence and properties of equilibrium strategies such as continuity and differentiability. The third is a central planner's formulation, which both highlights the incentive compatibility constraints which emerge under risk aversion and facilitates welfare analysis.

When confronting the market microstructure literature with existence results from the signalling literature, a puzzle emerges. Noise, or anonimity, seems originally to have been introduced in market microstructure models more to assure existence than to provide a realistic description of financial markets. In the words of Black [1986]: "Noise makes financial markets possible, ..." (p. 530). In the standard signalling model, equilibrium exists even in the absence of noise. Confronted with this puzzle, we will investigate the role of noise in market microstructure models. Anticipating some results. in Kyle. etc.. noise enters to avoid autarky in a world of common posterior beliefs (see e.g. Milgrom and Stokey [1982]). Heterogeneity of beliefs could substitute for noise. In Glosten and Madhavan's case. noise is necessary only because of the unboundedness of the signal

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[2]Copeland and Galai [1983] and Glosten and Milgrom [1985] also introduce market microstructure models where the market makers move first. The size of the orders that can be submitted are constrained. however.

space: there is no "worst" type of insider. We show that introduction of boundedness through, for example, limited liability, can substitute for noise.[3] From a practical point of view, this implies that dealership markets do *not* necessarily break down when there is little noise, contrary to what is claimed by Glosten and Madhavan.

Subsequently, we introduce noise, not in order to guarantee equilibrium existence. but as a way to provide anonimity. We extend the definition of a separating Bayes-Nash equilibrium. Intuitively, separation obtains if market makers' posterior beliefs about the signal that insiders must have received changes as a function of the quantity offered in the market. We analyze in depth a case where the first moment of the posterior distribution is sufficient to describe the equilibrium strategies of the market makers. The definition of a seperating Bayes-Nash equilibrium is reformulated as a generalization of Mailath's analysis and as the solution to a central planner's problem. Again, the latter facilitates computation. which is illustrated with an example. Moreover, it provides a benchmark to perform welfare analysis.

The formulation of the Bayes-Nash equilibria to our model with or without noise as solutions to central planner's problems indicate clearly that equilibrium strategies are solutions to an initial value problem. In contrast, ex-ante Pareto-efficient allocations are the solutions to a control problem. A comparison of the two leads to the conclusion that our market microstructure model does not lead to ex-ante Pareto-efficient allocations. Allocations which are ex-ante Pareto-efficient cannot be obtained, but we illustrate how they can be approximated as Bayes-Nash equilibria of a repeated version of our game.

The empirical implications of our model are very different from those in the traditional literature. In particular, price schedules are generically non-linear, due to the incentive compatibility constraint. In addition, potential non-differentiabilities make parametric estimation of the pricing function difficult.

The paper is organized as follows. Section 2 summarizes the existing noisy signalling models in the market microstructure literature, and investigates the role of noise. Section 3 analyzes market microstructure models without noise, i.e. without anonymity. In section 4, we extend the concept of a separating equilibrium to signalling models with noise. Section 5 investigates issues of welfare and market design. Section 6 concludes. Lemmas and propositions are proved in the appendix.

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[3]This observation also applies to Kyle [1989].

2 Market Microstructure Models and the Role of Noise.

The purpose of this section is to briefly describe existing market microstructure models and to investigate the role that noise plays in their specification. Consider, first, the models of Kyle [1985], Admati and Pfleiderer [1988], Caballé and Krishnan [1990], etc., where risk-neutral insiders hide their information behind noise traders. An aggregate quantity is announced, lumping both informed and uninformed trades, and risk-neutral, competitive market makers bid for it. Informed traders and market makers have common priors and agree on the interpretation of the signals the informed traders might obtain. Uninformed trades, signals and final payoffs are normally distributed, such that unique linear equilibria exist. The role of noise seems to be small. In the absence of noise, autarky would obtain because of the market makers' charging absurdly high prices for any positive demand or absurdly low prices for any positive supply. Noise is not necessary to avoid non-existence problems; it only prevents trivial equilibria from arising. An alternative to noise as a way to generate nontrivial equilibria would be to introduce differences in beliefs about the distribution of end-of-period asset payoffs conditional on the signal. This is a straightforward generalization of Morris [1990], who considers a case where traders decide whether to trade a predetermined quantity before observing prices, and hence, cannot condition on them.

Risk-averse insiders are introduced into Kyle's environment by Subrahmanyam [1990]. There, the insider's signal is not observed by the specialist but his endowment is known to the specialist and equal to zero. Because the risk-averse insiders trade only for speculative reasons, incentive issues associated with risk sharing are avoided. The noise is due to uninformed traders, and is normally distributed, as is the signal and the final payoff. Without noise, autarky would obtain.

In Glosten and Madhavan, however, risk-averse insiders face competitive, risk-neutral market makers who compete with price schedules. Noise appears in the form of a stochastic endowment, which is unobserved by the market makers. Insiders have exponential utility, and both signal and endowment noise are normally distributed. The model is set up as an extension of a continuous version of Rothschild and Stiglitz [1975] to noisy endowments. The market makers first compete with arbitrary price schedules, and insiders subsequently choose optimal quantities given the pricing function.

Glosten and Madhavan's results seem to indicate that a linear Nash equilibrium exists provided there is sufficient noise. We should contrast this with Riley's analysis. Riley [1979] analyzes the Nash equilibrium of the extension of Rothschild and Stiglitz (without noise) to continuous signal and payoff spaces, and shows that a Nash equilibrium does not exist. Therefore, we could interpret Glosten and Madhavan's results as indicating that noise is necessary for existence in these models. However, Glosten and Madhavan limit the strategy space for the market makers to that of continuous, twice-differentiable

functions. It is an open question whether their linear equilibrium would survive competition with discontinuous (or merely non-differentiable) price schedules. An affirmative answer would indicate that noise is indeed necessary for continuous screening models to have an equilibrium.[4]

One could also view the models of Glosten and Madhavan as signalling, as opposed to screening models, where the insider selects a quantity before market makers competitively quote a price. Notice the difference: the insider observes points on the optimal price schedule rather than the price schedule itself. Glosten and Madhavan's model, then, can be interpreted as linear equilibria to this "quantity first" model where the market makers behave as Stackelberg leaders rather than Nash (followers). Market makers determine the optimal price quote given the *reaction* of the insiders to their strategy.

Viewing their work as an extension of Kyle, it appears that Glosten and Madhavan have shown existence of a linear Stackelberg equilibrium provided sufficient noise is present. We shall confirm this shortly, after defining precisely the equilibrium concept we have in mind. We refrain from directly citing Glosten and Madhavan's results, since they do not explicitly define an equilibrium concept. They must have had in mind a Bayes equilibrium, as in solving for the equilibrium, they use the rules of conditional probability. While Glosten and Madhavan's results can be upheld when interpreting their model as an extension of Kyle, this does not imply that noise is crucial for the equilibrium to exist. We shall postpone a discussion to sections 3 and 4, where Nash equilibria for the same model will be investigated. The results for Nash solutions will obviously carry over to Stackelberg solutions.

We analyze the Glosten and Madhavan model as an extension of Kyle with Stackelberg market makers. Assume that an informed agent trades claims to a single risky asset with an end-of-period payoff f which is normally distributed with mean ϕ and variance ψ^2, and a risk free bond with price and payoff equal to one. The insider owns w units of the risky asset (and no bonds), where w is normally distributed with mean zero and variance ρ^2; w is independent of f. He observes a signal σ, independent of w, about the final value of the risky asset. Conditional on f, σ is normally distributed, with mean f and variance v^2. Hence unconditionally, σ is normally distributed with mean ϕ and variance $\psi^2 + v^2$. The insider chooses to hold b bonds and offer to sell q risky assets to risk neutral, uninformed market makers. His final wealth is: $W = b + (w - q)f$. Given price p for the risky asset, his budget constraint is: $b + (w - q)p \leq wp$. The insider exhibits negative exponential utility: $u(W) = -\frac{1}{a}e^{-aW}$. He behaves Nash: given the market makers' reply to his offer, $p(\cdot)$, he chooses q to maximize his expected utility.

The assumption of negative exponential preferences is not crucial in what follows, except when we calculate equilibria explicitly. Likewise, the normality assumptions will not be critical until we deal with existence.

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[4]Glosten [1989, p. 221] mentions that Ailsa Roell raised the same point.

Competitive risk-neutral market makers with common beliefs quote prices p after they observe q. They behave as Bertrand competitors; their equilibrium expected profits conditional on q will be zero. The market makers are Stackelberg leaders. They do not directly observe the signal σ, but will infer it from q using the rules of conditional probability and the insider's reaction function $Q(\sigma, w; p(\cdot))$.[5]

Let $h(\sigma|q, Q(\sigma, w; p(\cdot)))$ be the conditional probability of the insider observing σ given the offer q and the strategy $Q(\sigma, w; p(\cdot))$. This can be obtained from the joint density of (σ, w) by performing a change-of-variables to (σ, q) using $q = Q(\sigma, w; p(\cdot))$ and computing the conditional probability. Let $n(\cdot)$ denote the normal density.

Define the mapping T from the product of the set of real-valued functions on R and R onto R:

$$T(p(\cdot), q) = E[f|q; Q(\sigma, w; p(\cdot))], \tag{1}$$

with

$$E[f|q; Q(\sigma, w; p(\cdot))] = \int \int f n(f|\sigma) h(\sigma|q; Q(\sigma, w; p(\cdot))) df d\sigma. \tag{2}$$

Because of the Bertrand competition, market makers will quote prices $p(q)$ such that $T(p(\cdot), q) = p(q)$. We call the model M_1 and its equilibrium a Bayes-Stackelberg equilibrium.

Definition 1 *A pure strategy Bayes-Stackelberg equilibrium to M_1 is a combination of an offer rule, $Q(\cdot, \cdot; p(\cdot))$: $(\sigma, w) \rightarrow Q(\sigma, w; p(\cdot))$, and a quote rule $p(\cdot)$: $q \rightarrow p(q)$, such that:*

1. *$Q(\sigma, w; p(\cdot)) = argmax_{q \in R} E[u(W)|\sigma, w]$ where $W = q[p(q) - f] + wf$,*

2. *$p(q) = T(p(\cdot), q)$, where the mapping T is given in (1).*

Unfortunately, it is difficult to investigate general properties or even existence of equilibrium. Standard fixed-point arguments do not apply. Nevertheless, as pointed out before, the equilibrium calculated by Glosten and Madhavan provides an explicit solution for a sufficiently large amount of noise. The following proposition is a direct consequence of the analysis in Glosten [1989] and Madhavan [1987].

Proposition 1 *For $\rho^2 > \frac{\psi^2}{a^2 v^4}$, M_1 has a linear Bayes-Stackelberg equilibrium. Nonexistence obtains when $\rho^2 \leq \frac{\psi^2}{a^2 v^4}$.*

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[5]If the market makers were Nash competitors, they would not condition the insider's strategy on their own response, $p(\cdot)$.

3   A Model of Non-anonymous Market Making.

Consider the model in the previous section. What we mean by *non-anonymous* market making is that the insider's endowment is common knowledge. Unlike in Glosten and Madhavan, his only private information is the signal, σ. We also drop the assumption that the market makers are Stackelberg leaders, and assume they play Nash. The insider offers a quantity q and subsequently, market makers quote prices p. The optimal response of the market makers takes as given the insider's strategy and does not contemplate how the former's strategy changes as a function of the response. Thus, the model is an extension of the standard signalling model used by Spence [1973], Milgrom and Roberts [1982], and Cho and Kreps [1987], among others. The insiders attempt to signal their information by offering appropriate quantities.

From the signalling literature, it is clear that a Nash equilibrium may exist without noise. To see why M_1 is a signalling model, assume the signal and the final asset value can have only two values. This, together with a lack of noise and our assumptions about the sequencing of events, is Rothschild and Stiglitz reversed. Here, the insider (insuree) *first* announces a quantity (deductible) and *then* the market maker (insurer) quotes a price. Equilibrium always exists (see Hellwig [1987]).[6] In the absence of noise, the continuous signal and payoff case is also a signalling model. Separating equilibria continue to exist as long as certain conditions are satisfied.

More formally, define M_2 as M_1 with the following changes in assumptions:

1. The endowment w is commonly known and fixed at 1.

2. The market makers play Nash.

The insider's strategy is now a function of the signal only, $Q(\cdot)$: $\sigma \to Q(\sigma)$. The market maker's strategy, $p(\cdot)$ is determined by the Bertrand competitive zero-profit condition, where the expectation is formed using the insider's strategy $Q(\cdot)$ and Bayes' rule.[7] We focus on separating equilibria, those for which $Q(\cdot)$ is invertible. Hence:

Definition 2 *A separating pure strategy Bayes-Nash equilibrium to M_2 is a combination of an offer rule, $Q(\cdot)$: $\sigma \to Q(\sigma)$, and a quote rule, $p(\cdot)$: $q \to p(q)$, such that:*

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[6]One may wonder about the properties of the finite signal and payoff game. When there are two states, and many potential signals, it can be shown that equilibria which are supported by particular off-equilibrium path beliefs that satisfy Grossman and Perry's [1986] perfection concept always exist. They may be separating, pooling, or partially pooling. If we generalize to more (still finite) states, equilibria continue to exist, and continue to be either separating, pooling, or partial pooling.

[7]Unlike Stackelberg market makers, Nash market makers do not consider the reaction of the insider to their strategy $p(\cdot)$. Hence, we need not write $Q(\sigma; p(\cdot))$.

1. $Q(\sigma) = argmax_{q \in R} E[u(W)|\sigma, w]$ where $W = q[p(q) - f] + wf$,

2. $p(q) = E[f|Q^{-1}(q)]$.

provided the inverse $Q^{-1}(\cdot)$ exists.

This equilibrium could also be called *perfect* as long as each interval in the image space of $Q(\cdot)$ occurs with positive probability. Unlike in the finite signal and payoff game, equilibria which satisfy definition 2 and span the image space of $Q(\cdot)$ are robust to standard refinements.

We work with two equivalent formulations of the above equilibrium notion. The first is due to Mailath [1987]. Notice that the market makers' strategies are mechanical: given an announced quantity q, they determine the signal $\hat{\sigma}$ that generates the proposal by inverting what they perceive the insider's strategy to be. Then, from $\hat{\sigma}$, they determine $p(q)$. One could as well limit the market makers' actions to the announcement of a signal they believe to have generated the offer. Prices are then calculated by setting $p(\hat{\sigma}) = E[f|\hat{\sigma}]$. Consequently, we can rewrite the insider's optimization problem as:

$$max_{q \in R} U(\sigma, \hat{\sigma}, q), \tag{3}$$

where

$$U(\sigma, \hat{\sigma}, q) = E[u(W)|\sigma],$$
$$W = q[p(\hat{\sigma}) - f] + wf,$$
$$p(\hat{\sigma}) = E[f|\hat{\sigma}].$$

The optimand to be maximized by choice of q is the utility of the insider given he receives signal σ, offers quantity q, and the market maker infers signal $\hat{\sigma}$. We now show the following equivalence:

Lemma 1 $Q(\cdot)$: $\sigma \to Q(\sigma)$, *is a separating Bayes-Nash equilibrium strategy for an insider in M_2 if and only if $Q(\cdot)$ is invertible, with inverse $Q^{-1}(\cdot)$, and satisfies $T(Q(\cdot), \sigma) = Q(\sigma)$, where the mapping T is defined by:*

$$T(Q(\cdot), \sigma) = argmax_{q \in R} U(\sigma, Q^{-1}(q), q). \tag{4}$$

Mailath provides sufficient conditions on $U(\cdot, \cdot, \cdot)$ for a separating equilibrium to (4) to exist and to be continuously differentiable. Unfortunately, exponential utility, together with normally distributed signals, are *not* sufficient for existence, as we will subsequently show.

The separating Bayes-Nash equilibrium can also be found by solving a central planner's problem (see Maskin and Tirole [1990] and Laffont and Tirole [1990]). This facilitates

both the computation of equilibrium strategies, and the welfare analysis (which is considered in section 5). The criterion functional in the central planner's problem is an integral defined with respect to some probability measure μ. That is, the expected utility of the different types of insiders (each type receives a different signal) is weighted by μ. First. the central planner announces an allocation rule. Then the insider announces a signal. In setting the allocation rule, the central planner ensures truthful revelation of the signal.

Lemma 2 *Let Y be the set of invertible real-valued functions on the signal space. $Q(\cdot)$: $\sigma \rightarrow Q(\sigma)$, is a separating Bayes-Nash equilibrium strategy for the insider in M_2 if and only if:*

$$Q(\cdot) = argmax_{y(\cdot) \in Y} \int U(\sigma, \sigma, y(\sigma)) d\mu(\sigma) \tag{5}$$

$$s.t. \ U(\sigma, \sigma, y(\sigma)) \geq U(\sigma, \hat{\sigma}, y(\hat{\sigma})) \ \forall \ \sigma, \hat{\sigma},$$

for some probability measure μ defined on the signal space.

The incentive compatibility constraint in the planner's problem states that. given allocation rule $y(\cdot)$, truthfully revealing σ maximizes the insider's utility. Thus. this formulation highlights that when extending Kyle's market microstructure model to risk-averse insiders, risk sharing becomes an issue and consequently, an incentive compatibility condition is necessary.

If the equilibrium $Q(\cdot)$ to M_2 is continuously differentiable, (5) can be rewritten as:

$$Q(\cdot) = argmax_{y(\cdot) \in Y} \int U(\sigma, \sigma, y(\sigma)) d\mu(\sigma) \tag{6}$$

$$s.t. \ \frac{d}{d\hat{\sigma}} U(\sigma, \hat{\sigma}, y(\hat{\sigma}))|_{\sigma=\hat{\sigma}} = 0. \tag{7}$$

provided that the second order conditions of the insider's problem, $Max_{\hat{\sigma}} U(\sigma, \hat{\sigma}, y(\hat{\sigma}))$, are satisfied at $\sigma = \hat{\sigma}$. The constraint (7) is the first order condition to this maximization problem, which is just a restatement of the incentive compatibility constraint. (7) can be rewritten as:

$$y' = -\frac{U_2(\sigma, \sigma, y)}{U_3(\sigma, \sigma, y)}, \tag{8}$$

where U_j is the first derivative of U with respect to the jth argument. Consequently. the solution to the planner's problem is a solution to an initial value problem.[8] We solve (8) and pick the solution which maximizes (6), provided it satisfies the second order conditions.

The planner's problem can be simplified by setting $\mu(\cdot) = \delta_0(\cdot)$, a probability measure that puts all mass on $\sigma = 0$. Let $Y = \{y(\cdot)|y(\cdot) \ solves \ (8)\}$. Then. $y(\cdot) \in Y$ solves

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[8]Mailath also shows that Nash equilibria to signalling games can be found by solving an ordinary differential equation.

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the central planner's problem if $y(0) = argmax_z U(0, 0, z)$ provided it also satisfies the second order conditions.

To show that M_2 has a separating Bayes-Nash solution, we would like to use the representation in lemma 2 and appeal to Mailath. Unfortunately, Mailath provides sufficient conditions for existence only for compact signal spaces, whereas the signal space in our example, and indeed, in the market microstructure literature in general, is the real line. Consequently, we require a different proof. It is sufficient to show that the central planner's problem has a solution that satisfies the second order condition of the incentive compatibility constraint. Unfortunately, it does not.

Proposition 2 *There does not exist a separating Bayes-Nash equilibrium to M_2.*

In brief, the reasoning behind this proposition is as follows. Set the central planner's weighting function, $\mu(\cdot)$, equal to a probability distribution with full mass on a particular $\bar{\sigma}$. Then there will exist a signal below $\bar{\sigma}$ such that if the insider receives that signal, the second order conditions of his incentive compatibility constraint are not satisfied. Furthermore, weighting functions that put full mass on a particular value of the signal essentially capture all possible weighting functions. We refer to the appendix for a detailed proof. Consequently, the problem with M_2 is the unbounded signal space. Once we bound the signal space from below, so there is a worst signal, existence follows.

For example, change M_2 to a model M_2' as follows. Let the signal be defined on $[0, \infty)$, and let the value of the asset f, conditional on σ, be normally distributed with mean $\frac{v^2}{v^2+\psi^2}\phi + \frac{\psi^2}{v^2+\psi^2}\sigma$ and variance $\frac{v^2\psi^2}{v^2+\psi^2}$ for some constants v^2 and ψ^2 (which lose their meaning from model M_2). Drop the other assumptions about the distributions of σ and f. We can now show the following:

Proposition 3 *There exists a separating non-linear Bayes-Nash equilibrium to M_2', with $Q(\cdot)$ solving:*
$$av^2 \ln |Q| - av^2 Q + av^2 + \sigma = 0$$

Note that Q is a *non-linear* function of σ. The non-linearity is due to the incentive compatibility constraint. This implies that the price schedule, $p(\cdot)$, is also non-linear. We have:

Corollary 1 $p(q) = \frac{v^2}{v^2+\psi^2}\phi + \frac{\psi^2}{v^2+\psi^2}av^2[q - 1 - \log|q|]$, $\frac{dp(q)}{dq} = \frac{\psi^2}{v^2+\psi^2}av^2[1 - \frac{1}{|q|}] < 0$. $\frac{d^2p(q)}{dq^2} = \frac{\psi^2}{v^2+\psi^2}\frac{av^2}{q^2} > 0$.

That is, the pricing function is decreasing in the quantity offered at a decreasing rate. Notice that only the insider with the worst signal ($\sigma = 0$) is able to fully insure himself.

Due to the incentive compatibility constraints, insiders with better signals do not wish to sell the entire endowment.

4   A Model of Anonymous Market Making.

In the previous section, we investigated M_2, where risk-averse insiders with known endowments trade with risk-neutral market makers. This was an application of the standard signalling model, and results such as existence of a non-linear equilibrium obtain, provided the signal space is bounded from below.

That model was one of non-anonymous market making: the market makers know the endowments of the insiders. While this considerably simplifies the analysis, it is not a reasonable assumption in all environments. We re-introduce noise not to obtain existence of equilibrium, since equilibrium exists in the absence of noise, but instead to enhance realism.

We continue to work with our example, which we extend to allow for unknown endowments. We shall elaborate where the analysis can be generalized. Consider M_2, but assume in addition that, as in section 2, w, the endowment, is normally distributed, with mean zero and variance ρ^2. Insiders know w; the market makers do not. Call this model M_3.

There is an immediate problem. We are interested in separating equilibria, but the strategy of the insider now becomes a function of both σ *and w, $Q(\cdot, \cdot)$. This function *cannot* be invertible in σ without knowledge of w.[9] Therefore, the definition of separation must be extended to cover M_3. Intuitively, separation means that the market makers' assessment of the likely signal (and hence, the price that they quote), must change with the quantity offered. Since we are interested in Bayes-Nash equilibria, we require that the market makers use Bayes' rule to form a posterior for the signal given the quantity offered. A natural extension of the definition is that separation obtains if this posterior changes with the quantity.

When the market makers are risk-neutral and the final values given the signals are normally distributed, we do not need the complete posterior distribution of σ given q, which in general is difficult to compute. In our example, we need only compute the mean of the conditional distribution:

$$p(q) = E[f|q; Q(\cdot, \cdot)],$$

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[9]The insider could still communicate both his signal and his endowment by *coding* his message q. The market makers would decode the announced quantity and charge a price accordingly. In what follows we assume that the market makers cannot perfectly invert the quantity for the signal and the endowment. Actually, it is not clear that the insider wants such perfect decoding to take place, and, if he does, how he could communicate the key to his code.

$$= E[\int f \, n(f|\sigma)df \, |q; Q(\cdot,\cdot)],$$

$$= \frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2}E[\sigma|q; Q(\cdot,\cdot)],$$

where the conditional distribution of σ given q is determined by (2), using a change-of-variables and conditioning.

Our analysis extends to the more general case of risk-averse market makers or non-normal signals. It is considerably messier, because the complete conditional distribution of σ given q needs to be known in order for the market makers to compute the competitive price. We believe that our example, which is the canonical example in the market microstructure literature, captures most of the important features of the general case.

Now, separation can be defined with respect to the conditional mean of σ given q. This will replace $Q^{-1}(\cdot)$ in the definition of a separating Bayes-Nash equilibrium given in section 3.

Definition 3 *A separating pure strategy Bayes-Nash equilibrium to M_3 is a combination of quantity offers $Q(\cdot,\cdot)$: $(\sigma, w) \rightarrow Q(\sigma, w)$, and price quotes $p(\cdot)$: $q \rightarrow p(q)$, such that:*

1. $Q(\sigma, w) = argmax_{q \in R} \, E[u(W)|\sigma, w]$ *where* $W = q[p(q) - f] + wf$.

2. $p(q) = \frac{v^2}{v^2+\psi^2}\phi + \frac{\psi^2}{v^2+\psi^2}E[\sigma|q; Q(\cdot,\cdot)]$,

3. $E[\sigma|q; Q(\cdot,\cdot)] \neq E[\sigma|\hat{q}; Q(\cdot,\cdot)], \quad \forall q \neq \hat{q}$.

As in the previous section, we consider two alternative formulations of the equilibrium. The first is an extension of Mailath to unknown endowments. The second is a central planner's problem.

While the former is straightforward, the latter is considerably more difficult. There, the insider is asked to reveal his information. He might have an incentive to misrepresent that information and announce a different signal from the one he receives. The central planner's problem is to find an allocation rule (quantities and prices) which maximizes the insider's utility subject to a truth-telling constraint. Because of the noise, the central planner can insist only that combinations of the unknown variables (signal and endowment) be revealed truthfully. Since he can allocate only one asset, he cannot enforce truthful revelation of the signal and endowment separately. He can only ensure truthful revelation of a variable $z = Z(\sigma, w)$. That is, all insiders that have signals and endowments which generate some z announce this truthfully in a separating equilibrium. Consequently, for the purpose of translating our Bayes-Nash equilibrium into the equivalent of Mailath's formulation or into a central planner's problem, we focus on equilibria which reveal particular combinations of the signal and endowment, $z = Z(\sigma, w)$, where $Z(\cdot, \cdot)$ is invertible in w for all σ.

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Definition 4 *A separating pure strategy Bayes-Nash equilibrium to M_3 that reveals $z = Z(\sigma, w)$ is a combination of a quantity offer $Q(\cdot, \cdot)$: $(\sigma, w) \to Q(\sigma, w)$, and a price quotes $p(\cdot)$: $q \to p(q)$, such that:*

1. *$Q(\sigma, w) = argmax_{q \in R}\, E[u(W)|\sigma, w]$ where $W = q[p(q) - f] + wf$.*

2. *$Q(\sigma, w) = \dot{Q}(Z(\sigma, w))$, $\dot{Q}(\cdot)$ is invertible, with inverse $\dot{Q}^{-1}(\cdot)$, $Z(\cdot, \cdot)$ is invertible in w for all σ, with inverse $Z^{-1}(\cdot, \cdot)$,*

3. *$p(q) = \frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2} E[\sigma|\dot{Q}^{-1}(q); Z^{-1}(\cdot, \cdot)]$.*

This leads to the equivalent of Mailath's formulation. Let $\hat{\sigma}$ denote the conditional mean of σ that the market makers infer from q. The insider's problem is:

$$max_{q \in R}\, U(\sigma, w, \hat{\sigma}, q) \qquad (9)$$

where:

$$U(\sigma, w, \hat{\sigma}, q) = E[u(W)|\sigma, w],$$
$$W = q[p(\hat{\sigma}) - f] + wf,$$
$$p(\hat{\sigma}) = \frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2}\hat{\sigma}.$$

Now the optimand to be maximized by choice of q is the utility of the insider given he receives signal σ, has endowment w, offers quantity q, and the market maker infers $\hat{\sigma}$.

Lemma 3 *$Q(\cdot, \cdot)$: $(\sigma, w) \to Q(\sigma, w)$, is a separating Bayes-Nash equilibrium strategy for the insider to M_3 that reveals $z = Z(\sigma, w)$, where $Z(\cdot, \cdot)$ is invertible in w for all σ, if and only if $Q(\sigma, w) = \dot{Q}(Z(\sigma, w))$, where $\dot{Q}(\cdot)$ is invertible, with inverse $\dot{Q}^{-1}(\cdot)$, and $Z(\cdot, \cdot)$ is invertible in w for all σ, with inverse $Z^{-1}(\cdot, \cdot)$, and $\dot{Q}(\cdot)$ and $Z(\cdot, \cdot)$ satisfy: $T(\dot{Q}(\cdot), Z(\cdot, \cdot), \sigma, w) = Q(\sigma, w)$, where the mapping T is defined by:*

$$T(\dot{Q}(\cdot), Z(\cdot, \cdot), \sigma, w) = argmax_{q \in R}\, U(\sigma, w, E[\sigma|\dot{Q}^{-1}(q); Z^{-1}(\cdot, \cdot)], q). \qquad (10)$$

We could proceed as in Mailath and provide sufficient conditions on $U(\cdot, \cdot, \cdot, \cdot)$ for the solution to (10) to exist and be continuously differentiable. We shall delegate this to future research. Rather, we will work with the alternative formulation and show existence for our particular example M_3. The equivalent central planner's problem is:

Lemma 4 *$Q(\cdot, \cdot)$: $(\sigma, w) \to Q(\sigma, w)$, is a separating Bayes-Nash equilibrium strategy for the insider in M_3 that reveals $z = Z(\sigma, w)$, where $Z(\cdot, \cdot)$ is invertible in w for all σ, if and only if $Q(\sigma, w) = \dot{Q}(Z(\sigma, w))$, where*

1. $Z(\cdot, \cdot)$ is invertible in w for all σ, with inverse $Z^{-1}(\cdot, \cdot)$, such that

$$
\begin{aligned}
& U(\sigma, Z^{-1}(\sigma, z), E[\sigma | z; Z^{-1}(\cdot, \cdot)], y(z)) \\
& \qquad \geq \ U(\sigma, Z^{-1}(\sigma, z), E[\sigma | \hat{z}; Z^{-1}(\cdot, \cdot)], y(\hat{z})) \qquad (11)
\end{aligned}
$$

has a single invertible solution $y(\cdot)$ for all $\hat{z}, \sigma$, and

2. $\dot{Q}(\cdot)$ solves:

$$
\dot{Q}(\cdot) = argmax_{y(\cdot) \in Y} \int U(\sigma, Z^{-1}(\sigma, z), E[\sigma | z; Z^{-1}(\cdot, \cdot)], y(z)) d\mu(\sigma, z).
$$

for some probability measure $\mu(\sigma, z)$ defined on the product of the signal space and the endowment space. Y is the set of functions that solve (11).

The central planner's formulation of the equilibrium allocation makes clear how $Z(\sigma, w)$ can be chosen. The choice must be such that (11), the incentive compatibility condition, has a solution $y(\cdot)$. Provided the second order condition is satisfied, we can write the incentive compatibility condition as a differential equation:

$$
y' = -\frac{U_3(\sigma, Z^{-1}(\sigma, z), E[\sigma | z; Z^{-1}(\cdot, \cdot)], y)}{U_4(\sigma, Z^{-1}(\sigma, z), E[\sigma | z; Z^{-1}(\cdot, \cdot)], y)} \frac{d}{dz} E[\sigma | z; Z^{-1}(\cdot, \cdot)] \qquad (12)
$$

where U_j is the first derivative of U with respect to the jth argument. Notice the restriction that (12) puts on $Z(\cdot, \cdot)$. $Z^{-1}(\cdot, \cdot)$ must be such that the right hand side does not depend on σ, since the left hand side does not. In M_3, this would be satisfied by:

Lemma 5 If $Z(\sigma, w) = \sigma - av^2 w$, then the right hand side of (12) does not depend on σ.

Because of the assumption of jointly normally distributed signals and endowments, we can write:

$$
E[\sigma | z; Z^{-1}(\cdot, \cdot)] = \gamma_0 + \gamma_1 z, \qquad (13)
$$

where:

$$
\gamma_0 = \phi(1 - \frac{\psi^2 + v^2}{\psi^2 + v^2 + a^2 v^4 \rho^2}),
$$

$$
\gamma_1 = \frac{\psi^2 + v^2}{\psi^2 + v^2 + a^2 v^4 \rho^2}.
$$

From lemma 5 we can show:

Proposition 4 If $\rho^2 > \frac{\psi^2 + v^2}{a^2 v^4}$, there exists a linear separating Bayes-Nash equilibrium to M_3 that reveals $z = \sigma - av^2 w$, namely:

$$
\dot{Q}(z) = \frac{\gamma_0(1 - 2\gamma_1)}{av^2(1 - \gamma_1)} + \frac{2\gamma_1 - 1}{av^2} z,
$$

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$$p(q) = \frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2}\left(\gamma_0 + \gamma_1\left(\frac{\gamma_0}{1 - \gamma_1} + \frac{av^2}{2\gamma_1 - 1}q\right)\right).$$

Notice that if $v^2 \to \infty$, then $\gamma_1 \to 0$, and hence, $Q(\sigma, w) = \dot{Q}(z(\sigma, w)) = w$: The insider is perfectly insured. Also, more noise is required for the existence of the linear equilibrium than in the Stackelberg case (cf. Proposition 1). For equilibrium to exist, γ_1 must be less than $\frac{1}{2}$. As $\gamma_1 \uparrow \frac{1}{2}$, the specialist is increasingly able to distinguish amongst signals. Hence, his reaction to observing a large quantity is to decrease the price by an increasing amount. At $\gamma_1 = \frac{1}{2}$, his reaction is infinite, and markets break down.[10]

Proposition 5 *For $\rho^2 \le \frac{\psi^2 + v^2}{a^2 v^4}$, a Bayes-Nash equilibrium to M_3 that reveals $z = \sigma - av^2 w$ does not exist.*

The reason for this negative result is clear. Because the signal and the endowment are normally distributed, z is unbounded, and there is no worst type. Any strategy one could propose that satisfies the second order conditions of the incentive compatibility constraint for types $z > \bar{z}$ is suboptimal for some type below $\bar{z}$, in the sense that it violates his second order conditions.

Consequently, an equilibrium fails to exist for exactly the same reason that it failed to exist in the exponential-normal model without noise, M_2. Indeed, it is the very reason that Glosten obtains non-existence for insufficent noise in his model. To recover existence, it is necessary to bound the types z from below. Since $z = \sigma - av^2 w$, this implies that the signal and the endowment should both be defined on a *compact* set.

As an example, consider the following changes to the assumptions of M_3.

1. σ is uniformly distributed on $[0, 1]$.

2. w is uniformly distributed on $[0, 1]$.

3. f conditional on σ is normally distributed with mean $\frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2}\sigma$ and variance $\frac{v^2 \psi^2}{v^2 + \psi^2}$.

Call this model M_3'. Then:

Proposition 6 *A linear Bayes-Nash equilibrium to M_3' that reveals $z = \sigma - av^2 w$ does not exist.*

---

[10]It is puzzling that the criterion function in the central planner's problem plays no role. It does not even determine the initial condition for the differential equation as in section 3. Given sufficient noise, there is a unique, linear solution to the central planner's incentive compatibility constraint.

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Proposition 7 *For $v^2 < \frac{1}{a}$, there exists a non-linear Bayes-Nash equilibrium to M_3' that reveals $z = \sigma - av^2 w$.*

Existence can also be shown for values of v^2 above $\frac{1}{a}$.

While the equilibrium solution does not have a closed form, we can calculate it numerically. Figure 1 provides an example for particular parameter values. Notice the non-differentiabilities in $\hat{Q}(\cdot)$, which arise due to the compactness of the signal and endowment space and the fact that the equilibrium reveals only *combinations* of the signal and the endowment. These non-differentiabilities carry over to the market makers' pricing function. Since $E[\sigma|z; Z^{-1}(\cdot, \cdot)]$ is increasing in z, the pricing function is also downward sloping.

An interesting question is whether the above equilibrium converges to that derived in section 3 where there is no noise (model M_2'). To answer this question, define a sequence of models $\{M_{3\delta}'\}$, indexed by δ, identical to M_3' except that w, the endowment, is uniformly distributed on $[(1 - \delta), 1]$.

Proposition 8 *For $v^2 < \frac{1}{a}$, the non-linear Bayes-Nash equilibrium to $M_{3\delta}'$ converges to that of M_2' as $\delta \downarrow 0$.*

Figure 2 illustrates the convergence for particular parameter values. Notice that this implies that *equilibrium exists, regardless* of the amount of noise. Thus Glosten's claim that a monopolist specialist is required to keep markets open if there is insufficient noise depends critically on the unboundedness of the signal and endowment spaces. This is also true for Madhavan's claim about the necessity of batch markets.

5   Welfare Aspects and Market Design

Now we turn to the issue of market design. Is our market setup optimal? If not, what changes need to be made for it to be optimal? What we mean by optimality is Pareto-efficiency, when allocations are generated by a benevolent central planner who maximizes some welfare criterion (weighted average utility). In our environment, the best we can hope for is constrained Pareto-efficiency, where the central planner faces an information constraint. In particular, she does not observe the insider's signal, and therefore must provide insiders the right incentive to truthfully reveal the signal and allocate resources accordingly.

We focus on the no-noise environment. Here, we determine the optimal allocation rule in a situation where an informed, risk-averse insider wishes to share risk with risk-neutral

uninformed market makers, who know the endowment of the insider. The allocation rule can be written as a pair of functions of the report of the insider, $\hat{\sigma}$, $\{q(\cdot), p(\cdot)\}$, where q is the quantity the central planner takes from the insider, and p is the price she charges the market maker.

We examine the social welfare functions which put zero weight on the market maker's utility. This corresponds to assuming the market makers are competitive and on average, their profits equal zero. We also must put weights on the insider's utility function for each possible signal σ. We take $v(\cdot)$, the prior probability of observing σ. This will ensure that the allocation rule $\{q(\cdot), p(\cdot)\}$ (now functions of σ, assuming truthful reporting) is ex-ante Pareto-efficient (efficient from the point of view of the insider *before* he observes the signal).

Let $V(\sigma, q(\sigma), p(\sigma)) = E[u(W)|\sigma]$ where $W = q(\sigma)[p(\sigma) - f] + wf$.

Definition 5 *The allocation rule $\{q(\cdot), p(\cdot)\}$ is ex-ante Pareto-efficient if it is the solution to:*

$$\{q(\cdot), p(\cdot)\} = argmax_{y(\cdot), x(\cdot) \in Y} \int V(\sigma, y(\sigma), x(\sigma)) dv(\sigma) \tag{14}$$

$$s.t. \quad V(\sigma, y(\sigma), x(\sigma)) \geq V(\sigma, y(\hat{\sigma}), x(\hat{\sigma})) \quad \forall \hat{\sigma}, \tag{15}$$

$$\int (x(\sigma) - E[f|\sigma]) dv(\sigma) = 0, \tag{16}$$

where Y is the set of real-valued functions defined on the signal space.

(15) is the incentive compatibility constraint, and (16) is the market makers' zero profit constraint.

The central planner's problem can be written as a control problem with σ playing the role of time. Unlike the central planner's problems of sections 3 and 4, the criterion function now plays a role beyond providing initial conditions for the corresponding differential equation. The difference emerges because the pricing rule $p(\cdot)$ is no longer a predetermined function, but could be any of a family of functions which satisfy (16).

It is clear that the separating Bayes-Nash equilibrium to M_2' will often be different from the Pareto-efficient solution. That is, the Pareto-efficient allocation cannot always be implemented as a Bayes-Nash equilibrium to M_2'.[11] The question is, how can we design a

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[11] Examples may be constructed based on Pontryagin's maximum principle.

market which does generate Pareto-efficient allocations? There is an easy way to proceed: We can embed our one-period model in a repeated framework. and appeal to folk-theorem like arguments.

Let the game M_2' be repeated identically for time $t = 1, \ldots, \infty$. One insider can choose each period from among a countably infinite number of market makers. Once he has made his choice, he pays a fee ϵ to the market maker for the right to trade with her over the next period. Then he observes his signal, announces a quantity, and she charges him a price. The following period, the insider can opt to either switch market makers, or stay with the same one.

In order to keep the market maker from exploiting her temporary monopoly position. the insider pays a fee ϵ each period such that the discounted value for the market maker from keeping the insider as a customer forever (if δ is the discount rate. this value is $\frac{\epsilon}{1-\delta}$) marginally exceeds her one-period monopoly profit plus the one-period fee ϵ.

Consequently, the insider can make the market maker behave as in the Pareto central planner's problem. It is not in the latter's interest to deviate, because she would lose a customer, making zero profits from period $t + 1$ on, as opposed to $\frac{\epsilon}{1-\delta}$. Notice that the Pareto-efficient allocation can only be approximated: the insider must pay a fee ϵ each period. This fee, however, will be small if the discount rate δ is high.

Were there more than one insider, the fee would be bigger because the market maker faces a positive probability of another insider contacting her after the former customer has departed due to her price gouging. Now the discounted sum of fees must exceed the sum of (i) the one-period monopoly profits, (ii) the one-period fee ϵ, (iii) the expected profits from future customers who might contact her. Note that if all insiders *observe* price gouging, this situation reduces to the single insider case.

6   Conclusion

Our analysis of noisy signalling models has important implications for empirical analysis of market microstructure models. First, as in signalling models with no noise. the incentive compatibility constraints for the risk-averse traders lead to non-linearities in the equilibrium price-quantity schedules. Second, since the equilibrium will reveal only combinations of several random variables, each defined on a compact set, non-differentiabilities might appear in the price schedule. Both properties complicate empirical analysis of the price impact of trade. Nonparametric estimation seems advisable, especially since the shape and location of the non-differentiabilities of the equilibrium price schedule depend on relatively arbitrary assumptions.

Non-linearities and non-differentiabilities might explain why in empirical studies of the

traditional linear model, the slope coefficient of the price-quantity schedule. i.e. the price impact parameter, is often economically insignificant (see e.g. Glosten and Harris [1989], Hughson and Bernhardt [1990], etc.). Figure 2 illustrates how, as the amount of noise increases ($\delta \uparrow 1$), the equilibrium quantity-signal combination schedule. and hence the price-quantity schedule, has increasingly steeper edges and a flatter interior. Consequently, we conjecture that the common stock of firms held by a limited number of well known insiders, because of the absence of noise, will exhibit a strong price impact of trade. Conversely, common stock of firms held by a dispersed group of insiders, because of the greater noise, will show relatively little price impact.

In this paper we have assumed that market makers are risk-neutral. If market makers are risk-averse, the situation is more complicated. In particular, inventory considerations can no longer be ignored (see also Biais and Hillion [1991]). We speculate that the following framework might lead to more precise conclusions. Consider a large number of risk-averse market makers. Each of them will have a different reservation value for the risky asset because of differences in inventory. Hence, their bidding for the quantity offered by an insider is reminiscent of first-price auctions. If there are enough market makers, each will bid her reservation value (as opposed to a value strictly below the reservation value). The model might become especially intriguing if put in a repeated framework. Among other things, it seems that transaction prices might fail to be autocorrelated. despite inventory issues, and contrary to a widely cited presumption. We leave the verification of this conjecture to future research.

Finally, a comment regarding the static nature of our model is warranted. Even the repeated version lacks genuine dynamics. It is not clear what results would emerge when allowing the insider to trade at various times before all uncertainty is resolved. Kyle analyzes this problem for a risk-neutral insider. Our insider is risk-averse. and consequently, does not trade merely for speculative reasons, but also to share risk. In addition to the multiperiod incentive compatibility problems this raises, recontracting possibilities substantially complicate the analysis. Further research should clarify these issues.

Appendix

Proof: (Lemma 1)

$$Q(\sigma) \quad = \quad argmax_{q\in R} E[u(W)|\sigma]$$
$$\text{where} \quad W = q(p(q) - f) + wf$$
$$p(q) = E[f|Q^{-1}(q)]$$
$$\Leftrightarrow Q(\sigma) \quad = \quad argmax_{q\in R} U(\sigma, \hat{\sigma}, q)$$
$$\text{where} \quad \hat{\sigma} = Q^{-1}(q)$$
$$\Leftrightarrow Q(\sigma) \quad = \quad argmax_{q\in R} U(\sigma, Q^{-1}(q), q),$$

where $Q(\sigma) = argmax_{q\in R} U(\sigma, Q^{-1}(q), q)$ is shorthand notation for: $Q(\sigma) = T(Q(\cdot), \sigma)$. where the mapping T is defined by $T(Q(\cdot), \sigma) = argmax_{q\in R} U(\sigma, Q^{-1}(q), q)$. ∎

To prove lemmas 2 and 4, we first show the following equivalence:

Lemma 6 $x(\cdot)$, *a function from R to R, is invertible, with inverse $x^{-1}(\cdot)$, and satisfies $T(x(\cdot), \theta) = x(\theta)$, where the mapping T is defined by $T(x(\cdot), \theta) = argmax_{z\in R} f(\theta, x^{-1}(z), z)$, if and only if $x(\cdot) = argmax_{z(\cdot)\in Y} \int f(\theta, \theta, z(\theta)) d\mu(\theta)$, s.t. $\forall \theta, \hat{\theta}: f(\theta, \theta, z(\theta)) \geq f(\theta, \hat{\theta}, z(\hat{\theta}))$, for some probability measure $\mu(\theta)$. Y is the set of invertible real-valued functions on R.*

Proof: We use shorthand notation. First, $x(\theta) = argmax_{z\in R} f(\theta, x^{-1}(z), z)$ if $x(\cdot)$, a function from R to R, is invertible, with inverse $x^{-1}(\cdot)$, and satisfies $T(x(\cdot), \theta) = x(\theta)$, where the mapping T is defined by $T(x(\cdot), \theta) = argmax_{z\in R} f(\theta, x^{-1}(z), z)$. Second, $\theta \in argmax_{\hat{\theta}\in R} f(\theta, \hat{\theta}, z(\hat{\theta}))$ if $\forall \theta, \hat{\theta}: f(\theta, \theta, z(\theta)) \geq f(\theta, \hat{\theta}, z(\hat{\theta}))$.

($\Rightarrow$)

(i) $x(\theta) = argmax_{z\in R} f(\theta, x^{-1}(z), z) \Rightarrow$
$\quad x(\theta) = argmax_{z\in R} f(\theta, \theta, z) \Rightarrow$
$\quad x(\theta) = argmax_{z(\cdot)\in Y} \int f(\theta, \theta, z(\theta)) d\mu(\theta)$, all $\mu(\theta)$.

(ii) $x(\theta) = argmax_{z\in R} f(\theta, x^{-1}(z), z)$.
$\quad$ Hence, $\max_{z\in R} f(\theta, x^{-1}(z), z) = \max_{\hat{\theta}\in R} f(\theta, x^{-1}(x(\hat{\theta})), x(\hat{\theta})) = \max_{\hat{\theta}\in R} f(\theta, \hat{\theta}, x(\hat{\theta}))$.
$\quad$ Hence, $\theta = argmax_{\hat{\theta}\in R} f(\theta, \hat{\theta}, z(\hat{\theta}))$, with $z(\cdot) = x(\cdot)$.

($\Leftarrow$) (By contradiction) Assume, for some $\mu(\theta)$,

$$x(\cdot) = argmax_{z(\cdot)\in Y} \int f(\theta, \theta, z(\theta)) d\mu(\theta)$$

$$s.t. \quad \theta = argmax_{\hat{\theta}\in R} f(\theta, \hat{\theta}, z(\hat{\theta})),$$

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and $\forall x(\cdot)$ invertible: $argmax_{z\in R}f(\theta, x^{-1}(z), z) = z^*(\theta) \neq x(\theta)$, some θ (i.e. $x^{-1}(z^*) \neq \theta$, some z^*). Let Z be the set of functions $z(\cdot)$ that satisfy: $\theta \in argmax_{\hat\theta\in R}f(\theta, \hat\theta, z(\hat\theta))$, for all θ. We must verify whether $x(\cdot)$ is an element of Z. If not, it cannot possibly solve

$$argmax_{z(\cdot)\in Y} \int f(\cdot, \theta, z(\theta))d\mu(\theta)$$

$$s.t. \quad \theta = argmax_{\hat\theta\in R}f(\theta, \hat\theta, z(\hat\theta)).$$

For any invertible $x(\cdot)$: $\max_{\hat\theta\in R}f(\theta, \hat\theta, x(\hat\theta)) = \max_{\hat z\in R}f(\theta, x^{-1}(\hat z), x(x^{-1}(\hat z)))$. Consequently, for $argmax_{\hat\theta\in R}f(\theta, \hat\theta, x(\hat\theta)) = \theta$ for all θ, it must be that:

$$argmax_{\hat z\in R}f(\theta, x^{-1}(\hat z), x(x^{-1}(\hat z))) = x(\theta).$$

Yet, for some θ, $argmax_{\hat z\in R} f(\theta, x^{-1}(\hat z), \hat z) = z^*(\theta) \neq x(\theta)$, for some θ. Hence, $x(\cdot) \notin Z$.
∎

Proof: (Lemma 2) Follows directly from lemmas 1 and 6. ∎

Proof: (Proposition 2) We solve the central planner's problem setting $w = 1$.

$$
\begin{aligned}
U(\sigma, \hat\sigma, y(\hat\sigma)) &= p(\hat\sigma)y(\hat\sigma) + (1 - y(\hat\sigma))E[f|\sigma] - \frac{1}{2}a(1 - y(\hat\sigma))^2 var(f|\sigma) \\
&= y(\hat\sigma)\frac{\psi^2}{v^2 + \psi^2}(\hat\sigma - \sigma) + (\frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2}\sigma) - \frac{1}{2}a(1 - y(\sigma))^2\frac{v^2\psi^2}{v^2 + v^2}.
\end{aligned}
$$

Consequently, $y(\cdot)$ has to solve:

$$y' = -\frac{y}{av^2(1 - y)}$$

All solutions to this (separable) ordinary differential equation can be written as:

$$av^2 ln|y| - av^2 y + av^2 + \sigma + k = 0, \tag{17}$$

for constants k. To determine k, set $\mu(\sigma) = \delta_{\bar\sigma}(\sigma)$, a probability distribution with unit mass at $\sigma = \bar\sigma$. It follows that $y(\bar\sigma) = 1$ and $k = -\bar\sigma$. This satisfies the second-order conditions for $\sigma = \bar\sigma$. To see this, notice that the second-order conditions are:

$$\frac{d^2}{d\hat\sigma^2}U(\sigma, \hat\sigma, y(\hat\sigma))|_{\hat\sigma=\sigma} < 0, \quad \text{i.e.} \quad y'(\sigma) < 0.$$

Since $y(\bar\sigma) = 1$, $lim_{\sigma\downarrow\bar\sigma}y'(\sigma) = -\infty$, $lim_{\sigma\downarrow\bar\sigma}\frac{d^2}{d\hat\sigma^2}U(\sigma, \hat\sigma, y(\hat\sigma))|_{\hat\sigma=\sigma} = -\infty$, and the second-order conditions hold. However, take any $\sigma \leq \bar\sigma - av^2$. From (17), $y(\sigma)$ must satisfy: $ye^{-y} > 1$, i.e., $y(\sigma) > 1$. But then $y'(\sigma) > 0$, violating the second-order conditions.

What about other probability distributions that can be used in the criterion function of the central planner? We have only used $\mu(\sigma) = \delta_{\bar{\sigma}}(\sigma)$. Any other distribution, however, will pick a strategy determined by a particular k. Such a strategy already turned out to be optimal for $\bar{\sigma} = -k$, i.e., $\mu(\sigma) = \delta_{-k}(\sigma)$. Hence, we have essentially analyzed all possible distributions $\mu(\sigma)$. ∎

Proof: (Proposition 3) Use the formulation of the equilibrium as a central planner's problem. As in the proof of proposition 2, the following equation solves the ordinary differential equation that provides the first-order condition for the incentive compatibility constraint:

$$av^2 ln|y| - av^2 y + av^2 + \sigma + k = 0. \tag{18}$$

Set $\mu(\sigma) = \delta_\bullet(\sigma)$ to determine k. It follows that $y(0) = 1$ and $k = 0$. This particular solution satisfies the second-order condition for all $\sigma \geq 0$. To see this, notice that from (18), y has to satisfy:

$$y e^{-y} = e^{-1} e^{-\frac{\sigma}{av^2}}$$

Since $y e^{-y}$ is continuous on $[0, 1]$ and reaches a maximum of e^{-1} and a minimum of 0, whereas $e^{-1} e^{-\frac{\sigma}{av^2}}$ is between 0 and e^{-1} for $\sigma \geq 0$, $y(\sigma)$ will be somewhere between 0 and 1. Consequently, $\frac{y(\sigma)}{1-y(\sigma)} > 0$ and $y'(\sigma) > 0$, which is required for the second-order conditions to be satisfied. ∎

Proof: (Lemma 3)

$$
\begin{aligned}
Q(\sigma, w) \quad &= \quad argmax_{q \in R} E[u(W)|\sigma, Z^{-1}(\cdot, \cdot)] \\
\text{where} \quad &W = q[p(q) - f] + wf \\
&p(q) = \frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2} E[\sigma|\mathring{Q}^{-1}(q); Z^{-1}(\cdot, \cdot)] \\
\text{where} \quad &\mathring{Q}(Z(\sigma, w)) = Q(\sigma, w) \\
\Leftrightarrow Q(\sigma, w) \quad &= \quad argmax_{q \in R} U(\sigma, w, \hat{\sigma}, q) \\
\text{where} \quad &\hat{\sigma} = E[\sigma|\mathring{Q}^{-1}(q); Z^{-1}(\cdot, \cdot)] \\
\text{with} \quad &\mathring{Q}(Z(\sigma, w)) = Q(\sigma, w) \\
\Leftrightarrow Q(\sigma, w) \quad &= \quad argmax_{q \in R} U(\sigma, w, E[\sigma|\mathring{Q}^{-1}(q); Z^{-1}(\cdot \cdot)], q) \\
\text{with} \quad &\mathring{Q}(Z(\sigma, w)) = Q(\sigma, w),
\end{aligned}
$$

where $Q(\sigma, w) = argmax_{q \in R} U(\sigma, w, E[\sigma|\mathring{Q}^{-1}(q); Z^{-1}(\cdot, \cdot)], q)$ is shorthand notation for:

$$Q(\sigma, w) = T(\mathring{Q}(\cdot), Z(\cdot, \cdot), \sigma, w),$$

where the mapping T is defined by

$$T(\mathring{Q}(\cdot), Z(\cdot, \cdot), \sigma, w) = argmax_{q \in R} U(\sigma, w, E[\sigma|\mathring{Q}^{-1}(q); Z^{-1}(\cdot, \cdot)], q).$$

∎

Proof: (Lemma 4) Follows from lemmas 3 and 6. (Instead of a single "objective" argument, σ, we now have two "objective" arguments: σ and w. $Z^{-1}(\sigma, z)$ substitutes for the latter.) ∎

Proof: (Lemma 5)

$$U(\sigma, Z^{-1}(\sigma, z), E[\sigma|z; Z^{-1}(\cdot, \cdot)], y(z)) =$$

$$(\frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2}E[\sigma|z; Z^{-1}(\cdot, \cdot)])y(z)$$

$$+(Z^{-1}(\sigma, z) - y(z))(\frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2}\sigma)$$

$$-\frac{1}{2}a(Z^{-1}(\sigma, z) - y(z))^2 \frac{v^2\psi^2}{v^2 + \psi^2}.$$

Hence:

$$U_3 = \frac{\psi^2}{v^2 + \psi^2}y(z),$$

$$U_4 = (\frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2}E[\sigma|z; Z^{-1}(\cdot, \cdot)]$$

$$- (\frac{v^2}{v^2 + \psi^2}\phi + \frac{\psi^2}{v^2 + \psi^2}\sigma)$$

$$+ a(Z^{-1}(\sigma, z) - y(z))\frac{v^2\psi^2}{v^2 + \psi^2}.$$

In order for $-\frac{U_3}{U_4}\frac{d}{dz}E[\sigma|z; Z^{-1}(\cdot, \cdot)]$ to be independent of σ, we need

$$-\frac{\psi^2}{v^2 + \psi^2}\sigma + aZ^{-1}(\sigma, z)\frac{v^2\psi^2}{v^2 + \psi^2}$$

to be independent of σ. To obtain this result, let $Z(\sigma, w) = \sigma - av^2w$. Therefore, $Z^{-1}(\sigma, z) = \frac{1}{av^2}\sigma - \frac{1}{av^2}z$. Independence of σ follows since:

$$-\frac{\psi^2}{v^2 + \psi^2}\sigma + aZ^{-1}(\sigma, z)\frac{v^2\psi^2}{v^2 + \psi^2}$$

$$= -\frac{\psi^2}{v^2 + \psi^2}\sigma + \frac{\psi^2}{v^2 + \psi^2}\sigma - \frac{\psi^2}{v^2 + \psi^2}z$$

$$= -\frac{\psi^2}{v^2 + \psi^2}z.$$

∎

Proof: (Proposition 4) Use the central planner's formulation of the Bayes-Nash equi-

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librium. Solve the first-order conditions to the incentive compatibility constraint (an ordinary differential equation):

$$U_3(\sigma, Z^{-1}(\sigma, z), E[\sigma|\hat{z}; Z^{-1}(\cdot, \cdot)], y(\hat{z})) \cdot \frac{d}{d\hat{z}} E[\sigma|\hat{z}; Z^{-1}(\cdot, \cdot)]$$

$$+ U_4(\sigma, Z^{-1}(\sigma, z), E[\sigma|\hat{z}; Z^{-1}(\cdot, \cdot)], y(\hat{z})) y'(\hat{z})|_{\hat{z}=z} = 0,$$

i.e.:

$$-av^2 y' y + y'(\gamma_0 + (\gamma_1 - 1)z) + \gamma_1 y = 0.$$

The following provides a (unique) *linear* solution:

$$y(z) = \frac{\gamma_0(1 - 2\gamma_1)}{av^2(1 - \gamma_1)} + \frac{2\gamma_1 - 1}{av^2} z.$$

The second-order conditions are:

$$\frac{d}{d\hat{z}}(U_3(\sigma, Z^{-1}(\sigma, z), E[\sigma|\hat{z}; Z^{-1}(\cdot, \cdot)], y(\hat{z})) \cdot$$

$$\frac{d}{d\hat{z}} E[\sigma|\hat{z}; Z^{-1}(\cdot, \cdot)] + U_4(\sigma, Z^{-1}(\sigma, z), E[\sigma|\hat{z}; Z^{-1}(\cdot, \cdot)], y(\hat{z})) \cdot$$

$$y'(\hat{z}))|_{\hat{z}=z} < 0.$$

Rearranging yields,

$$\frac{2\gamma_1 - 1}{av^2} < 0.$$

For the second-order conditions to hold, it must be that $\gamma_1 < \frac{1}{2}$, or:

$$\rho^2 > \frac{\psi^2 + v^2}{a^2 v^4}.$$

$\blacksquare$

Proof: (Proposition 5) From the proof of proposition 4, it follows that the second-order conditions are violated for $\gamma_1 > \frac{1}{2}$, i.e., for $\rho^2 \leq \frac{\psi^2 + v^2}{a^2 v^4}$. $\blacksquare$

We prove proposition 7 before proposition 6.

Proof: (Proposition 7) Again, use the central planner's formulation of the Bayes-Nash equilibrium. First, determine $E[\sigma|z; \boldsymbol{Z}^{-1}(\cdot, \cdot)]$, where $Z(\cdot, \cdot)$ is given in lemma 5 (which continues to hold for M_3'). A tedious change-of-variables from (σ, w) to (σ, z) using $z = Z(\sigma, w)$ provides (assuming $v^2 < \frac{1}{a}$):

$$f(\sigma, z) = \frac{1}{1_{\{z < \bullet\}}(z + av^2) + 1_{\{z \in [\bullet, 1 - av^2)\}} av^2 + 1_{\{z \geq 1 - av^2\}}(1 - z)}$$

Consequently,

$$
\begin{aligned}
E[\sigma|z; Z^{-1}(\cdot\cdot)] &= \frac{1}{2}av^2 + \frac{1}{2}z, \quad (z \in [-av^2, 0)), \\
&= \frac{1}{2}av^2 + z, \quad (z \in [0, 1 - av^2)), \\
&= \frac{1}{2} + \frac{1}{2}z, \quad (z \in [1 - av^2, 1]).
\end{aligned}
$$

The first-order conditions of the incentive compatibility condition can, as before, be written as an ordinary differential equation:

$$
-av^2 y'y + y'(\gamma_0 + z(\gamma_1 - 1)) + \gamma_1 y = 0,
$$

where $\gamma_\bullet, \gamma_1$ take on values depending on z. The second-order conditions are:

$$
2\gamma_1 y'(z) - av^2(y'(z))^2 + (\gamma_\bullet + z(\gamma_1 - 1) - av^2 y(z))y''(z) < 0.
$$

(i) For $z \in [0, 1 - av^2), \gamma_\bullet = \frac{1}{2}av^2$ and $\gamma_1 = 1$. The ordinary differential equation that represents the first-order condition is separable, and the solution is given by:

$$
\frac{1}{2}av^2 ln|y| - av^2 y + z + k = 0,
$$

for constants k, where k is determined by the criterion function of the central planner's problem. The second-order conditions are satisfied for $y'(z) < 0$. i.e., $y \in (0, \frac{1}{2}]$.

(ii) For $z \in [-av^2, 0), \gamma_0 = \frac{1}{2}av^2$ and $\gamma_1 = \frac{1}{2}$. The first-order conditions become:

$$
-2av^2 y'y + y'(av^2 - z) + y = 0.
$$

This ordinary differential equation is not separable, and needs to be solved numerically. The second-order conditions are satisfied for $y'(z) < 0$. i.e., $y < \frac{1}{2}(1 - \frac{z}{av^2})$.

(iii) For $z \in [1 - av^2, 1], \gamma_\bullet = \gamma_1 = \frac{1}{2}$. the first-order conditions become:

$$
-2av^2 y'y + y'(1 - z) + y = 0,
$$

again a nonseparable differential equation. The second-order conditions are satisfied for $y'(z) < 0$, i.e., $y < \frac{1-z}{2av^2}$.

Next, set $\mu(\sigma, z) = \delta_{-av^2}(z)$ in the central planner's criterion function. The member of the family of solutions to the above ordinary differential equations that maximizes the resulting central planner's criterion function will be anchored at $\{z, y(z)\} = \{-av^2, 1 - \epsilon\}$ for some small number $\epsilon > 0$. (At $y(-av^2) = 1$, the corresponding second-order conditions are violated.) Consequently, the "worst type", i.e., the insider with $z = -av^2$

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$(\sigma = 0, w = 1)$, will be almost fully insured. k, the constant of integration in the solution for $z \in [0, 1 - av^2)$ will be determined so the solution for $z \in [-av^2, 0]$ matches up at $z = 0$. Similarly, the solution for $z \in [1 - av^2, 1]$ should match up with that for $z \in [0, 1 - av^2)$ at $z = 1 - av^2$. Figure 1 displays equilibria for various values of av^2. We have checked the numerical solutions against the second-order conditions, $y'(z) < 0$, for various values of av^2; they were never violated for initial values $(z, y(z)) = (-av^2, 1 - \epsilon)$.

■

Proof: (Proposition 6) Take $z \in [0, 1 - av^2)$. The linear solution to the ordinary differential equation that represents the first-order conditions of the incentive compatibility constraint has slope: $\frac{1}{av^2}$, which is greater than zero, violating the second-order conditions.

■

Proof: (Proposition 8) Replicating the proof of proposition 7, we obtain the following:

$$
\begin{aligned}
E[\sigma|z; Z^{-1}(\cdot, \cdot)] &= \frac{1}{2}av^2 + \frac{1}{2}z, \quad (z \in [-av^2, -(1-\delta)av^2)), \\
&= (1 - \frac{\delta}{2})av^2 + z, \quad (z \in [-(1-\delta)av^2, 1 - av^2)), \\
&= \frac{1}{2} + \frac{1}{2}(1-\delta)av^2 + \frac{1}{2}z, \quad (z \in [1 - av^2, 1 - (1-\delta)av^2)).
\end{aligned}
$$

(i) For $z \in [-av^2, -(1-\delta)av^2)$, the solution is identical to the one of proposition 7, except for the shorter support.

(ii) For $z \in [-(1-\delta)av^2, 1 - av^2)$, the solution becomes:

$$(1 - \frac{\delta}{2})av^2 ln|y| - av^2 y + z + k = 0$$

   The second-order conditions ($y'(z) < 0$) are satisfied for $y \in (0, 1 - \frac{\delta}{2}]$.

(iii) For $z \in [1 - av^2, 1 - (1-\delta)av^2]$, the ordinary differential equation that represents the first-order conditions of the incentive compatibility condition becomes:

$$2av^2 y'y - y'(1 + (1-\delta)av^2) + zy' - y = 0.$$

   This equation needs to be solved numerically. The second-order condition, $y'(z) < 0$, is satisfied for
$$y < \frac{1 + (1 - \delta)av^2 - z}{2av^2}.$$

Setting $\mu(\sigma, z) = \delta_{-av^2}(z)$, the same optimal supply schedule an $[-av^2, -(1-\delta)av^2)$ as before with $\delta = 1$ is obtained. It is anchored at $(z, y(z)) = (-av^2, 1 - \epsilon)$, for some small

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$\epsilon > 0$. The optimal schedules for $z \in [-av^2, -(1-\delta)av^2)$ and $z \in [1 - av^2, 1 - (1 - \delta)av^2]$ are matched with the one for $z \in [-(1-\delta)av^2, 1 - av^2)$. Figure 2 displays the optimal supply schedules on $[-av^2, 1 - (1-\delta)av^2]$ for various values of δ. In all cases, the second-order conditions are satisfied.

As $\delta \downarrow 0$, the first interval, $[-av^2, -(1-\delta)av^2)$, vanishes. The solution on $[-(1-\delta)av^2, 1 - av^2)$ converges to that of proposition 3. The last interval, $z \in [1 - av^2, 1 - (1 - \delta)av^2]$ also vanishes. ∎

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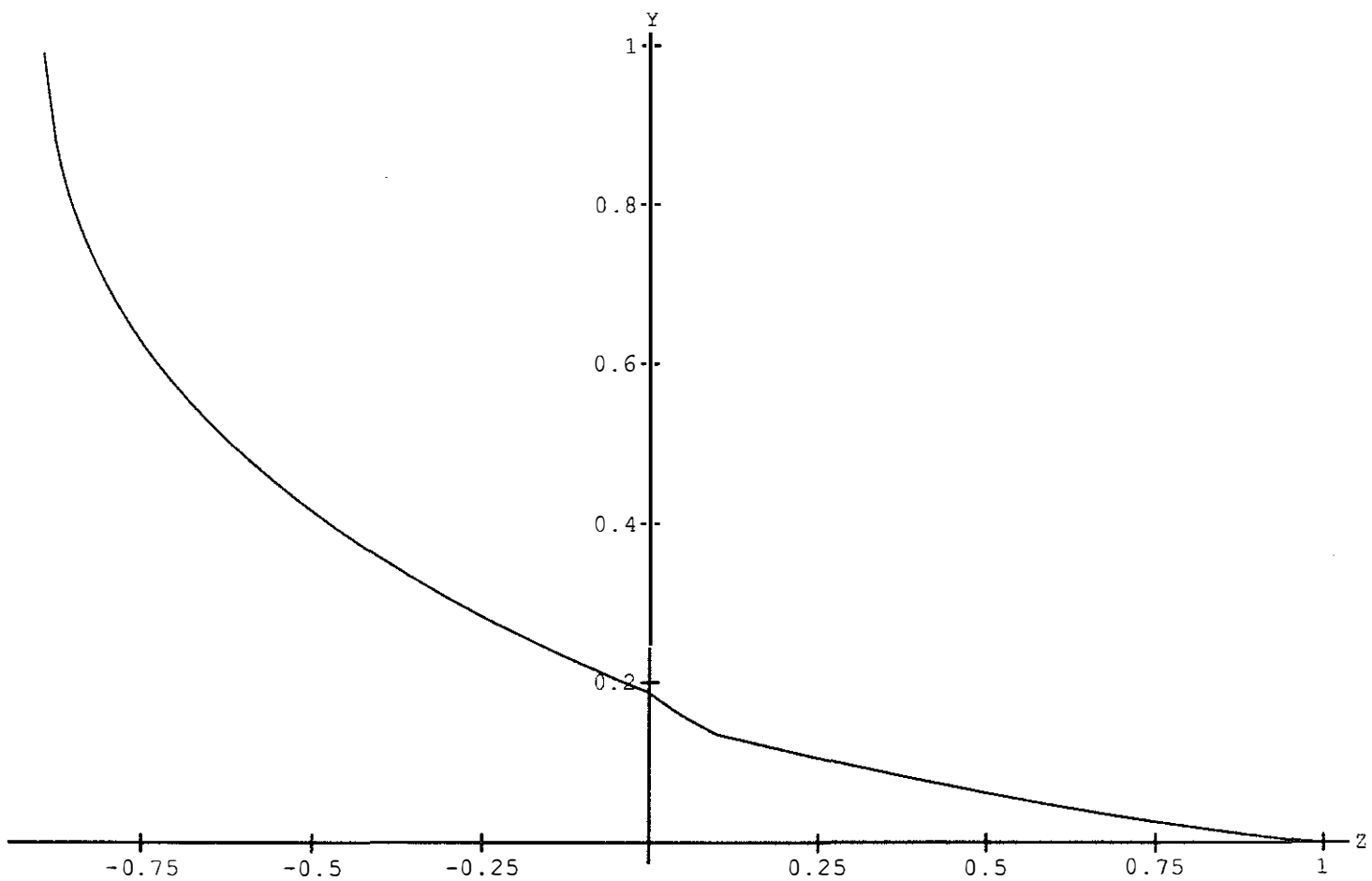
Figure 1: Quantity (Y) as a function of the market maker's imperfect information (Z) when $av^2 = .9$.
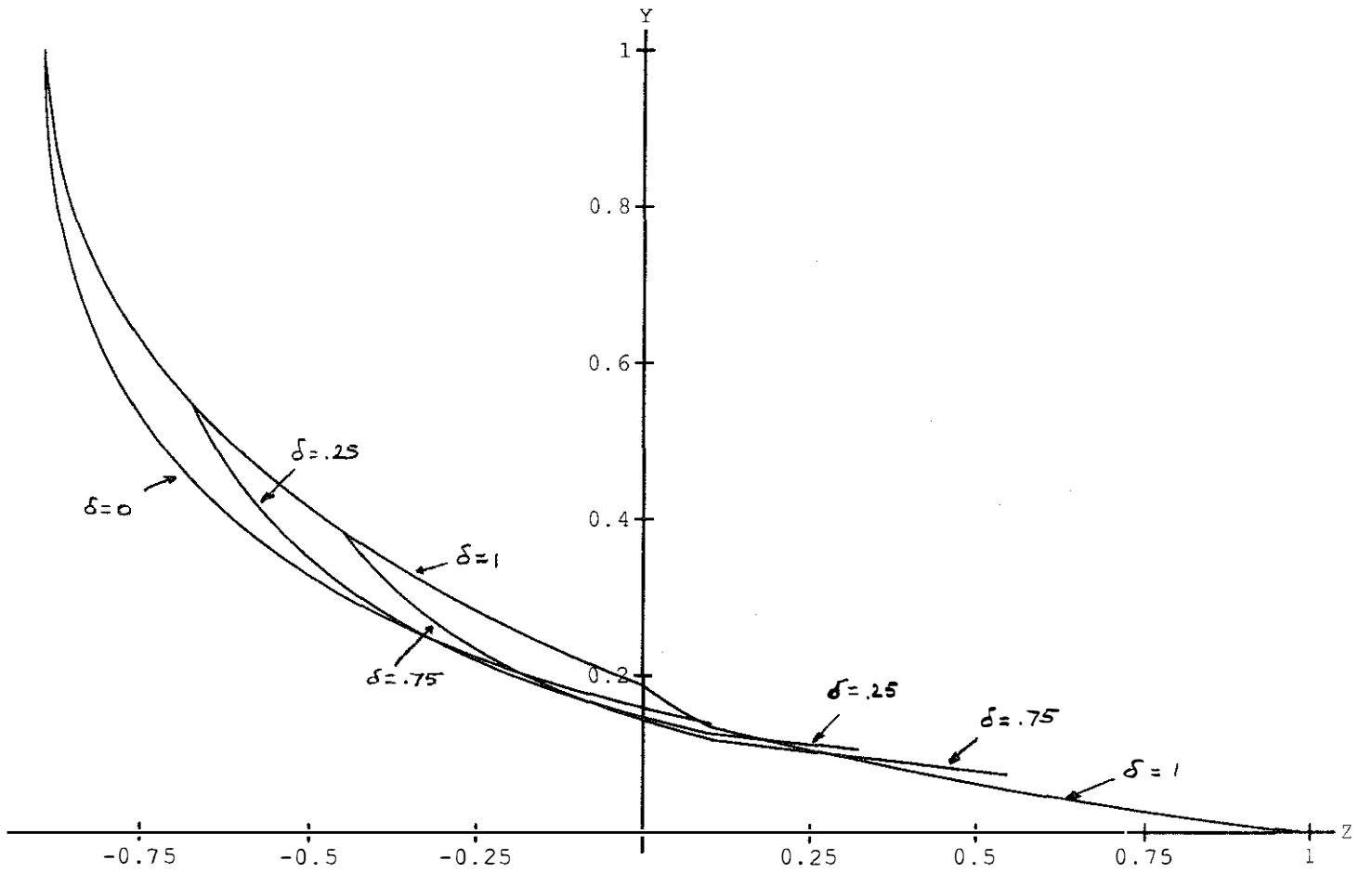
Figure 2: Quantity (Y) as a function of the market maker's imperfect information (Z) when $av^2 = .9$ and the amount of noise approaches zero ($\delta = [1, .75, .25, 0]$).

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