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Abstract

This paper derives testable restrictions on equilibrium prices when capital gains and losses are taxed only when realized. We use the Generalized Method of Moments (GMM) procedure to estimate and test the restrictions. The empirical results show evidence of capital gains tax effects on the pricing of common stock. The restrictions are not rejected by the data and estimates of the coefficient of risk aversion and the dividend tax rate are precise and economically plausible. Estimates of the capital gains tax rate, however, are often imprecise and economically implausible. Further results indicate that this can be attributed to the fact that our model does not accomodate differential long and short-term tax rates. The data appear to favor the martingale hypothesis for after-tax asset returns over a before-tax consumption-based asset pricing model.

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I. Introduction

The effect of taxes on the equilibrium pricing of financial assets is an important theoretical and empirical issue. In the late 1970's, the discussion focused primarily on the effect of dividend yield on the equilibrium before-tax expected rates of return on common stock (e.g., Brennan [1973], Litzenberger and Ramaswamy [1979, 1982], Black and Scholes [1974] and Miller and Scholes [1978, 1982]). More recently, however, attention has shifted to the importance of the capital gains tax in determining asset values. The particular feature of the tax code that makes capital gains taxation interesting is the tax-timing option available to investors. Capital gains and losses on stock, for example, are not taxed until the investor sells the stock. This gives the investor the option to time his asset sales so as to minimize the present value of the net tax payments made to the government. In particular, the investor has the option to realize losses and defer gains. To the extent that these tax timing options are valuable to investors, they should be reflected in equilibrium asset prices.

Constantinides [1983, 1984] and Constantinides and Scholes [1980] discuss the optimal trading of stocks in the presence of personal taxes and explore the effect of optimal realization decisions on equilibrium stock prices. Constantinides and Ingersoll [1984] and Litzenberger and Rolfo [1984] extend the analysis to the trading and pricing of government bonds. The results of these studies indicate that tax-timing options can represent a large fraction of the total benefits associated with holding capital assets and, therefore, should not be ignored when estimating and testing asset pricing models.

The purpose of this paper is to explore the implications of general equilibrium on asset prices when capital gains and losses are taxed only when realized. For tractability, we assume that the long- and short-term tax rates are equal. This assumption is consistent with the model of Constantinides

[1983] and with the current U.S. tax code. As in Hansen and Singleton [1982], we derive restrictions on asset returns assuming the existence of a representative consumer. This allows us to use aggregate consumption as a benchmark for pricing after-tax cashflows and thus avoids the difficulties associated with identifying portfolios that span the marginal rate of substitution. In equilibrium, asset returns depend not only upon the covariability of the asset's pretax rate of return with aggregate consumption, but also upon the covariability of the tax payments and rebates on the asset with aggregate consumption. The relevant after-tax cashflows for determining an asset's price at any date are independent of the past history of prices. This allows us to test the asset pricing restrictions without reference to the distribution of basis values across investors.

There have been a number of empirical studies of the traditional consumption-based asset pricing model without taxes. While the traditional consumption-based asset pricing model survives tests on equity returns alone, it generally fails in joint tests of equity and Treasury bill returns (e.g., Hansen and Singleton [1982]) or term structure data (e.g., Dunn and Singleton [1986]). Several attempts have been made to improve the fit of the model by acknowledging measurement errors in the consumption data (e.g., Grossman, Melino and Shiller [1987]), time non-separabilities (e.g., Dunn and Singleton [1986], Ferson and Constantinides [1989] and Heaton [1989]) or state non-separabilities (e.g., Epstein and Zin [1990]). The results have been mixed. Notice, however, that all attempts to ameliorate the fit of the traditional consumption-based asset pricing model have focused on problems with the way in which consumption or utility are measured. While we do not question the validity and insights of these approaches, we alter the focus somewhat by considering whether returns are properly measured. In particular, we acknowledge the fact that taxes alter the returns that assets provide investors. Previous studies have examined consumption-based models using after-tax measures of return (e.g., Grossman and Shiller [1981], Mankiw, Rotemberg and Summers [1985] and Rotemberg [1984]), but these studies ignore the option feature associated with capital gains taxation and assume that all capital gains are taxed each period. Not suprisingly, the results of these studies are virtually identical to those from studies that completely ignore taxes.

We estimate and test the tax-induced intertemporal restrictions on asset returns using Hansen's [1982] Generalized Method of Moments (GMM) procedure. Using monthly consumption and return data over the period from March 1959 to December 1986, the results provide reliable evidence of capital gains tax effects on the relative pricing of common stocks and Treasury bills. Although our model is not rejected and the estimates of the coefficient of risk aversion and the dividend tax rate are reasonable, the results fail to provide a reliable estimate of the capital gains tax rate. Our empirical results also indicate that after-tax returns, unlike their before-tax counterparts, are unpredictable and follow a martingale process.

One possible explanation for the imprecise estimates of the capital gains tax rate is that our theoretical model assumes symmetric taxation of long- and short-term capital gains and losses, whereas over the time period studied the long- and short-term tax rates differed. While a theoretical model that allows for asymmetric long- and short-term tax rates is warranted, there are well-known difficulties in deriving the optimal tax trading strategies of investors for this case.[1] In particular, when the long-term tax rate is less than the short-term tax rate, the optimal tax trading strategy may involve the sale and repurchase of assets with long-term capital gains to reestablish short-term status and restart the option to realize potential future losses short term. The difficulty arises in deriving the capital gains level below

which it is optimal to realize long-term capital gains and above which it is optimal to defer long-term capital gains. Our empirical results indicate that such a model may prove fruitful in fitting the data, but the complications involved dictate that we leave this for future research.

The paper is organized as follows. In Section II, we derive necessary restrictions on equilibrium asset prices given the optimal tax realization policy described by Constantinides [1983]. In Section III, we develop the empirical tests of the pricing restrictions. In Section IV, we describe our data set, discuss our choice of instrumental variables, and present the empirical results. Section V summarizes the paper.

II. The Model

Consider a multiperiod securities market economy under uncertainty that has T+1 trading dates indexed by $t = 0,\dots,T$. There are J financial assets indexed by $j = 1,\dots,J$. The financial assets are in positive net supply and are characterized by their exogenous stochastic dividend processess $\{d_j(t);\ t = 1,\dots,T\}$, where $d_j(t)$ is the random dividend on security j at time t. We assume that the dividend payments are nonnegative and made in units of a single consumption good. The ex-dividend price of security j at time t is denoted $P_j(t)$ and is determined through competitive trading at time t. We assume that all securities pay a liquidating dividend at date T and, hence, $P_j(T) = 0$ for all $j = 1,\dots,J$. Since the equilibrium only determines relative prices, the ex-dividend prices of the financial assets are stated in units of the single consumption good. There is a representative consumer with a time-additive and state-independent von Neumann-Morgenstern utility function given by

$$\sum_{t=0}^{T} \beta^t U(c(t)),$$

where $c(t)$ represents consumption at date t, β is the personal rate of time preference and $U(\cdot)$ is differentiable, strictly increasing and concave. We denote the derivative of U with respect to $c(t)$ as $U_c(t)$.

The tax environment is a simplification of the actual U.S. tax code and is similar in many respects to the tax environment in Constantinides [1983]. Dividend income is fully taxable at the constant rate of τ_d, where $\tau_d \in (0,1)$, and realized capital gains and losses are taxable at the constant rate of τ_c, where $\tau_c \in (0,1)$. To be as general as possible, we leave the relationship between τ_d and τ_c unspecified. As with the actual tax code, we assume that all unrealized capital gains and losses remain untaxed. This feature of the tax code gives investors the option to optimally time the realization of their capital gains and losses for tax purposes. No distinction is made between long-term and short-term status of capital gains and losses. Throughout the analysis, we will ignore the capital loss limit (currently $3,000 per year) imposed by the actual U.S. tax code.

Constantinides [1983] has shown that under these conditions the optimal tax-trading policy is to realize losses as soon as they occur and to defer capital gains. This has important implications for the valuation of financial assets. In tax-free economies (e.g., Hansen and Singleton [1982]), the price of a security is obtained by discounting all future dividends by the marginal rate of substitution of the representative consumer. With taxes, however, the payoffs on the security will include not only the future after-tax dividends but also the future tax rebates on the optimal realization of capital losses. This implies that, if a security market equilibrium exists, the price of security j must satisfy the following Euler equation:

$$P_j(t) = E_t\Big\{ \sum_{s=t+1}^{T} m(s,t)\Big[(1-\tau_d)d_j(s) + \tau_c \max\big[0,\ \min[P_j(t),\ldots,P_j(s-1)] - P_j(s)\big]\Big]\Big\} \quad (1)$$

$$j = 1,\ldots,J;\ t = 0,\ldots,T-1,$$

where $m(s,t)$ represents the marginal rate of substitution between consumption at date s and consumption at date t,

$$m(s,t) = \frac{\beta^{s-t} U_c(s)}{U_c(t)} \qquad \forall s > t. \quad (2)$$

The future tax rebates are represented in Equation (1) as a sequence of one-period put options, where the exercise price at date s is equal to the minimum price reached by the security over the period from date t to date s-1. This minimum price also equals the investor's tax basis at date s under the optimal realization policy.[2]

In equilibrium, the price of the security at date t must make the investor indifferent at the margin between time-t consumption and time-t investment. This requires a price at date t that reflects only the cashflows beyond date t, including the future tax rebates on capital losses from a newly established position in the security. As a result, the equilibrium price is independent of the past history of prices. This is in direct contrast to the common notion that the distribution of basis values across investors has an important effect on the market price of the security. The independence of the market price of the security from the basis values of investors is also important from an empirical standpoint since it allows the model to be tested without knowledge of the basis values of investors.[3]

To gain further insights into the equilibrium price process given by Equation (1), we rewrite it in the following convenient form:

$$P_j(t) = E_t\left\{ \sum_{s=t+1}^{T} m(s,t)x_j(s,t)\right\} \qquad t = 0,\dots,T-1, \tag{3}$$

where

$$x_j(s,t) = d_j(s)(1-\tau_d) + \tau_c \max\left[0,\ \min[P_j(t),\dots,P_j(s-1)] - P_j(s)\right] \tag{4}$$

is the total after-tax cashflow on security j at date s from an investment made at date $t < s$. According to Equation (4), $x_j(s,t) \le x_j(s,t+1)$ since $\min[P_j(t),\dots,P_j(s-1)] \le \min[P_j(t+1),\dots,P_j(s-1)]$. Therefore, the price of security j at date t, $t = 0,\dots,T-1$, must satisfy:

$$P_j(t) = E_t\left\{m(t+1,t)\left[x_j(t+1,t) + P_j(t+1) - z_j(t+1,t)\right]\right\} \tag{5}$$

where $P_j(t)$ and $P_j(t+1)$ conform to Equation (3) and $z_j(t+1,t)$ is a nonnegative random variable given by

$$z_j(t+1,t) = E_{t+1}\left\{ \sum_{s=t+2}^{T} m(s,t+1)\left[x_j(s,t+1) - x_j(s,t)\right]\right\}. \tag{6}$$

From Equations (4) and (6), $z_j(t+1,t)$ can be interpreted as the present value, at date t+1, of the differential tax rebates on future capital losses (beyond date t+1) from one share of security j purchased at date t+1 compared to date t. Therefore, the differential $P_j(t+1) - z_j(t+1,t)$ appearing in Equation (5) can be interpreted as the investor's <u>personal</u> <u>valuation</u> of a position in one share of security j at date t+1 with basis $P_j(t)$. We denote this personal valuation by $v_j(t+1,t)$.[4] The value of $z_j(t+1,t)$ is increasing in the difference between $P_j(t+1)$ and $P_j(t)$ and is equal to zero only if $P_j(t+1) \le P_j(t)$. In this case, the investor optimally realizes the capital loss and reestablishes a new position in the security at date t+1.

Equation (5) can also be written in terms of the rates of return on

security j. Dividing both sides of Equation (5) by $P_j(t)$, subtracting one from both sides, and rearranging terms gives

$$0 = E_t\left\{m(t+1,t)\left(1 + r_j^d(t+1,t)(1-\tau_d) + r_j^c(t+1,t) - \tau_c r_j^z(t+1,t) - \tau_c \min[0,\ r_j^c(t+1,t)]\right) - 1\right\} \tag{7}$$

$$\equiv E_t\left(m(t+1,t)R_j(t+1,t) - 1\right)$$

where $r_j^c(t+1,t)$ is the pretax capital gain rate of return on security j from date t to date t+1, $r_j^d(t+1,t)$ is the pretax dividend yield on security j from date t to date t+1, and $r_j^z(t+1,t) \equiv z_j(t+1,t)/\tau_c P_j(t)$ is given by

$$r_j^z(t+1,t) = \frac{P_j(t+1) - v_j(t+1,t)}{\tau_c P_j(t)} \tag{8}$$

$$= E_{t+1}\left\{ \sum_{s=t+2}^{T} m(s,t+1)\left(\max\left[0,\ \min[r_j^c(t+1,t),\dots,r_j^c(s-1,t)] - r_j^c(s,t)\right] - \max\left[0,\ \min[0, r_j^c(t+1,t),\dots,r_j^c(s-1,t)] - r_j^c(s,t)\right]\right)\right\}$$

where the second line of Equation (8) follows from Equations (4) and (6) and $r_j^c(s,t)$ denotes the pretax capital gain rate of return on security j from date t to date s. According to the first line of Equation (8), $\tau_c r_j^z(t+1,t)$ can be interpreted as the differential rate of capital appreciation from date t to date t+1 between the market price of security j and the investor's personal valuation (where $v_j(t,t) = P_j(t)$). As with $z_j(t+1,t)$, the value of $r_j^z(t+1,t)$ is always nonnegative and is equal to zero only if $r_j^c(t+1,t) \le 0$.

An important feature of the equilibrium pricing relation (7) is that it allows for time-varying risk premia. Using the definition of conditional covariance, Equation (7) implies that in equilibrium

$$E_t\left[R_j(t+1,t)\right] = R_f(t+1,t)\left(1 - \text{cov}_t\left[m(t+1,t),\ R_j(t+1,t)\right]\right), \tag{9}$$

where $R_f(t+1,t)$ is one plus the tax-exempt riskfree interest rate from date t to date t+1 and $\text{cov}_t(\cdot,\cdot)$ is the conditional covariance. As in Breeden's [1979] model, risk is measured by the conditional covariance of the "returns" with the marginal rate of substitution of consumption. The only difference is that our measure of "returns" includes the effect of taxes. In the absence of taxes, $R_j(t+1,t) = 1 + r_j(t+1,t)$, where $r_j(t+1,t)$ is the total rate of return on security j from date t to date t+1, and Equation (9) collapses to the familiar pricing relationship first derived by Rubinstein [1976],

$$E_t\left[1 + r_j(t+1,t)\right] = R_f(t+1,t)\left(1 - \text{cov}_t\left[m(t+1,t),\ r_j(t+1,t)\right]\right). \tag{10}$$

III. Testable Restrictions of the Model

In this section, we derive the testable restrictions implied by the equilibrium pricing relations of the previous section and describe our empirical methodology. According to Equation (7), a securities market equilibrium requires $[m(t+1,t)R_j(t+1,t) - 1]$ to be orthogonal to all the elements in the investor's information set at date t, y_t. That is,

$$E\left([m(t+1,t)R_j(t+1,t) - 1]y_t\right) = 0, \tag{11}$$

where $E[\cdot]$ denotes the unconditional expectation. The orthogonality condition (11) restricts the comovements between aggregate consumption and the "returns" on the J financial assets. In particular, Equation (11) restricts the mean of $[m(t+1,t)R_j(t+1,t)y_t]$ to equal the mean of y_t for all $j = 1,\ldots,J$.

To exploit these restrictions empirically, we must either specify the functional form of the investor's utility (e.g., Hansen and Singleton [1982, 1984]) or restrict $m(t+1,t)$ and $R_j(t+1,t)$ to be drawn from a particular family of distributions (e.g., Hansen and Singleton [1983] and Ferson [1983]). We take the former approach and assume that the investor possesses a power utility function. That is,

$$U(c(t)) = \frac{[c(t)]^{\gamma}}{\gamma}, \tag{12}$$

where $1 - \gamma > 0$ is the coefficient of relative risk aversion. Under power utility, the marginal rate of substitution $m(s,t)$ is given by

$$m(s,t) = \beta^{s-t}[c(s)/c(t)]^{\gamma-1} \tag{13}$$

and, therefore, Equation (11) becomes

$$0 = E\left(\left[\beta[c(t+1)/c(t)]^{\gamma-1}R_j(t+1,t) - 1\right]y_t\right). \tag{14}$$

The tests of the equilibrium relation (14) conducted in this paper rely on the Generalized Method of Moments (GMM) procedure proposed by Hansen [1982] and Hansen and Singleton [1982]. A brief description of this procedure is provided below.[5]

Let Λ be an n-vector of unknown parameters to be estimated from the model. For our model, $n = 4$ and $\Lambda = \{\beta,\ \gamma,\ \tau_c,\ \tau_d\}$. Define

$$u_{jt+1} \equiv \beta[c(t+1)/c(t)]^{\gamma-1}R_j(t+1,t) - 1 \qquad j = 1,\ldots,J \tag{15}$$

to be the disturbances for our econometric analysis. Equation (14) implies that $E[u_{jt+1}y_t] = 0$ for all y_t and for all $j = 1,\ldots,J$. Therefore, elements of the investor's information set at date t can be used as instrumental variables for the disturbance u_{jt+1}. After selecting m ($m > n$) instruments to be used in estimating Λ, the parameter estimates are chosen by GMM to minimize a quadratic form in the sample means of $u_{jt+1}y_{it}$, $i = 1,\ldots,m$. By choosing the parameter estimates in this way, these sample means are made close (in terms of the distance measure defined by the quadratic form) to their population value of zero under the null hypothesis. However, with more orthogonality conditions than parameters to be estimated, the model is overidentified and not all orthogonality conditons will be set equal to zero in the estimation. Nevertheless, the (m-n) overidentifying restrictions should be close to zero if the model is "correct". Hansen [1982] and Hansen and Singleton [1982] have shown how a chi square goodness-of-fit statistic can be used to test these overidentifying restrictions.

An advantage of the GMM procedure is that it allows for the conditional variance of the disturbances u_{jt+1} to be an arbitrary function of the elements of the investor's information set at date t. This means that the procedure allows for the possibility that conditional variances and covariances of returns and consumption vary over time and change sign. Thus, the GMM procedure is capable of testing asset pricing models that allow for time-varying risk premia. Moreover, it is not necessary to specify how these variances and covariances change over time as a function of the investor's information set. It is also not necessary to make any particular distributional assumptions about returns and consumption, unlike the log-linear models of asset prices (e.g., Hansen and Singleton [1983] and Ferson [1983]).

IV. The Empirical Tests

A. The Data

Various versions of the model were estimated using monthly data on consumption, stock returns and Treasury bill returns, covering the period

March 1959-December 1986. The consumption data were taken from the CITIBASE tape. Consumption of nondurables and services was divided by total civilian population to obtain estimates of per capita consumption. The stock return series were taken from the CRSP monthly returns tape. Returns with and without dividends were collected for the 388 NYSE firms with continuous records on the CRSP file. These stocks were ranked by their market values as of March 1959 and placed into quintiles, with the smallest firms in quintile 1 and the largest firms in quintile 5. Fama's Treasury bill file on the CRSP bond tape was the source of the return on a one- and two-month Treasury bill. Where appropriate, returns were deflated using the Consumption Price Index implicit in the data on consumption of nondurables and services from CITIBASE. Table 1 provides descriptive statistics (means, medians, standard deviations, minima, maxima, and autocorrelation coefficients) on the monthly growth rate in real per capita consumption, the monthly real return on an equally weighted portfolio of the 388 NYSE stocks, the real return on a one-month Treasury bill and the monthly inflation rate.

B. Choice of Instrumental Variables

GMM directly estimates and tests moment conditions, such as Equation (11), for different instrumental variables, y_t. The particular instruments chosen should have power to reject the null hypothesis. Asymptotically, it does not matter how many instruments are chosen, but Tauchen [1986] has shown that there is a tradeoff between bias and the number of instruments in small samples. Consequently, instead of choosing any instrument in the investor's information set, we limit our attention to those instruments that can be shown to have power to reject the model. The procedure for choosing instruments is described below.

We first collected data on instruments that have traditionally been used to test consumption-based asset pricing models, namely: the unit vector, lagged consumption growth, c_t/c_{t-1}, and lagged stock (or T-bill) returns, $R_j(t,t-1)$. The results from estimating and testing the first-order conditions of the representative consumer in a world without taxes will be used as a benchmark. Therefore, we investigate the power of the aforementioned instruments to reject Equation (11) with $R_j(t+1,t)$ equal to one plus the real return on an equally weighted portfolio of the 388 NYSE firms, $1+r_p(t+1,t)$, and one plus the real return on a one-month T-bill, $1+r_f(t+1,t)$. Accordingly, the set of traditional instruments that we consider for estimating and testing the model are: the unit vector, c_{t-1}/c_t, and $1+r_p(t,t-1)$. The lagged real return on a one-month Treasury bill is not considered as an instrument.

To this list of traditional instruments, we added variables that are known to be good predictors of future stock returns and/or Treasury bill returns. As will be explained shortly, predictability of future returns is the most important determinant of the power of an instrument. We included as additional instruments: one plus the contemporaneous nominal return on a one-month T-bill, $1+\mu_f(t+1,t)$, and one plus the lagged nominal return on a two-month T-bill in excess of the lagged nominal return on a one-month T-bill, $1+\mu_f^2(t,t-1)-\mu_f(t,t-1)$. The former should be a particularly good predictor of the real return on a one-month T-bill and the latter has been shown to be a good predictor of future stock returns by Campbell [1987].

As a way of investigating the power of an instrument, we shall determine whether it provides some potentially conflicting information in the GMM estimator beyond that given by the moment conditions using the unit vector as an instrument (i.e., $y_t = 1$ for all t). It will later become clear what is meant when we write down and compare moment conditions, but, first, let us simplify them using a linear approximation of $c_{t+1}^{\gamma-1}$ about c_t, as in Singleton [1989]:

$$c_{t+1}^{\gamma-1} \cong c_t^{\gamma-1} + (\gamma-1)c_t^{\gamma-2}(c_{t+1} - c_t). \tag{16}$$

Using this linear approximation, the marginal rate of substitution of consumption tomorrow for consumption today becomes:

$$m(t+1,t) = \beta\left(\frac{c_{t+1}}{c_t}\right)^{\gamma-1} \cong \beta + \beta(\gamma-1)\left(\frac{c_{t+1}}{c_t}\right) - \beta(\gamma-1). \tag{17}$$

Substituting the above linearization for $m(t+1,t)$ into Equation (11) yields the following moment condition,

$$(2-\gamma)E[R_j(t+1,t)y_t] + (\gamma-1)E\left(\frac{c_{t+1}}{c_t}R_j(t+1,t)y_t\right) = \beta^{-1}E[y_t]. \tag{18}$$

When using the unit vector as an instrument, Equation (18) becomes:

$$(2-\gamma)E[R_j(t+1,t)] + (\gamma-1)E\left(\frac{c_{t+1}}{c_t}R_j(t+1,t)\right) = \beta^{-1}. \tag{19}$$

Comparing Equations (18) and (19), it is clear that an instrument will not add any restriction beyond the one provided by the unit vector if it is uncorrelated with both $R_j(t+1,t)$ and $[c_{t+1}/c_t]R_j(t+1,t)$. Hence, for an instrument to be powerful (in the sense of being able to add potentially conflicting information not present in the moment condition with the unit vector as the instrument), it must be correlated with either variable. In other words, an instrument is powerful if either

$$\text{cov}\left(R_j(t+1,t),\ y_t\right) \tag{20a}$$

or

$$\text{cov}\left(\frac{c_{t+1}}{c_t}R_j(t+1,t),\ y_t\right) \tag{20b}$$

differ substantially from zero in absolute value. Thus, we shall use these covariances, scaled by one over the standard deviation of $R_j(t+1,t)$ times the standard deviation of y_t, as a measure of the power of an instrument.[6] Scaling the first covariance by $\mathrm{std}(R_j(t+1,t))\mathrm{std}(y_t)$ yields the correlation between $R_j(t+1,t)$ and y_t. In this respect, an instrument has power if it is a good predictor of future returns, $R_j(t+1,t)$.

It should be emphasized that our procedure to select instruments suffers from data mining biases. They are, however, different from the ones investigated in Lo and MacKinlay [1990]. Instead of using cross-sectional information in cross-sectional tests, we use time-series information in order to generate instruments to be used to test cross-sectional restrictions (in particular, across stock and Treasury bill returns). The data mining biases we introduce are to offset the small sample biases of GMM when instruments are picked at random. However, the extent to which this procedure mitigates the small sample biases remains an open issue.

Table 2 reports both measures of power for the proposed instruments. It is clear that $1+r_p(t,t-1)$ is not a very powerful instrument since it has low correlation with both future stock and Treasury bill returns and consumption. In contrast, c_t/c_{t-1} is a powerful instrument because it is highly (negatively) correlated with $[c_{t+1}/c_t][1+r_f(t+1,t)]$ (see Panel B). Thus, lagged consumption growth will be a good instrument for the moment condition involving the real return on a one-month Treasury bill. Likewise, $1+\mu_f(t+1,t)$ will also be a good instrument for this moment condition since it is highly correlated with $1+r_f(t+1,t)$ (see Panel A). The variable $1+\mu_f^2(t,t-1)-\mu_f(t,t-1)$, on the other hand, will be a good instrument for the moment conditions involving the real return on the stock portfolio because the corresponding measures of power are relatively high (see Panels A and B). This analysis suggests that we should use the following instruments to

estimate and test our model: the unit vector, c_t/c_{t-1}, $1+\mu_f(t+1,t)$ and $1+\mu_f^2(t,t-1)-\mu_f(t,t-1)$. In contrast to traditional tests of the consumption-based asset pricing model, we do not use as instruments the lagged real returns on stocks or Treasury bills.[7]

C. Parameter Estimates and Test Results

We ran three sets of joint tests. The first set of tests involve Euler equations for the one-month Treasury bill and an equally weighted portfolio of the 388 NYSE stocks. The second set of tests involve Euler equations for the one-month Treasury bill and an equally weighted portfolio of the 80 largest stocks (quintile 5) ranked by market value as of March 1959. The third set of tests involve Euler equations for the one-month Treasury bill and an equally weighted portfolio of the 77 smallest stocks (quintile 1) ranked by market value as of March 1959. The last two sets of tests are used as a check for robustness.[8]

As a benchmark, we tested the first-order conditions for a representative consumer in a tax-free world. In other words, we jointly tested the following stochastic Euler equations:

$$0 = E_t\left\{m(t+1,t)\left(1 + \frac{1}{J}\sum_{j=1}^{J} r_j(t+1,t)\right) - 1\right\} \tag{21}$$

and

$$0 = E_t\left\{m(t+1,t)[1 + r_f(t+1,t)] - 1\right\} \tag{22}$$

where J is the number of stocks in the stock portfolio, $r_j(t+1,t)$ is the real return on stock j ($j=1,\dots,J$) from date t to date t+1, $r_f(t+1,t)$ is the real return on a one-month Treasury bill from date t to date t+1, and $m(t+1,t)$ is the marginal rate of substitution given by Equation (13). This particular version of the model is the subject of the Hansen and Singleton [1982, 1984] studies.

Four instruments are used to estimate the parameters of the model. Hence, there are eight orthogonality conditions but only two parameters (β and γ) to be estimated. This leaves us with six overidentifying restrictions (degrees of freedom). The first column of Panels A, B and C of Table 3 report the estimation results.[9] As in Hansen and Singleton [1982, 1984], the model is rejected (at the 1 percent level in Panels A and C and at the 2 percent level in Panel B). The estimate of γ is above 1.0 in Panels A and B and below 1.0 in Panel C, although the standard errors are large. A value of γ above 1.0 indicates that the representative consumer is risk loving, while a value of γ below 1.0 indicates that the representative consumer is risk averse. Notice also the high negative correlation between the estimates of β and γ in all three panels.

We next estimate the Euler equations for the equally weighted stock index and the one-month Treasury bill for a representative consumer in a world where dividends, interest and capital gains and losses are taxed every month. Although this model ignores the tax timing option available to investors, it will provide us with a second useful benchmark against which our model can be compared. Dividends and interest are assumed to be taxed at a rate of τ_d and capital gains and losses are assumed to be taxed at a rate of τ_c. Complications arise in a world with taxes, however, since investors are taxed on nominal quantities, yet will deflate their after-tax returns to determine their optimal consumption plans. Consequently, we calculate tax payments and rebates on nominal dividends, interest and capital gains and losses before deflating these payoffs. Let π_t denote the consumer price index at date t. We jointly test the following stochastic Euler equations:

$$0 = E_t\left\{m(t+1,t)[\pi_t/\pi_{t+1}]\ \frac{1}{J}\sum_{j=1}^{J}\left(1 + \mu_j^d(t+1,t)(1-\tau_d) + \mu_j^c(t+1,t)(1-\tau_c)\right) - 1\right\} \tag{23}$$

and

$$0 = E_t\left(m(t+1,t)[\pi_t/\pi_{t+1}][1 + (1-\tau_d)\mu_f(t+1,t)] - 1\right). \tag{24}$$

where $\mu_j^d(t+1,t)$ is the nominal dividend yield on stock j from date t to date t+1, $\mu_j^c(t+1,t)$ is the nominal capital gain or loss on stock j from date t to date t+1 and $\mu_f(t+1,t)$ is the nominal return on the one-month Treasury bill from date t to date t+1.

We use the same four instruments to test Equations (23) and (24) that were used to test the no-tax model. Since there are eight orthogonality conditions and four parameters to be estimated (β, γ, τ_d and τ_c), we are left with four overidentifying restrictions (degrees of freedom). The estimation results are reported in the second column of Panels A, B and C of Table 3.[10] The estimates of the risk aversion parameter, γ, are now below 1.0 in all three panels (although not significantly so) and have lower standard errors than those in the no-tax model. The estimates of the dividend tax rate, τ_d, appear reasonable (in the neighborhood of 41-44 percent) and are significantly different from zero. The estimates of the capital gains tax parameter, τ_c, are significantly different from zero but appear too high (ranging from 72.9 percent to 97.3 percent). Finally, notice that the high negative correlation between $\hat{\beta}$ and $\hat{\gamma}$ in the no-tax model (see column 1) has now declined somewhat and that the correlation between $\hat{\beta}$ and $\hat{\tau}_d$ is highly positive.

Model 2 is not rejected, but in the case of Panel A, and to a lesser extent Panels B and C, the model can be rejected on economic grounds. When capital gains and losses are taxed every period, a capital gains tax rate close to 1.0 (as in Panel A) eliminates nearly the entire return that is due to price appreciation (or depreciation). This reduces both the average equity risk premium on an after-tax basis and the variability and predictability of the after-tax equity return. The former makes the moment conditions with the unit vector as instrument fit better for an estimate of γ close to 1.0. The latter has two effects. First, it makes the other moment conditions involving the equity return fit better. Second, it leads to relatively lower weights on the moment conditions involving the Treasury bill return, which is known to be more predictable. Overall, a higher goodness-of-fit (i.e., a lower χ^2 statistic) results.[11]

We next estimate the Euler equations for the equally weighted stock index and the one-month Treasury bill assuming that investors have the option to optimally time the realization of their capital gains and losses. The Euler equations for the one-month Treasury bill are again given by Equation (24). The Euler equations for the equally weighted stock index are given by:

$$0 = E_t\left\{ m(t+1,t)[\pi_t/\pi_{t+1}]\ \frac{1}{J}\sum_{j=1}^{J}\Big(1 + \mu_j^d(t+1,t)(1-\tau_d) + \mu_j^c(t+1,t) - \tau_c\mu_j^z(t+1,t) - \tau_c\min[0,\ \mu_j^c(t+1,t)]\Big) - 1\right\}, \tag{25}$$

As Equation (25) indicates, in estimating the Euler equations for the stock index we assume that capital gains and losses are computed separately for each individual stock in the index, as opposed to computing capital gains and losses for the index as a whole. This maximizes the value of the investor's tax options for essentially the same reason that a portfolio of options is more valuable than an option on a portfolio. Consequently, the after-tax rate of return on the index may include tax rebates on capital losses even though the before-tax rate of return on the index itself is positive.

Before the above model can be tested, there is one more complication that

must be resolved. Notice that Equation (25) involves the nominal quantity $\tau_c\mu_j^z(t+1,t)$, which is the differential rate of capital appreciation between the market price of the security and the investor's personal valuation of his position in the security with a basis of $P_j(t)$. This differential return captures the value of the higher tax rebates on future capital losses (beyond date t+1) that are available to the investor when his basis is $P_j(t+1)$ rather than $P_j(t)$. The value of $\mu_j^z(t+1,t)$ is given by:

$$\mu_j^z(t+1,t) = \tag{26}$$

$$E_{t+1}\left\{\sum_{s=t+2}^{T} m(s,t+1)[\pi_{t+1}/\pi_s]\Big(\max\big[0,\ \min[\mu_j^c(t+1,t),\ldots,\mu_j^c(s-1,t)] - \mu_j^c(s,t)\big] - \max\big[0,\ \min[0,\mu_j^c(t+1,t),\ldots,\mu_j^c(s-1,t)] - \mu_j^c(s,t)\big]\Big)\right\},$$

where $\mu_j^c(s,t)$ is the nominal capital gain return on security j from date t to date s. If security j suffers a capital loss between dates t and t+1 (i.e., $\mu_j^c(t+1,t) \le 0$), the value of $\mu_j^z(t+1,t) = 0$ by Equation (26). However, if security j experiences a capital gain between dates t and t+1 (i.e., $\mu_j^c(t+1,t) > 0$), the value of $\mu_j^z(t+1,t)$ is positive and it is then necessary to know the investor's horizon date T to estimate its value. Intuitively, however, as the capital gain return between dates t and t+1 increases, the value of $\mu_j^z(t+1,t)$ increases at a decreasing rate. Consequently, $\mu_j^z(t+1,t)$ is an increasing, concave function of $\mu_j^c(t+1,t)$.[12]

We first consider a simplified version of the model by assuming that returns, inflation and consumption growth are independently and identically distributed random variables. In this case, the value of $\mu_j^z(t+1,t)$ is solely a function of $\mu_j^c(t+1,t)$. We approximate this function by

$$\mu_j^z(t+1,t) = \Big(1 + \max[0,\ \mu_j^c(t+1,t)]\Big)^{1/2} - 1. \tag{27}$$

Equation (27) captures the essential features of the relationship between $\mu_j^z(t+1,t)$ and $\mu_j^c(t+1,t)$, including concavity and the fact that $\mu_j^z(t+1,t) = 0$ whenever $\mu_j^c(t+1,t) \le 0$. We jointly estimate Equations (24) and (25) using Equation (27) to approximate $\mu_j^z(t+1,t)$. Since there are eight orthogonality conditions and four parameters to be estimated (β, γ, τ_d and τ_c), we are left with four overidentifying restrictions (degrees of freedom). The results are reported in the third column of Panels A, B and C of Table 3.[13]

As is the case with models 1 and 2, the estimates of the risk aversion parameter, γ, are not significantly different from 1.0. The estimates of the dividend tax rate, τ_d, are again plausible (in the neighborhood of 36-40 percent) and are significantly different from zero. The estimates of the capital gains tax rate, τ_c, are lower than those for model 2, but the standard errors are large and the estimates are not significantly different from zero. Although the model is rejected (at about the one percent level in Panels A and C and at the five percent level in Panel B), the parameters are tightly estimated (with the exception of the capital gains tax rate) and economically plausible. Finally, notice that the high correlations are still present between some of the parameter estimates.

We subsequently dropped the assumption that returns, inflation and consumption growth are independently and identically distributed random variables and estimated our model without the approximation given in Equation (27). Instead, we substituted Equation (26) directly into Equation (25) and applied the law of iterated expectations to produce the following stochastic Euler equation for the equity portfolios:

$$
\begin{aligned}
0 = E_t\Big\{ & m(t+1,t)[\pi_t/\pi_{t+1}]\ \frac{1}{J}\sum_{j=1}^{J}\Big(1 + \mu_j^d(t+1,t)(1-\tau) + \mu_j^c(t+1,t) \\
& - \tau_c \sum_{s=t+2}^{T} m(s,t+1)[\pi_{t+1}/\pi_s]\Big(\max\big[0,\ \min[\mu_j^c(t+1,t),\ldots,\mu_j^c(s-1,t)] - \mu_j^c(s,t)\big] \\
& \quad - \max\big[0,\ \min[0,\mu_j^c(t+1,t),\ldots,\mu_j^c(s-1,t)] - \mu_j^c(s,t)\big]\Big) \\
& - \tau_c \min[0,\ \mu_j^c(t+1,t)]\Big) - 1\Big\}.
\end{aligned}
\tag{28}
$$

These moment conditions, together with the ones involving the Treasury bill (see Equation (24)), can be estimated directly provided the representative consumer's horizon date T is fixed. We report the results for T = t+13. Since the random variables in Equation (28) now overlap in time, we adjusted the GMM weighting matrix accordingly. For instance, setting T = t+13 creates a 12th order moving average process. To adjust the weighting matrix, the procedure described in Newey and West [1987] was employed. We tried several lag lengths in the Newey-West procedure, but it did not alter the results very much. Whereas the overlapping variables theoretically generate a moving average, very little of it can be picked up in the data. Consequently, we set the Newey-West lag length equal to 12.[14]

The results of estimating Equations (24) and (28) are reported in column 4 of Panels A, B and C of Table 3. Since there are four parameters to be estimated (β, γ, τ_d and τ_c) from eight orthogonality conditions, we are left with four overidentifying restrictions (degrees of freedom). The model is not rejected, but some of the parameter estimates are disappointing.[15] The estimates of the dividend and capital gains tax parameters are negative in some cases, although the standard errors are large. The estimate of the dividend tax rate is positive and significant in Panel A, but insignificant in Panels B and C. The estimate of the capital gains tax rate is insignificant in all three panels. The estimates of the risk aversion parameter, γ, are below 1.0 (although not significantly so). Notice also that the high correlations between some of the parameter estimates are still present.

The disappointing parameter estimates for model 4 cannot be attributed to the choice of the investor's horizon date T. As mentioned earlier, a change in T did not materially affect the results. In search of an explanation, we considered the possibility that the tax environment we assumed is incorrect. Specifically, in contrast to the actual tax code in existence over the sample period, our model does not distinguish between long- and short-term capital gains and losses. In reality, long-term capital gains and losses were taxed at lower rates than short-term capital gains and losses over the time period covered by our sample. Once the distinction between long- and short-term capital gains and losses is made, it may be optimal for investors to realize some of their smaller long-term capital gains to reset their tax bases and restart the option to realize potential future losses short term. In our model, however, it is suboptimal for investors to realize any capital gains. Perhaps our representative agent is realizing too many losses and too few gains relative to investors in the real world.

Unfortunately, our model becomes analytically intractable once a distinction is made between the long- and short-term tax rates. The difficulty arises in solving simultaneously for the optimal realization policies and the equilibrium stock price process. Intuitively, the decision to realize long-term capital gains in order to reestablish short-term status will depend upon the size of the gain and the volatility of the stock. The optimal capital gain cutoff level, however, is difficult to find analytically. Constantinides [1984] has solved for the optimal realization policy for a world in which annual stock price changes follow an exogenous binomial process and investors are allowed to trade only once per year. With the long-term tax

rate equal to 40 percent of the short-term tax rate, Constantinides [1984] shows that for high and medium variance stocks it is optimal to realize all long-term capital gains each year. With trading and stock price changes occurring more frequently than once per year (e.g., monthly), Dammon and Spatt [1990] solve analytically for the optimal long-term realization policy in the presence of differential long- and short-term tax rates. They find that there exists an optimal cutoff level, which depends upon the parameters of the stock price process and the level of transaction costs, above which all long-term capital gains are deferred and below which all long-term capital gains are realized.

In order to determine whether the absence of capital gains realizations is the source of the disappointing results reported earlier, we reestimated Equations (24) and (28) assuming that the representative agent is forced to realize all embedded capital gains at the horizon date T. We assume that these embedded capital gains are taxed at 40 percent of the capital gains tax rate, τ_c, and discount the resulting taxes using the marginal rate of substitution between dates t and T, m(t,T). The results are reported in column 5 of Panels A, B and C of Table 3. The model is not rejected and the parameters are tightly estimated. The estimates of γ are again less than 1.0 (although not significantly so) and the estimates of the dividend tax rate, τ_d, are significant and economically plausible (in the neighborhood of 38-39 percent). The estimates of the capital gains tax rate, τ_c, while low (ranging from 5.4 percent to 8.7 percent), are now significantly above zero. Notice also that the high correlation between $\hat{\beta}$ and $\hat{\tau}_d$ is again present. These results clearly indicate that forcing the representative agent to realize capital gains at the horizon date T improves the estimation results.

We conclude from these results that taxes are important for determining asset returns and improve the fit of the consumption-based asset pricing model. In intuitive terms, after-tax consumption betas, as opposed to before-tax consumption betas, determine the required rates of return on financial assets. After-tax betas should account for changes in the value of the option to optimally time the realization of capital gains and losses. Our results further indicate that it may be important to allow for differential long- and short-term tax rates so that investors find it optimal to realize capital gains more frequently. When investors are forced to realize capital gains and losses every month (see model 2), the estimate of the capital gains tax rate is too high, and when investors optimally defer their capital gains until the horizon date, and realize only capital losses before the horizon date (see model 5), the estimate of the capital gains tax rate is too low. This suggests that a model in which it is optimal for investors to realize some of their smaller long-term capital gains prior to the horizon date to reestablish short-term status may further improve the fit of the consumption-based asset pricing model.[16]

One interesting feature of the results in Table 3 is that the estimates of γ are not significantly different from 1.0 in any of the models. This suggested to us a potentially interesting test: fix $\gamma = 1.0$ and reestimate the models to see whether the results are sensitive to this restriction. The reason this test is interesting is that it can be informative about the time series properties of equity returns. When $\gamma = 1$, the representative consumer is risk neutral and his marginal rate of substitution for consumption between dates t and t+1, $m(t+1,t)$, is equal to his subjective discount factor, β. In this case, the stochastic Euler equation for any asset j is:

$$E_t\Big(\beta R_j(t+1,t) - 1\Big) = E_t\Big(R_j(t+1,t) - \beta^{-1}\Big) = 0 \tag{29}$$

where $R_j(t+1,t)$ is the "return" on security j from date t to date t+1. Equation (29) requires the conditional (and unconditional) expectation of the

"return" on security j, $R_j(t+1,t)$, to equal a constant, β^{-1}. Equation (29) also requires the "excess return" on security j, $[R_j(t+1,t) - \beta^{-1}]$, to be orthogonal to all elements in the investor's information set at date t. That is,

$$E\Big([R_j(t+1,t) - \beta^{-1}]y_t\Big) = 0. \tag{30}$$

Equation (30) restricts the "excess return" on all securities to be uncorrelated with all elements in the investor's information set.

To explore these restrictions, we reestimated models 1-5 with γ set equal to 1.0. The results are reported in Panels A, B and C of Table 4. The tax-free model is rejected at extremely high significance levels. This confirms the empirical finding that expected <u>before-tax</u> returns change predictably over time. The results for models 2-5, however, suggest a different story for the <u>after-tax</u> returns. Notice that the results for models 2-5 are essentially the same as those reported in Table 3, except that rejections occur less frequently in Table 4. The failure to reject some of the models suggests that the <u>after-tax</u> returns are not predictable. In other words, the evidence suggests that <u>after-tax</u> returns, unlike <u>before-tax</u> returns, follow a martingale process.

As discussed earlier, this martingale result has little economic content in the case of model 2. Similarly, because model 3 is rejected in Table 4, the after-tax returns given by this model do not exhibit the martingale property. In models 4 and 5, however, the martingale property has some economic content. Both models postpone capital gain realizations (indefinitely in model 4 and until the horizon date T in model 5) and realize all future capital losses as soon as they occur. Consequently, the instruments in the GMM estimation are asked to predict capital gains and losses, not one month in the future, but <u>several</u> months in the future. While the instruments are good at predicting equity returns one month in the future, they fail to predict monthly capital gains and losses beyond one month. Thus, the after-tax equity returns in models 4 and 5 appear to be martingales, despite the fact that their before-tax counterparts are predictable. The failure to reject the martingale hypothesis in models 4 and 5 occurs even though Treasury bill returns, both before and after tax, are thought to be more predictable than before-tax equity returns. However, our results are obtained with a multivariate statistic that tests whether after-tax equity and Treasury bill returns are jointly a martingale.

We interpret the results of our empirical tests as an indication that the data favors an after-tax martingale model over a before-tax consumption-based model. The role of consumption in previous tests of the consumption-based asset pricing model seems to have been to merely provide noise that dampens the predictability of before-tax returns. This interpretation is supported by the higher estimate of the coefficient of relative risk aversion in the before-tax model when restricting attention to the equity returns of small firms, which are known to be more predictable than equity returns of large firms (see Table 3, model 1).

V. Summary

In this paper, we tested tax-induced intertemporal restrictions on asset returns using the Generalized Method of Moments (GMM) procedure. The model explicitly considers the fact that investors are taxed on capital gains and losses only when the asset is sold. We found reliable evidence of capital gains tax effects on the pricing of common stock. The tax-adjusted asset pricing model developed in this paper was not rejected and provided reasonable estimates for the risk aversion parameter (consistently in the concave area) and the dividend tax rate (consistently in the neighborhood of 35-40 percent).

Unfortunately, our model was not able to reliably estimate the capital gains tax parameter, presumably because our model does not accomodate differential long- and short-term tax rates. We were unable to reject the hypothesis that the representative consumer is risk neutral and that after-tax returns, unlike before-tax returns, are a martingale.

In our model, investors optimally defer the recognition of all capital gains. Our empirical results, however, indicate that capital gains realizations may be important for determining asset returns. This is not surprising in a world where long-term capital gains are taxed at a lower rate than short-term capital gains. Occasional long-term capital gains realization becomes optimal in order to enjoy higher tax rebates on subsequent short-term capital losses. The extension of our model to differential long- and short-term tax rates may prove fruitful for estimating the capital gains tax rate more precisely.

Footnotes

1 See, for example, Constantinides [1984], Dammon, Dunn and Spatt [1989], and Dammon and Spatt [1991] for a discussion of the optimal tax trading strategies for the case in which the long- and short-term tax rates are different.

2 Under the optimal realization policy, the investor sells the asset only when he has a loss. Consequently, at date $s > t$, the investor's tax basis will be $\min[P_j(t),\ldots,P_j(s-1)]$. This would seem to require wash sales in which the investor sells and immediately repurchases the security to reestablish his tax basis. However, the price at which a <u>new</u> buyer is willing to purchase the security will reflect the same <u>future</u> after-tax dividends and tax rebates as those that the seller would experience in a wash sale. Consequently, even if wash sales are prohibited, the price can still be written as in Equation (1) provided there exists a competitive market for the asset.

3 Nevertheless, the overall level of basis values in the economy can be important in determining aggregate consumption and, therefore, the general level of securities prices.

4 In general, the investor's personal valuation of a position in one share of security j at date t, $t = 0,\ldots,T-1$, that was initially established at date $k \le t$, $v_j(t,k)$, can be written as

$$v_j(t,k) = P_j(t) - z_j(t,k)$$

$$= E_t\left\{\sum_{s=t+1}^{T} m(s,t)x_j(s,k)\right\} \qquad \text{(from Equation (3))}$$

where $z_j(t,k)$ conforms to Equation (6). Obviously, the higher the capital gain $P_j(t) - P_j(k)$, the higher is the value of $z_j(t,k)$ and, therefore, the larger is the difference between the current price of the security, $P_j(t)$, and the investor's personal valuation, $v_j(t,k)$.

5 In estimating the model, GQOPT's Davidson-Fletcher-Powell algorithm was used to search for the optimum. The criterion to decide whether an optimum was reached was solely based on the length of the gradient. The time preference parameter, β, was transformed to $\beta' = \exp\{10\beta\}$ in the optimization. This transformation improved convergence speed markedly. Analytical derivatives were used in the calculations of the gradients and the standard errors. In addition to improving the convergence over numerical derivatives, analytical derivatives often lead to smaller standard errors.

6 The scaled measures of power are obtained after dividing both Eqs. (18) and (19) by $\mathrm{std}[R_j(t+1,t)]\mathrm{std}[y_t]$. This scaling should proxy for the weighting of the moment conditions in the GMM estimation by the inverse of the asymptotic variance-covariance matrix of $T^{1/2}$ times the sample moment conditions.

7 In an earlier version of this paper, we reported the results of GMM estimation using only the traditional instruments. Specifically, we used the unit vector, the consumption growth rate lagged once and twice, and the real return on the stock portfolio lagged once and twice. These instruments yielded unacceptably high coefficients of relative risk aversion, $1-\gamma$, in the joint estimation of the Euler equations on the stock portfolio and the one-month Treasury bill in the absence of taxes. The anomaly disappeared only after iterating a number of times on the weighting matrix. When estimating the model with taxes, these instruments yielded implausibly high and imprecise estimates of the tax parameters. In the results reported below, high coefficients of relative risk aversion disappear after the second iteration and the estimates of the tax parameters are reasonable and more precisely estimated.

8 We also ran similar tests for the other three intermediate quintiles of stocks, but do not report these results in the paper because they are qualitatively similar to those that are reported.

[9] While qualitatively the same, the point estimates in the first column of Panel A of Table 3 do differ from the ones found in Hansen and Singleton [1982, 1984]. The differences are due to:

i) Differences in the instuments used to estimate and test the model.

ii) A longer time series is used in the present paper (extending beyond 1978).

iii) The CITIBASE and CRSP pre-1978 data have been updated substantially.

iv) We use a different population measure. Instead of the total population, we look only at total civilian population, which excludes the military. A great deal of the consumption of military personnel comes out of the defense budget, which is excluded from the consumption series reported by CITIBASE.

v) We match consumption over a given month by the end-of-month population, not by the population at the beginning of the month. The matching of consumption and returns, on the other hand, does not differ from that in Hansen and Singleton [1982, 1984]. Consumption over the month of January, for example, is matched with the January stock return.

[10] The estimates reported in Table 3 for models 2-5 are sensitive to the starting value for β, the time preference parameter. Two optima emerged: one with β below 1.0 and the other with β above 1.0. The criterion function was generally lower for the latter. Consequently, only the optima with β above 1.0 are reported in Table 3. The results for the optima with β below 1.0 are different in one important respect: the dividend tax rate, τ_d, is estimated with high standard error. The point estimate often turned out to be (insignificantly) negative. The difference in point estimates from those reported in Table 3 is to be expected, however, since the estimates of β and τ_d are highly positively correlated in all the models.

[11] Predictably, model 2 was rejected when the tax parameters were fixed at $\tau_d = .50$ and $\tau_c = .20$. At these tax rates, model 2 differs very little from model 1.

[12] $\mu_j^z(t+1,t)$ is proportional to the difference in the value of two put options, one with an exercise price of $P_j(t+1)$ and the other with an exercise price of $P_j(t)$. Since $P_j(t) = P_j(t+1)/(1+\mu_j^c(t+1))$ is convex in $\mu_j^c(t+1)$ and the value of a put is convex in its exercise price, $\mu_j^z(t+1,t)$ is concave in $\mu_j^c(t+1)$.

[13] We also estimated the model assuming that $\mu_j^z(t+1,t)$ is given by:

$$\mu_j^z(t+1,t) = \delta\left\{\left(1 + \max[0,\ \mu_j^c(t+1,t)]\right)^{1/2} - 1\right\}$$

where δ is parameter to be estimated. The results, however, are qualitatively indistinguishable from those for model 2, in which capital gains and losses are taxed every period. The reason these two models provide similar results stems from the fact that the above expression for $\mu_j^z(t+1,t)$ is approximately equal to:

$$\mu_j^z(t+1,t) \cong \delta\max[0,\ \mu_j^c(t+1,t)]/2$$

when $\mu_j^c(t+1,t)$ is small (as is typical with monthly returns). The point estimate of δ is close to 2.0, which reduces the above expression to:

$$\mu_j^z(t+1,t) \cong \max[0,\ \mu_j^c(t+1,t)].$$

If we then substitute this approximation for $\mu_j^z(t+1,t)$ into the Euler equation for the stock index (i.e., Equation (25)), we find that it collapses to the corresponding Euler equation for model 2 (i.e., Equation (23)). This explains the similarities in the results of the two models when δ is treated as an unknown parameter. We also allowed the exponent appearing in the expression for $\mu_j^z(t+1,t)$ to differ from 1/2, but this did not improve the fit of the model.

[14] We also ran the estimation for T = t+19 and T = t+31, and estimated the weighting matrix without Newey-West dampening, but the results were virtually unchanged.

[15] Model 4 was rejected when we fixed the tax parameters at $\tau_d = .50$ and $\tau_c = .20$. However, it is difficult to interpret this rejection without knowing whether these were the true tax rates that the representative consumer faced. We concluded that estimating the tax parameters would be more informative.

[16] This conclusion is underscored by the results we obtained when extending the horizon date in model 5 beyond 13 months to T = t+19 and T = t+31. While still significantly positive, the estimate of the capital gains tax rate declined as T increased, suggesting that capital gains realizations prior to date T may produce a more reasonable estimate of the capital gains tax rate.

Table 1

Descriptive statistics of selected variables, 3/59 to 12/86.[a]

	Mean	Median	St. Dev.	Minimum	Maximum	Autoc.
c_{t+1}/c_t	1.00168	1.00173	0.00442	0.98838	1.01704	-0.235
$r_p(t+1,t)$	0.00677	0.00739	0.04385	-0.14913	0.20018	0.105
$r_f(t+1,t)$	0.00078	0.00072	0.00282	-0.00751	0.01080	0.400
π_{t+1}/π_t	1.00412	1.00370	0.00310	0.99520	1.01372	0.534

[a] c_{t+1}/c_t = growth in real per capita consumption of nondurables and services over month t+1; $r_p(t+1,t)$ = real return over month t+1 on an equally weighted index of 388 NYSE stocks; $r_f(t+1,t)$ = real return over month t+1 on a one-month Treasury bill; and π_{t+1}/π_t = one plus the inflation rate over month t+1. Since 13 leads were used in the calculation of future tax rebates for some of the tax models of Table 3, we included only the first 321 observations in the computation of the descriptive statistics.

Table 2

Descriptive statistics of the power of various instruments, 3/59 to 12/86.[a]

Panel A: $\dfrac{\text{cov}(R_j(t+1,t), y_t)}{\text{sd}(R_j(t+1,t))\text{sd}(y_t)}$

y_t equals:	$R_j(t+1,t)$ equals:	
	$1+r_p(t+1,t)$	$1+r_f(t+1,t)$
c_t/c_{t-1}	0.019	-0.040
$1+r_p(t,t-1)$	0.105	0.014
$1+\mu_f(t+1,t)$	-0.083	0.322
$1+\mu_f^2(t,t-1)-\mu_f(t,t-1)$	0.239	0.114

Panel B: $\dfrac{\text{cov}\left(\dfrac{c_{t+1}}{c_t}R_j(t+1,t),\ y_t\right)}{\text{sd}(R_j(t+1,t))\text{sd}(y_t)}$

y_t equals:	$R_j(t+1,t)$ equals:	
	$1+r_p(t+1,t)$	$1+r_f(t+1,t)$
c_t/c_{t-1}	-0.005	-0.409
$1+r_p(t,t-1)$	0.108	0.051
$1+\mu_f(t+1,t)$	-0.096	0.121
$1+\mu_f^2(t,t-1)-\mu_f(t,t-1)$	0.236	0.069

[a] The entries in the table are measures of the power of various instruments to reject the first-order conditions of the optimal consumption and investment decisions of a representative consumer, where the variables y_t are candidate instruments. An instrument has power if at least one of the entries in the corresponding row is large in absolute value (see section V.B. for a detailed explanation). c_{t+1}/c_t = growth in real per capita consumption of nondurables and services over month t+1; $r_p(t+1,t)$ = real return over month t+1 on an equally weighted index of 388 NYSE stocks; $r_f(t+1,t)$ = real return over month t+1 on a one-month Treasury bill; $\mu_f(t+1,t)$ = nominal return over month t+1 on a one-month Treasury bill; $\mu_f^2(t,t-1)$ = nominal return over month t on a two-month Treasury bill; $\text{cov}(\cdot,\cdot)$ is the covariance and $\text{std}(\cdot)$ is the standard deviation. Since 13 leads were used in the calculation of future tax rebates for some of the tax models of Table 3, we included only the first 321 observations in the computation of the power measures.

Table 3

Estimation results using return data on indices of common stock and a Treasury bill over the period 3/59 - 12/86.[a]

Panel A: Results for an equally weighted index of 388 NYSE stocks and a one-month Treasury bill

	Asset Pricing Model[b]				
	1	2	3	4	5
Parameter estimates:					
$\hat{\beta}$	0.998	1.001	1.001	1.001	1.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\hat{\gamma}$	1.294	0.948	0.985	0.881	0.906
	(0.466)	(0.138)	(0.133)	(0.135)	(0.121)
$\hat{\tau}_c$		0.973	-0.124	-0.612	0.087
		(0.037)	(0.274)	(0.748)	(0.023)
$\hat{\tau}_d$		0.413	0.396	0.371	0.379
		(0.086)	(0.075)	(0.137)	(0.134)
Correlations[c]:					
$\hat{\beta},\hat{\gamma}$	-0.812	-0.757	-0.794	-0.474	-0.506
$\hat{\beta},\hat{\tau}_d$		0.856	0.865	0.863	0.870
$\hat{\tau}_c,\hat{\tau}_d$		0.143	-0.046	0.060	-0.060
$\hat{\tau}_c,\hat{\gamma}$		-0.050	0.063	0.184	0.105
Goodness-of-fit:					
χ^2 [d]	23.157	6.135	12.600	6.834	7.011
	(0.001)	(0.189)	(0.013)	(0.145)	(0.135)
dof[e]	6	4	4	4	4

Table 3 continued

Panel B: Results for an equally weighted index of the 80 largest stocks (quintile 5) and a one-month Treasury bill

	Asset Pricing Model[b]				
	1	2	3	4	5
Parameter estimates:					
$\hat{\beta}$	0.999	1.001	1.001	1.000	1.001
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
$\hat{\gamma}$	1.193	0.936	1.007	0.826	0.900
	(0.845)	(0.222)	(0.135)	(0.216)	(0.119)
$\hat{\tau}_c$		0.729	0.207	0.248	0.054
		(0.245)	(0.318)	(0.677)	(0.026)
$\hat{\tau}_d$		0.427	0.362	0.151	0.391
		(0.140)	(0.078)	(0.154)	(0.137)
Correlations[c]:					
$\hat{\beta},\hat{\gamma}$	-0.836	-0.667	-0.776	-0.602	-0.486
$\hat{\beta},\hat{\tau}_d$		0.837	0.861	0.828	0.872
$\hat{\tau}_c,\hat{\tau}_d$		0.310	-0.194	-0.093	0.433
$\hat{\tau}_c,\hat{\gamma}$		-0.118	0.147	0.271	-0.026
Goodness-of-fit:					
χ^2 [d]	15.108	2.981	9.742	5.879	6.765
	(0.019)	(0.561)	(0.045)	(0.208)	(0.149)
dof[e]	6	4	4	4	4

Table 3 continued

Panel C: Results for an equally weighted index of the 77 smallest stocks (quintile 1) and a one-month Treasury bill

	Asset Pricing Model[b]				
	1	2	3	4	5
Parameter estimates:					
$\hat{\beta}$	1.058 (0.051)	1.002 (0.001)	1.001 (0.001)	1.000 (0.001)	1.001 (0.001)
$\hat{\gamma}$	-33.056 (25.315)	0.931 (0.242)	0.991 (0.141)	0.819 (0.385)	0.926 (0.108)
$\hat{\tau}_c$		0.841 (0.115)	0.076 (0.413)	0.120 (1.001)	0.079 (0.019)
$\hat{\tau}_d$		0.435 (0.149)	0.375 (0.082)	-0.030 (0.213)	0.389 (0.125)
Correlations[c]:					
$\hat{\beta}, \hat{\gamma}$	-0.770	-0.677	-0.761	-0.685	-0.625
$\hat{\beta}, \hat{\tau}_d$		0.835	0.859	0.771	0.877
$\hat{\tau}_c, \hat{\tau}_d$		0.395	-0.102	0.371	0.135
$\hat{\tau}_c, \hat{\gamma}$		-0.123	0.069	0.321	0.111
Goodness-of-fit:					
χ^2 [d]	21.160 (0.002)	3.546 (0.471)	13.374 (0.010)	4.696 (0.320)	7.706 (0.103)
dof[e]	6	4	4	4	4

[a] Generalized Method of Moments estimation and testing results for the moment conditions representing various models using four instrumental variables (the unit vector, one plus lagged consumption growth, one plus the contemporaneous yield on a one-month Treasury bill and one plus the lagged nominal return on a two-month Treasury bill in excess of the lagged nominal return on a one-month Treasury bill). Standard errors are in parentheses under the parameter estimates. Probability values are in parentheses under the χ^2 values. Since 13 leads were used in the calculation of future tax rebates for Models 4 and 5, we included only the first 321 observations in the estimation of the other models.

[b] Model 1 is the tax-free consumption-based asset pricing model of Hansen and Singleton [1982]. The moment conditions are:

$$0 = E_t\left\{m(t+1,t)\left(1 + \frac{1}{J}\sum_{j=1}^{J} r_j(t+1,t)\right) - 1\right\}$$

and

$$0 = E_t\left(m(t+1,t)[1 + r_f(t+1,t)] - 1\right),$$

where J equals 388 (Panel A), 80 (Panel B), or 77 (Panel C), $r_j(t+1,t)$ is the real return on stock j from date t to date t+1, $r_f(t+1,t)$ is the real return on a one-month Treasury bill from date t to date t+1 and $m(t+1,t)$ is given by Equation (13).

Models 2 through 5 are after-tax models. The moment conditions for the Treasury bill are:

$$0 = E_t\left(m(t+1,t)[\pi_t/\pi_{t+1}][1 + (1-\tau_d)\mu_f(t+1,t)] - 1\right),$$

where π_t is the price index at time t and $\mu_f(t+1,t)$ is the nominal return on a one-month Treasury bill from date t to date t+1. In model 2, capital gains and losses are assumed to be taxed every month at the capital gains tax rate, τ_c. Therefore, the moment conditions for the stock indices in model 2 are:

$$0 = E_t\left\{m(t+1,t)[\pi_t/\pi_{t+1}]\ \frac{1}{J}\sum_{j=1}^{J}\left(1 + \mu_j^d(t+1,t)(1-\tau_d) + \mu_j^c(t+1,t)(1-\tau_c)\right) - 1\right\}$$

where $\mu_j^d(t+1,t)$ is the nominal dividend yield on asset j and $\mu_j^c(t+1,t)$ is the nominal capital gain return. In models 3-5, capital losses are realized each month and capital gains are deferred. Therefore, the moment conditions for the stock indices in models 3-5 are:

$$0 = E_t\left\{m(t+1,t)[\pi_t/\pi_{t+1}]\ \frac{1}{J}\sum_{j=1}^{J}\left(1 + \mu_j^d(t+1,t)(1-\tau_d) + \mu_j^c(t+1,t) - \tau_c\mu_j^z(t+1,t) - \tau_c\min[0,\ \mu_j^c(t+1,t)]\right) - 1\right\},$$

where $\mu_j^z(t+1,t) \geq 0$ is given by Equation (26). In model 3, we approximate $\mu_j^z(t+1,t)$ by

$$\mu_j^z(t+1,t) = \left(1 + \max[0,\ \mu_j^c(t+1,t)]\right)^{1/2} - 1.$$

In models 4 and 5, we estimate $\mu_j^z(t+1,t)$ directly by setting $T = t+13$ in Equation (26). In model 4, we assume that all unrealized capital gains at the horizon date $T = t+13$ are untaxed, whereas in model 5 we assume that all unrealized capital gains at the horizon date $T = t+13$ are taxed at 40 percent of the capital gains tax rate τ_c.

[c] Correlation between the parameter estimates.

[d] χ^2 value of the null hypothesis that the orthogonality conditions for each of the models are correct.

[e] Number of degrees of freedom (equal to the number of orthogonality conditions minus the number of parameters to be estimated).

Table 4

Estimation results when the coefficient of relative risk aversion is constrained to equal zero (γ = 1)[a]

Panel A: Results for an equally weighted index of 388 NYSE stocks and a one-month Treasury bill

	Asset Pricing Model[b]				
	1	2	3	4	5
Parameter estimates:					
$\hat{\beta}$	0.999 (<0.001)	1.001 (<0.001)	1.001 (<0.001)	1.001 (0.001)	1.001 (0.001)
$\hat{\gamma}$	1.000 (fixed)	1.000 (fixed)	1.000 (fixed)	1.000 (fixed)	1.000 (fixed)
$\hat{\tau}_c$		0.972 (0.035)	-0.122 (0.274)	-0.574 (0.760)	0.088 (0.023)
$\hat{\tau}_d$		0.405 (0.076)	0.395 (0.064)	0.419 (0.127)	0.412 (0.128)
Correlations[c]:					
$\hat{\beta}, \hat{\tau}_d$		0.916	0.891	0.873	0.882
$\hat{\tau}_c, \hat{\tau}_d$		0.127	-0.030	-0.092	-0.010
Goodness-of-fit:					
χ^2 [d]	47.316 (<0.001)	6.493 (0.261)	12.255 (0.031)	6.292 (0.279)	6.943 (0.225)
dof[e]	7	5	5	5	5

Table 4 continued

Panel B: Results for an equally weighted index of the 80 largest stocks (quintile 5) and a one-month Treasury bill

	Asset Pricing Model[b]				
	1	2	3	4	5
Parameter estimates:					
$\hat{\beta}$	0.999 (<0.001)	1.001 (<0.001)	1.001 (<0.001)	1.001 (0.001)	1.001 (0.001)
$\hat{\gamma}$	1.000 (fixed)	1.000 (fixed)	1.000 (fixed)	1.000 (fixed)	1.000 (fixed)
$\hat{\tau}_c$		0.700 (0.174)	0.187 (0.331)	-0.221 (0.918)	0.057 (0.026)
$\hat{\tau}_d$		0.403 (0.076)	0.374 (0.065)	0.344 (0.133)	0.422 (0.131)
Correlations[c]:					
$\hat{\beta},\hat{\tau}_d$		0.916	0.891	0.869	0.884
$\hat{\tau}_c,\hat{\tau}_d$		0.199	-0.144	-0.337	0.460
Goodness-of-fit:					
$\chi^{2\ d}$	30.805 (<0.001)	5.212 (0.391)	8.486 (0.131)	4.515 (0.478)	6.754 (0.240)
dof[e]	7	5	5	5	5

Table 4 continued

Panel C: Results for an equally weighted index of the 77 smallest stocks (quintile 1) and a one-month Treasury bill

	Asset Pricing Model[b]				
	1	2	3	4	5
Parameter estimates:					
$\hat{\beta}$	0.999 (<0.001)	1.001 (<0.001)	1.001 (<0.001)	1.001 (0.001)	1.001 (0.001)
$\hat{\gamma}$	1.000 (fixed)	1.000 (fixed)	1.000 (fixed)	1.000 (fixed)	1.000 (fixed)
$\hat{\tau}_c$		0.824 (0.077)	0.022 (0.430)	-0.308 (1.529)	0.081 (0.019)
$\hat{\tau}_d$		0.412 (0.075)	0.388 (0.065)	0.401 (0.121)	0.426 (0.122)
Correlations[c]:					
$\hat{\beta},\hat{\tau}_d$		0.917	0.892	0.874	0.784
$\hat{\tau}_c,\hat{\tau}_d$		0.232	-0.107	-0.124	0.199
Goodness-of-fit:					
$\chi^{2\ d}$	52.135 (<0.001)	6.347 (0.274)	11.742 (0.038)	3.956 (0.556)	7.743 (0.171)
dof[e]	7	5	5	5	5

[a] Generalized Method of Moments estimation and testing results for the moment conditions representing various models using four instrumental variables (the unit vector, one plus lagged consumption growth, one plus the contemporaneous yield on a one-month Treasury bill and one plus the lagged nominal return on a two-month Treasury bill in excess of the lagged nominal return on a one-month Treasury bill) when the coefficient of relative risk aversion is constrained to equal zero ($\gamma = 1$). Standard errors are in parentheses under the parameter estimates. Probability values are in parentheses under the χ^2 values. Since 13 leads were used in the calculation of future tax rebates for Models 4 and 5, we included only the first 321 observations in the estimation of the other models.

[b] Model 1 is the tax-free consumption-based asset pricing model of Hansen and Singleton [1982] with the additional constraint that $\gamma = 1$. The moment conditions are:

$$0 = E_t\left\{\beta\left(1 + \frac{1}{J}\sum_{j=1}^{J} r_j(t+1,t)\right) - 1\right\}$$

and

$$0 = E_t\left(\beta[1 + r_f(t+1,t)] - 1\right),$$

where J equals 388 (Panel A), 80 (Panel B), or 77 (Panel C), $r_j(t+1,t)$ is the real return on stock j from date t to date t+1, $r_f(t+1,t)$ is the real return on a one-month Treasury bill from date t to date t+1 and β is the time-preference parameter.

Models 2 through 5 are after-tax models. When the constaint $\gamma = 1$ is imposed, the moment conditions for the Treasury bill are:

$$0 = E_t\left(\beta[\pi_t/\pi_{t+1}][1 + (1-\tau_d)\mu_f(t+1,t)] - 1\right),$$

where π_t is the price index at time t and $\mu_f(t+1,t)$ is the nominal return on a one-month Treasury bill from date t to date t+1. In model 2, capital gains and losses are assumed to be taxed every month at the capital gains tax rate, τ_c. Therefore, setting $\gamma = 1$, the moment conditions for the stock indices in model 2 are:

$$0 = E_t\left\{\beta[\pi_t/\pi_{t+1}]\ \frac{1}{J}\sum_{j=1}^{J}\left(1 + \mu_j^d(t+1,t)(1-\tau_d) + \mu_j^c(t+1,t)(1-\tau_c)\right) - 1\right\}$$

where $\mu_j^d(t+1,t)$ is the nominal dividend yield on asset j and $\mu_j^c(t+1,t)$ is the nominal capital gain return. In models 3-5, capital losses are realized each month and capital gains are deferred. Therefore, setting $\gamma = 1$, the moment conditions for the stock indices in models 3-5 are:

$$0 = E_t\left\{\beta[\pi_t/\pi_{t+1}]\ \frac{1}{J}\sum_{j=1}^{J}\left(1 + \mu_j^d(t+1,t)(1-\tau_d) + \mu_j^c(t+1,t) - \tau_c\mu_j^z(t+1,t) - \tau_c\min[0,\ \mu_j^c(t+1,t)]\right) - 1\right\},$$

where $\mu_j^z(t+1,t) \geq 0$ is given by Equation (26). In model 3, we approximate $\mu_j^z(t+1,t)$ by

$$\mu_j^z(t+1,t) = \left(1 + \max[0,\ \mu_j^c(t+1,t)]\right)^{1/2} - 1.$$

In models 4 and 5, we estimate $\mu_j^z(t+1,t)$ directly by setting T = t+13 in Equation (26). In model 4, we assume that all unrealized capital gains at the horizon date T = t+13 are untaxed, whereas in model 5 we assume that all unrealized capital gains at the horizon date T = t+13 are taxed at 40 percent of the capital gains tax rate τ_c.

[c] Correlation between the parameter estimates.

[d] χ^2 value of the null hypothesis that the orthogonality conditions for each of the models are correct.

[e] Number of degrees of freedom (equal to the number of orthogonality conditions minus the number of parameters to be estimated).

References

Black, F. and M. Scholes, 1974, "The Effects of Dividend Yield and Dividend Policy on Common Stock Prices and Returns", Journal of Financial Economics, 1, 1-22.

Breeden, D., 1979, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities", Journal of Financial Economics, 7, 265-296.

Brennan, M., 1973, "Taxes, Market Valuation and Corporate Financial Policy", National Tax Journal, 23, 417-427.

Campbell, J., 1987, "Stock Returns and the Term Structure", Journal of Financial Economics, 18, 373-400.

Constantinides, G., 1983, "Capital Market Equilibrium with Personal Tax", Econometrica, 51, 611-636.

Constantinides, G., 1984, "Optimal Stock Trading with Personal Taxes: Implications for Prices and the Abnormal January Returns", Journal of Financial Economics, 13, 65-89.

Constantinides, G. and J. Ingersoll, 1983, "Optimal Bond Trading with Personal Taxes", Journal of Financial Economics, 13, 299-335.

Constantinides, G. and M. Scholes, 1980, "Optimal Liquidation of Assets in the Presence of Personal Taxes: Implications for Asset Pricing", Journal of Finance, 35, 439-449.

Dammon, R., K. Dunn and C. Spatt, 1989, "A Reexamination of the Value of Tax Options", Review of Financial Studies, 2, 341-372.

Dammon, R. and C. Spatt, 1991, "Optimal Trading Policies and the Pricing of Securities with Personal Taxes", manuscript, Carnegie Mellon University.

Dunn, K. and K. Singleton, 1986, "Modeling the Term Structure of Interest Rates under Non-Separable Utility and Durability of Goods", Journal of Financial Economics, 17, 27-56.

Epstein, L. and S. Zin, 1991, "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis", Journal of Political Economy, 99, 263-286.

Ferson, W., 1983, "Expectations of Real Interest Rates and Aggregate Consumption: Empirical Tests", Journal of Financial and Quantitative Analysis, 18, 477-497.

Ferson, W. and G. Constantinides, 1989, "Habit Persistence and Durability in Aggregate Consumption: Empirical Tests", manuscript, University of Chicago.

Grossman, S., A. Melino and R. Shiller, 1987, "Estimating the Continuous Time Consumption Based Asset Pricing Model", Journal of Business and Economic Statistics, 5, 315-327.

Grossman, S. and R. Shiller, 1981, "The Determinants of the Variability of Stock Market Prices", American Economic Review, 71, 222-227.

Hansen, L., 1982, "Large Sample Properties of Generalized Method of Moment Estimators", Econometrica, 50, 1029-1054.

Hansen, L. and K. Singleton, 1982, "Generalized Instrumental Variables Estimation of Nonlinear Behavior of Asset Returns", Econometrica, 50, 1269-1286.

Hansen, L. and K. Singleton, 1983, "Stochastic Consumption, Risk Aversion and the Temporal Behavior of Asset Returns", Journal of Political Economy, 91, 249-265.

Hansen, L. and K. Singleton, 1984, "Addendum", Econometrica, 52, 267-268.

Heaton, J., 1989, "The Interaction Between Time-Nonseparable Preferences and Time Aggregation", manuscript, University of Chicago.

Litzenberger, R. and K. Ramaswamy, 1979, "The Effect of Personal Taxes and Dividends on Capital Asset Prices", Journal of Financial Economics, 7, 163-195.

Litzenberger, R. and K. Ramaswamy, 1982, "The Effects of Dividends on Common Stock Prices: Tax Effects or Informational Effects", Journal of Finance, 37, 429-443.

Litzenberger, R. and J. Rolfo, 1984, "Arbitrage Pricing, Transaction Costs and Taxation of Capital Gains: A Study of Government Bonds with the Same Maturity Date", Journal of Financial Economics, 13, 337-351.

Lo, A. and C. MacKinlay, 1990, "Data-Snooping Biases in Tests of Financial Asset Pricing Models", Review of Financial Studies,

Mankiw, N., J. Rotemberg and L. Summers, 1985, "Intertemporal Substitution in Macroeconomics", Quarterly Journal of Economics, 100, 225-252.

Miller, M. and M. Scholes, 1978, "Dividends and Taxes", Journal of Financial Economics, 6, 333-364.

Miller, M. and M. Scholes, 1982, "Dividends and Taxes: Some Empirical Evidence", Journal of Political Economy, 90, 1118-1141.

Newey, W. and K. West, 1987, "A Simple, Positive Semi-definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix", Econometrica, 55, 703-708.

Rotemberg, J., 1984, "Interpreting the Statistical Failures of Some Rational Expectations Macroeconomic Models", American Economic Review, 74, 188-193.

Rubinstein, M., 1976, "The Valuation of Uncertain Income Streams and the Pricing of Options", Bell Journal of Economics, 7, 407-425.

Singleton, K., 1989, "Modeling the Term Structure of Interest Rates in General Equilibrium: Discussion", in: Theory of Valuation, edited by S. Bhattacharya and G. Constantinides.

Tauchen, G., 1986, "Statistical Properties of Generalized Method of Moments Estimates of Utility Function Parameters Based on Financial Market Data", Journal of Business and Economic Statistics, 397-417.