DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

SENIORITY IN LEGISLATURES

Richard D. McKelvey and Raymond Riezman

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SENIORITY IN LEGISLATURES[1]

Richard D. McKelvey
California Institute of Technology

Raymond Riezman
California Institute of Technology
and
University of Iowa

ABSTRACT

We construct a stochastic game model of a legislature with an endogenously determined seniority system. We model the behavior of the legislators as well as their constituents in an infinitely repeated divide the dollar game. Each legislative session must make a decision on redistributional issues, modeled as a divide the dollar game. However, each session begins with a vote in which the legislators decide, by majority rule, whether or not to impose on themselves a seniority system. Legislative decisions on the redistributional issues are made by the Baron-Ferejohn rule: an agenda setter is selected by a random recognition rule (which in our model is a function of the seniority system selected), the agenda setter makes a proposal on redistributional issues, and the legislature then votes whether to accept or reject the agenda setters proposal. If the legislature rejects the proposal, another agenda setter is randomly selected, and the process is repeated. If the legislature accepts the proposal, the legislative session ends, and the voters in each legislative district vote whether to retain their legislator or throw it out of office. The voters' verdict determines the seniority structure of the next period legislature. We find a stationary equilibrium to the game having the property that the legislature imposes on itself a non trivial seniority system, and that legislators are always reelected.

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1. INTRODUCTION

Why do legislatures have seniority systems? Why do incumbent legislators tend to be reelected by wide margins? These are questions that have engaged legislative scholars for some time.

On the issue of the incumbency advantage, there is a large empirical literature which has advanced a number of explanations for this effect. Jacobson [1983] gives a good review of this literature. The explanations range from the increased access of incumbents to money and the media (see eg., Mayhew [1974]), to the effects of gerrymandering (Jacobson [1983, pp. 13-15], Erikson [1972]), to the decline of the party system and consequent increased use of incumbency rather than party as a voting cue (Ferejohn [1977]), to constituency service and expertise built up by veteran legislators (eg, Mayhew [1974], Fiorina [1977a, b]).

Although the question of incumbency advantage and its relation to legislative organization have received considerable attention in the empirical literature on Congress, we know of no attempt to see if any of these explanations can be derived from a full equilibrium, dynamic model. All of the above explanations of the incumbency effect are non-dynamic, partial equilibrium explanations. In other words, it is not clear that all individuals, at all points in time are behaving rationally. For example, the explanations of the incumbency effect in terms of money and the media typically do not explain why it is that voters should be swayed repeatedly by advertising and campaign literature. The explanation based on gerrymandering assumes that voters' behavior can be determined by certain socioeconomic characteristics of the voters, such as party identification, race, sex, income and religion. It ignores the possibility that both voters and candidates may have incentives to alter their behavior based on the new district characteristics. The explanation based on the decline of parties has no well worked out theory as to why voters should use cues such as party or incumbency in the first place. The explanation based on constituency service has some weaknesses when one considers the timing of voter and candidate decisions. For example, why should voters vote for candidates who have done a lot for them in the past if the voters have already collected the rewards of the candidate's behavior? The above models are a rich source of ideas, and undoubtedly, some of the ideas could be made part of a consistent theory, in which all participants are behaving rationally, and timing issues are dealt with explicitly. However, this has not yet been done.[2]

[2]There have been partial attempts in this direction. Austen-Smith and Banks [1988] develop a full equilibrium model of voter and legislative behavior in a parliamentary system. However, their model is not dynamic since it deals with a one shot game. Kramer [1977], Baron and Ferejohn [1989] and Baron [1989] have developed dynamic models of policy formation and legislative organisation, but these models are not full equilibrium since they do not explicitly consider voter and legislative interactions.

From our perspective, the most interesting observation in the above literature is that many of the above variables are determined endogenously by the legislature. It has been argued persuasively by Mayhew and Fiorina that *Congress organizes itself to serve the reelection goals of its members.* Thus, the franking privilege, the specialized committee system, the norm of reciprocity, etc., are all seen as ways in which Congress advances the reelection goals of its members. Fiorina has taken this argument to its extreme in his thesis that big government is partially a result of the fact that Congressmen benefit from the increased opportuniuties to intervene in the bureaucracy on the behalf of their constituents.

In this paper, we consider one particular aspect of legislative organization, namely the seniority system, and build a theoretical model connecting the seniority system with the reelection goals of the legislators: we formulate a full equilibrium, dynamic model of policy formation in a representative system in which a seniority system emerges endogenously.[3] Our contribution is to develop a model in which both voters and legislators are acting rationally both on and off the equilibrium path. Voters take into account the fact that their representive is only a member of a legislative body and legislators realize that their actions will affect voters' behavior in subsequent elections. All agents take into account the dynamic effects of all of their actions.

The approach we take to accomplish the above objectives is to model the representative process as an $\mathcal{L}$ + n player stochastic game, where $\mathcal{L}$ is the number of legislators, and n is the number of voters, partitioned into $\mathcal{L}$ distinct districts. The game alternates back and forth between the voter game and the legislative game. The voter game will consist of a game in which all the voters in each of the $\mathcal{L}$ legislative districts vote to determine who will be their representative for the next legislative session. The legislative game will consist of a game in which the legislators decide whether or not to have a seniority system for the current session and then proceed to select a policy. We will model the legislative game using the approach of Baron and Ferejohn [1989], who consider the legislative game as a form of a Rubinstein bargaining game: There is a random recognition rule, which depends on seniority, which determines the legislator who makes a proposal. The legislators then vote, by majority rule, whether to accept or reject the proposal. The process continues until the legislature accepts a

[3]On the issue of seniority, there has been remarkably little formal work in the political science literature. One exception is Shepsle [1990], who develops a model explaining the existence of seniority systems in the group provision of public or private goods. His explanation is based on a model of overlapping generations, in which agents need to have incentives to participate throughout their lifetime. This explanation does not depend on any characteristics of the group that are unique to legislative bodies, and hence is equally applicable to firms as to legislative bodies. Although there has not been a lot of work explicitly on seniority, there has been a substantial body of formal work looking at the role of specialized committees in legislative organization (for example, see Shepsle [1979] and Gilligan and Krehbiel [1988]).

proposal, at which time the legislature adjourns, and new elections are held (i. e., we return to the voter game).

We show that an equilibrium exists in which the legislature always votes to impose on itself a non trivial seniority system. In the proposal stage, the proposer selects a minimum winning coalition, retaining $\frac{\mathcal{L}+1}{2\mathcal{L}}$ for its own district and allocating $\frac{1}{\mathcal{L}}$ to the districts of the remaining coalition members. Districts that are not part of the winning coalition get nothing. This proposal passes and the game proceeds to the voter game. Voters always reelect incumbents. The intuition behind the results is that voters, understanding the incentives in the legislative game, realize that their representative will be disadvantaged if it does not have seniority.

These results contrast with those found in most formal models of voting. Most formal voting models predict tied elections, with no incumbency effects. In our model the incumbent always wins by a unanimous margin. In addition, we have an endogenously chosen seniority system. These two phenomena are related to each other, in that the seniority system and the incumbency effect support each other in equilibrium.

It is tempting to interpret the equilibrium of this model as a situation in which legislators blackmail voters to reelect them through the imposition of the seniority system. However, note that that is not exactly what happens in the model. In our model, the legislators cannot commit future legislatures to adopt a seniority system. The future legislature is free to vote against the seniority system if it is not in the interest of the legislators in that legislature to do so. What drives the incumbency effect in our model is the recognition by voters that self interested legislators with seniority will vote for a seniority system. If a sufficient number of the other legislators have seniority, then it is in the self interest of a district to make sure that its legislator does also, since the legislature will undoubtedly impose a seniority system. If all voters think this, it becomes a self fulfilling prophecy.

2. THE GENERAL FRAMEWORK

Before introducing the model we work with, we develop some general notation for stochastic games. Our model will be a special case of such a general model.

Assume that there is a set N of ***players***, a set X of ***alternatives***, and for each player $i \in N$, a Von Neumann Morgenstern ***utility function*** u_i: $X \to \mathbb{R}$ over the set of alternatives. We assume that X contains a ***null outcome***, x_0 with $u_i(x_0) = 0$ for all $i \in N$. Let T be a finite set of ***states***. We now define a ***stochastic game***, $\Gamma = \{\Gamma^t: t \in T\}$ to be a collection of ***game elements*** $\Gamma^t = (S^t, \pi^t, \psi^t)$. Here S^t

$= \Pi_{i\epsilon N} S_i^t$ is an n tuple of ***pure strategy sets.*** Next $\pi^t\colon S^t \to \mathcal{M}(T) = \Delta^{|T|}$ is a ***transition function*** specifying for each $s^t \in S^t$ a probability distribution, $\pi^t(s^t)$ on T, which determines for each $s^t \in S^t$ and $y \in T$, the probability $\pi^t(s^t)(y)$ of proceeding to game element Γ^y. Finally, $\psi^t\colon S^t \to X$ is an ***outcome function*** which specifies for each $s^t \in S^t$ an outcome $\psi^t(s^t) \in X$. We let $S = \prod_{t\epsilon T} S^t$ be the collection of pure strategy n tuples, one for each game element. We write $\Sigma_i^t = \mathcal{M}(S_i^t)$, where $\mathcal{M}(S_i^t)$ is the set of probability disributions over S_i^t, and then define $\Sigma_i = \prod_{t\epsilon T} \Sigma_i^t$ to be the set of ***stationary strategies*** for player i. Elements of Σ are written in the form $\sigma = (\sigma_1, \sigma_2, \ldots , \sigma_n)$. We also use the abusive notation $\sigma^t(s^t) = \prod_{i\epsilon N} \sigma_i^t(s_i^t)$, and $\sigma(s) = \prod_{t\epsilon T} \sigma^t(s^t)$ to represent the probability under σ of choosing the pure strategy profile $s^t \in S^t$, and $s \in S$, respectively.

For stationary strategies, we can define the payoff function $M^t\colon \Sigma \to \mathbb{R}^n$ by

$$M_i^t(\sigma) = \sum_{\tau=1}^{\infty} \sum_{r\epsilon T} \pi_\tau^t(\sigma)(r) \cdot u_i(\psi^r(\sigma^r)), \tag{2.1}$$

where $\pi_\tau^t(\sigma)(r)$ is defined inductively by

$$\pi_1^t(\sigma)(r) = \pi^t(\sigma^t)(r) = \sum_{s^t \epsilon S^t} \sigma^t(s^t) \cdot \pi^t(s^t)(r),$$

$$\pi_\tau^y(\sigma)(r) = \sum_{y\epsilon Y} \pi_{\tau-1}^y(\sigma)(y) \cdot \pi_1^y(\sigma^t)(r),$$

and $u_i(\psi^t(\sigma^t))$ is defined by

$$u_i(\psi^t(\sigma^t)) = \sum_{s^t \epsilon S^t} \sigma^t(s^t) \cdot u_i(\psi^t(s^t)).$$

Thus Note that the above is only well defined if the sum in (2.1) converges for all σ, t, and i.

A strategy n-tuple, $\sigma \in \Sigma$ is said to be a ***Nash equilibrium*** if $M_i(\sigma_i', \sigma_{-i}) \le M_i(\sigma)$ for all $\sigma_i' \in \Sigma_i$. It follows from standard results of stochastic games, that if all the S^y are finite and if there is an absorbing state $t \in T$ with $\psi^t(s^t) = x_0$ for all $s^t \in S^t$, then (2.1) converges and there exists a stationary equilbrium to the game Γ (See Sobel [1971]). Applying Bellman's optimality principle (eg. see Sobel, Theorem 3), it follows that any stationary Nash equilibrium can be characterized by a collection $\{v^t\}_{t\epsilon T} \subseteq \mathbb{R}^n$ of values for each game element Γ^t, and a strategy profile, $\sigma \in \Sigma$ satisfying:

(a) For all $t \in T$, σ^t is a Nash equilibrium to the game with payoff function $G^t\colon \Sigma^t \to \mathbb{R}^n$ defined by:

$$\begin{aligned} G^t(\sigma^t) &= u(\psi^t(\sigma^t)) + \sum_{y\epsilon T} \pi^t(\sigma^t)(y) \cdot v^y \\ &= E_{\sigma^t}\left[u(\psi^t(s^t)) + \sum_{y\epsilon T} \pi^t(s^t)(y) \cdot v^y\right] \end{aligned}$$

$$= \sum_{s^t \epsilon S^t} \sigma^t(s^t) \cdot \left[u(\psi^t(s^t)) + \sum_{y \epsilon T} \pi^t(s^t)(y) \cdot v^y \right].$$

(b) For all $t \in T$, $v^t = G^t(\sigma^t)$.

We will use the above result to characterize equilibria in the stochastic game we consider. Finally, it also follows from results in Sobel that a Nash equilibrium in the set of stationary strategies is also a Nash equilibrium in the larger class of non-stationary strategies.

3. THE LEGISLATIVE SENIORITY GAME

We consider an infinitely repeated game between legislators and their constituents. The legislative game consists of three parts: a vote on the seniority structure, a proposal by a randomly selected member, and a vote on the proposal. The legislative session starts with a vote on the seniority structure. If a majority vote for a seniority system, it passes, otherwise there is no seniority system. Next, a random recognition rule, like that of Baron and Ferejohn [1989] is used to select a legislator as an agenda setter. If no seniority system was passed, all legislators have equal probability of being selected. On the other hand, if a seniority system was passed, then the probability of recognition is an increasing function of i's relative seniority. The agenda setter proposes a division of the dollar by legislative district. The legislature then votes on the proposal. If the proposal is defeated, a new agenda setter is selected and the game continues as before, except that in the second round and thereafter seniority is ignored in selecting the proposer.[4] Once a proposal passes the legislature the legislative session ends.

After each legislative session there is an election. The voters can choose to re-elect their incumbent legislator, in which case the legislator has seniority in the next session and receives a salary of c, or the voters can vote not to re-elect the incumbent, in which case their legislator receives no salary and goes to the next session with no seniority. While this is not completely realistic it at least

[4]It is important to note that in our formulation, the seniority system only matters on the initial proposal. An interesting variation to consider would be the case in which seniority counts not only on the first proposal, but on all successive proposals as well. We believe that our formulation makes sense for two reasons. First, it captures an aspect of the way in which Congressional rules operate: namely, seniority is embodied in the committee system, which gives higher than average influence to ranking committee members to specify the proposed legislation. But if a majority of the legislators oppose a committee proposal on the floor, then the committee effectively loses its power, and the proposal of the committee can be amended by the full legislature at will. Secondly, the solution of our model is equivalent to the solution of a model in which there is a status quo in which all districts get $1/\mathcal{L}$, and the failure of a proposal leads to a reversion to the status quo.

captures the idea that voters can punish their representatives if they feel that they are not acting in their best interests. Our formulation allows more limited punishments than would be the case if voters could remove the legislator from office permanently. After each election the legislative game begins again with the new seniority structure.All agents have utility functions which are the discounted present value of their lifetime stream of utility. For the legislators, in each period, payoffs consist of a salary, which depends on whether they are re-relected, and a percentage $(1 - \theta)$ of what they secure for their disrict. Thus, they skim some exogenously given portion of their district's payoff. For the voters, in each period they get θ times their share of what their legislator is able to secure for the district.

We now define the legislative seniority game more formally as a special kind of stochastic game. We let $N = L \cup V$, where L is the set of ***legislators***, with $\mathcal{L} = |L| \geq 3$ odd, and V is the set of ***voters.*** We assume that $X' = \Delta^{\mathcal{L}} \times \{0,1\}^{\mathcal{L}}$, and $X = X' \cup x_0$. Elements of X' are written in the form $x = (z, q)$, where $z = (z_1, \dots, z_{\mathcal{L}}) \in Z = \Delta^{\mathcal{L}}$and $q = (q_1, \dots, q_{\mathcal{L}}) \in Q = \{0,1\}^{\mathcal{L}}$. We assume that there is a function ϕ: $V \to L$ identifying the legislative districts, such that voter v is in legislator ℓ's district if $\phi(v) = \ell$. We assume that $n_\ell = |\phi^{-1}(\ell)|$ is odd for all $\ell \in L$. We assume that utility functions over X' are of the form $u_i(x) = (1 - \theta)z_i + cq_i$ for $i \in L$, and $u_i(x) = (\theta/n_{\phi(i)})z_{\phi(i)}$ for $i \in V$.

So $Q = \{0, 1\}^{\mathcal{L}}$ repesents the seniority structure of the legislature, with typical element $q = (q_1, \dots, q_{\mathcal{L}})$. Thus, $q_i = 1$ indicates that legislator i has seniority, whereas $q_i = 0$ indicates it does not have seniority. Let $T = \{0\} \cup (Q \times \{0,1,2\}) \cup L \cup (Z \times \{L, V\})$ be the set of states (of the system, not of the union). Let $0 < \delta < 1$ be a fixed discount rate, and q^* be the element of Q satisfying $q_i^* = 1$ for all i. We assume p: $Q \to \Delta^{\mathcal{L}}$ is strictly monotonic in each component: for all $q \in Q$, and $i \in L$, $q_i > q_i' \Rightarrow p_i(q) > p_i(q_i', q_{-i})$, and that $q_i = q_j \Rightarrow p_i(q) = p_j(q)$. Thus, more seniority means a higher probability that a legislator is selected as the proposer, and legislators with the same seniority have equal probability of being selected.

The strategy sets and transition functions for the game elements are defined as follows:

For $t = 0$: Termination Game

$$S_i^t = \{0\} \text{ if } i \in N,$$
$$\pi^t(s^t)(0) = 1,$$
$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

For $t \in Q \times \{0\}$: The Discounting Game

$$S_i^t = \{0\} \text{ if } i \in N,$$
$$\pi^t(s^t)(y) = \begin{cases} \delta & \text{if } y = (t_1, 1) \\ 1 - \delta & \text{if } y = 0, \end{cases}$$

$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

The above two games determine the termination conditions of the game. They are a formal way of introducing discounting into the model. It is assumed that there is a probability 1 - δ of termination after each round of the game. Note that the entire game terminates when this occurs. This is equivalent to assuming that players discount future payoffs by an amount δ.

For $t \in Q \times \{1\}$: $$S_i^t = \begin{cases} \{0,1\} & \text{if } i \in L \\ \{0\} & \text{if } i \in N - L, \end{cases}$$ The Seniority Game

$$\pi^t(s^t)(t_I, 2) = 1 \text{ if } \Sigma_{i \in L}\, s_i^t > \frac{\ell}{2},$$
$$\pi^t(s^t)(q^*, 2) = 1 \text{ if } \Sigma_{i \in L}\, s_i^t \leq \frac{\ell}{2},$$

$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

The first decision the legislature makes is whether or not to have seniority for the current session. The vote determines if seniority is used in the Random Recognition Game below. If a majority of the legislators vote for seniority, then the current seniority vector, t_I, is used in the Random Recognition Game. If there is not a strict majority for, then the seniority vector q^*, which assigns equal weight to all legislators, is used in the Random Recognition Game.

For $t \in Q \times \{2\}$: $S_i^t = \{0\}$ if $i \in N$, Random Recognition Game

$$\pi^t(s^t)(y) = p_y(t_1) \text{ if } y \in L,$$
$$\psi^t(s^t) = x_0 \text{ for all } s^t \in S^t.$$

The Random Recognition Game is the second stage of the legislative session. In this game, t_I is a vector of dimension $\mathcal{L}$ indicating the seniority of each legislator. If seniority passed, the seniority vector t_I from the Seniority Game is used. If seniority failed then q^* is used for the seniority vector. A legislator is selected by a random recognition rule to make a proposal for consideration by the legislature. This rule is similar to the Baron Ferejohn recognition rule, except we let the recognition rule be a function of seniority. Assumptions made above guarantee that higher seniority leads to higher probability of being selected.

For $t \in L$: $$S_i^t = \begin{cases} Z & \text{if } i = t \\ \{0\} & \text{if } i \in N - \{t\}, \end{cases}$$ The Proposal Game

$\pi^t(s^t)(s_t^t, L) = 1,$
$\psi^t(s^t) = x_0$ for all $s^t \in S^t$.

The Proposal Game is the third stage of the legislative session. In this game, the legislator who has been selected as the proposer in the Random Recognition Game makes a proposal for a division of the dollar between the legislative districts. If the legislator proposes the division z, then we proceed to the Legislative Voting Game (z, L).

For $t \in Z \times \{L\}$: $S_i^t = \begin{cases} \{0, 1\} & \text{if } i \in L \\ \{0\} & \text{if } i \in V, \end{cases}$ The Legislative Voting Game

$\pi^t(s^t)(t_1, V) = 1$ if $\Sigma_{i\in L}\, s_i^t > \frac{\ell}{2}$,
$\pi^t(s^t)(q^*,2) = 1$ if $\Sigma_{i\in L}\, s_i^t \leq \frac{\ell}{2}$,

$\psi^t(s^t) = x_0$ for all $s^t \in S^t$.

The Legislative Voting Game is the fourth stage of the legislative session. In this game, the proposal t_1 is before the legislature, and the legislators must vote whether to accept it or reject it. If the legislators vote to accept the proposal, the legislative session ends, and we proceed to the Voter Game. If the legislators reject the proposal, then we return to the Random Recognition Game, with the exception that seniority is ignored in selecting the proposer.

For $t \in Z \times \{V\}$: $S_i^t = \begin{cases} \{0, 1\} & \text{if } i \in V \\ \{0\} & \text{if } i \in L, \end{cases}$ The Voter Game

$\pi^t(s^t)(q(s^t), 0) = 1,$

$\psi^t(s^t) = (t_1, q(s^t)),$

where $q(s^t) = (q_1(s^t), q_2(s^t), \dots, q_{\mathcal{L}}(s^t)) \in Q$ is defined by

$$q_i(s^t) = \begin{cases} 1 & \text{if } \sum_{j\epsilon\phi^{-1}(i)} s_j^t > \frac{n_\ell}{2} \\ 0 & \text{if } \sum_{j\epsilon\phi^{-1}(i)} s_j^t \leq \frac{n_\ell}{2}, \end{cases}$$

and where $0 < \theta < 1$ and $0 < c$ are constants.

The Voter Game consists of a set of simultaneous elections in all of the legislative districts. In each legislative district, the voters of that district vote whether or not to reelect their legislator. In the version of the game as it is presented here, there is only one legislator in each district, and no challenger. So the effect of a negative vote in a given district is that the legislator from that district does not get a salary for the next period, and loses its seniority.

This completes the description of the stochastic game. Note that there are no payoffs except in the voter game. At that point policy $x = (t_1, q(s^t))$ is implemented. Thus, the pie is divided up among the districts according to $z = t_1 \in \Delta^{\mathcal{L}}$, and $q(s^t) \in Q$ determines which legislators get reelected, and which do not. Given the utility functions we have specified, it follows that the output $t_{1\ell}$ to district ℓ is first divided up with $\theta t_{1\ell}$ actually delivered to the voters, and $(1 - \theta)t_{1\ell}$ being skimmed off by legislator ℓ. The voters each get an even share of the delivered output. The legislators, in addition to their share of the output get a salary which is dependent on whether they are reelected or not.

4. RESULTS

PROPOSITION 1: The following is a stationary equilibrium to the legislative seniority game defined in section 2.

For $t \in Q \times \{1\}$, and $i \in L$: $\quad \sigma_i^t(t_{Ii}) = 1$

For $t \in L$: $$\sigma_t^t = \frac{1}{|\Omega_t|}\sum_{w\in\Omega_t}\delta_{z_t(w)},$$

where $\Omega_t = \{\omega\in \{0, 1\}^{\mathcal{L}}: \sum_i\omega_i=\frac{\mathcal{L}+1}{2}, \omega_t=1\}$, δ_x is the Dirac delta at x, and z_t: $\Omega_t \to \mathbb{R}^{\mathcal{L}}$ is defined by:

$$z_{ti}(\omega) = \begin{cases} \frac{\mathcal{L}+1}{2\mathcal{L}} & \text{if } i = t \\ \frac{1}{\mathcal{L}} & \text{if } i \neq t,\ \omega_i = 1 \\ 0 & \text{otherwise.} \end{cases}$$

For $t \in Z \times \{L\}$, and $i \in L$:

$$\sigma_i^t(1) = \begin{cases} 1 \text{ if } t_{1i} \geq \frac{1}{\mathcal{L}} \\ 0 \text{ if } t_{1i} < \frac{1}{\mathcal{L}}. \end{cases}$$

For $t \in Z \times \{V\}$, and $i \in V$: $\sigma_i^t(1) = 1$ for all i.

REMARKS: The proposition gives equilibrium strategies for both the legislators and voters in the above stochastic game. In the seniority stage all legislators who have seniority vote in favor of the seniority system, those who do not have seniority vote against the seniority system. Since in equilibrium all legislators get reelected the seniority system always passes.

In the proposal stage, the proposer will select a minimal winning coalition of legislators which includes itself. The proposer retains $\frac{\mathcal{L} + 1}{2\mathcal{L}}$ for its own district, leaving $\frac{1}{\mathcal{L}}$ to be allocated to the districts of each of the remaining members of the coalition. Districts that are not a part of the winning coalition are allocated 0. Thus the proposer obtains a premium of $\frac{\mathcal{L} + 1}{2\mathcal{L}} - \frac{1}{\mathcal{L}} = \frac{\mathcal{L} - 1}{2\mathcal{L}}$ due to its proposal power. As $\mathcal{L} \rightarrow \infty$ the premium goes to one half.

In the voting stage of the legislative session, a legislator votes for a proposal if and only if the legislator receives at least $\frac{1}{\mathcal{L}}$. Thus, if the proposer has proposed an equilibrium proposal, it will pass.

Finally, in the voting game, the voters always vote to reelect their legislators. It should be noted that although the proof shows only that this is a Nash equilibrium for the voters, in fact the strategy of voting for the incumbent is a dominant strategy for the voters in any given legislative district.

The conclusions of the above model stand in sharp contrast to the results that come out of the traditional voting literature. Most voting models predict tied elections, with no incumbency effects. Here, we obtain instead equilibrium behavior by the voters in which the incumbent wins by a large (unanimous) margin. The intuition behind the result is simple: The voters know that in equilibrium the seniority system will pass, hence it is in the voters' best interest to reelect the incumbent, since a senior legislator will be more easily able to serve the constituency than a junior legislator. Note that voters do not know that there will be a seniority system in the next session, but rather know that in the steady state equilibrium, seniority will be voted in each session.

PROOF OF THE PROPOSITION: We first specify the values, v^t, associated with these strategies.

We then verify that for these values, conditions (a) and (b) are satisfied. For the following equations, we set $w_1 = \frac{\mathcal{L}+1}{2\mathcal{L}}$, and $w_2 = \frac{1}{\mathcal{L}}$. Also, we define $Z^1 = \{z \in Z: |\{j \in L: z_j \geq w_2\}| > \frac{\mathcal{L}}{2}\}$, and $Z^0 = Z - Z^1$. Similarly, define $Q^1 = \{q \in Q: |\{i \in L: q_i = 1\}| > \frac{\mathcal{L}}{2}\}$, and $Q^0 = Q - Q^1$.

The values of the games are defined below. To interpret these values go to the definitions of the individual games above. For example, for $t \in Q \times \{1\}$ (see below) you are in the Seniority Game. $v_i^t = v_i^{(t_1,2)}$ means that the value of the seniority game given that seniority has passed ($t_1 \in Q^1$) is the value in the Random Recognition Game with seniority vector t_1. If seniority does not pass ($t_1 \in Q^0$) then the value of the game is given by the value in the Random Recognition Game with seniority vector q^*. Other values are defined in a similar way.

For $t \in \{0\}$: $\quad v_i^t = 0$ for all $i \in N$.

For $t \in Q \times \{0\}$: $\quad v_i^t = \delta v_i^{(t_1,1)}$ if $i \in N$.

For $t \in Q \times \{1\}$: $\quad v_i^t = v_i^{(t_1,2)}$ if $t_1 \in Q^1$,

$v_i^t = v_i^{(q^*,2)}$ if $t_1 \in Q^0$.

For $t \in Q \times \{2\}$:

$$v_i^t = (1-\theta)[p_i(t_1)w_1 + \tfrac{1}{2}\sum_{y \in \mathcal{L}-\{i\}} p_y(t_1)w_2] + c + \delta v_i^* \quad \text{if } i \in L,$$

$$v_i^t = \frac{\theta}{n_{\phi(i)}}[p_{\phi(i)}(t_1)w_1 + \tfrac{1}{2}\sum_{y \in \mathcal{L}-\{\phi(i)\}} p_y(t_1)w_2] + \delta v_i^* \quad \text{if } i \in V,$$

$$\text{where } v_i^* = v_i^{(q^*,2)} = \begin{cases} \frac{1}{1-\delta}\left[\frac{1}{\mathcal{L}}(1-\theta) + c\right] & \text{if } i \in L, \\ \frac{1}{1-\delta}\left[\frac{\theta}{\mathcal{L} n_{\phi(i)}}\right] & \text{if } i \in V. \end{cases}$$

For $t \in L$:

$$v_i^t = (1-\theta)w_1 + c + \delta v_i^* \quad \text{if } i = t,$$

$$v_i^t = \tfrac{1}{2}(1-\theta)w_2 + c + \delta v_i^* \quad \text{if } i \neq t,$$

$$v_i^t = \frac{\theta}{n_{\phi(i)}} w_1 + \delta v_i^* \quad \text{if } \phi(i) = t,$$

$$v_i^t = \tfrac{1}{2}\ \frac{\theta}{n_{\phi(i)}} w_2 + \delta v_i^* \qquad \text{if } \phi(i) \neq t,$$

For $t \in Z^1 \times \{L\}$: $\quad v_i^t = (1 - \theta)t_{1i} + c + \delta v_i^* \qquad$ if $i \in L$,

$\quad v_i^t = \theta t_{1\phi(i)} + \delta v_i^* \qquad$ if $i \in V$.

For $t \in Z^0 \times \{L\}$: $\quad v_i^t = v_i^* \qquad$ if $i \in N$.

For $t \in Z \times \{V\}$: $\quad v_i^t = (1 - \theta)t_{1i} + c + \delta v_i^* \qquad$ if $i \in L$,

$$v_i^t = \frac{\theta}{n_{\phi(i)}} t_{1\phi(i)} + \delta v_i^* \qquad \text{if } i \in V.$$

The next step in the proof is to verify condition (b), which requires that for each game and each player the payoffs correspond to the values we have specified above. To do this we start with the definition of G, then using the definitions of the game elements and the equilibrium strategies show that the payoffs equal the appropriate values.

For $t \in \{0\}$:

$$G^t(\sigma^t) = E_{\sigma^t}\left[u(\psi^t(s^t)) + \sum_{y \epsilon T} \pi^t(s^t)(y)v^y\right] = u(x_0) + \pi^t(\sigma^t)(t)v^t = v^t.$$

For $t \in Q \times \{0\}$:

$$G^t(\sigma^t) = E_{\sigma^t}\left[u(\psi^t(s^t)) + \sum_{y \epsilon T} \pi^t(s^t)(y)v^y\right] =$$

$$= u(x_0) + \delta v^{(t_1,1)} + (1 - \delta)v^0 = \delta v^{(t_1,1)} = v^t.$$

For $t \in Q \times \{1\}$:

$$G_i^t(\sigma^t) = E_{\sigma^t}\left[u_i(\psi^t(s^t)) + \sum_{y \epsilon T} \pi^t(s^t)(y)v_i^y\right]$$

$$= u_i(\psi^t(t_1)) + \sum_{y \epsilon T} \pi^t(t_1)(y)v_i^y$$

$$= \begin{cases} v_i^{(t_1,2)} & \text{if } \sum_{i \epsilon L} t_{1i} > \frac{\ell}{2} \\ v_i^{(q^*,2)} & \text{if } \sum_{i \epsilon L} t_{1i} \leq \frac{\ell}{2} \end{cases}$$

$$= \begin{cases} v_i^{(t_1,2)} \text{ if } |\{i \in L\text{: } t_{1i} = 1\}| > \frac{\ell}{2} \\ \\ v_i^{(q^*,2)} \text{ if } |\{i \in L\text{: } t_{1i} = 1\}| \leq \frac{\ell}{2} \end{cases}$$

$$= \begin{cases} v_i^{(t_1,2)} \text{ if } t_1 \in Q^1 \\ v_i^{(q^*,2)} \text{ if } t_1 \in Q^0. \end{cases}$$

$$= v^t$$

For $t \in Q \times \{2\}$:

$$G^t(\sigma^t) = E_{\sigma^t}\left[u(\psi^t(\sigma^t)) + \sum_{y \in T} \pi^t(\sigma^t)(y)v^y\right] = u(x_0) + \sum_{y \in L} p_y(t_1)v^y.$$

So, for $i \in L$,

$$G_i^t(\sigma^t) = p_i(t_1)v_i^i + \sum_{y \in \mathcal{L}-\{i\}} p_y(t_1)v_i^y$$

$$= p_i(t_1)[(1 - \theta)w_1 + c + \delta v_i^*]$$

$$+ \sum_{y \in \mathcal{L}-\{i\}} p_y(t_1)[\tfrac{1}{2}(1 - \theta)w_2 + c + \delta v_i^*]$$

$$= (1 - \theta)[p_i(t_1)w_1 + \tfrac{1}{2} \sum_{y \in \mathcal{L}-\{i\}} p_y(t_1)w_2] + c + \delta v_i^* = v_i^t$$

and for $i \in V$,

$$G_i^t(\sigma^t) = p_{\phi(i)}(t_1)v_i^{\phi(i)} + \sum_{y \in \mathcal{L}-\{\phi(i)\}} p_y(t_1)v_i^y$$

$$= p_{\phi(i)}(t_1)[\frac{\theta}{n_{\phi(i)}}w_1 + \delta v_i^*]$$

$$+ \sum_{y \in \mathcal{L}-\{\phi(i)\}} p_y(t_1)[\frac{\theta}{2 \cdot n_{\phi(i)}}w_2 + \delta v_i^*]$$

$$= \frac{\theta}{n_{\phi(i)}}[p_{\phi(i)}(t_1)w_1 + \sum_{y \in \mathcal{L}-\{\phi(i)\}} \tfrac{1}{2} p_y(t_1)w_2] + \delta v_i^* = v_i^t$$

For $t \in L$:

$$G^t(\sigma^t) = E_{\sigma^t}\left[u(\psi^t(s^t)) + \sum_{y\epsilon T} \pi^t(s^t)(y)v^y\right] = u(x_0) + E_{\sigma^t}[v^{(s_t^t,L)}]$$

But, since $\sigma_t^t(Z^1) = 1$, we have, for $i \in L$,

$$G^t(\sigma^t) = E_{\sigma^t}[(1 - \theta)s_t^t + c + \delta v^*] = (1 - \theta)E_{\sigma^t}[s_t^t] + c + \delta v^*$$

But

$$E_{\sigma^t}[s_t^t] = E_{\sigma^t}\left[\frac{1}{|\Omega_t|}\sum_{w\epsilon\Omega_t} \delta_{z_t(w)}\right] = \frac{1}{|\Omega_t|}\sum_{w\epsilon\Omega_t} E_{\sigma^t}\left[\delta_{z_t(w)}\right]$$

$$= \frac{1}{|\Omega_t|}\sum_{w\epsilon\Omega_t} z_t(\omega).$$

So

$$E_{\sigma^t}[s_{ti}^t] = \frac{1}{|\Omega_t|}\sum_{w\epsilon\Omega_t} z_{ti}(\omega) = \frac{1}{|\Omega_t|}\cdot|\Omega_t| \frac{\mathcal{L} + 1}{2\mathcal{L}} = \frac{\mathcal{L} + 1}{2\mathcal{L}} = w_1 \text{ if } i = t$$

$$E_{\sigma^t}[s_{ti}^t] = \frac{2\cdot\left[\frac{\mathcal{L} - 1}{2}\right]!}{(\mathcal{L} - 1)!}\cdot\frac{(\mathcal{L} - 2)!}{2\cdot\left[\frac{\mathcal{L} - 3}{2}\right]!}\cdot\frac{1}{\mathcal{L}} = \frac{\left[\frac{\mathcal{L} - 1}{2}\right]}{(\mathcal{L} - 1)}\cdot\frac{1}{\mathcal{L}} = \frac{1}{2\mathcal{L}} = \tfrac{1}{2}w_2 \text{ if } i \neq t$$

Thus,

$$G_i^t(\sigma^t) = \begin{cases} (1 - \theta)w_1 + c + \delta v_i^* & \text{if } i = t \\ \tfrac{1}{2}(1 - \theta)w_2 + c + \delta v_i^* & \text{if } i \neq t \end{cases}$$

$$= v_i^t$$

and for $i \in V$,

$$G_i^t(\sigma^t) = E_{\sigma^t}\left[\frac{\theta}{n_{\phi(i)}}s_{ti}^t + \delta v_i^*\right] = \frac{\theta}{n_{\phi(i)}}E_{\sigma^t}[s_{ti}^t] + \delta v_i^*$$

$$= \begin{cases} \frac{\theta}{n_{\phi(i)}}w_1 + \delta v_i^* & \text{if } \phi(i) = t \\ \tfrac{1}{2}\frac{\theta}{n_{\phi(i)}}w_2 + \delta v_i^* & \text{if } \phi(i) \neq t \end{cases}$$

$$= v_i^t$$

For $t \in Z^1 \times \{L\}$:

Since $t_1 \in Z^1$, it follows that $\sigma^t(\Sigma_{i\in L}\, s_i^t > \frac{\mathcal{L}}{2}) = 1$. So $\pi^t(\sigma^t)(t_1,V) = 1$. Hence,

$$G_i^t(\sigma^t) = E_{\sigma^t}\left[u_i(\psi^t(s^t)) + \sum_{y\epsilon T} \pi^t(s^t)(y)v_i^y\right]$$

$$= u(x_0) + \pi^t(\sigma^t)(t_1,V)v_i^{(t_1,V)} + \pi^t(\sigma^t)(q^*,2)v_i^{(q^*,2)}$$

$$= v_i^{(t_1,V)} = \begin{cases} (1 - \theta)t_{1i} + c + \delta v_i^* & \text{if } i \in L \\ \theta t_{1\phi(i)} + \delta v_i^* & \text{if } i \in V \end{cases}$$

$$= v_i^t.$$

For $t \in Z^0 \times \{L\}$:

Since $t_1 \in Z^0$, it follows that $\sigma^t(\Sigma_{i \in L}\, s_i^t \leq \frac{\ell}{2})$, so $\pi^t(\sigma^t)(q^*,2) = 1$. Hence,

$$G_i^t(\sigma^t) = E_{\sigma^t}\left[u_i(\psi^t(s^t)) + \sum_{y\epsilon T} \pi^t(s^t)(y)v_i^y\right]$$

$$= u(x_0) + \pi^t(\sigma^t)(t_1, V)v_i^{(t_1,V)} + \pi^t(\sigma^t)(q^*,2)v_i^{(q^*,2)}$$

$$= v_i^{(q^*,2)} = v_i^* = v_i^t.$$

For $t \in Z \times \{V\}$:

$$\pi^t(s^t)(q(s^t), 0) = 1,$$

$$\sigma_i^t(1) = 1 \text{ for all i.}$$

$$G^t(\sigma^t) = E_{\sigma^t}\left[u(\psi^t(s^t)) + \sum_{y\epsilon T} \pi^t(s^t)(y)v^y\right]$$

$$= E_{\sigma^t}[u(t_1, q(s^t))] + \pi^t(\sigma^t)(q(s^t), 0)v^{(q(s^t),0)}$$

$$= u(t_1, q^*) + v^{(q^*,0)} = u(t_1, q^*) + \delta v^*$$

So, for $i \in L$,

$$G_i^t(\sigma^t) = (1 - \theta)t_{1i} + cq_i^* + \delta v_i^* = (1 - \theta)t_{1i} + c + \delta v_i^* = v_i^t$$

and for $i \in V$,

$$G_i^t(\sigma^t) = \frac{\theta}{n_{\phi(i)}} t_{1\phi(i)} + \delta v_i^* = v_i^t$$

We next verify that (a) is satisfied, that is, σ^t is a Nash equilibrium. For each game element we show that no player can benefit from playing a different strategy.

For $t \in Z \times \{V\}$:

We want to show that σ^t is a Nash equilibrium to the game with payoff function G^t, where $\sigma_i^t(1) = 1$ for all $i \in V$. It suffices to show that for each $i \in V$, σ_i^t is at least as good as any pure strategy $s_i' \in S_i^t$. So,

$$\begin{aligned} &G_i^t(\sigma^t) \geq G_i^t(\sigma_i', \sigma_{-i}^t) \\ &\Leftrightarrow E_{\sigma^t}[u_i(\psi^t(s^t)) + \sum_{y\in T} \pi^t(s^t)(y)v_i^y] \\ &\qquad\qquad \geq E_{\sigma_{-i}^t}[u_i(\psi^t(s_i', s_{-i}^t)) + \sum_{y\in T} \pi^t(s_i', s_{-i}^t)(y)v_i^y] \end{aligned}$$

for all $s_i' \in S_i^t$. Writing $\underline{1}$ for the $|V|$ component vector of ones, we can rewrite this inequality as

$$u_i(\psi^t(\underline{1})) + \sum_{y\in T} \pi^t(\underline{1})(y)v_i^y \geq u_i(\psi^t(s_i', \underline{1}_{-i})) + \sum_{y\in T} \pi^t(s_i', \underline{1}_{-i})(y)v_i^y$$

$$\Leftrightarrow v_i^{(q(\underline{1}),0)} \geq v_i^{(q(s_i', \underline{1}_{-i}),0)} \Leftrightarrow v_i^* \geq v_i^{(q(s_i', \underline{1}_{-i}),1)}$$

But $q(s_i', \underline{1}_{-i}) \geq \underline{1} - \epsilon_\ell$, where $\ell = \phi(i)$, and ϵ_ℓ is the ℓ^{th} standard basis vector. Further, since $\underline{1} - \epsilon_\ell \in Q^1$, we have $q(s_i', \underline{1}_{-i}) \in Q^1$. So $v_i^{(q(s_i', \underline{1}_{-i}),1)} = v_i^{(q(s_i', \underline{1}_{-i}),2)}$. Hence the above inequality can be written

$$v_i^* \geq \frac{\theta}{n_{\phi(i)}}\left[p_{\phi(i)}(q(s_i', \underline{1}_{-i}))w_1 + \frac{1}{2} \sum_{y\in \mathcal{L}-\{\phi(i)\}} p_y(q(s_i', \underline{1}_{-i}))w_2\right] + \delta v_i^*$$

$$\Leftrightarrow \frac{\theta}{\mathcal{L}n_{\phi(i)}} \geq \frac{\theta}{n_{\phi(i)}}\left[p_{\phi(i)}(q(s_i', \underline{1}_{-i}))w_1 + \frac{1}{2}(1 - p_{\phi(i)}(q(s_i', \underline{1}_{-i})))w_2\right]$$

$$\Leftrightarrow \frac{1}{\mathcal{L}} = p_i(q_0)w_1 + \frac{1}{2}(1 - p_i(q_0))w_2 \geq p_i(q(s_i', \underline{1}_{-i}))w_1 + \frac{1}{2}(1 - p_i(q(s_i', \underline{1}_{-i})))w_2$$

$$\Leftrightarrow [p_i(q_0) - p_i(q(s_i', s_{-i}))](w_1 - \frac{1}{2}w_2) \geq 0$$

Now if $n_{\phi(i)} = |\phi^{-1}(\phi(i))| > 1$ (there is more than one voter in district i), then $q_0 = q(s_i', \underline{1}_{-i})$, so one voter changing their vote does not affect the outcome. Hence the above expression equals 0, and it follows that σ^t is a Nash equilibrium for G^t. If $n_{\phi(i)} = 1$, (there is a single voter in district i) then since $\sigma_i^t(1) = 1$, and $s_i' \leq 1$, it follows that $q_{0i} = 1$ and $q_i(s_i', \underline{1}_{-i}) = s_i' \leq 1$. In this case one voter changing their vote changes the outcome. Hence, monotonicity of p implies that $[p_i(q_0) - p_i(q(s_i', \underline{1}_{-i}))] \geq 0$, and the last inequality holds if and only if

$$[w_1 - \frac{1}{2}w_2] = \frac{\mathcal{L}+1}{2\mathcal{L}} - \frac{1}{2\mathcal{L}} = \frac{1}{2} \geq 0.$$

Since all terms in the last expression are positive, this inequality holds, and it follows that s^t is a Nash equilibrium for G^t. What this demonstrates is that from the voter's point of view changing their vote either does not change the outcome or changes the outcome in a way which makes that voter worse off.

For $t \in Q^1 \times \{1\}$:

We want to show that σ^t is a Nash equilibrium to the game with payoff function G^t, where $\sigma_i^t(t_{1i}) = 1$ for all $i \in L$. It suffices to show that for each $i \in L$, σ_i^t is at least as good as any pure strategy $s_i' \in S_i^t$. So,

$$G_i^t(\sigma^t) \geq G_i^t(\sigma_i', \sigma_{-i}^t)$$
$$\Leftrightarrow E_{\sigma^t}[u_i(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v_i^y]$$
$$\geq E_{\sigma_{-i}^t}[u_i(\psi^t(s_i', s_{-i}^t)) + \sum_{y \in T} \pi^t(s_i', s_{-i}^t)(y)v_i^y].$$

for all $s_i' \in S_i^t$. Using $\sigma_i^t(t_{1i}) = 1$, this can be reduced to

$$u_i(\psi^t(t_1)) + \sum_{y \in T} \pi^t(t_1)(y)v_i^y \geq u_i(\psi^t(s_i', (t_1)_{-i})) + \sum_{y \in T} \pi^t(s_i', (t_1)_{-i})(y)v_i^y.$$

Since $t_1 \in Q^1$, it follows that $\Sigma_{j \in L} t_{1j} > \frac{\ell}{2} \Rightarrow \pi^t(t_1)(t_1, 2) = 1$. So we get

$$v_i^{(t_1,2)} \geq \pi^t(s_i', (t_1)_{-i})(t_1,2)v_i^{(t_1,2)} + \pi^t(s_i', (t_1)_{-i})(q^*,2)v_i^{(q^*,2)}$$

Clearly, if $\pi^t(s_i', (t_1)_{-i})(t_1, 2) = 1$, legislator i is not pivotal and the above is an equality. So we consider the case when legislator i is pivotal, $\pi^t(s_i', (t_1)_{-i})(t_1, 2) \neq 1$. In this case, we must have $\Sigma_{j \in L} t_{1j} > \frac{\ell}{2}$ and $s_i' + \sum_{j \in L-\{i\}} t_{1j} < \frac{\ell}{2}$. So $t_{1i} = 1$, and $s_i' = 0$. Thus, $\pi^t(s_i', (t_1)_{-i})(q^*, 2) = 1$, and the above inequality can be rewritten

$$v_i^{(t_1,2)} \geq v_i^{(q^*,2)}$$

$$\Leftrightarrow (1-\theta)[p_i(t_1)w_1 + \tfrac{1}{2} \sum_{y \in \mathcal{L}-\{i\}} p_y(t_1)w_2] + c + \delta v_i^* \geq (1-\theta)[q_i^* w_1 + \tfrac{1}{2} \sum_{y \in \mathcal{L}-\{i\}} q_y^* w_2] + c + \delta v_i^*$$

$$\Leftrightarrow p_i(t_1)w_1 + \tfrac{1}{2}(1 - p_i(t_1))w_2 \geq q_i^* w_1 + \tfrac{1}{2}(1 - q_i^*)w_2$$

$$\Leftrightarrow p_i(t_1)(w_1 - \tfrac{1}{2} w_2) \geq q_i^* (w_1 - \tfrac{1}{2} w_2)$$

$$\Leftrightarrow\ p_i(t_1)\ \geq q_i^*.$$

Now $p_i(t_1)$ is the probability i is selected given that seniority is used and that $\frac{\mathcal{L}+1}{2}$ members (including i) have seniority and $\frac{\mathcal{L}-1}{2}$ do not have seniority. The seniority assumption implies that, for all $q \in Q$, and $i,j \in L$, $q_i > q_i' \Rightarrow p_j(q) < p_j(q_i', q_{-i})$, $i \neq j$. That is, higher (lower) seniority for legislator i means that every other legislator now has a lower (higher) probability of being selected as the proposer. Now begin at q^* (assume every legislator has seniority) and remove seniority for $\frac{\mathcal{L}-1}{2}$ legislators (not including i). At each step p_i increases. Therefore, the last inequality is satisfied. Hence, σ^t is a Nash equilibrium for G^t.

For $t \in Q^0 \times \{1\}$:

As above, we have ,

$$G_i^t(\sigma^t) \geq G_i^t(\sigma_i', \sigma_{-i}^t)$$

$$u_i(\psi^t(t_1)) + \sum_{y \in T} \pi^t(t_1)(y)v_i^y \geq u_i(\psi^t(s_i', (t_1)_{-i})) + \sum_{y \in T} \pi^t(s_i', (t_1)_{-i})(y)v_i^y$$

for all $s_i' \in S_i^t$. Since $t_1 \in Q^0$, it follows that $\Sigma_{j \in L} t_{1j} \leq \frac{\mathcal{L}}{2} \Rightarrow \pi^t(t_1)(q^*, 2) = 1$. So we get

$$\Leftrightarrow v_i^{(q^*,2)} \geq \pi^t(s_i', (t_1)_{-i})(t_1,2)v_i^{(t_1,2)} + \pi^t(s_i', (t_1)_{-i})(q^*,2)v_i^{(q^*,2)}$$

Clearly, if $\pi^t(s_i', (t_1)_{-i})(q^*,2) = 1$, legislator i is not pivotal and the above is an equality. So we consider the case when legislator i is pivotal, $\pi^t(s_i', (t_1)_{-i})(q^*,2)) \neq 1$. In this case, we must have $\Sigma_{j \in L} t_{1j} \leq \frac{\mathcal{L}}{2}$ and $s_i' + \Sigma_{j \in L-\{i\}} t_{1j} > \frac{\mathcal{L}}{2}$. So $t_{1i} = 0$, and $s_i' = 1$. Thus, $\pi^t(s_i', t_{1-i})(t_1, 2) = 1$, and the above inequality can be rewritten

$$v_i^{(q^*,2)} \geq v_i^{(t_1,2)}$$

$$\Leftrightarrow (1 - \theta)[q_i^* w_1 + \tfrac{1}{2} \sum_{y \in \mathcal{L}-\{i\}} q_y^* w_2] + c + \delta v_i^* \geq (1 - \theta)[p_i(t_1)w_1 + \tfrac{1}{2} \sum_{y \in \mathcal{L}-\{i\}} p_y(t_1)w_2] + c + \delta v_i^*$$

$$\Leftrightarrow q_i^* w_1 + \tfrac{1}{2}(1 - q_i^*)w_2 \geq p_i(t_1)w_1 + \tfrac{1}{2}(1 - p_i(t_1))w_2$$

$$\Leftrightarrow q_i^* (w_1 - \tfrac{1}{2} w_2) \geq p_i(t_1)(w_1 - \tfrac{1}{2} w_2)$$

$$\Leftrightarrow q_i^* \geq p_i(t_1).$$

But $p_i(t_1)$ is the probability i is selected given that seniority is used and that $\frac{\ell-1}{2}$ members have seniority and $\frac{\ell-1}{2}$ (including i) do not have seniority. Using reasoning similar to that above, begin at q^* (assume no legislator has seniority) and add seniority for $\frac{\ell-1}{2}$ legislators (not including i). At each step p_i decreases. Therefore, the last inequality is satisfied. Hence, σ^t is a Nash equilibrium for G^t.

For $t \in Z^1 \times \{L\}$:

We want to show that σ^t is a Nash equilibrium to the game with payoff function G^t, where

$$\sigma_i^t(1) = \begin{cases} 1 \text{ if } t_{1i} \geq \frac{1}{\ell} \\ 0 \text{ if } t_{1i} < \frac{1}{\ell}. \end{cases}$$

for all $i \in L$. It suffices to show that for each $i \in L$, σ_i^t is at least as good as any pure strategy $s_i' \in S_i^t$. So

$$\begin{aligned} & G_i^t(\sigma^t) \geq G_i^t(\sigma_i', \sigma_{-i}^t) \\ \Leftrightarrow\ & E_{\sigma^t}[u_i(\psi^t(s^t)) + \sum_{y \in T} \pi^t(s^t)(y)v_i^y] \\ & \quad \geq E_{\sigma_{-i}^t}[u_i(\psi^t(s_i', s_{-i}^t)) + \sum_{y \in T} \pi^t(s_i', s_{-i}^t)(y)v_i^y] \end{aligned}$$

for all $s_i' \in S_i^t$. Since $t_1 \in Z^1$, $|\{j \in L\colon t_{1j} \geq w_2\}| > \frac{\ell}{2}$. But $\sigma_i^t(1) = 1$ if $t_{1i} \geq \frac{1}{\ell} = w_2$. So, define $r \in \{0, 1\}^\ell$ by $r_i = 1$ if $t_{1i} \geq w_2$, and $r_i = 0$ if $t_{1i} < w_2$. Then $\sigma(r) = 1$ and $\sigma_{-i}(r_{-i}) = 1$. Since $\Sigma_{i \in L} r_i > \frac{\ell}{2}$, $\pi^t(r)(t_1, V) = 1$, and the above equation can be reduced to

$$u_i(x_0) + \sum_{y \in T} \pi^t(r)(y)v_i^y \geq u_i(x_0) + \sum_{y \in T} \pi^t(s_i', r_{-i})(y)v_i^y$$

$$\Leftrightarrow \sum_{y \in T} \pi^t(r)(y)v_i^y \geq \sum_{y \in T} \pi^t(s_i', r_{-i})(y)v_i^y$$

$$\Leftrightarrow v_i^{(t_1,V)} \geq \pi^t(s_i', r_{-i})(t_1,V)v_i^{(t_1,V)} + \pi^t(s_i', r_{-i})(q^*,2)v_i^{(q^*,2)}$$

Clearly, if $\pi^t(s_i', r_{-i})(t_1,V) = 1$, legislator i is not pivotal and the above is an equality. So we consider the case when legislator i is pivotal, $\pi^t(s_i', r_{-i})(t_1,V) \neq 1$. In this case, we must have $\Sigma_{j \in L} r_j > \frac{\ell}{2}$ and $s_i' + \Sigma_{j \in L-\{i\}} r_j < \frac{\ell}{2}$. So $r_i = 1$, and $s_i' = 0$. Thus, $\pi^t(s_i', r_{-i})(q^*, 2) = 1$, and the above inequality can be rewritten

$$v_i^{(t_1,V)} \geq v_i^{(q^*,2)} \Leftrightarrow (1 - \theta)t_{1i} + c + \delta v_i^* \geq v_i^*$$

$$\Leftrightarrow t_{1i} \geq \frac{1}{(1 - \theta)}[(1 - \delta)v_i^* - c].$$

$$\Leftrightarrow t_{1i} \geq \frac{1}{(1 - \theta)}[\left[\frac{1}{\ell}(1 - \theta) + c\right] - c].$$

$$\Leftrightarrow t_{1i} \geq \frac{1}{\ell}.$$

But $s_i^t = 1 \Rightarrow t_{1i} \geq w_2 = \frac{1}{\ell}$. Hence, the above inequality holds, and we have shown that $G_i^t(\sigma^t) \geq G_i^t(\sigma_i', \sigma_{-i}^t)$, so σ^t is a Nash equilibrium for G^t.

For $t \in Z^0 \times \{L\}$:

Define r as above. Since $t_1 \in Z^0$, $|\{j \in L\colon t_{1j} \geq w_2\}| \leq \frac{\ell}{2}$, we get $\Sigma_{i \epsilon L} r_i \leq \frac{\ell}{2}$, implying $\pi^t(r)(q^*, 2) = 1$. Then arguing as above,

$$G_i^t(\sigma^t) \geq G_i^t(\sigma_i', \sigma_{-i}^t)$$
$$\Leftrightarrow v_i^{(q^*,2)} \geq \pi^t(s_i', r_{-i})(t_1,V)v_i^{(t_1,V)} + \pi^t(s_i', r_{-i})(q^*,2)v_i^{(q^*,2)}$$

for all $s_i' \in S_i^t$. Clearly, if $\pi^t(s_i', r_{-i})(q^*,2) = 1$, legislator i is not pivotal and the above is an equality. So we consider the case when legislator i is pivotal, $\pi^t(s_i', r_{-i})(q^*,2) \neq 1$. In this case, we must have $\Sigma_{j \epsilon L} r_j < \frac{\ell}{2}$ and $s_i' + \Sigma_{j \epsilon L-\{i\}} r_j > \frac{\ell}{2}$. So $r_i = 0$, and $s_i' = 1$. Thus, $\pi^t(s_i', r_{-i})(t_1,V) = 1$, and the above inequality can be rewritten

$$v_i^{(q^*,2)} \geq v_i^{(t_1,V)} \Leftrightarrow v_i^* \geq (1 - \theta)t_{1i} + c + \delta v_i^*$$

$$\Leftrightarrow \frac{1}{(1 - \theta)}[(1 - \delta)v_i^* - c] \geq t_{1i}.$$

$$\Leftrightarrow \frac{1}{(1 - \theta)}[\left[\frac{1}{\ell}(1 - \theta) + c\right] - c] \geq t_{1i}.$$

$$\Leftrightarrow \frac{1}{\ell} \geq t_{1i}.$$

But $s_i^t = 0 \Rightarrow t_{1i} < w_2 = \frac{1}{\ell}$. Hence, the above inequality holds, and it follows that σ^t is a Nash equilibrium for G^t.

For $t \in L$:

We want to show that σ^t is a Nash equilibrium to the game with payoff function G^t, where

$$\sigma_t^t = \frac{1}{|\Omega_t|}\sum_{w\in\Omega_t}\delta_{z_t(w)},$$

where $\Omega_t = \{\omega\in\{0, 1\}^{\mathcal{L}}: \sum_i\omega_i=\frac{\mathcal{L}+1}{2}, \omega_t=1\}$, and $z_t: \Omega_t\to\mathbb{R}^{\mathcal{L}}$ is defined by:

$$z_{ti}(\omega) = \begin{cases} \frac{\mathcal{L}+1}{2\mathcal{L}} & \text{if } i = t \\ \frac{1}{\mathcal{L}} & \text{if } i \neq 1,\ \omega_i = 1 \\ 0 & \text{otherwise.} \end{cases}$$

for all i $\in$ L. It suffices to show that σ_t^t is at least as good as any pure strategy $s_i' \in S_i^t$. So For all s_t' $\in S_t^t$,

$$\begin{aligned} & G_t^t(\sigma^t) \geq G_t^t(\sigma_t', \sigma_{-t}^t) \\ \Leftrightarrow\ & E_{\sigma^t}[u_t(\psi^t(s^t)) + \sum_{y\in T}\pi^t(s^t)(y)v_t^y] \\ & \quad \geq E_{\sigma_{-i}^t}[u_t(\psi^t(s_t', s_{-t}^t)) + \sum_{y\in T}\pi^t(s_t', s_{-t}^t)(y)v_t^y] \\ \Leftrightarrow\ & u_t(x_0) + \frac{1}{|\Omega_t|}\sum_{w\in\Omega_t}\sum_{y\in T}\pi^t(z_t(\omega), \underline{0}_{-t})(y)v_t^y \\ & \quad \geq u_t(x_0) + \sum_{y\in T}\pi^t(s_t', \underline{0}_{-t})(y)v_t^y \\ \Leftrightarrow\ & \frac{1}{|\Omega_t|}\sum_{w\in\Omega_t}v_t^{(z_t(\omega),L)} \geq v_t^{(s_t',L)}. \end{aligned}$$

But now for all ω, $\omega' \in \Omega_t$, $z_{tt}(\omega) = z_{tt}(\omega')$. So, writing $z_{tt} = z_{tt}(\omega)$, then the above inequality becomes

$$\frac{1}{|\Omega_t|}\sum_{w\in\Omega_t}v_t^{(z_t(\omega),L)} = (1 - \theta)z_{tt} + c + \delta v_t^* \geq v_t^{(s_t',L)}$$

Now if $s_t' \in Z^0$, which means that the proposal will not pass, then $v_t^{(s_t',L)} = v_t^*$. So the above inequality becomes

$$\begin{aligned} & (1 - \theta)\,\frac{\mathcal{L}+1}{2\mathcal{L}} + c \geq (1 - \delta)v_t^* = (1 - \theta)\,\frac{1}{\mathcal{L}} + c \\ \Leftrightarrow\ & \frac{\mathcal{L}-1}{2\mathcal{L}} \geq 0 \Leftrightarrow \mathcal{L} \geq 1. \end{aligned}$$

Since this inequality holds, σ^t is a Nash equilibrium for G^t in this case.

On the other hand, if $s'_t \in \mathbf{Z}^1$, the proposal is one which will pass, then in order to have $|\{j \in L: s'_{tj} > w_2\}| \geq \frac{\ell}{2}$, we must have $s'_{tt} \leq s^t_{tt}$. But then

$$\frac{1}{|\Omega_t|}\sum_{w\epsilon\Omega_t} v_t^{(z_t(\omega),L)} = (1 - \theta)s^t_{tt} + c + \delta v^*_t \geq (1 - \theta)s'_{tt} + c + \delta v^*_t = v_t^{(s'_t,L)}.$$

Hence, σ^t is a Nash equilibrium for G^t.

5. CONCLUDING COMMENTS

We have developed a formal model of voter behavior and legislative decision making in which the seniority system and the incumbency effects emerge as an equilibrium. There are a number of weaknesses in the above model. We have assumed an unrealistically simple model of the legislative session, and of how seniority plays a role. Namely, the legislative session is characterized by a random recognition voting game similar to the Baron Ferejohn model, and the only effect of seniority is to change the probability of recognition on the first round. Secondly, we assume that the only decision made by the legislature is a decision on the division of a fixed pie. We also assume that legislators preferences are a function of how much they get for their constituents, rather than just being a function of whether they are reelected. We hope to remedy some of these weaknesses in the future. Despite these obvious weaknesses of the model, we feel that the model illustrates that it is possible to construct consistent formal models which connect legislative organization with reelectoral goals of legislators.

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