No Cosmological D=11 Supergravity

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Abstract

We show, in two complementary ways, that D=11 supergravity—in contrast to all its lower dimensional versions—forbids a cosmological extension. First, we linearize the putative model about an Anti de Sitter background and show that it cannot even support a "global" supersymmetry invariance; hence there is no Noether construction that can lead to a local supersymmetry. This is true with the usual 4−form field as well as for a "dual", 7-form, starting point. Second, a cohomology argument, starting from the original full nonlinear theory, establishes the absence of deformations involving spin 3/2 mass and cosmological terms. In both approaches, it is the form field that is responsible for the obstruction. "Dualizing" the cosmological constant to an 11-form field also fails.

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The recent revival of $D = 11$ supergravity [1] is connected to its role as a sector of M-theory unification. Of the many special properties of D=11 supergravity, one of the most striking is that it is unique and seems to forbid a cosmological term extension, which *is* allowed in all lower ($D \leq 10$) dimensions. In view of the importance of this question to lower-D structures in the duality context, we propose to establish this obstruction in a concrete physical way[1].

We will proceed from two complementary starting points. The first will be the Noether current approach, in which we attempt—and fail!—to find a linearized, "globally" supersymmetric model about an Anti de Sitter (AdS) background upon which to construct a full locally supersymmetric theory. Since a Noether procedure is indeed a standard way to obtain the full theory, in lower dimensions, the absence of a starting point for it effectively forbids the extension. In contrast, the second procedure will begin with the full (original) theory of [1] and attempt, using cohomology techniques, to construct—also unsuccessfully—a consistent deformation of the model and of its transformation rules that would include the desired fermion mass term plus cosmological term extensions. In both cases, the obstruction is due to the $4-$ (or $7-$) form field necessary to balance degrees of freedom.

First, we recall some general features relevant to the linearized approach. It is well-known that Einstein theory with cosmological term linearized about a background solution of constant

[1] To our knowledge, there have been two previous approaches to this result. One [2] consists in a classification of all graded algebras and consideration of their highest spin representations. Although we have not found an explicit exclusion of the cosmological extension in this literature, it is undoubtly implied there under similar assumptions. The second [3] considers the properties of a putative "minimal" graded Anti de Sitter algebra and shows it to be inconsistent in its simplest form. While one may construct generalized algebras that still contract to super-Poincaré, these can also be shown to fail, using for example some results of [4]. In [3], a Noether procedure, starting from the full theory of [1], was also attempted; as we shall show below, there is an underlying cohomological basis for that failure.

curvature retains its gauge invariance and degree of freedom count, with the necessary modification that the vielbein field's gauge transformation is the background covariant $\delta h^a_\mu = D_\mu \xi^a$. Similarly it is also known that the free spin 3/2 field's gauge invariance in this space is no longer $\delta\psi_\mu = \partial_\mu \alpha(x)$ or even $D_\mu \alpha(x)$, but rather the extended form [5]

$$\delta\psi_\mu = \mathcal{D}_\mu\, \alpha(x) \equiv (D_\mu + m\gamma_\mu)\alpha(x) \tag{1}$$

where $\mathcal{D}_\mu$ has the property that $[\mathcal{D}_\mu, \mathcal{D}_\nu] = 0$ when the mass m is "tuned" to an AdS cosmological constant: $2m = \sqrt{-\Lambda}$ (in $D = 11$). The modified transformation (1) then keeps the degree of freedom count for ψ_μ the same as in flat space, provided—as is needed for consistency—that the ψ's action and field equations also involve $\mathcal{D}_\mu$ rather than D_μ. [This is of course the reason for the "mass" term $m\bar{\psi}_\mu \Gamma^{\mu\nu}\psi_\nu$ acquired by the spinor field to accompany the cosmological one for gravity.] Given the above facts, the 3-form potential $A_{\mu\nu\rho}$ still balances fermi/bose degrees of freedom here. [For now, we keep the same field content as in the flat limit.] Unlike the other two fields, its action only involves curls and so it neither needs nor can accomodate any extra terms in the background to retain its gauge invariance and excitation count; indeed, the only possible quadratic addition would be a – true – mass term $\sim \Lambda\, A^2$ that would destroy both (there would be 120, instead of the 84 massless, excitations). One can therefore expect, with reason, that the problem will lie in the form (rather than gravity) sector's transformation rules. In the AdS background, the desired "globally" supersymmetric free field starting point involves the Killing spinor $\epsilon(x)$, $\mathcal{D}_\mu \epsilon(x) = 0$, which is unrelated to the general gravitino gauge spinor $\alpha(x)$ in (1). [Note that we can neither use $\partial_\mu \epsilon = 0$ because space is curved, nor $D_\mu \epsilon = 0$ because only $\mathcal{D}_\mu$'s commute.] The rules are essentially fixed from the known flat background ones (to which they must reduce for $\Lambda = 0$),

$$\begin{aligned}
\delta\,\psi_\mu &= \delta_h \psi_\mu + \delta_A \psi_\mu = \left(\frac{1}{4} X_{\mu ab}(h)\Gamma^{ab} - m\gamma^a h_{\mu a}\right)\epsilon + i/144\;(\Gamma^{\alpha\beta\gamma\delta}{}_{\mu} - 8\,\Gamma^{\beta\gamma\delta}\delta^\alpha_\mu)\epsilon\, F_{\alpha\beta\gamma\delta} \\
\delta\, h_{\mu a} &= -i\;\bar{\epsilon}\,\Gamma_a \psi_\mu \qquad\quad \delta\, A_{\mu\nu\rho} = 3/2\,\bar{\epsilon}\,\Gamma_{[\mu\nu}\psi_{\rho]}.
\end{aligned} \tag{2}$$

The linearized connection $X(h)$ is derived by a linearized "vanishing torsion" condition $D_\mu h_{\nu a} + X_{\mu ab} e^b_\nu - (\nu\mu) = 0$; throughout, the background vielbein is $e_{\mu a}$ and its connection is $\omega_{\mu ab}(e)$. Now vary the spinorial action $I[\psi] = -1/2 \int (dx) \psi_\mu \Gamma^{\mu\alpha\beta} \mathcal{D}_\alpha \psi_\beta$ (world Γ indices are totally antisymmetric and $\Gamma^\mu = e^\mu{}_a \gamma^a$ etc.). It is easily checked that although $[\Gamma, \mathcal{D}] \neq 0$, varying $\bar{\psi}$ and ψ does yield the same contribution, and using (2) we find

$$\delta I[\psi] = \delta_h I[\psi] + \delta_A I[\psi] =$$
$$- \quad i/8 \int (dx) E^{\mu b} (-i\kappa \bar{\epsilon} \Gamma_a \psi_\mu) - i/8 \int (dx) [D_\alpha F^{\alpha\mu\rho\sigma} (\bar{\epsilon} \Gamma_{[\mu\nu} \psi_{\rho]}) + m \bar{\psi}_\mu (\Gamma^{\mu\alpha\beta\rho\sigma} F_{\alpha\beta\rho\sigma}) \epsilon] \ . \quad (3)$$

Here $E^{\mu b}$ is the variation of the Einstein cosmological action linearized about AdS. The form-dependent piece of (3) has a first part that behaves similarly, namely it is proportional to the form field action's variation $D_\alpha F^{\alpha\mu\rho\sigma}$ (the Chern–Simons term, being cubic, is absent at this level). With the transformation choice (2), the variation of the Einstein plus form actions almost cancels (3). There remains $\bar{\psi} F \epsilon$, the $A-$variation of the gravitino mass term. What possible deformations of the transformation rules (2) and of the actions might cancel this unwanted term? The only dimensionally allowed change in (2) is a term $\bar{\delta}\psi_\mu \sim m \not{A}_\mu \epsilon$; however, it will give rise to unwanted gauge-variant contributions from the $m\bar{\psi}\Gamma\psi$ term $\sim m^2 \bar{\psi} \Gamma A \epsilon$, that would in turn require a true mass term $I_m[A] \sim m^2 \int (dx) A^2$ to cancel, thereby altering the degree of freedom count. Indeed these two deformations, $\bar{\delta}\psi_\mu$ and $I_m[A]$, are the only ones that have nonsingular $m \to 0$ limits. A detailed calculation reveals, however, that even with these added terms, the action's invariance cannot be preserved. In particular, there are already variations of the A^2 term that cannot be compensated. A completely parallel calculation starting with a dual, 7-form, model yields precisely the same obstruction[2]: defining the $4-$form dual of the $7-$form, we have the same structure as the

[2]The $7-$ form variant was originally considered by [6], who argued that it was excluded in the non-cosmological case, but the possibility for a cosmological extension was not entirely removed; the latter was considered and rejected at the Noether level in [3].

4−form case, up to normalizations, and face the same non-cancellation problem; also here a mass term is useless.

Our second approach analyses the extension problem in the light of the master equation and its consistent deformations [7, 8, 9]; see [10] for a review of the master equation formalism appropriate to the subsequent cohomological considerations. One starts with the solution of the master equation $(S, S) = 0$ [10, 11] for the action of an undeformed theory (for us that of [1]). One then tries to perturb it, $S \to S' = S + g\Delta S^{(1)} + g^2\Delta S^{(2)} + ...$, where g is the deformation parameter, in such a way that the deformed S' still fulfills the master equation $(S', S') = 0$. As explained in [7] any deformation of the action of a gauge theory and of its gauge symmetries, consistent in the sense that the new gauge transformations are indeed gauge symmetries of the new action, leads to a deformed solution S' of the master equation. Conversely, any deformation S' of the original solution S of the master equation defines a consistent deformation of the original gauge invariant action and of its gauge symmetries. In particular, the antifield–independent term in S' is the new, gauge-invariant action; the terms linear in the antifields conjugate to the classical fields define the new gauge transformations [7, 12] while the other terms in S' contain information about the deformation of the gauge algebra and of the higher-order structure functions. To first order in g, $(S', S') = 0$ implies $(S, \Delta S^{(1)}) = 0$, *i.e.*, that $\Delta S^{(1)}$ (which has ghost number zero) should be an observable of the undeformed theory or equivalently $\Delta S^{(1)}$ is "BRST-invariant" - recall that the solution S of the master equation generates the BRST transformation in the antibracket. To second order in g, then, we have $(\Delta S^{(1)}, \Delta S^{(1)}) + 2(S, \Delta S^{(2)}) = 0$, so the antibracket of $\Delta S^{(1)}$ with itself should be the BRST variation of some $\Delta S^{(2)}$.

Let us start with the full nonlinear 4-dimensional $N = 1$ case, where a cosmological term *can* be added, for contrast with $D = 11$. The action is [13]

$$I_4[e^a_\mu, \psi_\lambda] = -\frac{1}{2}\int (dx)(\frac{1}{2} e e^{a\mu} e^{b\nu} R_{\mu\nu ab} + \overline{\psi}_\mu \Gamma^{\mu\sigma\nu} D_\sigma \psi_\nu), \tag{4}$$

where $e \equiv \det(e_{a\mu})$ and D_μ here is of course with respect to the full vierbein; it is invariant under the local supersymmetry (as well as diffeomorphism and local Lorentz) transformations

$$\delta e^a_\mu = -i\bar{\epsilon}\Gamma^a\psi_\mu\ , \quad \delta\psi_\lambda = D_\lambda \epsilon(x)\ , \tag{5}$$

and under those of the spin connection ω^{ab}_μ. The solution of the master equation takes the standard form

$$S = I_4 + \int\int (dx)(dy)\varphi^*_i(x) R^i_A(x,y) C^A(y) + X, \tag{6}$$

where the φ^*_i stand for all the antifields of antighost number one conjugate to the original (antighost number zero) fields $e_{a\mu}$, ψ_λ, and where the C^A stand for all the ghosts. The $R^i_A(x,y)$ are the coefficients of all the gauge transformations leaving I_4 invariant. The terms denoted by X are at least of antighost number two, i.e. contain at least two antifields φ^*_i or one of the antifields C^*_α conjugate to the ghosts. The quadratic terms in φ^*_i are also quadratic in the ghosts and arise because the gauge transformations do not close off-shell [14]. We next recall some cohomological background [10] related to the general solution of the "cocycle" condition $(S, A) \equiv sA = 0$ for A with zero ghost number. If one expands A in antighost number $A = A_0 + \bar{A}$, where $\bar{A}$ denotes antifield-dependent terms, one finds that the antifield-independent term A_0 should be on-shell gauge-invariant. Conversely, given an on-shell invariant function(al) A_0 of the fields, there is a unique, up to irrelevant ambiguity, solution A (the "BRST invariant extension" of A_0) that starts with A_0. Below we shall obtain the required A_0. The relevant property that makes the introduction of a cosmological term possible in four dimensions is the fact that a gravitino mass term $m\int(dx)e\overline{\psi}_\lambda\Gamma^{\lambda\rho}\psi_\rho$ defines an observable; one easily verifies that it is on-shell gauge invariant under (5). Hence, one may complete it with antifield-dependent terms, to define the initial deformation $m\Delta S^{(1)}$ that satisfies $(\Delta S^{(1)}, S) = 0$. The antifield-dependent contributions are fixed by the coefficients of the field equations in the gauge variation of the mass term. Specifically, since one must use the *undeformed* equations for the gravitino and the spin connection in order to verify the invariance of the mass term under supersymmetry transformations, these contributions will be of the form $\psi^* C$ and $\omega^* C$, where C is the commuting supersymmetry ghost. They then lead to the known [15] modification of the supersymmetry transformation rules for the gravitino and the spin connection when the mass term is turned on[3]. Having obtained an acceptable first order deformation, $m\Delta S^{(1)}$, we must in principle proceed to verify that $(\Delta S^{(1)}, \Delta S^{(1)})$ is the BRST variation of some $\Delta S^{(2)}$; indeed it is , with $\Delta S^{(2)} = 3/2 \int (dx) e$, as expected. There are no higher order terms in the deformation parameter m because the antibracket of $\Delta S^{(1)}$ with $\Delta S^{(2)}$ vanishes ($\Delta S^{(1)}$ does not contain the antifields conjugate to the vierbeins), so the complete solution of the master equation with cosmological constant is $S + m\Delta S^{(1)} + m^2 \Delta S^{(2)}$, the action of [15]. [The possibility of introducing the gravitino mass term as an observable deformation hinged on the availability of a dynamical curved geometry in the sense that while $(S, \Delta S^{(1)}) = 0$ is always satisfied, only then is $(\Delta S^{(1)}, \Delta S^{(1)})$ BRST exact, i.e. is there a second order –gravitational– deformation.]

To summarize the analysis of the four-dimensional case, we stress that the cosmological term appears, in the formulation without auxiliary fields followed here, as the second order term of a consistent deformation of the ordinary supergravity action whose first order term is the gravitino mass term, with the mass as deformation parameter; it is completely fixed by the requirement that the deformation preserve the master equation and hence gauge invariance. This means, in particular, that the cosmological constant itself must be fine-tuned to the value $-4m^2$, as explained

[3] A complete investigation of the BRST cohomology of $N = 1$ supergravity has been recently carried out in [16].

in [5].[4]

Let us now turn to the action I_{CJS} of [1] in $D = 11$. The solution of the master equation again takes the standard form[5]

$$S = I_{CJS} + \int\int (dx)(dy)\varphi_i^*(x)R_A^i(x,y)C^A(y) + \int (dx)C^{*\mu\nu}\partial_\mu\eta_\nu + \int (dx)\eta^{*\mu}\partial_\mu\rho + Z, \qquad (7)$$

where the η_ν and ρ are the ghosts of ghosts and ghost of ghost of ghost necessary to account for the gauge symmetries of the 3-form $A_{\lambda\mu\nu}$, and where Z (like X in (6)) is determined from the terms written by the $(S,S) = 0$ requirement. As in $D = 4$, we seek a first-order deformation analogous to

$$\Delta S^{(1)} = \frac{1}{2}m\int (dx)e\overline{\psi}_\lambda\Gamma^{\lambda\rho}\psi_\rho + \text{ antifield-dep.} \qquad (8)$$

However, contrary to what happened at $D = 4$, the mass term no longer defines an observable, as its variation under local supersymmetry transformations reads

$$\delta(e\overline{\psi}_\lambda\Gamma^{\lambda\rho}\psi_\rho) \approx -\frac{i}{18}\overline{\psi}_\mu\Gamma^{\mu\alpha\beta\gamma\delta}\epsilon F_{\alpha\beta\gamma\delta} + O(\psi^3) \qquad (9)$$

where $\approx$ means equal on shell up to a divergence. Indeed, the condition that the r.h.s. of (9) also weakly vanish is easily verified to imply, upon expansion in the derivatives of the gauge parameter ϵ, that $\overline{\psi_\mu}\Gamma^{\mu\alpha\beta\gamma\delta}\epsilon F_{\alpha\beta\gamma\delta}$ must vanish on shell, which it does *not* do.

Can one improve the first-order deformation (8) to make it acceptable? The cosmological term will not help because it does not transform into F. The only possible candidates would be

[4]We emphasize that in this procedure, one cannot start with the cosmological term as a $\Delta S^{(1)}$. Indeed, the variation of the cosmological term under the gauge transformations of the undeformed theory is algebraic in the fields and hence does not vanish on-shell, even up to a surface term. Hence it is not an observable of the undeformed theory, and so cannot be a starting point for a consistent deformation: adding the cosmological term (or the sum of it and the mass term) as a $\Delta S^{(1)}$ to the ordinary supergravity action is a much more radical (indeed inconsistent !) change than the gravitino mass term alone.

[5]Many of the features of (7) were anticipated in [17].

functions of the 3-form field. In order to define observables, these functions must be invariant under the gauge transformations of the 3-form, at least on-shell and up to a total derivative. However, in 11 dimensions, the only such functions can be redefined so as to be off-shell (and not just on-shell) gauge invariant, up to a total derivative. This follows from an argument that closely patterns the analysis of [18], defining the very restricted class of on-shell invariant vertices that cannot in general be extended off-shell. [The above result actually justifies the usual assumptions, *e.g.*, those of [3] that "on–" implies "off–".] Thus, the available functions of A may be assumed to be strictly gauge invariant, i.e., to be functions of the field strength F (which eliminates A^2; also, changing the coefficient of the Chern-Simons term in the original action clearly cannot help). But it is easy to see that no expression in F can cancel the unwanted term in (9), because of a mismatch in the number of derivatives. Hence, there is no way to improve the mass term to turn it into an observable in 11 dimensions. It is the A-field part of the supersymmetry variation of the gravitino that is responsible for the failure of the mass term to be an observable, just as it was also responsible for the difficulties described in the first, linearized, approach. Since the cohomology procedure saves us from also seeking modifications of the transformations rules, we can conclude that the introduction of a cosmological constant is obstructed already at the first step in $D = 11$ supergravity from the full theory end as well.

In our discussion, we have assumed (as in lower dimensions) both that the limit of a vanishing mass m is smooth[6] and that the field content remains unchanged in the cosmological variant. Any "no-go" result is of course no stronger than its assumptions, and ours are shared by the earlier treatments [2, 3] that we surveyed. There is one (modest) loosening that can be shown not to work either, inspired by a recent reformulation [20] of the $D = 10$ cosmological model [19]. The idea

[6] This restriction is not necessarily stringent: in cosmological $D = 10$ supergravity [19], there is m^{-1} dependence in a field transformation rule, but that is an artefact removable by introducing a Stuckelberg compensator.

is to add a deformation involving a nonpropagating field, here the 11-form $G_{11} \equiv dA_{10}$, through an addition $\Delta I \sim \int (dx)[G_{11} + b\bar{\psi}\Gamma^9\psi]^2$. The A_{10}-field equation states that the dual, $\epsilon^{11}[G_{11} + b\bar{\psi}\Gamma^9\psi]$ is a constant of integration, say m. The resulting supergravity field equations look like the "cosmological" desired ones. However, while this "dualization" works for lower dimensions, in $D = 11$ we are simply back to the original inconsistent model with supersymmetry still irremediably lost, as can be also discovered –without integrating out– in the deformation approach.

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