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TESTING THE DEMOCRATIC HYPOTHESIS IN THE  
PROVISION OF LOCAL PUBLIC GOODS

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## ABSTRACT

The financing of local public goods in French communities can be viewed, until 1980, as a one dimensional choice. We propose a model to formalize this choice which results in the best choice of the "median" agent in a population in which two types of citizens have been distinguished. Those who pay and those who do not pay the "taxe professionnelle". A translog specification of the model is estimated using data about 36 communities near the city of Toulouse, France. The democratic hypothesis according to which both types of agents mentioned above have the same weight in the decision process is rejected. Moreover, we do not reject the hypothesis that this non democratic bias decreases with the size of the city.

### 1. Introduction

The decision processes used in the provision of local public goods are in general very complex given the various forms of financing and the number of goods involved. However, the political-economic theory available for empirical research is basically reduced to the median agent-paradigm<sup>1</sup> which requires both the unidimensionality of decisions and single peaked preferences.

Curiously enough, it happens that the methods of financing local public goods used in French communities until 1980 can be, despite their apparent complexity, assimilated to unidimensional decisions. Even though four different taxes were used, the structure of these taxes were fixed by law so that only a general level had to be chosen by the community<sup>2</sup>.

Decisions are not taken by direct voting as in some countries, but by the council of the community which is democratically elected : If citizens' preferences are single peaked we can postulate the assumption that the

<sup>1</sup> See Bowen (1943), Black (1948), Downs (1957), Comanor (1976) and Romer and Rosenthal (1979) for the theory and Bergstrom and Goodman (1973), Inman (1978), Holcombe (1980) and Gramlich and Rubinfeld (1982) for empirical implementations.

<sup>2</sup> We are not concerned here with the allocation of this budget among the different types of local expenditures. We make the assumption that this imputation does not influence the choice of the budget's amount. This could be the case for example if the detailed choice of public goods was made by the median agent (or the generalized median agent of this paper) with Cobb-Douglas preferences across local public goods

decision taken by the council is as if it corresponded to the best choice of the median agent. We pursue this line of research in this paper. However, two types of agents must be distinguished, those who only pay taxes on their housing expenses and those who, in addition, have a professional activity requiring the payment of the "taxe professionnelle" (a trade income tax). With respect to these categories, two observations are in order. First it is acknowledged that communities do make efforts to attract firms on their area. Second, trade income tax payers constitute a small and usually well informed group able to influence other groups' choices. From these observations one can suspect that the trade income tax payers group has a larger weight than its demographic weight in the decisions about public goods. Moreover this overweight might be greater in small communities than in large ones. Indeed in small communities notabilities often play a leading role and such neighbouring communities are competing with each other when seeking after firms whereas the larger ones do not face this problem to the same extent.

French local taxes are described in section 2. In section 3 we provide a model of preferences implying indirect utility functions which are single peaked simultaneously for both types of agents. We then assume that decisions are the outcome of a weighted majority rule in which the two types of agents have different weights. Section 4 describes the econometric model we derive from this theory and the estimation method we use. We test two types of hypotheses within this theory : first, the democratic assumption that both types have the same weight against the alternative that the voting procedure is a weighted majority rule, second that the non democratic bias decreases with the size of the community. Section 4 describes the various sets of data we need to undertake the tests. Results are given in section 5. Technicalities are relegated in appendices and a few concluding comments are offered in section 6.

### 2. French Local Taxes

Public goods are mainly provided at four levels : state, regions (22), departments (101), communes (36400). In the budget of a community appear two types of expenses (current expenses and investment) which are balanced by four kinds of resources : Subsidies from superior levels (on average 36 % in 1981), loans (15 %), local taxes (34 %), other resources among which mainly tariffs for special local public goods for which exclusion of use is easy (supply of water, swimming pools...).

There are four main local taxes : The "taxe d'habitation" which is collected from households and is based on a value of their dwelling called the "valeur locative". This value has been determined administratively in 1973 for the last time and is now actualised each year by multiplying with a coefficient provided nationally. The "taxe foncière sur les propriétés bâties" which is collected from owners of houses. It is a property tax based on a value of houses, called the "revenu cadastral", which is half the sum of the "valeurs locatives" of the dwellings composing the house. The "taxe foncière sur les propriétés non bâties" which is collected from owners of non constructed pieces of land and is based on their values. The "taxe professionnelle" which is collected from corporate and uncorporate firms and is based on the value of their productive capital and, according to activities, on wages paid or sales.

There are other specific taxes, such as a tax on mines, a tax on electricity... which represent less than 9 % of the budget in cities of more than 10 000 inhabitants and that we include here in non fiscal resources.

Until 1980, a single decision was taken to fix the rates of the four main taxes, as follows : The local assembly decided the level of total expenditures and, taking into account other resources, it derived the total proceeds needed from those taxes. This total amount was divided into four parts corresponding to the four taxes, using predetermined coefficients setting the fiscal burdens of the four categories of tax-payers. Roughly, the coefficients were such that the share of each tax payer group remained what it would have been with the 1917 tax-rates and the actual basis. Finally, the rates of taxes to be applied to each tax payer's fiscal bases were determined by dividing each part by the total basis of the tax.

So until 1980 and also for some communities in 1981<sup>1</sup>, the ordinary budget of a city can be written :

$$z = h B_h + f B_f + g B_g + t B_t + T$$

where  $h, f, g, t$  are the four tax rates,  $B_h, B_f, B_g, B_t$  are the four corresponding tax bases and  $T$  is the total of the other ordinary resources.

$T$  being assumed predetermined, the vote on  $z$  is a vote on  $(h B_h + f B_f + g B_g + t B_t)$  which, as seen before, can be written :

$$h B_h \left( 1 + \frac{f B_f}{h B_h} + \frac{g B_g}{h B_h} + \frac{t B_t}{h B_h} \right) = h B_h K$$

where  $K$  is a constant since the relative shares of each tax-payer group are fixed. Thus,  $B_h$  being exogenous, the vote on  $z$  turns out to be a vote on  $h$  (one of the four tax rates) and  $z$  can be written :

$$z = h B + T$$

where

$$B = B_h \left( 1 + \frac{f B_f}{h B_h} + \frac{g B_g}{h B_h} + \frac{t B_t}{h B_h} \right).$$

In fact, in our study, we take :

$$B = (1 + 0.38) B_h \left( 1 + \frac{f B_f}{h B_h} + \frac{g B_g}{h B_h} \right) + B_h \frac{t B_t}{h B_h}$$

where  $t$  refers to the "taxe professionnelle", because one of the subsidies received by the city is proportional to the amount of taxes collected from households (0.38 F of subsidy for 1 F of such taxes).

As the sample refers only to urban communities the "taxe foncière sur les propriétés non bâties" is neglected. There is a final difficulty due to the fact that the "taxe foncière sur les propriétés bâties" is payed by owners and it is not clear to which extent it is transferred to renters. We will take the two extreme attitudes of including it in the other resources of the community and of assuming that it is entirely transferred. We get in this case :  $z = h B_f + \frac{f}{2} B_f + t B_t + T = (h + \frac{f}{2}) B_f + t B_t + T = h' B' + T$ . In every expression, including  $\delta, h$  is then replaced by  $h'$ . The results appear robust to these changes of specification.

### 3.The Model

Let us consider a consumer  $i$  in an economy with  $L$  private goods and one public good. Let  $x^i \in \mathbb{R}_+^L$  be his consumption of private goods and  $z \in \mathbb{R}_+$  his consumption of public good. Let  $p \in \mathbb{R}_+^L$  the price vector of private goods and  $R^i$  consumer  $i$ 's income.

We define consumer  $i$ 's semi indirect utility function  $V^i(p, R^i, z)$  as :

<sup>1</sup> Cities which chose to keep a proportional growth of the four tax-rates.

$$V^i(p, R^i, z) = \max_{x^i} U^i(x^i, z) \\ \text{S.T. } p \cdot x^i \leq R^i$$

Using standard arguments (see Varian (1978)) we can show that  $V^i$  is continuous, decreasing in  $p$ , increasing in  $R^i$ , strictly quasi-concave in  $p$ , homogeneous of degree 0 in  $(p, R^i)$  and from our assumptions it is strictly quasi-concave in  $z$ .

In our work we assume that there are only two private goods, the numeraire which represents an aggregate of private goods and housing. Housing is measured by an aggregate of characteristics and let us denote  $p \in \mathbb{R}_+$  its price.

For the econometric implementation, we postulate a translog family of semi indirect utility functions such that :

$$\begin{aligned} \text{Log } V^i &= \alpha_1 \text{Log } p + \alpha_2 \text{Log } R^i + \\ &\frac{1}{2} (\beta_{11} (\text{Log } p)^2 + 2 \beta_{12} \text{Log } p \text{Log } R^i + \beta_{22} (\text{Log } R^i)^2) \\ &+ \gamma_1 \text{Log } z + \frac{1}{2} \gamma_2 (\text{Log } z)^2 \end{aligned}$$

We wish to use the median-voter paradigm to determine the chosen level of the tax. However, we must distinguish two types of agents. First, we have the citizens who pay only taxes on housing, type 1 agents. In community  $j$ , they face a price of housing<sup>1</sup> :

$$(1 + h_j) p_j$$

The total budget of community  $j$  used to finance local public goods is (see appendix I) :

$$z_j = h_j B_j + T_j$$

where  $h_j$  is the rate of housing tax in community  $j$ ,  
 $B_j$  is the generalized basis of the tax in community  $j$ ,  
 $T_j$  are other incomes of community  $j$ .

<sup>1</sup> For one of these agents living in community  $j$  and whose housing has a surface of  $s$ , the cost of the public good is  $h_j p_j s$ .

The semi indirect utility function of an agent  $i$  of type 1 is :

$$\begin{aligned} \text{Log } V^i &= \alpha_1 \text{Log } (1 + h_j) p_j + \alpha_2 \text{Log } R^i + \\ &\frac{1}{2} (\beta_{11} (\text{Log } (1 + h_j) p_j)^2 + 2 \beta_{12} \text{Log } (1 + h_j) p_j \text{Log } R^i + \\ &\beta_{22} (\text{Log } R^i)^2) \\ &+ \gamma_1 \text{Log } (h_j B_j + T_j) + \frac{1}{2} \gamma_2 (\text{Log } (h_j B_j + T_j))^2 \end{aligned}$$

Agents of type 2 pay in addition to housing taxes the "taxe professionnelle" that we assimilate<sup>1</sup> to an income tax at the rate  $\delta_j h_j$ . The semi indirect utility function of an agent  $i$  of type 2 is<sup>2</sup> :

$$\begin{aligned} \text{Log } V^i &= \alpha_1 \text{Log } (1 + h_j) p_j + \alpha_2 \text{Log } (1 - \delta_j h_j) R^i + \\ &\frac{1}{2} (\beta_{11} (\text{Log } (1 + h_j) p_j)^2 + 2 \beta_{12} \text{Log } (1 + h_j) p_j \text{Log } (1 - \delta_j h_j) R^i + \\ &\beta_{22} (\text{Log } (1 - \delta_j h_j) R^i)^2) \\ &+ \gamma_1 \text{Log } (h_j B_j + T_j) + \frac{1}{2} \gamma_2 (\text{Log } (h_j B_j + T_j))^2 \end{aligned}$$

We next assume that in each community  $j$  the level  $h_j$  is determined by a weighted majority rule. We will impose conditions on the parameters such that the objective functions of both types of agents are single peaked in  $h_j$ <sup>3</sup>. Then, we know that the weighted majority rule yields a well defined social choice decision<sup>4</sup> corresponding to the choice of the generalized median voter.

<sup>1</sup> The "taxe professionnelle" is actually rather complex but it is legitimate as a first approximation to treat it as a tax on income (see appendix I and the section on data).

<sup>2</sup> For one of these agents living in community  $j$ , whose housing has a surface of  $s$ , and whose income is  $R$ , the cost of the public good is  $h_j p_j s + h_j \delta_j R$ .

<sup>3</sup> In a first step we imposed constraints on the coefficients ensuring single peakedness. Later we discovered that single peakedness was automatically achieved from unconstrained estimations. The results given below are those obtained without such constraints.

<sup>4</sup> See Moulin (1980).

To be able to identify this agent we need a monotonic relationship<sup>1</sup> between incomes and desired tax levels for both types of agents. It turns out that the simple constraint  $\beta_{22} = 0$  produces the desired result (see appendix 1). We will impose this restriction below. Let  $R = R_1(h)$  this relationship for a type 1 agent and  $R = R_2(h)$  the relationship for a type 2 agent.

Let us call  $N_1^j$  and  $N_2^j$  respectively the sizes of type 1 and type 2 populations in community  $j$ , and  $g_1^j(R)$ ,  $g_2^j(R)$  the density functions of income distributions for type 1 and type 2 agents in city  $j$ .

Then, the level  $h^j$  of tax determined by weighted majority in which a type 1 agent has one vote and a type 2 agent  $m$  votes is obtained from the equation :

$$(1) \quad \frac{N_1^j}{N_1^j + m N_2^j} \int_0^{R_1(h^j)} g_1^j(R) dR + \frac{m N_2^j}{N_1^j + m N_2^j} \int_0^{R_2(h^j)} g_2^j(R) dR - \frac{1}{2} = 0$$

The democratic hypothesis corresponds to  $m = 1$ .

#### 4. Estimation Method and Test

We assume that for each community  $j$  the income distributions for the two types of agents depend only on average incomes  $\bar{R}_1^j$ ,  $\bar{R}_2^j$  and standard-errors  $S_1^j$  et  $S_2^j$ .

The model to estimate is then written :

$$(2) \quad g(h^j, \left(\frac{T}{B}\right)^j, \delta^j, \bar{R}_1^j, \bar{R}_2^j, N_1^j, N_2^j, S_1^j, S_2^j; a) = \epsilon^j \quad j = 1, \dots, J$$

<sup>1</sup> About the need for this relationship see Bergstrom and Goodman (1973) and for critical discussions see Stiglitz (1974), Romer and Rosenthal (1979) and Groves and Todo-Rovira (1987).

where  $g$  is the nonlinear function given in (1),  $a$  is the vector of parameters  $(\alpha_1, \alpha_2, \beta_{11}, \beta_{12}, \gamma_1, \gamma_2, m)$ ,  $h^j$  is the endogenous variable,  $\left(\frac{T}{B}\right)^j, \delta^j, \bar{R}_1^j, \bar{R}_2^j, N_1^j, N_2^j, S_1^j, S_2^j$  are the exogenous variables, and  $\epsilon^j$  is an error  $N(0, \sigma_\epsilon^2)$  distributed<sup>1</sup>.

We use maximum likelihood estimation. The likelihood function is maximized with respect to  $\sigma_\epsilon^2$  and the concentrated likelihood function  $L_C(a)$  obtained in appendix 3 is numerically maximized with respect to  $a$ .

The test of the democratic hypothesis,  $m = 1$ , is realized using the estimated asymptotic standard deviation of  $m$ 's estimator.

#### 5. Data

The data which are necessary to estimate the model are the following, for each city  $j$ .

- 1) density functions of income distributions of type 1 and type 2 agents,
- 2) counts of type 1 and type 2 agents,
- 3) rate ( $h^j$ ) of the "taxe d'habitation" and price of housing ( $p^j$ ),
- 4) values of  $B^j$  (the generalized tax base) and  $T^j$  (non fiscal resources of the city),
- 5) coefficient  $\delta^j$ .

We were able to get these datas for 51 observations corresponding to 36 cities (Toulouse and 35 cities around it) in 1980 and 15 among them in 1981.

##### 5.1 Distributions of income

We make the assumption that incomes are log-normally distributed with parameters  $\mu$  and  $\sigma^2$ , where  $\mu = \text{Log } \bar{R} - \frac{1}{2} \sigma^2$  and  $\sigma^2 = \text{Log} \left( \frac{S^2}{\bar{R}} + 1 \right)$

where  $\bar{R}$  and  $S^2$  are the mean and the variance of incomes, in each community and each type.

<sup>1</sup> Heteroscedastic specifications with  $\sigma^2$  decreasing with the population size were tested but we could not reject homoscedasticity.

So we need to know the mean and the variance of the income distribution for each city and each group of agents 1 and 2. The French fiscal administration gave us  $\bar{R}_1$  and  $\bar{R}_2$  for each city  $j$  and the two years 1980 and 1981, but  $S_1^j$  and  $S_2^j$  for Toulouse only and the two years. We decided to replace the variance of each city and each group by the corresponding variance of Toulouse for the same year.

## 5.2 Other data.

We give their basic statistics in the following table.

Variables	Units	Minimum	Average	Maximum	Standard-Error	Source
$\bar{R}_1$	F	44262.00	66515.39	109455.0	11259.11	Note <sup>1</sup>
$\bar{R}_2$	F	74372.00	108187.43	197755.0	20683.39	"
$N_1$	count	139	3019.20	98464	13677.73	"
$N_2$	count	18	352.16	11313	1568.67	"
$h$	rate	0.02	0.08	0.16	0.03	Note <sup>2</sup>
$\delta$	ratio	0.004	0.10	0.94	0.16	Note <sup>3</sup>
$B_h/N$	KF	4674.40	6872.86	10006.33	1247.43	Note <sup>4</sup>
$T/B$	ratio	0.02	0.13	0.29	0.07	"

## 6. Results

In table I we give the maximum likelihood estimation of the coefficients for different situations.

<sup>1</sup> French Ministry of Finance.

<sup>2</sup> Local fiscal administration.

<sup>3</sup> Local fiscal administration.

We assume that all the agents of a same city have the same  $\delta_j$ .

So  $\delta_j = \frac{t^j}{h^j} \times \frac{\text{total basis of the "taxe professionnelle"}}{N_2^j \bar{R}_2^j}$

where  $t^j$  is the rate of the "taxe professionnelle",  $h^j$  is the rate of the "taxe d'habitation",  $N_2^j$  the number of households of the second group in city  $j$  and  $\bar{R}_2^j$  the average income of the second group of agents in city  $j$ .

<sup>4</sup> Local fiscal administration.

$B_h$  is the basis of the "taxe d'habitation".  $B_h/N$ , where  $N$  is the total number of households, has been used as a proxy for the price of housing :  $p$ .

The utility function is defined up to a monotonic increasing transformation. Therefore a parameter may be arbitrarily fixed to a non zero value ; we choose  $\beta_{12} = 2300$ . We also set  $\beta_{22} = 0$  to impose the monotonicity of the functions  $R_1(h)$  and  $R_2(h)$  (see appendix 1).

Moreover we have dealt with two specifications for the model. In the first one the utility function is that given in section 3 ;  $\gamma_2$  is seen to be non significantly different from zero. In the second one  $\gamma_2$  is fixed at zero.

The following observations can be made from the table.

The  $m$  coefficient is seen to be significantly different from 1 for all situations. Therefore, we reject the democratic hypothesis that all agents have the same weight in the decision process.

When the size effect is taken into account by setting :

$$m = m_0 + m_1 \frac{\text{Log } N}{\text{Log } N_{\max}}$$

where  $N$  is the population size of the city and  $N_{\max}$  the size of the largest city in the sample, one can observe that  $m$  decreases with the size of the city (lines c, e, h).

These conclusions remain valid when : (i) the property tax, instead of being incorporated in the other taxes  $T$ , is added to the housing tax for every agent (lines f, g, h), (ii) cities with larger  $\delta$  are rejected from the sample (line d). Indeed the approximation introduced by expressing the trade income tax as part of the income tax ( a  $\delta$  ratio for each community) may be hazardous for communities containing a few large firms which usually do not play a crucial role in the decisions about local public goods.

## 7. Concluding comments

Despite a relatively limited sample we obtain two robust conclusions. There appears to be a bias in favor of the agents paying the "taxe professionnelle" in the decision process leading to the choice of the level of local taxes. Moreover this bias decreases with the size of the city. The decrease is sufficiently important so that one city in the sample ( the largest) would have an  $m < 1$ . In that city the weight of each trade income tax payer is smaller than that of the pure housing tax payer.

We think that the estimations of the coefficients in the semi indirect utility function are remarkably stable across our various tests of robustness. We should however point out a weakness of the current results. According to the theory the semi indirect utility function is decreasing in  $p$  and this is not always the case. The reason may come from the fact that  $p$

is, in our specification, always associated with  $(1 + h)$ . But  $(1 + h)$  plays two opposite roles which create a problem of "identification"<sup>1</sup>. On the one hand  $h$ , through its effect on the price of housing, has a negative effect. On the other hand, through its effect on the level of public goods, it has a positive effect. This "identification problem" affects the coefficients  $\alpha_1$ ,  $\beta_{11}$ ,  $\beta_{12}$  which determine also the sign of variation in  $p$ .

We hope to solve this difficulty by the use of a richer data basis, an unfortunately unreachable goal now.

### Appendix 1 : Monotonic Relationship between Incomes and Desired Tax Levels

Each type of agent obtains his desired level of tax by maximizing with respect to  $h$  his semi indirect utility function. For an agent  $i$  of type 2 (those who pay the professional tax) we obtain from the first order condition (dropping the index  $j$  of the community  $j$ ).

$$(A 1.1) \quad \text{Log } R_2^i = \frac{A_2}{B_2}$$

$$\begin{aligned} \text{with } A_2 = & -\frac{1}{1+h} (\alpha_1 + \beta_{11} \text{Log}(1+h)p + \beta_{12} \text{Log}(1-\delta h)) \\ & + \frac{\delta}{1-\delta h} (\alpha_2 + \beta_{12} \text{Log}(1+h)p + \beta_{22} \text{Log}(1-\delta h)) \\ & - \frac{1}{h + \frac{T}{B}} (\gamma_1 + \gamma_2 \text{Log}(h B + T)) \\ B_2 = & \frac{\beta_{12}}{1+h} - \frac{\delta}{1-\delta h} \beta_{22} \end{aligned}$$

For an agent  $i$  of type 1 we obtain :

$$(A 1.2) \quad \text{Log } R_1^i = \frac{A_1}{B_1}$$

where  $A_1$  and  $B_1$  are obtained from  $A_2$  and  $B_2$  by setting  $\delta = 0$ .

To obtain a one to one relationship between incomes and desired levels of tax, it is convenient to impose  $\beta_{22} = 0$ .

The first order condition is :

$$A_2 - B_2 \text{Log } R = 0$$

$$\begin{aligned} \text{Hence} \\ \frac{dh}{dR} = & \frac{B_2 / R}{\frac{\partial A_2}{\partial h} - \frac{\partial B_2}{\partial h} \text{Log } R} \end{aligned}$$

with  $\frac{\partial A_2}{\partial h} - \frac{\partial B_2}{\partial h} \text{Log } R < 0$  from the local second order condition of an interior maximum.

<sup>1</sup> The " " refers to the fact that it is not an identification problem asymptotically because of the nonlinearity of the model.

Hence, under  $\beta_{22}=0$ , the relationship is monotonic for both types of agents and the direction of the relationship is determined by the sign of  $\beta_{12}$ . These relationships are respectively called  $R_1(h)$ ,  $R_2(h)$  in the text.

## Appendix 2 : The Likelihood Function

We give below the likelihood function corresponding to the best specification of heteroscedasticity that we found.

Under the lognormality assumption of income distributions for both types of agents the probabilistic model is :

$$(A.2.1) \frac{N_1^j}{N_1^j + m N_2^j} \Phi\left(\frac{\text{Log } R_1(h^*j) - \mu_1^j}{\sigma_1}\right) + \frac{m N_2^j}{N_1^j + m N_2^j} \Phi\left(\frac{\text{Log } R_2(h^*j) - \mu_2^j}{\sigma_2}\right) - \frac{1}{2} = \epsilon^j$$

where  $\Phi$  is the cumulative distribution function of the standardized normal distribution and  $\epsilon^j$  is  $N(0, \sigma_\epsilon^2)$ .

(A.2.1) can be summarized as :

$$g(h^j, X^j, a) = \epsilon^j \quad j = 1, \dots, J$$

where  $h^j$  is the endogenous variable,  $X^j$  the vector of exogenous variables and  $a$  the vector of parameters.

The density function of the endogenous variable  $h$  is :

$$f(h) = \frac{\left| \frac{\partial g}{\partial h} \right|}{\sigma_\epsilon \sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma_\epsilon^2} g(h)^2\right).$$

$$\text{Let } \varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} x^2}.$$



$$\text{then } \frac{\partial g}{\partial h} = \frac{1}{\sigma_1} \left( \frac{N_1^j}{N_1^j + m N_2^j} \right) \frac{d \text{Log } R_1(hj)}{d h} \varphi \left( \frac{\text{Log } R_1(hj) - \mu_1^j}{\sigma_1} \right) +$$

$$\frac{1}{\sigma_2} \left( \frac{m N_2^j}{N_1^j + m N_2^j} \right) \frac{d \text{Log } R_2(hj)}{d h} \varphi \left( \frac{\text{Log } R_2(hj) - \mu_2^j}{\sigma_2} \right)$$

where  $\frac{d \text{Log } R_1(hj)}{d h}$  and  $\frac{d \text{Log } R_2(hj)}{d h}$  are obtained by differentiating (A 1.1) and (A 1.2) in appendix 1.

The log-likelihood function L is then :

$$L = - \frac{1}{2} \sum_{j=1}^J \left( \text{Log } 2\pi \sigma_\epsilon^2 + \frac{g(hj)^2}{\sigma_\epsilon^2} - 2 \text{Log } \left| \frac{\partial g}{\partial h}(hj) \right| \right)$$

By solving  $\frac{\partial L}{\partial \sigma_\epsilon^2} = 0$  we obtain the maximum likelihood estimator of

$\sigma_\epsilon^2$  :

$$\hat{\sigma}_\epsilon^2 = \frac{1}{J} \sum_{j=1}^J g(hj)^2.$$

Finally the concentrated log-likelihood function  $L_c$  is given by :

$$L_c = - \frac{J}{2} \left[ \text{Log } 2\pi \hat{\sigma}_\epsilon^2 + 1 - \frac{2}{J} \sum_{j=1}^J \text{Log } \left| \frac{\partial g}{\partial h}(hj) \right| \right]$$

This expression is then maximized numerically with respect to a, using a quasi-Newton algorithm (routine f04jbf of the NAG library).

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Table I - : Translog utility function . (  $\beta_{12} = 2300$  )

	$\alpha_1$	$\alpha_2$	$\beta_{11}$	$\gamma_1$	$\gamma_2$	$m_0$	$m_1$
a : Every community without size effect	-1923.7 <i>0.9</i>	16819.5 <i>5.1</i>	-2765.7 <i>10.7</i>	238.1 <i>1.4</i>	8.75 <i>0.4</i>	4.7 <i>3.5</i>	0
a' : Every community with linear size effect	-3957.7 <i>2.55</i>	17061.6 <i>2.8</i>	-2552.3 <i>15.02</i>	747.5 <i>3.7</i>	-23.32 <i>-0.84</i>	17.02 <i>1.32</i>	-20.5 <i>-0.93</i>
b : Same as a : $\gamma_2 = 0$ without size effect	-2143.7 <i>1.4</i>	16767.7 <i>4.0</i>	-2743.7 <i>16.1</i>	374.4 <i>13.0</i>	0	4.7 <i>3.9</i>	0
b' : with linear size effect.	-2884.8 <i>1.1</i>	16796.2 <i>4.2</i>	-2667.7 <i>8.2</i>	383.8 <i>10.8</i>	0	14.71 <i>3.9</i>	-16.3 <i>2.4</i>
c : Without largest $\delta$ community, without size effect.	-10172.7, <i>2.5</i>	23617.5 <i>6.6</i>	-1892.3 <i>4.1</i>	517.6 <i>10.8</i>	0	6.1 <i>3.9</i>	0
c : Same as c - with linear size effect.	-14908.4 <i>1.7</i>	22563.0 <i>6.1</i>	-1363.1 <i>7.2</i>	519.4 <i>10.6</i>	0	18.7 <i>3.1</i>	-20.1 <i>2.05</i>
e : Every community. "Impôt foncier Bâti" paid by every one - without size effect.	-2049.3 <i>1.0</i>	35043.8 <i>6.1</i>	-2728.0 <i>9.8</i>	623.0 <i>3.4</i>	-9.69 <i>0.31</i>	5.48 <i>3.4</i>	0
e' : Same as e - with linear size effect.	-4009.4 <i>1.3</i>	34663.0 <i>7.4</i>	-2523.1 <i>7.2</i>	1183.4 <i>4.4</i>	-45.7 <i>1.3</i>	20.93 <i>3.9</i>	-25.4 <i>3.7</i>
f : Same as e : $\gamma_2 = 0$ without size effect.	-1801.1 <i>0.9</i>	34931.6 <i>6.1</i>	-2756.3 <i>9.7</i>	470.8 <i>9.3</i>	0	5.54 <i>3.4</i>	0
f' : Same as f with linear size effect	-2446.3 <i>0.9</i>	34447.4 <i>7.2</i>	-2683.2 <i>8.1</i>	481.8 <i>9.8</i>	0	17.95 <i>3.01</i>	-19.8 <i>2.6</i>

In the estimations we impose  $\alpha_1 < 0$ ,  $\beta_{11} < 0$ ,  $\alpha_2$  and  $\gamma_1 > 0$ .

The *F-statistics* correspond to the test with respect to 0 for all parameters and with respect to 1 for  $m_0$ .