DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA 91125

THE MULTIPLE UNIT DOUBLE AUCTION

Charles R. Plott and Peter Gray

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ABSTRACT

The note outlines institutional features of an open outcry market that permit multiple unit or "block" trades. The purpose of the note is to introduce the process as a tool to be used in economics experiments. The detailed rules governing the multiple unit double auction (MUDA) are stated for the case where offers are tendered by voice as opposed to through a computer. The results of markets that use the rules are reported as a demonstration that convergence to equilibrium price traditionally observed in experimental markets remains with the use of the MUDA process.

THE MULTIPLE UNIT DOUBLE AUCTION

Charles R. Plott* and Peter Gray
California Institute of Technology

This note is limited to a discussion of a market instrument which was designed for experimental economics. The institutional features of a new market process are outlined. The process, which is called the multiple unit double auction (MUDA) permits multiple unit or "block" trades in the framework of a double auction. Because the process allows volume to move faster than the single unit counterparts, it allows researchers greater flexibility in experimental design. The single unit double auction (DA) has been an important tool for the development of an experimental methodology and it has also served as a background "constant" against which institutional influences on market performance could be measured. The MUDA has the same potential as serving as a background process that operates in real time to clear large volumes.

Within the traditional DA organization, markets with large volumes[1] and many traders are difficult to study. A primary feature of the traditional DA is that both buyers and sellers tender bids and asks for a *single* unit of the commodity. Such tenders stand until accepted, canceled, or replaced by a better bid or ask. In computerized versions of the traditional double auction, confirmation of trades is required. Thus, the market proceeds as a series of auctions for single units. In each auction bids ascend and asks decline until a contract (acceptance) occurs. Typically, in an oral double auction market, approximately thirty seconds per unit of equilibrium volume is the time allowed for a market period. The time is cut to a minimum of eight seconds per equilibrium trade in computerized versions with a standby queue. Since each traded unit is accompanied by bids and asks, the time used for trading will vary according to the number of

traders and the number of units each trader attempts to move. The time allocated to move a large volume of units in a given period reduces the number of periods that markets can be opened due to the practical constraint on subjects' time for participation. The purpose of the multiple unit double auction introduced here is to facilitate studies in which large volumes are an important feature.

Many experiments would benefit from a real time process that can move large quantities rapidly. (1) Small numbers of units can cause rather severe discontinuities in incentive functions held by individual subjects. The discontinuities can induce the existence of unwanted (local) equilibria. The capacity to move large volumes gives greater flexibility in experimental design to "smooth out" the incentive functions. (2) Frequently studies require the separation of theories in terms of equilibrium quantities. If the total number of units per period must be "small" then the separation is "small" relative to natural noise. (3) Studies of multiple interdependent markets frequently involve agents who wish to trade rapidly to secure a position in one market while an opportunity exists in another market. If trading is slow, the operation of a multiple market system is affected. (4) If a market has multiple equilibria, those requiring high volume will not be observed if the period is not sufficiently long. However, if the period is very long, the propensity of subjects to play to relieve boredom might bias the process in favor of the large volume equilibria. (5) Studies in which information becomes available during a period need the capacity for large volumes due to before-release volume and after-release volume. Other reasons may exist but these seem sufficiently important to justify the introduction of a possible solution to the real time volume problem.

Various means of increasing volume can be imagined. At least three different features of institutions are readily apparent. (1) Each bid/ask can be accompanied by a quantity. This reduces the number of tenders that must be made by each agent in order to move a given volume. Since tenders require time, the time needed to move a given volume is reduced. (2) Bids/asks can be entered into a "book" and automatically pulled to the floor after a contract is made. This allows

"slack time" to be utilized to make tenders that would otherwise be precluded by the floor tender.[2] (3) Replaced tenders can go back to the book if a book is maintained. This eliminates the need to repeat a tender after it has been exposed to the floor and replaced by a better tender. This is similar to a "bulletin board" market.[3] (4) More than one market can be open for the same commodity. This allows more than one bid/ask to occupy the floor at the same time similar to security markets in which more than one specialist deals in a single security. This paper deals with only the first of these four possibilities.

The study has four brief parts. The first is devoted to a documentation of the institutional details for experimenters. The second part demonstrates two important features of the process: (1) markets organized by MUDA will converge to the competitive equilibrium; and (2) units can move at speeds considered impossible under the single unit double auction. The concluding section contains remarks and observations that can be of use to experimenters.

How does the process compare with other processes that have the capacity to allow multiple unit trades such as the sealed bid-offer, posted prices, or tâtonnement? How does the process compare with the single unit double auction? How do special institutional features like computerization, standby queues, tie breaking rules, or the method of market access (voice or recognition by the auctioneer) impact on comparative behavior? Does the comparison change with differing market parameters such as shapes of curves, number of traders, equilibrium volume, and time period for which markets are open? What is the impact on relative profits, market efficiency, speed of adjustment, price variability, bidding behavior, timing of contracts, etc.? A staggering number of questions can be posed. However, at this time the relevance of these questions has not been established. No doubt as theory advances and use of the process occurs, issues can be posed, which will serve as priority guides through the maze of potential questions and comparative analyses will develop. For now the investigation is restricted to a discussion of a potential tool.

THE MUDA RULES

The process can be implemented orally or by computer. Rules for the computerized version are the same but since screen displays, information, etc., are important, the interested reader should request the appropriate software package (Johnson, Lee, Plott, 1988). The rules for the oral process are as follows.[4] (1) Traders verbally tender "bids" to buy and "asks" to sell. (2) If the trader is a buyer, the tender is submitted in the following sequence: "(trader number) BIDS (price) FOR (quantity) UNITS." If the trader is a seller, the word BIDS is replaced by ASKS. (3) A trader is recognized by the auctioneer and the bid/ask is formally tendered and becomes the floor bid/ask. (4) The replacement rule is lexicographic. If a bid price is higher than that of the tender on the floor, the floor bid is replaced by the new bid regardless of the quantity of units. If the bid price is not higher than the floor bid, the bid is rejected by the auctioneer. A replaced bid is considered canceled and is dropped from further consideration. Similarly, if the price of a tendered ask is lower than the floor ask, it replaces the floor ask regardless of the quantities. (5) Contracts can be made for all or part of the floor bid by saying, "ACCEPT: (*number*) ACCEPTS (*quantity*) UNITS OF THE BID." An ask is accepted similarly by replacing the word "bid" with "ask." (6) If the total quantity of bid or ask is accepted, then the floor is open for both new bids and asks at any level. (7) If part of the bid quantity is accepted, the remainder stands as the floor bid which can be accepted or replaced by a higher bid. However, no ask is outstanding and the floor is open for new asks at any level. (8) If part of an ask quantity is accepted, the remainder stands as the floor ask, which can be accepted or replaced by a lower ask. However, no bid is outstanding and the floor is open for new bids at any level.

The logistics of the auctioneer involve a sequence. Bids and asks are tendered verbally. The auctioneer recognizes a trader by verbally repeating the trader number but not the quotation, and then enters the bid or ask on the chalkboard. The bid or ask is then repeated verbally by the auctioneer. After this voiced quotation, the floor is open for new bids or asks. An acceptance can

take place at any time after the bid or ask is written on the chalkboard. This procedure permits a space for acceptances and a brief period of standing for a bid or ask before new tenders can replace them.

DEMONSTRATIONS OF OPERATION

The results of three experimental markets are included here as a demonstration of three important points. First, the logistics of the process work. The institutional features are internally consistent, understandable to subjects and experimenters, and foster exchange. Second, convergence of prices to near the competitive equilibrium occurs. Third, the process can move units more rapidly than can the single unit double auction. Together the three constitute a demonstration that this new tool does the job that has been mastered by the traditional tool and does something important in addition.

The general parametric configurations chosen for the experiments were "difficult" for convergence. In particular, individual demands and supplies changed every period. Even though market parameters are constant, except for a single shift, the constantly changing individual parameters are believed to contribute to a high variance in market prices and thus a slow convergence to equilibrium.[5]

In market 1 the aggregate parameters were constant for six periods; then demand shifted upward. The theoretical equilibrium volume increased from fourteen to nineteen units. In market 2 the aggregates were constant for six periods and then both demand and supply shifted downward keeping the theoretical equilibrium constant at fourteen units. The third market was a computerized market while the first two were oral. In the third market demand and supply were stationary for the first ten periods at an equilibrium volume of fifty and then demand became flat for thirty-five units and supply became flat for fifty units. The shift in the curves in all experiments occurred with no announcement of parameter changes.

In Figures 1, 2, and 3 the equilibrating tendencies can be seen in all three markets. All trades are shown for the first two experiments. Because volume was so large in the third experiment, individual trades ecannot be shown. The volume is recorded on the figure for each period. In all three experiments prices converge to near the competitive equilibrium both before and after the shift. Volumes in all three experiments are also near the competitive equilibrium. Together these three experiments demonstrate the first two points. The MUDA facilitates trades, and convergence to near the competitive equilibrium can be expected.

The third point, that MUDA does something new, can be demonstrated by a study of the time involved. A natural differentiation here exists between oral auctions and computerized auctions. The periods in the first two oral auction experiments were four and one-half minutes long after the first two periods which were seven minutes and six minutes respectively. The rule of thumb for determining the time of periods for the single unit oral double auction is thirty seconds per equilibrium transaction. Thus, when the theoretical equilibrium volume was nineteen in market one, the market period would have been about nine minutes in the traditional single unit oral auction. According to this standard the oral version of MUDA can result in periods that have been reduced by 50 percent. This means that experiments with nearly twice the number of periods for a given amount of subject time can be conducted.

Experiment 3 was a computerized MUDA. Table 1 shows the time pattern of units sold three periods before and three periods after the parameter change in experiment 3. In the table, the five minute period is divided into fifteen second intervals. The first thing to notice is that approximately fifty units are moved during the periods 9 and 10. Furthermore, the trading is not "cramped" in the sense that a few of the fifteen second intervals contain no trades and many intervals contain only one trade. In retrospect the period could have been reduced to three and one-half minutes. By comparison the rule of thumb limit for period lengths for the (PLATO) computerized single unit double auction is eight seconds per equilibrium trade so the shortest

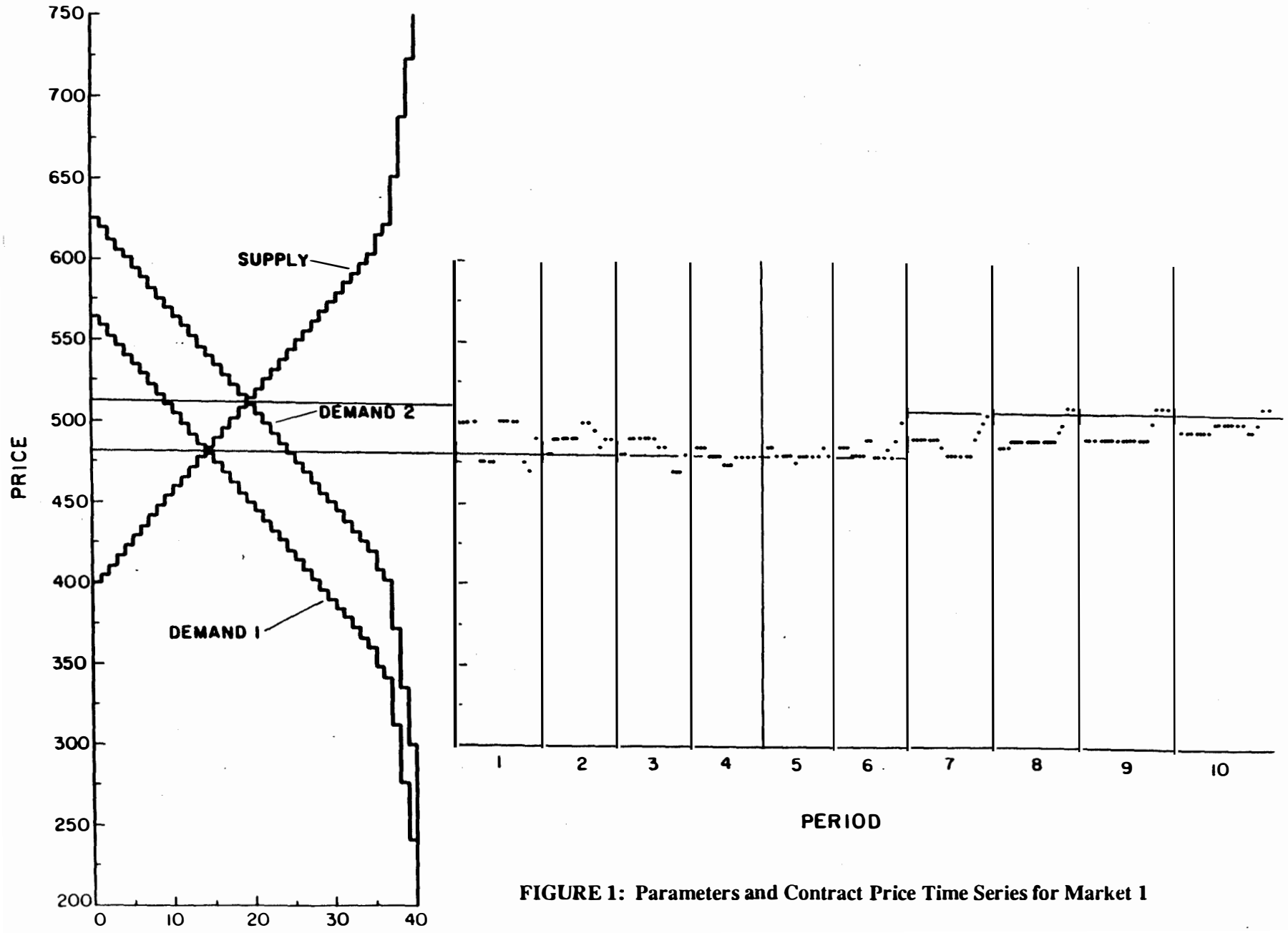

FIGURE 1: Parameters and Contract Price Time Series for Market 1

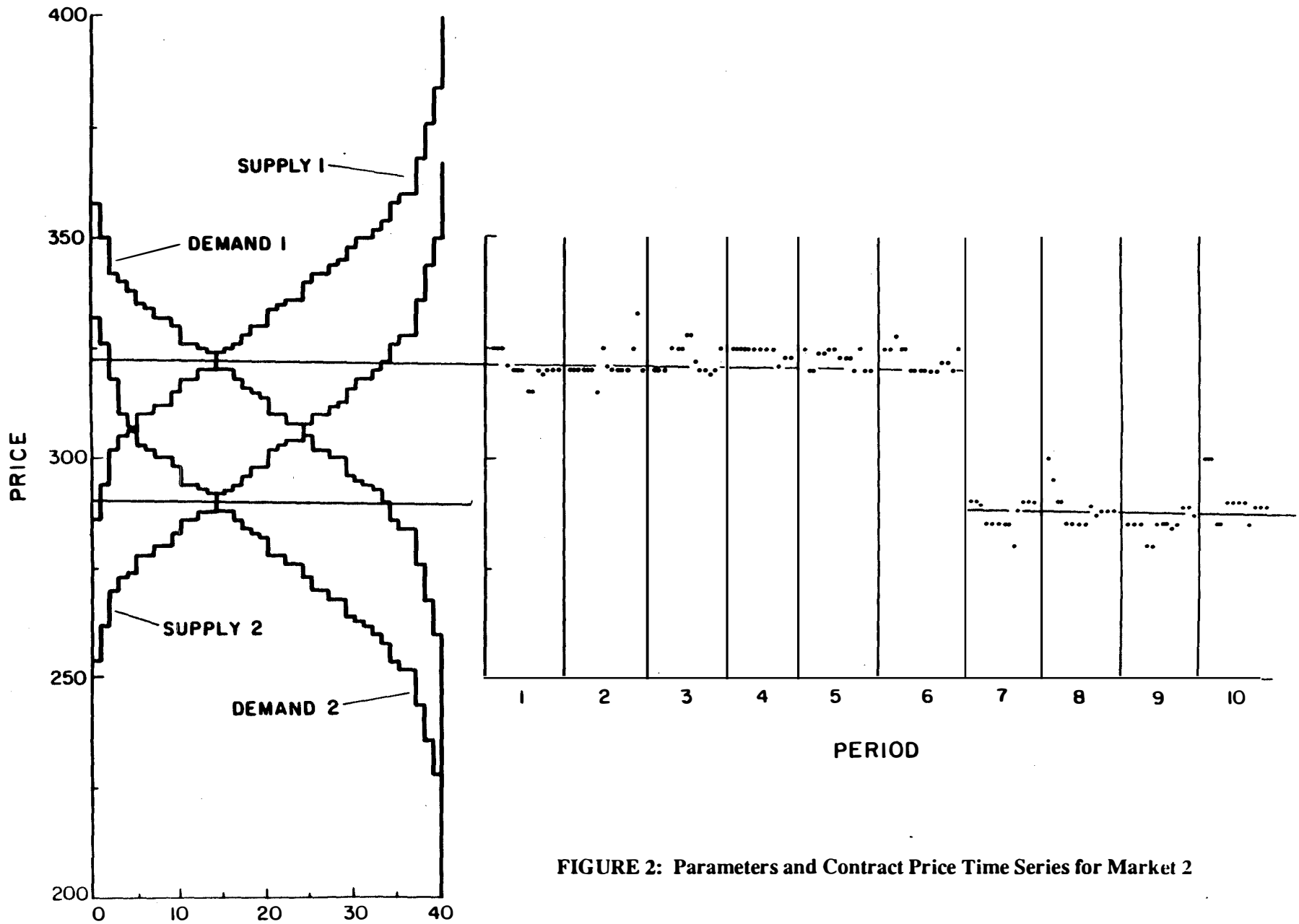

FIGURE 2: Parameters and Contract Price Time Series for Market 2

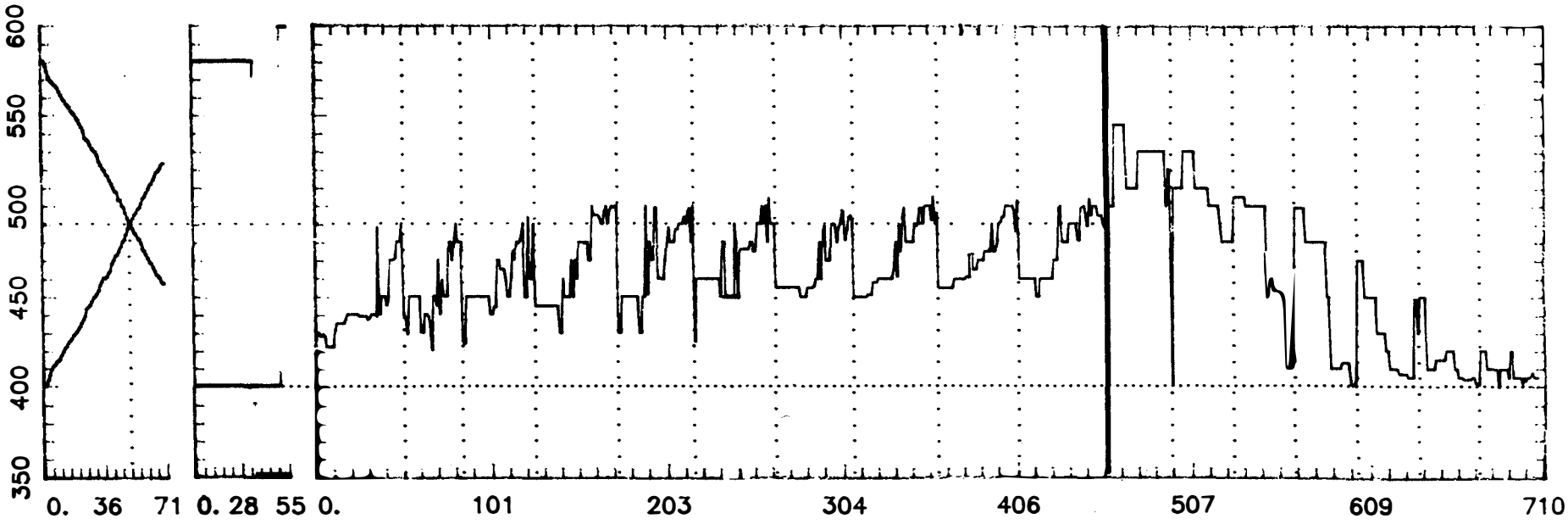

FIGURE 3: Parameters and Contract Price Time Series for Market 3, Computerized MUDA

TABLE 1: Number of Units Traded by Period and by Fifteen Second Intervals

Number of Seconds Elapsed	Period					
	8	9	10	11*	12	13
15	11	18	17	28	22	19
30	10	5	6	8	6	7
45	3	2	5	2	1	2
60	3	6	6	1	2	0
75	2	1	1	1	6	2
90	6	2	1	0	0	3
105	0	0	3	0	0	0
120	2	1	2	0	0	0
135	3	2	1	0	0	1
150	0	1	1	0	0	1
165	1	1	1	0	0	0
180	2	2	1	0	0	0
195	2	0	0	0	0	0
210	0	0	1	0	0	0
225	0	1	0	0	0	0
240	0	1	1	0	0	0
255	1	1	2	0	0	0
270	1	0	0	0	0	0
285	0	2	1	0	0	0
300	1	1	1	0	0	0

*A change in demand and supply occurred this period.

possible time allocated to move fifty units would have been about six and one-half minutes per period. Here again the periods can be shortened by a factor of approximately 0.5.

Perhaps the most dramatic demonstration of the difference occurs in the first fifteen seconds of period 11 of experiment 3. Notice in Table 1 that twenty-eight units are traded in this fifteen seconds. Twelve of these trades occurred in the first three seconds and involved two different buyers and two different sellers. These twelve trades would have been allotted over one and one-half minutes in the single unit computerized process and six minutes in the single unit oral process. Clearly compressing periods to three seconds might not result in interesting economics but the example demonstrates the relative speed of transactions. The MUDA is capable of moving volume faster than the single unit process.

CONCLUSION

The purpose of this note is somewhat different from traditional articles published in economics. The focus is on instrumentation, and research technology as opposed to general behavioral principles. The body of the note introduced a new tool and provided a demonstration which shows that the tool is capable of doing the job to which it is intended to apply. This concluding section contains some anecdotal and impressionistic accounts of things that might prove useful to those who might use the tool.

When the process is oral, the experimenter should pay particular attention to the wording of tenders as described above. The sequence presented in the text has been successful in eliminating errors by the auctioneer and allowing the auctioneer to function smoothly.

Subjects have a tendency to confuse the redemption value of a marginal trade with the value of a block trade. They sometimes think that the redemption value of the kth unit is the value they receive for a block trade of k units. Subjects can also become confused by the per unit price quotation and think the price is for the total of the units offered. Relative to the single unit double

auction there is a tendency for subjects to buy or sell too much. They can lose track of the number they have traded. After a few rounds such mistakes are infrequent.

Computerized versions have special problems. Typos can occur which with multiple units can cause immediate bankruptcy and possible termination of the experiment. One solution to this problem is to tell subjects that no gains can be made from seizing the typo of someone else. They are encouraged to announce "typo" if they see one or make one. They are then allowed to cancel. If someone "grabs" one, the action can be appealed with the experimenter as a judge. If it appears to be a typo, the contract is canceled.

The source of the speed of the multiple unit process is not obvious. The current computerized one-unit processes (PLATO) require confirmation after a trade. This alone requires a few seconds per transaction. As it turns out, a large proportion of trades in the computerized MUDA process are for one unit. Tenders are frequently in blocks but acceptances tend to be in single-unit blocks because multiple unit acceptances are at risk of being rejected.[6] It could be that standby queues together with no requirement for confirmation in the PLATO markets would be just as fast as computerized MUDA markets.

Aside from the fact that the MUDA can move units faster than single unit processes, it is not obvious that the processes differ. Differences might be observable under specific circumstances. For example, if the periods were made very short, the single unit process would be limited to a fixed volume which is known to have effects (Plott 1983). The MUDA leaves more room for subject confusion and mistakes, so experimental procedures that have an impact on learning or instructions could have an effect. The quantities associated with tenders might carry information thereby effecting the dynamics especially in markets with asymmetric information. The fact that the NY rule allows a single seller to "lock in" at a price might have some effects on convergence—especially if other institutions like price floors are operative. However, nothing of behavioral differences between the single and multiple unit process has been documented to date.

Features of the MUDA invite questions. Why is the bid cancelled after only part of an ask is accepted? One answer is that this feature reflects a type of consistency in rules. The acceptance of the ask can be interpreted as a new bid which is also an improvement (increase) over the standing bid and therefore calls for the cancellation of the standing bid. A second answer reflects considerations other than consistency. If the ask is accepted by the same person who held the standing bid, as is frequently the case, that person may no longer be willing to be committed to the bid after having just traded. In order to deal with this new problem a technology for cancelling bids and asks must be added but cancellation can be used as a strategy to limit the tenders of others.[7] Even if cancellation is possible, the original bidder must remember to cancel or must cancel quickly before someone else accepts the bid. On balance, the benefits of noncancellation over cancellation rules are not obvious.

Why does price take priority over quantity? We are open for suggestions of alternative processes. The priority of price over quantity is a distinguishing feature of MUDA. Perhaps some alternative that gives more weight to quantity in the bidding process would perform better.

Why are agents required to accept a partial filling of orders? Clearly MUDA could be altered to incorporate "all or none" offers. Our *conjecture* is that markets characterized by only "all or none" tenders would operate less efficiently. Such contracts would appear to be less compatible with adjustments of tenders to the marginal units. Clearly tests of this conjecture or tests of related conjectures are possible should they be deemed of sufficient interest to justify the research expenses.

The floors of the commodity markets and the stock markets are characterized by a host of detailed rules and customs. Some of these rules are in response to legal requirements as shaped by the beliefs of legislators and administrative law judges. Others might have evolved in response to the underlying economic characteristics of the goods and firms that are represented by the documents being traded. Still other rules might serve no functions at all. Elements of similarity

exist with the MUDA and the rules of exchanges but the MUDA was not developed as an attempt to simulate some existing market. However, existing markets might hold suggestions of modifications to MUDA or alternative rules that perform better than MUDA.

APPENDIX 1

INSTRUCTIONS

GENERAL

This is an experiment in the economics of market decision making. The instructions are simple and if you follow them carefully and make good decisions you might earn money which will be paid to you in cash.

In this experiment we are going to conduct a market in which some of you will be buyers and some of you will be sellers in a sequence of market days or trading periods. Attached to the instructions you will find a sheet labeled Buyer or Seller, which describes the value to you of any decisions you might make. **YOU ARE NOT TO REVEAL THIS INFORMATION TO ANYONE.** It is your own private information.

The currency in these markets is francs. Each franc is worth _____ dollars to you.

SPECIFIC INSTRUCTIONS TO BUYERS

During each market period you are free to purchase from any seller or sellers as many units as you might want. For the first unit that you buy *during a trading period* you will receive the amount listed in row (1) marked *1st unit redemption value*; if you buy a second unit you will receive the additional amount listed in row (4) marked *2nd unit redemption value*, etc. The profits from each purchase (which are yours to keep) are computed by taking the difference between the redemption value and purchase price of the unit bought. That is,

[your earnings = (redemption value) – (purchase price)].

Suppose, for example, that you buy two units and that your redemption value for the first unit is 200 and for the second unit is 180. If you pay 150 for your first unit and 160 for the second unit, your earnings are:

earnings from first = 200 – 150 = 50

earnings from second = 180 – 160 = 20

total earnings = 50 + 20 = 70

The blanks on the table will help you record your profits. The purchase price of the first unit you buy during the first period should be recorded on row (2) *at the time of purchase*. You should then record the profits on this purchase as directed on row (3). At the end of the period record the total of profits on the last row on the page. Subsequent periods should be recorded similarly.

SPECIFIC INSTRUCTIONS TO SELLERS

During each market period you are free to sell to any buyer or buyers as many units as you might want. The first unit that you sell *during a trading period* you obtain at a cost of the amount listed on the attached sheet in row (2) marked *cost of 1st unit*; if you sell a second unit you incur the cost listed in row (5) marked *cost of 2nd unit*; etc. The profits from each sale (which are yours to keep) are computed by taking the difference between the price at which you sold the unit and the cost of the unit. That is,

[your earnings = (sale price of unit) – (cost of unit)].

Suppose, for example, your cost of the first unit is 140 and your cost of the second unit is 160. For illustrative purposes we will consider only a two-unit case. If you sell the first unit at 200 and the second unit at 190, your earnings are:

earnings from first = 200 – 140 = 60

earnings from second = 190 – 160 = 30

total earnings = 60 + 30 = 90

The blanks on the table will help you record your profits. The sale price of the first unit you sell during the first period should be recorded on row (1) *at the time of sale*. You should then record

the profits on this sale as directed on row (3). At the end of the period, record the total of profits on the last row on the page. Subsequent periods should be recorded similarly.

MARKET ORGANIZATION (MULTIPLE UNIT ODA)

The market for units is organized as follows. The trading period is open for _____ minutes. Any person is free to bid, to buy (ask to sell) at any time that recognition is gained from the auctioneer. The bid (ask) is tendered by giving the sequence: name, bid price per unit (ask price per unit), quantity. The bid (ask) will be written on the chalkboard and will remain there until accepted, canceled or replaced by a higher (lower) bid (ask). Anyone is free to accept any part of a standing bid (ask) and the remainder continues to stand. If a person accepts all or part of a bid (ask) a binding contract has been closed and both parties must record the transaction.

APPENDIX 2

BUYER AND SELLER RECORD SHEETS

Record of Purchases and Earnings, Buyer No.____

Unit Purchased		Trading Period Number	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1st unit redemption value												
	2	Purchase price												
	3	Profit (row 1 - row 2)												
10	28	10th unit redemption value												
	29	Purchase price												
	30	Profit (row 28 - row 29)												
	31	Total per period												

Name_____ Soc. Sec. No._____ Total Payment_____

Address_____

Record of Sales and Profits, Seller No._____

Unit Sold		Trading Period Number	1	2	3	4	5	6	7	8	9	10	11	12
1	1	Selling price												
	2	Cost of 1st unit												
	3	Profit (row 1 - row 2)												
10	28	Selling price												
	29	Cost of 10th unit												
	30	Profit (row 28 - row 29)												
	31	Total per period												

Name_____ Soc. Sec. No._____ Total Payment_____

Address_____

FOOTNOTES

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1. Volume refers to the number of smallest tradable units that are traded in a time period.

2. Computerized markets sometimes have this feature. See Smith and Williams (1982).

3. This market has been used by Lynch, Miller, Plott, and Porter (1986) and by Miller and Plott (1985).

4. The language used to convey these rules to subjects is included as Appendix 1.

5. Individual parameters for each period are not included because of space limitations. These will be supplied by the authors upon request.

6. Part of an order might be accepted before a multiple unit acceptance reaches the market. If the amount of the order remaining is less than the amount listed in the acceptance, then the acceptance is rejected.

7. Suppose someone tenders a "high" bid that someone else would prefer to see withdrawn. An even higher bid followed by an immediate cancellation will in effect cancel the original bid with a minimum exposure of having the higher bid accepted. This strategy is called "flashing" in some versions of MUDA and rules have evolved to control it.

REFERENCES

Johnson, Alonzo; Lee, Hsing Yang; and Plott, Charles R. "Multiple Unit Double Auction User's Manual." Social Science Working Paper no. 650. Pasadena, California Institute of Technology, June 1988.

Lynch, Michael; Miller, Ross M.; Plott, Charles R.; and Porter, Russell. "Product Quality, Consumer Information, and 'Lemons' in Experimental Markets." In *Empirical Approaches to Consumer Protection Economics*, edited by P. M. Ippolito and D. T. Scheffman. Proceedings of a Conference Sponsored by the Bureau of Economics, Federal Trade Commission, April 26-27, 1984. FTC, March 1986.

Miller, Ross M. and Plott, Charles R. "Product Quality Signaling in Experimental Markets." *Econometrica* 53 (July 1985): 837-872.

Plott, Charles R. "Externalities and Corrective Policies in Experimental Markets." *Economic Journal* 93 (March 1983): 106-127.

Smith, Vernon L. and Williams, Arlington W. "An Experimental Comparison of Alternative Rules for Competitive Market Exchange." In *Auctions, Bidding and Contracting: Uses and Theory*, edited by M. Shubik. New York: New York University Press, 1982.