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AGENDAS, STRATEGIC VOTING, AND SIGNALING WITH INCOMPLETE INFORMATION

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Abstract

The literature on agendas with sincere and strategic voting represents an important contribution to our understanding of committees, of institutions, and of the opportunities to manipulate outcomes by the manipulation of institutions. That literature, though, imposes an assumption that may be unrealistic in many situations; namely, that everyone knows the preferences of everyone else. In this essay we apply Bayesian equilibrium analysis to show that the properties of agendas that others derive assuming complete information do not hold necessarily under incomplete information. First, a Condorcet winner need not be selected, even if nearly everyone on the committee most prefers it. Second, the "2 step theorem," that any outcome reachable in n voting stages via some amendment agenda is reachable in two stages under sophisticated voting, need not hold. Third, nonbinding votes such as straw polls, can critically effect final outcomes.

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Agendas, Strategic Voting, and Signaling with Incomplete Information

Formal study of the idea that members of a committee might not vote sincerely in legislative ballot procedures was motivated by Farquharson's (1969) insight that voting is a strategy and that strategies ought to be chosen carefully to attain preferred ends. Often, the best strategy is to vote insincerely, to misrepresent one's preference in early ballots to eliminate alternatives that would otherwise defeat preferred outcomes in later ballots. The result that best summarizes the research stimulated by Farquharson's insight in the context of voting procedures in which a committee incrementally narrows the set of possible decisions by sequential pairwise voting (called binary procedures), is: Any motion that defeats something can prevail in some agenda if everyone votes sincerely -- if everyone fails to gauge the future consequences of their actions and instead votes at each stage for the most preferred of the two alternatives being considered. But if a Condorcet winner exists, if there is an alternative on the agenda that defeats everything else, then regardless of the details of the agenda, that winner is chosen if everyone votes strategically -- if everyone identifies the future consequences of their actions and votes throughout the agenda in accordance with an appropriate strategy (for a survey see Moulin, 1985).

Building on subsequent research by McKelvey and Niemi (1978), Moulin (1979), and Gretlein (1981), which shows how working backwards up the extensive form representation of a voting procedure reduces a dynamic voting problem to one that is static, other results have been established for special types of binary agendas. For example, if everyone is strategic, then any amendment agenda -- any agenda in which the winner of

one vote automatically enters the voting in the next stage and the balloting proceeds until all alternatives are considered -- is equivalent to some two-stage agenda in the sense that if an alternative, α, can be reached via some amendment agenda from the status quo, β, then α can be reached from β in two votes (Miller, 1977). And if Ω is the set of all alternatives in an amendment agenda, then $\alpha \in \Omega$ prevails only if α is uncovered in Ω, which means that there isn't another alternative in Ω that defeats α and defeats everything that α defeats (Miller, 1978, and for additional restrictions see Banks, 1985). With spatial preferences, majority rule, and sincere voting, if there is no Condorcet winner, then there is an amendment agenda that leads from any initial point, α, to any other point, β (McKelvey, 1976, 1979). The results about the uncovered set, however, show that this implied power of agendas is reduced by strategic voting (Miller, 1978, and Shepsle and Weingast, 1984). Strategic voting in non-amendment agendas, including variants used in the U.S. Congress, have also been studied, with somewhat different results than those established for amendment agendas (Ordeshook and Schwartz, 1986). Indeed, even if all voters are strategic, we can construct binary agendas of a special sort (sequential elimination agendas, in which balloting ends whenever an alternative is approved) that lead to any outcome in the top cycle set (Moulin, 1985).

Thus, we have a considerable literature that analyzes binary agendas in the contexts of sincere and strategic voting. Nevertheless, this research suffers from a restriction that severely limits its applicability -- it supposes that people are completely informed about each other's preferences. Everyone knows everyone elses' preferences, everyone knows that everyone knows this, and therefore everyone knows when it is in the interest of others to be strategically insincere. But even if the quality of information in committees sometimes approximates this assumption, a general theory of agendas must consider situations when information is incomplete. For example, the complex agendas that congressional rules admit (c.f. Bach 1981) suggest that agendas serve purposes necessitated by incomplete information. Although the two-step reduction theorem is not always applicable because congress is not required to use an amendment agenda, multiple stage agendas seem needlessly complex under complete information. However, with incomplete information, agendas do more than record votes in serial order: members can use early ballots to mislead others about their intentions and preferences.

Although the assumption of complete information is analytically convenient, it thus precludes consideration of important features of committee voting, including how legislators learn about their colleagues' preferences, when it is in the interest of those colleagues to disguise their preferences, and how legislators anticipate and respond to the strategic revelation of preferences both in their voting and in the design and implementation of specific agendas. And it also precludes the possibility of studying those formal and informal procedural details, such as straw votes, "position taking," and pre-vote discussions, that committee members use either to estimate how others will vote or to effect how others will vote.

This view of procedures in general and of agendas in particular compels us to revise the way in which agendas are analyzed. Previous

research views strategic voting outcomes as a conjunction of strategies in which no person has an incentive to alter unilaterally his or her actions after the fact. That is, outcomes correspond to a (perfect) Nash equilibrium of voting strategies. But notice that with complete information, nothing can be learned as the voting proceeds, because everyone is informed about the situation's relevant parameters (the alternatives to be voted on, peoples' preferences over these alternatives, and the agenda). Thus, everyone can map out everyone else' strategies, as well as their own, and deduce the equilibrium outcome. So the final outcome can be inferred beforehand, and the actual voting is the mere recording of dominant strategies.

Another way to view this classical approach is to notice that since peoples' beliefs about the preferences of others are fixed (everyone knows the truth), there is no mention of beliefs in the statement of equilibrium. But with incomplete information, beliefs about preference can change as the voting proceeds. If I observe you voting one way in some early stage of an agenda, I may infer one thing about your preferences, whereas if I observe a different choice, I may believe something else. And if I believe something different, then I may vote differently in subsequent ballots. Thus, voting serves a dual purpose, to affect outcomes via the selection and rejection of outcomes at a particular stage and to affect beliefs and, thus, future votes. In this world of incomplete information, the usual definitions of equilibrium must be augmented with conditions on the stability and consistency of beliefs. Since both strategies and beliefs can vary, an equilibrium in a world of incomplete information consists of a set of beliefs and strategies such that no one has any incentive to change their strategies given their beliefs at any stage of the game and all beliefs are consistent with the strategies of other voters and prior assessments about their preferences.

This essay, via several carefully selected examples, explores the implications of incomplete information in amendment agendas. Section 1 outlines our basic approach, and provides the notation required for a definition of an appropriate equilibrium for binary voting games. Section 2 illustrates the application of this equilibrium notion for a simple 3-alternative, 3-voter, 2-stage amendment agenda. To show, however, how agendas with incomplete information differ from those with complete information, Section 3 offers an example in which a Condorcet winner is not selected: Although one alternative is almost certainly a Condorcet winner and thus it would be selected if information were complete, if information is incomplete, then it never corresponds to the equilibrium outcome for any symmetric pure strategy strategic voting equilibrium. Section 4 considers a 3-stage agenda. Three-stage agendas permit us to explore the Bayesian approach to signaling and the strategic revelation of preferences. In addition, the likelihood of outcomes in this example cannot be matched by any two-stage agenda, which suggests that incomplete information provides at least a partial explanation for why lengthy agendas are sometimes observed. Section 5 extends the analysis of agendas to include an initial communication stage (in this case a nonbinding straw vote). We show how this can undo the Condorcet paradox that we construct in Section 3.

We emphasize that we offer no general theorems, and that our examples are "carefully selected" to illustrate a general theoretical approach.

Games with incomplete information are complex, and we are only beginning to learn how to treat the problems they pose. But our examples do more than simply illustrate an alternative concept of equilibrium. They also show why many of the conclusions about agenda manipulation deduced with complete information must be scrutinized and modified (even abandoned) when information is incomplete and endogenous.

1. The Basic Approach

The most useful approach to studying strategic voting in binary agendas relies on extensive-form game theory. To represent this form and to develop an appropriate equilibrium concept for the corresponding noncooperative incomplete information game, let a committee, C, consist of n (odd) members, $C = \{1,2,\ldots,n\}$, and let X be the set of all feasible outcomes. The purpose of most agendas, of course, is to winnow out alternatives until only one outcome in X remains. Suppose at stage s ($s = 1,2,\ldots,k$) of this process that only the outcomes X_s remain feasible. Then in any binary agenda, the vote in stage s must be between two subsets of X_s, say X_s^1 and X_s^2, where the intersections of these two sets need not be empty, but where these two subsets together exhaust all remaining possibilities (that is, $X_s^1 \cup X_s^2 = X_s$). In general, the content of these two sets depend on the history of previous votes. Letting $\eta_\tau = (\eta_\tau^1,\ldots,\eta_\tau^n)$ identify how each member of C votes in stage τ, then the history up to stage s, $h_s = (\eta_1,\ldots,\eta_{s-1})$ completely summarizes the outcomes of the first s-1 stages of the agenda, which we assume is common knowledge at stage s. Thus, we let X_s^1 and X_s^2 both be functions of h_s.

Amendment agendas are a special case of binary agendas. The distinguishing assumption for them is that at any stage s, $X_s - X_s^1 = \{\alpha(h_s)\}$

and $X_s - X_s^2 = \{\beta_s\}$, where $\alpha(h_s)$ and β_s are two alternatives in X -- specifically, $\alpha(h_s)$ corresponds to the winner of the previous ballot (and, hence, is a function of the history to stage s) and β_s is a prespecified alternative introduced into the voting at stage s, independent of h_s. The winner of this vote then enters the next round against the prespecified alternative, $\beta_{s+1} \in X_s^1 - \{\alpha, \beta\}$. (Although we can define winning any number of ways to admit, say, the chair's power to veto decisions or to allow for weighted voting, we assume that something wins if it receives a majority of votes.)

Now, given an agenda and a committee, we must define the associated extensive-form noncooperative game, since the analysis of it tells us how members vote at each stage and the final outcome. First, we must specify whether voting is by open or secret ballot. We assume that voting occurs in an open ballot setting, that all members know how every other member voted in earlier stages. We impose this assumption merely as a matter of convenience, though, since our results apply as long as committee members at least know how many votes each alternative received. Next, we complete the description of the extensive form by specifying the preferences of the members of C and the information each member has about his or her environment. With respect to preferences, we follow a standard procedure in game theory (c.f. Myerson, 1983) by viewing each member $i \in C$ as being randomly drawn from a set, T_i, of possible "types.," Thus, the set of all possible realizations of committee environments is $\underline{T} = T_1 \times T_2 \times \ldots \times T_n$, and a specific committee environment is a vector, t, of member "types." Since we equate types with preference orders over the finite set X, $\underline{T}$ is finite. Let each $i \in C$ have a utility function $u^i : T_i \times X \to R$ that takes each type in

T_i and outcome in X and defines a cardinal utility number. Now suppose that there is a common prior distribution over the possible types and denote the probability of environment $t \in \underline{T}$ by $q(t)$. Thus, although the members of C may not know each other's type, they all share the same prior probability as to what environment they confront.

Even though our notation may be unfamiliar to readers unacquainted with Bayesian game-theoretic models, to this point our formal development follows the traditional approach, in which everyone knows everyone else's type. But here we suppose that although each person knows his own type, t_i, he only has probabilistic information about the remaining members of C (he is uncertain about the types of others). To distinguish formally between the traditional approach to the study of agendas and the approach we take here, notice that generally one's knowledge of t_i can be used to update q. Specifically, letting t_{-i} denote a vector of types of all persons except i, member i can establish the conditional distribution

$$q^i(t_{-i} \mid t_i) = \frac{q(t_{-i}, t_i)}{\sum_{t'_{-i} \in T_{-i}} q(t'_{-i}, t_i)}$$

We then have two alternative assumptions about information structure:

<u>Complete Information</u>: All conditional probability distributions are degenerate. That is, for all i and all t, $q^t(t_{-i} \mid t_i)$ is either 0 or 1, and everyone's types are common knowledge (i.e., everyone knows everyone else's type, everyone knows this, etc).

<u>Private Information</u>: The conditional distributions are not degenerate. Members of C are uncertain about the types of other members. In the

simplest model, knowledge of one's own type provides no useful information about other member's types, and we refer to such situations as independent private information.

To study agendas in which information about types is private, we must apply concepts drawn from Bayesian equilibrium analysis. Recall that the plausibility of the Farquharson reduction method for identifying sophisticated voting strategies (as well as Mckelvey and Niemi's backward reduction procedure) relies on the assumption that all players know the preferences of all other players and, hence, that everyone can predict the outcome of every ballot. But here, information is not complete and players may choose certain strategies because they reveal one thing about their preferences while different strategies reveal something else. And at the same time voters can learn about the preferences of others as the agenda unfolds. Hence, our equilibrium must consider not only strategies but what voters believe about others.

The first step is to specify each voter's strategies, so letting H_s be the set of all possible histories of votes up to stage s, a pure s-strategy for each player i is a function that takes each history, $h_s \in H_s$, and each type $t_i \in T_i$ and specifies a decision for i at stage s of the agenda to either vote for $X^1_s(h_s)$ or for $X^2_s(h_s)$. A strategy, then, is a decision rule that maps types and histories at each stage into vote choices. Formally, a pure strategy for voter i is a set of K functions, $\delta^i = (\delta^i_1, \ldots, \delta^i_k)$, such that $\delta^i_s: H_s \times T_i \rightarrow \{X^1_s(h_s), X^2_s(h_s)\}$. Because in principle we want to allow mixed strategies, let the strategy set $\sum^i$ denote the set of all probability distributions over pure strategies, with

the typical element $\sigma^i \in \sum^i$. Thus, in terms of the voting tree, $\sigma^i_s(h_s, t_i) = p$ is interpreted thus: If voter i is a type t_i and if the history of the first s-1 stages of the agenda is h_s, then i votes for $X^i_s(h_s)$ with probability $p \in [0,1]$.

Finally, voters make their decisions based on their <u>beliefs</u> about other members of C. Formally, the belief of $i \in C$ at stage s, denoted b^i_s, is a mapping from $H_s \times T_i$ into the set of probability distributions on T_{-i}. That is, conditional on the voting history up to stage s and on what i knows about himself (his own type), b^i_s denotes what i thinks is the likelihood that the remaining members of the committee have preferences that correspond to the various possibilities (alternative types). Because i is conditioning these beliefs on the voting history and because this history changes as the committee proceeds through the agenda, this notation anticipates beliefs that change. For a K-stage amendment agenda, let $b^i = (b^i_1, \ldots, b^i_s, \ldots, b^i_K)$ denote i's belief sequence, which we assume is consistent with Bayes's rule about probabilities.

We are now in a position to define an equilibrium appropriate to binary agendas. First, following Moulin [1979] and McKelvey and Niemi [1978], we define a game's <u>dominance reduction</u> by sequentially eliminating dominated strategies. Notice that even the first stage of this elimination process leaves us with a greatly simplified game: Because on any final stage of an agenda, no person should vote for their second choice between the two remaining alternatives, all strategies that admit a positive probability of such a decision are dominated. If player i's strategies are thus reduced $\sum^i_1 \subset \sum^i$, we can apply this reduction procedure to $\sum^i_1$ to yield $\sum^i_2$, and so forth until no further reduction is possible. The product of this reduction procedure, though, demonstrates the clear difference between agendas with complete and incomplete information. We know that with complete information and with strict preferences, this reduction continues until each member of C is left with strategy choices that are equivalent in the sense that they all yield the same, unique outcome, the sophisticated voting equilibrium. At this point the game is solved and there is no strategic ambiguity. But with incomplete information this procedure does not result in unambiguous strategy choices for all voters, and thus, it does not solve the game.

To refine our predictions, let Γ' be the binary voting game that results from Γ after all dominated strategies are successively eliminated. Since voters may have more than one strategy in Γ', we must apply an equilibrium notion that is weaker than dominance solvability. We use Kreps and Wilson's (1982) sequential equilibrium, which, when applied to the reduced game Γ', we call a strategic voting equilibrium (SVE). Briefly, in a sequential equilibrium the part of each player's strategy that remains at any point from any given history, maximizes that player's expected utility over outcomes. The probabilities in these calculations use the player's current beliefs and strategy, where these beliefs are consistent with Bayesian probabilities and with an additional technical requirement. Rather than digress to a lengthy technical exposition, however, we turn now to our first example, which illustrates our definitions and the steps that must be taken to analyze an incomplete information agenda game.

2. Illustrating a Strategic Voting Equilibrium

Because we merely want to illustrate a method of solving incomplete

information agenda games and because nothing fundamentally changes with more elaborate possibilities (aside from a considerable increase in analytic complexity), we keep our example simple. Thus, we consider a situation in which there are only three voters, three alternatives, and three types of preferences. Suppose that the set of alternatives is $\{a,b,c\}$, in which case we can summarize preferences over three alternatives by two pieces of information: (1) the rank order of the alternatives, and, letting the utility of the first and last ranked alternative be 1 and 0 respectively, (2) a utility number $v \in (0,1)$ assigned to the middle ranked alternative (we assume throughout that all preferences are strict, so v must be less than 1 and greater than zero). Suppose these three preference orders are possible:

Preference Order Type:	1	2	3	u
	a	b	c	1
	b	c	a	v
	c	a	b	0

Again, to keep the example simple, we assume that v is identical for all three voters, and that the prior probability that any voter has Type j preferences, q_j, is common to all voters (the generalization of the analysis with respect to these two assumptions is straightforward.) Next, we assume the amendment agenda: "a vs b, the winner against c," which can be portrayed by the voting tree in Figure 1. The following remarks assist us in calculating an equilibrium for this voting game:

Remark 1: In any binary procedure, everyone votes sincerely on the last ballot. Regardless of one's beliefs, no strategic purpose is served by misrepresenting preferences on the last vote. (This follows from the assumption that dominated strategies are eliminated from Γ.)

Remark 2: To verify that a particular strategy n-tuple is a SVE of a 2-stage agenda, we need only examine those cases in which a person's vote is critical for one alternative as against another. If your vote is irrelevant at any stage in the voting owing, say, to a unanimous choices by others, you cannot effect the outcome and, thus, any choice can be part of an equilibrium n-tuple. In deducing best response strategies, then, we condition probabilities on the decisiveness of a voter. Notice, though, that this remark applies only for 2-stage agendas such as the one shown in Figure 1. With more than two stages, voters may condition their beliefs in later stages by margins of victory observed on previous ballots (see the example in Section 4) as well as by what wins or loses.

Remark 3: Regardless of their beliefs, voters with Type 2 preferences -- those who prefer *b* to *c* to *a* -- have voting sincerely on both ballots as a dominant strategy. For voters of this type any lottery between *b* and *c* is preferred to any lottery between *c* and *a*. If both of the other voters vote for *b* over *a* or *a* over *b*, any choice at this stage is an equilibrium choice since the voter is not pivotal (remark 2). If only one of the other voters chooses *b* over *a*, the voter in question cannot reject the hypothesis that there is some non-zero probability that *b* defeats *c*, in which case voting for *b* over *a* is referred.

Remark 4: Type 3 voters -- those who prefer *c* to *a* and *a* to *b* -- should also vote sincerely. There are two cases: (i) neither of the remaining voters is a Type 3; and (ii) at least one of the others is a Type 3 voter. Under case (i), a type 3 voter prefers to end up with (*a* vs

rather than (*b* vs *c*) since, in this last situation with everyone voting sincerely in the last stage (remark 1) *b* wins and *b* is this type's last choice. Under case (ii), such a voter is indifferent between (*a* vs *c*) and (*b* vs *c*) since *c* always wins. Hence, voters of this type have a dominant strategy of voting for *a* on the first ballot as long as other voters abide by dominant strategies. Since there is some non-zero probability that case i prevails, type 3's are strictly better off voting for *a*. (Despite an obvious abuse of technical language, for convenience we refer to this strategy as "dominant".)

The numbers alongside the branches in Figure 1 summarize the sincere voting patterns described by Remarks 1, 3 and 4. Note that these remarks do not depend on the specification of prior beliefs (as long as $q_j > 0$ for all $j = 1,2,3$ and $q_1 + q_2 + q_3 = 1$) or upon the choice of a von Neumann--Morgenstern utility function to represent preferences. Sincere voting for Type 2 and Type 3 voters is dominant (or nearly so), given their ordinal preferences (regardless of v). Hence, we assume that the "partial" strategies described in Remarks 3 and 4 are used by all voters: If a voter has Type 2 or Type 3 preferences, then his voting decision necessarily follows these strategies, and no further analysis is required for these two types. Correspondingly, each voter knows how others act if others have either of these two types of preferences.

What remains is to identify the t_1-strategy profiles -- a specification of the first-stage strategy for each player when he has type 1 preferences that yield a SVE . Limiting ourselves to pure t_1-strategies, there are at least these two possibilities:

Case 1: All three voters vote sincerely when they have type 1 preferences (all vote for a over b in stage 1)

Case 2: All three voters vote insincerely in stage 1 if they have type 1 preferences (they vote for b over a)

In addition to these equilibria, which are symmetric in the sense that all voters of the same type are assumed to abide by identical strategies, we also have two nonsymmetric possibilities:

Case 3: One of the voters votes sincerely (as in case 1) and the other two voters are insincere (as in case 2).

Case 4: Two of the voters vote sincerely (as in case 1) and one voter votes insincerely (as in case 2).

Again, to keep the example as simple as possible, we look only at the two symmetric cases (see Ordeshook and Palfrey, 1985, for the analysis of the nonsymmetric equilibria).

To see how we approach each case, consider the first. Looking at the agenda from the perspective of a single voter, suppose this voter has Type 1 preferences and, given the beliefs generated by his priors and his conjecture that everyone else always votes sincerely, suppose that he concludes that his best response is to vote insincerely. Then the case 1 strategies, together with the appropriate set of beliefs cannot be an equilibrium, because equilibrium strategies are necessarily best responses. Hence, to establish that case 1 corresponds to an SVE, we must show that if a Type 1 voter's beliefs are derived from conjectures corresponding from this case, then his best response is to vote sincerely.

By symmetry, this will be sufficient to show that case 1 strategies are an SVE for Γ'.

To check for the conditions under which a Type 1 voter, say voter *i*, who conjectures that everyone is sincere, will indeed choose to vote sincerely, notice from Remark 2 that we need only compute best response strategies under the contingency that *i* is decisive on the first ballot. Decisiveness requires that the other two voters split between voting for *a* and *b*. But if everyone is sincere, then the single other voter choosing *b* must be of Type 2, and the single other voter choosing *a* is either of Type 1 or 3. If *i* knew the "*a* voter" is Type 1, then *i* would vote sincerely, because with two Type 1 voters *a* defeats *c* on the second ballot and *a* is *i*'s most preferred alternative. On the other hand, if *i* knew that the "*a* voter" was of Type 3, then *i* would vote (insincerely) for *b* since with one voter of each type *a* loses to *c*, but *i*'s second choice, *b*, defeats *c*.

In deducing an optimal strategy, then, the question is whether or not the "*a* voter" is Type 1 or Type 3. Given that he cannot be Type 2, the Bayesian posterior probability that the "*a* voter" is a Type 1 voter is $q_1/(q_1+q_3)$, and the posterior that he is a Type 3 voter is $q_3/(q_1+q_3)$. Thus, if *i* votes sincerely, *i*'s expected payoff is

$$1[q_1/(q_1+q_3)] + 0[q_3/(q_1+q_3)] = q_1/(q_1+q_3).$$

On the other hand, if *i* votes insincerely, then he receives *b* with certainty since one other voter is known to be a Type 2 voter, and this voter and *i* will join to choose *b* over *c* in the second (final) ballot. Since the utility *i* receives from *b* is v, it follows that all voters voting sincerely is an equilibrium if

$$q_1/(q_1 + q_3) > v \qquad (1.1)$$

That is, if expression (1.1) holds, each voter's sincere voting strategy is optimal given that voter's correct conjectures about the equilibrium strategies of others and beliefs that are consistent with these strategies (i.e., beliefs that are derived from q and σ using Bayes's rule).

Reviewing the steps in our analysis, they are:

(1) In accordance with the dominance reduction procedure, delete the dominated strategies of all players. For the present example, this requires that all types vote sincerely on the last ballot and that Types 2 and 3 vote sincerely on the first ballot.

(2) For those types whose strategies are not completely specified by step (1) -- Type 1 voters in the example -- identify the circumstance under which a voter of that type is decisive.

(3) Conjecture a specific equilibrium strategy for all types whose strategies are not completely specified. In this instance, Case 1 provides the conjecture.

(4) Calculate the posterior beliefs (probabilities) conditional on being decisive and on the circumstance of the hypothesized equilibrium as described by (1) and (3).

(5) Calculate the conditions under which the strategies hypothesized in (3) are in fact best responses to the assume strategies of all other voters. This condition corresponds to expression (1.1).

With these steps in mind we can now turn to Case 2 to learn if other equilibria exist. Suppose then that each voter conjectures that other voters vote insincerely if they are Type 1 but vote sincerely otherwise, in accordance with Remarks 3 and 4. To establish an equilibrium we must

show that, given this belief, voting insincerely is indeed a best response for a Type 1 voter. As before, we look only at those instances in which your vote is decisive; but now, given the conjecture that Type 1 voters are strategic and thus vote for *b* in the first ballot, the "*a* voter" must be of Type 3. Voter i's belief about the "*b* voter," given that such a voter cannot be a Type 3, is that he is a Type 2 with posterior probability $q_2/(q_1 + q_2)$ and a Type 1 with posterior $q_1/(q_1 + q_2)$. If i votes insincerely, then *b* wins for sure since Types 1 and 2 both vote for *b* over *c* in the second round and since i knows, from the fact of being decisive on the first ballot, that at least one voter is a Type 2. Strategic voting, then, yields the payoff v. But if i votes sincerely, *a* wins over *c* if the "*b* voter" is Type 1 whereas *c* wins if this voter is a Type 2. Hence, i's expected utility from sincere voting is $q_1/(q_1 + q_2)$, so insincere voting for all Type 1 voters is an equilibrium if,

$$q_1/(q_1 + q_2) < v \qquad (1.2)$$

That is, if condition (1.2) holds, being insincere if you are a Type 1 voter is optimal, given beliefs derived from Bayes's rule and the correct conjecture that other Type 1 voters are insincere.

Notice that conditions (1.1) and (1.2) have logical interpretations. First, if we hold v constant, then if q_1 is sufficiently large -- if it is likely that there is more than one Type 1 voter -- expression (1.1) states that voting sincerely is an equilibrium strategy for Type 1's. This is only reasonable. because insincere voting helps i only if i is the <u>only</u> Type 1 voter. Indeed, intuition alone suggests that if the preferences of others are "similar" to yours, then sincere voting is generally a reasonable strategy. Condition (1.1) confirms this view. But in addition, this condition adds an important qualification to this assumption: It is not the absolute magnitude of q_1 that dictates a choice of strategy, but rather the relative magnitudes of q_1 and q_3. Thus, even if q_1 is quite small, sincere voting is an equilibrium as long as q_3 is small as well. The reason for this, although less obvious, can be made intuitive. Because a voter's decision has no effect unless he is pivotal, the probabilities relevant to his decision are conditionals, not marginals. That is, a Type 1 voter only needs to know the likelihood that alternative *a* wins in the second stage, conditional on his voting for it in the first stage and conditional on the actual vote split being 2-to-1 for *a*. Being pivotal in the first stage is the only circumstance under which a Type 1 voter affects the outcome, but voting for *a* is counter productive if the other "*a* voter" has too great a probability of voting for *c* subsequently. And similar reasoning supplies the intuition behind expression (1.2)

Turning now to the role of the cardinal utility number v, recall that in the standard sophisticated voting model, results depend only on ordinal preferences. But since v measures preference intensity, the dependence of our results on v accords with the intuition that voting decisions themselves depend on preference intensity. Indeed, the dependence that expressions (1.1) and (1.2) reveal make sense. First, as v increases, the relative preference between a voter's first and second choice decreases, while the relative preference between the second and third ranked alternatives increases. Thus, if v is sufficiently large so that the inequality in (1.2) is satisfied, then a Type 1 voter "settles" for his second choice, *b*, rather than risking the possibility of having his least

preferred outcome, *c*, prevails as a result of voting sincerely for *a*. (Recall that in equilibrium, conditional on being pivotal in the first stage, voting for *b* in the first stage guarantees *b*, but voting for *a* amounts to a gamble between *a* and *c*.) Analogously, if v is sufficiently small so that condition (1.1) is satisfied, then the gamble is worthwhile.

The preceding discussion supplies the intuition behind expressions (1.1) and (1.2), but notice in addition that these two conditions may be satisfied simultaneously if $q_2 > q_3$, in which case there are two symmetric pure strategy equilibria, where by "symmetric" we mean that either all Type 1 voters are sincere or all are strategic. On the other hand, it is also possible that neither (1.1) nor (1.2) is satisfied. In this instance, either an equilibrium exists only in mixed strategies or the equilibrium is asymmetric. The appendix to this essay details the analysis of the two possible asymmetric equilibria involving pure strategies, Cases 3 and 4, and shows that, depending on the q's and v, these asymmetric equilibria can exist as well. Indeed, this multiplicity of equilibria is a common feature of games with incomplete information, and certainly many more kinds of equilibria may become possible if we admit mixed strategies or additional preference types.

Our objective in this section, though, is to illustrate a method of analysis and to show that complete and incomplete information games yield qualitatively different conclusions. To see this difference more clearly with our example, suppose that $(q_1, q_2, q_3) = (1/3, 1/3, 1/3)$, in which case expression (1.1) is satisfied if $v < 1/2$ and (1.2) is satisfied if $v > 1/2$. With these priors, then, sincere voting (Case 1) and insincere voting (Case 2) are equilibrium strategies for Type 1 voters, depending on the intensity of a type 1 voter's preferences for alternative *b*. Table 1, now, compares for every possible preference profile the outcomes that prevail under complete information to the outcomes that prevail with incomplete information, for the two cases $v < 1/2$ and $v > 1/2$.

The numbers in this table yield several interesting comparisons. First, notice that regardless of the value of v, it is never the case that the outcomes listed under complete information match, for all ten possible profiles, the outcomes prevailing with incomplete information. For example, then, if the q's are the actual probabilities with which preference types are drawn and "assigned" to voters, and if this assignment is revealed before the voting to everyone (complete information), then *a* and *c* will each prevail with probability $1/27 + 1/9 + 1/9 = 7/27$, and *b* with probability 13/27. But if this assignment is not revealed (incomplete information) and if $v < 1/2$, then *a* and *b* each prevail with probability 7/27, and *c* prevails with probability 13/27. Similarly, if $v > 1/2$, *a* cannot occur, *b*'s probability is 20/27, and *c*'s is 7/27. Hence, with preference types assigned equiprobably in this fashion, neither the exact pattern of outcomes nor the a priori probability of outcomes under complete information will match the likelihood under incomplete information. While this is only an example, this conclusion is true generally, in the sense that either "exact" of "on average" matching of complete and incomplete information outcomes is merely coincidental and not to be expected.

The are also some interesting distributional consequences of incomplete information that Table 1 makes apparent. First, for any value of v, type 1 voters are the "victims" of incomplete information: for all

possible profiles, they are never better off and sometime they are worse of with incomplete as compared to complete information. Second, type 2's are victims if $v < 1/2$ and "winners" if $v > 1/2$, with the converse holding true for type 3 voters. Thus, perhaps not surprisingly incomplete information has distributional effects.

Finally, the most striking disparity between complete and incomplete information in this example occurs when $v > 1/2$ (so type 1's vote insincerely) <u>and</u> type 1's are a majority (the first three profiles of the table). In this instance, *a* is a Condorcet winner, but *b* is selected. Owing to the seeming paradoxical nature of this possibility, and to its profound consequence for the study of agendas, we examine it more closely in the next section.

3. <u>Condorcet Winners and Incomplete Information</u>

The previous example illustrates the nature of equilibrium with incomplete information, but it does not show fully how the assumptions we make about information alter the general conclusions that have been drawn about agendas. Again, because we want to keep the analysis as simple as possible, we consider the same agenda as before and the same three preference types. Rather than assume that there are only three voters, however, we now assume that the number of voters, n, is odd and exceeds three.

We begin the analysis by noticing that the same dominance reduction argument applies as before to Type 2 and Type 3 voters, which is to say that such types vote sincerely. Turning then to Type 1 voters and limiting the analysis to symmetric pure strategy equilibria, the first possibility is that Type 1 voters all vote sincerely for *a* over *b* in the first ballot. We claim that as n increases, these sincere strategies cannot be sustained as an equilibrium. To see this, consider a specific Type 1 voter. As in the previous section, we only need to look at those cases in which this voter is decisive in the first vote (Remark 2), in which case there must be exactly (n-1)/2 Type 2 voters (these are the only "*b* voters" for this case). Hence, if this voter is decisive, then he can guarantee the outcome *b* by voting for *b* in the first stage, because he and the (n-1)/2 Type 2 voters all vote for *b* over *c* in the second stage. On the other hand, if he votes for *a* in the first ballot, *a* is then paired against *c* and the outcome there depends on whether Type 1 voters are in the majority (since Type 2 and 3 voters choose *c* over *a*). Since we already know that there are (n-1)/2 Type 2 voters (recall that we condition on a Type 1 voter being pivotal in the first ballot), a vote for *a* yields *a* only if <u>all</u> remaining voters are Type 1. That is, even if only one of the remaining voters has Type 3 preferences, voting for *a* in the first stage will ultimately yield *c*.

Given that a randomly chosen "first ballot *a*-voter" cannot be a Type 2, the probability that he is a Type 1 (as against Type 3) is $q_1/(q_3 + q_1)$. Therefore, the probability that voting for a yields *a* when you are decisive on the first ballot equals the probability that all other "first ballot *a*-voters" are Type 1 voters, or

$$[q_1/(q_3 + q_1)]^{(n-1)/2}$$

This term is the probability that if a decisive Type 1 voter votes for *a* in the first stage, *a* ultimately wins. If the value of *b* to this voter is v, then he should vote for *a* over *b* in the first ballot only if,

$$[q_1/(q_3 + q_1)]^{(n-1)/2} > v \qquad (2.1)$$

But since v is strictly greater than zero, for sufficiently large n this inequality cannot hold for <u>any</u> strictly positive q_1, q_2 and q_3. Therefore, all Type 1 voters voting sincerely for *a* over *b* cannot be a symmetric equilibrium for sufficiently large n regardless of the actual distribution of preferences across types.

Now consider the second possible symmetric equilibrium: All Type 1 voters strategically choose *b* over *a* on the first ballot. By similar reasoning, if a particular Type 1 voter is pivotal, there must be exactly (n-1)/2 Type 3 voters. Hence, if decisive, a vote for *b* in the first ballot guarantees that *b* beats *c* in the second vote and that *b* is the outcome. But a vote for *a* results in *c* beating *a* in the second stage only if there are no Type 2 voters, which occurs with probability

$$[q_1/(q_1 + q_2)]^{(n-1)/2}$$

So Type 1 voters vote insincerely for *b* in the first ballot if and only if

$$v > [q_1/(q_1 + q_2)]^{(n-1)/2} \qquad (2.2)$$

For sufficiently large n, this inequality necessarily holds. Hence, for large n, the only symmetric pure strategy equilibrium is: In the first ballot, Type 1 and 2 voters choose *b* and Type 3 voters choose *a*; If *b* wins, Type 1 and 2 voters choose *b*, Type 3 voters choose *c*; If *a* wins, Type 2 and 3 voters choose *c*, Type 1 voters choose *a*.

Notice that for large n, the q_j's approximate the actual proportions of voter types in the population, so if $q_1 + q_2 > 1/2$, then *b* wins with almost certainty: Type 1 and 2 voters constitute a majority, and they are sufficient to pass A. But if $q_3 > 1/2$, then *c* wins with almost certainty. This shows that <u>for sufficiently large n, regardless of priors, alternative *a* never prevails in a symmetric pure strategy equilibrium</u>.

Since this is true for q_1 arbitrarily close to 1, this generates a range of situations in which *a* is almost certainly a Condorcet winner, but the probability that *a* is selected is zero!

We emphasize that by "sufficiently large n" we do not mean necessarily that our results apply only to very large committees. Suppose, for example, that the committee has only five members but that $q_1 = .9$, $q_2 = .09$ and $q_3 = .01$. Hence, expressions (2.1) and (2.2) are both satisfied and the only symmetric equilibria is when Type 2's vote strategically if $[.9/.91]^2 > v > [.9/.99]^2$ or, equivalently, if $.98 > v > .83$. If the committee has 9 members, the constraints on v expand to $.96 > v > .69$.

The question naturally arises as to whether this example illustrates a possibility that is generic to agenda games with incomplete information in that it is not a pathological consequence of the parameters we chose. In fact, the example is robust in this sense. First, the assumption that only three preference order types are possible is not essential to the result. For example, we could also assume that the remaining three orders have ϵ-probability and that the priors on types 1, 2 and 3 are $q_1-\epsilon$, $q_2-\epsilon$, and $q_3-\epsilon$. The formal argument is naturally more complicated since we cannot reduce the problem to the strategic choices of only one preference type. Nevertheless, the equilibrium strategies of the three primary types are the same as in our analysis if ϵ is small. Second, the result does not rely on certainty (or symmetry) regarding v. Third, the implication of the example does not depend on there only being three alternatives -- although an analysis with more alternatives is considerably more complex -- nor does it depend on the use of a binary amendment agendas.

4. Communication and Signalling: Three Voting Stages

Deriving equilibria with incomplete information is complicated, both for the analyst and for committee members, because voters must simultaneously do two things: (1) secure the best possible outcome, and (2) learn what outcomes <u>are</u> possible -- what outcomes can win. With complete information, both tasks are accomplished simultaneously by a backwards solution of the voting tree (McKelvey and Niemi, 1978), but with incomplete information, the second task requires anticipating how others will vote, and to make accurate predictions requires learning as much as possible about the preferences of others and their anticipations of future votes. We might conjecture, then, that one reason why we can secure a seemingly perverse outcome in the previous section is that in a two stage agenda, although voters form expectations in the first stage based on a combination of their priors and their conjectures about the strategies of others, any revision of beliefs after the first ballot comes too late for voters to act on these revised beliefs: everyone votes sincerely on the second (last) ballot, regardless of what is learned from the first ballot. That is, there is no real communication or useful learning possible in a two-stage agenda.

In this and the next sections we present an analysis of two different types of communication that correspond to important possibilities for signaling and learning. The first type arises when, with more than two stages, early votes are used by committee members to convey information (or misinformation) that affects the decisions of other in later stages. The second type of communication, which we discuss in Section 5, arises prior to the actual voting on the agenda, and is modeled here as a nonbinding straw vote. The first type illustrates the informational role of multistage agendas in committees, and the second type illustrates the role of procedural rules that permit non-binding acts such as straw votes or "position taking."

Returning to the case of a three-person committee, we provide the opportunity for the communication of useful information by adding a stage to the voting. Specifically, we suppose that the committee must first choose between the agendas (a,c,b) and (b,c,a), after which the appropriate agenda is implemented. Figure 2 illustrates the corresponding voting tree. We proceed by supposing that preferences are restricted to the same three orders as in the previous examples. However, rather than describe all symmetric pure strategy equilibria as in Section 2, we concentrate on the specific possibility that Figure 2 describes. The underlined numbers there alongside branches denote dominant choices for voters of that type, and the remaining numbers denote the specific conjectured strategies that are part of our example. The numbers in brackets indicate that if the agenda (a,c,b) is chosen by a vote margin of 2-to-1, then Type 3 voters vote for a whereas if the margin is 3-to-0, then these voters choose c. Given these conjectures, we deduce the optimal strategies for voters of each type and check whether these strategies and the conjectured strategies coincide.

First, we must confirm that the underlined strategies in Figure 2 are dominant. Looking at the two subgames in Figure 2 that begin with the second vote -- the "left" subgame being the one in which (a,c,b) is the chosen agenda and the "right" subgame being the one in which (b,c,a) wins -- notice that in the left subgame, player 1 prefers any lottery between a

and b to one between b and c. Hence, 1 voting for a in the first ballot of the agenda (a,c,b) is dominant. For a Type 2 voter, if no one else is a Type 2 voter, then choosing a yields the least preferred outcome (a) whereas choosing c yields c or b. And if one or both of the other voters is also a Type 2, then b prevails regardless of how you vote. Hence, choosing c in the first ballot of the left subgame is sometimes better for a Type 2 voter and it is never worse than choosing a. Equivalent reasoning establishes the dominance of the strategies as indicated in the right subgame of the voting tree.

We assume as before that these dominant strategies as well as the conjectured strategies in Figure 2 are common knowledge, that any particular voter is of Type i with probability q_i, and that the cardinal utility of the intermediate alternative is v. For purposes of a specific numerical example, however, we also let $v = .6$, $q_1 = .1$, and $q_2 = q_3 = .45$. Our next step is to show that these numbers and the conjectured strategies portrayed in Figure 2 are an equilibrium.

Consider the situation of a Type 2 voter in the right subgame. Notice that such a voter believes that the only way to reach this part of the agenda is if one or both of the other voters is of Type 1. If he himself voted "left," there must be two Type 1 voters and a wins, regardless of what he does. But if he voted for the right subtree and the vote margin is 2-1, then there is only one Type 1 voter (the other "right" voter). If the third voter is a Type 2 like himself, then voting for c yields c, whereas voting for b yields b. But if the third voter is a Type 3, then voting for c yields c as before, whereas voting for b yields a. Thus, a Type 2 voter's choice at this node depends on how he evaluates "c with certainty," which yields him $v = .6$, as against the lottery "*b* if the third voter is Type 2, *a* if he is Type 3." The conditional probability that a voter is of Type 2, given that he cannot be of Type 1, is $q_2/(q_2 + q_3)$. Since this term is also the expected utility of the lottery resulting from a choice of *b*, a Type 2 voter will choose *c* if $v \geq q_2/(q_2 + q_3) = .45/(.45 + .45) = .5$. This requirement is satisfied since $v = .6$, so the strategy depicted in Figure 2 is consistent with a Type 2 voter's beliefs and conjectures whenever the agenda (*b*, *c*, *a*) wins.

Now consider a Type 3's decision in the left subgame. If the initial margin of victory is 3-0, then the voter in question believes that there are no Type 1 voters, since all such voters are conjectured to choose the right subgame. In this event, from Figure 2, voting for *c* yields *c* in precisely the same circumstances as voting for *a* yields *a*. Hence, voting for *c*, as conjectured, is dominant. Suppose, then, that the margin is 2-1. There are two possibilities: either the Type 3 voter in question (1) chooses the right subtree, or (2) he choose the left subtree. In case (1), he believes that there are no Type 1 voters, so the analysis proceeds as before. In case 2 there must be exactly one Type 1 voter, so voting for *c* yields *c* only if the third voter is also of Type 3, which has a probability of $q_3/(q_2 + q_3)$. Voting for *a*, on the other hand yields *a* with certainty. Thus, this voter chooses *a* over *c* if $v \geq q_3/(q_2 + q_3)$, which, in terms of the numbers in the example, requires that $.6 \geq .5$.

This establishes that the conjectured strategies for Type 3 voters in the left subgame and Type 2 voters in the right subgame are best responses given the parameters of the example, beliefs, and other conjectures. What remains is to show that the conjectured decisions in the first ballot

correspond to those that Figure 2 indicates. Earlier we analyzed such votes assuming that the voter in question is decisive, but we must be careful here, because decisions can affect margins of victory and, thus, the decisions of Type 3 voters. That is, votes are signals, and voters might seek to manipulate those signals.

With this strategic complication in mind, consider a Type 1 voter. If such a voter is pivotal in the first ballot or if both other voters vote "right," then voting "right" or "left" yields *a*. But if the left subtree wins regardless of this voter's decision (if there are no other Type 1 voters), his decision depends on the margin of victory he prefers to convey. If he votes "left" he will induce Type 3 voters to choose *c* -- his least preferred alternative -- whereas if he chooses "right" and signals a 2-1 vote, then Type 3 voters will choose *a* on the second ballot -- his most preferred alternative. Hence, his choice is to choose "right" and signal a 2-1 vote, which is what Figure 2 indicates.

Next, consider the decision confronting a Type 2 voter in the first ballot. If such a voter is pivotal (i.e., if there is precisely one Type 1 voter), then choosing "right" yields *c*, because he and the third voter (who is either Type 2 or Type 3) will choose *c* on the next two ballots. Choosing "left," on the other hand, yields *b* if the third voter is of Type 2, and *a* otherwise. The lottery occasioned by choosing "left" is preferred to the certainty of *c*, and is therefore consistent with the conjectured choice, if,

$$v < 1 - [q_3/(q_2 + q_3)]^2,$$

which is satisfied with the parameters of our example. Suppose now that there are no Type 1 voters, in which case "left" necessarily wins but the question becomes whether the Type 2 voter whose decision we are analyzing prefers to signal a 2-1 or a 3-0 split. If one or both of the other voters is a Type 2, then *b* prevails regardless of what signal he generates, because he and other Type 2's vote for *c*, then *b*. But if both are Type 3's, then a 3-0 split induces Type 3's to choose *c* whereas a 2-1 split induces both Type 3's to choose *a*, which gives a Type 2 his least preferred outcome. Hence, a Type 2 prefers to signal the 3-0 split occasioned by voting "left" rather than a 2-1 split, which proves that the conjectured strategy is consistent with the other features of the example.

Our final task concerns the analysis of Type 3 voters, and here we must show that the expected utility of voting "left" exceeds that of voting "right." As before, if both of the other voters are of Type 1, it does not matter how a Type 3 voter votes. But if there is exactly one Type 1 voter, then choosing "right" yields *c* and a payoff of 1 with certainty: The voter in question plus the other non-Type 1 voter will choose *c* on the next two ballots. The probability that there is precisely one Type 1 voter among the other two voters is $2q_1(1-q_1) = .18$. Now suppose that neither of the other two voters is Type 1, in which case there are three possibilities: both are Type 2 voters (probability $.45^2$); both are Type 3 voters (probability $.45^2$); and one is a Type 2 and the other is a Type 3 (probability $2(.45^2)$). If both are Type 2, then *b* prevails and the voter in question receives 0. If both are Type 3, they observe a 2-1 vote and choose *a* on the second and third ballots, and the voter in question realizes a payoff of .6. Finally, if one is Type 2 and the other is Type 3, then the voter in question knows that he caused the 2-1 split, and if he votes for *c*, he and the other Type 3 will elect *c* on

the final ballot, for a payoff of 1. Hence, the expected utility of voting "right" is $1(.18) + 0(.45^2) + .6(.45^2) + 1(2(.45^2)) = .7065$.

To compute the expected utility of voting "left," notice that if the initial vote is 3-0 because there are no Type 1 voters (probability $.9^2$), then other Type 3's will choose *c* on the second ballot and *c* will prevail unless both of the other voters are Type 2 (conditional probability $.5^2$). So *c* prevails with probability $.9^2(1-.5^2) = .6075$. If precisely one of the other voters is a Type 1 (probability .18), then the voter in question should vote for *a*: This yields *a* and a payoff of .6 with certainty, as against an even chance lottery between *c* and *b* with expected payoff of .5, depending on the type of the third voter, who is either Type 2 or Type 3. Hence, the expected utility of choosing "left" is $.9^2(1 - .5^2) + .6(.18) = .7155$, which shows, in accordance with the conjectured strategies in Figure 2, that the expected utility of voting "left" exceeds that of voting "right."

This example reveals another important difference between incomplete and complete information agendas. Notice that with complete information, regardless of whether there are two or three Type 3 voters and regardless of the 2-stage agenda used, *c* prevails. But with incomplete information, if there are two Type 3's and one Type 1 voter, then *a* prevails instead. In this instance, Type 3's key off the 2-1 vote signal and choose *a* rather than *c*. This establishes directly that, contrary to the result under complete information that any outcome that can be reached in n stages with an amendment agenda can be reached in two stages, our example induces a probability distribution over the outcomes that cannot be matched by any 2-stage agenda. Our example also suggests that the rules governing the precision with which the votes on earlier ballots are reported can affect equilibria. If a secret ballot permits announcement of the winner, but not its plurality, then Type 3 voters cannot key off the actual vote split, and the equilibrium will be different.

5. Communication and Signaling: Straw Votes:

Voting on an agenda provides an important signal, but we should not suppose from our example that only formal procedures and binding votes can perform this function. Consider straw votes, the existence of which seems, intuitively, to indicate that committees members are attempting to learn something about each others' preferences prior to making the "actual" decision. We are accustomed, for example, to an endless parade of T.V. Network polls of delegates prior to the actual balloting in the closely contested national party nominating conventions. What we can show here is that such nonbinding votes are more than mere window dressing, that they can affect profoundly the outcomes that eventually prevail.

Before proceeding, it is useful to reexamine the result obtained in Section 3 in which the unique symmetric pure strategy noncooperative outcome generally fails to correspond to the Condorcet winner. This event corresponds to an SVE, because no Type 1 voter is certain that his type is a majority in the committee even though q_1 is significantly greater than .5. Due to this uncertainty, the optimal decision criterion for such a voter is to condition his prior probabilities on being decisive, regardless of the likelihood of such a situation. Only if this situation is impossible -- only if the voter assigns a zero probability to the possibility of being pivotal -- is such conditioning unnecessary. Thus,

it seems to be clearly in the interest of Type 1 voters to learn before the first ballot whether or not this unlikely event is possible. That is, it seems only reasonable to suppose that if Type 1 members are confronted with the agenda in Figure 1 and if they share the belief that q_1 is nearly one, then they should reduce the uncertainty beforehand with a straw vote that signals how many voters there are of each type.

To be certain that this argument is correct, consider the voting tree in Figure 3, which represents the agenda (a,b,c) preceded by a nonbinding poll between a and b. Hence, regardless of the preference people reveal on the poll, the same agenda is implemented thereafter. Consider now the following specification of strategies as a candidate for a symmetric equilibrium:

> All types are sincere in the straw poll and Type 2 and 3 voters are always sincere in the agenda. Type 1 voters are sincere in the agenda if the number indicating a preference for b in the poll, r, is less than $R^* < (n-1)/2$, whereas if $r \geq R^*$, then Type 1 voters are insincere in the agenda's first ballot.

R^* depends on the q's, v, and n in a manner to be deduced shortly. But to verify that this conjecture is an SVE, let us consider first how a Type 1 voter uses the information of the straw poll in deciding how to vote in the first ballot of the agenda. Notice that if $r = (n-1)/2$, a Type 1 voter, i, is pivotal in the actual agenda, in which case the analysis in Section 3 tells us that for sufficiently large committees, i is insincere. If $r > (n-1)/2$ -- if the poll reveals that a majority of the committee are Type 2 voters -- then, by weak dominance, i should again be insincere.

But if $r < (n-1)/2$, i cannot be pivotal; furthermore, i does not possess a dominant (weak or strong) strategy from this point on. Thus, we have some freedom in choosing voting decisions for Type 1 voters at this information set in constructing an equilibrium. For reasons that will become clear shortly, we let i vote sincerely if and only if $r < R^* \leq (n-1)/2$.

To verify that there is an equilibrium of this sort, we must show that there exists some $R^* < (n-1)/2$ such that each voter type prefers to vote sincerely in the straw poll, conditional on being pivotal in the poll. Beginning with Type 1 voters, notice that such a voter, i, is pivotal in the poll only in the event that $r = R^*-1$, because then i can determine whether all other Type 1 voters choose sincerely or insincerely in the agenda. Naturally, i's optimal strategy in the poll depends on how many Type 1 and 3 voters there are among the remaining $n-r-1$ who are revealed by the poll to not be Type 2's. First, if i is sincere, he gets *a* rather than *c* only if at least $(n-1)/2$ of these voters are Type 1's; otherwise there are enough Type 3's to join with the r Type 2's to yield *c*. The probability that there are at least $(n-1)/2$ Type 1's among these $n-r-1$ voters is given by the following binomial expression (which also corresponds to the expected utility of being sincere since i's utility from *a* is 1 and from *c* it is 0):

$$\sum_{j=(n-1)/2}^{n-r-1} \binom{n-r-1}{j} (q_1/(q_1+q_3))^j (q_3/(q_1+q_3))^{n-r-1-j} \tag{4.1}$$

But if i is insincere when pivotal in the poll, then all Type 1's vote insincerely in the agenda's first ballot. Thus, by being insincere when pivotal, *c* prevails unless the r Type 2's and the Type 1's among the remaining $n-r$ voters are a majority. If they are a majority, *b* prevails

and i gets v. Since i knows that he is Type 1, *b* prevails if $(n-1)/2 - r$ or more of the "non-Type 2 voters" are Type 1's. Thus, the expected value of inducing insincerity is

$$v \sum_{j=(n-1)/2-r}^{n-r-1} \binom{n-r-1}{j} (q_1/q_1+q_3))^j (q_3/q_1+q_3))^{n-r-1-j} \tag{4.2}$$

Letting A denote the term after the summation, the requirement for sincere voting in the straw poll, that expression (4.1) exceed (4.2), becomes

$$\sum_{j=(n-1)/2}^{n-r-1} A \quad > \quad v \left[\sum_{j=(n-1)/2-r}^{(n-1)/2} A \quad + \quad \sum_{j=(n-1)/2}^{n-r-1} A \right] \tag{4.3}$$

If we set $r = 0$, then (4.3) is satisfied, because the term in brackets equals the left-most term, and $1 > v$. But if we let r be as large as possible, $(n-1)/2-1$, the inequality is more difficult to satisfy. Indeed, if we set $r = (n-1)/2$, which is the maximum value for R^*, (4.3) is identical to (2.1) since the term in brackets is 1; and we know that (2.1) cannot be satisfied for sufficiently large n. From the monotonicity properties of each term as a function of r, then for fixed q's, n, and v, the maximum value of r for which the inequality is satisfied, say R^*, lies in the interval $(0,(n-1)/2))$. This argument can also be used to establish that R^* is unique.

We must check, however, that R^* does not become zero as n approaches infinity. This concern arises from the fact that our conclusion about Condorcet winners not prevailing depends to some extent on n. We want to be certain that, regardless of n, the straw poll solves the dilemma arising in the example in Section 3.

Recall then that as n approaches infinity, the actual proportion of Type j voters approaches q_j, $j = 1,2,3$. Recall also that if $q_1 < 1/2$, then our Condorcet paradox does not arise, because a majority always prefers *b* to *a*. So let $q_1 > q_3$, and consider the infinite sequence $\{R_N^*\}_{N=3, N\ odd}^{\infty}$, letting R* be any limit point of that sequence. What we want to show is that if $q_1 > 1/2$ (i.e., if *a* is always the Condorcet winner for large committees), then

$$\lim_{N\to\infty} R_N^*/N = f^* > q_1 \qquad (4.4)$$

This inequality guarantees that in large committees, Type 1 voters never vote strategically in the second stage (i.e., with almost certainty, less than R* voters choose *b* in the straw poll).

To see that (4.4) is always satisfied for large n, notice that $q_1 > 1/2$ implies that $q_1/(q_1+q_3) > 1/2$, so that expression (4.2) converges to v, because $R_N^*/N < 1/2$ for all N. Therefore, R* must have the property that (4.1) evaluated at R_N^* converges to v. But this can only happen if the probability that there are exactly (N-1)/2 Type 1 voters out of $N - R_N^* - 1$ Type 1 or Type 3 voters converges to 1. That is, we must have,

$$\lim_{N\to\infty} [(N-R_N^*-1)/(N-1)]/[q_1/(q_1+q_3)] = 1/2,$$

which implies that,

$$\lim_{N\to\infty} R_N^*/N = f^* = (q_1-q_3)/2q_1 = 1/2 - q_3/q_1.$$

It is now easily verified that if $q_1 > 1/2$, then $f^* > q_2$, which is what we wanted to show.

The final step in the analysis is to show that Type 2 and Type 3 voters should be sincere in the straw poll. First, if a Type 2 voter is pivotal in the poll (i.e., if there are precisely R*-1 other Type 2 voters), then voting sincerely induces Type 1's to be insincere. If Type

1's and Type 2's are a majority, then *b* prevails, which is a Type 2's ideal. Otherwise, *c* prevails. So a Type 2 always prefers to have Type 1's vote insincerely. If a Type 3 is pivotal, then being sincere induces Type 1's to be sincere and being insincere induces them to be insincere. If type 3's are a majority, then *c* prevails, regardless. But if this type is not a majority, then inducing sincerity yields a lottery between *a* and *c* (since Type 2's are not a majority either) whereas inducing insincerity causes Type 1 and 2 to vote alike, which leads to a Type 3's least preferred alternative, *b*. Hence, whenever pivotal, Types 2 and 3 are sincere in the straw vote. This establishes the equilibrium.

The implication of expressions (4.1) - (4.3) can be made more intuitive by noticing that the critical value R* decreases as v increases, which means that a Type 1 voter can tolerate fewer "*b* voters" in the straw poll before he attempts to induce insincerity on the part of all Type 1 voters in the binding portion of the agenda by voting insincerely in the poll. That is, as his second preference becomes more attractive, then, ceteris paribus, he is willing to entertain riskier lotteries to get this preference, where these lotteries are riskier in the sense that there are fewer Type 2 voters willing to joint Type 1's to secure *b*.

We emphasize again that such conclusions pertain only to the specific conjectured equilibrium, and there are certainly a great many other SVE's. Also, our arguments are no doubt sensitive to the preferences and to the number of alternative outcomes that the example admits. Nevertheless, the example does show how a procedure that is purely informational, and thus irrelevant if information is complete -- in this case, a nonbinding poll -- becomes relevant if information is incomplete.

5. Conclusions

Our intent is to present a new approach to the formal study of agendas, an approach that is more promising for examining the role and importance of procedural rules in committees than traditional approaches that require complete information. Although examples cannot be a last word on a theory, at least three conclusions are evident. First, voting with incomplete information in binary voting systems is significantly different than voting with complete information. With complete information, Condorcet winners necessarily prevail, but no such guarantee is possible with incomplete information. Furthermore, although we use an especially simple example to show this, the application of the methodology that we review would reveal that this example is robust (albiet, with a considerable increase in the complexity of the analysis). Second, although strict preferences guarantee a unique full information equilibrium outcome with traditional assumptions, there may be a multiplicity of equilibria outcomes if voters are imperfectly informed about the preferences of others. Third, we already know that the formal procedural structure of committees can influence outcomes profoundly, and the analytic approach presented here permits further refinements of this conclusion. Specifically, our examples reveal the importance in incomplete information environments of such institutional details as pre-vote discussions, straw votes, and secret vs. open balloting.

It is also clear that the analysis of agendas with incomplete information requires considerably more effort than with full information. There are a number of issues that should be addressed, but we cannot explore examples endlessly. Nevertheless, important lessons about the

role of information could be learned by arbitrarily assigning one voter the role of being the agenda setter and asking: What agenda should a setter choose given that his choice reveals something about his preferences? A setter must take such inferences into account before choosing, which, of course, makes the problem of setting agendas a more interesting strategic issue. Indeed, this suggestion only scratches the surface of the problems associated with analyzing the formation of agendas. For example, agendas often are formed sequentially in committee, in which case strategies encompass decisions to second a motion, to introduce a new motion, when to introduce a motion, and when to move and to vote to terminate the process of adding motions to the agenda. Because informational and strategic concerns interact in complex and subtle ways, our argument is that a framework such as the one presented here in which beliefs are endogenously determined is required to study this class of problems realistically.

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Table 1

Preference Profile			Probability	Incomplete Info.		Complete Info.
$\#T_1$'s	$\#T_2$'s	$\#T_3$'s	of Profile	$v < 1/2$	$v > 1/2$	
3	0	0	1/27	*a*	*b*	*a*
2	1	0	1/9	*a*	*b*	*a*
2	0	1	1/9	*a*	*b*	*a*
1	2	0	1/9	*b*	*b*	*b*
1	1	1	2/9	*c*	*b*	*b*
1	0	2	1/9	*c*	*c*	*c*
0	3	0	1/27	*b*	*b*	*b*
0	2	1	1/9	*b*	*b*	*b*
0	1	2	1/9	*c*	*c*	*c*
0	0	3	1/27	*c*	*c*	*c*

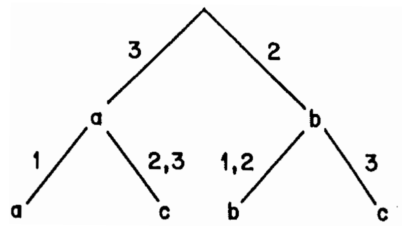

Figure 1

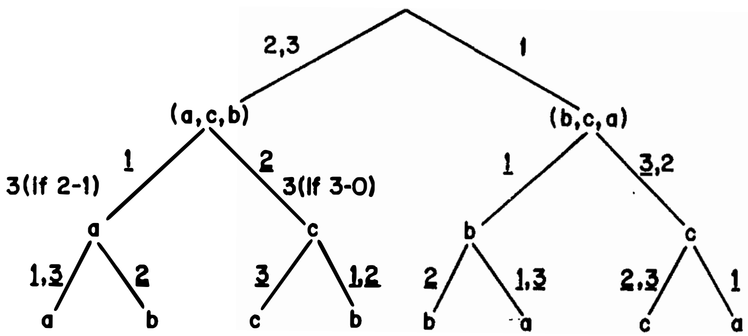

Figure 2

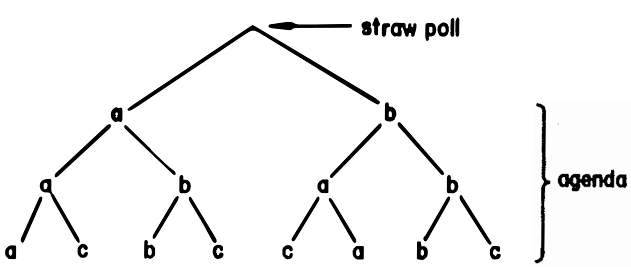

Figure 3