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THEORIES OF PRICE FORMATION AND EXCHANGE IN DOUBLE ORAL AUCTIONS

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ABSTRACT

We provide a theory to explain the data generated by Double Oral Auctions. The primary conclusion suggested by Double Oral Auction experiments is that the quantities exchanged and the prices at which transactions take place converge to, or near to, the values predicted by the competitive equilibrium model. Our theory predicts convergence to the competitive equilibrium and provides an explanation of disequilibrium behavior. The predictions of our theory fit the data better than do the predictions of Walrasian, Marshallian or game theoretic models.

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David Easley and John Ledyard*

1. INTRODUCTION

One of the main justifications for the use of equilibrium models in economics is the argument that there are forces which tend to drive agents and their decisions towards an equilibrium, if they are not at one already. Market equilibrium models have proven to be extremely powerful in the analysis of many situations; however, attempts to model and explain the forces that do drive an economy to equilibrium have met with little success. Most of the literature on the stability of equilibrium uses the fiction of a disinterested auctioneer who adjusts a single known price for each good in response to stated excess demand resulting from agents' equilibrium plans. The limitations and defects of this approach are well-known; for a survey of the literature see Arrow and Hahn [1]. In addition, as far as we know, the *only* institutional arrangement that even approximates this idealized model of price formation is the London gold market (see Jarecki [5]).

Now, however, a body of data has been generated which provides detailed information on the disequilibrium behavior of traders in auction markets similar to those of organized commodity or stock exchanges. These data are difficult to ignore since they are generated experimentally under controlled conditions, and cannot be explained away by reference to measurement error, unobserved variables, or other fudge factors. In the experiment, a small number of traders, each with

limited imperfect information, determine prices and quantities transacted through interactive bargains. There is neither a single price nor a single price quoter. Nonetheless, the quantities exchanged and the prices at which transactions take place typically converge to, or near to, the values predicted by the competitive equilibrium model. In spite of the fact that the traditional demand-supply model appears to yield reasonably accurate predictions of the long-run average prices and quantities in these markets, it fails to yield any insights into the process by which these prices and quantities are obtained.

In this paper we consider several positive theories of the price formation and exchange process for the class of experimental exchange markets called Double Oral Auctions.[1] We examine three of these theories in detail and argue that one of them seems to be the most consistent with the data. The ability of this theory also to explain price formation and exchange in other markets such as the New York Stock Exchange depends, of course, on the degree of parallelism that exists between the two (see Smith [11]). An astronomer's maintained hypothesis is that the physics of the lab is the same as that of the sun; our working hypothesis is that behavior in experimental markets is similar to that in other markets, and that insights discovered in the evidence generated in the lab are potentially transferable to non-experimental markets with similar institutional structures. Thus, we view the theory in this paper as a first step towards constructing a positive theory of the process of exchange and price formation in many other markets.

2. THE EXPERIMENTAL MARKET

In a Double Oral Auction (DOA) experiment, a pool of subjects (usually eight to twelve) is divided at random into a group of buyers and a group of sellers. The buyers are given value schedules telling them the amount in cents that they will receive from the experimenter for each unit of the good they purchase. Buyers keep the difference between their value and price they pay for that unit. The sellers are given cost schedules telling them their cost in cents for each unit of the good they sell. Sellers keep the difference between their selling price and their cost on each unit they sell. Each subject knows his own payoff schedule, but is given no information about the others' payoffs. Smith [9] shows how these payoff schedules induce demand and supply schedules. An example of the induced supply and demand schedules for one experiment is provided in Appendix A. A description of this experiment, including the instructions given to subjects, and their payoff schedules is provided in Appendix B.

After they receive their payoff schedules, subjects are allowed to trade during a market period of some fixed length. Buyers can make bids to buy a unit of the good and sellers can make offers to sell a unit. If a bid or offer is accepted, a binding trade occurs and all traders are informed of the contract price. Once a trade is completed, bids and offers can be made for another unit of the good. No information other than bids, offers, acceptances, and contract prices is transmitted or known by the participants.

When a market period ends, the subjects are given new payoff schedules, identical to their schedules for the previous period, and

the experiment is repeated.[2] Market demand and supply conditions are typically held constant across periods so that any equilibrating process that exists has a chance to establish an equilibrium. For a more detailed explanation of auction experiments and the usual results see Williams [14] and Smith and Williams [13].

These experiments provide a unique opportunity to examine price formation for two reasons. The first is that, unlike non-experimental markets, the actual competitive equilibrium prices and quantities are known. Second, complete data on bids, offers, contracts, and their timing is available. An example of a typical design and the data generated is provided in Appendices A and B. Demand and supply functions can be calculated from the subjects' valuations, and competitive equilibrium prices and quantities can then be computed. The first obvious fact from these experiments is that *actual exchange prices are not equal to those predicted by the competitive model.* In a strict sense, demand-supply theory is rejected by these data. The second obvious fact, however, is that after a very few replications, *transaction prices and quantities converge to near those predicted by the competitive model.* These observations have been replicated many times.[3] The only conclusion one can draw is that the traditional theory needs refining before one has a compelling explanation of the observed behavior in these markets. Not only must "equilibrium" be explained, but we must also explain the "disequilibrium" values, the sequence in which they occur, and the process by which participants are "learning."

The methodology we use to evaluate alternative theories of price adjustment consists of three steps. First, a good theory must predict convergence of quantities transacted and the prices at which transactions take place to the values predicted by the competitive equilibrium model. Any theory that fails this test will be at odds with the data and will be a poor theory of price adjustment. Second, the subset of theories which pass the first test are evaluated according to the frequency of violation of their predictions of disequilibrium data. A good theory must not predict disequilibrium behavior that is substantially at odds with the data. Those theories which predict convergence to competitive equilibrium, but whose predictions about the disequilibrium sequence are inconsistent with data are rejected. Third, those theories which pass the above tests are ranked according to the precision of their predictions. Better theories yield more precise predictions about bids, offers and contracts.

3. TWO POSSIBLE THEORIES

Our goal is to understand how the actual dynamics of these markets work, not how they should work. We recognize that there are a variety of models which purport to explain price adjustments, but we view the existence of experimental data as an opportunity to reject the subset of those theories which are obviously incorrect. The set of reasonable theories for these markets can now be constrained by the data in a way that has been unusual for economics but common in other sciences. To see what this means let us consider two obvious candidates for a theory

of market dynamics.

Since both the institutional description and the data from the experimental DOA markets reject the Walrasian tatonnement auctioneer as the appropriate model of price formation, a natural alternative might be a Marshallian theory. In a naive version of this theory, the trading sequence depends on the differences in buyers' prices (willingness to pay) and sellers' prices. In particular, this theory predicts that the first trade will occur between the buyer with the highest induced value (Buyer 1 in the example in Appendix A) and the seller with the lowest induced cost (Seller 1 in the example in Appendix A). The second trade occurs between the buyer and seller with the second values and costs, and so on. This theory does not predict which prices will occur, but it does predict that the total quantity transacted will be the competitive equilibrium quantity. When we look closely at the microdata we see that the theory is soundly rejected. A cursory glance at the summary data of Appendix B should convince even the most skeptical reader that the predictions of the naive Marshallian theory are not at all consistent with the data. (In IPDA14, the rank correlation coefficient between the order of the true values and the order of the transactions is .369 in week 1 and .273 in week 2.) This is an excellent example of a case in which the experimental setup allows us to test more hypotheses than would be possible if we only had access to non-experimental market data. Testing the prediction concerning the order in which participants are involved in transactions would be impossible without explicit knowledge of the individual valuations.

A second candidate for a theory might be a model based on game theoretic considerations. For most of the DOA markets there is a Nash equilibrium (with price-quantity offers or bids as strategies) in which all trades take place at the competitive equilibrium price. However, the use of a Nash equilibrium concept to describe the experimental market has two difficulties. First, as Appendix B illustrates, the data are not consistent with this equilibrium. Second, the participants in the experiments do not have enough information to calculate the strategies required to support this equilibrium. (They would have to be able to calculate the competitive equilibrium price.) Thus, we must consider a further complication.

In the experiments which have been run, details on others' payoffs (and thus on the competitive equilibrium) may only be inferred by the subjects from the public data on bids, offers and contracts. Thus, the structure in which subjects find themselves is a repeated game with incomplete information. If an equilibrium were calculated for this game its predictions could then be compared with the data. We feel that there are at least three reasons why this approach would be inadequate as a positive theory of Double Oral Auctions. First, as common knowledge about the distribution of valuations and the strategies selected is not controlled for in the experiments it is not clear how to apply game theory, as it currently exists, to the experiments. One could try to ignore this problem and assume that there is, at some level, common knowledge. However, this leads to a second difficulty.

If the subjects are risk neutral, we know from Gresik and Satterthwaite [3] and the revelation principle that any Bayes-Nash equilibrium has the property that no extra-marginal units are traded when subjects only own one unit of the commodity. Yet in the experiments extra-marginal units are often traded (see, e.g., the data from IPDA14 in Appendix B). If the subjects are risk averse then we know from Ledyard [6] that virtually anything can be an equilibrium. If risk attitudes are not controlled for (see Roth and Malouf [9]) then the game theoretic model explains everything.

Our third difficulty with the game theoretic approach is that, as far as we know, no one has solved for an equilibrium of the appropriate game. Wilson [15] has found strategies for a non-repeated version of the DOA which satisfy the necessary conditions for a Bayes-Nash equilibrium. But the DOAs are repeated, common knowledge is not controlled for and subjects may not be risk neutral as Wilson assumes. Under these circumstances, it is not fair to compare Wilson's predictions with the data. Wilson's model predicts that the rank correlation coefficient between the order of true values and trades is one which, as we noted above, is strongly at odds with the data. Thus, at least one of repetition, common knowledge, or risk attitudes seem to be crucial.

Friedman [2] takes an alternative game theoretic approach to the problem by redefining the game. He studies one day of a DOA with traders who are allowed to resell or repurchase the good being traded. Under a no-congestion condition which requires that at the day's end no trader wants to reset the closing bid or ask prices or accept the outstanding bid or ask, he shows that the final allocation is at most one transaction away from being Pareto-optimal. No-congestion implies that the final ask be no more than the second lowest cost of selling a unit, the final bid be no less than the second highest value of buying a unit, and that no one wants to accept the final prices. With resale and repurchase allowed this insures that all but perhaps one infra-marginal unit has traded and that no more than one extra-marginal unit has traded. Beyond the question of the appropriateness of the no-congestion assumption, the difficulty in applying this theory to the DOA experiments is that the theoretical conclusion relies heavily on the agents' ability to retrade, while retrading is not allowed in the experiments.

Since neither a Marshallian nor a game theoretic model appears to be appropriate as a positive theory we must develop an alternative theory.

4. A POSITIVE THEORY

a. Preliminaries

A participant in a Double Oral Auction experiment has a complex decision problem. He must decide when to bid, how much to bid, and whether or not to accept the trades offered by other subjects. Further, all of these decisions must be made with very imperfect information. The subject does not know the payoffs or expectations of other agents, he does not know the terms of trade that will be available to him in the future, and he does not know the effect of his actions on the actions of others. This is a very complex interactive

decision problem with incomplete information in which individuals must choose bidding and acceptance strategies. To place some structure on this problem, we first introduce some notation and definitions concerning the data known to both the experimenter and us.

The *true payoffs or values given to buyers* are integers, and are ranked as $V^1 \geq V^2 \geq . . . \geq V^n \geq 0$, where V^i is the ith highest value and there are n units. A buyer b will be assigned a subset of these units $V^{b1} \geq V^{b2} \geq . . . \geq V^{bB}$ and will trade them one at a time in the sequence b1, b2, . . . bB. No recontracting is allowed. The *true costs given to sellers* are integers, and are ranked as $0 \leq M^1 \leq M^2 \leq . . . \leq M^m$, where M^j is the cost of the jth unit and there are m units. It should be noted that in all of the hundred or so experimental double oral auction markets that we know of the values, V^i, and costs, M^i, are assigned once and remain fixed across a number of days. Each buyer (seller) knows only his own values (costs) and no participant is given any information as to how these values and costs were chosen. There is no basis for common knowledge assumptions about independence of values or their distributions. Consequently we neither make such assumptions nor use these concepts in our theory.

Market periods or days for an experiment are indexed by d = 1, 2, . . . The *time remaining in any given day* is indexed by t = 0, 1, . . . , T. Contract prices, bids, and offers are in integer units in the interval $[0,\overline{P}]$, where $\overline{P} < \infty$ is some arbitrarily selected upper bound above V^1 and M^m, and during any particular day, d, each participant observes all contract prices, bids, and offers.[4]

To summarize, each buyer knows the rules of the auction; the value of his own units; as well as the sequence, timing, amount, and identity of all past bids, offers, and contracts. It is these data alone on which the buyer can base his decisions to bid and to accept. A symmetric remark applies to each seller.

b. An Intuitive Look

We adopt the spirit of both revealed preference theory and demand-supply analysis by placing assumptions on individual behavior, which we believe are consistant not only with optimal behavior but also with a vast range of "boundedly rational" rules of thumb. We do not model how agents should make their decisions. Instead, we provide criteria which we believe sensible individuals in these markets act as if they satisfy. We do this by decomposing the decision problem into three main elements; expectations, reservation prices, and bidding strategies. These are most easily explained in reverse order.

Assume that at each instant of time there is for each buyer (seller) a reservation price, possibly different from his true value, which summarizes his willingness to bid up (offer down) to that price or to accept any offer up (bid down) to that price.[5] If each participant has such a reservation price as a function of time, the auction can then be thought of as proceeding like an English auction, *with these reservation prices substituting for the true values*. After some period of time the outstanding bid will always be held by the buyer with the highest reservation price (not necessarily the highest untraded value), and that bid will be at least as high as the second

highest reservation price. Otherwise, the holder of the second highest reservation price will bid, causing the holder of the highest reservation price to rebid, and so on. We find it unnecessary to explicity model this process, and we assume that it occurs instantaneously. Thus, all observed bids will be the reduced form results of the above English auction. This intuitive view of the bidding is formalized in Assumption 1 below. Sellers' offers are viewed symmetrically in Assumption 1'.

Since bids and offers depend on reservation prices, they ultimately depend on the relationship between reservation prices and the data observed by each agent. This relationship is assumed to depend on two principles of learning. First, it is true that whenever the bids and acceptance prices of a buyer are higher than were necessary to complete a transaction, the buyer completes a trade but overpays. We assume that a buyer will realize that he overpaid and will, during the next auction, lower his reservation price. If it is not lowered too much the buyer should still be able to complete a transaction but at a better price. Second, it is true that if a buyer waits too long to bid or, what is the same thing, maintains too low a reservation price during the day, then that buyer may not complete a transaction even though profitable ones are available. We assume that if a buyer could have purchased a unit at less than its value to him, V^i, but did not, then that buyer will realize he underbid and will, either that day or during the next auction, raise his reservation price at each time of day. It is the delicate balance between "paying too much" and "not offering to pay enough" which the buyers must learn in order to be successful in the auction. We do not explicitly model this learning process; instead, we provide assumptions about reservation price behavior which, if satisfied, reflect these learning principles. We summarize this rather simple intuition in Assumption 2 below.

c. Bidding Behavior

We start our description of the formal theory with the introduction of a hypothesis concerning the existence of the key unobservable of our model. It is important to realize that we treat reservation prices in this paper in the way that preferences are generally treated in economics. We cannot observe whether subjects really compute reservation prices; we can only assume they act as if they do. For a coherent theory, the reservation prices may need to be related in a systematic way to the true values but, *a priori*, do not need to be.

Assumption 0: Reservation Prices

For each buyer unit and seller unit there is an (unobservable) reservation price at each day d and time t, denoted $r_d^i(t) \in R^1$ for buyers and $s_d^j(t) \in R^1$ for sellers.

Assumption 0 only contains notation. To link the unobservable reservation prices to the data, we need to tie the bids and acceptances to them, and then to tie reservation prices to the true values and costs. As we indicated in the previous section, this is done by assuming that, given reservation prices, bids and acceptances are the reduced form of English auction behavior.

Assumption 1: Buyers' Bids and Acceptances

i) $b_d(t)$, the current outstanding bid in day d, with time t left, is held by buyer i^* where $r_d^{i^*}(t) \geq r_d^i(t)$, for all $i = 1, \ldots, n$.

ii) $b_d(t) \leq r_d^{i^*}(t)$.

iii) $b_d(t) \geq r_d^i(t)$, for all $i \neq i^*$.

iv) Buyer i^* accepts the current outstanding offer, $o_d(t)$, if and only if $o_d(t) \leq r_d^{i^*}(t)$. No other i accepts $o_d(t)$.

Simply stated, at each point in time, the current bid is held by the buyer with the highest reservation price--*not necessarily the buyer with the highest true value*. This bid lies below that reservation price and above the second highest reservation price. Under Assumption 1, and 1' below, trades always occur between the buyer with the highest reservation price and the seller with the lowest reservation price. We emphasize that *these need not be the buyer with the highest value and the seller with the lowest cost* since the English auction is based on reservation prices and not on the "true values," V^i and M^i.

For completeness, we make an assumption on the offers and acceptances of sellers that is symmetric with that made for buyers. The only difference is that we have arbitrarily assumed that if seller j^* is willing to accept $b_d(t)$ and buyer i^* is willing to accept $o_d(t)$ then the buyer accepts first.

Assumption 1': Sellers' Offers and Acceptances

i) $o_d(t)$, the current outstanding offer in day d, with time t left, is held by seller j^* where $s_d^{j^*}(t) \leq s_d^j(t)$, for all

$j = 1, \ldots m$.

ii) $o_d(t) \geq s_d^{j*}(t)$.

iii) $o_d(t) \leq s_d^j(t)$, for all $j \neq j^*$.

iv) seller j^* accepts the outstanding bid $b_d(t)$ if and only if $b_d(t) \geq s_d^{j*}(t)$ and i^* does not accept $o_d(t)$. No other j accepts $b_d(t)$.

We do not yet have a testable theory since, given any sequence of bids and contracts, it is possible to construct a sequence of reservation prices which, under Assumption 1, would imply the given data precisely. Unless we place some restrictions on the reservation prices, we can explain anything, and therefore nothing.

d. Reservation Price Formation

We now tie the theory down by restricting reservation price behavior in a way which relates it to observable data. This is the way in which we connect bids, contract prices, and the sequence of trades to the initial data known by the experimenter and, thus, provide testable propositions about these auctions.

Reservation prices are assumed to be formed in accordance with the intuitive principles outlined in Section 4.b. We begin by assuming that a buyer's expectations in any period are based on the prices of last period. In particular, we assume that the support of the buyer's expectations is the set of prices bounded by the maximum of last period's highest contract price or highest bid, and the minimum of last period's lowest contract price or lowest offer. Based on these expectations, reservation prices are formed over time as follows: (a)

for most of a trading day, one's reservation price lies below V^i and within the support of the expectations (when this is feasible), (b) if possible, the reservation price is actually below the maximum price in the support since the buyer does not want to "overpay," (c) eventually, if no contract is agreed to, buyers will cave-in and let the reservation price approach the maximum price in the support, and (d) if still no contract is completed, the reservation price will rise higher than even the maximum in the support of the expectations. This sequence of actions includes behavior known to be optimal in finite time stochastic search models and should, therefore, be uncontroversial.

Before formalizing our assumption on reservation prices we need to introduce some notation. If a trade occurs at time t of day d we let $c_d(t)$ be the contract price. Then for each day $d > 1$, let $\underline{P}_d = \text{Min}\{o_{d-1}(t), c_{d-1}(t) : t = 0, \ldots, T\}$ and $\overline{P}_d = \text{Max}\{b_{d-1}(t), c_{d-1}(t) : t = 0, \ldots, T\}$. We assign $[\underline{P}_1, \overline{P}_1] = [0, \overline{P}]$. The interval $[\underline{P}_d, \overline{P}_d]$ is interpreted as the support of traders' price expectations in day d. Let $\Delta P_d = \overline{P}_d - \underline{P}_d$.

Assumption 2: Buyer's Reservation Price Formation

For all buyers $i = 1, \ldots, n$:

i) If i has traded (accepted an offer or had a bid accepted) in day d before time t then $r_d^i(t) = 0$.

ii) For each day d there is time $\hat{t}_d^i > 1$ such that, if i has not traded in d before t, then:

a) For all $t > \hat{t}_d^i$;

$\text{Min}\{V^i, \overline{P}_d\} > r_d^i(t) \geq \text{Min}\{V^i, \underline{P}_d\}$ if $\Delta P_d > 1$ and $V^i \geq \underline{P}_d$,

$\mathrm{Min}\{V^i, \overline{P}_d\} \geq r_d^i(t) \geq \mathrm{Min}\{V^i, \underline{P}_d\}$ otherwise.

b) $r_d^i(\hat{t}_d^i) = \mathrm{Min}\{V^i, \overline{P}_d - 1\}$.

c) For all $t < \hat{t}_d^i$;

$r_d^i(t) = \mathrm{Min}\ \{V^i, b_d(t-1) + 1\}$ if

$b_d(t-1) \in \{\overline{P}_d, \overline{P}_d - 1\}$ and $b_d(t-1)$ unaccepted,

$r_d^i(t) \in \{r_d^i(t-1), \mathrm{Min}\ \{V^i, b_d(t-1) + 1\}\}$ if

$b_d(t-1) > \overline{P}_d$ and $b_d(t-1)$ unaccepted,

$r_d^i(t) = r_d^i(t-1)$ otherwise.

Assumption 2(i) sets the reservation price for traded units to zero to indicate that they have left the market. The conditions in assumptions 2(ii)(a) embody the intuition that, as a result of learning, reservation prices will not be "too high" early in the trading day. The conditions in assumption 2(ii)(b) and 2(ii)(c) embody the intuition that, towards the end of the day, if the buyer has not completed a transaction then that buyer will learn to raise his reservation price slowly. Towards the end of the day, reservation prices will not be "too low."

To complete the model we make a symmetric assumption about sellers' reservation prices which we call Assumption 2'.

<u>Assumption 2'</u>: Sellers Reservation Price Formation

For all sellers j = 1, . . . , m:

i) If j has traded (accepted a bid or had an offer accepted) in day d before time t then $s_d^j(t) = \overline{P}$.

ii) For each day d there is a time $\overline{t}_d^j > 1$ such that, if j has not traded in day d before time t, then:

a) For all $t > \overline{t}_d^j$;

$\mathrm{Max}\{M^j, \overline{P}_d\} \geq s_d^j(t) > \mathrm{Max}\{M^j, \underline{P}_d\}$ if $\Delta P_d > 1$ and $M^j \leq \overline{P}_d$,

$\mathrm{Max}\{M^j, \overline{P}_d\} \geq s_d^j(t) \geq \mathrm{Max}\{M^j, \underline{P}_d\}$ otherwise.

b) $s_d^j(\overline{t}_d^j) = \mathrm{Max}\{M^j, \underline{P}_d + 1\}$.

c) For all $t < \overline{t}_d^j$;

$s_d^j(t) = \mathrm{Max}\{M^j, o_d(t-1) - 1\}$ if

$o_d(t-1) \in \{\underline{P}_d, \underline{P}_d + 1\}$ and $o_d(t-1)$ unaccepted,

$s_d^j(t) \in \{s_d^j(t-1), \mathrm{Max}\ \{M^j, o_d(t-1) - 1\}\}$ if

$o_d(t-1) < \underline{P}_d$ and $o_d(t-1)$ unaccepted

$s_d^j(t) = s_d^j(t-1)$ otherwise.

We have made two implicit assumptions which should be recognized. First, we assume that each buyer's and seller's behavior is independent of the total number of participants in the market. That is, a buyer's choices of bids and acceptances is the same whether he is a monopolist or one of 100 buyers. Although this runs counter to conventional economics, experimental evidence suggests that if the number of buyers and the number of sellers are both greater than two then this assumption is satisfied. Further, even if there is a single seller, what little evidence there is suggests that the model we propose may still be appropriate. We leave as an open empirical question just how few participants, if any, are needed before our theory is not applicable.

The second implicit assumption is that buyers and sellers with multiple units to purchase or sell will decide on strategies for each unit separately. That is, the bids and acceptances a buyer makes for his, say, highest valued (first) unit are assumed to be independent of the total number of units he may want to buy. This is not "rational behavior" but the interaction effects are difficult to model (we know of no literature which does this). The simplicity this assumption gives the theory is, we feel, well worth the price.[6]

A question that naturally arises is whether our behavioral rules are consistent with optimal behavior for a game theoretic formulation of the DOA. Since we have not modeled the repeated incomplete information game, and since the common knowledge assumptions that are an integral part of game theory are not controlled for in the experiments, there is no one correct model of the game actually being played. So we have no formal way to address the optimality of our behavioral rules. However, we know from Ledyard [6] that the implications of our rules for the observable data on trades are consistent with the implications of a Bayes-Nash equilibrium in the repeated incomplete information game for some specification of the traders' levels of risk aversion and common knowledge. So our behavioral rules are not necessarily inconsistent with optimal behavior.

We believe that there are traders in the experiments whose behavior is, at least for a few iterations, vastly different from the behavior which would be consistent with our assumptions. In particular there are traders, such as seller 2 in IPDA57 (see Section 6) who continually hold out for a highly profitable trade even though they never complete one. These traders usually modify their behavior after a few iterations. Those who don't lose a considerable amount of "opportunity" income. We can't "explain" this "irrationality," but

neither can any other sensible theory.

We turn now to the derivation of a number of testable implications of the theory. We then confront these with the data from a small number of representative experiments. At that point, the reader should be able to decide whether or not our model offers a realistic description of actual behavior in double auctions.[7]

5. THEOREMS

In this section we trace through some of the implications of our theory. As will become apparent, most of the action will occur when there is an "excess demand or supply" of two or more units remaining in the auction, as then there are competitive pressures on bids and offers. Thus we are interested in the following concepts.

<u>Definition</u>: Let $D^c(P) = \#\{V^i \geq P\}$, $D^o(P) = \#\{V^i > P\}$, $S^c(P) = \#\{M^j \leq P\}$ and $S^o(P) = \#\{M^j < P\}$. Let $P_* = \min\{P : D^c(P) \leq S^o(P) - 2\}$ and $P^* = \max\{P : S^c(P) \leq D^o(P) - 2\}$.

To see the role of P_* and P^* we consider the following propositions. All results are stated under Assumptions 0, 1, 1' 2, and 2'.

<u>Lemma 1</u>:

a) If $\underline{P}_d \geq P_*$ then $\underline{P}_{d+1} < \underline{P}_d$. If $\underline{P}_d < P_*$ then $\underline{P}_{d+1} < P_*$.

b) If $\overline{P}_d \leq P^*$ then $\overline{P}_{d+1} > \overline{P}_d$. If $\overline{P}_d > P^*$ then $\overline{P}_{d+1} > P^*$.

Proof of Lemma 1:

We prove (a); the proof of (b) is symmetric.

Suppose $\underline{P}_d \geq P_\star$ and $\underline{P}_{d+1} \geq \underline{P}_d$. As $\underline{P}_d \geq P_\star$ we have $D^c(\underline{P}_d) \leq S^o(\underline{P}_d) - 2$. The number of trades in day d is no more than $D^c(\underline{P}_d)$ as by hypothesis all trades have been at price $\underline{P}_d$ or above. Thus at $t = 2$ there are at least two sellers j and j' with M^j, $M^{j'} < \underline{P}_d$ who have not yet traded. Then by applying Assumptions 2'(ii)(c) and 1' repeatedly, we have $o_d(0) \leq \underline{P}_d - 1$. But then $\underline{P}_{d+1} < \underline{P}_d$ which contradicts $\underline{P}_{d+1} \geq \underline{P}_d$.

Suppose $\underline{P}_d < P_\star$ and $\underline{P}_{d+1} \geq P_\star$. Then all trades have been at prices at or above $P_\star$. A minor modification of the argument above then yields a contradiction.

Thus there are competitive forces driving minimum contract prices below $P_\star$ and keeping them below $P_\star$. These same forces drive maximum contract prices above $P^\star$ and keep them above $P^\star$. There are also competitive pressures driving maximum prices down and minimum prices up.

Lemma 2: Suppose $\Delta P_d > 1$.

a) If $D^o(\underline{P}_d) \geq S^o(\overline{P}_d)$ then $\underline{P}_{d+1} > \underline{P}_d$.

b) If $S^o(\overline{P}_d) \geq D^o(\underline{P}_d)$ then $\overline{P}_{d+1} < \overline{P}_d$.

Proof of Lemma 2:

We prove (a); the proof of (b) is symmetric.

Suppose that $D^o(\underline{P}_d) \geq S^o(\overline{P}_d)$ and $\underline{P}_{d+1} \leq \underline{P}_d$. Then there exists a time t' such that either $o_d(t') = \underline{P}_{d+1}$ or $c_d(t') = \underline{P}_{d+1}$. Since $\underline{P}_{d+1} \leq \underline{P}_d$ and $\Delta P_d > 1$ it follows from Assumption 2'(ii)(a), (c) that there exists a time $\hat{t} > t'$ such that $o_d(\hat{t}) = \underline{P}_d + 1$ was not accepted.

Therefore, as $\Delta P_d > 1$ Assumption 1(ii)(a) implies that all units $V^i > \underline{P}_d$ have been traded before time $\hat{t}$. So the number of trades before time $\hat{t}$ is at least $D^o(\underline{P}_d) \geq S^o(\overline{P}_d)$. Then as Assumption 2'(ii)(a) implies that if $\Delta P_d > 1$ all $M^j < \overline{P}_d$ trade before any $M^j \geq \overline{P}_d$ we know that all $M^j < \overline{P}_d$ have been traded before time $\hat{t}$. So all $M^j \leq \underline{P}_d$ have been traded before time $\hat{t}$. Then by Assumption 2', $s_d^j(t)$, $o_d(t) > \underline{P}_d$ for all $t \leq \hat{t}$ and all j. This contradicts $[o_d(t') = \underline{P}_{d+1} \leq \underline{P}_d$ or $c_d(t') = \underline{P}_{d+1} \leq \underline{P}_d]$.

Finally, buyers' reluctance to pay too much and sellers' reluctance to accept too little eventually force minimum and maximum contract prices closer together. Of course, the difference between maximum and minimum contract prices does not necessarily decrease every day. In a day where there is excess demand at the upper bound $\overline{P}$, prices may rise, but they will not go above the cost of unit $D^o(\underline{P})$. This occurs because units up to $D^o(\underline{P})$ trade first (if $\Delta P > 1$) and these can all be traded at prices no more than $M^{D^o(\underline{P})}$. Thus the statistic that falls, or at least does not rise, in every period is the maximum of $\overline{P}$ and $M^{D^o(\underline{P})}$.

Definition: Let $u_d = \max\{\overline{P}_d, M^{D^o(\underline{P}_d)}\}$ and $\ell_d = \min\{\underline{P}_d, V^{S^o(\overline{P}_d)}\}$.

Lemma 3: If $\Delta P_d > 1$ then $u_{d+1} \leq u_d$, $\ell_{d+1} \geq \ell_d$ and

$$|u_{d+1} - \ell_{d+1}| < |u_d - \ell_d|.$$

Proof of Lemma 3:

There are two cases to consider: (1) $S^o(\overline{P}_d) \geq D^o(\underline{P}_d)$, and (2) $D^o(\underline{P}_d) \geq S^o(\overline{P}_d)$. We prove the lemma under case 1; the proof under case 2 is symmetric.

We first need to establish that if $\Delta P_d > 1$ and $S^o(\overline{P}_d) \geq D^o(\underline{P}_d)$ then $\underline{P}_{d+1} \geq \ell_d$.

Claim 1: If $\Delta P_d > 1$ and $S^o(\overline{P}_d) \geq D^o(\underline{P}_d)$ then $\underline{P}_{d+1} \geq \ell_d$.

Proof of Claim 1: Suppose that $\Delta P_d > 1$, $S^o(\overline{P}_d) \geq D^o(\underline{P}_d)$ and $\underline{P}_{d+1} < \ell_d$. From the definition of $\underline{P}_{d+1}$ we know that there is a time t' in day d such that $o_d(t') = \underline{P}_{d+1}$ or $c_d(t') = \underline{P}_{d+1}$. Then as $\underline{P}_{d+1} < \ell_d \leq \underline{P}_d$ there must be a time $\hat{t}$ in day d such that $o_d(\hat{t}) = \ell_d$ was not accepted. This implies that all $V^i \geq V^{S^o(\overline{P}_d)}$ have traded before time $\hat{t}$. So the number of units traded before time $\hat{t}$ is at least $S^o(\overline{P}_d)$. By Lemma 2(b) we have $\overline{P}_{d+1} < \overline{P}_d$. So the number of units traded in day d is no more than $S^o(\overline{P}_d)$. Thus the number of units traded in day d, before time $\hat{t}$, is $S^o(\overline{P}_d)$. So all $M^j < \overline{P}_d$ have traded before time $\hat{t}$. Then there does not exist a seller unit $M^j \leq \underline{P}_{d+1} < \ell_d$ to offer $o_d(t') = \underline{P}_{d+1}$ or accept a contract at $c_d(t') = \underline{P}_{d+1}$. This contradicts $\underline{P}_{d+1} < \ell_d$.

The proof of Lemma 3 follows directly from the claims below.

Claim 2: If $\Delta P_d > 1$ and $S^o(\overline{P}_d) \geq D^o(\underline{P}_d)$ then $\ell_{d+1} \geq \ell_d$.

Proof of Claim 2: By Lemma 2, $\overline{P}_{d+1} < \overline{P}_d$. So $S^o(\overline{P}_{d+1}) \leq S^o(\overline{P}_d)$. This implies that $V^{S^o(\overline{P}_{d+1})} \geq V^{S^o(\overline{P}_d)}$. By Claim 1, $\underline{P}_{d+1} \geq \ell_d$. Now $\ell_{d+1} = \text{Min}\{\underline{P}_{d+1}, V^{S^o(\overline{P}_{d+1})}\} \geq \text{Min}\{\underline{P}_{d+1}, V^{S^o(\overline{P}_d)}\} \geq \ell_d$.

Claim 3: If $\Delta P_d > 1$ and $S^o(\overline{P}_d) \geq D^o(\underline{P}_d)$ then $u_{d+1} < u_d$.

Proof of Claim 3: By Lemma 2(b), $\overline{P}_{d+1} < \overline{P}_d$ and by Claim 1, $\underline{P}_{d+1} \geq \ell_d = \text{Min}\{\underline{P}_d, V^{S^o(\overline{P}_d)}\}$. Thus $D^o(\underline{P}_{d+1}) \leq \text{Max}\{D^o(\underline{P}_d), S^o(\overline{P}_d)\} = S^o(\overline{P}_d)$. So $M^{D^o(\underline{P}_{d+1})} \leq M^{S^o(\overline{P}_d)} < \overline{P}_d$. Then $u_{d+1} = \text{Max}\{\overline{P}_{d+1}, M^{D^o(\underline{P}_{d+1})}\} < \overline{P}_d \leq u_d$.

So $u_{d+1} < u_d$.

The forces embodied in Lemma's 1, 2, and 3 serve to drive contract prices together and into the interval $[P^*, P_*]$. If supply and demand balance at this point, prices will stay in this interval.

Theorem 1: If $D^c(P^*) = S^c(P_*)$ then there exists a day $d^* < \infty$ such that $P^* \leq \underline{P}_d < P_*$ and $P^* < \overline{P}_d \leq P_*$ for all $d \geq d^*$.

Before proceeding to the proof of Theorem 1 we need to show that the interval $[P^*, P_*]$ is well defined.

Claim 4: $P_* \geq P^*$.

Proof of Claim 4: Suppose $P^* \geq P_*$. Then $S^c(P_*) \leq D^o(P_*) - 2$ and $D^c(P_*) \leq S^o(P_*) - 2$. So $D^c(P_*) + 2 \leq S^o(P_*) \leq S^c(P_*) \leq D^o(P_*) - 2$. This implies $D^c(P_*) < D^o(P_*)$ which is false.

Proof of Theorem 1:

As the price set, the integers in $[0, \overline{P}]$, is finite, Lemma 1 implies that there is finite day $\overline{d}$ such that $\underline{P}_d < P_*$ and $\overline{P}_d > P^*$ for all $d \geq \overline{d}$. Then by Lemma 3 there is a finite day $d^* \geq \overline{d}$ such that $[\underline{P}_d^*, \overline{P}_d^*] \subseteq [P^*, P_*]$ and $\Delta P_d^* \leq 1$.

We now prove the theorem by an induction argument. Suppose $[\underline{P}_d, \overline{P}_d] \subseteq [P^*, P_*]$ for some day $d \geq d^*$. We need to show that this implies $[\underline{P}_{d+1}, \overline{P}_{d+1}] \subseteq [P^*, P_*]$. Suppose not, say $\underline{P}_{d+1} < P^*$. Then there is a time t' in day d such that $o_d(t') < P^*$ or $c_d(t') < P^*$. As $P^* \leq \underline{P}_d$ there is a time $\hat{t} \geq t'$ such that $o_d(\hat{t}) = P^*$ was not accepted. As $P^* \leq \underline{P}_d$ this implies that all units $V^i \geq P^*$ have traded before time $\hat{t}$. So the number of units traded is at least $D^c(P^*) = S^c(P_*)$. As $P_* \geq \overline{P}_d$ this implies that all units $M^j \leq P_*$ have traded before time $\hat{t}$. Then there is no seller with a unit $M^j < P^* < P_*$ to offer $o_d(t') < P^*$ or to accept $c_d(t') < P^*$. The proof that $\overline{P}_{d+1} \leq P_*$ is symmetric.

By the induction argument above and Lemma 1, we have a day $d^* < \infty$ such that $P^* \leq \underline{P}_d < P_*$ and $P^* < \overline{P}_d \leq P_*$ for all $d \geq d^*$.

Theorem 1 applies to experiments which have a Walrasian equilibrium price P^e and quantity Q^e. These experiments fall into three groups. First, if there are multiple units at the Walrasian equilibrium price (and if $D^c(P^e - 1) = S^c(P^e + 1)$), then Theorem 1 predicts that prices will eventually remain within one cent of P^e (as $P_* = P^e + 1$, $P^* = P^e - 1$) and that quantity traded will be at least $Q^e - 1$ and no more than the maximum of $S^c(P_* + 1)$ and $D^c(P^* - 1)$. Second, if there is only one unit at the Walrasian equilibrium price and $D^c(P^*) = S^c(P_*)$, then Theorem 1 predicts that eventually the maximum price will be no more than one cent above the minimum of the value of the first infra-marginal buyer (V^{Q^e-1}) and the cost of the first extra-marginal seller (M^{Q^e+1}). The prediction for the minimum price is symmetric. In the limit, prices tend to keep out extra-marginal units and to keep in infra-marginal units. For this class of experiments the prediction is again that quantity traded will eventually remain in the interval $[Q^e - 1, \max\{S^c(P_* + 1), D^c(P^* - 1)\}]$. Finally, there may be an interval of prices any of which can be a Walrasian equilibrium with no units at any of these prices. If $D^c(P^*) = S^c(P_*)$ the predictions of Theorem 1 are again that prices eventually remain in $[P^*, P_*]$.

However, it is possible to design payoff schedules with one unit at the Walrasian equilibrium or with no units at any Walrasian equilibrium so that $D^c(P^*) \neq S^c(P_*)$. This case and cases where there is no Walrasian equilibrium are addressed by the following theorem.

<u>Theorem 2</u>:

a) If $D^c(P^*) > S^c(P_*)$ then there exists a day $d^* < \infty$ such that $P^* \leq \underline{P}_d < P_*$ and $P^* < \overline{P}_d \leq M^{D^c(P^*)}$ for all $d \geq d^*$.

b) If $S^c(P_*) > D^c(P^*)$ then there exists a day $d^* < \infty$ such that $V^{S^c(P_*)} \leq \underline{P}_d < P_*$ and $P^* < \overline{P}_d \leq P_*$ for all $d \geq d^*$.

<u>Proof of Theorem 2</u>:

We prove part (a); the proof for (b) is symmetric.

We first need to establish the relationship between P^*, P_*, $M^{D^c(P^*)}$ and $V^{S^c(P_*)}$.

<u>Claim 5</u>: If $D^c(P^*) > S^c(P_*)$ then $M^{D^c(P^*)} > P_* > V^{S^c(P_*)} \geq P^*$.

<u>Proof of Claim 5</u>:

i) Suppose $P_* \geq M^{D^c(P^*)}$. Then $S^c(P_*) \geq D^c(P^*)$. A contradiction.

ii) Suppose $V^{S^c(P_*)} \geq P_*$. Then $D^c(P_*) \geq S^c(P_*) \geq S^o(P_*)$. But by definition, $D^c(P_*) + 2 \leq S^o(P_*)$.

iii) Suppose $P^* > V^{S^c(P_*)}$. Then $D^c(P^*) < S^c(P_*)$. A contradiction.

By the argument in Theorem 1 we know that there is a day $d^* < \infty$ such that $[\underline{P}_d^*, \overline{P}_d^*] \subseteq [P^*, P_*]$. By Claim 5, $M^{D^c(P^*)} > P_*$. So $[\underline{P}_d^*, \overline{P}_d^*] \subset [P^*, M^{D^c(P^*)}]$. The proof now proceeds by induction. We need to show that if $[\underline{P}_d, \overline{P}_d] \subseteq [P^*, M^{D^c(P^*)}]$ then $[\underline{P}_{d+1}, \overline{P}_{d+1}] \subseteq [P^*, M^{D^c(P^*)}]$.

There are two cases to consider: (1) $\overline{P}_d \le P_*$, and (2) $\overline{P}_d > P_*$.

Case 1: $\overline{P}_d \le P_*$. As $M^{D^c(P_*)} > P_* \ge \overline{P}_d$ and $V^{S^c(P_*)} \ge P^*$ by Claim 5, an argument similar to the proof of Theorem 1 shows that $\overline{P}_{d+1} \le M^{D^c(P_*)}$ and $\underline{P}_{d+1} \ge P^*$.

Case 2: $\overline{P}_d > P_*$. We know that $\underline{P}_d < P_*$ for all $d \ge d^*$, so $\overline{P}_d > P_*$ implies that $\Delta P_d > 1$. So by Lemma 3, $u_{d+1} \le u_d$. By definition $u_d = \mathrm{Max}\{\overline{P}_d, M^{D^0(\underline{P}_d)}\}$ and by hypothesis $\underline{P}_d \ge P^*$. So $D^0(\underline{P}_d) \le D^0(P^*)$. Thus, $M^{D^0(\underline{P}_d)} \le M^{D^0(P_*)} \le M^{D^c(P_*)}$. By hypothesis $\overline{P}_d \le M^{D^c(P_*)}$. So $u_d \le M^{D^c(P_*)}$. By definition $u_{d+1} = \mathrm{Max}\{\overline{P}_{d+1}, M^{D^0(\underline{P}_{d+1})}\}$. Now $u_{d+1} \le u_d \le M^{D^c(P_*)}$. So $\overline{P}_{d+1} \le M^{D^c(P_*)}$.

We also need to show that $\underline{P}_{d+1} \ge P^*$. Suppose not. Then $\underline{P}_{d+1} < P_*$. This requires $S^0(\underline{P}_d) > D^c(\underline{P}_d)$. Thus, $S^c(P_*) \ge S^0(P_*) \ge S^0(\underline{P}_d) > D^c(\underline{P}_d) \ge D^c(P^*)$. This contradicts $D^c(P^*) > S^c(P_*)$. So $\underline{P}_{d+1} \ge P^*$.

Theorem 2 now follows from the induction argument above and Lemma 1.

Although Lemmas 1, 2, and 3 imply that prices are eventually contained in the interval $[P^*, P_*]$ they need not stay in this interval if supply and demand are not equal there. For example, if $D^c(P^*) > S^c(P_*)$ and low value buyers (those with $P^* - 1 \le V^i < P_*$) trade first, the remaining high value buyers may bid prices up. However, they need not and so will not, bid more than $M^{D^c(P^*)}$ in order to complete a trade. So the range of prices could expand to be $[P^*, M^{D^c(P^*)}]$. In subsequent days it will shrink until it is again contained in $[P^*, P_*]$. It seems unlikely that this process would continue, and our theory does not predict that it will, only that it might. In fact Lemma 3 implies that

for all supply and demand configurations if all extra-marginal units are excluded by $[\underline{P}_d, \overline{P}_d]$ then the interval will shrink to at most one cent and then remain fixed.

6. COMPARISONS OF THE PREDICTIONS WITH THE DATA

Our prediction of convergence seems consistent with the experimental data, but it is not directly testable with these data as the number of repetitions necessary for convergence is not specified. In any case, obtaining the competitive equilibirum in the limit is only a first test of a theory of price formation in Double Oral Auctions. We have rejected the models considered in section 3, at least in part, on the basis of their incorrect predictions about dynamics. In this section we compare the predictions of our model with experimental data. There are three categories of data for which our theory has implications: the sequence of minimum and maximum prices, the sequence of trading partners, and the number of units traded.

The three lemmas in section 5 directly yield predictions about the dynamics of minimum and maximum prices. Lemma 1 implies that prices move to bracket the competitive equilibrium price and that once this is accomplished the equilibrium price remains in the interval $[\underline{P}, \overline{P}]$. Lemmas 2 and 3 imply that minimum and maximum prices respond to the forces of demand and supply. The prediction is that the minimum price will rise if demand at $\underline{P}$ exceeds supply at $\overline{P}$ and that the maximum price will fall if supply at $\overline{P}$ exceeds demand at $\underline{P}$. In the excess demand case ($D^o(\underline{P}) \ge S^o(\overline{P})$) the maximum price may rise, but the prediction is that it will go no higher than the level necessary to allow the $D^o(\underline{P})^{th}$

unit to trade ($M^{D^\circ}(\underline{P})$). For the excess supply case the prediction is that although the minimum price may fall it will not go below $V^{S^\circ}(\overline{P})$.

<u>Prediction 1</u>: Prices

i) If $\underline{P}_d \geq P_*$ then $\underline{P}_{d+1} < \underline{P}_d$. If $\underline{P}_d < P_*$ then $\underline{P}_{d+1} < P_*$.

ii) If $\overline{P}_d \leq P^*$ then $\overline{P}_{d+1} > \overline{P}_d$. If $\overline{P}_d > P^*$ then $\overline{P}_{d+1} > P^*$.

iii) If $\Delta P_d > 1$ and $D^\circ(\underline{P}_d) \geq S^\circ(\overline{P}_d)$ then $\underline{P}_{d+1} > \underline{P}_d$ and $\overline{P}_{d+1} \leq u_d$.

iv) If $\Delta P_d > 1$ and $S^\circ(\overline{P}_d) \geq D^\circ(\underline{P}_d)$ then $\overline{P}_{d+1} < \overline{P}_d$ and $\underline{P}_{d+1} \geq \ell_d$.

Our prediction about the sequence of trading partners follows from the proofs of the Lemmas. It is essentially that sellers below $\overline{P}$ trade before those above $\overline{P}$ and buyers above $\underline{P}$ trade before those below $\underline{P}$.

<u>Prediction 2</u>: Trading Sequence

i) If $\Delta P_d > 1$:

All $M^j < \overline{P}_d$ trade before any $M^j \geq \overline{P}_d$.

All $V^i > \underline{P}_d$ trade before any $V^i \leq \underline{P}_d$.

ii) If $\Delta P_d \leq 1$:

All $M^j \leq \overline{P}_d$ trade before any $M^j > \overline{P}_d$.

All $V^i \geq \underline{P}_d$ trade before any $V^i < \underline{P}_d$.

Our prediction about the number of units traded is that it will be at least the competitive equilibrium for demand and supply curves truncated at $\overline{P}$ and $\underline{P}$, respectively, less one unit.

<u>Prediction 3</u>: Quantity Traded

The quantity traded in day d will be $Q_d \geq \mathrm{Max}\{K : \hat{V}^K_d \geq \hat{M}^K_d\} - 1$ where

$\hat{V}^K_d = \mathrm{Min}\{\underline{P}_d, V^K\}$ for $K = 1, \ldots, n$ and

$\hat{M}^K_d = \mathrm{Max}\{\underline{P}_d, M^K\}$ for $K = 1, \ldots, m$.

The following table summarizes violations of our predictions about prices, sequence of trades, and number of units traded as a percentage of total possible violations for nine DOA experiments. This table is based on data from Williams [14], and on unpublished data which were made available by Vernon Smith.

Experiment	Exp	Marg	Com	Units	Q^e	NYSE	Que	Price	Seq	Quant
IPDA8	No	1	5	8	6	Yes	No	13.9	0	11.1
IPDA9	No	1	5	8	6	Yes	No	9.4	1.6	0
IPDA10	Yes	1	5	10	8,6	Yes	No	6.3	0	0
IPDA11	No	1	5	10	8	Yes	No	18.8	1.3	0
IPDD14	Yes	1	5	8	6	Yes	No	9.4	0	0
IIPDA14	No	3	10	21	15	Yes	No	12.5	0	0
IIPDA22	No	0	10	16,11	11	No	No	3.1	NA	12.5
IIPDA25	Yes	2	10	12	7	No	Yes	21.9	0.8	12.5
IIPDA57	No	3	10	21	15	Yes	Yes	16.7	1.0	8.3
Average								12.7	0.7	5.3

Exp = experienced subjects (have participated in another PDA)

Marg = number of marginal units

Com = commission in cents

Units = number of units on each side of the market

Q^e = competitive equilibrium quantity

NYSE = New York Stock Exchange rules (new bids and offers must improve on outstanding bids and offers)

Que = electronic queuing of bids and offers (see Smith and Williams [13])

Price = % violation of price predictions

Seq = % violation of trading sequence predictions

Quant = % violation of quantity traded predictions

Table 1: VIOLATION PERCENTAGE[8]

To see the total number of price violations in perspective, the following table illustrates the margins of error. This table reports the total number of violations of our price predictions (over all nine experiments) which were more than x cents, as a percentage of the total number of possible violations of our price predictions.

Price violations of x¢ or less not counted:	Percentage of price violations over eight DOAs:
x = 1	6.3
x = 5	3.3
x = 10	1.3

Table 2: Price Violations

To put sequence and quantity violations in context it is useful to compare them with the violations of the sequence and quantity predictions of the Marshallian theory and the sequence predictions of the game theory approach. The Marshallian theory predicts that units will trade in the order of value and that all profitable trades will occur. The violations of this prediction as a percentage of possible violations in IPDA8 is 42.5%. The game theory approach (Wilson [15]) predicts that units will trade in the order of value but yields no further prediction on the number of trades. The violations of this prediction as a percentage of possible violations in IPDA8 is 29.4%.

7. FURTHER EXPERIMENTS

There is now a role for further interaction between theory and experiments. The class of experiments described in Section 6 motivated our theory, and it in turn suggests several experiments which could lead to refinements or rejection of the theory. There are several aspects of our theory which could be tested. First, we do not assume that traders' reservation prices and bids or offers converge to their true values at the end of each day. The data that we have seems to reject such an assumption. However, without this assumption we can establish convergence only to an interval determined by P_* and P^*. Our theory admits as an equilibrium a situation in which one extra-marginal unit is included or in which one infra-marginal unit is excluded. For example, Theorem 1 applies to the demand and supply configuration in Figure 1 of Appendix C to predict equilibrium in the interval [114, 148]. The placement of the first extra-marginal units in that figure has no effect on our equilibrium prediction. Charles Plott and Chris Worrell have run a DOA experiment using the configuration of Figure 1. Their data suggest that prices converge into the interval [133, 139] determined by the first extra-marginal units. This conclusion is consistent with our theory, but it does suggest that the theory might be refined to produce sharper convergence results.

Second, we have refrained from placing any direct assumption on the relative (between agent) rankings of true values and reservation prices. Possible ranking hypotheses on buyers reservation prices include (1) if $V^i > V^j$ then $r_d^i(t) > r_d^j(t)$, and (2) if $V^i > \overline{P}_d$ and $V_j \leq \underline{P}_d$ then $r_d^i(t) > r_d^j(t)$. Hypothesis 1 is clearly rejected by the data,

but whether hypothesis 2 is rejected depends on one's standard of acceptance. The absence of a ranking hypothesis is responsible for the relatively weak prediction of Theorem 2. In DOAs where $D^c(P^*) \neq S^c(P_*)$ our theory predicts convergence into the interval $[P^*, P_*]$, but it then admits the possibility of cycles between prices in this interval and prices as low as $V^{S^c(P_*)}$ if $S^c(P_*) > D^c(P^*)$ or prices as high as $M^{D^c(P^*)}$ if $D^c(P^*) > S^c(P_*)$. In the presence of either ranking hypothesis (and a symmetric hypothesis on sellers' reservation prices) cycles would not occur and prices would remain in $[P^*, P_*]$. We have some data about experiments where our theory admits the possibility of cycles. In both IPDA8 and IPDA9 (reported in Section 6), $S^c(P_*) > D^c(P^*)$. In neither of these experiments do we see cycles, prices seem to remain approximately in $[P^*, P_*]$. However, the extra-marginal seller unit at $P_* - 1$ is occasionally traded, so it is possible that cycles would have arisen had the experiments continued beyond ten days.[9] This suggests two possible further experiments. First, IPDA8 could be run for more days to decide whether cycles will appear. Second, an experiment with a design more likely to produce cycles could be run. The supply and demand configuration of Figure 2 in Appendix C is such a design. The prediction of Theorem 2 for this configuration is that prices will remain in the interval $[V^{S^c(P_*)}] = [70, 101]$. Our conjecture is that in the experiments any cycles would eventually disappear, with prices remaining in $[P^*, P_*]$ and perhaps following a time path during each day starting at P^*, and then rising during the day. By offering a small discount (to P^*) early in the trading day, infra-marginal sellers could insure that they complete a trade. Prices would then rise by one or two cents as marginal traders complete their trades. If this occurs it suggests that the theory might be further refined.

There are several other experiments which could lead to refinements or rejection of our theory. First, our theory is silent about the fine details of organizing a DOA. All that counts is that traders can make bids or offers and acceptances, and that they are informed of others' bids, offers and acceptances. Thus the predictions of the theory are unchanged by the use of New York Stock Exchange rules, electronic queues, or other details. However, the data are not unchanged by these details (see Smith and Williams [13]). It may be that sharper predictions would result if these details were taken into account. Second, the theory does not apply to experiments in which one side of the market is not allowed to bid or offer. See Plott and Smith [8] for some experiments. It would be easy, however, to modify the theory and to compare its predictions to the outcome of such experiments. Third, the theory does not yield predictions about the affect of shifts in supply and demand curves.[10] There is now data from experiments in which supply and demand curves are shifted systemmatically. The theory would need to be refined to yield useful predictions about the effect of such changes in market conditions.

The methodology of using experiments to test the predictions of theory can also be applied to the alternative theories that we have described. For instance, Wilson's game theoretic model of DOAs does not directly apply to the existing DOA experiments, but a DOA experiment could be designed to test the theory. Trader's values and

costs could be drawn independently across days from distributions which the traders know, risk attitudes could be controlled for as in Roth and Malouf [9] and the experiment could be repeated for a number of days to allow for learning about the game and about strategies. Wilson's predictions could then be compared to data from the final day of the experiment.

8. CONCLUSION

The theory presented here is deterministic and, although it does not completely describe precise paths of bids, offers and contracts, it does place fairly tight bounds on these data. One observation not in accord with these bounds is grounds for rejection of the theory, and in fact there are a number of such observations. However, the percentage of observations which violate the crucial implications of the theory is amazingly low.

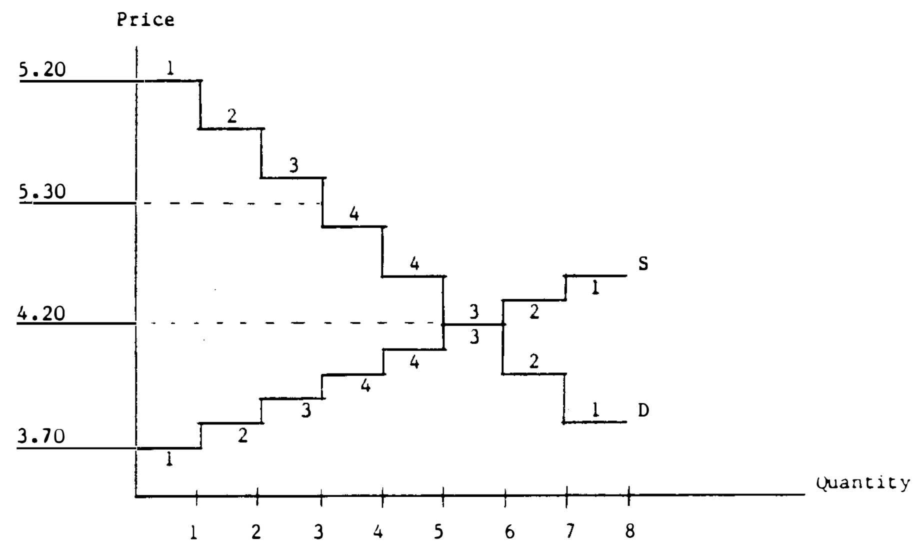
The potential importance of this theory is not just that it seems to describe what happens in DOA experiments, but also that it is the beginning of a positive theory of how market prices are formed and of how they adjust to changes in demand and supply conditions. The question of price formation has a long history of ad hoc and unsuccessful attempts at an answer. Our theory is also ad hoc in the sense that we make assumptions on individual behavior which are not derived from an optimizing model. However, our assumptions seem sensible and, more important, they seem to do a reasonable job of describing actual bids, offers, and contracts. There is now a target for experimentalists to reject with data or for theorists to improve on by obtaining a better fit with the data or a better explanation of observed behavior.

APPENDIX A

Induced Demand-Supply Schedule

DOA #IPDA14 (10/18/77)

Week 1



The number indicates the holder of the unit with that value or cost.

APPENDIX B

The following information is provided for a typical design:

1. Instructions from the standard DOA experiments run without the aid of a computer.
2. The values, V_i, and costs, M_j, from a Plato computer-assisted experiment IPDA14 run on 10/18/77 at the University of Arizona (a week is five days).
3. The record sheets of Buyer 4 and Seller 1 from IPDA14.
4. The data saved by the computer for Day 9 (MKR = maker of bid or offer, TM = time left in day in seconds, TKR = acceptor of that bid or affer, a "*" in the TKR column indicates that bid or offer occured before the acceptance).
5. A list of contracts in the order agreed to each day including price, buyer, and seller.

INSTRUCTIONS

General

This is an experiment in the economics of market decision making. Various research foundations have provided funds for this research. The instructions are simple and if you follow them carefully and make good decisions you might earn a considerable amount of money which will be paid to you in cash.

In this experiment we are going to simulate a market in which some of you will be buyers and some of you will be sellers in a sequence of market days or trading periods. Attached to the instructions you will find a sheet, labeled Buyer or Seller, which describes the value to you of any decisions you might make. You are not to reveal this information to anyone. It is your own private information.

Specific Instructions to Buyers

During each market period you are free to purchase from any seller or sellers as many units as you might want. For the first unit that you buy during a trading period you will receive the amount listed in row 1 marked 1st unit redemption value; if you buy a second unit you will receive the additional amount listed in row 5 marked 2nd unit redemption value. The profits from each purchase (which are yours to keep) are computed by taking the difference between the redemption value and purchase price of the unit bought. Under no conditions may you buy a unit for a price which exceeds the redemption value. In addition to this profit you will receive a five cent commission for each purchase. That is

[your earnings = (redemption value) - (purchase price) + .05]

Suppose, for example, that you buy two units and that your redemption value for the first unit is $200 and for the second unit is $180. If you pay $150 for your first unit and $160 for the second unit, your earnings are:

$ earnings from 1st = 200 - 150 + .05 = 50.05

$ earnings from 2nd = 180 - 160 + .05 = 20.05

total % earnings = 50.05 + 20.05 = 70.10

The blanks on the table will help you record your profits. The purchase price of the first unit you buy during the first period should be recorded on row 2 at the time of purchase. You should then record the profits on this purchase as directed on rows 3 and 4. At the end of the period record the total of profits and commissions on the last row, 9, of the page. Subsequent periods should be recorded similarly.

Specific Instructions to Sellers

During each market period you are free to sell to any buyer or buyers as many units as you might want. The first unit that you sell during a trading period you obtain at a cost of the amount listed on the attached sheet in the row, 2, marked cost of 1st unit; if you sell a second unit you incur the cost listed in the row, 6, marked cost of the 2nd unit. The profits from each sale (which are yours to keep) are computed by taking the difference between the price at which you sold the unit and the cost of the unit. Under no conditions may you sell a unit at a price below the cost of the unit. In addition to this profit you will receive a five cent commission for each sale. That is

[your earnings = (sale price of unit) - (cost of unit) + .05]

Your total profits and commissions for a trading period, which are yours to keep, are computed by adding up the profit and commissions on sales made during the trading period.

Suppose, for example, your cost of the 1st unit is $140 and your cost of the second unit is $160. If you sell the first unit at $200 and the second unit at $190, your earnings are

$ earnings from 1st = 200 - 140 + .05 = 60.05

$ earnings from 2nd = 190 - 160 + .05 = 30.05

total $ earnings = 60.05 + 30.05 = 90.10

The blanks on the table will help you record your profits. The sale price of the first unit you sell during the 1st period should be recorded on row 1 at the time of sale. You should then record the profits on this sale as directed on rows 3 and 4. At the end of the period record the total of profits and commissions on the last row, 9, of the page. Subsequent periods should be recorded similarly.

Market Organizations

The market for this commodity is organized as follows. We open the market for a trading period (a trading "day"). You will be warned when the end of the trading period is approaching. Any buyer (or seller) is free at any time during the period, to raise his hand and make a verbal bid (offer) to buy one unit of the commodity at a specified price. Any seller (or buyer) is free to accept or not accept the bid of any buyer (or seller). If a bid is accepted a binding contract has been closed for a single unit and the buyer and seller will record the contract price to be included in their earnings. Any ties in bids or

acceptances will be resolved by a random choice of buyer or seller. Except for the bids and their acceptance you are not to speak to any other subject. There are likely to be many bids that are not accepted, but you are free to keep trying, and as a buyer or seller you are free to make as much profit as you can.

Are there any questions?

	Week 1		Week 2	
	Unit 1	Unit 2	Unit 1	Unit 2
BYR 1	5.20	3.80	3.70	3.60
BYR 2	5.00	4.00	3.80	3.50
BYR 3	4.80	4.20	3.90	3.40
BYR 4	4.60	4.40	4.00	3.30
SLR 1	3.70	4.40	3.10	3.30
SLR 2	3.80	4.30	2.90	3.50
SLR 3	3.90	4.20	2.70	3.70
SLR 4	4.00	4.10	2.50	3.90

Touch the parameter to be changed.

BACK for last page; BACK1 to replot this page;
LAB for monitor; DATA to update specific vc
HELP1 for Week 2 D-shift;
HELP to shift back to original

Record Sheet for Buyer 4

	Trading Period	1	2	3	4	5	6	7	8	9	10
1	1st Unit Resale Value	4.60	4.60	4.60	4.60	4.60	4.00	4.00	4.00	4.00	4.00
2	Purchase Price	4.20	4.30	4.25	4.30	4.30	3.35		3.40	3.40	3.45
3	Profit (row 1 - row 2)	0.40	0.30	0.35	0.30	0.30	0.65		0.60	0.60	0.55
4	Profit + $.05 Commission	0.45	0.35	0.40	0.35	0.35	0.70	0.00	0.65	0.65	0.60
5	2nd Unit Resale Value	4.40	4.40	4.40	4.40	4.40	3.30	3.30	3.30	3.30	3.30
6	Purchase Price	4.40	4.30	4.30	4.30	4.25					
7	Profit (row 5 - row 6)	0.00	0.10	0.10	0.10	0.15					
8	Profit + $.05 Commission	0.05	0.15	0.15	0.15	0.20	0.00	0.00	0.00	0.00	0.00
9	Total Profit (row 4 + row 8)	0.50	0.50	0.55	0.50	0.55	0.70	0.00	0.65	0.65	0.60

Subject: ott

Profit: periods 1 - 5 = $ 2.60
periods 6 - 10 = $ 2.60
TOTAL PROFIT = $ 5.20

Experiment finished: 10/18/77

Record Sheet for Seller 1

Trading Period		1	2	3	4	5	6	7	8	9	10
1	Selling Price of 1st Unit	4.20	4.35	4.25	4.30	4.30	3.35	3.35	3.40	3.43	3.45
2	Cost of 1st Unit	3.70	3.70	3.70	3.70	3.70	3.10	3.10	3.10	3.10	3.10
3	Profit (row 1 - row 2)	0.50	0.65	0.55	0.60	0.60	0.25	0.25	0.30	0.33	0.35
4	Profit + $.05 Commission	0.55	0.70	0.60	0.65	0.65	0.30	0.30	0.35	0.38	0.40
5	Selling Price of 2nd Unit						3.32	3.38	3.40	3.45	3.50
6	Cost of 2nd Unit	4.40	4.40	4.40	4.40	4.40	3.30	3.30	3.30	3.30	3.30
7	Profit (row 5 - row 6)						0.02	0.08	0.10	0.15	0.20
8	Profit + $.05 Commission	0.00	0.00	0.00	0.00	0.00	0.07	0.13	0.15	0.20	0.25
9	Total Profit (row 4 + row 8)	0.55	0.70	0.60	0.65	0.65	0.37	0.43	0.50	0.58	0.65

Subject: **mecham**

Profit: periods 1 - 5 = $ 3.15
periods 6 - 10 = $ 2.53
TOTAL PROFIT = $ 5.68

Experiment finished: 10/18/77

PERIOD 9

	MKR	TM	BIDS	OFFERS	TKR	TM
1	B2	297	3.30			
2	B1	295	3.34			
3	B2	292	3.35			
4	S4	289		3.45	B3	285
5	B2	281	3.30			
6	B4	278	3.35			
7	B2	275	3.36			
8	S3	274		3.50		
9	B1	268	3.39			
10	B4	258	3.40		S2	252
11	S1	254		3.45	*	
12	B2	247	3.30			
13	B1	245	3.40			
14	S3	244		3.50		
15	S1	236		3.45		
16	B2	231	3.41			
17	S3	191		3.44		
18	S1	181		3.43	B2	154
19	B4	151	3.30			
20	B2	147	3.31			
21	S2	146		4.50		
22	B1	146	3.40			
23	B2	140	3.41			
24	S1	136		3.45	B1	68
25	B2	122	3.42		*	
26	B1	112	3.43		*	
27	B2	106	3.44		*	
28	S2	63		4.50		
29	B4	62	3.30			
30	B1	61	3.40			
31	S3	57		3.49		
32	B2	55	3.41			
33	B1	45	3.43			
34	B2	41	3.44			
35	B2	23	3.45		S3	17
36	B4	11	3.30			
37	S2	10		3.50	B1	0
38	B1	8	3.40		*	

SUMMARY DATA FROM IPDA14

	Contract Price	Buyer	Seller
Day 1	4.25	2	3
	4.20	4	1
	4.50	3	2
	4.40	4	4
	4.30	1	2
Day 2	4.35	3	1
	4.30	4	2
	4.30	4	2
	4.30	1	4
	4.25	2	4
Day 3	4.25	4	1
	4.35	3	3
	4.30	4	2
	4.27	2	4
	4.25	1	3
Day 4	4.30	4	1
	4.39	3	4
	4.30	4	4
	4.26	2	2
	4.25	1	3
	4.20	3	2
Day 5	4.30	4	1
	4.26	2	2
	4.35	3	4
	4.26	1	3
	4.25	4	4
Day 6	3.35	4	1
	3.30	3	2
	3.35	3	3
	3.32	2	1
	3.35	1	4
Day 7	3.31	2	2
	3.35	3	1
	3.40	3	4
	3.38	2	1
	3.45	1	3

Day 8	3.40	4	2
	3.40	1	1
	3.40	1	1
	3.40	3	4
	3.40	2	3
Day 9	3.45	3	4
	3.40	4	2
	3.43	2	1
	3.45	1	1
	3.45	2	3
	3.50	1	2
Day 10	3.41	2	2
	3.44	1	4
	3.45	4	1
	3.45	1	3
	3.50	3	1

APPENDIX C

Supply and Demand Schedules for Further Experiments

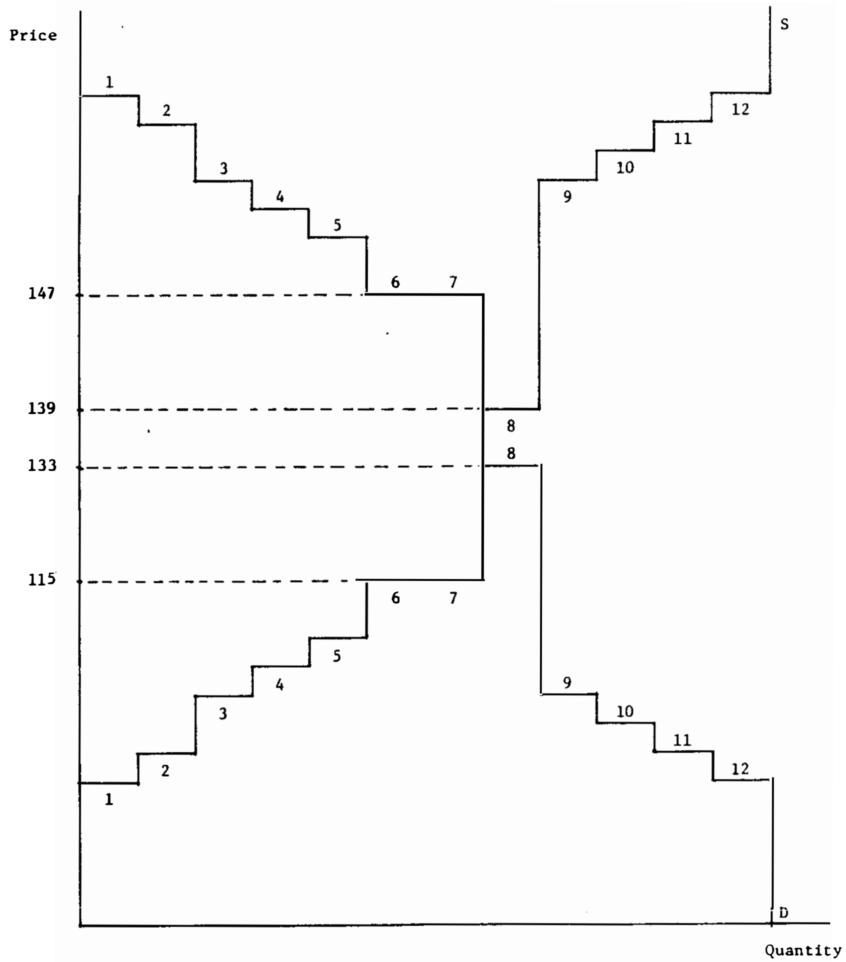


Figure 1

(From unpublished experiment by Charles Plott and Chris Worrell)

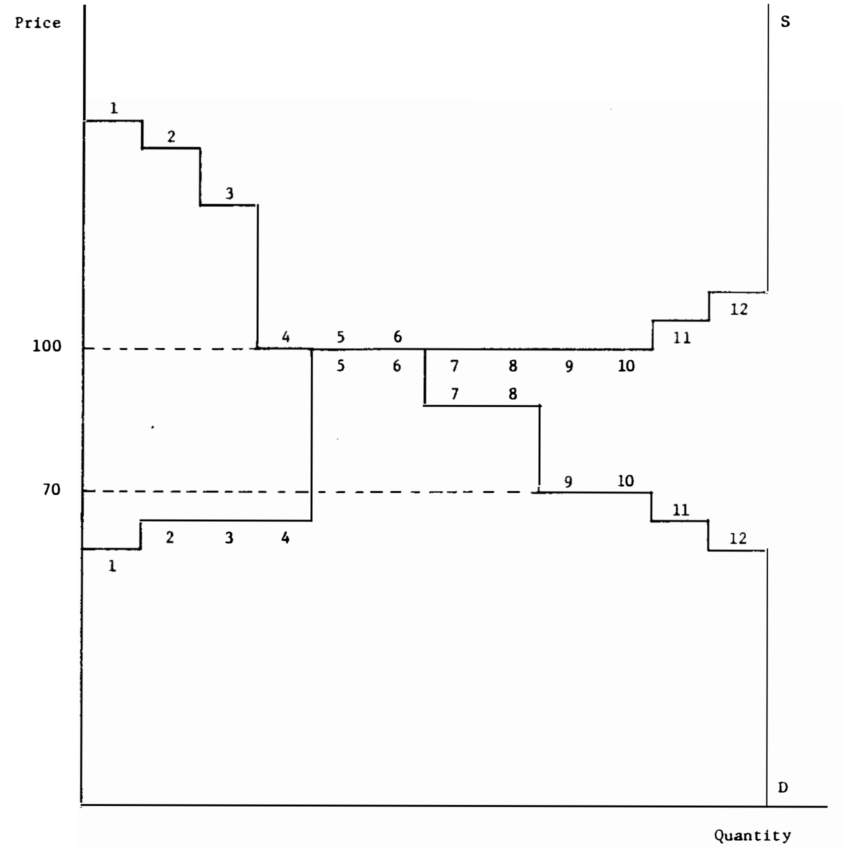


Figure 2

($P_* = 101$, $P^* = 99$, $V^{s^c}(P_*) = 70$)

FOOTNOTES

* This paper benefited from discussions in seminars at Cornell, Northwestern, Stonybrook, and an NSF Conference on Experimental Economics at the University of Arizona. This version is significantly different from earlier versions. We would like to thank Vernon Smith and Arlington Williams for making data on their Plata DOA experiments available to us.

1. These markets are described in detail in Appendix B. In fact some of the auctions are computerized rather than oral. All that matters is that participants can make bids or offers and acceptances, and are informed of others' bids or offers and acceptances.

2. Other designs are also used. See Smith [10] for some of these.

3. See Smith and Williams [13] for a description of the usual results.

4. Each participant also observes the timing of each contract, bid, and offer. It is highly probable that the timing of these events is an important piece of information which affects the actions of the buyers and sellers. However, the level of complexity required to incorporate timing into the model seems to outweigh the gains to be achieved. Thus, we ignore it throughout the paper.

5. A more sophisticated theory might distinguish between the amount a buyer is willing to bid and the lowest offer he would accept. In particular, buyers may not be willing to bid up to their reservation price (see Wilson [15]). This distinction could be easily incorporated into our model, but it is not clear how it would add to the explanatory power of the model.

6. Holt, Langan and Villamil [4] report a series of experiments in which traders had multiple units with payoffs structured to give some traders market power. Their data are nonetheless reasonably consistent with the predictions of our theory. So our implicit assumption that traders decide on strategies for each unit separately seems not to be at odds with the facts.

7. We make no direct assumptions about the relative rankings of true values and reservation prioes. In our model an individual can determine his reservation price using only his own value and past bids, offers and contracts.

8. Our model yields no predictions for day 1 of any experiment. In any experiment in which supply or demand was shifted we treat the first day after the shift as day 1 of a new experiment. In each non-initial day of an experiment we have four possible violations of price predictions: violations of 1(i), 1(ii) and the two predictions of either 1(iii) or 1(iv). So, in an experiment running for ten days with no shifts there are 36 possible price

violations. The entry for price violations is the number of violations divided by the number of possible violations. In each non-initial day the number of possible trading sequence violations is the number of buyer plus seller units (n + m). The number of actual violations is the number of units traded out of order. In each non-initial day there is one quantity prediction, and thus one possible quantity violation.

9. In IPDA8 the extra-marginal unit at P_{*} - 1 is seller 2's second unit. This unit is traded in both days 9 and 10 of the experiment.

10. For experiments in which shifts occur we have treated the day of the shift as day 1 of a new experiment.

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