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A THEORY OF AUDITING AND PLUNDER*

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ABSTRACT

Taxpayers know their income but the IRS does not. The IRS can audit taxpayers to discover their true income, but auditing is costly. We characterize optimal policies for the IRS when it is free to choose tax levies, audit probabilities and penalties. The main results are that optimal policies involve taxes which are monotonically increasing in reported incomes and audit probabilities are monotonically decreasing in reported income. In general optimal schemes involve stochastic auditing of reports and rebates for telling the truth. A theory of optimal plundering is described.

A THEORY OF AUDITING AND PLUNDER

Kim C. Border and Joel Sobel

The problem of inducing voluntary compliance with income-tax laws is a major public policy issue that has recently attracted the attention of the economics profession (see Reinganum and Wilde [1984a] and their references). This paper derives properties of audit policies when the taxing authority has the freedom to choose the tax schedule, the schedule of penalties, and the probability with which any return is audited. The results are then applied to discuss a rudimentary theory of plunder. We adopt the following model. The taxing authority, which we call the IRS, knows the probability distribution of incomes for the population from which taxpayers are drawn, but does not know the income of any particular taxpayer. It may audit taxpayers in order to discover their income; auditing is perfect, but costly. Taxpayers, treating the policy of the IRS parametrically, act to maximize their expected net income. The IRS chooses its policy in order to maximize some function of its gross revenue and its auditing activity.

It is not clear what the appropriate objective function for the IRS is. One possibility is to maximize expected revenue net of audit costs. Another reasonable objective might be to minimize audit costs subject to a net revenue constraint. A third possibility is to

maximize the sum of taxpayers' utility subject to a net revenue constraint. For the case of risk-neutral taxpayers, the sum of utilities is just total expected income minus gross revenue. For each of these objective functions, if auditing is costly, then an optimal auditing scheme will have the property that it is not possible to raise the same gross revenue with less auditing. If the objective function is increasing in gross revenue, then an optimal scheme will in addition have the property that revenue cannot be increased with the same amount of auditing. We call such schemes efficient auditing schemes and characterize them in our framework. Our major findings are: optimal auditing schemes need not exist unless taxes and penalties are bounded below; optimal auditing schemes typically involve stochastic auditing; and in an efficient direct scheme, in which the IRS induces truthful reporting, taxes are monotonically increasing and audit probabilities are monotonically decreasing in reported income, and rebates are given to taxpayers who are audited and found to be telling the truth, so that taxpayers prefer to be audited.

Several assumptions limit our analysis. We assume that the distribution of income is fixed and unaffected by the IRS's policies. Thus, we abstract from the distortions that an income tax creates on the labor-leisure decisions made by wage earners (see Mirrlees [1971]) or on the decision to allocate effort between taxable and nontaxable sources of income (see Kramer and Snyder [1983]). There are two reasons for this. One is to be able to attack the problem of

compliance per se, the other is that the solution of this problem is a necessary condition for solving the more general case in which policies affect the distribution of income.

We also assume that the IRS can commit to an audit policy and that taxpayers respond optimally to this policy. Alternatively, we could have assumed that the IRS's policy must also be a best response to the taxpayer's behavior. Reinganum and Wilde [1984] study this problem.

Another major assumption of our analysis is that taxpayers seek to maximize their expected net income. Thus, taxpayers are risk neutral. When taxpayers are risk neutral, compliance is a more serious problem than when taxpayers are risk averse; our results provide lower bounds on the revenue that the IRS could raise with risk-averse taxpayers.

We also assume that taxpayers cannot pay more than their income. This restriction limits the types of messages that taxpayers can send to the IRS since they must be able to pay the tax associated with any message. On the other hand, this assumption implies that the IRS has no greater threat than taking away the taxpayer's income. Without this assumption, the IRS could extract all of the income of the taxpayers at an arbitrarily small cost by auditing rarely and levying enormous fines for misreported income.

The last restrictive assumption that we discuss relates to the auditing technology. We assume that there is a fixed cost per audit and that an audit discovers true income without error. Baron and

Besanko [1984], Laffont and Tirole [1985], and others present models in which auditing is possible, but cannot be done perfectly.

Finally, we assume that there is a finite set of income levels and that the cost of auditing a taxpayer is the same for all taxpayers. Allowing a continuum of income levels would not change our results, provided the support of the distribution is bounded.

Our analysis generalizes the work of Reinganum and Wilde (1985) who restrict attention to lump sum taxes and audit cutoff rules, that is, rules with only two different audit probabilities. In their framework the optimal choices for the audit probabilities are 0 and 1. Call an auditing policy deterministic if all the audit probabilities are either 0 or 1. We provide an example to show that these sorts of policies are not optimal in the class of more general audit rules and tax schedules. Townsend [1979] makes the point that stochastic auditing dominates deterministic auditing in a somewhat different model. We also extend the analysis of Matthews [1984] who obtains partial results in a framework that is identical to ours.¹

MODEL

We assume that there is a finite set $\{x_1, \dots, x_n\}$ of income levels with $h_1 > 0$ taxpayers of income level x_1 . We label the incomes so that $0 \leq x_1 < \dots < x_n$. The IRS chooses a set M of messages (reports) that are available to a taxpayer. The IRS also chooses a tax function $t : M \rightarrow \mathbb{R}$ so that a taxpayer who reports message m sends a payment $t(m)$ to the IRS. Since we assume that taxpayers are incapable

of paying more than their income, we limit the choice of message spaces and tax functions by assuming that for each i , there is a message m_i satisfying $t(m_i) \leq x_i$. After receiving the report m , the IRS audits it with probability $p(m)$. If the IRS audits a taxpayer, that taxpayer's income is revealed and the IRS returns $t(m)$ and instead collects a payment of $f(x, m)$, where x is the taxpayer's income and m is his report. We assume that $f(x, m) \leq x$ for all x and m .

Given M , t , p , and f , let m_i^* maximize

$$[1 - p(m)][x_i - t(m)] + p(m)[x_i - f(x_i, m)]$$

over $\{m \in M : t(m) \leq x_i\}$. I.e., m_i^* is a report that maximizes the net expected income of a taxpayer of income level x_i . The IRS's objective function is assumed to depend only on gross revenue and auditing activity and can thus be written in the form

$$F\left(\sum_{i=1}^n \{[1 - p(m_i^*)]t(m_i^*) + p(m_i^*)f(x_i, m_i^*)\}h_i, p(m_1^*), \dots, p(m_n^*)\right).$$

We assume that F is decreasing in each $p(m_i^*)$.

Consider the message space $M^* = \{1, \dots, n\}$, the tax function t^* defined by $t_i^* = t(m_i^*)$, the audit function p^* defined by $p_i^* = p(m_i^*)$ and penalty function f^* defined by $f_{ij}^* = f(x_i, m_j^*)$. For this choice of M , t , p , and f setting $j = i$ maximizes

$$[1 - p_j^*][x_i - t_j^*] + p_j^*[x_i - f_{ij}^*]$$

over $\{j : t_j^* \leq x_i\}$.

Thus without loss of generality we may assume that the IRS uses $\{x_1, \dots, x_n\}$ as its message space and that any scheme (t, f, p) it chooses satisfies the incentive constraints

$$(1 - p_i)(x_i - t_i) + p_i(x_i - f_{ii}) \geq (1 - p_j)(x_i - t_j) + p_j(x_i - f_{ij})$$

for all i, j satisfying $t_j \leq x_i$.

This argument is a slight modification of the standard proof of the Revelation Principle, which asserts that the IRS can restrict attention to direct mechanisms in which taxpayers have incentives to tell the truth. The modification is necessary because, in contrast to the typical mechanism-design problem, the IRS's mechanism must satisfy the constraint that taxpayers cannot pay more than their income. The Revelation Principle need not apply to situations in which the set of reports available to an agent depends on the agent's type.²

The incentive constraints are weakest when $f_{ij} = x_i$ for $i \neq j$. Thus we set $f_{ij} = x_i$ for $i \neq j$ and write f_i for f_{ii} . Since we assume that we can only consider schemes with $t_i \leq x_i$ and $f_i \leq x_i$, we can write the IRS's problem as

$$\max_{t_i, p_i, f_i} F\left(\sum_{i=1}^n [(1 - p_i)t_i + p_i f_i]h_i, p_1, \dots, p_n\right)$$

subject to

$$\begin{aligned} 0 \leq p_i \leq 1 & \quad i = 1, \dots, n \\ t_i \leq x_i & \quad i = 1, \dots, n \end{aligned}$$

$$f_i \leq x_i \quad i = 1, \dots, n$$

$$(1 - p_i)(x_i - t_i) + p_i(x_i - f_i) \geq (1 - p_j)(x_i - t_j) \quad i, j = 1, \dots, n.$$

If the IRS maximizes net revenue, the objective function is

$$F = \sum_{i=1}^n z_i h_i - \sum_{i=1}^n c_i p_i h_i$$

where $z_i = (1 - p_i)t_i + p_i f_i$ and c_i is the cost of auditing a report of x_i . If the IRS minimizes auditing costs subject to a revenue requirement, the objective function can be written

$$F = \begin{cases} -\sum_{i=1}^n c_i p_i h_i & \text{if } \sum_{i=1}^n z_i h_i - \sum_{i=1}^n c_i p_i h_i \geq R \\ -\sum_{i=1}^n c_i p_i h_i - M & \text{if } \sum_{i=1}^n z_i h_i - \sum_{i=1}^n c_i p_i h_i < R \end{cases}$$

where R is the revenue requirement and M is chosen large enough so that the revenue requirement will be met if it is feasible to do so. Finally, if the IRS maximizes the sum of taxpayers' utilities, its objective function is

$$F = \begin{cases} \sum_{i=1}^n x_i h_i - \sum_{i=1}^n z_i h_i & \text{if } \sum_{i=1}^n z_i h_i - \sum_{i=1}^n c_i p_i h_i \geq R \\ \sum_{i=1}^n x_i h_i - \sum_{i=1}^n z_i h_i - M & \text{if } \sum_{i=1}^n z_i h_i - \sum_{i=1}^n c_i p_i h_i < R \end{cases}$$

Note that these latter objective functions are not increasing in gross revenue.

We denote the incentive constraints collectively by the symbol IC; IC(i, j) denotes the constraint that an individual with income x_i does as well to report x_i as to report x_j . The reason that the above problem requires IC(i, j) to hold for all i and j rather than for just those for which $x_i \geq t_j$ is notational. If $x_i < t_j$, then the constraint holds anyway.

Note that the constraint set is not compact as t_i and f_i are not bounded below. This may not seem crucial since the objective function is increasing in t_i and f_i , but it does matter. The intuition is this. Any solution to the above problem must force taxpayers to want to tell the truth. There are two ways to do this. One is to punish them for lying ($f_{ij} = x_i, i \neq j$), the other is to reward them for telling the truth ($f_i < 0$). It may be that a large reward for telling the truth can be offset by a tiny audit probability and thus economize on audit costs.

EXAMPLE

The example given in the table below demonstrates the above intuition. The example uses three income levels and presents a family of schemes indexed by $\epsilon > 0$.

$h_1 = 2$	$x_1 = 1$	$p_1 = 1 - \frac{(1 - \epsilon)}{2}$	$t_1 = 1$	$f_1 = 1$
$h_2 = 1$	$x_2 = 2$	$p_2 = \epsilon$	$t_2 = 2$	$f_2 = 2 - \frac{1 - \epsilon}{2\epsilon}$
$h_3 = 1$	$x_3 = 3$	$p_3 = 0$	$t_3 = 2 + \epsilon$	$f_3 = 3$

Simple calculations show that the incentive constraints are satisfied and that the expected net revenue is equal to $4 \frac{1}{4} - \varepsilon/2$. Note that $f_2 \rightarrow -\infty$ as $\varepsilon \rightarrow 0$. In fact, tedious computation shows that the supremum of expected net revenue is indeed $4 \frac{1}{4}$ and is not achieved by any scheme.

The observation that solutions do not exist relates to two familiar problems in the principal-agent literature. If perfect, ex post observations are feasible, then it is typically true that enforcement costs can be made arbitrarily small by forcing agents to pay large penalties with arbitrarily small probabilities if the agents fail to do what the principal prefers. This result is often associated with Becker [1968] and Stigler's [1970] work on the economics of crime prevention.³ In general, Mirrlees [1975] shows that even when observations are not perfect, it may be possible to approximate (but not attain) full-information optima with incentive schemes that require some agents to pay large penalties with small probabilities. We restrict attention to fines and taxes that do not exceed income. Consequently, approximating full-information optima with large fines is not feasible in our model. However, the benefits of paying arbitrarily large rewards with infinitesimal probability in order to weaken incentive constraints is not surprising in light of the earlier results.

Thus, in order to guarantee the existence of optimal schemes, an additional ad hoc constraint is necessary. Since there is always the danger that an arbitrarily large reward for telling the truth

might have to be paid, a natural constraint is to prohibit any bonuses for telling the truth. This leads to the following problem.

$$\max_{t_i, p_i, f_i} F\left(\sum_{i=1}^n [(1 - p_i)t_i + p_i f_i] h_i, p_1, \dots, p_n\right)$$

subject to

$$\begin{aligned} 0 &\leq p_i \leq 1 & i = 1, \dots, n \\ 0 &\leq t_i \leq x_i & i = 1, \dots, n \\ 0 &\leq f_i \leq x_i & i = 1, \dots, n \end{aligned}$$

$$(1 - p_i)(x_i - t_i) + p_i(x_i - f_i) \geq (1 - p_j)(x_i - t_j) \quad i, j = 1, \dots, n$$

Note that the incentive constraint IC(i,i) reduces to $f_i \leq x_i$ unless $p_i = 0$, in which case f_i is irrelevant. This constraint set is compact and since the objective function is continuous in all the variables an optimum does exist.

CHARACTERIZATION OF OPTIMAL AUDITING SCHEMES

This section investigates properties of the optimum. Set $q_i = 1 - p_i$, $z_i = q_i t_i + (1 - q_i) f_i$, and $u_i = x_i - z_i$. Then q_i is the probability that taxpayer i is not audited; z_i is the expected payment of a taxpayer of income i ; and u_i is the expected utility of taxpayer i . If $q_i = 1$, then $z_i = t_i$ and f_i is irrelevant. If $q_i < 1$, then

$$f_i = \frac{z_i - q_i t_i}{1 - q_i}.$$

When $q_i < 1$, $f_i \geq 0$ is equivalent to $z_i - q_i t_i \geq 0$. When $q_i = 1$, $z_i = t_i$. Thus an equivalent maximization problem is

$$\max_{q_i, z_i, t_i} \hat{F}\left(\sum_{i=1}^n z_i h_i, q_1, \dots, q_n\right) \quad (1)$$

subject to

$$0 \leq q_i \leq q \quad i = 1, \dots, n \quad (2.1)$$

$$0 \leq t_i \leq x_i \quad i = 1, \dots, n \quad (2.2)$$

$$q_i t_i \leq z_i \quad i = 1, \dots, n \quad (3)$$

$$u_i \geq q_j(x_i - t_j) \quad i, j = 1, \dots, n \quad (4)$$

When $q_i = 1$, the incentive constraint $IC(i, i)$ implies $t_i \geq z_i$ and the constraint $p_i t_i \leq z_i$ implies then that $t_i = z_i$, so all the constraints are included. This objective function is increasing in each q_i .

Call a scheme (z, t, q) feasible if it satisfies the constraints (2)-(4) and also satisfies the two conventions that

$$\text{if } q_i = 0, \text{ then } t_i = x_i \quad (5.1)$$

and

$$\text{if } q_i = 1, \text{ then } f_i = t_i. \quad (5.2)$$

Call a scheme (z, t, q) audit efficient if it is feasible and there is no other feasible scheme (z', t', q') satisfying $\sum_{i=1}^n z'_i h_i = \sum_{i=1}^n z_i h_i$ and $q'_i \geq q_i$ for all i with strict inequality for at least one i . A scheme (z, t, q) is called revenue efficient if it is feasible and there is no other feasible scheme (z', t', q') with $q'_i \geq q_i$ for all i and $z'_i \geq z_i$ for all i with at least one strict inequality. Any optimal scheme clearly satisfies the last part of the definition of audit efficiency (6) and is equivalent to a scheme satisfying (5.1) and (5.2). If the objective function is strictly increasing in gross revenue, then any optimal scheme will be both audit efficient and revenue efficient. Conditions (5.1) and (5.2) have been appended to avoid awkward circumlocutions in the statement of the theorem below.

Another useful definition is the following. We say that report j attracts i , denoted $i \rightarrow j$ if $i \neq j$ and $IC(i, j)$ binds:

$$u_i = q_j(x_i - t_j).$$

Let $A(i) = \{j : i \rightarrow j\}$ and $A^{-1}(j) = \{i : j \in A(i)\}$.

The following theorem lists several properties of efficient (and hence optimal) schemes. The proof may be found in the appendix.

THEOREM: If (z, t, q) is either audit efficient or revenue efficient, then

1. If $i > j$, then

a. $z_i \geq z_j$, with equality if and only if $q_i = q_j = 1$.

- b. $u_i \geq u_j$, with equality if and only if $u_i = 0$.
2. If $i > j$, then
- a. $q_i \geq q_j$, with equality if and only if $q_i = 0$ or $q_j = 1$.
- b. $t_i \geq t_j$, with equality if and only if $q_j = 1$.
3. For each i , $t_i \geq z_i \geq f_i$.
4. a. If $q_j < 1$, then $A^{-1}(j) \neq \emptyset$.
- b. If $0 < q_i < 1$, then $i \in A(i)$.
- c. If $i > j$, then $A(i) \supseteq A(j)$.
- d. $q_n = 1$
- e. $n - 1 \in A(n)$.
- f. If $\bar{i} = \max\{j : q_j < 1\}$, then $\bar{i} \in A(\bar{i} + 1)$.

If (z, t, q) is revenue efficient then

5. a. If $i > 1$, then $A(i) \neq \emptyset$.
- b. If $\underline{i} = \min\{j : q_j > 0\}$, then $u_{\underline{i}} = 0$ if and only if $i \leq \underline{i}$.

Efficient schemes have a variety of not very surprising properties. Part 1 states that higher income levels expect to pay more to the IRS and attain higher levels of utility than lower income levels. A simple consequence of Parts 1a and 2a is that if the cost of auditing all reports is the same, then first-order stochastically dominating shifts in income increase the revenue of the IRS. Part 2 states that the auditing probabilities and taxes are monotonic in income. That fact that taxes should increase with reported income in a revenue-maximizing solution is intuitively clear. That the probability of an audit falls as income rises is a necessary measure

to prevent high-income taxpayers from reporting low incomes.

Part 3 of the theorem states that honest taxpayers would rather be audited than not be audited. While this does not meet with our intuition about the real world, the reason for the result in our model is straightforward. When q_i is fixed, the IRS has two instruments to raise a given amount of revenue, z_i , from a taxpayer with income i : the tax, t_i and the penalty, f_i . Increasing t_i while reducing f_i in a way that holds z_i constant is beneficial to the IRS because it weakens the incentive constraints $IC(k, i)$. Therefore, the IRS can always improve on a tax scheme in which $f_i > t_i$ for some i by increasing t_i and reducing f_i . There are several reasons, not captured by our model, that would cause taxpayers to prefer not to be audited. First, auditing might impose a cost to the taxpayer in addition to any monetary payments required by the IRS. For example, the audit might involve gathering and/or manufacturing records and taking time to provide the IRS with information. In this case, taxpayers might prefer not to be audited even if the expected payment to the IRS given an audit is less than the expected payment to the IRS when there is no audit. Second, even in our model, taxpayers who do not report their income honestly prefer not to be audited. This fact is not important in our model since honesty is preferred by all taxpayers. However, situations in which it is in the best interest of some taxpayers to misrepresent their income arise and can be incorporated into our model. Specifically, if some taxpayers always tell the truth regardless of the incentive scheme, but the IRS cannot

directly observe honesty, then dishonesty, in spite of grave consequences if discovered, could be optimal for some taxpayers.⁴ Alternatively, audits could be imperfect and taxpayers could have different information regarding the probability that the IRS will discover a lie.

Parts 4 and 5 of the theorem list technical results regarding which constraints bind in efficient schemes. Part 5a states that every taxpayer except those with the lowest income is indifferent between reporting true income and some other income. If this were not true, then the IRS could obtain more revenue from the taxpayer. It need not hold for the lowest income class only because these taxpayers obtain no surplus (Part 5b). Part 4a states that there is no reason to audit reports that are attractive only to taxpayers who report their true income; the IRS audits to encourage compliance. Part 4b implies that the only incentive constraints that need be included in the optimization problem are the downward constraints, that is, $IC(i,j)$ for $i > j$.⁵ Part 4c states that the set of reports that attract an income class increases with income. Part 4d states that the IRS need not audit those taxpayers with the highest reported income.

Part 5 states that if the IRS wants to maximize revenue, ceteris paribus, the taxpayers with the lowest income always receive no surplus as does any other taxpayer who cannot receive positive surplus by underreporting income.

It may be worthwhile to note two directions in which the

theorem does not extend. First, if the IRS has a utilitarian objective function, then Part 5 of the theorem does not hold. The reason to extract all of the income of low-income taxpayers is to increase revenue. If the utility of these taxpayers entered positively in the objective function of the IRS, then there is no reason in general for them to be persecuted. Second, the monotonicity results need not hold if the IRS seeks to redistribute income. An extreme case convinces one that this must be so. Consider a situation in which auditing is costless and that the IRS cares only about revenue and the utility of intermediate income levels. In this situation, it is plain that the IRS will leave all but the intermediate income levels with no income, but not tax the intermediate groups at all.

While the theorem are for the most part intuitively appealing and quite consistent with results in related types of mechanism-design problems, we should remark that the results do not go as far as results in similar models and our proofs, while elementary, are delicate. For a concrete comparison, consider the auction-design problem (see Maskin-Riley [1984a,b] and Myerson [1981]) with risk-neutral buyers, which our model resembles. In the auction-design problem, the seller faces a fixed number of buyers. The seller knows the distribution from which the values of the buyers are drawn, but not their exact value. The seller's objective is to find a mechanism, which consists of a probability that a given bid will win the item as well as a purchase price, that maximizes his expected profit. The

standard approach in this type of problem is to solve a modified optimization problem in which local downward incentive constraints, those constraints of the form $IC(i, i - 1)$, bind, but all other incentive constraints do not bind. It is often possible to obtain a solution to the modified problem. This solution is a solution to the original problem if two conditions hold. First, the local downward incentive constraints must bind in the solution to the original problem. This condition guarantees that the modified problem has fewer constraints than the original problem. Second, the solution to the modified problem must satisfy the additional incentive constraints of the original problem. The second condition need not be true in general. However, in the auction-design problem it is possible to find conditions on the underlying variables of the model under which, if the local downward incentive constraints bind, then all constraints are satisfied. This approach does not help us analyze our problem because there is no guarantee that the local downward incentive constraints bind. It is straightforward, but tedious, to construct examples in which local incentive constraints do not bind at the optimum. While the fact that only the downward incentive constraints may bind at the optimum allows us to simplify our problem along the lines of Moore [1984], we are unable to explicitly characterize optima.

We represent some of the qualitative feature of efficient schemes in Figure 1. There is a (possibly empty) group of taxpayers who report low incomes and are always audited. Taxpayers who make

these reports pay all of their income in taxes. There is a nonempty group of high reports that are never audited. The IRS audits intermediate reports with a probability strictly between zero and one. While we do not have a simple, general condition that guarantees that the revenue-maximizing scheme requires selecting $0 < q_1 < 1$ for some i , these schemes are necessary in general.⁶ A routine verification, aided by the results in the theorem, shows that the net revenue maximizing scheme for the example is the element in the family that we described in which $\varepsilon = 1/5$. Thus, for the example, $q_1 = 2/5$ and $q_2 = 4/5$ in the optimal scheme.

As a final remark about revenue-maximizing schemes we note that it can be shown that the nonnegativity constraint on the penalties binds if and only if there is no optimum in which $q_1 = 0$ or 1 for all i .

LUMP SUM TAXES AND DETERMINISTIC AUDITING

In this section, we present results that relate lump-sum taxation to deterministic auditing. We do this because tax policies in this restrictive class may be of interest in their own right and also to further relate our work to Reinganum and Wilde [1985].

Formally, we call taxes lump sum if the tax function satisfies $t(x) = \min[x, T]$ for some $T > 0$. We present three propositions about lump-sum taxation. In this section we assume the IRS seeks to maximize expected net revenue.

Proposition 1: If the distribution of incomes has a continuous distribution with an interval support and if the optimal taxation scheme involves lump-sum taxation, then auditing is deterministic.

Proof: Let $t(x) = \min[x, T]$. By Part 2b of the theorem all taxpayers with incomes greater than or equal to T are never audited. Therefore, $u_T = 0$ and hence $q_x = 0$ for all $x \leq T$. □

Proposition 1 depends on the assumption that there are a continuum of income levels. The example that we presented earlier makes this point. The fact that the result depends on the nature of the distribution of income levels indicates that lump-sum tax policies are less likely to be optimal as the number of income levels increases.

Proposition 2: If the revenue-maximizing tax scheme is deterministic, then taxes are lump sum.

Proof: If the optimal tax scheme is deterministic, then

$$\underline{i} = \min \{j : q_j > 0\} = \min \{j : q_j = 1\}$$

and, by Part 5 of the Theorem, $u_{\underline{i}} = 0$ if and only if $i \leq \underline{i}$. It follows from Part 2 of the theorem that $t(x) = \min[x, x_{\underline{i}}]$. □

The final result of this section gives restrictions on penalties under which deterministic auditing is optimal.

Proposition 3: If the IRS is restricted to schemes in which all audited taxpayers must pay their entire income to the IRS, that is, if $f_i = x_i$ for all i , then there exists an optimal auditing scheme that is deterministic.

When the condition of Proposition 3 is met our problem reduces to a standard mechanism-design problem. In particular, local downward incentive constraints bind; we can use this fact, and familiar arguments (see, for example Myerson [1981] and Maskin and Riley [1984a]) to show that we can take the auditing scheme to be deterministic.

A THEORY OF PLUNDER

The model of income taxation has other interpretations. For instance, the "IRS" might be a conglomerate manager and the "taxpayers" might be his subsidiary managers. The manager does not know the profits of each subsidiary, but can send auditors to verify the managers' reports. Such a manager would be interested in maximizing the net revenue to the parent company.

In both of the IRS and conglomerate interpretations, there is arguably a legitimate claim on the part of the principal to the income of the agents. This need not be the case. Kurosawa [1954] describes the problem faced by a roving band of brigands which assails a peasant village and demands tribute. Upon receiving the tribute, the brigands may either move on or plunder the village. The latter course is more costly, particularly if the village harbors masterless samurai. The

brigands' problem is to choose two schedules; $p(t)$, the probability of plundering the village if they receive an amount of tribute t , and $b(x,t)$ the amount of booty they carry off if they plunder and find wealth x and were offered tribute t ; so as to maximize their expected revenue net of plundering costs, given that the peasants respond optimally. The peasants are risk-neutral so that if the brigands can make commitments, then this problem is formally very close to the model of tax compliance analyzed earlier. That is, the peasants choose the amount of tribute to offer, $t^*(x)$ so as to maximize $(1 - p(t))(x - t) + p(t)(x - b(x,t))$. The difference is that the IRS could choose a scheme in which two different income reports paid the same tax, but were audited with different probabilities. The brigands' schedule is a function only of the offered tribute. However, Part 2b of the theorem states that the IRS would never want to do this. Letting \hat{t} , \hat{p} , \hat{f} denote the IRS's expected net revenue maximizing scheme, we can write the brigands' optimal scheme as

$$p(t) = \hat{p}(\hat{t}^{-1}(t)) \quad (6.1)$$

and

$$b(x,t) = \begin{cases} \hat{f}(x,x) & \text{if } t = \hat{t}(x) \\ x & \text{otherwise.} \end{cases} \quad (6.2)$$

The fact that \hat{t}^{-1} is not well defined is not essential, because if $\hat{t}(x) = \hat{t}(y)$ and $x \neq y$ then $\hat{p}(x) = \hat{p}(y) = 0$ and $\hat{f}(x,x)$ is irrelevant. The peasants' optimal response to these schedules is to offer $\hat{t}(x)$ when the village wealth is x . Part 3 of our theorem says that

$b(x, \hat{t}(x)) \leq \hat{t}(x)$ for optimally chosen policies. That is, after plundering, the brigands never take more than they were offered! This underscores the need for the brigands to be able to make commitments. If they are unable to make commitments, the only believable schedule is $b(x,t) \equiv x$, which puts them in the suboptimal world of Proposition 3. That is, there would be some level of tribute which would buy them off and any lesser amount would cause them to plunder. Thus it is in the brigands' interest to design an institution that makes the schedules believable. One way the schedules might be believable is if it is common knowledge that there is a strong and vindictive god that the brigands could swear by.⁷ Another possibility for the brigands is to somehow create an institution like the state, which would have codified laws which presumably have more force than just the word of a roving band of brigands.

APPENDIX: PROOFS

The proof of the theorem is divided into several lemmas. The following table indicates which lemmas are used directly in the proof of any part of the theorem. There are some statements in the theorem which do not follow immediately from any lemma, but the details are easily filled in.

Part of Theorem	Lemmas that Apply
1	7
2	12, 13, 14
3	3
4a	1
4b	15
4c	19
4d	18a
4e	18c, 14
4f	18c
5	20, 6, 7b

Throughout this section the statements of all results are understood to be preceded by the phrase, "If (z, t, q) is either audit efficient or revenue efficient, then. . . ." Lemmas that apply only to revenue efficient schemes will carry a separate label. Both kinds of efficiency imply that it is impossible to increase some q_j , holding everything else fixed.

Lemma 1: If $q_j < 1$, then there exists $i \in A^{-1}(j)$ such that $x_i \geq t_j$.

Proof: If the lemma fails to hold, then there exists j with $q_j < 1$ such that for all $i \neq j$,

$$u_i > q_j(x_i - t_j) \quad \text{or} \quad t_j > x_i. \quad (1)$$

Moreover, if the first inequality in (1) fails to hold, then the second inequality implies that $q_j = u_j = 0$. Consequently, for all $i \neq j$,

$$u_i > q_j(x_i - t_j) \quad \text{or} \quad q_j = 0 \text{ and } t_j > x_i. \quad (2)$$

It follows from (2) that we can increase q_j and reduce t_j in such a way that $q_j t_j$ does not change and $IC(i, j)$ continues to hold for $i \neq j$. Since the constraint $IC(j, j)$ reduces to $x_j \geq f_j$ when $q_j < 1$, increasing q_j in this way is feasible. This result contradicts efficiency and therefore establishes the lemma. \square

Lemma 2: For each j , at least one of the constraints $x_j \geq t_j$, $z_j \geq q_j t_j$ holds with equality.

Proof: If $q_j = 1$, then $z_j = t_j$. If $q_j = 0$, then $t_j = x_j$ by convention. If $0 < q_j < 1$, then Lemma 1 implies that there exists some $i \in A^{-1}(j)$. Unless $x_j = t_j$ or $z_j = q_j t_j$, we could increase t_j . However, increasing t_j weakens $IC(i, j)$. Consequently we could make $A^{-1}(j)$ empty, contradicting Lemma 1, so either $x_j = t_j$ or $z_j = q_j t_j$. \square

Lemma 3: For each j , $t_j \geq z_j \geq f_j$.

Proof: If $t_j = x_j$, then the lemma follows since $x_j \geq z_j$ and z_j is a convex combination of t_j and f_j . Otherwise, by Lemma 2, $z_j = q_j t_j$ and the lemma follows since $t_j \geq 0$ and z_j is a convex combination of t_j and f_j . \square

Lemma 4: If $q_i \geq q_j$ and $q_j > 0$, then $t_i \geq t_j$.

Proof: Suppose that the result fails and $t_i < t_j$. Since $q_j > 0$, then for any $x \geq t_i$,

$$q_i(x - t_i) > q_j(x - t_j). \quad (1)$$

If $q_j = 1$, then (1) and $IC(j,i)$ imply $t_j > z_j$, which contradicts the definition of z_j . If $q_j < 1$, then by Lemma 1 there exists $x_k \geq t_j > t_i$ such that $IC(k,j)$ binds. But then (1) implies that $IC(k,i)$ is violated. This contradiction establishes the lemma.

Lemma 5: If $q_j = 1$, $t_j > 0$, $i \rightarrow j$ and $z_i = q_i t_i$, then $q_i = 1$ and $t_i = t_j$.

Proof: Since $i \rightarrow j$ and $q_j = 1$,

$$x_i - z_i = x_i = t_j \quad (1)$$

so

$$t_j = z_i = q_i t_i. \quad (2)$$

Since $t_j > 0$, it follows from (2) that $q_i > 0$. Then (2) and Lemma 4 imply $q_i = 1$ and $t_j = t_i$. \square

Lemma 6: For every i , there is some j such that $IC(i,j)$ binds.

Proof: If the scheme is revenue efficient the result is immediate. Assume that the scheme is audit efficient. If $q_i = 1$, then $IC(i,i)$ binds. The argument for the case $q_i < 1$ is by contradiction. Suppose that $IC(i,j)$ binds for no j . Then z_i can be increased without violating any incentive constraints. The revenue increase generated by increasing z_i can be offset by lowering some other z_k 's in such a way that q_i can be increased, which contradicts efficiency. If $z_k > q_k t_k$ for some $k \neq i$, then that z_k can be decreased without violating any constraints and offset the revenue increase from increasing z_i . Thus without loss of generality assume that $z_k = q_k t_k$ for all $k \neq i$. If $k \in A^{-1}(i)$ implies that $x_k = 0$, then q_i can be increased without violating $IC(k,i)$. Thus without loss of generality we may assume that for some $k \in A^{-1}(i)$, $x_k > 0$.

First consider the case $q_i = 0$ and let $k \in A^{-1}(i)$. Then

$$x_k - z_k = q_i(x_k - t_i) = 0. \quad (1)$$

Since $z_k = q_k t_k$, (1) implies that $q_k = 1$ and $t_k = x_k$. In order to decrease z_k we must also decrease t_k . The obstacles to lowering t_k are the constraints $IC(j,k)$. Let $j \rightarrow k$. Then

$$x_j - z_j = x_j - t_k. \quad (2)$$

Since $z_j = q_j t_j$, Lemma 5 implies that $q_j = 1$, $t_j = t_j$, and furthermore if $IC(j',j)$ binds then $IC(j',k)$ binds.

Thus if we increase q_i to $\eta > 0$, reduce z_k and t_k by $\eta(x_k - t_i)$ for all $k \in A^{-1}(i)$ for which $x_k \neq 0$, and then reduce $z_j (= t_j)$ by $\eta(x_j - t_i)$ for all $j \in A^{-1}(k)$ no constraints will be violated. By proper choice of η any small increase in z_i can be offset so as to hold gross revenue constant and q_i may be increased, contradicting efficiency.

Next consider the case $0 < q_i < 1$. First increase z_i and then increase q_i and t_i so that each $IC(k,i)$ continues to hold.

(Increasing q_i by ε and t_i by $\frac{\varepsilon(x_k - t_i)}{\varepsilon + q_i}$ for $k = \max_{A^{-1}(i)}$ works.)

Now let $z^* = \max_{j \neq i} \{z_j\}$ and let z_m satisfy $z_m = z^*$. We will show how to

decrease z_m without violating any constraints. To do this we must decrease t_m and again the obstacles are the $IC(j,m)$ constraints.

Suppose

$$x_j - z_j = q_m(x_j - t_m). \quad (3)$$

By hypothesis, $j \neq i$, so by the choice of m , $z_m \geq z_j$. Then using $z_m = q_m t_m$, (3) can be rewritten as

$$(1 - q_m)x_j = z_j - z_m \leq 0. \quad (4)$$

There are two ways that (4) might hold: either $x_j = z_j = z_m = 0$ or $q_m = 1$ and $z_j = z_m$.

If $z_m = 0$, then $z_k = 0$ for all $k \neq i$. Since $0 < q_i < 1$, for any $k \neq i$ satisfying $x_k = q_i(x_k - t_i)$, it follows that $x_k = t_i = 0$, which contradicts the existence of $k \in A^{-1}(i)$ with $x_k > 0$.

If $z_m > 0$, then $q_m = 1$ and $z_j = z_m$. Since $q_m = 1$ implies that $z_m = t_m$, Lemma 5 implies that $q_j = 1$ and $z_j = t_j = t_m$. Also, if $IC(j',j)$ binds, then $IC(j',m)$ binds, so so we may reduce z_m and z_j for $j \in A^{-1}(m)$ without violating any constraints. This leads to a contradiction of efficiency and establishes the lemma. \square

Lemma 7: If $i > j$, then

- a. $z_i \geq z_j$.
- b. $u_i \geq u_j$ and if $u_i > 0$, then $u_i > u_j$.

Proof: It follows from IC that

$$(1 - q_k)x_i + q_k t_k \geq z_i \quad (1)$$

for all i and k . Thus, Lemma 6 implies that

$$z_i = \min_k \{(1 - q_k)x_i + q_k t_k\} \quad (2)$$

for all i . Therefore, since $x_i > x_j$,

$$z_i = \min_k \{(1 - q_k)x_i + q_k t_k\} \geq \min_k \{(1 - q_k)x_j + q_k t_k\} = z_j. \quad (3)$$

Similarly,

$$u_i = \max_k q_k(x_i - t_k) \geq \max_k q_k(x_j - t_k) = u_j \geq 0, \quad (4)$$

where the last inequality follows since $x_i \geq t_i$. Plainly, the first inequality is strict if and only if $u_i > 0$. \square

Lemma 8: If for some $x_i > 0$, $t_i = 0$; then for all j , $t_j = 0$ and $q_j = 1$.

Proof: If $t_i = 0$ it follows from Lemma 2 that $z_i = q_i t_i = 0$. Lemma 6 implies that for some k , IC(i,k) binds, i.e.,

$$x_i = q_k(x_i - t_k). \quad (1)$$

Since $x_i > 0$, (1) implies that $q_k = 1$ and $t_k = 0$. Then for any j , IC(j,k) becomes

$$x_j - z_j \geq x_j, \quad (2)$$

so that for all j , $z_j = 0$ and hence $t_j = 0$. Then for any j and m , IC(m,j) reduces to $x_m \geq q_j x_m$, so efficiency implies that $q_j = 1$ for all j . □

Lemma 9: If $i > j$ and $t_i \leq t_j$, then $q_i \geq q_j$.

Proof: Since $x_j \geq t_j$, we have

$$x_i > x_j \geq t_j \geq t_i. \quad (1)$$

Therefore,

$$q_i t_i = z_i \geq z_j \geq q_j t_j \quad (2)$$

where the equality follows from Lemma 2 and (1), the first inequality follows from Lemma 7a and the second inequality follows from

feasibility. If $t_i > 0$, then (1) and (2) imply that $q_i \geq q_j$. If $t_i = 0$, it follows from (1) and Lemma 8 that $q_i = q_j = 1$. □

Lemma 10: If $t_i = t_j$, then $q_i = q_j$.

Proof: Without loss of generality we may assume that $q_i \geq q_j$. Then if $q_j = 1$, $q_i = q_j = 1$. If on the other hand $q_j < 1$, then by Lemma 1 there is some $k \in A^{-1}(j)$ satisfying $x_k \geq t_j$. Combining then IC(k,j) and IC(k,i) yields

$$u_k = q_i(x_k - t_i) = q_j(x_k - t_j), \quad (1)$$

which implies either $q_i = q_j$ or

$$x_k = t_i = t_j. \quad (2)$$

If $q_i \neq q_j$, then (2) implies $u_k = 0$, which in turn implies

$$x_k = t_k = z_k. \quad (3)$$

Thus

$$t_i = t_j = t_k. \quad (4)$$

However, since $k \neq j$, $x_j \geq t_j$ and (2) imply

$$x_j > x_k = t_j. \quad (5)$$

It follows that

$$q_j t_j = z_j \geq z_k = t_k = t_j \quad (6)$$

where the first equality follows from (5) and Lemma 2, the inequality follows from (5) and Lemma 7a, and the next two equalities follow from (3) and (4). From (6) it follows that $q_j = 1$, contrary to assumption, or that $t_j = 0$. Then (5) and Lemma 8 imply $q_j = q_i = 1$. In any event, $q_i = q_j$. \square

Lemma 11: If $i > j$ and $q_i = 0$, then $q_j = 0$.

Proof: If $q_i = 0$, then by convention, $x_i = t_i$. Lemma 1 implies the existence of an $x_k \geq x_i$ with $k \in A^{-1}(i)$. Therefore,

$$0 = q_i(x_k - x_i) = u_k \geq q_j(x_k - t_j), \quad (1)$$

where the inequality follows from IC(k,j). However,

$$x_k > x_i > x_j \geq t_j \quad (2)$$

since $k \neq i$ and $i > j$. Combining (1) and (2) yields $q_j = 0$. \square

Lemma 12: If $i > j$, then $t_i \geq t_j$ and $q_i \geq q_j$.

Proof: We first show that $q_i \geq q_j$. If $t_i < t_j$, then Lemma 9 implies $q_i \geq q_j$. If $t_i = t_j$, then Lemma 10 gives the result. If $t_i > t_j$, suppose that $q_i \geq q_j$ is false, i.e., that $q_i < q_j$. Then the contrapositive of Lemma 4 (exchanging the roles of i and j) implies $q_i = 0$, so $q_j = 0$ by Lemma 11. Thus $q_i \geq q_j$.

If $q_j > 0$, then Lemma 4 implies $t_i \geq t_j$. If $q_j = 0$, then Lemma 1 implies that there is some $k \in A^{-1}(j)$ with $x_k \geq t_j$ and so

$$0 = u_k \geq q_i(x_k - t_i) \quad (1)$$

where the inequality follows from IC(k,i). It follows from (1) that either $x_k = t_i$ or $q_i = 0$. If $t_i = x_k$, then $x_k \geq t_j$ implies $t_i \geq t_j$. If $q_i = 0$, then $t_i = x_i$ by convention, so $i > j$ implies $t_i > t_j$. \square

Lemma 13: If $0 < q_i = q_j < 1$, then $i = j$.

Proof: It follows from Lemma 4 that $t_i = t_j$. We now assume $i > j$ and argue to a contradiction. We have $x_i > x_j \geq t_j = t_i$; therefore

$$z_j \geq q_j t_j = q_i t_i = z_i \quad (1)$$

where the last equality follows from Lemma 2. Since $i > j$, Lemma 7a and (1) imply that

$$z_i = z_j. \quad (2)$$

Lemma 6 implies that for some k ,

$$\begin{aligned} x_i - z_i &= q_k(x_i - t_k) = q_k(x_i - x_j + x_j - t_k) \\ &\leq q_k(x_i - x_j) + x_j - z_j \end{aligned} \quad (3)$$

where the inequality follows from IC(j,k). Since $x_i - x_j > 0$, (2) implies that (3) holds only if $q_k = 1$ and therefore

$$t_k = z_i = z_j \quad (4)$$

and thus

$$q_i t_i = z_i = t_k \geq t_i \quad (5)$$

where, in (5), the first equality follows from (1), the second equality

from (4) and the inequality from Lemma 12 since $q_k = 1 > q_i$. Thus (5) implies $t_i = 0$, so by Lemma 8, $q_i = q_j = 1$, a contradiction. \square

Lemma 14: If $i \neq j$, then $t_i = t_j$ if and only if $q_i = q_j = 1$.

Proof: From Lemma 10, $t_i = t_j$ implies $q_i = q_j$. Thus Lemma 13 implies that if $i \neq j$ and $t_i = t_j$, then either $q_i = q_j = 0$ or $q_i = q_j = 1$. However, if $i \neq j$ and $q_i = q_j = 0$, then $t_i = x_i \neq x_j = t_j$. This proves that if $i \neq j$ and $t_i = t_j$, then $q_i = q_j = 1$.

Conversely if $q_i = q_j = 1$, then $z_i = t_i$ and $z_j = t_j$. Thus IC(i,j) and IC(j,i) combine to imply that $t_i = t_j$. \square

Lemma 15: If $0 < q_j < 1$ and $i \rightarrow j$, then $i > j$.

Proof: Assume that $i < j$ and argue to a contradiction. Then

$$x_i - z_i = q_j(x_i - t_j). \quad (1)$$

Since $x_i - z_i \geq 0$ and $q_j > 0$, $x_i \geq t_j$. It follows that

$$x_j > x_i \geq t_j > t_i \quad (2)$$

where the last inequality follows from Lemma 14. Therefore

$$z_j = q_j t_j > q_i t_i = z_i \quad (3)$$

where the equalities both follow from (2) and Lemma 2 and the inequality follows from (2) and the fact that $q_j \geq q_i$ which is a consequence of Lemma 12. Rearranging (1) and using $z_j = q_j t_j$ yields

$$0 \leq (1 - q_j)x_i = z_i - z_j, \quad (4)$$

contradicting (3). \square

Lemmas 1 and 15 combined have the following immediate consequence.

Lemma 16: If $q_j < 1$, then for some $i > j$, $i \rightarrow j$.

Lemma 17: If $\bar{i} = \max\{j : q_j < 1\}$, then

- a. $\bar{i} < n$.
- b. For all $i \neq j$, if $i, j > \bar{i}$, then $i \rightarrow j$.
- c. $\bar{i} + 1 \rightarrow \bar{i}$.

Proof: Part a follows directly from Lemma 16. To prove the remainder of the lemma, recall that Lemma 14 implies that for all $i, j > \bar{i}$

$$t_i = t_j \quad (1)$$

Part b follows directly from $q_i = q_j = 1$ and (1). Finally, Lemma 16 guarantees that there exists an $i > \bar{i}$ such that $i \rightarrow \bar{i}$. Therefore, by IC, we have

$$\begin{aligned} x_i - z_i &\geq \frac{q_i}{i}(x_i - t_i) && \text{or} \\ x_i &\geq (z_i - q_i t_i) / (1 - \frac{q_i}{i}), \end{aligned} \quad (2)$$

with equality for some $i > \bar{i}$. However, the right hand side is independent of i for $i > \bar{i}$ because of (1) and the definition of \bar{i} . Therefore, (2) holds as a strict inequality if $i > \bar{i} + 1$. \square

Lemma 18: If $\bar{i} \geq j$ and $i > j$, where \bar{i} is as defined in Lemma 17, then $z_i > z_j$.

Proof: Lemma 6 implies

$$z_j = \min_k \{(1 - q_k)x_j + q_k t_k\}. \quad (1)$$

Lemma 14 implies that if $k > \bar{i}$, then $t_i = t_n$. Lemma 17c implies that

$$t_n = (1 - q_{\bar{i}})x_{\bar{i}} + q_{\bar{i}} t_{\bar{i}}. \quad (2)$$

Thus for $j \leq \bar{i}$,

$$z_j = \min_{k \leq \bar{i}} \{(1 - q_k)x_j + q_k t_k\}. \quad (3)$$

Then if $i > j$ and $j \leq \bar{i}$, since $x_i > x_j$ and $q_k < 1$ for $k \leq \bar{i}$,

$$z_i = \min_k \{(1 - q_k)x_i + q_k t_k\} > \min_{k \leq \bar{i}} \{(1 - q_k)x_j + q_k t_k\} = z_j \quad (4)$$

□

Lemma 19: If $i \rightarrow k$, and $j \rightarrow m$, $0 < q_m < 1$ and $i > j$, then $k \geq m$.

Proof: Together $i \rightarrow k$ and $IC(i,m)$ imply that

$$q_k(x_i - t_k) = u_i \geq q_m(x_i - t_m). \quad (1)$$

Similarly, $j \rightarrow m$ and $IC(j,k)$ imply that

$$q_m(x_j - t_m) = u_j \geq q_k(x_j - t_k). \quad (2)$$

Combining (1) and (2) and rearranging terms yields

$$x_i(q_k - q_m) \geq q_k t_k - q_m t_m \geq x_j(q_k - q_m). \quad (3)$$

Consequently, if $i > j$, then $q_k \geq q_m$. The lemma now follows from Lemmas 12 and 13. □

Lemma 20: Suppose that (z,t,q) is revenue efficient. Let

$\underline{i} = \min \{j : q_j > 0\}$. If $i \leq \underline{i}$, then $u_i = 0$.

Proof: If $j < \underline{i}$, then $q_j = 0$, so we can replace $IC(\underline{i},j)$ by $u_{\underline{i}} \geq 0$. If $j > \underline{i}$, then by Lemma 15, $IC(\underline{i},j)$ can bind only if $q_j = q_{\underline{i}} = 1$. In that case, Lemma 14 implies that $t_{\underline{i}} = t_j$ and $IC(\underline{i},j)$ holds automatically. Finally, $IC(\underline{i},\underline{i})$ reduces to $f_{\underline{i}} \leq x_{\underline{i}}$, which is independent of $u_{\underline{i}}$. Thus revenue efficiency is obtained by setting $u_{\underline{i}} = x_{\underline{i}} - z_{\underline{i}} = 0$. The lemma now follows from Lemma 7b. □

FOOTNOTES

1. Matthews [1984] has some results for the case in which taxpayers are risk averse and the IRS seeks to maximize a weighted sum of taxpayer's utilities subject to a revenue constraint.
2. This point was made by Green and Laffont [1983] who restricted attention to revelation mechanisms. It should be noted that our restriction on the message space and tax function satisfies their nested range condition.
3. Polinsky and Shavell [1979] show that these results depend on the assumption of risk neutrality.
4. In the context of a model in which the IRS chooses only the audit policy and is not able to make commitments, Graetz, Reinganum and Wilde [1983] show that if some taxpayers are honest, then there is a mixed strategy equilibrium which involves high income taxpayers misreporting their income with some positive probability.
5. Part 4c implies that we can delete, without loss of generality, constraints of the form $IC(j,i)$ for $i > j$ from the optimization problem provided that $0 < q_j < 1$. If $q_j = 0$, then $t_j = x_j$. Thus, for $i > j$, $t_i > t_j = x_j$ and $IC(j,i)$ is automatically satisfied. If $q_j = 1$, then an efficient scheme involves $q_i = q_j$, $t_i = t_j$ for all $i > j$ whether or not we include $IC(j,i)$ in the set of constraints.

6. If the spacing between income levels is uniform, say $x_{i+1} - x_i = 1$, and the distribution of incomes is decreasing and exponential, so that $h_{i+1} = ah_i$ for $0 < i < n$, then there always exists a nontrivial interval of costs for which stochastic auditing dominates deterministic policies.
7. This idea was suggested once by Ed Green.

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