

**DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES
CALIFORNIA INSTITUTE OF TECHNOLOGY**

PASADENA, CALIFORNIA 91125

ECONOMIC ANALYSIS OF BRAIN DRAIN

Da-Hsiang Donald Lien



SOCIAL SCIENCE WORKING PAPER 556

February 1985

ABSTRACT

In this paper, we consider the possibility of signaling and individual's two-stage decision procedures within an asymmetric information framework to investigate the brain drain phenomena. The results indicate the relationship between an individual's ability and the signal he will choose at rational expectations equilibrium. Also, the persons who will remain abroad are identified. Where the ranking of the universities provides the signal to domestic employers, we can therefore interpret these results in such a way that there is an association between students of a particular quality and corresponding qualities of universities they will choose to attend to attain Ph.D.'s. Moreover, we can predict whether these graduating Ph.D.'s choose to return home or remain abroad.

ECONOMIC ANALYSIS OF BRAIN DRAIN

Da-Hsiang Donald Lien

1. INTRODUCTION

Although conventional economic theory would treat international migrations as analogous to international commodity exchange, and hence mutually beneficial, yet in reality there exist many reasons to question this proposition. Particularly, persons in the source countries complain that too many of the brighter people remain abroad, which yields the so-called brain drain problem.¹ Traditionally, among economic factors, different wage schedules were assumed to generate the incentives leading to this problem. Nonetheless, a recent study by Kwok and Leland [3] indicated another possibility, mainly the asymmetric information structure (i.e., the source country employers cannot observe the abilities of returning persons), which can explain the brain drain problem without assuming difference in wage schedules.² Their results showed that, owing to the asymmetric information structure, only lower ability people will return to their home countries while higher ability people remain abroad.

In this paper, we further extend Kwok-Leland's framework in two directions: (1) the possibility of signaling, (2) the explicit considerations of two-stage decision procedures. By (1), we mean that, while the source countries may not observe the true ability of returning individuals, they may well try to infer it from other sources in which case these sources become signals. By (2), we mean

that, before deciding whether to go abroad or not, each individual will consider explicitly whether he will return to his home country or remain abroad once he has decided to go abroad. The model is describe in Section 2.

From the model, we derive the properties of feasible rational expectations equilibria in Section 3. The results indicate the relationship between individual's ability and the signal he will choose. As a natural interpretation of the signal would be the ranking of all universities (a "lower" signal being equivalent to getting a Ph.D. from a lower ranked university), we can therefore interpret these results as indicating what types of students will choose what levels of universities to pursue their Ph.D.'s, and whether they will return home after getting their Ph.D.'s. These implications are described in Section 4.

Warning: since, in reality, the brain drain problem is determined by many social factors [1], the conclusions of this paper should be treated with caution.

2. A SIMPLE MODEL

Consider a source country, say Taiwan³, endowed with individuals graduating from colleges (in Taiwan) with ability $\theta \in [\underline{\theta}, \bar{\theta}]$, $0 < \underline{\theta} < \bar{\theta}$. Knowing his own θ , each individual has to make a choice over $\alpha \in \{0\} \cup [\underline{\alpha}, \infty)$, $\underline{\alpha} > 0$. A choice of $\alpha \geq \underline{\alpha}$ for the θ -ability person will cost $c(\theta, \alpha)$ where $c_\theta < 0$, $c_\alpha > 0$, $c_{\theta\alpha} < 0$, $c_{\alpha\alpha} \geq 0$. On the other hand, after α -investment is completed, his ability will be increased from θ to $\hat{\theta}(\theta, \alpha) = f(\theta, \alpha) + u$ where

$f_{\alpha} > 0$, $f_{\theta} > 0$, $f_{\theta\alpha} \geq 0$, $f_{\alpha\alpha} \leq 0$, $f_{\theta\theta} \leq 0$, and u is a random variable with finite support $[a, b]$.

The interpretations are as follows: " $\alpha = 0$ " represents staying and working in Taiwan while " $\alpha \geq \theta$ " represents going abroad, say U.S.A., to study for Ph.D. degrees. Furthermore, as α increases, he goes to better universities (assuming α is common knowledge). In this model, we assume everyone may receive admissions and (finally) Ph.D. degrees from any university in U.S.A. However, there will be differences in cost terms and ability improvement capacities: (i) Smarter persons (with higher θ) pay less cost; (ii) Better universities (with higher α) cost more; (iii) Both higher θ and higher α increase more of the abilities (in average); (iv) Marginal cost of α decreases as α decreases (or θ increases); (v) Marginal ability improvement of α decreases as α increases (or θ decreases).

After α -investment is completed, an individual may choose to work in U.S.A. or in Taiwan. Assume the wage schedules prevailed in both countries are the same: $W(\theta) = \theta$, when θ is observable. Nonetheless, since in general individuals prefer to live in their home country, there is a discount factor $0 < k < 1$ which applies to the income received from working in U.S.A. Under this assumption, clearly everyone from Taiwan would return if his final ability $\hat{\theta}$ is observable to both employers. However, asymmetric information structure is now imposed. Specifically, because α -investment is made and completed in U.S.A., we assume the individual's $\hat{\theta}(\theta, \alpha)$ could be perfectly observed by U.S. employers. Yet, Taiwanese employers only observe α and pay

$\bar{W}(\alpha)$, which is their common beliefs of the mean ability for those returning α -level Ph.D.'s.

The causes of asymmetric information are mainly of institutional characteristics.⁴ The assumption that α is common knowledge may be justified by the fact that there are censuses conducted in U.S.A. about the ranks of universities. Although this classification is discrete, to simplify mathematical considerations, we assume α is continuous. The reasons for assuming wage schedules depend upon α also has some institutional flavors,⁵ but it may be simply that (i) $\hat{\theta}$ is unobservable, therefore Taiwanese employers have to infer $\hat{\theta}$ from some other sources and (ii) α is commonly believed to be highly correlated with $\hat{\theta}$.

Lastly, for those choosing $\alpha = 0$, we assume Taiwanese employers can perfectly observe their ability θ (i.e., the only imperfect information in this model is knowledge of the ability of those returning Ph.D.'s). The model is now complete and we want to further study the equilibrium states, which satisfy (1) Each individual maximizes his expected net wage income by choosing appropriate α and deciding whether to return Taiwan or not thereafter (if $\alpha \geq \theta$); and (2) The wage paid by Taiwanese employers to those returning α -level Ph.D.'s actually reflects their average ability. As in the literatures, this is a typical rational expectations equilibrium (REE). Mathematically, REE is characterized by $\alpha^*(\theta)$ and $\bar{W}(\alpha)$ such that the following two conditions are met:

- (1) Given $\bar{W}(\alpha)$, $\alpha^*(\theta)$ maximizes $N(\theta, \alpha)$ for every θ where $N(\theta, \alpha)$ is

the

net payoff for θ ability individuals from choosing a .

$$(2) \int_{\theta \in \Theta(a)} \int_{u \in \Omega(\theta, a)} p(\theta) g(u) \bar{W}(a) du d\theta$$

$$= \int_{\theta \in \Theta(a)} \left\{ \int_{u \in \Omega(\theta, a)} \hat{\theta}(\theta, a) g(u) du \right\} \Pr(u \in \Omega(\theta, a)) p(\theta) d\theta, \forall a \geq \bar{\theta}$$

where $p(\theta)$ is the p.d.f. of θ ; $g(u)$ is the p.d.f. of u ; $\Theta(a)$ is the set of all possible θ 's who will choose a ; $\Omega(\theta, a)$ is the set of possible realizations of u such that those persons characterized by (θ, a) will return to Taiwan. This is a "Rational Expectation" condition. In other words, the average wage, $\bar{W}(a)$, paid to returning students is "correct," given the distribution of those who elect to return.

3. SOME PROPERTIES OF FEASIBLE REE

Now, given $\bar{W}(a)$ with $\bar{W}'(a) \geq 0$, an individual with ability θ may choose a from the following three different regimes:

$$\text{Regime 1: } k[f(\theta, a) + a] \geq \bar{W}(a)$$

Here, he never returns Taiwan, and a is thus chosen to maximize $N_1(\theta, a) = kE_u[f(\theta, a) + u] - c(\theta, a)$. Assume interior solution exists, then the optimal choice of a in this regime, $a_1(\theta)$, satisfies:

$$\frac{\partial N_1(\theta, a)}{\partial a} = kf'_a(\theta, a) - c_{aa}(\theta, a) = 0, \quad (1)$$

$$\frac{\partial^2 N_1(\theta, a)}{\partial a^2} = kf''_{aa}(\theta, a) - c_{aaa}(\theta, a) < 0. \quad (2)$$

$$\text{Regime 2: } k[f(\theta, a) + b] \leq \bar{W}(a)$$

Contrast to Regime 1, now the individual always returns Taiwan after a -investment is completed. Therefore, a is chosen to maximize $N_2(\theta, a) = \bar{W}(a) - c(\theta, a)$. Assume again interior solution exists, the optimal choice of a in the regime, $a_2(\theta)$, satisfies:

$$\frac{\partial N_2(\theta, a)}{\partial a} = \bar{W}'(a) - c_a(\theta, a) = 0, \quad (3)$$

$$\frac{\partial^2 N_2(\theta, a)}{\partial a^2} = \bar{W}''(a) - c_{aa}(\theta, a) < 0. \quad (4)$$

$$\text{Regime 3: } k[f(\theta, a) + a] \leq \bar{W}(a) \leq k[f(\theta, a) + b]$$

Now, depending upon the realization of u , the individual may remain in U.S.A. or return Taiwan. Specifically, if $\bar{W}(a) \geq k[f(\theta, a) + u]$, or equivalently $u \leq \frac{\bar{W}(a)}{k} - f(\theta, a)$, he will return Taiwan; otherwise he will remain in U.S.A. Therefore, by choosing a in this regime, he tries to maximize

$$N_3(\theta, a) = \Pr \left[u \leq \frac{\bar{W}(a)}{k} - f(\theta, a) \right] \bar{W}(a) + E_u \left[k(f(\theta, a) + u) \mid u \geq \frac{\bar{W}(a)}{k} - f(\theta, a) \right] - c(\theta, a)$$

$$= G\left[\frac{\bar{w}(\alpha)}{k} - f(\theta, \alpha)\right] \bar{w}(\alpha) + \int_{\frac{\bar{w}(\alpha)}{k} - f(\theta, \alpha)}^b k(f(\theta, \alpha) + u)g(u)du - c(\theta, \alpha) \quad (5)$$

where $G(\cdot)$ [resp. $g(\cdot)$] is the probability distribution [resp. density] function of u . Assume interior solution exists, the optimal choice of α in this regime, $\alpha_3(\theta)$, satisfies:

$$\frac{\partial N_3(\theta, \alpha)}{\partial \alpha} = G\left[\frac{\bar{w}(\alpha)}{k} - f(\theta, \alpha)\right] \bar{w}'(\alpha) + kf_{\alpha}(\theta, \alpha) \left\{ 1 - G\left[\frac{\bar{w}(\alpha)}{k} - f(\theta, \alpha)\right] \right\} - c_{\alpha}(\theta, \alpha) = 0 \quad (6)$$

$$\frac{\partial^2 N_3(\theta, \alpha)}{\partial \alpha^2} = G\left[\frac{\bar{w}(\alpha)}{k} - f(\theta, \alpha)\right] \bar{w}''(\alpha) + kg\left[\frac{\bar{w}(\alpha)}{k} - f(\theta, \alpha)\right] \left(\frac{\bar{w}'(\alpha)}{k} - f_{\alpha}(\theta, \alpha)\right)^2 + kf_{\alpha\alpha}(\theta, \alpha) \left\{ 1 - G\left[\frac{\bar{w}(\alpha)}{k} - f(\theta, \alpha)\right] \right\} - c_{\alpha\alpha}(\theta, \alpha) < 0 \quad (7)$$

From the above considerations, the final optimal choice of $\alpha \in [0, \infty)$ will be $\alpha^*(\theta) \in \{\alpha_1(\theta), \alpha_2(\theta), \alpha_3(\theta)\}$, assuming $\alpha^*(\theta)$ is the interior solution for some regimes.⁶ Given this, a person will be said to be of type 1 iff he sets $\alpha^*(\theta) = \alpha_1(\theta)$, $i = 1, 2, 3$.

Obviously, type 1 persons will never return while type 2 persons will

always return and type 3 persons observe the realizations of $\hat{\theta}$ (or u) to determine whether to return or not.

Lemma 1: If θ_1, θ_2 and θ_3 all correspond to the same α , where θ_i belongs to type i , $i = 1, 2, 3$, then (i) $\theta_2 \leq \theta_3 \leq \theta_1$,⁸ (ii) $\hat{\theta}(\theta_1, \alpha) \geq \hat{\theta}(\theta_2, \alpha)$ with probability 1.

Lemma 2: Given θ_3 , if there exists θ_1 or θ_2 such that they correspond to the same α , where θ_i belongs to type i , then $kf_{\alpha}(\theta_3, \alpha) \leq \bar{w}'(\alpha)$.

Lemma 3:

- (i) At Rational Expectations Equilibrium, if θ belongs to type 2 and $\bar{w}''(\alpha) \leq 0$ for all α , then $\alpha_2(\theta) \geq \alpha_3(\theta)$.
- (ii) Assume $\bar{w}''(\alpha) \leq 0$, if θ belongs to type 3 such that $\alpha_3(\theta) = \alpha_2(\theta') = \alpha^*(\theta')$ for some θ' , then $\alpha_2(\theta) \geq \alpha_3(\theta) \geq \alpha_1(\theta)$.
- (iii) If θ belongs to type 3 such that $\alpha_3(\theta) = \alpha_1(\theta'') = \alpha^*(\theta'')$ for some θ'' , then $\alpha_1(\theta) \leq \alpha_3(\theta) \leq \alpha_2(\theta)$.

Lemma 4:

- (i) $\frac{d\alpha_1(\theta)}{d\theta} \geq 0, \forall \theta;$
- (ii) $\frac{d\alpha_2(\theta)}{d\theta} \geq 0, \forall \theta;$
- (iii) $\frac{d\alpha_3(\theta)}{d\theta} > 0$ if and only if

$$\bar{w}'(\alpha) - kf_{\alpha}(\theta, \alpha) = \frac{< kf_{\theta\alpha}(\theta, \alpha) \cdot [1 - G\left(\frac{\bar{w}(\alpha)}{k} - f(\theta, \alpha)] + c_{\theta\alpha}(\theta, \alpha) >}{g\left[\frac{\bar{w}(\alpha)}{k} - f(\theta, \alpha)\right]f_{\theta}(\theta, \alpha)} \quad (8)$$

(The Proofs of Lemma 1 - 4 are presented in the Appendix.)

Given the above results, assume $\bar{w}'(\alpha) \leq 0$, the only rational expectations equilibrium where three types of persons coexist⁹ can be shown as Figure 1, of which B and C (also, D and E) correspond to the same α . Other configurations, such as "U" shape or downward sloping curve within type 3 will violate Lemma 3. Nevertheless, as a special case, if $\bar{w}(\alpha)$ is a constant function, then $\bar{w}'(\alpha) = 0 < kf_{\alpha}(\theta_3, \alpha)$, $\forall \theta_3$, and hence there is no overlapping α across different types (by Lemma 2). Furthermore, the relationship between θ and α is continuous and monotonically increasing as shown in Figure 2.

Although existence of equilibrium wage schedules of the form of Figure 1 has not been shown in general, for certain cases, such schedules can be shown to exist. Mathematically, the equilibrium wage schedule $\bar{w}(\alpha)$ associated with Figure 1 may be solved from the following differential equations system:

$$(1) \quad f(\theta_2, \alpha)p(\theta_2) + E_u \left[f(\theta_3, \alpha) + u \mid u \leq \frac{\bar{w}(\alpha)}{k} - f(\theta_3, \alpha) \right] - G \left[\frac{\bar{w}(\alpha)}{k} - f(\theta_3, \alpha) \right] - p(\theta_3) = \left[p(\theta_2) + p(\theta_3)G \left[\frac{\bar{w}(\alpha)}{k} - f(\theta_3, \alpha) \right] \right] \bar{w}(\alpha), \quad \forall \alpha \geq \bar{\alpha};$$

$$(2) \quad \bar{w}'(\alpha) = c_{\alpha}(\theta_2, \alpha);$$

$$(3) \quad G \left[\frac{\bar{w}(\alpha)}{k} - f(\theta_3, \alpha) \right] \bar{w}'(\alpha)$$

$$+ kf_{\alpha}(\theta_3, \alpha) \left\{ 1 - G \left[\frac{\bar{w}(\alpha)}{k} - f(\theta_3, \alpha) \right] \right\} = c_{\alpha}(\theta_3, \alpha).$$

As an example, Assume (i) $c(\theta, \alpha) = c_0 + c_1\theta + c_2\alpha + c_3\theta\alpha$ with $c_{\theta} < 0$, $c_{\alpha} > 0$, $c_{\theta\alpha} = c_3 < 0$; (ii) $f(\theta, \alpha) = \theta + \delta\alpha$, $\delta > 0$; (iii) u is uniformly distributed over $[-b, b]$; (iv) the distribution of θ is uniform over $[\underline{\theta}, \bar{\theta}]$. Then, it can be shown that if $\theta \geq (k\delta - c_2)/c_3$, $\alpha_1(\theta) = \alpha^*(\theta) = \infty$; otherwise, if $\theta < (k\delta - c_2)/c_3$, then only the boundary points in Regime 1 will matter. Also, type 2 and type 3 persons' choices are characterized, respectively, by:

$$c_2 + c_3\theta_2 = \bar{w}'(\alpha) \quad (9)$$

$$\theta_3 = \frac{\bar{w}'(\alpha) - k\delta}{\bar{w}'(\alpha) - k\delta + 2bc_3} \left[b - \delta\alpha + \frac{\bar{w}(\alpha)}{k} + \frac{2b(\delta\alpha - c_2)}{\bar{w}'(\alpha) - k\delta} \right]. \quad (10)$$

On the other hand, REE will require:

$$\theta_2 + \delta\alpha + \int_{-b}^k \frac{\bar{w}(\alpha) - \theta_3 - \delta\alpha}{k} \frac{\theta_3 + \delta\alpha + u}{2b} du \cdot \int_{-b}^k \frac{\bar{w}(\alpha) - \theta_3 - \delta\alpha}{k} \frac{1}{2b} du = \bar{w}(\alpha) \times \left[1 + \int_{-b}^k \frac{\bar{w}(\alpha) - \theta_3 - \delta\alpha}{k} \frac{1}{2b} du \right]. \quad (11)$$

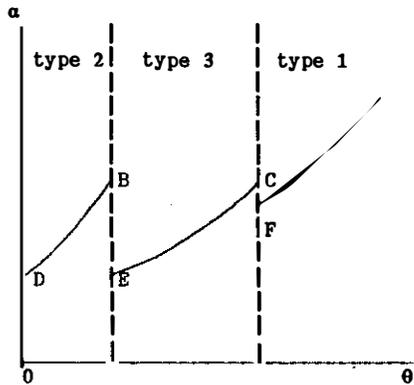


Figure 1

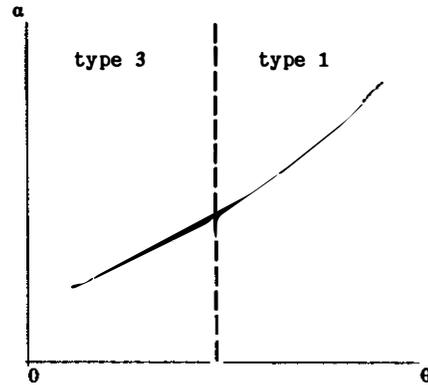


Figure 2

Substituting eqs. (9) and (10) into (11), after algebraic manipulations, we have

$$\begin{aligned} & \delta\alpha + \frac{\bar{W}'(\alpha) - c_2}{c_3} + \frac{1}{4b} \left[\frac{\bar{W}(\alpha)}{k} \right]^2 - \left[\frac{2b\gamma + \frac{\bar{W}(\alpha)}{k}(\bar{W}'(\alpha) - k\delta)}{\bar{W}'(\alpha) - k\delta + 2bc_3} \right]^2 \\ & \times \left[\frac{\bar{W}(\alpha)}{2bk} - \frac{2b\gamma + \frac{\bar{W}(\alpha)}{k}(\bar{W}'(\alpha) - k\delta)}{2b(\bar{W}'(\alpha) - k\delta + 2bc_3)} \right] \\ & = \bar{W}(\alpha) \times \left[1 + \frac{\bar{W}(\alpha)}{2bk} - \frac{2b\gamma + \frac{\bar{W}(\alpha)}{k}(\bar{W}'(\alpha) - k\delta)}{2b(\bar{W}'(\alpha) - k\delta + 2bc_3)} \right] \end{aligned} \quad (12)$$

where $\gamma = k\delta - c_2 + c_3(\delta\alpha - b)$. Note that, given finite α , if $\bar{W}'(\alpha) \rightarrow +\infty$, then $\bar{W}(\alpha) \rightarrow +\infty$. Also, if there exist possibilities such that $\bar{W}'(\alpha) \rightarrow k\delta - 2bc_3$, then eq. (12) reduces to $2b\gamma + \frac{\bar{W}(\alpha)}{k}(-2bc_3) = 0 \Rightarrow \frac{\bar{W}(\alpha)}{k} = \frac{\gamma}{c_3} = \frac{k\delta - c_2}{c_3} + \delta\alpha - b \Rightarrow \bar{W}'(\alpha) = k\delta$, which leads to a contradiction. Therefore, by specifying the initial condition for $\bar{W}(\alpha)$, we can expand $\bar{W}(\alpha)$ pointwisely from eq. (12), i.e., the solutions for eq. (12) exist.

4. IMPLICATIONS

From Figure 1, we may find that, within the same level universities, there may exist three types of students. Those who decided to remain (before observing the realization of u) have higher

prior ability (θ) than those who decided to return Taiwan (before observing the realization of u). Furthermore, whatever the realizations of u may be, the former group always has higher final abilities ($\hat{\theta}$) than the latter one (by Lemma 1). Also, within type 3, as θ increases, α also increases, which implies $0 \leq \bar{W}'(\alpha)$ - $k f_{\alpha}(\theta, \alpha) \leq \beta$ for some constant β (by Lemma 2 and 4). Therefore, there will be both an upper bound and a lower bound for $\bar{W}'(\alpha)$.

However, unlike Figure 2, since α affects the wage income, it possesses signal values. (This can be seen from Lemma 3 where given θ , if the probability to return Taiwan increase, then α should also increase). Hence, lower ability people will have more incentives to exploit these values, particularly type 2 persons will exploit more of these values through the externalities provided by type 3 persons.

More importantly, as shown in Figure 1 or 2, there will be a $\bar{\theta}$ (or, equivalently, an $\bar{\alpha}$) such that all the persons with ability higher than $\bar{\theta}$ (or, all the persons attend universities with ranks higher than $\bar{\alpha}$) will never return. This means, the best persons will never return Taiwan which may be the most serious brain drain problem. On the other hand, for type 3 persons, only those with worse realizations of u (or $\hat{\theta}$) will return Taiwan while the others remain. This constructs another brain drain problem which was indicated by Kwok-Leland [3].

Finally, as a remark, note that there are several ways to increase wage incomes in this model. For those lower ability type 2 person, they simply use the signal values of α and exploit the externalities provided by type 3 persons. Type 3 persons, however,

can only exploit the advantages from asymmetric information structure by which externalities are thus introduced. As for type 1 persons, they will only use their high ability levels to increase wage incomes, foregoing the asymmetric information structure.

FOOTNOTES

- * I am grateful to Richard McKelvey for long helpful discussions, to Leonid Hurwicz and Joel Sobel for constructive suggestions, to Robert Bates and James Quirk for valuable comments on an earlier draft. Of course, all errors remain mine.
1. From the host country side, the influx of unskilled immigrants (legal or illegal) is said to distort the labor market and delay technological innovations [5].
 2. By assuming opposite asymmetric information structure (i.e., the uninformed agents are those in the host countries), Katz-Stark [2] generated the conclusion that only lower ability people will go abroad. As shown in their papers, it seems that they were dealing with different issues. Specifically, Kwok-Leland considered brain drain problem while Katz-Stark dealt with unskilled immigrants. Nevertheless both papers indicated the importance of asymmetric information structure in international migrations studies [4].
 3. Although Taiwan was selected by Kwok-Leland to justify the assumption of the same wage schedules (which, more or less, raises some objections), yet here it was selected since more institutional characteristics could be provided.
 4. If a Ph.D. tries to enter U.S. job markets, interviews, job talks and recommendation letters, etc. will be required. However, if he applies a job in Taiwan (from U.S.A.), interviews or job talks usually won't be conducted. Although recommendation letters or samples of research work might be requested, yet due to lack of

- connections and gaps of academic research levels (i.e., for example, some new ideas which are highly appreciated in U.S.A. may be still unfamiliar to Taiwanese academic circles), their values as references to abilities will be seriously reduced. Hence, U.S. employers have much more information about job candidates' abilities to access while Taiwanese employers are poorly informed. As an extreme case, we assume the abilities are perfectly observed by U.S. employers, yet Taiwanese employers have no information at all.
5. I have been told some economic research institutes pay the salaries according to which universities the employees get Ph.D. from. Also, an oversea job candidate list circulated in Taiwan includes only the following information: sex, age, degrees from which universities and expected salaries, etc. Although further information may be acquired, yet the school ranking still plays important roles in the first step of recruitment procedures. On the other hand, the special case when $\bar{w}(\alpha)$ is a constant function is also studied in Section 3.
 6. Any boundary solution, except $\alpha^*(\theta) = \bar{\alpha}$, can only appear in the interaction points of type frontiers. For example, if $\alpha^*(\theta)$ is the boundary solution for type 1, then it must be that the individual with ability θ is indifferent between being type 1 or type 3. Therefore, the relationship of θ and α will be continuous at this "type" intersection point, i.e. $C = F$ in Figure 1 (assume every type 3 person always has unique maximum point). Note that,

if there is also $\theta' < \theta$ such that the individual with ability θ' is indifferent between being type 2 or type 3, then B also equals E in Figure 1 which renders the figure the same as Figure 2.

Therefore, if $a^*(\theta)$ is never a boundary solution, then Figure 1 will be established and this is a particular property that the assumption of $\bar{W}(a) = \text{constant}$ cannot generate.

7. Alternatively, we may only deal with type 3, and then those type 3 people who will never return to Taiwan become "type 1" persons while those type 3 people who always return become "type 2."
8. If there are several different ability levels, say, $\theta_{31}, \theta_{32}, \dots, \theta_{3n}$, such that θ_{3i} belongs to type 3 for all i and they all correspond to the same a , then the following property holds:
 $\theta_{3i} \geq \theta_{3j}$, iff the probability for θ_{3i} to return Taiwan is lower than or equal to that of θ_{3j} .
9. There exists another possibility for the REE where three types of persons coexist, i.e., when $a_3(\theta)$ and θ display "∩" shape. However, in this case, there will be some low level universities which only very smart students attend. Obviously, this phenomenon deems unreasonable.

APPENDIX

Proof of Lemma 1: Since $\bar{W}(a) \leq k[f(\theta_1, a) + a]$, $\bar{W}(a) \geq k[f(\theta_2, a) + b]$ and $k[f(\theta_3, a) + a] \leq \bar{W}(a) \leq k[f(\theta_3, a) + b]$, it can be easily shown $\theta_2 \leq \theta_3 \leq \theta_1$ and $f(\theta_1, a) \geq f(\theta_2, a) + (b - a) \Rightarrow \hat{\theta}(\theta_1, a) \geq \hat{\theta}(\theta_2, a)$ with probability 1, since $a \leq u \leq b$ and $\hat{\theta}(\theta, a) = f(\theta, a) + u$.

Q.E.D.

Proof of Lemma 2: If θ_1 exists, then $kf_a(\theta_1, a) = c_a(\theta_1, a)$ and

$$G\left[\frac{\bar{W}(a)}{k} - f(\theta_3, a)\right] \bar{W}'(a) + kf_a(\theta_3, a) \left\{1 - G\left[\frac{\bar{W}(a)}{k} - f(\theta_3, a)\right]\right\} = c_a(\theta_3, a) \quad (A1)$$

$$\Rightarrow k[f_a(\theta_1, a) - f_a(\theta_3, a)] + [c_a(\theta_3, a) - c_a(\theta_1, a)]$$

$$= [\bar{W}'(a) - kf_a(\theta_3, a)] G\left[\frac{\bar{W}(a)}{k} - f(\theta_3, a)\right]$$

$$\Rightarrow \bar{W}'(a) \geq kf_a(\theta_3, a), \text{ since } f_{a\theta} \geq \theta, c_{a\theta} \leq 0 \text{ and } \theta_1 \geq \theta_3.$$

Similarly, if θ_2 exists, then $\bar{W}'(a) = c_a(\theta_2, a)$ and (A1) hold which imply

$$\left\{1 - G\left[\frac{\bar{W}(a)}{k} - f(\theta_3, a)\right]\right\} (\bar{W}'(a) - kf_a(\theta_3, a)) = c_a(\theta_2, a) - c_a(\theta_3, a)$$

$\Rightarrow \bar{W}'(\alpha) \geq kf'_\alpha(\theta_3, \alpha)$, since $c_{\theta\alpha} \leq 0$ and $\theta_3 = \theta_2$.

Q.E.D.

Proof of Lemma 3: Note that, at REE, every type 2 person must be supported by (at least) one type 3 person with $\theta' \geq \theta$, where θ is the ability of type 2 person, θ' is the ability of corresponding type 3 person. By Lemma 2, we have $\bar{W}'(\alpha) \geq kf'_\alpha(\theta', \alpha) \Rightarrow \bar{W}'(\alpha) \geq kf'_\alpha(\theta, \alpha)$, since $f_{\theta\alpha} \geq 0$ and $\theta' \geq \theta$ (by Lemma 1). Now,

$$\begin{aligned} & \frac{\partial N_2(\theta, \alpha)}{\partial \alpha} - \frac{\partial N_3(\theta, \alpha)}{\partial \alpha} \\ &= [\bar{W}'(\alpha) - kf'_\alpha(\theta, \alpha)] \cdot \left\{ 1 - G\left[\frac{\bar{W}(\alpha)}{k} - f(\theta, \alpha)\right] \right\} \geq 0. \end{aligned} \quad (A2)$$

Hence when $\frac{\partial N_3(\theta, \alpha)}{\partial \alpha} = 0$, $\frac{\partial N_2(\theta, \alpha)}{\partial \alpha} \geq 0 \Rightarrow a_2(\theta) \geq a_3(\theta)$ since

$\bar{W}''(\alpha) \leq 0$. If θ belongs to type 3, (A2) directly applies if there exists θ' or θ'' such that $a_3(\theta) = a_2(\theta') = a^*(\theta')$ or $a_3(\theta) = a_2(\theta'') = a^*(\theta'')$. On the other hand,

$$\frac{\partial N_1(\theta, \alpha)}{\partial \alpha} - \frac{\partial N_3(\theta, \alpha)}{\partial \alpha} = [kf'_\alpha(\theta, \alpha) -]G\left[\frac{\bar{W}(\alpha)}{k} - f(\theta, \alpha)\right] \leq 0$$

when the above assumption holds. Hence, when $\frac{\partial N_3(\theta, \alpha)}{\partial \alpha} = 0$, $\frac{\partial N_1(\theta, \alpha)}{\partial \alpha} \leq 0 \Rightarrow a_1(\theta) \leq a_3(\theta)$. Combining the above results, we have $a_1(\theta) \leq a_3(\theta) \leq a_2(\theta)$.

Q.E.D.

Proof of Lemma 4: Applying comparative statics to eqs. (1), (3), and

(6), the results will then be established, since $c_{\theta\alpha} < 0$ (by assumption).

REFERENCES

- Johnson, H. G. (1967). "Some Economic Aspects of Brain Drain."
Pakistan Development Review 7:379-411.
- Katz, E. and Stark, O. (1984). "Migration and Asymmetric
Information: Comment." American Economic Review 74:533-534.
- Kwok, V. and Leland, H. (1982). "An Economic Model of the Brain
Drain." American Economic Review 72:91-100.
- _____ (1984). "Migration and Asymmetric Information: Reply."
American Economic Review 74:535.
- Stahl, C. W. (1982). "Labor Emigration and Economic Development."
International Migration Review 16:869-899.