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THE TAXATION OF RISKY INVESTMENTS: AN ASSET PRICING APPROACH

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ABSTRACT

Some economists have argued that offsetting effects on risk and return may make capital income taxes nondistorting. This paper performs three tasks. First, the conditions under which the argument is true are studied in an asset pricing model that unlike earlier models allows the timing of depreciation deductions to vary and incorporates the effectiveness and distribution of government expenditures. One result is that it is plausible that the nondistortion result holds regardless of that timing or of the distribution and effectiveness of expenditures if the pre-tax riskless rate is zero.

A second task concerns the cases where the pre-tax riskless rate is not negligible and the nondistortion result does not hold. Then the degree and pattern of distortion depends on the general equilibrium impact of taxes and expenditures on average risk aversion and on the pre-tax riskless rate. An interesting result emerges concerning the impact of the timing of depreciation allowances. When average risk aversion stays constant, the conventionally expected effect that faster write-offs result in more investment will occur if and only if the pre-tax riskless rate falls when timing is accelerated. This is true because in the absence of any change in the pre-tax riskless rate, changes in depreciation timing cause changes in risk and expected return that exactly offset each other.

Finally, the paper shows that the failure to add a premium for "capital risk" to the standard economic depreciation allowance based on expected decline in asset value does not change that result unless the income tax system has the pathology of allowing used asset sales to be tax free. The current U.S. tax system seems to be free of that pathology.

The Taxation of Risky Investments:

An Asset Pricing Approach

Jeff Strnad

I. INTRODUCTION

Over the past few years, several economists have studied the taxation of income from risky capital investments using asset pricing models. Striking conclusions have emerged from two of those studies. Gordon (1981) argues that taxation of income from risky investment has two opposing effects. First, taxation of such income lowers the expected return from the investments therefore making them less attractive. Second, the government shares in the income and losses from the investments and thus absorbs some of the risk of the investments. To the extent of the reduction in risk the investments are more attractive to investors. Gordon argues that these two effects are "largely offsetting" so that taxes on income from capital leave "investment incentives basically unaffected, despite the sizable tax revenues collected."

Bulow and Summers (1984) argue that although taxation of the income from capital reduces the risk with respect to the income, it does not reduce the risk involved with the value of the capital. As an empirical matter, this latter risk, "capital risk," appears to be large compared to the former risk, "income risk." As a result, Bulow and Summers conclude that the negative effects of taxes on the

expected rate of investment return are not fully offset by the reduction in risk due to taxation since most of the risk is "capital risk." They also argue that for a depreciation allowance given ex ante to be equivalent to economic depreciation given ex post the allowance should not be merely the expected decline in value of the asset. A premium should be added that compensates the investor for capital risk. Use of a depreciation rate consisting of expected decline in value plus this premium reestablishes the Gordon effect: the negative effect of taxes on expected rate of return will be approximately fully offset by a corresponding reduction in risk.

After examining the results in the two papers in more detail in part II, this paper performs three tasks. First, a portion of section III determines the conditions required for Gordon's result to be true. Despite the intuitive appeal of his idea, the models in Gordon (1981) have unnecessarily severe limitations. He derives his results in two ways: an intuitive argument based on the capital asset pricing model ("CAPM") and a rigorous argument based on a two-period consumption model. The result of his two-period consumption model is that income taxes have no effect on investment or consumption if the tax revenues are distributed lump-sum when collected. One important innovation in this model is that the model accounts for the expenditure of tax revenues. Gordon himself notes that the earlier writers on the effects of taxes on risk bearing "almost all assume that individuals no longer bear what risk is passed on to the government." Unfortunately, as shown in the Appendix to this paper, when his two-period consumption model is simplified to a single-level tax on income from capital in a non-inflationary world, this result is a consequence of each taxpayer knowing that he or she will receive back exactly the taxes he or she will pay. In addition, the intuitive arguments based on the CAPM turn out to be significantly qualified when that model is applied rigorously to gauge the effect of taxes on asset prices as this paper does in part III.

The CAPM-based approach in part III differs from Gordon's approach in two fundamental ways. First, some of the limitations in his two-period consumption model are removed by providing a much richer model of government expenditure. The distribution of expenditure benefits is a variable, and there is a variable representing the anticipated total effectiveness of expenditures. This second variable allows for the fact that individuals may anticipate total government expenditure benefits to be more or less valuable than the benefits from spending the same total dollars privately. The "more" case might arise where the government spends to provide goods more efficiently than the private sector can. The "less" case includes the situation where government spending is at least partially "wasteful" as well as the situation where individuals myopically ignore government expenditure wholly or in part.

The second fundamental way in which the model of part III differs from Gordon's approach is that explicit allowance is made for the timing of depreciation deductions. Traditionally, this timing has been considered to have a potentially very important impact on

investment incentives. In Gordon's models depreciation deductions are delayed until the end of the final period. An important issue is whether this delay affects his result. Finally, if some depreciation deductions are allowed before taxes on profits are levied, the government must fund the ensuing revenue loss. In the model in part III this is done explicitly by government borrowing against future tax payments.

What emerges from part III is that Gordon's result depends on two parameters. One is a measure of changes in average risk aversion due to taxes and expenditures. The second is a measure of the difference between the riskless rate in the world with no taxes and the pre-tax riskless rate in the world with taxes. Changes in average risk aversion arise from the wealth effects of the tax and expenditure package and affect Gordon's results by changing risk premia. The intuition is straightforward. Suppose, for example, that people anticipate that government expenditure benefits will be more valuable than private expenditure of the tax revenues. This makes people feel wealthier and, under standard assumptions about behavior under uncertainty, makes them less risk averse. The market risk premium demanded for a given risk therefore falls. This phenomenon is in addition to the effect of taxes of absorbing part of the risk in investment since the government shares in gains and losses. If this second effect by itself cancels out the disincentive to investment from the drop in rate of return caused by taxes, adding the first effect may cause risky investment to increase over no tax world levels.

Despite the potential for wealth effects to change Gordon's results, his results are plausible if one assumes the real pre-tax riskless rate of return to be negligible. This assumption has some empirical support. Ibbotson and Sinquefield (1977), for example, find that the average real pre-tax return on U.S. Treasury bills (presumably nearly riskless) has been about 0.2 percent, while the average risk premium in the stock market has been about 9 percent.

The implications of pre-tax riskless rate being negligible are that:

(1) the net present value of tax revenues, and thus of expenditure, is negligible;

(2) asset prices remain nearly the same in the world with taxes as in the no tax world;

(3) the effect on investment incentives is negligible.

These implications follow independent of the timing of depreciation allowances. The first two implications ensure that wealth effects can be neglected. As a consequence, the third implication, Gordon's main result, holds true. Finally, note that implications (1) and (3) go together: although taxes have little effect on investment incentives, the net present value of tax revenues is also negligible. This confirms one argument in Bulow and Summers (1984): the lack of investment disincentives in Gordon (1981) is not accompanied by the "free lunch" of substantial tax revenues.

This set of implications is no longer plausible if the pre-tax riskless rate is nontrivial. A real pre-tax riskless rate of even one or two percent will result in tax revenues with substantial net present value and in substantial potential effects on investment incentives. This leads to the second task of this paper: a study of the effect of taxes on investment incentives when Gordon's result does not apply. Proposition 1 in part III shows that taxes impact on investment incentives only if taxes either change average risk aversion or make the pre-tax riskless rate differ from the no tax world riskless rate. The effect of changes in average risk aversion and in the pre-tax riskless rate may depend on asset characteristics. For example, if average risk aversion is unchanged but the pre-tax riskless rate rises so that the after-tax riskless rate equals the no tax world riskless rate, then riskier assets experience more of a disincentive effect from taxes.

Many different conclusions about incentive affects can be obtained depending upon what one postulates about the impact of taxes on average risk aversion and on the pre-tax riskless rate. One important area in which the model creates new insights is in the perennial dispute about the impact of the timing of depreciation deductions on investment. Timing differences may affect average risk aversion through various wealth effects. These wealth effects occur because accelerating depreciation deductions means that the government must spend more of the net present value of tax revenues to fund those deductions than on other expenditures. There is no a priori direction of the effects: accelerating deductions may increase, decrease or not affect average risk aversion depending on the effectiveness of government expenditure and on its distribution.

According to Proposition 2 in part III, in the absence of changes in average risk aversion the conventionally expected effects of timing can occur only if the acceleration of deductions reduces the required pre-tax riskless rate below the riskless rate in the no tax world. That reduction would cause all security prices to rise in the same proportion so that investing in physical assets and then selling ownership of the assets in securities markets would be more lucrative. Proposition 2 has a striking corollary. If the economy-wide pre-tax riskless rate of return is unaffected by the timing of depreciation deductions for investments, then the timing pattern has no effect on investment incentives. This is true even in cases where the after-tax riskless rate varies greatly with the timing pattern.

The intuition behind this can be made clearer by considering two polar cases: "expensing" and "recovery at retirement." Assume that average risk aversion is the same in both cases. This allows a focus on the conventional effects of timing through the pre-tax riskless rate. "Expensing" is where there is an immediate 100% write-off for investment. "Recovery at retirement" allows no write-off until the investment asset is sold or abandoned as worthless.

Expensing in effect amounts to government purchase of the proportion T (where T is the tax rate) of any asset at the same cost

as the taxpayer bears. The government then obtains the proportion T of all gains and losses. The investor effectively owns the portion (1 - T) of the asset and receives the pre-tax rate of return tax free on that portion. With recovery at retirement the government does not purchase part of the asset at the outset. Instead, it simply takes part of the returns at the end and properly accounts for costs at that time. Both risk and expected return are reduced from pre-tax levels. But the pre-tax rate of return is the same as in the case of expensing so long as the pre-tax riskless rate is not affected by timing changes. (Pre-tax risk premia can be shown to be independent of timing regime so long as the pre-tax riskless rate and average risk aversion are). As a result, the basic Gordon effect makes recovery at retirement equivalent to expensing: the reduction in risk exactly compensates for the reduction in rate of return. In order for more rapid depreciation to increase the incentive to invest the economy-wide pre-tax riskless rate must fall.

The third and final task of this paper is a critique of arguments in Bulow and Summers (1984). These arguments, if correct, would greatly alter the results discussed so far. Furthermore, Bulow and Summers' notion that ex ante depreciation should include a premium for capital risk has considerable intuitive appeal. Unfortunately, their result is based on the unstated assumption that the gains and losses from the sale of used assets will be ignored by the tax system. If gains from such sales are taxed and losses from such sales are deductible, as is the case under current U.S. tax law, then the government <u>does</u> share in the capital risk element at the time of sale. Not surprisingly, in this case the Bulow and Summers result disappears. On the other hand, if the owner never finds it profitable to sell used assets, the capital risk never materializes. It is only in the case where for some reason sales of used assets are exempted from tax that the Bulow and Summers notion that depreciation allowances should carry a risk premium is applicable. In effect, such a risk premium "corrects" for a defect in the tax system and is not appropriate in an income tax system such as the current U.S. system that does not have the defect.

Section IV of this article establishes these points using a simple discounted present value analysis. Before doing that or constructing the asset pricing model in section III, section II makes the arguments and approach in the previous literature more precise.

II. PRIOR ARGUMENTS AND APPROACHES

The CAPM analysis in Gordon (1981) and all of the analysis in Bulow and Summers (1984) gauges the effect of taxes on investment incentives by comparing the required rate of return in the no-tax world to the required pre-tax rate of return in the tax world. The idea is that if there are no general equilibrium effects on pre-tax operating profits and costs, there will be no change in the incentive to invest if these rates remain the same. On the other hand, if the required pre-tax rate of return in the tax world substantially exceeds the required rate of return in the no-tax world, then taxes make some

investments nonviable that would be profitable in a world without taxes.

Suppose that in a no tax world an additional marginal unit of asset j returns revenues (net of operating costs except depreciation) at an expected rate f_j^e during a particular period and depreciates in value at rate d_j during that period. The required minimum expected rate of return for this project will consist of the sum of two quantities. First, there is the riskless rate of return, r', that investors require in order to invest in projects with revenues and costs not subject to uncertainty. Second, there is a premium, a_j', that reflects the additional increment in rate of return required to compensate for the fact that the actual revenues and costs for the period are uncertain. In the no tax world, then, a marginal investment will have the property that

$$f_j^e - d_j = r' + a_j'. \tag{1}$$

<u>I.e.</u>, the expected revenues net of costs including depreciation are equal to the minimal expected rate of return that investors will require for a project with those risk characteristics.

When an "income tax" at rate T on net profits during that period is added, Gordon (1981) argues that a marginal investment will have the following property:

$$(f_j^e - d_j)(1 - T) = r' + a_j'(1 - T). \tag{2}$$

<u>I.e.</u>, profits, $f_j^e - d_j$, are taxed at rate T, the risk premium is

reduced to reflect the fact that the government has absorbed 100T percent of the risk, but investors require a riskless return of r' after-tax in order to invest.

In section III, I will examine the argument for reducing a' by the factor (1 - T), but leaving r' the same. In the rest of this section that approach, common to both Gordon (1981) and Bulow and Summers (1984), will be assumed to be correct. Gordon's basic argument can be seen by rearranging (2) to obtain:

$$f_j^e - d_j = \frac{1}{1 - T} r' + a_j'. \tag{3}$$

The right hand side of (3) is $\frac{T}{1 - T} r'$ more than the right hand side of (1). Thus, due to taxes investors require an increase by that amount in the no tax rate of return. Thus, projects with returns between $r' + a_j'$ and $\frac{1}{1 - T} r' + a_j'$ will not be undertaken in the tax world although they are in the no tax world. In this simplified version, Gordon's thesis relies on the presumption that $\frac{T}{1 - T} r'$ is very small compared to $r' + a_j'$ so that not much change in investment behavior results from the income tax. This presumption is reasonable given that empirical evidence strongly suggests that r' would be negligible compared to a_j' for the average stock market asset.

Bulow and Summers' argument relies on the assumption that none of the capital risk is absorbed by the government and that capital risk far exceeds income risk. In the framework above they present the core of their argument by assuming that all of the risk in the investment is capital risk. Since there is no income risk, revenues

net of all costs except depreciation are certain and for asset j can be represented by using f_j, now a constant, instead of f_j^e. Bulow and Summers make the point that in a real world tax system depreciation is set ex ante, i.e., before the actual decline in asset value is known. There is some amount of actual depreciation that will occur. This amount is known only ex post and usually is called "economic depreciation." Traditionally, the best ex ante approximation of economic depreciation has been taken to be the expected amount of depreciation. For asset j call this amount d_j^e (rather than d_j).

Now since risk is not reduced due to taxes, the after-tax equation changes from (2) to the following:

$$(f_j - d_j^e)(1 - T) = r' + a_j'. \tag{4}$$

Thus, the minimum before-tax rate of return required by investors is $\frac{1}{1 - T}(r' + a_j')$ which is $\frac{T}{1 - T}(r' + a_j')$ larger than $(r' + a_j')$, the minimum rate of return required by investors in the no tax world. Bulow and Summers argue that since a_j' is large, $(r' + a_j')$ is not negligible. As a result, a tax will cause a significant reduction in investment in risky assets.

One of Bulow and Summers major theses is that the best ex ante approximation of economic depreciation for asset j would consist of $d_j^e + a_j'$, expected depreciation for the period plus the risk premium that investors would demand with respect to the risk that actual depreciation will differ from d_j^e. With that amount of depreciation allowance equation (4) becomes:

$$(f_j - d_j^e - a_j')(1 - T) = r'. \quad (5)$$

This gives a result similar to Gordon's in that (5) is almost identical to (2). In particular, the pre-tax return must increase by $\frac{T}{1 - T}$ r' over the return in the no tax world to compensate investors for the effects of taxation. Thus, if depreciation of $d_j^e + a_j'$ is used, the tax on capital income will have very little effect on investment behavior.

III. AN ASSET PRICING MODEL WITH GOVERNMENT EXPENDITURES

The literature discussed in part II tests the incentive effects of taxes on investment by comparing the required rate of return in the no-tax world to the required pre-tax rate of return in the tax world. This part makes the same comparison but makes it through a rigorous asset pricing model. Section A sets up a model and gives special attention to the treatment of government expenditures, to the timing of taxes and deductions and to government budget constraints. Section B derives as equation (42) the difference between the pre-tax and no tax required rates of return. Section C discusses the implications of that equation and of some of the other results in section B.

A. The Model

In order to analyze the separate effects of income taxes on the riskless rate of return and on risk premia, it is convenient to use the Sharpe-Lintner version of the CAPM that includes a riskless

asset. The framework used here is similar to that of Brennan (1970). There are m risk averse investors (indexed by i) who select portfolios of securities with a single-period horizon. There are n + 1 securities (indexed by j) in the economy. The asset labelled j = 0 is riskless, and the other n securities (j = 1, ..., n) are risky. Take the total number of shares in each security to be fixed during the period. Define P_j as the value per share of security j at the beginning of the period in a world with taxes on income from capital. Let P_j' be the value per share of security j at the beginning of the period in a world with no taxes on income from capital. The value of security j at the end of the period in the no tax world is a random variable g_j. Finally, g_j is also the pre-tax terminal value of security j in the world with taxes. The paradigm here is of a return of g_j at the end of the period to an asset that is priced differently at the beginning of the period under different tax regimes. Denote by $\bar{g}_j$ the expected value of g_j and let $s_{jk} = \mathrm{cov}(g_j, g_k)$.

The basic model is completed with the following assumptions:

(A1) Individual preferences are characterized by von-Neumann-Morgenstern utility functions that are monotone increasing strictly concave functions of after-tax end of period wealth.

(A2) The returns on securities are distributed multivariate normal.

(A3) Individuals are price takers and have homogeneous expectations about $\bar{g}_j$ and s_{jk} for all j, k.

(A4) There are no transactions costs, no restrictions on borrowing and no restrictions on short sales of securities.

(A5) All assets are marketable.

(A6) At both the beginning of the period and the end of the period, all owners face the same marginal tax rate T and full loss offsets are available.

(A7) When trading ends at the beginning of the period each owner of asset j receives a depreciation deduction of DP_j per share. This deduction has immediate effect so that given full loss offsets it is equivalent to a cash payment of TDP_j per share from the government to each owner of asset j when trading ends. Since the proportion D of the asset cost has been deducted at the beginning of the period, only the proportion (1 - D) may be deducted in calculating gains at the end of the period. End of period gains for tax purposes are thus $g_j - (1 - D)P_j$ per share or $(1 - T)(g_j - (1 - D)P_j)$ after tax.

(A8) The price of the riskless asset in the no-tax world is one. *I.e.*, $P_0' = 1$.

(A9) Define G as the beginning-of-period net present value of tax revenues minus the cost the government incurs to cover beginning-of-period depreciation allowances. Individual i anticipates that expenditures will benefit him or her by d_iE times G where $\sum_{i=1}^{m} d_i = 1$. *I.e.*, expenditures will have an effect

equivalent to distributing d_iE times G lump-sum to i at the beginning of the period.

Assumptions (A1) and (A2) imply that preferences in a model of optimal portfolio choice can be described by a utility function over the mean and variance of after-tax wealth. See Baron (1977). Assumptions (A1) through (A5) taken together are a standard set of assumptions for the CAPM with a riskless asset. See Litzenberger and Ramaswamy (1979). Assumptions (A6) and (A7) imply that there is a single-level tax (*i.e.*, no tax at an entity level added to individual taxes), no special rate on capital gains, no realization requirement as a trigger to taxation, and no progressivity in rates.[1]

The variable D captures the timing of depreciation deductions. When D = 1, the asset is "expensed." The owner receives a full deduction of asset cost in advance of any decline in value. When D = 0, the owner must wait until the end of the period to deduct any portion of the asset cost. If the asset is expected to decline in value over the period so that the before-tax portion of g_j that represents residual asset value is less than P_j, then the standard analysis would suggest that some D such that $0 < D < 1$ would result in a time 0 depreciation deduction equal to the present value of the expected decline in asset value. Of course, it is this method of setting depreciation allowances that Bulow and Summers (1984) suggest results in inadequate deductions. Nonetheless, various values of D represent various alternatives for the timing of depreciation allowances.

The CAPM gives only the relative prices of the n + 1 securities under each tax regime. Since we want to see whether and how the effect of taxes on the riskless rate relates to the effect of taxes on risk premia, the model is solved with a parameter b specifying the relation between riskless rates. The riskless asset returns g_0 before taxes (regardless of tax regime) at the end of the period. In the no tax world the riskless asset price is set at one by assumption (A8) so that the return per share of riskless asset is $r' = g_0 - 1$. Let the riskless asset be priced at

$$P_0 = \frac{g_0 b(1 - T)}{g_0 - 1 + b(1 - T)} \tag{6}$$

in a tax world with tax rate T so that in that world the pre-tax riskless rate of return is

$$r = \frac{r'}{b(1 - T)}. \tag{7}$$

In Bulow and Summers (1984) and Gordon (1981) b = 1 is the assumption. I.e., the pre-tax riskless rate of return in the tax world rises just enough from the no-tax riskless rate of return so that, $r(1 - T)$, the after-tax riskless rate of return when D = 0, equals the no-tax riskless rate of return. Summers (1981) justifies this assumption principally by the argument that if the U.S. corporate capital stock is the focus, this capital stock is small compared to the world capital stock available for investment so that the after-tax riskless rate of return can be assumed to be given exogenously.

Assumption (A9) treats government expenditures through the summary statistic, G. G represents net government revenues that will be spent during the period or distributed at the end of the period. If an individual anticipates benefiting from such expenditures or distributions, it is as if the individual is given an additional security representing the right to those benefits at the beginning of the period. Government revenues are simply a slice of asset returns. Since there are no short sale restrictions, each individual can realize the wealth increment inherent in his or her expenditure benefits by taking the appropriate short positions. Thus, it is reasonable to summarize government expenditures by adding a portion of the net present value of tax revenues to each individual's endowment at the beginning of the period.

Assumption (A9) specifies a set of homogeneous beliefs about the efficacy of government expenditures through E and about the distributional impact of those expenditures through the d_i. Thus, $\sum_{i=1}^{m} d_i = 1$ so that the d_i indicate how the expenditure benefits are divided. E can be thought of as the anticipated constant marginal effectiveness of government expenditure. When E = 1, individuals view government expenditure as equivalent to lump-sum distribution of the full amount of tax revenues with individual i receiving the proportion d_i. When E = 0, individuals either myopically ignore government expenditures or value the expenditures at zero.

Intuition might suggest that the marginal effectiveness of expenditures would decline with the total amount of expenditure.

Furthermore, a rational welfare-maximizing government would spend only to the point where E = 1. Nonetheless, it greatly simplifies the model to take E to be constant, and the results are still quite rich. In addition, $E \neq 1$ is realistic under certain views of government functioning. For example, the government may "overspend" in response to politicians who are maximizing the probability of reelection as described in Fiorina (1977). In that case $E < 1$. Conversely, if the government buys public goods with its revenues, there may be substantial unsatisfied demand for such goods due to the inability of the political process to charge costs to the true beneficiaries. $E > 1$ will be the result.

To complete the model, it is necessary to specify government financial behavior. The government receives revenues at the end of the period but must pay the after-tax value of depreciation allowances at the beginning of the period. There is an additional problem. A traditional approach, adopted here, is to take the net expenditure amount G to be fixed and set in advance under each tax regime. But revenues are risky and may turn out to be higher or lower than expected if total gains at the end of the period are higher or lower than expected. In fact, under assumption (A2) about the returns on securities, end of period losses might exceed end of period gains so that revenues may be negative at that time.

The problem of paying for the after-tax value of depreciation allowances at the beginning of the period is easy to handle within the model. The government can borrow or sell short against its end-of-

period tax revenues to raise the money for the payments. However, this will change the securities comprising the private market at the beginning of the period, and market-clearing prices may depend on the pattern of government borrowing and short sales. The possibility of a "shortfall" or "surplus" in revenues at the end of the period is harder. In a one period model it is inappropriate to assume that this will be handled by running a government deficit or surplus at the end of the period. The effects of such a policy extend into later periods. Another solution is to assume that the government will levy additional taxes at the end of the period if there is a revenue "shortfall" and will distribute any excess revenues if there is a "surplus." The result will be to force certain individuals to own securities representing the risk of a shortfall or surplus. Unless they are myopic, these individuals will adjust their portfolios at the beginning of the period in anticipation. Thus, the additional taxes and distributions required at the end of the period to balance the government's budget may affect security prices at the beginning of the period.

Fortunately, there is an easy way to model both the requirement that the government pay the after-tax value of beginning-of-period depreciation allowances and the need to take into account the additional end-of-period taxes and distributions required to balance the budget. The end-of-period taxes and distributions make government revenues riskless. The government transfers the downside and upside to private parties. Exactly the same effect would result if the government engaged in a pattern of short sales at the beginning of the period that eliminated the riskiness from its "portfolio" consisting of revenues and the short positions. Private traders would perceive exactly the same increase in the supply of securities to the market from those short sales as they would from the government announcing at the beginning of the period the pattern of distributions and taxes it will engage in at the end of the period to balance the budget. Given no restrictions on individual short sales, it does not matter whether the government explicitly engages in short sales at the beginning of the period or "creates" the same set of securities by announcing end-of-period taxes and distributions. Any distributional differences between the two schemes can be eliminated by imposing beginning-of-period transfers between individuals that create the same initial wealth distribution as the announcement of the taxes and distributions would.

More formally suppose that there are x_j^o total shares of security j. The government buys a portfolio during trading at the beginning of the period. This portfolio is represented by $(\alpha_0, \alpha_1, \ldots, \alpha_n)$, where α_j is the proportion of total shares x_j^o sold short by the government. Thus, private market traders have $(1 + \alpha_j)x_j^o$ shares of security j available to them during the trading at the beginning of the period. Prices in private markets must adjust such that there is no excess demand or supply with that total number of shares rather than the lower number x_j^o. The government chooses its

portfolio such that

(1) both upside and downside tax revenue risk is eliminated by transfering it to private parties;

and (2) short sales and borrowing at the beginning of the period completely cover payouts required on depreciation allowances.

It is straightforward to specify both the portfolio required to satisy these two conditions and G, the net present value of government expenditures other than "expenditure" to cover beginning-of-period depreciation allowances. At the end of the period the government will receive revenues in the amount

$$Q_t = T\sum_{j=0}^{n}(g_j - (1 - D)P_j)(1 + \alpha_j)x_j^o. \tag{8}$$

The income that is taxed per share is $g_j - (1 - D)P_j$ instead of $g_j - P_j$ since the taxpayer's basis in the shares is reduced by DP_j, the depreciation allowance at the beginning of the period. The amount $(1 + \alpha_j)x_j^o$ is the total shares of security j held by the private sector. At the end of the period, the government must pay

$$Q_s = \sum_{j=0}^{n} g_j\alpha_j x_j^o \tag{9}$$

on its short positions so that net revenues will be

$$Q = Q_t - Q_s = \sum_{j=1}^{n} g_j(T(1 + \alpha_j) - \alpha_j)x_j^o + g_0(T(1 + \alpha_0) - \alpha_0)x_0^o - T(1 - D)\sum_{j=0}^{n} P_j(1 + \alpha_j)x_j^o. \tag{10}$$

Since returns are normally distributed, the possibility of Q being risky can be ruled out only if the coefficients of g_j in the first term of (10) are all zero.[2] This requires that

$$\alpha_j = \frac{T}{1 - T} \quad \text{for } j = 1, 2, \ldots, n. \tag{11}$$

Now suppose that the government sets α_0 such that payments on its short positions exactly equal tax revenues. This amounts to solving (10) for α_0 given (11) and $Q = 0$. Now using the fact that $g_0 - P_0 = rP_0$ we obtain

$$P_0x_0^o\alpha_0 = \frac{-(1 - D)\frac{T}{1 - T}\sum_{j=1}^{n} P_jx_j^o + T(r + D)P_0x_0^o}{1 + r(1 - T) - DT}. \tag{12}$$

Now the government receives at time 0 the amount

$$\sum_{j=0}^{n} \alpha_jP_jx_j^o = \frac{T(r + D)}{1 + r(1 - T) - DT}\sum_{j=0}^{n} P_jx_j^o \tag{13}$$

in revenues from short sales that represent the value of all future tax revenues. The government must pay $DT\sum_{j=0}^{n} P_jx_j^o(1 + \alpha_j)$ in after-tax depreciation benefits at the beginning of the period. Subtracting

this from (13) yields G:

$$G = \frac{Tr(1 - D)}{1 + r(1 - T) - DT}\sum_{j=0}^{n} P_jx_j^o. \tag{14}$$

B. The Effect of Taxes on the Riskless Rate and Risk Premia

This section uses the model to compute the impact of income taxes on the required rate of return. The result is stated as equation (42). Several relations developed as steps to that result express the impact of taxes on the riskless rate and on risk premia. These relations are of interest independent of equation (42), and section C discusses them.

If v_i is the random after-tax return of investor i's portfolio and S_i^2 the variance of that return, then the following equations describe $\bar{v}_i$ (the expected value of v_i) and S_i^2:

$$\bar{v}_i = \sum_{j=1}^{n} [\bar{g}_j(1 - T) + (1 - D)TP_j]x_{ij} + [r(1 - T) + (1 - DT)]P_0x_{i0} \tag{15}$$

$$S_i^2 = \sum_{j=1}^{n}\sum_{k=1}^{n} s_{jk}x_{ij}x_{ik}(1 - T)^2 \tag{16}$$

where individual i holds x_{ij} shares of security j at the end of the period, and there are x_j^o shares in total of security j.[3] The individual investor maximizes $U_i(\bar{v}_i, S_i^2)$ subject to a budget constraint:

$$\sum_{j=1}^{n} P_j(x_{ij}(1 - DT) - x_{ij}^o) + P_0(x_{i0}(1 - DT) - x_{i0}^o) - Ed_iG = 0 \tag{17}$$

where x_{ij}^o denotes i's initial holdings in shares of security j.

The ultimate goal is to compare security prices (and thus required rates of return) in the tax world to those in the no tax world. A first step is to allow portfolio adjustment consistent with individual maximization. The resulting equations will then be subject to market clearing conditions.

Form the Lagrangean:

$$L_i = U_i(\bar{v}_i, S_i^2) - \lambda[\sum_{j=1}^{n} P_j(x_{ij}(1 - DT) - x_{ij}^o) + P_0(x_{i0}(1 - DT) - x_{i0}^o) - Ed_iG]. \tag{18}$$

The first order conditions are the following:

$$\frac{\partial L_i}{\partial x_{i0}} = U_{i1} \frac{\partial \bar{v}_i}{\partial x_{i0}} - \lambda P_0(1 - DT) = 0 \tag{19}$$

and

$$\frac{\partial L_i}{\partial x_{ij}} = U_{i1} \frac{\partial \bar{v}_i}{\partial x_{ij}} + U_{i2} \frac{\partial S_i^2}{\partial x_{ij}} - \lambda P_j(1 - DT) = 0 \quad \text{for } j \neq 0 \tag{20}$$

where U_{in} is the first partial derivative with respect to the nth argument of U_i.[4] Furthermore, it is true that:

$$\frac{\partial \bar{v}_i}{\partial x_{ij}} = \begin{cases} [r(1 - T) + (1 - DT)]P_0 & \text{for } j = 0 \\ \bar{g}_j(1 - T) + (1 - D)TP_j & \text{for } j \neq 0 \end{cases} \tag{21}$$

$$\frac{\partial S_i^2}{\partial x_{ij}} = \begin{cases} 0 & \text{for } j = 0 \\ 2\sum_{i=1}^{n} s_{jk}x_{ik}(1 - T)^2 & \text{for } j \neq 0. \end{cases} \tag{22}$$

Substituting (21) and (22) into (19) and (20) and simplifying yields for each $j \geq 1$:

$$\sum_{k=1}^{n} s_{jk}x_{ik} = \frac{w_i}{(1 - T)}(\bar{g}_j - (1 + r)P_j) \tag{23}$$

where $w_i = -U_{i1}/2U_{i2}$ is proportional to the marginal rate of substitution of investor i between portfolio expected return and portfolio variance. Combined with the budget constraint (17), for a given i the n equations in (23) yield n + 1 equations in the n + 1 unknowns x_{ij} (j = 1, ..., n). Under assumption (A3) of price-taking behavior by investor i, these equations determine the x_{ij} for all j.

Market clearing requires the following:

$$\sum_{i=1}^{m} x_{ij} = (1 + \alpha_j)x_j^o \tag{24}$$

where x_j^o is the total number of shares of security j. Summing both sides of (23) over the m investors yields an aggregate equation:

$$\sum_{k=1}^{n} s_{jk}x_k^o = \sum_{i=1}^{m} w_i(\bar{g}_j - (1 + r)P_j). \tag{25}$$

since $1 + \alpha_j = 1/(1 - T)$ for $j \geq 1$.

Make the following definitions. $\tilde{R}_j = (g_j - P_j)/P_j$ is the rate of return on security j. $W = \sum_{k=1}^{n} P_k x_k^o$ is the total market value of all securities at the beginning of the period and $P_k x_k^o/W$ is the share of security k by value in that total at that time. Finally let $(1/\sum_{i=1}^{m} w_i) = C$. Using that fact that $\text{cov}(\tilde{R}_j, \tilde{R}_k) = s_{jk}/(P_j P_k)$ it is straightforward to transform (25) into:

$$CW\ \mathrm{cov}(\tilde{R}_j,\ \tilde{R}_m) = \bar{R}_j - r \tag{26}$$

where $\tilde{R}_m$ is the rate of return on the market portfolio with initial value W and $\bar{R}_j = (\bar{g}_j - P_j)/P_j$ is the expected rate of return on security j. Setting $\tilde{R}_j = \tilde{R}_m$ in equation (26) yields $\bar{R}_m - r = CW\ \mathrm{var}(\tilde{R}_m)$ so that (26) becomes:

$$\bar{R}_j - r = \beta_j(\bar{R}_m - r) \tag{27}$$

where $\beta_j = \mathrm{cov}(\tilde{R}_j,\ \tilde{R}_m)/\mathrm{var}(\tilde{R}_m)$. This relation is the classic relation between the expected return on a risky security, market expected return and riskless return in a world with no taxes under assumptions (A1) - (A5). Here the relation holds between pre-tax expected returns in the tax world.

This result can be transformed easily into a relation between after-tax expected returns. Where the superscript t denotes after tax, the following relations hold for D = 0:

$$\begin{aligned} \bar{R}_j^t &= (1 - T)\bar{R}_j \\ r^t &= (1 - T)r \\ \beta_j^t &= \beta_j. \end{aligned} \tag{28}$$

As a result, an after-tax CAPM equation follows from (27) by multiplying both sides by (1 - T):

$$\bar{R}_j^t - r^t = \beta_j(\bar{R}_m^t - r^t). \tag{29}$$

In the case where D = 0, for each security j the after-tax risk premium is just (1 - T) times the before-tax risk premium.

As has been noted, Bulow and Summers (1984) and Gordon (1981) reduce the basic pre-tax risk premium by (1 - T) to arrive at an after-tax risk premium for the tax world. Under the CAPM in the simple framework here use of the (1 - T) factor is correct if one is relating the pre-tax and after-tax risk premia in the tax world and D = 0.[5] But even leaving aside the cases where $D \neq 0$, assessing the effect of taxation on corporate investment, ostensibly the goal of the two articles, would seem to require comparing the no tax world to the after-tax results in the tax world. One would expect that pre-tax prices in the tax world are affected by the anticipation of the taxes. _I.e._, there is no reason to believe that $P_j' = P_j$ for any given security j. If $P_j' \neq P_j$, then where the rate of return in the no tax world for asset j is $R_j' = (g_j - P_j')/P_j'$ it will be true that $\bar{R}_j' \neq \bar{R}_j$. Consequently, even if $r' = r(1 - T)$, $\bar{R}_j - r'$ will not be the risk premium in the no-tax world that is to be compared with $\bar{R}_j^t - r^t$, the after-tax risk premium in the tax world.

As discussed in section II, we want to compare the required rate of return in the no tax world to the required pre-tax rate of return in the tax world. This relation is developed as equation (42). To describe that relation it is necessary to relate no tax world risk premia to pre-tax risk premia in the tax world. As a side product, the connection between no tax world risk premia and after-tax risk premia in the tax world will become clear.

Deriving the relations between risk premia is straightforward given that the term $\sum_{k=1}^{n} s_{jk}x_k^o$ in (25) is the same in both the tax and the no tax world. Letting w_i' denote $-U_{i1}/2U_{i2}$ in the no tax world and recalling that r' is the no tax world riskless rate while P_j' is the no tax world price of asset j, (25) implies

$$(\bar{g}_j - (1 + r')P_j')\frac{\sum_{i=1}^{m} w_i'}{\sum_{i=1}^{m} w_i} = (\bar{g}_j - (1 + r)P_j). \tag{30}$$

Define F as

$$F = \frac{\sum_{i=1}^{m} w_i'}{\sum_{i=1}^{m} w_i}. \tag{31}$$

Now (30) can be rewritten as

$$a_j'\frac{P_j'}{P_j}F = a_j \tag{32}$$

where

$$a_j' = \frac{\bar{g}_j}{P_j'} - (1 + r') \tag{33}$$

is the no tax world risk premium on asset j while

$$a_j = \frac{\bar{g}_j}{P_j} - (1 + r) \tag{34}$$

is the pre-tax risk premium on asset j in the tax world.

In the tax world the pre-tax riskless rate is defined as $r = \frac{g_0 - P_0}{P_0}$ so that $g_0 = P_0(r + 1)$. The after-tax riskless rate is

$$r^t = \frac{(g_0 - P_0)(1 - T)}{(1 - DT)P_0} = \frac{r\ (1 - T)}{(1 - DT)} \tag{35}$$

since the after-tax cost of investing in the riskless asset is $(1 - DT)P_0$.[6] Now the overall after-tax rate of return on asset j in the tax world is

$$r^t + a_j^t = \frac{(\bar{g}_j - P_j)(1 - T)}{(1 - DT)P_j} \tag{36}$$

where a_j^t is the after-tax risk premium in the tax world. Equations (35) and (36) yield

$$a_j^t = \frac{(1 - T)}{(1 - DT)}[\bar{g}_j - (1 + r)P_j]/P_j \tag{37}$$

so that from (34)

$$a_j^t = \frac{(1 - T)}{1 - DT}a_j \tag{38}$$

as established previously for the special case D = 0.

From (30), (31), and (32) it follows that

$$\frac{\bar{g}_j}{P_j} = \frac{(1 + r')a_j}{(F - 1)a_j'}\ . \tag{39}$$

Now using (34) and $r = r'/b(1 - T)$ from (7)

$$a_j = \frac{Fa_j'[1 + r'/b(1 - T)]}{1 + r' + a_j'(1 - F)} \tag{40}$$

so that from (38)

$$a_j^t = \frac{Fa_j'(1 - T + r'/b)}{[1 + r' + a_j'(1 - F)](1 - DT)}. \tag{41}$$

Now we can calculate $\Delta_j = (r + a_j) - (r' + a_j')$, the difference between the required rate of return pre-tax in the tax world and the required rate of return in the no tax world. As discussed in part II, the idea is that if this is small, there is not much impact of taxes on investment. Using (7) and (41)

$$\Delta_j \equiv (r + a_j) - (r' + a_j') =$$

$$\frac{r'[1 - b(1 - T)]}{b(1 - T)} + \frac{[F - 1 + \frac{r'[F - b(1 - T)]}{b(1 - T)}]a_j' - a_j'^2(1 - F)}{1 + r' + a_j'(1 - F)}. \tag{42}$$

This equation is the central result of this part of the paper since it expresses how the required rate of return in the no tax world differs from the required pre-tax rate of return in the tax world.

C. Discussion of the Results

The difference, Δ_j, expressed in (42) depends on the parameters b and F as well as on the no tax world riskless rate and risk premium. As indicated in equation (7) the parameter b connects the no tax world riskless rate and the pre-tax riskless rate in the tax world. Before discussing the implications of equation (42), it is important to specify the meaning of F.

The w_i' and w_i that comprise F in (31) indicate the degree of individual risk aversion in an inverse way. Twice w_i or twice w_i' is simply the ratio of the marginal utility of a unit of expected return divided by the absolute value of the marginal disutility of a unit of variance. A higher w_i or a higher w_i' thus indicates less risk aversion in the sense that at the margin the person values increases in expected return more highly relative to a given decrease in variance. In fact, the w_i and w_i' correspond inversely to Arrow-Pratt absolute risk aversion: when w_i or w_i' increase (decrease) Arrow-Pratt absolute risk aversion decreases (increases).[7]

Defining individual "risk aversion" as $U_{i1}/2U_{i2}$, F measures how average risk aversion changes between the no-tax world and the tax world.[8] The larger F is, the higher average risk aversion is in the tax world compared to the no-tax world. If F is greater than (less than) one, average risk aversion is higher (lower) in the tax world than in the no tax world. Since the w_i and w_i' correspond inversely to Arrow-Pratt absolute risk aversion, they may depend on the individual's wealth. Under the standard assumption of decreasing absolute risk aversion, the w_i and w_i' increase with wealth.

For the time being consider the case where F = 1 so that taxes and expenditures have not changed average risk aversion. After considering this case, it will be easier to discuss both the impact on the results of $F \neq 1$ and the wealth effects that might cause F to deviate from one. Let us start by examining the case F = 1 with the assumption b = 1 employed by both Gordon (1981) and Bulow and Summers (1984). Now from (42)

$$\Delta_j = \frac{r'T}{1 - T} + \frac{r'T}{(1 - T)(1 + r')}a_j'. \tag{43}$$

From (1) and (3) it can be seen that the literature result for Δ_j is $r'T/(1 - T)$. Careful application of the CAPM produces a second term that depends on the risk characteristics of the investment. In particular, for $r' > 0$ and $0 < T < 1$ investments that are riskier in the no tax world suffer a larger increase in the required rate of return due to taxes. Nonetheless, the result in (43) is similar to Gordon's result in the sense that if r (and thus r') is assumed to be very small then the right hand side is close to zero. In fact, if we had taken $b = 1/(1 - T)$ when F = 1, then the right hand side would be exactly zero. Taxes would have no effect on investment.

This last observation generalizes into the following proposition:

<u>Proposition 1</u>: When F = 1, then r = r' is a necessary and sufficient condition to insure that taxes and expenditures do not affect investment incentives for investments of any risk class.

<u>Proof</u>: Taking r = r' is equivalent to taking $b = 1/(1- T)$. When b has this value and F = 1 then $\Delta_j = 0$ regardless of a_j'. Thus, r = r' is a sufficient condition.

Suppose $r \neq r'$. Then from (7) $r' \neq 0$. From (42) with $r' \neq 0$ and $b \neq 1/(1 - T)$, it is easy to choose an a_j' such that $\Delta_j \neq 0$.

Q.E.D.

Thus, in the case F = 1 taxes and expenditures will have no allocative effect on investment if and only if the taxes and expenditures result in a pre-tax riskless rate that is the same as the no tax riskless rate. Finally, note that r = 0 implies r' = 0 from (7) so that r = r' in that case also.

Proposition 1 has an intuitive interpretation. When F = 1, it follows from (6), (7) and (30) that

$$\frac{P_j}{P_j'} = \frac{1 + r'}{1 + r} \tag{44}$$

for all j including j = 0. This expression indicates that when F = 1, r = r' is a necessary and sufficient condition for $P_j' = P_j$. *I.e.*, r = r' ensures that security prices that correspond to any type of asset do not change due to taxes. As a result, if the cost of the physical asset remains the same, the incentive to invest remains the same. By buying physical assets and selling stock representing ownership of the assets, one will obtain the same profit or loss as would have been the case without taxes. Put in the parlance of general equilibrium investment models such as the one in Summers (1981), the marginal "q" (stock market value divided by replacement cost) for each asset is unchanged by taxes. As a result, investment incentives are unchanged.

All the results so far are independent of the value of D. Depreciation timing can only affect the allocative impact of taxes on investment through b or F. But it turns out that D does affect the relation between after-tax and no tax risk premia even when F = 1,

$r' = r$ and there is no allocative effect. Aside from being interesting in itself, explaining the coexistence of these results is a building block for understanding the general results about the impact of depreciation timing that will follow.

With $F = 1$ and $r' = r$, we obtain from (41)

$$a_j^t = \frac{a_j'(1 - T)}{1 - DT}. \tag{45}$$

Thus, the idea in Gordon (1981) that the after-tax risk premia will be $(1 - T)$ times the no tax world risk premia holds only when $D = 0$. I.e., the government absorbs the proportion T of the risk per after-tax dollar invested only under a specific timing regime for depreciation deductions: the deductions are not taken until all the profits are realized and the asset is retired. In contrast, when $D = 1$ and the asset is expensed, $a_j^t = a_j$ so that taxes do not result in any reduction in the risk premium per after-tax dollar of investment.

When security prices remain unchanged from their no tax values (as they do when $F = 1$ and $r = r'$) and $D = 0$, it is easy to see why the government "absorbs" a proportion T of the no tax world risk premium per after-tax dollar invested. The government is simply taking the proportion T of the profits and losses per after-tax dollar invested thus cutting both return and risk by that proportion. When $D = 1$, however, the government in effect purchases a proportion T of the outstanding shares. Thus, it reduces price per share by $(1 - T)$ but also takes $(1 - T)$ of all returns. The remaining investment has

the same risk and return characteristics per dollar of after-tax investment as in the no tax world. Thus, risk premia per after-tax dollar invested are unaffected by taxes. This reflects the idea of Graetz (1981) and others that expensing with full loss offsets effectively makes the government a partner in each investor's operations.

When $r' = r$ and $F = 1$ investment incentives remain unaffected by taxes regardless of the value of D, and yet D determines the proportion of the risk premium that the government absorbs per after-tax dollar. This apparent paradox is easy to resolve. When $D > 0$, the government still absorbs the proportion T of the risk per pre-tax dollar of investment. This can be verified by multiplying equations (35), (36) and (37) by $(1 - DT)$. This changes r^t and a_j^t into the riskless rate and risk premium per pre-tax dollar of investment respectively. Then (45) becomes

$$a_j^t = a_j'(1 - T). \tag{46}$$

Thus, the government absorbs the proportion T of the risk premium per pre-tax dollar of investment regardless of the value of D. When $D = 1$ it does so by purchasing the proportion T of each investor's portfolio. As a result, the net present value of tax revenues is zero, as can be seen from (14). The government is simply purchasing securities expected to yield the market rate of return. The purchase price is equal to the expected value of ownership so the net present value of the transaction is zero at the time of purchase.[9] When

$D = 0$, given that the government adjusts its portfolio to eliminate revenue risk, it will earn a return of Tr times the total value of all securities. This is because the government has appropriated the proportion T of all end-of-period net revenues and has sold its right to the risky portion of those revenues at market value. End-of-period government revenues must be discounted by $1 + r(1 - T)$ to reflect the value to the public of those revenues given that the public faces an after-tax riskless discount rate of $r(1 - T)$. The result is exactly equation (14) with $D = 0$. The present value of government revenues is positive (assuming $r > 0$) since the government has appropriated a proportion T of market returns without having to pay for the securities generating those returns.

Now the intuitive picture can be completed for the case $r' = r$ and $F = 1$. Whether $D = 0$ or $D = 1$, the government is taking a proportion T of all security returns. Since security prices do not change, the reduction in risk per pre-tax dollar of investment exactly "cancels" the reduction in return. The incentive to invest each pre-tax dollar is thus unchanged from the no tax world. When $D = 1$, however, the government is paying for the returns it takes at the market price. This leaves the net present value of tax revenues at zero. When $D = 0$, the government is not paying anything for the returns it takes, and the net present value of the revenues is positive.

The results so far depend on assuming $r = r'$ and $F = 1$. Assuming $r = r'$ has some empirical justification. Since the empirical

evidence suggests that $r \simeq 0$ and since from (7) r' is proportional to r, it is not unreasonable to assume that both r and r' are zero. Consider, however, the case where $r \neq 0$ and $r' \neq r$. Then D will have an impact on investment incentives through its impact on b. This is best illustrated with an example. Suppose that the taxed investment sector is small compared to the total set of available investments. Then it is reasonable to assume that $r^t = r'$. I.e., the after-tax riskless rate will be equal to the no tax world riskless rate, and both r^t and r' will be equal to the riskless rate prevailing outside of the taxed investment sector. As the argument in Summers (1981) suggests, this is a nontrivial case. In studying corporate taxes in that paper Summers takes the required after-tax rate of return on U.S. corporate investment to be fixed given the availability of a large pool of noncorporate investments and foreign investments.

Using (7) and (35) $r^t = r'$ implies

$$b = \frac{1}{1 - DT}. \tag{47}$$

From this expression and (7) it is clear that r moves inversely with D. From (30) it follows that

$$\left.\frac{\delta P_j}{\delta D}\right|_{F = 1,\ r^t = r'} > 0. \tag{48}$$

Since $b = 1/(1 - T)$, $r^t = r' = r$ and $P_j = P_j'$ when $D = 1$ and $F = 1$, $P_j < P_j'$ when $D < 1$ and $|P_j - P_j'|$ increases as D falls from one. Thus, more rapid depreciation allowances (a higher D) stimulate investment by lowering the pre-tax riskless rate. This raises security prices and thus marginal q.

Using equation (44) the effects of changes in D can be generalized beyond the case $r^t = r'$ to yield the following proposition.

Proposition 2. When F = 1, an across-the-board change in D decreases (increases) security prices if and only if it increases (decreases) r.

Assuming $r^t = r$ simply allows us to specify through equation (47) exactly the impact of D on r.

We have seen that the conventionally expected effects of the timing of depreciation allowances on investment incentives will occur if the pre-tax riskless rate falls when the allowances are accelerated. Other effects of that timing can occur if the timing affects the value of F. It is convenient to explore the determinants and the effect of F generally rather than focusing only on how D may affect investment incentives through F.

The possibility that $F \neq 1$ raises several questions. First, how sensitive are the results to deviations from F = 1? Second, if there is some sensitivity, how would the condition F = 1 emerge from individual preferences and government behavior and is that condition plausible? Finally, if the deviation of F from one affects the results, how would that deviation come about?

To answer the first question, consider the effect of F on $\Delta_j \equiv (r + a_j) - (r' + a_j')$, the gap between the required pre-tax rate of return and the required no tax world rate of return:

$$\frac{\partial \Delta_j}{\partial F} = \frac{(1 + r' + a_j')a_j'b(1 - T) + (a_j')^2 r' + r'a_j' + (r')^2 a_j'}{b(1 - T)[1 + r' + a_j'(1 - F)]^2}. \tag{49}$$

For the empirically plausible case where $r' \simeq 0$, this simplifies to

$$\left.\frac{\partial \Delta_j}{\partial F}\right|_{r' = 0} \simeq \frac{(1 + a_j')a_j'}{[1 + a_j'(1 - F)]^2}. \tag{50}$$

In the range $0 \lesssim F \lesssim 2$ this derivative is approximately a_j' for $0 < a_j' \ll 1$. Since $\Delta_j = 0$ when $r' = 0$ and $F = 1$, we can take

$$\Delta_j \simeq (F - 1)a_j' \tag{51}$$

for $r' = 0$, $0 < a_j' \ll 1$ and $0 \lesssim F \lesssim 2$. This sensitivity to F is high enough to make exploration of the causes and potential extent of deviations of F from 1 worth considering.

Taxes and expenditures may impact on F through two kinds of wealth effects. First, security prices may change from the no tax world to the tax world thereby affecting the value of individuals' endowments. Second, as a result of taxes, the government in effect has removed wealth in amount G from the economy at the beginning of the period. This is the net present value of government expenditures other than expenditure to cover beginning-of-period depreciation allowances. The government spends G to the benefit of individuals, and this is represented by individual i receiving a lump sum amount Ed_iG at the beginning of the period. If E, the effectiveness of government expenditure, is greater than (less than) one, then in aggregate wealth increases (decreases) at the beginning of the period.

Now suppose that taxes cause security prices to increase or that government expenditures are more effective than private expenditures. The increase in wealth would tend to decrease average risk aversion so that $F < 1$ results. For simplicity assume that $r = r'$. Then from (6) and (7) $P_0 = 1$, and the riskless security has the same price in the tax world as in the no tax world. From equation (30), it is clear that $P_j > P_j'$ and that $P_j - P_j'$ is larger for securities carrying larger risk premiums (in either the tax or the no tax world). This makes sense. The decrease in average risk aversion means that risky assets must increase in price for the market to clear and the increase must be greater the riskier the asset. As a result, the tendency of the absorption of risk by the government to reduce required pre-tax return is augmented by the reduction in risk premia due to the reduction in average risk aversion. In the extreme case, $F = 0$ so that the market prices assets as if the average person was risk neutral. Taking for simplicity $r = r' = 0$, equation (42) then yields $a_j - a_j' = -a_j'$. I.e., the tax world risk premia a_j (and also a_j^t) are zero for all assets since individuals have become risk neutral.

Is it reasonable to assume $F = 1$? This assumption implies either that individual risk aversion is unresponsive to wealth or that changes in risk aversion tend to cancel out on average. The former possibility is ruled out under the standard assumption of decreasing absolute risk aversion.[10] The latter possibility is not implausible when $E = 1$ and r is close to r'. By equation (44) if $r = r'$ and

$F = 1$, security prices do not change so that no wealth effects arise from the revaluation of endowments. When $E = 1$, the present value of net tax payments is simply redistributed with none lost. This corresponds to the marginal expenditures of a rational government: the value of the expenditures (less all costs including administrative costs) is exactly the value of leaving the money in the private sector. This redistribution may change the distribution of wealth in society, but the gains of winners will exactly offset the losses of losers. If everyone has risk aversion that decreases with wealth, $w_i > w_i'$ will hold for winners, but $w_i < w_i'$ will hold for losers. Therefore $\sum_{i=1}^{m} w_i$ might end up being about the same as $\sum_{i=1}^{m} w_i'$.

Only if the redistribution is systematically in favor of those with risk aversion particularly responsive or particularly unresponsive to wealth will deviations of F from 1 assume potential importance. There is no obvious assumption to make about how the responsiveness of w_i to wealth varies with wealth or with the initial value of w_i.[11] Under the circumstances ($E = 1$, $r' = r$), it may not be unreasonable to assume that $F = 1$. Another way to reach this result is the assumption that r and r' are close to zero. Then the net present value of government revenues is small and any wealth effects due to anticipated government expenditures can be ignored independent of the value of E.

In summary, it is clear that the conclusion in Gordon (1981) that income taxes have negligible effects on investment holds up in a CAPM framework in at least one empirically plausible environment.

That is where both r and r′ are close to zero. In that situation there is no free lunch since by equation (14) the net present value of government receipts will be close to zero. Where r and r′ are significantly greater than zero so that the net present value of government revenues is nontrivial, more complicated conditions are required to reach Gordon's result. For example, $r' = r$ and $E = 1$ is an environment where Gordon's result is believable. But it is unclear why $r' = r$ would hold. The argument in Summers (1981), for example, would suggest that $r^t = r'$ is a reasonable assumption if the after-tax riskless rate in the taxed sector must equal the riskless rate in a much larger untaxed or foreign sector. But from (35) $r^t = r$ only when $D = 1$. More generally (when $r' \neq r$ or $E \neq 1$), wealth effects arising through either the revaluation of endowments due to taxes or the pattern of expenditure may cause investment incentives to change in either direction. At the same time, the net present value of tax revenues will depend solely on the pre-tax riskless rate, the tax rate and the timing of depreciation deductions.

There remains the task of determining whether these results would be different if a distinction were drawn between capital risk and income risk as in Bulow and Summers (1984). That is the task of the next part.

IV. CAPITAL RISK VERSUS INCOME RISK: THE CRUCIAL ROLE OF THE TAX TREATMENT OF ASSET SALES

When a capital asset is purchased, there are two main possibilities for its use. First, the asset may be retained until it is worthless at which point it will be discarded. This possibility includes the case where the owner leases the asset for use by others during part or all of its life. Second, it may at some point become more profitable to sell the asset. This possibility includes sale of the asset for scrap when no one can continue to use it profitably.

In advance, the owner may not know which case will prevail. The discounted present value of the asset at the time of investment will include the possibility that the asset will be sold at certain times in the future. Rather than explicitly modelling the decision to sell or retain the asset, we will look at the consequences of sale and retention separately. This approach suffices to describe the circumstances under which the effects that Bulow and Summers describe will take place.

We will proceed by considering a simple paradigmatic asset investment in a discrete time framework. For convenience, any sale of the asset will take place at "time 1" exactly one period after purchase at "time 0." The asset will have the following characteristics:

X: time 0 cost of the asset;

Y: total revenues net of all costs except depreciation -- all of those revenues are assumed to be received at time 1;

Z: sale value of the asset at time 1.

The expected values of Y and Z are denoted Y^e and Z^e respectively.

A tax at a single rate T will be levied on both operating income and on gains and losses from sales. Thus, there will be no special capital gains rate on asset sales versus the ordinary income rate on operating income. Under current U.S. tax law, this is a reasonable assumption for assets that decline substantially in value with use.[12] As in section III, the tax will be a single-level tax and the tax rate is independent of the taxpayer's income. This tax can be thought of as the corporate tax and appropriately so, since much of the depreciable capital stock is held by corporations. Another assumption shared with section III is full loss offsets. Thus, any deductions can be used immediately at a value of T per dollar of deduction. This avoids the complexities modelled in Auerbach (1983) of assessing the impact of the possible postponement of the benefit of deductions. Finally, following Bulow and Summers (1984) inflation is ignored.

The following assumptions and notation for discount rates shall be used:

r': riskless rate in the no tax world and after-tax riskless rate in the tax world;

a': risk premium for the revenue stream Y or any portion of it in the tax and no tax worlds;

c': risk premium for the sale value Z or any portion of it in the tax and no tax worlds.

Equality of the no tax world riskless rate and the after-tax riskless rate in the tax world simply follows the assumptions in Gordon (1981) and Bulow and Summers (1984). In the framework of the previous section, taking the discount rates a' and c' to be invariant to the tax structure is equivalent to assuming F = 1. I.e., taxes do not affect average risk aversion so that market risk premia for any given risky stream remain the same.

There are three situations to consider:

A: the asset is retained until it is worthless at time 1;

B: the asset is sold at time 1 before it is worthless and the sale results in a tax on gain or in a deductible loss;

C: the asset is sold before it is worthless and the sale has no tax consequences.

In each situation we want to compare the required pre-tax expected rate of return to the no tax world expected rate of return. In situation j let NPV_j be the net present value at time 0 of the investment in the no tax world and NPV_j^t be the after-tax net present value at time 0 of the investment in the tax world. Now, given that $Z = Z^e = 0$ in situation A, define for situation j:

$$\Delta_j = \left.\frac{Z^e + Y^e - X}{X}\right|_{NPV_j^t = 0} - \left.\frac{Z^e + Y^e - X}{X}\right|_{NPV_j = 0} \tag{53}$$

where Δ_j is the difference between the required pre-tax rate of return in the tax world and the required no tax world rate of return.

To test Bulow and Summers' claims, tax depreciation will be set at the expected decline in asset value from time 0 to time 1. This is the amount that they suggest is inadequate because it does not take into account capital risk. Furthermore, the entire depreciation deduction will be allowed at time 1. This simplifies the computations, and no change in the qualitative results would occur if some other timing regime were used.

In situation A $Z^e = Z = 0$ so that

$$NPV_A = -X + \frac{Y^e}{1 + r' + a'} \tag{54}$$

and

$$NPV_A^t = -X + \frac{Y^e(1 - T)}{1 + r' + a'} + \frac{TX}{1 + r'}. \tag{55}$$

The third term on the right hand side of (55) reflects the present value at time 0 of the depreciation deduction of X allowed at time 1. Some arithmetic yields

$$\left.\frac{Y^e - X}{X}\right|_{NPV_A = 0} = r' + a' \tag{56}$$

and

$$\left.\frac{Y^e - X}{X}\right|_{NPV_A^t = 0} = \frac{(1 + r' - T)a'}{(1 + r')(1 - T)} + \frac{r'}{1 - T} \tag{57}$$

so that

$$\Delta_j = \frac{r'T}{1 - T} + \frac{r'Ta'}{(1 + r')(1 - T)}. \tag{58}$$

Comparing equation (43), this is exactly the result we obtain in the asset pricing model in section III when F = 1 and b = 1, the assumptions of this section.[13] Assuming r' is negligible, Δ_j is negligible and taxes have very little impact on investment behavior.

In situation B

$$NPV_B = -X + \frac{Y^e}{1 + r' + a'} + \frac{Z^e}{1 + r' + c'} \tag{59}$$

and

$$NPV_B^t = -X + \frac{Y^e(1 - T)}{1 + r' + a'} + \frac{Z^e(1 - T)}{1 + r' + c'} + \frac{TX}{1 + r'}. \tag{60}$$

The fourth term on the right hand side of (60) reflects the fact that the total cost of the asset will be deducted for certain at time 1 either as a depreciation deduction or as basis in computing gain on sale of the asset. Now we have

$$\left.\frac{Z^e + Y^e - X}{X}\right|_{NPV_B = 0} = r' + a' + \frac{Z^e}{X}\,\frac{c' - a'}{1 + r' + c'} \tag{61}$$

and

$$\left.\frac{Z^e + Y^e - X}{X}\right|_{NPV_B^t = 0} = \frac{(1 + r' - T)a'}{(1 + r')(1 - T)} + \frac{r'}{(1 - T)} + \frac{Z^e}{X}\frac{c' - a'}{1 + r' + c'} \quad (62)$$

so that

$$\Delta_B = \Delta_A = \frac{r'T}{1 - T} + \frac{r'Ta'}{(1 + r')(1 - T)}. \quad (63)$$

Thus, the existence of capital risk as well as income risk does not change the outcome if sales of assets are taxed. This makes sense. Comparing NPV_B^t in (60) to NPV_A^t in (55), effectively all we have done is to split the income stream into two components with different risk premia. Distinguishing income risk and capital risk involves nothing else if income from the sale of assets is taxed at the same rate as income from operations.

NPV_C is the same as NPV_B and we have

$$NPV_C^t = -X + \frac{Y^e(1 - T)}{1 + r' + a'} + \frac{Z^e}{1 + r' + c'} + \frac{T(X - Z^e)}{1 + r'} \quad (64)$$

where the final term on the right hand side allows for a depreciation deduction of $X - Z^e$, the expected decline in asset value. Now

$$\left.\frac{Z^e + Y^e - X}{X}\right|_{NPV_C^t = 0} = \frac{(1 + r' - T)a'}{(1 + r')(1 - T)} + \frac{r'}{1 - T}$$

$$+ \frac{Z^e}{X}\left[\frac{c' - a'}{(1 - T)(1 + r' + c')} + \frac{Ta'}{(1 + r')(1 - T)}\right] \quad (65)$$

and using (61)

$$\Delta_C = \frac{r'T}{1 - T} + \frac{r'Ta'}{(1 + r')(1 - T)} + \frac{Z^e}{X}\frac{Tc'(1 + r' + a')}{(1 + r')(1 + r' + c')(1 - T)}. \quad (66)$$

This is the type of result that Bulow and Summers have in mind. For sufficiently large T and an asset expected to have a significant value at sale compared to original cost, the last term is on the order of c'. But c' is the capital risk premium that Bulow and Summers argue is large compared to r' or even a'. As a result, the required pre-tax rate of return will be significantly larger than the required no tax world rate of return, and taxes will significantly burden investment. The reason for this outcome is easy to see. Because total depreciation deductions at time 1 equal the expected decline in asset value, the undeducted costs equal expected asset value. Taxing the sale produces a risky loss in return at time 1 with expected value $-TZ^e$ but a certain deduction with time 1 value of TZ^e. If the risk premium is nontrivial, the investor is better off if the sale is taxed. From the ex ante time 0 perspective, taxation results in valuable risk reduction with no loss in expected return.

In summary, the effects claimed in Bulow and Summers (1984) arise only when used asset sales have no tax consequences. That tax treatment would be an anomaly under current U.S. tax law. Not only do depreciable asset sales receive ordinary income and loss treatment for the most part,[14] but there are special provisions in the tax code and administrative regulations that prevent evasion of tax when assets are disposed of through liquidations or distributions that would otherwise be tax free.[15]

V. CONCLUSION

The provocative argument in Gordon (1981) that offseting effects on risk and return may make capital income taxes largely nondistorting must be taken seriously. Section III shows that argument to be plausible in an asset pricing model with government expenditure as long as one makes the empirically believable assumption that the pre-tax riskless rate of return in the tax world is negligible. Furthermore, the failure to add a premium for "capital risk" to the standard economic depreciation allowance based on expected decline in asset value does not change that result unless the income tax system has the pathology of allowing used asset sales to be tax free. The current U.S. tax system seems to be free of that pathology.

FOOTNOTES

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1. These assumptions greatly simplify the model. Including tax rates that differ among individuals, for example, requires that somewhat arbitrary restrictions on short sales and borrowing be included in the model. Schaefer (1976) shows that otherwise there may be no equilibrium because of unlimited opportunities for arbitrage. Sometimes it is necessary to include such restrictions in a model. Auerbach and King (1983), for example, introduce short sale and borrowing restrictions in the Brennan (1970) framework in order to study issues such as how portfolio composition varies according to tax bracket.

2. If there were a perfect correlation between two different linear combinations of securities, Q could be made free of risk without

setting all the coefficients of g_j in (10) to zero. But this situation corresponds to the case where there is at least one redundant security. By relabeling securities the set can be reduced to one where no distinct linear combinations of securities are perfectly correlated. The statement in the text presumes that has been done. This assumption, equivalent to assuming that the covariance matrix for risky securities is nonsingular, is convenient for the development of theory in a CAPM framework. See, for example, Long (1977) and Roll (1977).

3. In equation (15) $\bar{g}_j(1 - T)$ is the after-tax value of the expected revenues $\bar{g}_j$ while $(1 - D)TP_j$ is the after-tax value of the end-of-period deduction in the amount $(1 - D)P_j$. See assumption (A7). The term $[r(1 - T) + (1 - DT)]P_0$ is equivalent to $g_0(1 - T) + (1 - D)TP_0$ since $g_0 = (1 + r)P_0$.

4. Following Brennan (1970), this analysis assumes that the second order conditions for a maximum are satisfied. That assumption is warranted if investors are assumed to be risk averse since the after-tax mean variance frontier will be concave (as long as the tax rate is not regressive over some range -- here it is constant). <u>See</u> Litzenberger and Ramaswamy (1979).

5. Litzenberger and Ramaswamy (1979) obtain a relation like (29) as the market equilibrium in a more complex model that includes income-related constraints on borrowing.

6. The numerator $(g_0 - P_0)(1 - T)$ in (35) is simply the after-tax return $g_0(1 - T) + (1 - D)TP_0$, see note 3, minus the after-tax cost $(1 - DT)P_0$.

7. It is straightforward but tedious to relate w_i (or w_i') to Arrow-Pratt absolute risk aversion. Given assumptions (A1) and (A2) we have some $U_i^*(Z_i)$ where Z_i is person i's wealth and $U_i(\bar{v}_i, S_i^2) = EU_i^*(Z_i)$ where E is the expectation operator over the distribution of wealth outcomes at the end of the period. (Since each individual selects a portfolio of securities that are distributed multivariate normal, end-of-period wealth will be normally distributed.) Arrow-Pratt absolute risk aversion is $A_i = -U_i^{*\prime\prime}/U_i^{*\prime}$ where primes denote derivatives with respect to wealth. It is straightforward to show that when A_i decreases (increases) in wealth, w_i increases (decreases) with wealth. <u>I</u>.<u>e</u>., changes in w_i correspond inversely to changes in absolute risk aversion. This is intuitively clear since w_i is proportional to the marginal trade-off between mean and variance, and thus would measure willingness to engage in any given marginal gamble.

8. Note that $U_{i1}/2U_{i2}$ is $-w_i$ or $-w_i'$. Thus, this measure of individual risk aversion increases when the individual is more risk averse in the usual sense of the term.

Since there are no limitations on short sales and no gap between the rates at which individuals can borrow (sell short) and lend (invest) in this model, in equilibrium every investor will have the same w_i or w_i'. Therefore, F can be conceptualized as the change in everyone's common marginal rate of substitution between risk and return. But it is convenient to refer to F as expressing a change in average risk aversion.

9. This would not be the case if some of the taxed assets were expected to yield greater than the market rate of return. Then the government as co-owner would obtain the proportion T of the value generated at the time of investment due to the expected return being greater than market rates. This result makes expensing the treatment that corresponds to the Haig-Simons ideal for an income tax: the government is taxing away the proportion T of the increase in wealth due to an investment expected to yield greater than market rates of return. That "tax" occurs exactly at the time when the wealth increases. This result that expensing results in Haig-Simons treatment at the statutory rate T is completely the opposite of the conventional wisdom that expensing corresponds to a zero tax rate income tax and is a form of a consumption tax. The result here depends on the assumptions $F = 1$ and $r' = r$. But it seems clear that the conventional wisdom is mistaken. For an extensive discussion see Strnad (1984).

The model in the present paper is of a perfectly competitive securities market. There are no investments available that produce greater than the market rate of return. As a result, the

possibility that there will be investment opportunities yielding an increase in wealth at the time of investment is excluded.

10. In addition, there are technical problems with assuming constant risk aversion. In a mean-variance portfolio model such as the one in this paper, a sufficient condition for w_i to be constant is that individuals have utility functions of the form $U_i(\bar{v}_i, S_i^2) = a_i + b\bar{v}_i - cS_i^2$ where the a_i, b and c are constants. The condition is in a sense also a necessary condition. Without this condition w_i will in general vary with wealth since wealth changes will affect the $\bar{v}_i$ and S_i^2 of each individual's chosen portfolio.

However, utility functions linear in $\bar{v}_i$ and S_i^2 are not possible under assumptions (A1) and (A2). If $U_i(\bar{v}_i, S_i^2)$ is the expected value of a von-Neuman-Morgenstern utility function, $U_i^*(Z_i)$, where Z_i is person i's wealth and securities are distributed multivariate normal, then it is well known that U_i must obey the following partial differential equation:

$$2\frac{\partial U_i}{\partial S_i^2} = \frac{\partial^2 U_i}{\partial \bar{v}_i^2}. \tag{52}$$

See Baron (1977). When $U_i(\bar{v}_i, S_i^2)$ is linear in $\bar{v}_i$ and S_i^2 with a nonzero coefficient on S_i^2, (52) is not satisified.

11. How the responsiveness of absolute risk aversion to wealth varies with wealth, for example, depends on the sign of the second

derivative of absolute risk aversion with respect to wealth. But from the expression for absolute risk aversion in note 7, it is clear that the sign of its second derivative depends on the signs of the third and fourth derivatives of the utility function. It is hard to make an intuitive judgment as to what these signs are and how they vary with wealth.

12. Suppose that a depreciable asset is purchased for an amount A, depreciation deductions totalling B have been taken on the asset, and the asset has current value V. The "adjusted basis" of the asset is thus $C = A - B$. *I.e.*, the adjusted basis is the portion of original cost not yet deducted. Except for a few special kinds of real property (primarily low-income housing and residential rental structures) the tax treatment under current U.S. law is roughly as follows (with reference to the appropriate Internal Revenue Code sections):

    (1) $V < C$: an ordinary loss under section 1231 in the amount $C - V$;

    (2) $A > V > C$: ordinary income under section 1245 in the amount $V - C$;

    (3) $V > A$: ordinary income under section 1245 in the amount $A - C$ and capital gains income in the amount $V - A$.

For assets that decline substantially in value with use (3) is unlikely to occur unless inflation rates are extremely high. As

a result, ordinary income and ordinary loss treatment can be taken to be the usual treatment on sale of a depreciable asset.

13. We can also verify that in the case $F = 1$ and $b = 1$ it is correct to use the same risk premium in discounting to present value as in the no tax world despite the fact apparent from (40) and (41) that this risk premium differs from both the after-tax risk premium and the pre-tax risk premium in the tax world. Equation (57) indicates that use of the no tax world risk premium in discounting yields $(1 + r' - T)a'/[(1 + r')(1 - T)]$ as a pre-tax risk premium and $r'/(1 - T)$ as a pre-tax riskless rate in the tax world. By equations (7) and (40) these are correct for the case $F = 1$ and $b = 1$.

14. See note 12.

15. Section 311(a)(2) of the Internal Revenue Code allows a corporation to ignore gains and losses for tax purposes when it distributes property with respect to its stock. Section 337 and (prior to 1982) section 336 allow corporate liquidations to be tax free in certain situations. These sections raise the possibility of "selling" an asset without any tax consequences. Treasury regulation 1.245-6(b), however, causes the corporation to realize gain equal to the amount by which value exceeds adjusted basis up to original cost less adjusted basis. Also, in 1984 Congress amended section 311(d)(1) to reverse the rule of section 311(a)(2) in the case of appreciated property. A

corporation recognizes gain when it distributes such property with respect to its stock, and the gain recognized is not limited to original cost less adjusted basis.

In a system (such as the current U.S. system) where depreciation allowances are at a much faster rate than expected decline in asset value, most depreciable assets have original cost > value > adjusted basis. See note 12. Thus, Treasury regulation 1.245-6(b) by itself insures that use of tax-free distributions and liquidations usually will not change the tax consequences that would result from direct sale of a depreciable asset.

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APPENDIX:

Gordon's Two-Period Consumption Model

At the core of his paper, Gordon (1981) does not use the CAPM but uses a two-period consumption model. Individual i maximizes expected utility over C_i^1 and C_i^2, consumption by i in periods 1 and 2 respectively. All after-tax return from investment is consumed in period 2. There are j firms that each invest in capital with a cost K_j in period 1 and a value V_j at that time. Individual i initially owns $\bar{x}_{ij}$ of firm j and that individual receives that proportion of the initial surplus, $V_j - K_j$. After some first period trading individual i emerges owning x_{ij} of firm j. The period 2 before-tax return in excess of capital costs for firm j is N_j. N_j is uncertain.

Gordon has five kinds of taxes in his model: a property tax, a tax at the corporate level, a tax on earnings from corporate equity, a tax on earnings from lending at the riskless rate, and a tax on inflationary gains. To make his results comparable to those in this paper, I simplify his model by assuming no inflation, no tax at the corporate level, no property tax, and a common tax at rate e on earnings from corporate equity and on earnings from lending at the riskless rate.

Gordon's model also includes government expenditure effects. The government distributes all of the taxes it has collected to individuals. Each individual i receives a distribution Y_i in the second period, the same period when the individual pays taxes on his or her earnings from capital. Gordon specifies that the distributions are lump sum. However, in proving his main result, Gordon implicitly assumes that individuals foresee the values of Y_i since they use these values in maximizing their expected utilities. Gordon argues that if transfers are set at a level so that C_i^1 and the distribution of C_i^2 are unchanged from the no tax world for all individuals i, then each individual i will have the same expected utility and derivatives thereof in the tax world as in the no tax world. (This follows from the fact that for each individual i utility and its derivatives are functions of C_i^1 and C_i^2 only). But in order for this to be true, each individual must anticipate the transfer Y_i, its distribution, and how its distribution relates to the taxes that i will pay. Thus, Gordon's model is like the case of government expenditure effects in section III: each individual knows in advance how the government distributions to him or her will depend on the outcomes of the investments in the world.

Gordon's main result in the simplified context here is that an income tax system will not have any effect on equilibrium values of the C_i^1 and K_j or on the distribution of C_i^2 if the following two conditions are met:

(1) $re = 0$

(2) $Y_i = e\sum_{j=1}^{J}[(V_j - K_j)(\bar{x}_{ij} - x_{ij}) + x_{ij}(N_j - rK_j)]$

where r is the no tax world return on the riskless asset and there are J firms. Condition (1) says that either r or e is zero. The case where e is zero is trivial since there are no taxes in a world where $e = 0$ is the tax rate on income from captial and that is the only tax.

Consider the case $r = 0$. Now (2) becomes:

$$(2') \quad Y_i = e \sum_{j=1}^{J} [(V_j - K_j)(\bar{x}_{ij} - x_{ij}) + x_{ij}N_j].$$

Consider the two cases for an asset j held by i: $\bar{x}_{ij} = 0$ and $\bar{x}_{ij} \neq 0$. Now for $\bar{x}_{ij} = 0$, the term in the square brackets in (2′) is just the profit realized by i by buying x_{ij} of firm j in period 1. $x_{ij}N_j$ is i's share of the profit of firm j above the firm's cost K_j. But investor i paid $x_{ij}V_j$ not $x_{ij}K_j$ for x_{ij} of firm j. As a result, $x_{ij}(V_j - K_j)$ must be subtracted from $x_{ij}N_j$ to yield i's profits from buying x_{ij} of firm j.

The analysis is the same when $\bar{x}_{ij} \neq 0$ except that there is an additional term in the square brackets in (2′). This is $\bar{x}_{ij}(V_j - K_j)$. But that term is equal to the initial gain that i realized by owning $\bar{x}_{ij}$ of firm j. Thus, the sum of the square bracket terms is the total income in both periods of person i from all of his or her investments in risky assets. Gordon shows in his equation (20a) that $r = 0$ implies that the tax world riskless rate is zero. Thus, there is no income from riskless investments. Furthermore, since individuals can borrow and lend without restriction in the model at this zero rate, income and transfers from different periods can properly be compared without discounting to allow for the time value of money. As a result, the transfer Y_i is equivalent to a transfer of all taxes paid by i on that income <u>back</u> to i.

There is one subtlety in asserting this equivalence. Part of the taxation of profits occurs through price adjustments in Gordon's model. In particular, from his equation (20b) $V_j^* = K_j + (1 - e)(V_j - K_j)$ where V_j^* is the initial value of firm j in the tax world. It is easy to see that this change in prices "taxes" initial owners who sell in the first period at exactly the rate e since their proceeds are reduced by $e(V_j - K_j)$ from those in the no tax world.

It is also the case that the adjustment of V_j to V_j^* provides the "proper" tax treatment of those who buy shares in the initial period. In Gordon's model, individuals who buy part of firm j do not get to deduct their cost but only their share of the firm's capital cost. <u>I.e.</u>, if individual i buys x_{ij} of firm j, then in period two that individual will pay eN_j in taxes where N_j is firm j's return over and above the capital cost K_j. Individual i's profit, however, is the excess of the firm's value in period two over V_j, its value in period one. But $V_j - V_j^* = e(V_j - K_j)$ so that the price that i has to pay is reduced by exactly the taxes on $V_j - K_j$. This compensates for being given a basis K_j instead of the actual cost V_j. With a zero after-tax riskless rate and unlimited borrowing and lending permitted with zero transactions costs at that rate, the compensation is exact. The individual is indifferent between paying V_j^* instead of V_j in the first period and having a basis of V_j instead of K_j for computing gains in the second period.

In summary, when the after-tax riskless rate is zero, the set of prices and taxes is equivalent to setting $V_j^* = V_j$ and taxing all profits on investments in the traditional way (respecting basis as cost) including the gains from sales by original owners. In fact, with $V_j^* = V_j$ and taxes set that way, the same equilibrium values of C_i^1, K_j and the same equilibrium distribution of C_i^2 would obtain in the no tax world as in the tax world. The taxes on profits would be exactly returned by the transfers Y_i.

Thus, when Gordon's model is simplified to include a single-level tax with no inflation and that tax is made to conform to the conventions currently used in income taxation, each taxpayer is given transfers that refund the taxes paid. Since the after-tax riskless rate is zero, the timing of the transfers does not matter. It is as if each person were instantaneously given a full refund of taxes at the time of payment. It is not surprising that where all taxes are transferred back to taxpayers and they know this will happen, the equilibrium in the tax world will be the same as in the no tax world. Individuals will behave as if there are no taxes.