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SEQUENTIAL ELECTIONS WITH LIMITED INFORMATION

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ABSTRACT

We develop theoretically and test experimentally a one dimensional model of two candidate competition with incomplete information. We consider a sequence of elections in which the same general issue predominates from election to election, but where the voters have no contemoporaneous information about the policy positions adopted by the candidates, and where the candidates have no contemporaneous information about the preferences of the voters. Instead, participants have access only to contemporaneous endorsement data of an interest group, and to historical policy positions of the previous winning candidates. We define a stationary rational expectations equilibrium to the resulting (repeated) game of incomplete information, and show that in equilibrium, all participants, voters and candidates alike, end up acting as if they had complete information: Voters end up voting for the correct candidate, and candidates end up converging to the median voter.

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> The widespread lack of familiarity with prominent issues of public policy, along with confusion on party position ... attests to the frailties of the political translation process. The fact that a policy question is subjected to intense partisan debate between elections does not assure us that at the next election the public will respond to the substance of the argument.
>
> [Campbell et al. The American Voter, p. 188]

1. Introduction

Despite the well established empirical generality expressed in the preceeding quote, spatial models of elections typically make severe and unrealistic assumptions about the information available to candidates and voters. Specifically, it is generally assumed that voters know the policy positions adopted by the candidates and that candidates know the preferences of voters. Because these assumptions contrast so sharply with empirical reality, many scholars, who might otherwise be sympathetic to the perspectives of such models, discount their political relevance.

To assess the extent to which the conclusions of the spatial modeling literature are dependent on such heroic information

assumptions, this paper develops and experimentally tests an alternative model of elections with decentralized and incomplete information assumptions. Specifically, we consider a sequence of elections in which voters have no contemporaneous information about candidate positions and candidates have no contemporaneous information about voter preferences. The only information available to the participants consists of contemporaneous endorsement data and historical data on the past election outcomes.

The model developed here is not necessarily more empirically accurate than the previous literature in its description of the information available to participants. Rather, we move to the opposite extreme from that literature by assuming that participants have less information than they might be expected to have on the basis of empirical evidence. Yet, in spite of this dearth of information in our model, our primary conclusion is that the actions of both voters and candidates in equilibrium approximate the actions one anticipates under full information. Thus, in the long run, all participants behave exactly as if they are fully informed: the candidates converge to the median voter ideal point, and each voter votes for the correct candidate--the candidate closest to his ideal point.

This paper is similar to another paper of ours (McKelvey and Ordeshook [1984a]) in which we also develop a model of elections with incomplete information. But this paper differs significantly from that one in that: (1) Here we model a sequence of elections rather than a single election. (2) Here the information source for uninformed voters (and candidates) is historical data rather than poll data. (3) Here all voters are initially uninformed, unlike our other paper, in which we assume asymmetric information, with some informed voters and some uninformed voters. However the basic conclusion of this paper is similar to our conclusion in [1984a]: Namely, equilibrium behavior in both models extracts full information.

2. Motivation for the Model

The low level of political awareness and sophistication of the average voter is, by now, a well established fact. The literature is replete with examples where the voter displays ignorance of such basic political currency as the number of senators from his state, the name of his congressman, etc. (eg., Greenstein [1963], Almond and Verba [1963], and Kinder and Sears [1985]). The major election studies have concluded that the typical voter is unaware of the positions of candidates on important issues and that he is unable to conceptualize politics in the ideological terms that are taken for granted by academics and politicians (Berelson et al [1954], Campbell et al [1960]). Empirical work points instead to a voter who, if he has any policy preferences, is much more narrowly self interested, and who relies on cues from the party and other interest groups to inform his vote (Repass [1971], Kuklinski et al [1982]). Our model is an attempt to model an electorate comprised of such voters.

The voter that we try and capture in our model is the voter whose interest in politics stems from a narrow concern about his own

economic well being. However, our voter is quite uninformed about the specific policy positions espoused by the candidates, either because he does not read the newspapers, or because he does not believe the promises made by the candidates, or because the candidates themselves are sufficiently ambiguous to make it impossible to discover their true positions. Instead, he associates (or dissociates) his interests with those of a particular group in society, and recognizing his common interest with that group, is therefore willing to trust that the leadership of that group speaks for him. This is the category of voter that is labeled "ideology by proxy" by Campbell et al [1960], and they estimate that such voters comprise about 45% of the electorate. Our voter also is aware of the party identification of the two candidates, and himself has a predisposition for one party or the other, based on his past experience with the policies that have been enacted by that party when it has been in power.

The candidates in our model are typical Downsian candidates: they just try and adopt policy positions to win each election as it rolls around. But since our voters are in a fog, the candidates are also in a fog. The voters cannot articulate their policy preferences, so candidates are unsure what policies they should adopt. Should they respond to the interest groups, or should they try and ferret out the preferences of the voters? The only clue they have to the preferences of the voters is the history of election outcomes, but given the nature of the voters in our model, that seems a pretty tenuous clue.

Can an electorate comprised entirely of voters and candidates satisfying the above description come anywhere near achieving the type of policy outcomes that would be expected from a fully informed electorate? We investigate this question for a simple model with only one interest group and a one dimensional issue space, but our answer is that over a sequence of elections, it is possible that a full information outcome can be achieved. Of course a realistic model would allow for multiple issues and interest groups--but because of the theoretical complexity, we leave this extension until later.

To formulate our model, we use the notion of "rational expectations equilibrium" which has been successfully applied in economics to models of markets with incomplete information (see Grossman [1982] for a review of this literature). Our approach is to assume that, when information is costly, people take cues from other endogenous sources that are easily observable, which they believe convey useful information. They use this data to make predictions about the parameters relevant to their welfare. In rational expectations equilibrium (which can be thought of as a steady state of the system) all individuals must be maximizing their utility conditional on the inferences they make from the data they observe, and all individuals must be making statistically correct inferences, which are not countermanded by the data they then actually observe.

Our model consists of a sequence of elections in which the same general (one dimensional) issue predominates in the public's mind from election to election. We model this as a repeated game where the

players are n citizens (voters) plus two candidates. The candidates may be thought of as political parties, who select new nominees (with different policy platforms) in each election. In addition to these candidates, there is an interest group whose whose policy preferences are known to the voters and whose sole function is to endorse the candidate that it prefers.

In each election, the candidates first adopt policy positions, the interest group then endorses the candidate it prefers, and then an election is held, in which the voters vote for one candidate or another, or abstain. We assume that the voters are all uninformed about the positions adopted by the candidates before the election. Instead, they have access to information of two general types: historical data and the endorsement. We assume that candidates do not know the preferences of voters before they adopt their policies. However, they have access to the same historical and endorsement data that is available to the voters.

The endorsement represents the truthful preferences of the interest group between the two candidate positions. But it is assumed to convey only ordinal information about which candidate is to the left and which candidate is to the right on the issue. Of course, in the real world, endorsements typically convey richer information than this. For example, the intensity of an endorsement might be used by voters to indicate the "distance" between the candidates or the distance between the candidate or the leadership of the interest group. In our model, though, we ignore the cardinal information conveyed by endorsements.

With regard to historical information, we assume voters have knowledge of the policies of past administrations of each candidate's party--and of the effects of those policies on their individual welfare. Theoretically and experimentally, we incorporate this information by assuming that the positions of winning (but not losing) candidates in earlier elections are known.

The equilibrium that we define to the above repeated game is what we call a stationary rational expectations equilibrium. Voters have beliefs about the likely positions that each candidate will adopt, which they arive at by making statistical projections based on the historical positions of past winning candidates. The voters then adopt voting strategies which maximize their expected utility given these beliefs. In equilibrium, their beliefs must be rational, in the sense that the distribution of actual positions that each candidate ends up adopting must agree with the distribution that the voter would predict. The candidates, on the other hand, do not know the voter preferences and voting strategies, so they treat the game as a repeated game with unknown payoff function. Hence they look for a stationary mixed strategy equilibrium to the game. I. e., an equilibrium strategy for the candidates is a mixed strategy such that if the candidate uses this mixed strategy in each election, then the distribution of his winning positions is the same as the distribution of positions he has adopted.

Before proceeding, we give some further discussion of the

above assumptions. With regards to the historical data, note that our model assumes that voters only learn the positions of _winning_ candidates. In our model, the candidate positions represent the positions that candidates actually adopt _if_ they win the election. Clearly, it is not necessary for the candidates to reveal this position to the voters until after the election. In fact they may have incentives to conceal their positions from the voters, and empirically, they indeed do seem to do this (Page [1978], Chapter 6). Furthermore, the voters have no particular reason to believe what the candidates do promise. Typically, then, it will be very difficult for even a well informed voter to be able to get good information on the likely candidate positions. However after the election, the winning candidate must implement a policy, and we assume that this policy becomes common knowledge.

Our model assumes that the voters know the policy position of the interest groups. Thus we trade voter knowledge of candidate positions for voter knowledge of the interest group position. At first glance, this may seem like a poor trade off--substituting one unrealistic assumption for another. However there are theoretical and empirical reasons justifying this assumption: (1) Candidates are transient actors on the political scene, with incentives to conceal rather than reveal their true policy positions, whereas interest groups are much more permanent. Because of their constant and selective base of support, interest groups are less able to conceal their policy preferences. (2) Empirical evidence shows that voters do, in fact have fairly good information about interest group positions, even on relatively technical issues. (See, e.g., Kuklinski, Metlay & Kay [1982]). (3) We do not need to assume that the voter knows the exact position of the interest group on the policy scale, but we need only assume that the voter knows enough about the interest group to be able to interperet an endorsement as being a "liberal" or "conservative" endorsement.

With respect to the sequential structure of our experiments and theory, several earlier commentators on this research objected to the assumption that the same candidates compete over the sequence of elections--referring to the analysis as a Harold Stassen model. However it is important to interpret the sequence of candidate positions as the positions of the nominees of the two major parties. The historical record, and voters' interpretations of that record, then, are viewed as the basis for partisian preferences. Admittedly, our theory and experiments ignore certain facts that permit drawing even stronger inferences from partisian labels, including the fact that candidates must secure nominations from a biased sample of the electorate, which predisposes them to support and implement certain policies. These facts, though, would almost certainly induce even more rapid convergence of voter beliefs to the candidates' true strategies. In contrast to the assumption that partisian labels and interest group endorsements are devices for reducing the costs to citizens of gathering the information necessary to cast an "approximately" informed vote, our analysis suggests that these

devices may be sufficient for a wholly informed vote.

Finally it should be noted that the type of voting behavior that we assume is related to the notions of retrospective voting developed by Fiorina [1981]. I. e., the voter does not vote on the basis of promised performance, but on the basis of delivered output. The voter thus develops expectations of future performance of candidates based on their average past performance, and he rewards or punishes candidates in future elections based on whether during their incumbency, they have done better or worse than expected. To this picture of voting behavior, we simply add the dependence also on interest group endorsement.

3. The Formal Model

We assume that there are two candidates, designated by $K = \{1,2\}$, n voters, designated by $N = \{a_1, a_2, \cdots a_n\}$, and a one dimensional convex policy space, $X \subseteq \mathbb{R}$. Each voter, $a_i \in N$, has a utility function $u_i : X \to \mathbb{R}$, which is symmetric and single peaked about an ideal point y_i^*. We let K_0 be the three element set consisting of the two candidates, plus a third element, "0", which is used to represent "no endorsement" or "abstention." Thus, $K_0 = \{1,2,0\} = K \cup \{0\}$. Let F be the set of functions from K_0 into K_0. (Elements of F will be used to represent voter strategies.) For any $k \in K$, we use $\bar{k}$ to denote the opposition candidate. I.e., $\bar{k} \in K - \{k\}$. We let $\mathbb{B}$ be the set of Borel measurable subsets of X.

We now define a sequence of identical games, where in each game, the players consist of the candidates, K, together with the voters, N. The strategy space for candidate k, and voter α_i are denoted S_k and B_i respectively. We assume $S_k = X$ and $B_i = F$ for all $k \in K$ and $\alpha_i \in N$. So candidate strategies are positions on the issue, while voter strategies are decisions as to who vote for as a function of which candidate is endorsed (see below). We write $S = S_1 \times S_2$, and $B = B_1 \times \cdots \times B_n$, and we denote specific strategy choices of the candidates and voters, respectively, by $s = (s_1,s_2) \in S$ and $b = (b_1,\ldots,b_n) \in B$. For any s, s' $\in$ S and b, b' $\in$ B, let $(s|s_k',b)$ denote the strategy n+2 tuple obtained by replacing the k'th candidate's strategy in s by s_k', and $(s,b|b_i')$ denote the strategy n+2 tuple resulting from replacing the i^{th} voter's strategy, b_i, by b_i'. Given a strategy n+2 tuple $(s,b) \in S \times B$, we now define the payoff function for the game, in several steps.

First, we define the <u>endorsement</u> to be a function, $e:S \to K$, of the candidates' positions on the issue. Specifically,

$$e(s) = \begin{cases} 1 \text{ if } s_1 < s_2 \\ 2 \text{ if } s_2 < s_1 \\ 0 \text{ otherwise} \end{cases} \tag{3.1}$$

Thus e(s) tells the voters which candidates is to the left and which is to the right on the election issue. For any specific vector of votes by the n voters, $p \in K_0^n$, and for any $k \in K_0$, we write

$$v_k(p) = |\{\alpha_i \in N | p_i = k\}| \tag{3.2}$$

and

$$w(p) = \begin{cases} 1 \text{ if } v_1(p) > v_2(p) \\ 2 \text{ if } v_2(p) > v_1(p) \\ 0 \text{ otherwise} \end{cases} \tag{3.3}$$

(Here we use the notation $|A|$ to represent the number of elements of a set A.) Then, for any $(s,b) \in S \times B$, the <u>vote for candidate k</u> is given by

$$v_k(s,b) = v_k(b(e(s))) \tag{3.4}$$

The <u>winning candidate</u>, or <u>election outcome</u> is

$$w(s,b) = w(b(e(s))) \tag{3.5}$$

where w(s,b) = 0 means a fair coin is tossed to determine if 1 or 2 wins. We can then define the payoff to voter $\alpha_i \in N$ by

$$M_i(s,b) = u_i(s_{w(s,b)}), \tag{3.6}$$

Here, we use the notation s_0 to represent the outcome resulting from a tie--namely a fair lottery between s_1 and s_2,--and we assume that the utility a voter associates with a tie is simply $u_i(s_0) = \frac{1}{2}u_i(s_1) + \frac{1}{2}u_i(s_2)$. Expression (3.6), then, simply states that voter i's payoff equals the utility he associates with the position (strategy) of the winning candidate, w(s,b). Finally, the payoff to candidate $k \in K$ is

$$M_k(s,b) = \begin{cases} 1 \text{ if } w(s,b) = k \\ -1 \text{ if } w(s,b) = \bar{k} \\ 0 \text{ otherwise} \end{cases} \tag{3.7}$$

So the candidate gains or loses a unit of utility depending or whether he wins or loses the election. These definitions, then, specify the normal form of an election game that is represented by the extensive form portrayed in Figure 1 (which we have simplified by considering only two voters and by letting the policy space, X, contain only three elements--denoted by x, y and z, where $x < y < z$). First the candidates have a simultaneous move in which they each choose a policy position. After the candidates have moved, voters then have a simultaneous move in which they vote for a candidate. The voters' move can be thought of as the "election." However, the strategies of the candidates are not revealed to the voters before the election. Instead, only endorsement information is revealed: that is, through e(s), voters are told which candidate's position is further to the left. For example, if the candidates have chosen identical positions, Voter 1 knows this but he does not know whether he is at his 1st, 5th or 9th decision node. I.e., he does not know whether s = (x,x), s = (y,y), or s = (z,z). Similarly, if candidate 1 is endorsed, the voter knows that $s_1 < s_2$, but he does not know whether this corresponds to s = (x,y), s = (x,z) or s = (y,z). The function $v_k(s,b)$ tallies the vote for each candidate and determines the winner, w(s,b). The payoff to a voter is the utility he associates with the position of the winning candidate, while a candidate's payoff is 1 if he wins, −1 if he loses, and 0 if the election is a tie.

The preceeding structure models the limited information that is available to voters in a <u>single play</u> of the game (namely, voters see only the endorsement, and not the candidate positions), but it does not model the information that is available to voters or

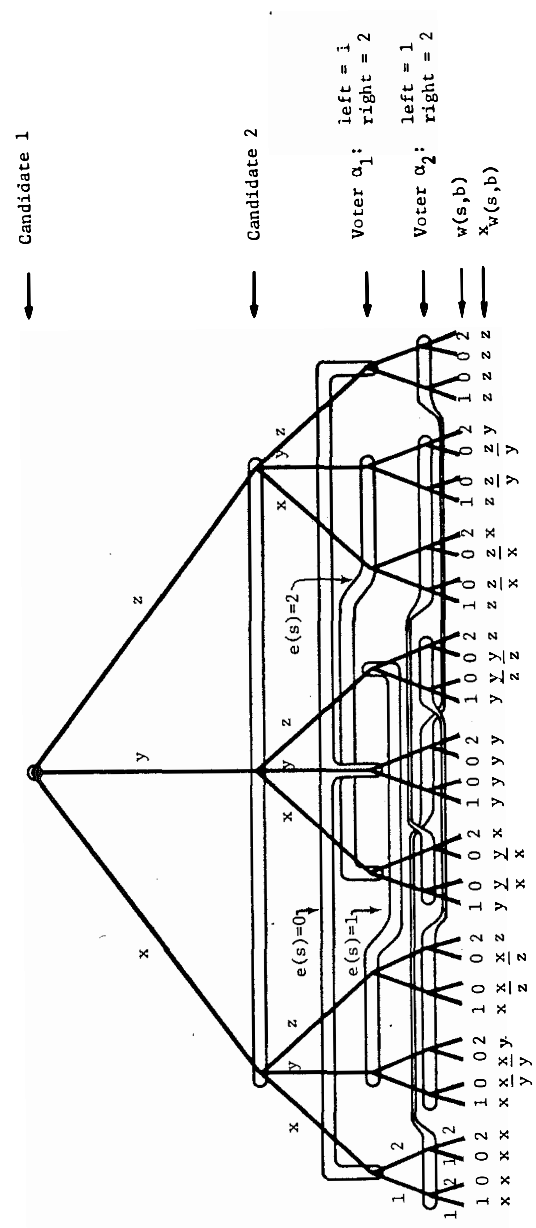


Figure 1

Extensive form for one stage of game.*‡

*For simplicity, only two strategy choices are diagrammed at each node for each voter. The full extensive form would also allow for abstention, with the obvious modifications in the last two tiers of the tree.

‡The notation $\frac{x}{y}$ means an even chance lottery between x and y, etc.

candidates from previous plays of the game. We assume that the candidates do not know the voter characteristics--i.e., they do not know the voter ideal points, y_i^*. However, all players observe the outcome w(s,b) and the position, $s_{w(s,b)}$, of winning candidates in previous plays of the game.

We now define an equilibrium for this repeated, incomplete information game. But first we need some further notation. Let Λ_k be a set of probability measures on S_k. So Λ_k represents the set of admissible mixed strategies of candidate k. We assume throughout that elements of Λ_k are either absolutely continuous (with respect to Lebesque measure on X), or degenerate point masses (i.e., $\lambda_k(\{x\}) = 1$ for some $x \in X$). We let $\Lambda = \Lambda_1 \times \Lambda_2$. So for $\lambda = (\lambda_1, \lambda_2) \in \Lambda$, λ represents the product measure of λ_1 and λ_2.

Given $\lambda = (\lambda_1, \lambda_2) \in \Lambda$, we define several derived measures on $\mathbb{B}$. For $k \in K$, $j \in K_0$, and $b \in B$, define

$\lambda_{kj}(\cdot)$: The distribution of s_k given that j is endorsed, (i.e. given e(s) = j). (3.8)

$\lambda_{kw}(\cdot|b)$: The distribution of s_k given that k wins (If k never wins, then $\lambda_{kw}(\cdot|b) = \lambda_{\underline{k}}(\cdot)$). (3.9)

$\lambda_w(\cdot|b)$: The distribution of winning positions, i.e., of $s_{w(s,b)}$. (3.10)

These probability measures are defined formally in Appendix A.

Finally, for any $\gamma \in \Lambda$, we define $B(\gamma) \subseteq B$ by

$$B(\gamma) = \{b^* \varepsilon B \mid \forall b \varepsilon B,\ b_i^* \ \varepsilon \ \arg \max_{b_i' \varepsilon B_i} \mathbb{E}_\gamma[M_i(s, b|b_i')]\}. \tag{3.11}$$

Here $\mathbb{E}_\gamma$ is the expectation under γ. Hence $B(\lambda)$ is the set of optimal voting strategies by the voters given beliefs $\gamma = (\gamma_1, \gamma_2)$ of the candidate strategies.

<u>Definition 3.1</u> A <u>Rational Expectations Equilibrium</u> (REE) for the game defined by (3.6)-(3.7) is a triple $(\lambda^*, \gamma^*, b^*) \in \Lambda \times \Lambda \times B$ satisfying

(V1) $b^* \in B(\gamma^*)$,

(V2) For all $k \in K$, $\gamma_k^* = \lambda_{kw}^*(\cdot|b^*)$,

(C1) For all $k \in K$, $\lambda_k^* = \lambda_{kw}^*(\cdot|b^*)$.

In short, letting $\lambda^* \in \Lambda$ be the actual distribution of strategies adopted by the candidates, and $\gamma^* \in \Lambda$ be each voter's belief about the distribution of a candidate's positions, this definition of equilibrium requires that: (V1) Each voter adopts a voting strategy, b_i^*, that maximizes his expected utility, conditional on his belief about the candidates' policy positions. (V2) The voters have rational expectations about the candidate positions: the distribution that voters believe a candidate's position follows must agree with the distribution of the observed winning positions of that candidate. That is, <u>in equilibrium</u>, voters cannot believe that the mixed strategy of a candidate differs from the observed distribution of that candidate's strategies (given our earlier assumptions, a candidate's position is observed only when he wins). (C1) Finally, the distribution of strategies adopted by each candidate is in equilibrium if this distribution equals the distribution of his own winning positions.

A Rational Expectations Equilibrium represents the type of behavior one would expect, in the long run, from the repeated play of the game (3.6)-(3.7), assuming that players have the limited information as described above. Thus, voters do not observe the candidates' policy positions before they vote, but they do observe the endorsement, along with the previous history of winning positions. A voter's equilibrium strategy takes his belief γ_k^* about the distribution of each candidate's position, based on the historical data, and, given this belief, maximizes his utility conditional on the endorsement. Note that as long as the candidates draw their positions from a stationary distribution, condition V2 is satisfied if voters perform an ordinary linear regression on the past winning positions of each candidate to arrive at their belief, γ_k^*, of candidate k's position. While this decision rule for voters appears straightforward, the implications of condition C1 are perhaps less clear. Since candidates do not know voter ideal points, they cannot know the payoff function of the single stage game. But, because we are interested in a stationary equilibrium to the repeated game, our definition of equilibrium supposes that the candidates assume that the payoff function for the game is the same from period to period and that they seek a mixed strategy equilibrium to this repeated game.

Because they must find this equilibrium _without_ _knowing_ _the_ _game's_ _payoff_ _function_. Condition C1 assumes that candidates follow a strategy of matching their mixed strategies to the observed distribution of their previous winning positions.

Now in any REE, $(\lambda^*, \gamma^*, b^*)$, we can show that voter beliefs must correspond to the distribution of strategies chosen by the candidates, _i.e._, $\lambda^* = \gamma^*$. Further, optimal voting behavior can be characterized in terms of the distributions λ^*_{jk} and $\lambda^*_{\bar{j}k}$. Hence a REE can be characterized more simply as,

Proposition _1_. A Rational Expectation Equilibrium $(\lambda^*, \gamma^*, b^*)$ can be characterized by a pair $(\lambda^*, b^*) \in \Lambda \times B$, where $\gamma^* = \lambda^*$, and (λ^*, b^*) satisfies:

(V1') $b^* \in B(\lambda^*)$,

(C1') for all $k \in K$, $\lambda_k^* = \lambda_{kw}^*(\cdot | b^*)$.

Further, for all $\alpha_i \in N$ and all $j \in K$, b_i^* satisfies

$$b_i^*(k) = j \quad \text{if} \quad \mathbb{E}_{\lambda^*_{jk}}[u_i(x)] > \mathbb{E}_{\lambda^*_{\bar{j}k}}[u_i(x)]$$

for all $k \in K$ with $\lambda^*(\{s \in S | e(s) = k\}) \neq 0$.

Here, λ^* represents the distribution of candidate positions as well as the voter beliefs. So (C1') requires that the distribution adopted by each candidate equals the distribution of his winning positions, and (V1') requires that each voter should adopt a voting strategy that for each possible endorsement, maximizes his expected utility with respect to λ^*, _conditional_ on the endorsement.

A probability distribution $\lambda_k \varepsilon \Lambda_k$ is said to be <u>symmetric</u> around $x^* \varepsilon \mathbb{R}$ if its cumulative density function $F_k: \mathbb{R} \to [0,1]$ satisfies $F_k(x^*+t) + F_k(x^*-t) = 1$ for all $t \varepsilon \mathbb{R}$. Clearly if λ_k is symmetric around x^*, then $F_k(x^*) = \frac{1}{2}$ and $x^* = E_{\lambda_k}(x)$. We let $\Lambda^s \subseteq \Lambda$ denote the set of measures $\lambda = (\lambda_1, \lambda_2)$ such that $\lambda_1 = \lambda_2$ and such that λ_k is symmetric around x^* for some $x^* \varepsilon X$. Our first theorem shows that any REE with λ^* in Λ^s must either be degenerate (<u>i.e.</u>, with $\lambda_1^* = \lambda_2^*$ both being point masses) or we must have the candidates' <u>expected</u> policy positions equaling the ideal point of the median voter. (Note that the assumption of symmetry is required only for Theorem 1, not for Theorem 2).

<u>Theorem 1</u>. If n is odd, and $(\lambda^*, b^*) \varepsilon \Lambda \times B$ characterizes a REE, with $\lambda^* \varepsilon \Lambda^s$, then either

(a) $\lambda_1^*(\{x\}) = \lambda_2^*(\{x\}) = 1$ for some $x \varepsilon X$ or

(b) $\mathbb{E}_{\lambda_1^*}(x) = \mathbb{E}_{\lambda_2^*}(x) = y^*$,

where y^* is the median of the ideal points, $\{y_i^*\}$.

By introducing an additional stability condition, we can narrow the class of admissible REE's further. For this result, we endow Λ_k with the weak topology.

<u>Definition</u>. If (λ^*, b^*) characterizes a REE, then it is <u>stable</u> if, for each $k \varepsilon K$, there is a neighborhood $N(\lambda_k^*)$ of λ_k^* such that whenever $(\lambda', b') \varepsilon \Lambda \times B$ satisfies $\lambda_k' \varepsilon N(\lambda_k^*)$, $\lambda_{\bar{k}}' = \lambda_{\bar{k}}^*$ and $b' \varepsilon B(\lambda')$, then

$\lambda_w'(\cdot | b') = \lambda_w^*(\cdot | b^*)$.

Thus, a REE is stable if and only if, whenever one candidate changes his distribution, and voters vote optimally (given the new distribution), the distribution of winning positions does not change. So in a stable REE, each candidate adopts the distribution of observed winning positions, and neither candidate by unilaterally deviating from this distribution, can change the observed distribution of winning positions. The following theorem shows that the only stable REE occurs when both candidates are at the median voter ideal point thus, any REE results in candidates (and voters) behaving as if they have complete information.

<u>Theorem 2</u>: There exists a stable REE. Further, if n is odd and $(\lambda^*, b^*) \varepsilon \Lambda \times B$ characterizes a stable REE, then $\lambda_1^*(\{y^*\}) = \lambda_2^*(\{y^*\}) = 1$ for $y^* \varepsilon X$, where y^* is a median ideal point of the electorate.

4. <u>A Dynamic Process</u>

Our definition of a Rational Expectations Equilibrium is static. However, it is possible to define a dynamic process by which the beliefs and strategies of the players are adjusted, which converges in some cases to a rational expectations equilibrium as we define it. The process is highly stylized and artificial, and as is evident from the experimental section that follows, actual convergence takes place much more quickly than is implied by the dynamic process that we present. Nevertheless this process illustrates the forces that drive the system towards equilibrium, so it is instructive to present it.

We define a sequence $\{(\lambda^t,\gamma^t,b^t)\}_{t=0}^{\infty} \subseteq \Lambda \times \Lambda \times B$ as follows: Pick arbitrary $(\lambda^0,\gamma^0,b^0) \in \Lambda \times \Lambda \times B$ and $i \in N$, such that $b^0 \in B(\gamma^0)$. Then, assuming $(\lambda^{t-1},\gamma^{t-1},b^{t-1})$ has been defined, we define (λ^t,γ^t,b^t) to satisfy

(a) $b^t \in B(\gamma^t)$

(b) For all $k \in K$, $\gamma_k^t = \lambda_k^t = \lambda_{kw}^{t-1}(\cdot|b^{t-1})$.

So the above dynamic process works as follows: Candidates start with arbitrary strategies, $\lambda^0 = (\lambda_1^0,\lambda_2^0)$, and voters with arbitrary beliefs $\gamma^0 = (\gamma_1^0,\gamma_2^0)$ about the candidate strategies. The voter strategies, $b^0 = (b_1^0,b_2^0,\ldots,b_n^0)$, maximize expected utility, in every contingency, subject to the voter beliefs. We can imagine this set of strategies being played for a large number of periods, at which point voters and candidates look back at the history of what has occurred. Candidates then revise their strategies by changing their mixed strategy to $\lambda^1 = (\lambda_1^1,\lambda_2^1)$. This corresponds to each candidate revising his strategy so that it agrees with the distribution of his past winning strategies. (If he never wins, he mimics his opponent.) Voters, similarly, revise their beliefs of candidate strategies to $\gamma^1 = (\gamma_1^1,\gamma_2^1)$, which is a belief which is consistent with the winning candidate positions that they have observed. Note $\gamma^1 = \lambda^1$. Finally, voters revise their voting strategies to $b^1 = (b_1^1,\ldots,b_n^1)$, to be

utility maximizing subject to their new beliefs.

Next, players adopt the strategies and beliefs (λ^1,γ^1,b^1) for a large number of periods, at which point they look at the history of winning positions generated by (λ^1,γ^1,b^1) and revise their beliefs and strategies to (λ^2,γ^2,b^2), etc.

Figure 2 illustrates the above dynamic process for a case with three voters with preferences of the form $u_i(x) = -|x - y_i^*|$, where $y_i^* \in \mathbb{R}$. We assume $y_1^* = -4$, $y_2^* = 0$, $y_3^* = 4$. Since for $t > 0$ we have $\lambda^t = \gamma^t$, we begin with $t = 1$ and assume $\gamma^1 = \lambda^1$, where $\lambda_1^1 \sim N(0,4)$ and $\lambda_2^1 \sim N(1.3,.25)$. Figure 2a illustrates the computation of (λ^2,γ^2,b^2). Voters vote on the basis of their beliefs. But voters also observe the endorsement before they vote. Hence, they can condition their vote on the endorsement. If the voters observe that Candidate 1 is endorsed, for example, then the distributions of Candidate 1 and Candidate 2's position conditional on the endorsement yields λ_{11}^1 and λ_{21}^1. The voter then computes his expected utility under λ_{11}^1 and λ_{21}^1, respectively, to determine which candidate to vote for. Similarly, if Candidate 2 is endorsed, the voter computes λ_{12}^1 and λ_{22}^1, and maximizes expected utility under these distributions. This yields the strategies $b^1 = (b_1^1,b_2^1,b_3^1)$ indicated in the table. So, for example, b_1^1 satisfies $b_1^1(1) = 1$, and $b_1^1(2) = 2$, whereas b_3^1 satisfies $b_3^1(1) = 2$ and $b_3^1(2) = 1$. Hence Voter 1 always votes for the endorsed candidate, whereas Voter 3 votes for the unendorsed candidate.

The above voting strategies yield a new distribution of winning candidates. In particular notice that the endorsed candidate

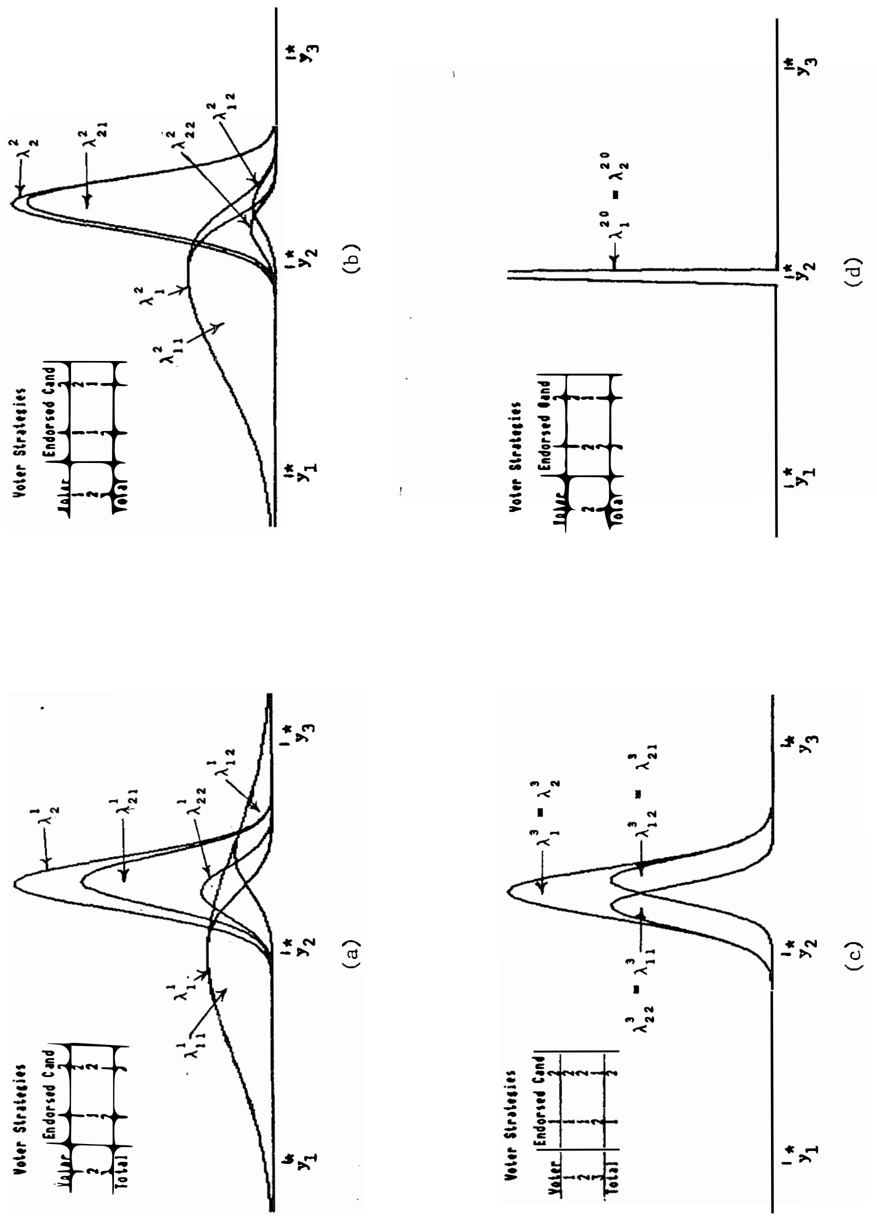


Figure 2
Illustration of dynamic process

always wins. So the distribution of winning positions for Candidate 1 is $\lambda^1_{1w}(\cdot|b^1) = \lambda^1_{11}$, whereas that for Candidate 2 is $\lambda^1_{2w}(\cdot|b^1) = \lambda^1_{22}$. Notice further that in this example $\lambda^1_{11} \neq \lambda^1_1$ and $\lambda^1_{22} \neq \lambda^1_2$. Hence after a while in this regime, voters find that their beliefs are not confirmed, and candidates find that the distributions of their winning positions is different from the distribution of their adopted positions. Thus, the next round, voters adopt the new beliefs, $\gamma^2 = (\lambda^1_{11}, \lambda^1_{22})$, and candidates adopt new strategies $\lambda^2 = (\lambda^1_{11}, \lambda^1_{22})$, as illustrated in Figure 2b. As before, voters adopt optimal voting strategies $b^2 = (b^2_1, b^2_2, b^2_3)$, which are listed in the accompanying table.

As the above process proceeds, we note that the distribution of candidate positions converges to a distribution which is a point mass at the ideal point of the median voter. See Figures (2b)-(2d). In accordance with Theorem 2, this is the only distribution which is stable--in the sense that when all individuals act on it, it is fulfilled.

It is worth noting the Darwinian nature of our dynamic model. The candidates adopt mixed strategies based on their current information with regard to the best winning strategies. The variance in policy positions provided by these mixed strategies supplies the raw material (the "gene pool") for the second stage--i.e., the natural selection which is performed by the election. This second stage selects and preserves only winning candidates, which then yields a new pool (due to the optimizing behavior of candidates for the next round.)

5. Experiments

This section describes a series of experiments designed to investigate the role of information in elections as modeled by the preceeding theoretical development. While that theory is limited presently to a one dimensional policy space, these experiments explore the possibility of a multidimensional extension by considering both one and two dimensional situations. We report on eleven 1-dimensional and three 2-dimensional experiments, in which we use students from Carnegie-Mellon University and California Institute of Technology as subjects. The instructions read to the subjects of our experiments are presented in Appendix B.

Each experiment involves from 5 to 21 subjects. Two subjects are randomly selected and assigned the role of "candidates", while the remaining subjects are designated "voters". Each voter is given a payoff chart depicting his or her monetary payoff as a function of the position adopted by the winning candidate. For a one dimensional experiment, the payoff function is single peaked, and a typical example is illustrated in Appendix B. The payoff charts are private information, and subjects are not permitted to reveal their chart to anyone else. Different voters are assigned different ideal points so that candidates do not know the payoff functions of any of the voters. Nor do they know the distribution of ideal points. Also, voters and candidates are told that there is a "dummy" player whose ideal point

is at zero, who does not vote, but who publicly "endorses" the candidate closest to its ideal point.[1]

Each experiment consists of a sequence of trials--representing elections--which take place under the limited information conditions described in the previous sections. The sequence of events in a specific trial is as follows: First, the two candidates select policy positions. This is done secretly and voters are not informed of these positions. Second, the "endorsement" is announced, informing voters which candidate is further to the left. Third, the voters must vote for one candidate or the other, and the ballots are collected and tallied. At this time voters and candidates learn the position of the winning candidate and the vote margin (the position of the losing candidate is not made public). Voters then compute their payoff and add this amount to what they might have won from previous trials. This sequence is then repeated a predetermined number of times, or until time expires (most experiments were limited to one hour), whichever comes first. Participants are not informed before the experiment about the total number of trials to be conducted.

Single Issue Experiments.

Figure 3 portrays sequences of outcomes in four of the one dimensional experiments. These figures illustrate the sequence of candidate positions, and for the three voter experiments they also give the vote of each voter in each period.

Experiment C2 is an example in which the candidates converge

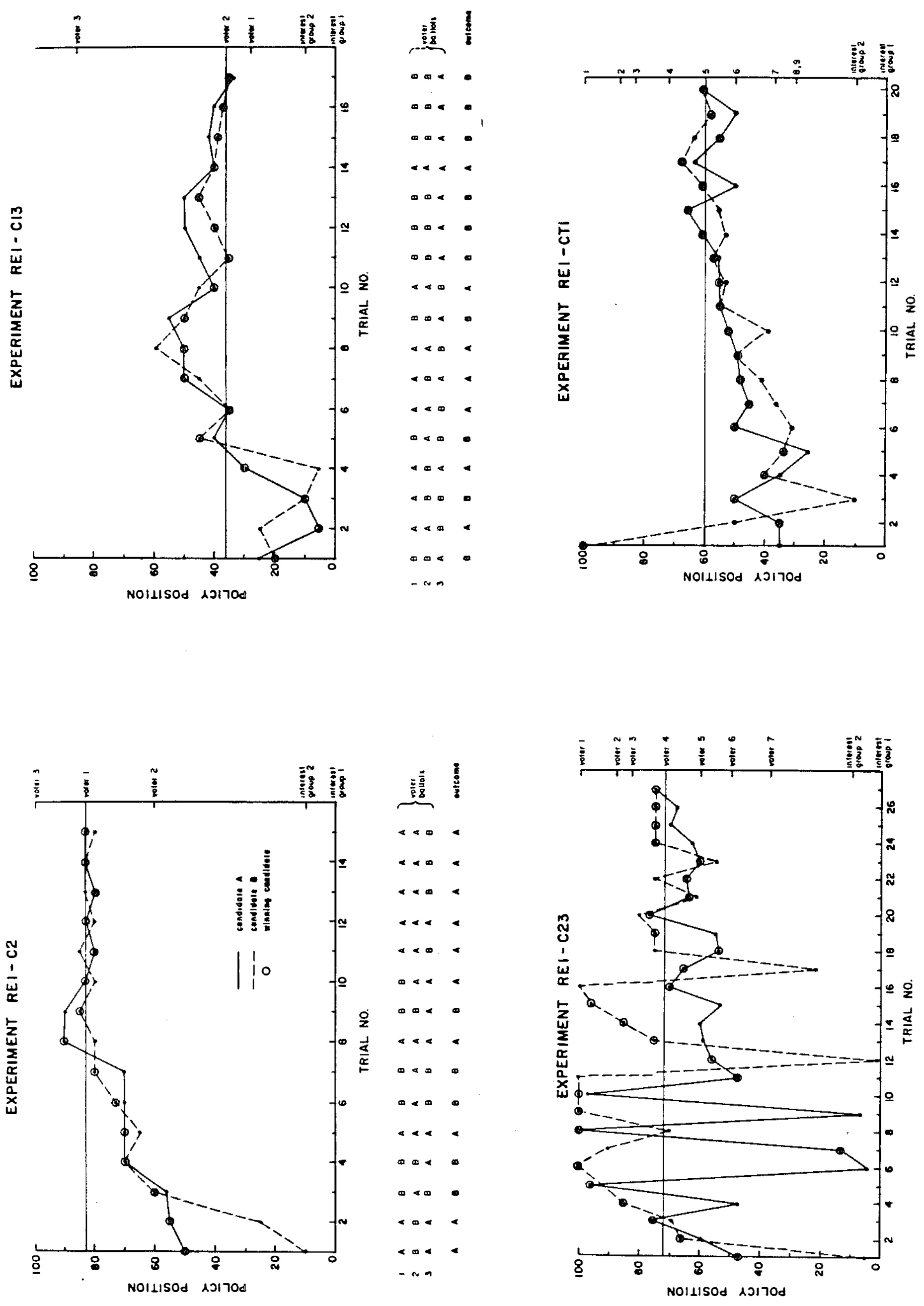


Figure 3
Typical Experimental Outcomes

to the correct equilibrium even though there is substantial error in voter behavior.[2] From trial 1 thru trial 8, the unendorsed candidate always wins, and the candidates correctly interperet this as a signal to move up. Finally, in trial 9, the endorsed candidate wins, and as far as the candidates are concerned, this pins down the location of the equilibrium position. From then on the candidates attempt to fine tune their positions around the correct equilibrium, even despite substantial error on the part of the voters (eg. voter 2 votes incorrectly in trials 10 and 12). Experiment C13 is a good example illustrating the way in which voters can learn how to use the information available to them to vote correctly. From trial 8 thru trial 16, all voters vote correctly in every trial. The reason voters can do this is that by this time, the general location of the candidates is established to be between, say, 35 and 55. But if the voters know that the candidate position will be in this range, then the endorsement is all the information that the voter needs to know in order to vote correctly.

Experiment C23 is illustrative of a pattern of candidate behavior that occurred in several of the experiments. Here, the candidates realised that the voters observe only the endorsement, so they simply fought to receive or avoid receiving the endorsement. This led to quite unstable behavior until the candidates learned that development of a reputation is important since the voters would punish extremist candidates in subsequent elections. Finally, Experiment CT1 is an experiment in which one of the candidates (candidate B) seems quite slow to respond to the persistent signals to move up. Thus Candidate A is able to run up a long series of victories by himself slowly moving in the direction the voters are urging.

In lieu of reviewing all eleven experiments in such detail, we provide a more compact summary by first looking at the voter behavior, and then at candidate behavior.

Voters

We first assess the extent to which the voters act as if they have complete information. We consider all noninitial trials of all one dimensional experiments in which the candidates adopt distinct positions. Voting is consistent with a perfectly informed choice 82% of the time. Furthermore, this correspondence rises from 76% in the first five trials to 83% in the last five. (See Row 1 of Table 1). It appears, then, that voting becomes more informed as the experiment progresses. Minimally, we can reject the hypothesis that subjects are choosing randomly.

Of course, it is unreasonable to expect voters to act as if they are completely informed, since whether or not they vote correctly is affected by strategic choices of the candidates. Thus, if the candidates err, and do not choose in accordance with the information they posess, we cannot expect subjects to vote as if they are informed. Further, we would only expect voters to vote in a completely informed manner once a REE in candidate strategies is reached. On the path to equilibrium, while participants are still

	All Trials (except first)		First 5 Trials		Trials 11-15		Last 5 Trials	
Full Information	.82	(1314)	.76	(217)	.84	(316)	.83	(300)
Partial Information Model I (J=1)	.85	(1363)	.80	(227)	.88	(324)	.87	(310)
Partial Information Model II (r=4.16)	.85	(1350)	.78	(221)	.88	(324)	.87	(311)
Total n		(1597)		(284)		(370)		(360)

Table 1

Percentage Correct Votes under Assumption of Full and Partial Information

	First 5 Trials	Trials 11-15	Last 5 Trials
CONST	11.5(11.3)	1.67(6.01)	2.63(4.15)
MED	.69*(.177)	.950*(.094)	.908*(.065)
SE	20.1‡#	10.6‡	7.35‡
R^2	.22	.66	.78

Table 2

Regression Estimates for Winning Candidate Positions (Standard Errors in Parentheses)

* indicates coefficient significant at level .05

‡ indicates rejection of $H_0:(\alpha,\beta) = (50,0)$ at level .05

# indicates rejection of $H_0:(\alpha,\beta) = (0,1)$ at level .05

learning, we would expect behavior more like that in Section 4. We therefore investigate the degree to which the voting is correct given a learning model as described in Section 4.

Specifically, we assume that voters perform a regression on the past positions of the winning candidates in order to formulate their beliefs of the likely positions at time t. Voters assume both candidates will adopt strategies drawn from this predicted distribution.

We let z_j represent the position of the winning candidate in period j. In period t, the voters have observed z_j for $j < t$, and we assume they use the model.

$$z_j = \mu_t + u_j, \tag{5.1}$$

where the u_j are assumed multivariate normal with

$$E(u_j) = 0$$
$$E(u_j u_\ell) = \begin{cases} 0 & \text{if } j \neq \ell \\ w_{t-j}\sigma_t^2 & \text{if } j = \ell \end{cases} \tag{5.2}$$

where $\sigma_t > 0$, $w_j > 1$ for all t,j. We will discuss assumptions on the w_j below. (Thus, the disturbances are assumed heteroskedastic, with different weight given to different observations). Estimating this model, for fixed r, yields estimates of the candidate distributions, for period t, of

$$\gamma_1^t = \gamma_2^t \sim N(\hat{\mu}_t, \hat{\sigma}_t^2), \tag{5.3}$$

where $\hat{\mu}_t$ and $\hat{\sigma}_t$ are the OLS estimates of μ_t and σ_t in (5.3). As discussed in the previous sections, we then assume that voters choose

$b^t \in B(\gamma^t)$. Since the γ_k^t are symmetric, Lemma 3 of Appendix A can be applied to give the prediction that for each t, and all $\alpha_i \in N$,

$$\begin{aligned} y_i^* < \hat{u}_t &\Rightarrow b_i^t(k) = k \\ y_i^* > \hat{u}_t &\Rightarrow b_i^t(k) = \bar{k}. \end{aligned} \tag{5.4}$$

In the data analysis that follows, we report how well the above model predicts individual voting behavior. We consider this model with two different assumptions on the form of the w_j:

$$\begin{aligned} &\text{Model 1:} \quad w_j = 1 \quad \text{for } j \leq J, \; w_j = \infty \quad \text{for } j > J. \\ &\text{Model 2:} \quad w_j = r^j \quad \text{for some } r > 1. \end{aligned} \tag{5.5}$$

So Model 1 assumes that the previous J periods enter into the regresssions with equal weight, and all periods previous to t − J are discarded. I.e., μ_t is estimated as a J period moving average of winning positions in the previous J periods. Model 2 assumes that all previous periods enter into the estimation, but that more recent data is given more weight (i.e., has less variance) than older data.

We pick J in Model 1 and r in Model 2 to maximize the number of correctly predicted votes. Using these models, the number of correct predictions is given in the second two rows of Table 1. As can be seen, both models are essentially equivalent in terms of their ability to predict votes. About 85% of all the votes cast are correct under both models, with this proportion rising to about 87–88% for the latter trials.

The second feature of these estimates is that for both Model 1 and Model 2, we see that voters tend to discard old data fairly quickly. The best fitting J for Model 1 yields J = 1, which implies that voters ignore all data except the last trial. Similarly in Model 2, the optimal value of r is 18.9 implying that data from period t − 2 is assigned a standard deviation $\sqrt{18.9} = 4.16$ that of period t − 1, implying that it is weighted only .23 as heavily as data from period t − 1 in computing $\hat{\mu}_t$.

We conclude that voters do not choose randomly, but use the limited information available to them to make understandable choices, and learn how to use this information better as the experiment proceeds. Further, there is some evidence to support the type of learning model postulated in Section 4--namely that voters use past winning positions to predict future candidate positions. However the evidence suggests that this learning model is not nearly as static as is assumed in Section 4. Rather, voters put much more weight on recent data than on past data. Thus, they perhaps attempt to take into account and anticipate the candidate learning which is taking place simultaneously to the voter learning. This may explain why the experiments converge towards a REE faster than would be expected under the dynamic story presented in Section 4.

Candidates

Under full information, we would expect candidates to converge to the ideal point of the median voter. To evaluate this prediction, the position of the median voter varies across experiments, which permits us to compare the candidates' strategies against the true

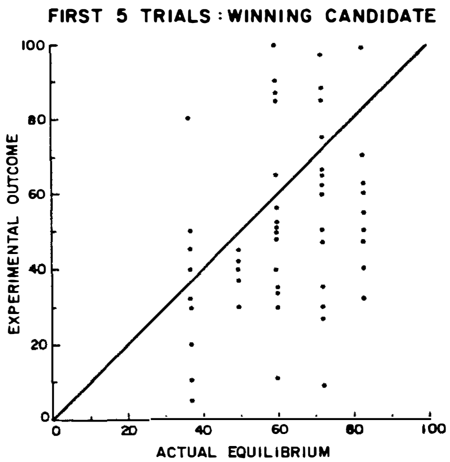
median voter ideal point. Figure 4 summarizes the strategies of winning candidates across all eleven one dimensional experiments. Figure 4c plots the final five trials of each experiment, and shows that the actual outcomes are closely scattered around the predicted outcomes. For purposes of comparison, Figures 4a and 4b, which graph outcomes for the first five trials and Trials 11 through 15, show the convergence to the predicted outcomes that occurs as the experiments proceed.

Table 2 presents the results of the simple regressions corresponding to Figure 4. We write MED_m for the actual median, y^*, in experiment m, and z_{mt} for the winning candidate's strategy, in period t of experiment m. We then estimate

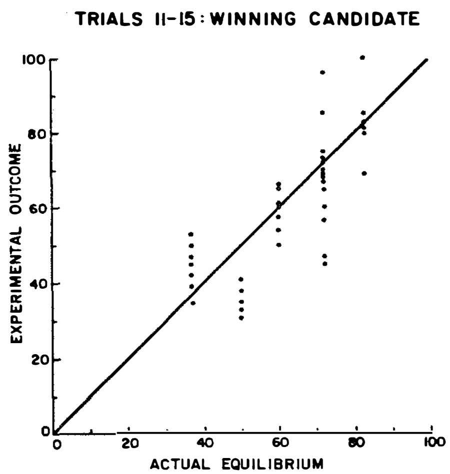
$$z_{mt} = \alpha + \beta MED_m + u_{mt}. \tag{5.6}$$

As these regressions show, in every case, we can reject the hypothesis of no association ($\alpha = 50, \beta = 0$), but in only the first five periods do we reject the hypothesis that the information is extracted ($\alpha = 0, \beta = 1$). The theoretical prediction that $\alpha = 0$ and $\beta = 1$ is more closely approximated in the latter trials, where r^2 and the t-statistics on α and β improve considerably. The standard error around the regression--an indication of how tightly the candidates have converged--decreases from the first five trials to the latter trials.

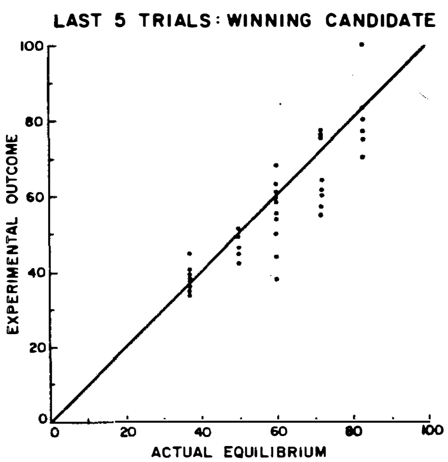
Figure 4 and Table 2 present the data only on winning candidates, so this measures how well the candidates <u>and</u> voters extract the full information. If we perform the same regressions as above using data for <u>both</u> the winning and losing candidates, we obtain



(a)



(b)



(c)

Figure 4

Winning Candidate Position
vs Median Voter

the results presented in Table 3a. Here, we see that the fit is worse in all cases than in Table 2, but the same general qualitative conclusion holds. We always reject the hypothesis of no effect, with tighter fits as time increases. However, note that we also reject the hypothesis that $\beta = 1$, so the candidates themselves do not extract as much information as is extracted by the combination of candidate strategies and voter behavior.

As with voters, we do not expect candidates to behave as if they have complete information for two reasons. First, if voters make errors, we would expect these errors to be compounded in voter behavior. Secondly, we do not expect perfectly informed behavior until all information is extracted. Rather, we expect a learning model as described in Section 4. We can test whether candidates behave according to such a learning model by assuming that they update their beliefs of the optimal strategies for time t in the same way that voters update their beliefs. They thus obtain estimates $\lambda_k^t \sim N(\hat{\mu}_t, \hat{\sigma}_t^2)$ for the optimal strategies at time t, and we would expect their actual strategy choices in period t to be distributed about $\hat{\mu}_t$. We write $MOD1_{mt}$ and $MOD2_{mt}$ for the estimate $\hat{\mu}$ in period t of experiment m according to model 1 and 2 respectively. We then estimate the following model on all candidates (winning and losing):

$$z_{mt} = \alpha + \beta MED_{mt} + \eta MOD2_{mt} + u_{mt} \qquad (5.7)$$

Because of the high colinearity between MOD1 and MOD2, we enter only the latter. Results for Model 1 are quite similar.

The results of estimating (5.7) are presented in Table 3b. As we see, the fit is considerably better than the bivariate model of Table 3a in all cases. Further, the variable MOD2 is always significant, and much of the apparent effect of MED in Table 3a is captured by MOD2 when it is entered in the equation. MED is only significant in Trials 11-15. As before the standard error drops as learning progresses, with MOD2 seemingly becoming stronger with time. Finally, note that while MED is not significant in Trials 2-6 or the last 5 trials, that the coefficient is always positive. The candidates' position seems to be almost a convex combination of MOD2 and MED, with MOD2 receiving most of the weight. This suggests that there may be some misspecification in the learning model, and that candidates converge to the median voter faster than predicted by MOD2. (It should be noted that these conclusions still hold when MOD1 is entered in the above equations).

We conclude that candidates, like voters, do a reasonable job of extracting the information available, converging quite rapidly towards strategies centered near the median voter. As with voters, we see that learning takes place faster than suggested by the dynamic of Section 4. Further there is some evidence that the candidate learning is faster than voter learning.

Two Dimensional Experiments

While our theoretical model is limited to one-dimensional contests, it is important to ascertain experimentally whether a

	First 5 Trials	Trials 11-15	Last 5 Trials
CONST	22.6(9.47)	8.42(5.44)	5.74(3.32)
MED	.444(.147)	.820(.085)	.853(.052)
SE	23.7	13.6	8.31
R^2	.07	.46	.72

(a)

	First 5 Trials	Trials 11-15	Last 5 Trials
CONST	12.9(8.48)	4.19(4.91)	1.86(2.85)
MED	.136(.140)	.304*(.122)	.197(.106)
MU2	.595*(.103)	.605*(.113)	.747*(.110)
SE	20.8	12.1	6.98
R^2	.30	.58	.80

(b)

Table 3

Regression Estimates for All Candidate Positions
(Standard Errors in parentheses)

* coefficient significant at level .05

multi-dimensional extension is promising empirically. In this section, then, we report briefly on three 2-dimensional experiments that replicate the information conditions of our one-dimensional experiments, except that candidates must now compete over two issues simultaneously (represented by a grid that varies between 0 and 100 on each dimension). To induce preferences, voters are given two payoff charts, one for each "issue". For a given position, say (x,y), voters must compute the payoff attributable to each issue $u_x(x)$ and $u_y(y)$, and add them together. The payoff functions are of the form $u_x(x) = -k(x - x_i^*)^2$, and $u_y(y) = -k(y - y_i^*)^2$. So preferences over the two dimensional issue space have circular indifference curves, with ideal point at (x_i^*, y_i^*). Hence, to guarantee the existence of a majority rule equilibrium point (a Condorcet winner), it is sufficient to construct a radially symmetric distribution of ideal points in the two-dimensional space. Candidates, in each trial, again must simultaneously adopt positions on both issues. Voters are supplied with two endorsements that tell them which candidate is to the left on issue x and which is to the left on issue y. This situation, then, might correspond to the endorsements of two interest groups that are each concerned with a single, different issue.

A sample 2-dimensional experiment is given in Figure 5, and a summary of the experimental outcomes for the winning candidates is given in Table 4. Table 4 parallels Table 2 for the 1-dimensional experiments and reveals a noticable convergence towards points near the Condorcet equilibrium in later trials. The standard error of the

	First 5 Trials	Trials 11-15	Last 5 Trials
CONST	-20.7(12.1)	-21.7(7.60)	-8.5(6.43)
MED β	1.42*(.235)	1.48*(.147)	1.24*(.125)
SE	16.19	10.16	8.60
R^2	.57‡#	.78‡#	.78‡

Table 4

Regression Estimates of Candidate Position vs. Median Voter in Two Demensional Experiments (Winning Candidates Only)

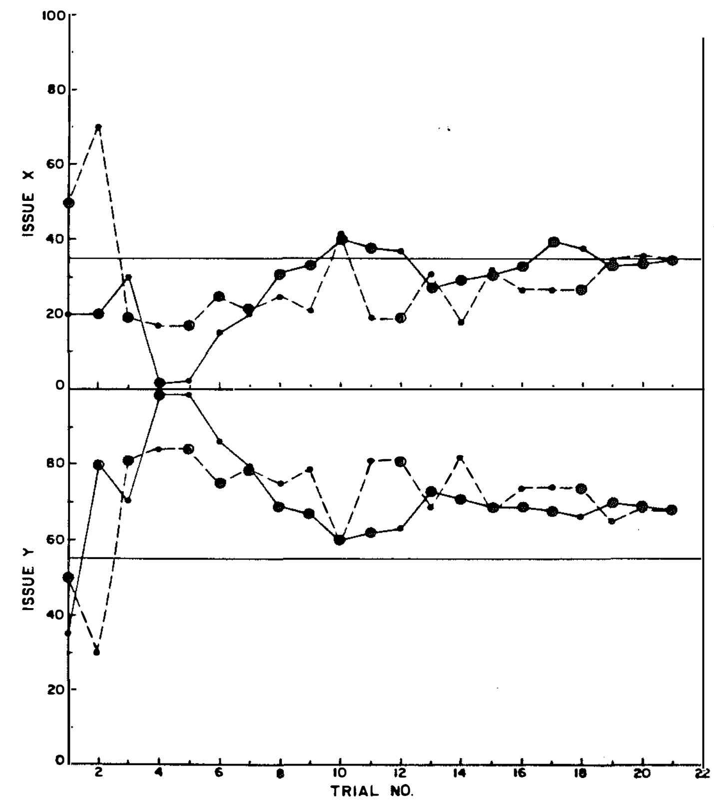


Figure 5

A Sample Two Dimensional Experiment

regression of observed winning positions, z_{mt}, against the actual issue median, MED_m, is noticeably smaller in the last five trials than in Trials 11-15 and in the first five.

Unquestionably, subjects found these 2-dimensional experiments more challenging than their 1-dimensional counterparts. We do not have any dynamic learning model which parallels our analysis of the one dimensional experiment. Yet, overall, these experiments suggest that voters and candidates can function with limited information in a 2-dimensional space. Somewhat surprisingly, in fact, the statistics reported in Table 4 are quite comparable to those reported in Table 2. Thus, these experiments give us some confidence that our theoretical work might be extended to the more general multi- dimensional context.

5. Conclusion

The preceeding theory and experiments show that the strong assumptions about voter and candidate information generally used in formal models of elections are not necessary to guarantee the convergence of candidates to majority dominant policies.

Our analysis has several implications for the more traditional literature in democratic theory. It has long been thought that a necessary condition for the successful operation of democratic systems under majority rule is an informed electorate. If voters are generally unaware of the candidates' positions on issues or if voters are unable to articulate their preferences in terms of the issues over which the election is contested, then there seems no guarantee that

"appropriate" policies will be choosen by candidates and implemented by winners. Hence, there has been a general lament by political scientists and journalists over the empirical fact that citizens come nowhere near the ideal of perfect information. A second concern, which has yielded numerous confrontations between politicians and the news media, is the degree to which the media and other "opinion elites" affect the outcome of elections. The concern is that such elites are not representative of the "majority" and that they exercise undue influence over public policy through their ability to endorse candidates for election (In the case of the news media, the argument goes further, of course, and concerns their ability to present biased or unfair accounts of candidates.)

Our experiments and the theory that structures their design, however, suggest that two candidate, democratic systems can work (in the sense of aggregating and responding to all relevant information about voter preferences) even if voters have no direct knowledge of candidate positions and candidates have no direct knowledge of voter preferences. Second, in equilibrium, interest groups need not affect policy outcomes. The voters use endorsements to gather information, but in equilibrium that is the only role such endorsements play insofar as candidates converge to the full information equilibrium-- the median electoral preference.

The limitations of these results, however, warrant emphasis. First, the attainment of an equilibrium supposes a sequence of elections in which voter preferences and the identity of the candidates are constant. Second, our theoretical model is limited so far to one dimensional contests. Third, we do not consider other aspects of a citizen's calculus, including the decision whether to vote or to abstain. Fourth, interest groups in our model are simply passive agents who do not attempt to influence the candidates directly (for example through campaign contributions or endorsements that are contingent on policy concessions). Each of these limitations can, in principal, be addressed, and in future research we will attempt to do so.

Elsewhere, we develop a single period election model that relies on public opinion polls instead of historical data as an information source for the uninformed voters [McKelvey and Ordeshook, 1984a,b]. We establish conditions in that model under which such polls are sufficient to induce full information equilibria. It should be pointed out that both the model developed here and the one developed in [1984a,b] are bare bones models, in which the information source for all voters is very limited. However, any more realistic model would have voters obtaining information from several different sources -- for example polls, interest group endorsements and the historical record. Perhaps different voters would obtain information from different sources. We would expect that in any such model the different informations sources would reinforce each other to increase the robustness of democratic institutions under incomplete information.

A rational expectations approach, then, may sharply alter the

way we view the relationship between political attitudes and voting. As distinct from the research that documents citizen ignorance about the policy positions of candidates on specific issues, we conclude that precise knowledge may not be required for voters to cast a correct vote, or for candidates to converge to the median voter. All that is required is an appropriate set of endogenous variables which voters can use to make inferences about candidate positions. In this paper we have shown that in the context of repeated elections, historical data together with contemporaneous endorsement data is sufficient.

FOOTNOTES

1. Actually, two endorsements are announced--one corresponding to a group with its ideal point at 0 and the other with its ideal point at 10--although only one is required theoretically since these endorsements are in general redundant. Two groups are included, however, in anticipation of some future experiments.

2. By "err" we mean the voter votes incorrectly given complete information (which the voter does not have). However the voter "errors" discussed here are also errors according to the weaker test--that the voter votes for the best candidate given a reasonable projection of the likely candidate positions. See (5.4) below.

Appendix A

This appendix contains proofs of the proposition and theorems of Section 3. The notation in this appendix follows that of Section 3. Before proving these results, we give some preliminary definitions and round up a herd of lemmas.

For any $A \subseteq S$, and $k \in K$, we let $\pi_k(A) = \{x \in S_k | x = s_k \text{ for some } s \in S\}$ be the projection of A on dimension k. For $k \in K_0$, we let $E_k = \{s \in S | e(s) = k\}$ be the set of strategy pairs where candidate k is endorsed, and for $b \in B$, $W_k(b) = \{s \in S | w(s,b) = k\}$ be the set of strategies where candidate k wins. Given $\lambda = (\lambda_1, \lambda_2) \in \Lambda$, we define some derived probability measures. For each $k \in K$ and $j \in K_0$, define the probability measures λ_{kj} as follows: For $C \in \mathbb{B}$,

$$\lambda_{kj}(C) = \lambda(\pi_k^{-1}(C) | E_j) = \frac{\lambda(\pi_k^{-1}(C) \cap E_j)}{\lambda(E_j)}. \tag{A1}$$

So λ_{kj} represents the distribution of k's positions when he is endorsed. We also define, for any $b \in B$, the measure $\lambda_{kw}(\cdot | b)$ by, for all $C \in \mathbb{B}$,

$$\lambda_{kw}(C|b) = \begin{cases} \dfrac{\lambda(\pi_k^{-1}(C) \cap W_k(b)) + \frac{1}{2}\lambda(\pi_k^{-1}(C) \cap W_0(b))}{\lambda(W_k(b)) + \frac{1}{2}\lambda(W_0(b))} & \text{if } \lambda(W_{\bar{k}}(b)) \neq 1 \\ \lambda_{\bar{k}}(C) & \text{otherwise} \end{cases} \tag{A2}$$

So λ_{kw} represents the distribution of k's positions when he <u>wins</u> the election--except if k never wins, then λ_{kw} is defined as the distribution of $\bar{k}$'s winning positions. Note that in defining $\lambda_{kw}(\cdot|b)$, we must take account of the possibility that k wins outright as well as the probability that k wins the coin toss when there is a tie. Given $\lambda \in \Lambda$, and $b \in B$, we let $\lambda_w(\cdot|b)$ denote the overall distribution of winning positions. So for $C \in \mathbb{B}$,

$$\lambda_w(C|b) = \sum_{k \in K} \lambda_{kw}(C|b)[\lambda(W_k(b)) + \tfrac{1}{2}\lambda(W_0(b))]$$

$$= \sum_{k \in K} [\lambda(\pi_k^{-1}(C) \cap W_k(b)) + \tfrac{1}{2}\lambda(\pi_k^{-1}(C) \cap W_0(b))] \qquad (A3)$$

<u>Lemma 1</u> If $(\lambda^*, b^*) \in \Lambda \times B$ characterizes a REE, then it satisfies the following conditions

(a) For all $\alpha_i \in N$ and all $j \in K$, all $k \in K_0$ with $\lambda^*(E_k) > 0$, b_i^* satisfies

$$\mathbb{E}_{\lambda_{jk}}^* [u_i(x)] > \mathbb{E}_{\lambda_{\_jk}^*} [u_i(x)] \Rightarrow b_i^*(k) = j.$$

(b) $w(s,b^*)$ is constant for λ^* a.e. $s \in S$.

(c) For each $k \in K_0$, if $\lambda^*(E_k) > 0$, then $\exists \alpha_i \in N$ for which

$$\mathbb{E}_{\lambda_{1k}}^* [u_i(x)] = \mathbb{E}_{\lambda_{2k}}^* [u_i(x)].$$

<u>Proof</u>

(a) Note that $w(s,b)$ depends on s only through the dependence of b on s. Hence we write $w(s,b) = w(b(e(s)))$. Then, by (V1),

$$b_i^* \in \arg\max_{b_i^* \in F} \mathbb{E}_{\lambda^*}[M_i(s,b|b_i^*)] \quad \text{for all } b \in \underline{B},$$

But

$$\mathbb{E}_{\lambda^*}[M_i(s,b|b_i^*)] = \mathbb{E}_{\lambda^*}[u_i(s_{w[(b|b_i^*)(e(s))]})]$$

$$= \sum_{k \in K_0} \int_{E_k} u_i(s_{(w(b|b_i^*)(k))} \, d\lambda^*(s) \qquad (A1)$$

so we maximize (A1) by picking b_i^* so that for each $k \in K_0$, with $\lambda^*(E_k) > 0$, then

$$b_i^*(k) \in \arg\max_{b_i^*(k) \in K_0} \int_{E_k} u_i(s_{w(b|b_i^*)(k)}) d\lambda^*(s) \qquad (A2)$$

but since w is monotonic, and (A2) must hold for all $b \in B$, this is equivalent to, for all $k \in K_0$, with $\lambda^*(E_k) > 0$

$$b_i^*(k) \in \arg\max_{b_i^*(k) \in K_0} \int_{E_k} u_i(s_{b_i^*(k)}) d\lambda^*(s)$$

or

$$b_i^*(k) \in \arg\max_{j \in K_0} \int_{E_k} u_i(s_j) d\lambda^*(s)$$

but for $j \in K$,

$$\int_{E_k} u_i(s_j) d\lambda^*(s) = \int_{S_j} u_i(s_j) \left[\int_{S_{E_k}} d\lambda_{\_j}^*(s_{\_j})\right] d\lambda_j^*(s_j)$$
$$= \int u_i(x) d\lambda_{jk}^*(x) = \mathbb{E}_{\lambda_{jk}}{}^*[u_i^k(x)]$$

so the result follows.

(b) We write $w(s,b^*) = w(b^*(e(s)))$, and for any $k \in K_0$, write $w_k = w(b^*(k))$. Also, we write $E_k = \{s|e(s) = k\}$ for $k \in K_0$. Thus,

$$s \in E_k \Rightarrow w(s,b^*) = w_k$$

If $\lambda^*(E_k) = 1$ for any $k \in K_0$, then the result follows trivially, so we assume that $\lambda^*(E_k) < 1$. But by the assumptions on Λ, $\lambda^*(E_0) = 0$ or

$\lambda^*(E_0) = 1$, hence we have $\lambda^*(E_0) = 0$ and $0 < \lambda^*(E_k) < 1$ for all $k \in K$. We now have two cases:

<u>Case 1</u> $w_k \neq 0$ for any $k \in K$

Let $k \in K$, and let $w_k = j$. We then show that $w_{\bar{k}} = j$. Suppose not. Then $w_k = j$ and $w_{\bar{k}} = \bar{j}$. Now from (C1), we have, for all $C \in \underline{B}$,

$$\lambda_j^*(C) = \frac{\lambda^*(\pi_j^{-1}(C) \cap E_k)}{\lambda^*(E_k)} = \lambda_{jk}^*(C)$$

and

$$\lambda_{\bar{j}}^*(C) = \frac{\lambda^*(\pi_{\bar{j}}^{-1}(C) \cap E_{\bar{k}})}{\lambda^*(E_{\bar{k}})} = \lambda_{\bar{j}\bar{k}}^*(C)$$

By Lemma 2, it follows that λ_j^* and $\lambda_{\bar{j}}^*$ are both degenerate point densities, implying that $\lambda^*(\{s\}) = 1$ for some $s \in S$. But this contradicts the fact that $\lambda^*(E_0) = 0$ and $\lambda^*(E_k) > 0$ for all $k \in K$. Hence we must have $w_k = w_{\bar{k}}$. So, since $\lambda^*(E_k \cup E_{\bar{k}}) = 1 - \lambda^*(E_0) = 1$, it follows that $w(s,b^*) = w_k$ for λ^* a.e. $s \in S$.

<u>Case 2</u> $w_k = 0$ for some $k \in K$.

Let $w_k = 0$, and let $\bar{k} \in K - \{k\}$. We will show that $w_{\bar{k}} = 0$. Assume $w_{\bar{k}} \neq 0$, say $w_{\bar{k}} = j \in K$, and let $\bar{j} \in K - \{j\}$. Then from (C1), we have, for all $C \in \underline{B}$,

$$\lambda_j^*(C) = \frac{\lambda^*(\pi_j^{-1}(C) \cap E_{\bar{k}}) + \frac{1}{2}(\pi_j^{-1}(C) \cap E_k)}{\lambda^*(E_{\bar{k}}) + \frac{1}{2}\lambda^*(E_k)}$$

$$= \frac{\lambda^*(\pi_j^{-1}(C)) - \frac{1}{2}\lambda^*(\pi_j^{-1}(C) \cap E_k)}{1 - \frac{1}{2}\lambda^*(E_k)}$$

$$= \frac{2\lambda_j^*(C) - \lambda^*(E)\lambda_{jk}^*(C)}{2 - \lambda^*(E_k)}.$$

Solving this for $\lambda_j^*(C)$ yields

$$\lambda_j^*(C) = \lambda_{jk}^*(C).$$

Also applying (C1) we have, for all $C \in \underline{B}$,

$$\lambda_{\bar{j}}^*(C) = \frac{\lambda^*(\pi_{\bar{j}}^{-1}(C) \cap E_k)}{\lambda^*(E_k)} = \lambda_{\bar{j}k}^*(C).$$

By Lemma 2, it follows that λ_j^* and $\lambda_{\bar{j}}^*$ are both degenerate point densities. As in Case 1, this yields a contradiction, unless $w_{\bar{k}} = 0$. But then $w(s,b^*) = 0$ for λ^* a.e. $s \in S$.

(c) If $\lambda^*(E_0) = 1$, then the result is trivial, so we assume $\lambda^*(E_0) = 0$. There are 2 cases.

<u>Case 1</u> $w(s,b^*) = k \neq 0$ for λ^* a.e. $s \in S$. In this case, it follows from (C1) that $\lambda_1^* = \lambda_2^*$. Hence $\lambda_{11}^* = \lambda_{22}^*$ and $\lambda_{12}^* = \lambda_{21}^*$. From Lemma 1a it follows that if $b_i^*(1) \neq 0$, then $b_i^*(1) = j \Rightarrow b_i^*(2) = \bar{j}$. So $b^*(1) = k \Rightarrow b^*(2) = \bar{k}$, a contradiction unless $b_i^*(1) = 0$ for some $\alpha_i \in N$. But then, for this i, by Lemma 1a, we have that

$$\mathbb{E}_{\lambda_{11}^*}[u_i(x)] = \mathbb{E}_{\lambda_{21}^*}[u_i(x)].$$

A similar argument shows that $b_i^*(2) = 0$ for some $\alpha_i \in N$, from which it follows that $\mathbb{E}_{\lambda_{12}^*}[u_i(x)] = \mathbb{E}_{\lambda_{22}^*}[u_i(x)]$ for some $\alpha_i \in N$.

<u>Case 2</u> $w(s,b^*) = 0$ for λ^* a.e. $s \in S$. In this case let $k \in K$ satisfy $\lambda^*(E_k) > 0$, and assume $b_i^*(k) \neq 0$ for all $a_i \in N$. Then since there are an odd number of voters, we must have $v_1(s,b^*) \neq v_2(s,b^*)$ for any $s \in E_k$. But then $w(s,b^*) \neq 0$ for $s \in E_k$, a contradiction. So we must have $b_i^*(k) = 0$ for some $a_i \in N$. But this implies, by Lemma 1a, that $E_{\lambda^*_{1k}}[u_i(x)] = E_{\lambda^*_{2k}}[u_i(x)]$.

Q.E.D.

For the next lemma, we need to define the notion of stochastic dominance. For any $c \in \mathbb{R}$, we define $L_c = \{t \in \mathbb{R} \mid t \leq c\}$. Given two measures λ, μ on the Borel sets of $\mathbb{R}$, we say that λ (<u>weakly</u>) <u>stochastically dominates</u> μ, written $\mu \preceq \lambda$ iff $\mu(L_c) \geq \lambda(L_c)$ for all $c \in \mathbb{R}$. We say λ (<u>strongly</u>) <u>stochastically dominates</u> μ, written $\mu \prec \lambda$ iff $\mu \preceq \lambda$ and it is not the case that $\lambda \preceq \mu$. Thus, λ weakly stochastically dominates μ whenever its cumulative density function is always less than or equal to that of μ. For strong domination, the two c.d.f.'s cannot be equal.

<u>Lemma 2</u> For each $k \in K$, $\lambda_{kk} \prec \lambda_k \prec \lambda_{k\bar{k}}$ whenever all measures are defined, unless $\lambda_k(\{t^*\}) = 1$ for some $t^* \in \mathbb{R}$. Further, for each $k \in K$, $\lambda_{kk} \prec \lambda_{\bar{k}k}$ whenever both measures are defined.

<u>Proof</u>: For $k \in K$, and $t \in \mathbb{R}$, let $F_k(t) = \lambda_k(L_t)$ be the cumulative density function of λ_k. Then from the definition of λ_{kj}, for any $C \in \mathbb{B}$, we have

$$\lambda_{k\bar{k}}(C) = \frac{1}{\lambda(E_{\bar{k}})}\int_C F_{\bar{k}}(t)\,d\lambda_k(t)$$

and

$$\lambda_{kk}(C) = \frac{1}{\lambda(E_k)}\int_C (1 - F_{\bar{k}}(t))\,d\lambda_k(t).$$

Now, since $F_{\bar{k}}(t)$ is a monotonic increasing function of t and $(1 - F_{\bar{k}}(t))$ is a monotonic decreasing function of t, the result that $\lambda_{kk} \prec \lambda_k \prec \lambda_{k\bar{k}}$ follows directly from Lemma 3.2 of McKelvey and Page [1984].

To see that $\lambda_{kk} \prec \lambda_{\bar{k}k}$, we note that, for $c \in \mathbb{R}$,

$$\lambda_{\bar{k}k}(L_c) = \frac{1}{\lambda(E_k)}\int_{-\infty}^{c} F_k(t)\,d\lambda_{\bar{k}}(t)$$

and

$$\lambda_{kk}(L_c) = \frac{1}{\lambda(E_k)}\int_{-\infty}^{c} (1 - F_{\bar{k}}(t))\,d\lambda_k(t)$$

But, writing $f_j(t)$ for the density function of λ_j, we can integrate by parts to obtain

$$\begin{aligned}\int_{-\infty}^{c}(1 - F_{\bar{k}}(t))\,d\lambda_k(t) &= F_k(c) - \int_{-\infty}^{c} F_{\bar{k}}(t) f_k(t)\,dt \\ &= F_k(c)[1 - F_{\bar{k}}(c)] + \int_{-\infty}^{c} f_{\bar{k}}(t) F_k(t)\,dt \\ &\geq \int_{-\infty}^{c} F_k(t)\,d\lambda_{\bar{k}}(t)\end{aligned}$$

Hence

$$\begin{aligned}\lambda_{kk}(L_c) &= \frac{1}{\lambda(E_k)}\int_{-\infty}^{c}(1 - F_{\bar{k}}(t))\,d\lambda_k(t) \\ &\geq \frac{1}{\lambda(E_k)}\int_{-\infty}^{c} F_k(t)\,d\lambda_{\bar{k}}(t) = \lambda_{\bar{k}k}(L_c)\end{aligned}$$

which proves $\lambda_{kk} \prec \lambda_{\overline{kk}}$

Q.E.D.

We let $\Lambda^S \subseteq \Lambda$ be the set of measures satisfying, for all $\lambda = (\lambda_1, \lambda_2) \in \Lambda$,

(a) $\lambda_1 = \lambda_2$

(b) λ_k is symmetric about t^* for some $t^* \in \mathbb{R}$ i.e. $\lambda_k(L_t) = 1 - \lambda_k(L_{2t^*-t})$ for all $t \in \mathbb{R}$.

Lemma 3 Let $u: X \to \mathbb{R}$ be symmetric and single peaked, with ideal point at 0, and let $\lambda = (\lambda_1, \lambda_2) \in \Lambda^S$, with λ not degenerate. For $k \in K$, define $\phi_k: X \to \mathbb{R}$ by

$$\phi_k(y) = \mathbb{E}_{\lambda_{\overline{kk}}}[u(x-y)] - \mathbb{E}_{\lambda_{kk}}[u(x-y)]$$

Then $\phi_k(y)$ has exactly one root at $y = x^*$, where $x^* = E_{\lambda_1}(x) = E_{\lambda_2}(x)$. Further $y < x^* \to \phi_k(y) > 0$ and $y > x^* \to \phi_k(y) < 0$.

Proof: We let f_1 and f_2 be the density functions of λ_1 and λ_2 respectively. Since $\lambda_1 = \lambda_2$, these density functions are identical, so we can write $f = f_1 = f_2$. We let g_1 and g_2 be the density functions for $\lambda_{\overline{kk}}$ and λ_{kk}. (Note that since $\lambda_1 = \lambda_2$, we have $\lambda_{11} = \lambda_{22}$ and $\lambda_{12} = \lambda_{21}$). Let $F(x) = \int_{-\infty}^{x} f(t)dt$ and $G_j(x) = \int_{-\infty}^{x} g(t)dt$ be the corresponding cumulative density functions. Then, writing u' for the first derivative of u,

$$\phi_k(y) = \mathbb{E}_{\lambda_{\overline{kk}}}[u(x-y)] - \mathbb{E}_{\lambda_{kk}}[u(x-y)]$$

$$= \int u(x-y)[g_1(x) - g_2(x)]dx$$
$$= \int u'(x-y)[G_2(x) - G_1(x)]dx$$

(The last step follows from integration by parts). We write $\varphi(x) = G_2(x) - G_1(x)$. Then we can write G_2 and G_1 as

$$G_2(y) = \int_{-\infty}^{x}[1 - F_1(t)]f_2(t)dt$$
$$G_1(y) = \int_{-\infty}^{x} F_2(t)f_1(t)dt$$

and then, using the fact that $F_1 = F_2 = F$, and $f_1 = f_2 = f$, we get

$$\varphi(x) = G_2(x) - G_1(x) = \int_{-\infty}^{x}[1 - 2F(t)]f(t)dt.$$

Now from Lemma 2, it follows that $\lambda_{\overline{kk}} \succ \lambda_{kk}$, so $G_2(x) - G_1(x) \geq 0$ for all x. Hence $\varphi(x)$ is nonnegative. Next, using the symmetry of $f(t)$ about x^*, and the fact that $F(x^*) = \frac{1}{2}$, it follows easily that $\varphi(x)$ is symmetric about x^*. I.e., $\varphi(x) = \varphi(2x^* - x)$ for all $x \in X$. Finally, since $\varphi'(x) = [1 - 2F(x)]f(x)$ is positive for $x < x^*$ and negative for $x^* < x$, it follows that $\varphi(x)$ is single peaked. Thus, we write

$$\phi_k(y) = \int u'(x - y)\varphi(x)dx \tag{A3}$$

where φ is nonnegative, symmetric and single peaked about x^*, and where $u'(t) = -u'(-t)$ for all t. But now, we can rewrite (A3) as

$$\phi_k(y) = \int_{-\infty}^{y} u'(t-y)\varphi(t)dt - \int_{y}^{\infty} u'(t-y)\varphi(t)dt$$
$$= -\int_{y}^{\infty} u'(y-r)\varphi(2y-r)dr - \int_{y}^{\infty} u'(t-y)\varphi(t)dt$$
$$= \int_{y}^{\infty} u'(t-y)[\varphi(2y-t) - \varphi(t)]dt$$

So if $x^* \leq y$, then for $y \leq t$, we have $2x^* - t \leq 2y - t \leq t$, so, using the symmetry and single peakedness of φ,

$$\varphi(2y-t) \geq \varphi(2x^*-t) = \varphi(t).$$

And if $y \leq x^*$, then for $y \leq t$, we have $2y - t \leq t \leq 2x^* - (2y-t)$ so

$$\varphi(t) \geq \varphi(2x^*-2y+t) = \varphi(2y-t)$$

In both cases, these become strict inequalities if $y \neq x^*$, and are equalities when $y = x^*$, hence, since $u'(t-y) < 0$ for $y \leq t$, we get

$$y < x^* \Rightarrow \phi_k(y) > 0$$
$$y = x^* \Rightarrow \phi_k(y) = 0$$
$$y > x^* \Rightarrow \phi_k(y) < 0$$

Hence, ϕ_k has a unique root at $y = x^*$, as we wished to show.

Q.E.D.

<u>Proof of Proposition 1</u>

That $\gamma^* = \lambda^*$ follows directly from (V2) and (C1) of the definition of a REE. Then (V1′) and (C1′) are immediate consequences of the fact that $\gamma^* = \lambda^*$. The last assertion of the proposition follows directly from Lemma 1a.

<u>Proof of Theorem 1</u> We show that not (a) implies (b).

If (a) does not hold, then $\lambda_1 = \lambda_2$, and neither is degenerate, hence $\lambda(E_k) \neq 0$ for $k \in K$. For each $\alpha_i \in N$, define $v_i(y) = u_i(y+y_i^*)$. So $u_i(x) = v_i(x-y_i^*)$, where v_i is symmetric and single peaked about 0. Write

$$\phi^i(y) = \mathbb{E}_{\lambda^*_{\bar{k}k}}[v_i(x-y)] - \mathbb{E}_{\lambda^*_{kk}}[v_i(x-y)]$$

By Lemma 3, it follows that

$$y < x^* \Rightarrow \phi^i(y) > 0$$
$$y = x^* \Rightarrow \phi^i(y) = 0$$
$$y > x^* \Rightarrow \phi^i(y) < 0.$$

But $\phi^i(y_i^*) = \mathbb{E}_{\lambda^*_{kk}}[u_i(x)] - \mathbb{E}_{\lambda^*_{kk}}[u_i(x)]$, and by Lemma 1a, it follows that

$$\phi^i(y_i^*) > 0 \Rightarrow b_i^*(k) = k$$
$$\phi^i(y_i^*) < 0 \Rightarrow b_i^*(k) = \bar{k}.$$

Hence

$$x_i^* < x^* \Rightarrow b_i^*(k) = k \quad \text{and} \quad b_i^*(\bar{k}) = \bar{k}$$
$$x_i^* > x^* \Rightarrow b_i^*(k) = \bar{k} \quad \text{and} \quad b_i^*(\bar{k}) = k$$

Since in equilibrium, we need $w(s,b^*)$ constant for λ^* a.e. $s \in S$ (see Lemma 1b), it follows that we need $x^* = y_m^*$, where y_m^* is the median of the y_i^*.

Q.E.D.

<u>Proof of Theorem 2</u> Let (λ^*,b^*) characterize a stable REE. We note, first, from the definition of stability, that for any $b' \in \underline{B}(\lambda^*)$, $\lambda_w^*(\cdot|b') = \lambda_w^*(\cdot|b^*)$, so (λ^*,b') characterizes a REE. But now, define $b' \in \underline{B}(\lambda^*)$ as follows

$$b_i'(k) = \begin{cases} b_i^*(k) & \text{if } E_{\lambda^*_{kk}}(u_i(x)) \neq E_{\lambda^*_{\bar{k}k}}(u_i(x)) \\ e(k) & \text{otherwise} \end{cases}$$

Clearly, $b' \in \underline{B}(\lambda^*)$. Further, we have $w(s,b') = e(s)$ for all $s \in S$.

Hence

$$\lambda_w^*(C|b') = \lambda^*(E_k)\lambda_{kk}^*(C) + \lambda^*(E_{\bar{k}})\lambda_{\bar{k}\bar{k}}^*(C).$$

But, for any $c \in \mathbb{R}$,

$$\begin{aligned}\lambda_w^*(L_c|b') &= \lambda^*(E_1)\lambda_{11}^*(L_c) + \lambda^*(E_2)\lambda_{22}^*(L_c)\\ &= \int_{-\infty}^{c}(1 - F_2(x))f_1(x)dx + \int_{-\infty}^{c}(1 - F_1(x))f_2(x)dx\\ &= F_1(c) + F_2(c) - [\int_{-\infty}^{c}F_2(x)f_1(x)dx + \int_{-\infty}^{c}F_1(x)f_2(x)dx]\\ &= F_1(c) + F_2(c) - F_1(c)F_2(c)\\ &= 1 - (1 - F_1(c))(1 - F_2(c)) \geq F_1(c)\\ &\qquad\qquad\qquad\qquad \geq F_2(c)\end{aligned}$$

with strict inequality when $F_1(c) \neq F_2(c)$ or $0 < F_k(c) < 1$. Hence $\lambda_w \prec \lambda_1$ and $\lambda_w \prec \lambda_2$ but then $\lambda_w \prec \frac{1}{2}\lambda_1 + \frac{1}{2}\lambda_2 = \lambda^*(w|b^*)$. Hence $\lambda_w^*(\cdot|b') \neq \lambda_w^*(\cdot|b^*)$, a contradiction, unless $\lambda_1^*(\{y\}) = \lambda_2^*(\{y\}) = 1$ for some $y \in X$.

We must now only show that $y = y^*$. Suppose not, assume w.l.o.g. that $y < y^*$. Then for any neighborhood $N(\lambda_k^*)$ of λ_k^*, we pick y' such that $y < y' < y^*$ and set $\lambda_k'(\{y'\}) = 1$. Then if y' is chosen so that $\lambda_k' \in N(\lambda_k^*)$, we have $b' \in \underline{B}(\lambda') \Longleftrightarrow$

$$y_i^* < \frac{y + y'}{2} \Rightarrow b'(k) = k$$
$$y_i^* > \frac{y + y'}{2} \Rightarrow b'(k) = \bar{k}.$$

but since $\frac{y + y'}{2} < y^*$, it follows that $w(s,b') = \bar{e}(s)$.

Hence

$$\lambda_w'(\{y'\}|b') = 1$$
$$\lambda_w^*(\{y\}|b^*) = 1$$

so the two are not equal, hence (λ^*,b^*) is not stable.

Finally, to prove existence, we let $(\lambda^*,b^*) \in \Lambda \times B$ satisfy $\lambda_k^*(\{y^*\}) = 1$. Let $N(\lambda_k^*)$ be any neighborhood of λ_k^*, and let $\lambda_k' \in N(\lambda_k^*)$, $\lambda_{\bar{k}}' = \lambda_{\bar{k}}^*$, and $b' \in \underline{B}(\lambda')$. Then we must have, for $k \in K$, if $\lambda'(E_k) \neq 0$,

$$y_i^* \geq y_i \Rightarrow b_i'(k) = \bar{k}$$
$$y_i^* \leq y_i \Rightarrow b_i'(k) = k$$

But then $w(s,b) = \bar{k}$ for all $s \in S$, hence $\lambda_w'(\{y^*\}) = 1$, so $\lambda_w(\cdot|b') = \lambda^*(\cdot|b^*)$.

Q.E.D.

Appendix B: Experimental Instructions

This experiment is a study of voting in two candidate elections. As subjects in the experiment, you will each be assigned to be either a voter or a candidate, and you will each be paid for your participation in the experiment on the basis of the decisions you make. If you are careful, and make good decisions, you can make a substantial amount of money.

In this experiment, there are two candidates, labeled A and B, while the rest of you are voters. The experiment itself will consist of a predetermined number of trials. In each trial the candidates will adopt positions in a one dimensional policy space. Voters will vote for the candidate they prefer, and the majority outcome of this vote will determine the winning candidate. Voters are paid for their participation on the basis of their payoff function--to be described in more detail below, and candidates are paid for their participation on the basis of the total number of elections they win.

Before describing the experiment in detail, let me describe the policy space and the payoff function of the voters.

At the beginning of the experiment, voters will each be given a payoff chart similar to the sample chart in front of you. This chart depicts the policy space and a sample payoff function for a voter. Candidates will be given a similar chart; however, the candidate chart will only contain the policy space, and will not have <u>any</u> voter payoff functions. The "policy space" is simply the set of all numbers between 0 and 100, and is represented on the horizontal

axis in the diagram. In each trial, candidates will adopt positions in the policy space. At the end of each trial, the position of the winning candidate will be announced, and each voter will be paid for his or her participation in that trial on the basis of his payoff function. Thus, with the sample payoff chart, if the winning candidate were to adopt the position 33, then the voter would earn $1.25 for that trial. At the end of the experiment, the voter will be paid, in cash, an amount equal to the cumulative amount he or she has earned on each trial.

In the actual experiment, the payoff charts for each of the voters will be different from the sample chart. Further, the payoff functions for different voters may also be different. Each voter will have a payoff function which has a peak, or ideal point at some point in the policy space, and decreases as we move in either direction, as in the example in the sample chart. However, different voters' ideal points, or peaks, may be at different points in the space, and they may decrease at different rates. One important rule in the experiment is that the information on your payoff chart is <u>private</u> information. None of the other voters or candidates will know the information on your chart. At no time should you show, talk about, or in any other way reveal any information about your apyoff chart to other subjects. Further, at no time during the experiment are you to have <u>any</u> communications with any of the other subjects except those explicitly provided for in the rules.

Are there any questions about the payoff chart? If not, I will proceed to a description of the experiment itself.

The experiment is divided into a number of "trials," each one of which consists of an election. At the beginning of each trial, the two candidates, A and B, will each adopt policy positions. The positions adopted by the two candidates <u>will not be made public</u>. Each candidate will write his position, a number between 0 and 100, on one of the cards provided, and hand it to the experimenter. This will be done in secrecy. The only information the voter will have about the candidates is the interest group endorsement information. This will require some further explanation. You will note, on the sample payoff chart, are marked the positions of two interest groups. After the two candidates have adopted their positions, the experimenter will record, on the blackboard, the endorsement of each interest group. You may interpret this information as follows. Each interest group will endorse the candidate which is closest to the position of that interest group. Thus, if candidate A adopted the point <u>33</u>, and candidate B adopted <u>53</u>, as marked on the sample payoff chart, it follows that interest group 1 would endorse A (since A is closer to interest group 1 than is B), but interest group 2 would endorse candidate B. Note that if this were part of the experiment, and you were a voter, the only information you would have is the endorsements of the two interest groups. You would know that interest group 1 endorses A and interest group 2 endorses B. You would <u>not</u> know that positions of either candidate. They are indicated on the diagram simply to illustrate how the interest group endorsements are computed

from the candidate positions.

After the candidates have adopted their positions and the interest group endorsements have been announced, the voters will then vote, on the ballot card provided for either candidate A or candidate B. These votes will also be cast in secret, and handed to the experiment. The experimenter will then tally the votes and announce the winning candidate, and the position of the winning candidate. The trial will end at this point. At this time, the voters should record the position of the winning candidate on the record sheet that will be provided and compute the payoff they receive for that trial. We will thjn proceed to the next trial. After a predetermined number of trials have elapsed, the experiment will terminate, and all subjects will be paid in accordance with their payoff charts. Candidates will be paid $1.00 for each election they win, and nothing otherwise.

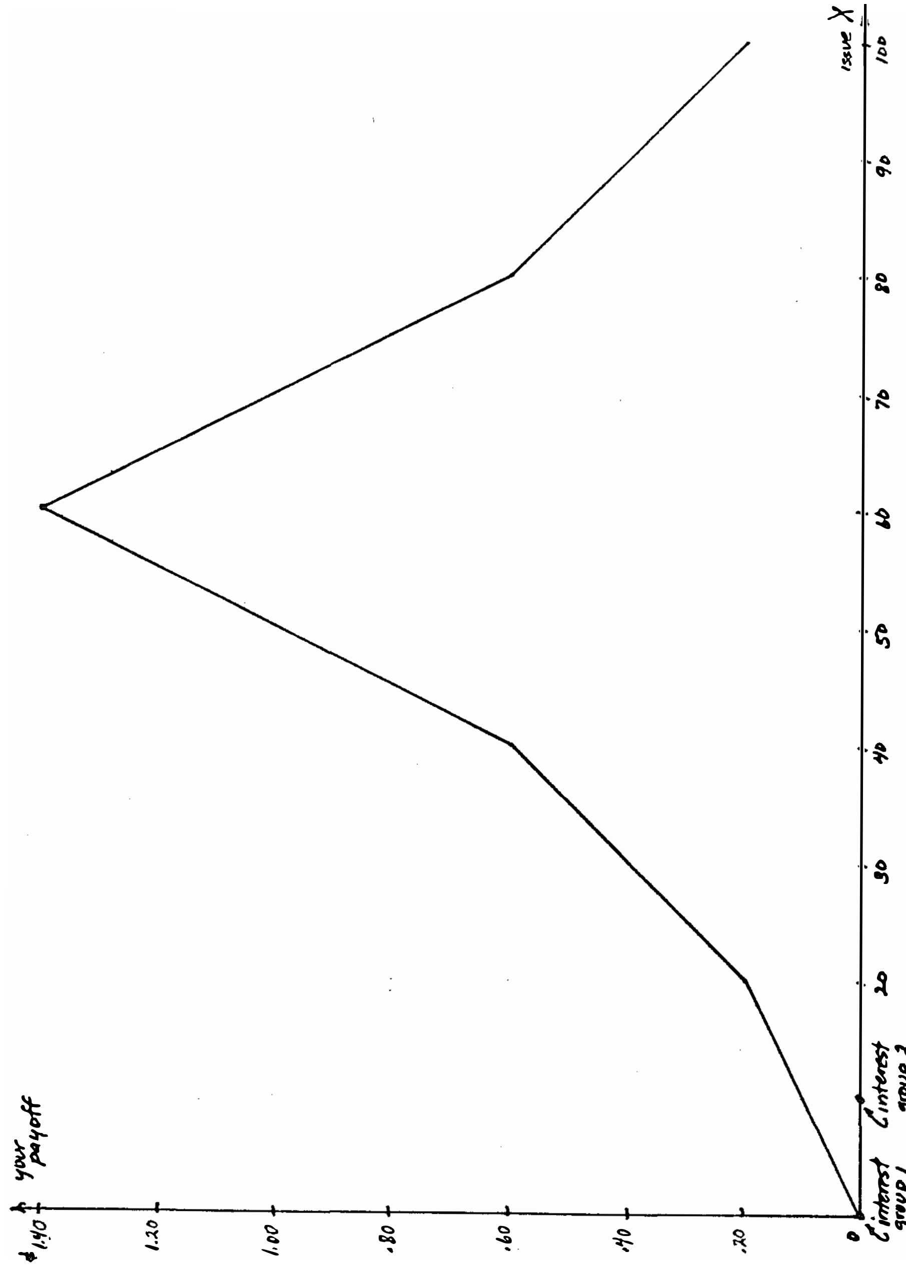
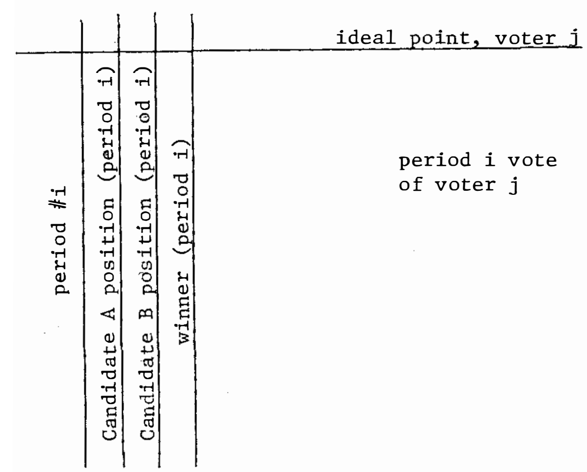


Figure B1
Sample Payoff Function

Appendix C

This Appendix contains the raw data for the experiments, plus figures for the experiments not contained in the body of the text. The raw data on the subsequent pages is in the following form:

Experiment #



C7

				37	72	100
1	10	50	A	A	B	A
2	9	35	B	B	B	A
3	27	20	A	B	A	A
4	27	35	B	B	B	B
5	88	41	A	A	A	A
6	100	10	B	B	B	A
7	100	70	A	A	A	A
8	40	50	A	A	A	B
9	40	80	B	A	B	B
10	45	20	B	B	B	A
11	45	70	B	B	B	B
12	45	65	B	A	B	B
13	60	20	A	B	A	A
14	45	70	A	A	A	B
15	45	70	A	A	A	B
16	45	10	A	A	A	A
17	45	50	B	A	B	B
18	53	45	A	B	A	A
19	53	40	A	B	A	A
20	53	60	B	A	B	B
21	53	40	A	B	A	A
22	55	55	A	B	A	A
23	55	30	A	B	A	A
24	55	60	B	A	B	B
25	57	65	A	A	A	B
26	57	45	A	B	A	A
27	57	40	A	B	A	A

C23

				100	68	83	72	60	50	37
1	47	5	A	A	A	A	A	A	A	A
2	59	66	B	B	B	B	A	A	B	B
3	75	69	A	A	A	A	A	B	B	B
4	47	85	B	B	B	B	B	B	B	A
5	97	92	A	A	A	A	B	B	A	A
6	4	100	B	B	A	B	A	A	B	B
7	13	90	A	B	A	B	A	A	A	A
8	100	70	A	A	A	A	A	B	B	A
9	7	100	B	B	B	B	B	B	A	A
10	98	100	B	B	B	B	B	B	A	A
11	47	100	A	B	A	B	A	A	A	A
12	56	1	A	A	A	A	A	A	A	B
13	59	75	B	B	B	B	A	B	A	A
14	60	85	B	B	B	B	B	A	A	A
15	51	96	B	B	B	B	B	A	A	A
16	70	100	A	B	A	B	A	B	A	A
17	66	22	A	A	A	A	A	A	B	B
18	54	75	A	B	B	B	A	A	A	A
19	55	75	B	B	B	B	B	B	A	A
20	77	80	A	B	B	B	A	A	A	A
21	64	61	A	A	A	A	A	B	B	B
22	65	75	A	B	B	B	A	A	A	A
23	60	55	A	A	A	A	A	B	B	B
24	63	75	B	B	B	B	B	B	A	A
25	70	75	B	B	B	B	B	A	A	A
26	68	75	B	B	B	B	B	A	A	A
27	76	75	B	A	A	A	B	B	B	B
28	77	75	B	A	A	A	B	A	B	B

C12

				100	83	60
1	63	15	A	A	A	A
2	25	34	B	B	B	B
3	99	2	A	A	A	A
4	40	59	A	B	A	A
5	1	47	B	B	B	B
6	76	42	A	A	A	A
7	50	67	B	B	B	A
8	69	58	A	A	A	B
9	100	53	A	A	A	B
10	55	25	A	A	A	B
11	81	79	A	A	A	A
12	95	100	B	B	A	B
13	85	85	A	A	A	B
14	99	69	A	A	A	B
15	100	74	A	A	A	A
16	0	74	A	B	A	A
17	83	100	B	B	A	B
18	100	49	B	A	B	B
19	55	68	B	B	B	B
20	99	75	B	A	B	B
21	78	60	B	A	B	B
22	95	80	A	A	A	B
23	76	75	B	A	B	B
24	88	70	B	A	B	B
25	74	83	B	B	B	B
26	91	77	B	A	B	B
27	55	79	B	B	B	B
28	69	75	B	B	A	B
29	100	70	A	A	A	B
30	76	70	B	A	B	B
31	90	77	B	A	B	B
32	77	75	B	B	B	B

C20

				18	37	60
1	80	5	A	B	A	A
2	65	40	B	B	B	A
3	50	30	B	B	B	B
4	40	45	B	B	B	B
5	10	50	B	A	B	B
6	70	45	B	B	B	A
7	80	55	B	B	B	A
8	60	50	B	B	B	A
9	50	50	A	A	B	A
10	50	40	A	A	B	A
11	50	55	B	A	A	B
12	50	55	A	B	A	B
13	50	40	B	B	B	A
14	50	42	B	B	B	A
15	47	44	A	B	A	A
16	45	50	B	A	B	B
17	47	42	A	A	B	A
18	48	52	A	A	B	A
19	47	46	B	B	B	A
20	46	47	A	A	A	B
21	45	50	A	A	A	B
22	44	40	B	B	B	A
23	40	36	B	B	B	A
24	38	35	B	B	B	A
25	33	36	B	A	B	B
26	40	34	B	B	B	A

CT1

				100	87	83	72	60	50	37	30	30
1	35	100	B	A	B	B	B	B	B	B	B	A
2	35	50	A	A	B	B	A	A	A	A	B	A
3	50	10	A	B	A	A	A	A	A	A	A	A
4	35	40	B	B	B	B	B	B	B	A	B	A
5	25	34	B	B	B	B	B	B	B	A	A	A
6	50	31	A	A	A	A	A	A	A	B	B	B
7	45	36	A	A	A	A	A	A	A	A	A	B
8	48	41	A	A	A	A	A	A	B	A	A	B
9	49	49	B	B	B	A	A	A	B	A	B	B
10	52	39	A	A	A	A	A	A	A	A	B	B
11	55	55	A	A	B	A	B	A	A	A	B	B
12	55	53	A	A	A	A	A	A	B	A	A	B
13	55	57	B	B	B	B	B	B	A	A	A	A
14	61	53	A	A	A	A	A	A	B	B	B	B
15	66	55	A	A	A	A	A	A	B	B	B	B
16	50	61	B	B	B	B	B	B	A	A	A	A
17	64	68	B	B	B	B	B	B	A	A	A	A
18	55	64	A	B	B	B	B	B	A	A	A	A
19	50	58	B	B	B	B	B	B	A	A	A	A
20	62	61	B	B	A	A	A	B	B	B	B	B

Table C1

Raw Data

				83	60	100
1	50	10	A	A	B	A
2	55	25	A	A	B	A
3	56	60	B	B	A	B
4	70	70	B	B	B	A
5	70	65	A	A	A	A
6	70	75	B	B	A	B
7	70	80	B	B	A	B
8	90	80	A	A	A	A
9	90	85	B	B	B	A
10	83	80	A	B	A	A
11	80	85	A	A	A	B
12	83	80	A	A	A	A
13	80	83	A	A	A	B
14	83	83	A	A	A	B
15	83	80	A	A	A	B

				30	37	88
1	25	20	B	B	B	A
2	5	25	A	A	B	B
3	10	10	B	A	B	B
4	30	5	A	A	B	A
5	40	45	B	B	A	B
6	35	35	A	A	A	B
7	50	45	A	A	B	A
8	50	60	A	A	A	B
9	55	50	B	B	B	A
10	40	45	A	A	A	B
11	45	35	B	B	B	A
12	50	40	B	B	B	A
13	50	45	B	B	B	A
14	40	40	A	A	A	A
15	42	39	B	B	B	A
16	40	37	B	B	B	A
17	40	37	B	B	B	A

1	5	30	B	A	B	B
2	51	50	A	B	A	A
3	48	100	A	A	A	A
4	35	52	B	A	B	B
5	49	56	B	A	B	B
6	55	59	B	A	B	B
7	62	61	B	B	B	A
8	90	61	B	B	B	A
9	61	60	B	B	B	A
10	59	62	B	A	B	B
11	60	63	A	A	A	B
12	61	63	A	A	A	B
13	60	55	B	B	B	A
14	30	55	B	A	B	B
15	57	54	A	B	A	B
16	63	60	A	B	A	A
17	64	59	B	B	B	A
18	61	58	B	B	B	A
19	58	58	A	A	A	A
20	58	60	B	A	B	B

				18	30	50	60	72	88	100
0	11	6	A	A	A	A	A	A	B	A
2	20	65	B	B	A	B	B	B	B	B
3	60	85	B	A	A	B	B	B	A	B
4	87	75	A	B	B	A	A	B	A	A
5	90	90	B	A	B	B	B	A	B	B
6	80	100	A	A	A	A	B	A	B	A
7	75	87	A	A	A	A	A	B	A	B
8	70	70	A	A	A	A	A	A	B	B
9	70	65	B	B	B	B	B	A	A	A
10	60	67	A	A	A	A	A	A	A	B
11	85	65	B	B	B	B	B	A	B	A
12	60	65	A	A	A	A	A	A	B	A
13	55	55	B	A	B	A	A	B	B	B
14	54	60	A	A	A	A	A	B	A	B
15	50	55	A	A	A	A	A	B	A	A
16	44	65	A	A	A	A	A	B	A	A
17	41	35	B	B	B	B	A	A	B	B

C27

				100	88	83	72	60	50	37	30	18	100	18	83	60	50	37	30	30	18	30
1	40	40	B	B	A	B	A	B	A	B	A	A	B	B	B	B	B	A	A	B	A	A
2	60	45	B	A	A	B	A	A	B	B	B	B	A	B	A	A	A	B	B	B	A	B
3	63	30	B	A	A	A	A	A	B	B	B	A	A	B	A	A	B	B	B	B	B	B
4	37	35	A	A	A	B	A	A	A	A	B	B	A	B	A	A	A	B	B	A	B	B
5	29	42	B	A	B	B	B	B	B	A	A	B	B	A	B	B	B	A	A	A	A	A
6	50	32	A	B	A	A	A	A	B	B	B	A	A	B	A	A	A	B	B	B	B	A
7	41	35	B	B	A	A	A	A	B	B	B	B	A	B	A	A	A	B	B	B	B	B
8	26	37	B	B	B	A	B	B	B	B	A	A	B	A	B	B	B	A	A	A	B	A
9	43	30	A	A	A	A	A	A	A	B	B	B	A	B	A	A	A	B	B	B	B	B
10	40	35	A	A	A	A	A	A	B	A	B	A	A	B	A	A	A	B	B	B	B	B
11	39	33	B	A	A	B	A	A	B	B	B	A	A	B	A	A	A	B	B	B	B	B
12	36	38	B	B	B	B	B	B	A	B	A	A	B	A	B	B	B	A	A	A	A	A
13	39	35	B	A	A	A	A	A	B	B	B	B	A	B	A	A	A	B	B	B	B	B
14	38	30	A	A	A	A	A	A	A	B	B	B	A	B	A	A	A	B	B	B	B	B
15	41	32	A	A	A	A	A	A	A	B	B	A	A	B	A	A	A	B	B	B	B	B
16	42	33	A	A	A	A	A	A	A	B	B	A	A	B	A	A	A	B	B	B	B	B
17	44	35	A	A	A	A	A	A	A	B	B	A	A	B	A	A	A	B	B	B	B	B
18	40	38	A	A	A	B	A	A	A	B	A	A	A	B	A	A	A	B	B	B	B	B
19	45	40	B	A	A	B	A	A	A	B	B	B	A	B	A	A	A	B	B	B	B	B
20	40	40	A	B	A	A	B	A	A	B	A	A	B	A	A	B	A	A	B	A	B	B
21	42	42	A	B	A	A	B	A	A	B	A	B	B	B	A	B	A	A	B	A	B	A
22	46	42	A	A	A	A	A	A	A	B	B	A	A	B	A	A	A	B	B	B	B	B
23	51	41	A	A	A	B	B	A	A	B	B	A	A	B	A	A	A	B	B	B	B	A
24	49	44	A	A	A	B	A	A	A	B	B	A	A	B	A	A	A	B	B	A	B	B
25	50	45	B	A	A	B	A	A	A	B	B	B	A	B	A	A	A	B	B	B	B	B

C26

				100	30	37	50	60	72	72	83	88	100	100	88	37	72	60	60	37	30	88
1	50	50	A	B	B	A	A	B	A	A	B	B	B	A	A	A	A	A	A	A	A	B
2	56	60	B	B	A	B	A	B	B	B	B	B	B	B	B	A	B	B	B	A	A	A
3	65	61	A	A	B	B	B	B	A	A	A	B	A	A	A	B	A	A	A	A	B	A
4	62	59	A	A	A	A	B	B	A	A	A	A	A	A	A	B	A	A	A	A	B	A
5	66	66	B	A	B	B	B	B	B	A	B	B	B	A	B	A	B	A	B	B	A	B
6	71	65	A	A	A	B	B	B	A	A	A	A	A	A	A	B	A	A	B	A	A	A
7	55	69	B	B	A	A	A	A	B	A	B	B	B	B	B	A	A	B	A	B	A	B
8	63	67	B	B	A	A	A	A	B	B	B	B	B	B	B	A	B	A	B	B	A	B
9	69	72	B	B	A	A	A	A	B	B	A	B	B	B	B	A	B	A	A	A	B	B
10	71	68	A	A	A	B	B	B	A	B	A	B	A	A	A	B	B	B	A	B	A	A
11	76	68	B	A	A	B	B	B	A	A	A	B	A	A	A	B	B	B	B	B	B	A
12	69	66	A	A	A	B	B	B	A	A	A	A	A	A	A	B	A	B	B	B	A	A
13	71	73	B	B	A	A	A	A	B	B	B	A	B	B	B	A	B	A	A	A	B	B
14	72	67	B	A	A	B	B	B	A	B	A	B	A	A	A	B	B	B	B	B	B	A
15	69	72	B	B	A	A	A	A	B	B	B	B	B	B	B	A	B	A	A	A	A	B
16	69	67	B	A	A	B	B	B	A	B	A	B	A	A	A	B	B	B	B	B	B	A
17	74	73	A	A	B	B	B	B	A	A	A	B	A	A	A	B	A	B	B	B	A	A
18	64	66	A	B	A	A	A	A	B	A	B	A	B	B	B	A	A	A	A	A	A	B
19	73	76	B	B	A	A	A	A	B	B	B	B	B	B	B	A	B	A	A	A	A	B
20	61	70	A	B	A	A	A	A	B	A	B	B	B	B	B	A	B	A	A	A	A	B
21	77	70	A	A	B	B	B	B	A	A	A	B	A	A	A	B	A	A	A	A	B	A
22	61	70	A	B	A	A	A	A	B	A	B	A	B	B	B	A	A	A	A	A	B	B

Table C2

Raw Data

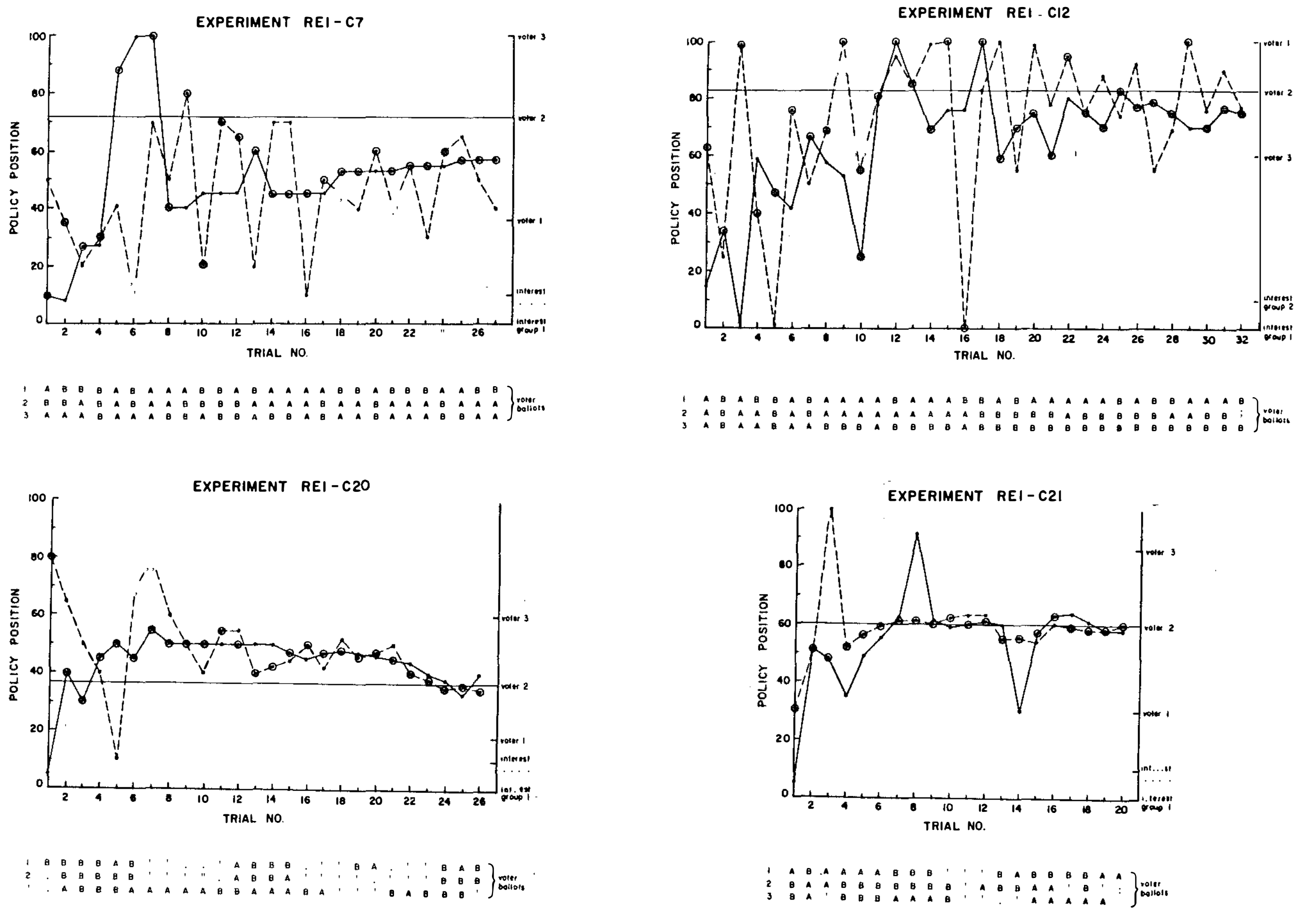


Figure C1

Experimental Outcomes

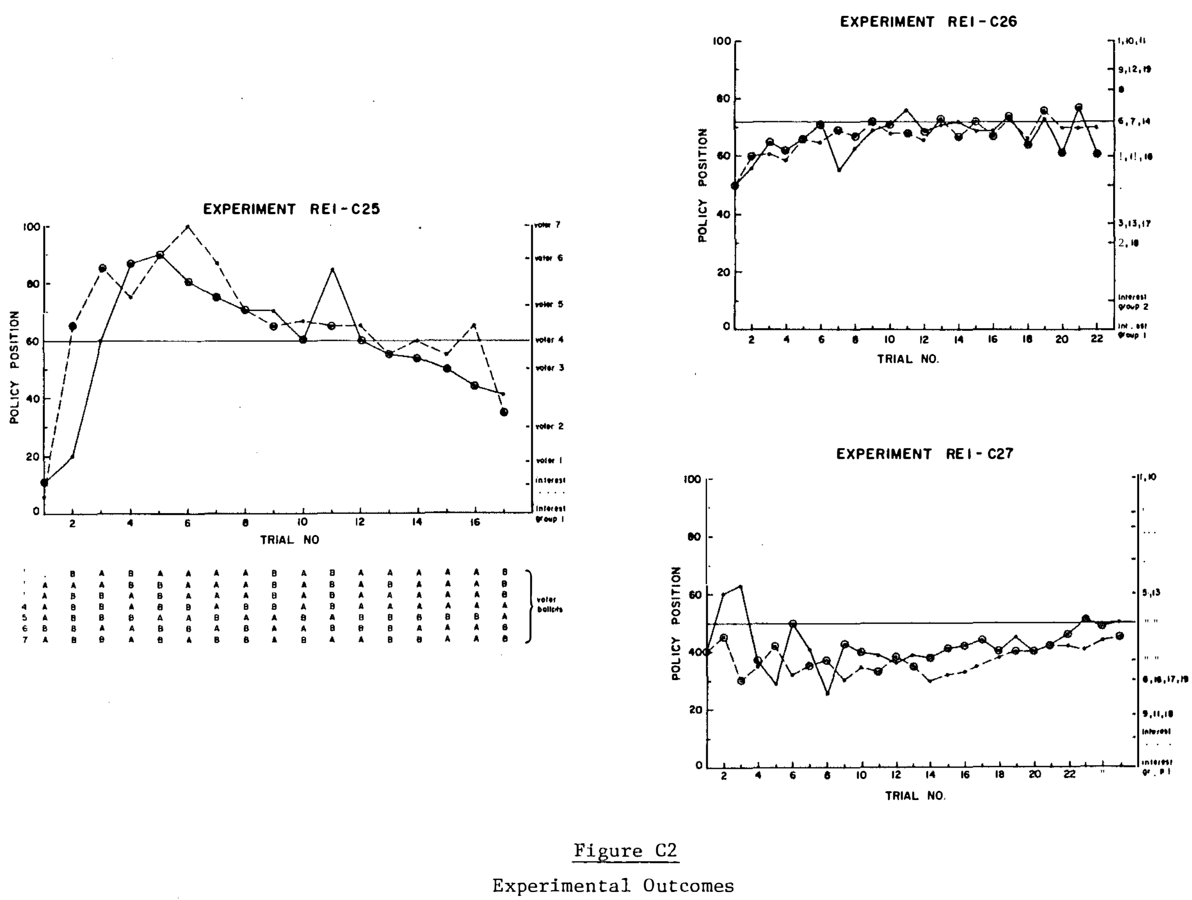


Figure C2

Experimental Outcomes

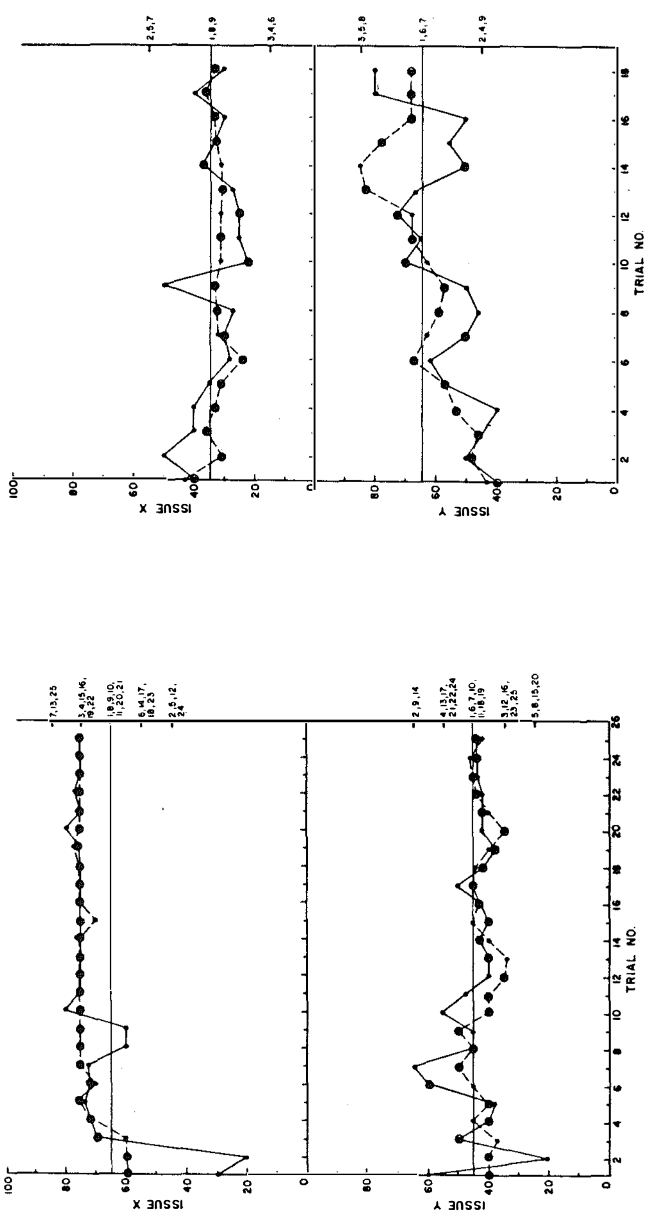


Figure C3

Two Dimensional Experiments

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