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PROFITABLE SPECULATION AND LINEAR EXCESS DEMAND

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Abstract

Since Friedman maintained that profitable speculation necessarily stabilizes prices, there had been many debates. Farrell concluded these debates by showing that (i) for a two-period model, any continuous negatively sloped non-speculative excess demand function would validate Friedman's conjecture if there is no lag structure, and (ii) for a T-period model with $T \geq 3$, negatively sloped linear non-speculative excess demand is necessary and sufficient for Friedman's conjecture to be true if there is no lag structure. Later, Schimmler generalized Farrell's results to lag-responsive non-speculative excess demand cases.

However, there are some problems in Farell's and Schimmler's approaches which invalidate their proofs. In this paper, we will point out these problems and show that after correcting these slips, Farrell's two results are in fact correct. Also, we will redo Schimmler's problem for time-independent non-speculative excess demand functions. The conclusions derived are (i) for two-period models, any continuously differentiable non-speculative excess demand $f(P_t,P_{t-1})$ with $f_1(P_t,P_{t-1}) < 0$, $f_2(P_t,P_{t-1}) \leq 0$ (where $f_{t-s+1}(P_t,P_{t-1}) = \frac{\partial f(P_t,P_{t-1})}{\partial P_s}$, $s = t-1, t$) will validate Friedman's conjecture; (ii) for T-period models ($T \geq 3$), within the class of twice-continuously differentiable functions, linear non-speculative excess demand functions $f(P_t,P_{t-1},\cdots,P_{t-T+1})$ satisfying $f_1 < 0$, $f_2 = f_3 = \cdots = f_{t-T+1} = 0$ represent necessary and sufficient conditions for Friedman's conjecture to be true.

Profitable Speculation and Linear Excess Demand*

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Introduction

In arguing the case for flexible versus fixed exchange values, it was maintained by Friedman [3, p. 175] that profitable speculation necessarily stabilizes prices. Thereafter, several studies tried to verify Friedman's conjecture. Baumol [1] constructed a theoretical counterexample to show that the conjecture isn't always true. Stein [6] used a real-life counterexample to invalidate Friedman's argument. On the other hand, both Kemp [4] and Telser [7] showed that when the non-speculative excess demand function is linear with no lag structure, and satisfies the law of demand, Friedman's conjecture is always true. At this point in the debate, it was clear there were only certain classes of non-speculative excess demand functions which can validate Friedman's conjecture. The problem is what classes, and whether these classes can generally describe market behavior.

Farrell [2] tackled these problems, and showed that, (i) for a two-period model, any continuous negatively sloped non-speculative excess demand function would validate Friedman's argument if there is no lag structure; and (ii) for a T-period model with $T \geq 3$, negatively sloped linear non-speculative excess demand is necessary and sufficient for Friedman's conjecture to be true if there is no lag structure. Schimmler [5] generalized Farrell's results to the case of lag-responsive non-speculative excess demand, showing again that linearity coupled with the law of demand are necessary and sufficient to validate Friedman's argument.

However, there are some problems in Farrell's and Schimmer's approaches which invalidate their proofs. In this paper we will show that after correcting these slips, Farrell's two results are in fact correct, and we will redo Schimmler's problem for time-independent non-speculative excess demand functions. The conclusions derived are (i) for two-period models, any continuously differentiable non-speculative excess demand $f(P_t, P_{t-1})$ with $f_1(P_t,P_{t-1}) < 0$, $f_2(P_t,P_{t-1}) \leq 0$ (where $f_1(P_t,P_{t-1}) = \frac{\partial f(P_t,P_{t-1})}{\partial P_t}$, $f_2(P_t,P_{t-1}) = \frac{\partial f(P_t,P_{t-1})}{\partial P_{t-1}}$) will validate Friedman's conjecture; (ii) for T-period models ($T \geq 3$), within the class of twice continuously differentiable functions, linear non-speculative excess demand functions $f(P_t,P_{t-1},\cdots,P_{t-T+1})$ satisfying $f_1 < 0$, $f_2 = f_3 = \cdots = f_{t-T+1} = 0$ represent necessary and sufficient conditions for Friedman's conjecture to be true.

Farrell's Framework and Associated Problems

Farrell considered a discrete time abstract market model, where the associated commodity is storable. Let $t = 1, 2, \cdots, T$ denote T periods. Within any period all transactions are assumed to take place at the same price. Also, let P_t^w, $t = 1, 2, \cdots, T$ denote the price in period t when there is no speculation, and let P_t^s, $t = 1,2,\cdots,T$ denote the price in period t given the speculation sequence $\{s_1, s_2, \cdots, s_T\}$, where s_t, $t = 1, 2, \cdots, T$, is the speculative sales in period t. To make the effects of speculation sequences well-defined, we need a clear-cut terminal date. Therefore, Farrell defined a complete speculation sequence[1] as a speculation sequence $\{s_1, s_2, \cdots, s_T\}$ such that

$$\sum_{t=1}^{T} s_t = 0 \qquad (1)$$

By sales and buys in the market, speculators' profits are

$$\pi = \sum_{t=1}^{T} P_t^S \cdot s_t \qquad (2)$$

The introduction of speculation changes the variance of prices according to

$$C' = \{ [\sum_{t=1}^{T} (P_t^S)^2 - \frac{1}{T}(\sum_{t=1}^{T} P_t^S)^2] - [(\sum_{t=1}^{T} (P_t^W)^2 - \frac{1}{T}(\sum_{t=1}^{T} P_t^W)^2] \} \cdot \frac{1}{T} \qquad (3)$$

where

$$C = TC' = [\sum_{t=1}^{T} (P_t^S)^2 - \frac{1}{T}(\sum_{t=1}^{T} P_t^S)^2] - [\sum_{t=1}^{T} (P_t^W)^2 - \frac{1}{T}(\sum_{t=1}^{T} P_t^W)^2] \qquad (3)'$$

is taken to be the measure of the stabilizing effect of speculation.[2] That is, if $c > 0$, we say the speculation sequence destabilizes prices; if $c < 0$, we say the speculation sequence stabilizes prices.

Since Farrell only considered complete speculation sequences, Friedman's conjecture can be formalized as follows:

$$\text{When } \sum_{t=1}^{T} s_t = 0, \text{ if } \pi > 0, \text{ then } c < 0 \qquad (4)$$

To derive his two results about (4), Farrell employed an independence assumption, i.e., he assumed that the non-speculative excess demand function has the following property:

$$P_t^S - P_t^W = h(s_t), \ \forall t, \text{ for some function } h(\cdot) \text{ such that } h(0) = 0.^3 \qquad (5)$$

In other words, suppose we have an non-speculative excess demand function $f(\cdot)$ such that $Q_t^W = f(P_t^W)$. When there are speculative sales s_t, P_t^W must be adjusted to P_t^S in order to clear the market, i.e., we must have $Q_t^W + s_t = f(P_t^S)$ which implies $s_t = f(P_t^S) - f(P_t^W)$. Therefore, we can rewrite (5) as

$$P_t^S - P_t^W = h(f(P_t^S) - f(P_t^W)) \qquad (5)'$$

Under eq. (5) (or equivalently eq. (5)'), Farrell derived the results: (i) for a two-period mode, any continuous negatively sloped non-speculative excess demand function will satisfy (4); (ii) for a T-period model ($T \geq 3$), negatively sloped linear non-speculative excess demand is necessary and sufficient for (4) to be true.

The problem with Farrell's proofs is that there is a tautology involved. To see this, we ask when can we write eq. (5)'? Equivalently, what functional form for non-speculative excess demand is consistent with eq. (5)'?

Theorem 1

Let $Q^S = f(P^S)$, $Q^W = f(P^W)$. Then, within the class of continuous, differentiable functions, the only functional form $h(\cdot)$ which can satisfy $P^S - P^W = h(Q^S - Q^W)$ for all $P^S \geq 0$, $P^W \geq 0$ is linear. Also, $f(\cdot)$ must be linear.

[Proof]

$$P^S - P^W = h(Q^S - Q^W) = h(f(P^S) - f(P^W)), \quad \forall P^S, P^W$$

Taking the partial derivative with respect to P^S, we have

$$1 = h'(f(P^S) - f(P^W))f'(P^S), \quad \forall P^S, P^W$$

Similarly, taking the partial derivative with respect to P^W,

$$-1 = h'(f(P^S) - f(P^W))(-f'(P^W)), \quad \forall P^S, P^W$$

Hence, $(f'(P^S) - f'(P^W))h'(f(P^S) - f(P^W)) = 0, \quad \forall P^S, P^W$

$\Rightarrow \quad f'(P^S) = f'(P^W), \quad \forall P^S, P^W$, i.e., $f(\cdot)$ is linear. Therefore $h'(Q^S - Q^W)$ is also a constant which implies $h(\cdot)$ is linear.

Q.E.D.

Theorem 1 shows that only linear non-speculative excess demand functions are consistent with (5)'. Therefore, Farrell's proofs, involve writing down a functional form (5) which can only be satisfied by a linear non-speculative excess demand function. Farrell then proved (4) is true only when we have linear non-speculative excess demand. Obviously, this involves a tautology.

Reevaluations of Farrell's two claims

While Farrell's proofs are incorrect, the remaining question is whether his two results are still true. The following three theorems show that both his two claims are in fact correct!

Theorem 2

For a two-period model, any differentiable, negatively sloped non-speculative excess demand function will satisfy Friedman's conjecture (i.e., Eq. (4)).

[Proof]

To simplify the notation, let P_t denote P_t^S, q_t denote P_t^W, t=1,2. Now, consider the following problem:

$$\min_{\{q_1, q_2\}} V(q_1, q_2) = (q_1^2 + q_2^2) - \tfrac{1}{2}(q_1 + q_2)^2$$

$$\text{Subect To : } [f(p_1) - f(q_1)]P_1 + [f(P_2) - f(q_2)]P_2 \geq 0 \quad (6)$$

$$f(P_1) - f(q_1) + f(P_2) - f(q_2) = 0 \quad (7)$$

where $f(\cdot)$ represents the non-speculative-excess demand function. If Eq. (4) is true, it must be that the minimum of $V(q_1, q_2)$ is greater than or equal to $(P_1^2 + P_2^2) - \tfrac{1}{2}(P_1 + P_2)^2$ for any given P_1, P_2. Since (P_1, P_2) satisfies (6), (7), this means the minimum is achieved at (P_1, P_2). To derive the minimum point, let the Lagrangian $\mathbb{L}$ be given by

$$\mathbb{L}(q_1, q_2, \lambda, \mu) = (q_1^2 + q_2^2) - \tfrac{1}{2}(q_1 + q_2)^2 + \lambda\{[f(P_1) - f(q_1)]P_1 + [f(P_2) - f(q_2)]P_2\} + \mu[f(P_1) + f(P_2) - f(q_1) - f(q_2)]$$

The first-order conditions are:

$$\frac{\partial \mathbb{L}}{\partial q_1} = 2q_1 - (q_1 + q_2) + \lambda[-f'(q_1)P_1] - \mu f'(q_1) = 0 \quad (8)$$

$$\frac{\partial \mathbb{L}}{\partial q_2} = 2q_2 - (q_1 + q_2) + \lambda[-f'(q_2)P_2] - \mu f'(q_2) = 0 \quad (9)$$

$$\lambda\cdot\frac{\partial \mathbb{L}}{\partial \lambda} = \lambda[(f(P_1) - f(q_1))P_1 + (f(P_2) - f(q_2))P_2] = 0 \quad (10)$$

$$\lambda \geq 0,\ (f(P_1) - f(q_1))P_1 + (f(P_2) - f(q_2))P_2 \geq 0 \quad (11)$$

$$\frac{\partial \mathbb{L}}{\partial \mu} = f(P_1) + f(P_2) - f(q_1) - f(q_2) = 0$$

Summing up (8) and (9), we have

$$\lambda[-f'(q_1)P_1 - f'(q_2)P_2] - \mu[f'(q_1) + f'(q_2)] = 0$$

$$\Rightarrow \quad \lambda = \mu\cdot\frac{f'(q_1) + f'(q_2)}{-f'(q_1)P_1 - f'(q_2)P_2} \quad (12)$$

Case A: Now, if $\mu = 0$, then $\lambda = 0$ (by (12)), which implies $q_1^* = q_2^* = q^*$ is the minimum point. Hence, by (7), we have:

$$f(P_1) - f(q^*) = f(q^*) - f(P_2)$$

and by (6), we have:

$$(f(P_1) - f(q^*))(P_1 - P_2) \geq 0$$

(i) If $P_1 > P_2$, then $f(P_1) \geq f(q^*)$ which implies $f(q^*) \geq f(P_2)$. Since $f(\cdot)$ has negative slope, we have $P_1 \leq q^* \leq P_2$, contradicting $p_1 > P_2$.

(ii) If $P_1 < P_2$, then $f(P_1) \leq f(q^*)$ which implies $f(q^*) \leq f(P_2)$. Hence $P_1 \geq q^* \geq P_2$.

(iii) If $P_1 = P_2$, then $q^* = P_1 = P_2$, which satisfies the reguirement.

Case B: If $\mu \neq 0$, then $\lambda \neq 0$ (by (12)). Therefore by (10), we have:

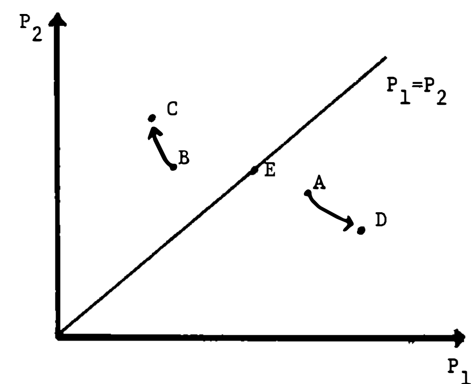
$$(f(P_1) - f(q_1^*))\,P_1 + (f(P_2) - f(q_2^*))P_2 = 0$$

$$\Rightarrow \quad [f(P_1) - f(q_1^*)]\,(P_1 - P_2) = 0 \qquad \text{(by (7))}$$

$$\Rightarrow \quad f(P_1) = f(q_1^*),\ (\text{if } P_1 \neq P_2)$$

$$\Rightarrow \quad P_1 = q_1^*,\ \text{hence } P_2 = q_2^* \ (\text{if } P_1 \neq P_2)$$

The case of $P_1 = P_2$ is the same as case A (iii). Combining case A and B, we know (P_1, P_2) is the only optimum point. To make sure V achieves a minimum, we can check the second order condition, or alternatively, consider the local properties of $V(q_1, q_2)$ at (P_1, P_2) while still satisfying (6) and (7). To derive this, consider the following diagram:



At point A, $P_1 > P_2$. Then by (6) and (7), we have:

$$(f(P_1) - f(q_1))\,(P_1 - P_2) \geq 0 \quad \Rightarrow \quad f(P_1) \geq f(q_1)$$

$$\Rightarrow \quad f(P_2) \leq f(q_2)$$

Hence, $P_1 \leq q_1$, $P_2 \geq q_2$. Therefore the AD curve is the feasible solution, and $V(P_1, P_2) \leq V(q_1, q_2)$.

Similarly, at B, $P_1 < P_2 \Rightarrow f(P_1) \le f(q_1) \Rightarrow f(P_2) \ge f(q_2)$, hence $P_1 \ge q_1$, $P_2 \le q_2$, which implies the BC curve is the feasible solution, and $V(P_1, P_2) \le V(q_1, q_2)$.

Finally, at E, $P_1 = P_2$ is the only feasible solution, and there is no local property to be considered. From the above, we can conlude (P_1, P_2) is the unique minimum and this completed our proof.

Q.E.D.

Theorem 3

For a three-period model, the only (nontrivial[4]) continuously differentiable functional form for the nonspeculative excess demand function which can satisfy Friedman's conjecture is linear with negative slope.

[Proof]

Using the same considerations as we did in proving Theorem 2, the minimization problem now is:

$$\min_{\{q_1,q_2,q_3\}} (q_1^2+q_2^2+q_3^2)-1/3(q_1+q_2+q_3)^2$$

Subject To: $f(P_1) - f(q_1) + f(P_2) - f(q_2) + f(P_3) - f(q_3) = 0$ (13)

$P_1[f(P_1) - f(q_1)] + P_2[f(P_2) - f(q_2)] + P_3[f(P_3) - f(q_3)] \ge 0$ (14)

The associated Langrangian is:

$$\mathbb{L}(q_1,q_2,q_3,\lambda,\mu) \equiv (q_1^2 + q_2^2 + q_3^2) - 1/3(q_1 + q_2 + q_3)^2 + \lambda\left(\sum_{i=1}^{3} f(P_i) - f(q_i)\right) + \mu\left(\sum_{i=1}^{3} P_i(f(P_i) - f(q_i))\right)$$

Therefore, the first-order conditions are:

$$\frac{\partial \mathbb{L}}{\partial q_1} = 2q_1 - 2/3(q_1 + q_2 + q_3) - \lambda f'(q_1) - \mu P_1 f'(q_1) = 0 \quad (15)$$

$$\frac{\partial \mathbb{L}}{\partial q_2} = 2q_2 - 2/3(q_1 + q_2 + q_3) - \lambda f'(q_2) - \mu P_2 f'(q_2) = 0 \quad (16)$$

$$\frac{\partial \mathbb{L}}{\partial q_3} = 2q_3 - 2/3(q_1 + q_2 + q_3) - \lambda f'(q_3) - \mu P_3 f'(q_3) = 0 \quad (17)$$

$$\frac{\partial \mathbb{L}}{\partial \lambda} = f(P_1) + f(P_2) + f(P_3) - f(q_1) - f(q_2) - f(q_3) = 0$$

$$\mu \cdot \frac{\partial \mathbb{L}}{\partial \mu} = \mu \cdot \left(\sum_{i=1}^{3} P_i(f(P_i) - f(q_i))\right) = 0 \quad (18)$$

$$\mu \ge 0, \ \frac{\partial \mathbb{L}}{\partial \mu} = \sum_{i=1}^{3} P_i(f(P_i) - f(q_i)) \ge 0$$

Summing up (15), (16) and (17), we have

$$\lambda = \frac{-P_1 f'(q_1) - P_2 f'(q_2) - P_3 f'(q_3)}{f'(q_1) + f'(q_2) + f'(q_3)} \cdot \mu \quad (19)$$

Case A: Now, if $\mu = 0$, then $\lambda = 0$ which implies $q_1^* = q_2^* = q_3^* = q^*$ (by (15), (16), (17)), so by (13), we have:

$$f(q_1^*) = f(q_2^*) = f(q_3^*) = 1/3\sum_{i=1}^{3} f(P_i), \text{ and}$$

$$\sum_{i=1}^{3} P_i(f(P_i) - f(q_i)) = \sum_{i=1}^{3} P_i(f(P_i) - 1/3\sum_{i=1}^{3} f(P_i))$$

Since we require the minimum to be achieved at (P_1,P_2,P_3) for any given (P_1,P_2,P_3), therefore we must require

$$\sum_{i=1}^{3} P_i(f(P_i) - 1/3\sum_{i=1}^{3} f(P_i)) \le 0, \quad \forall P_1,P_2,P_3 \quad (20)$$

Case B: If $\mu \ne 0$, then $\lambda \ne 0$. Substituting (19) into (15), (16), and (17), we have:

$$2P_1 - 2/3\sum_{i=1}^{3} P_i + \mu f'(P_1)\left[\frac{\sum_{i=1}^{3} P_i f'(P_i)}{\sum_{i=1}^{3} f'(P_i)} - P_1\right] = 0 \quad (21)$$

$$2P_2 - 2/3\sum_{i=1}^{3} P_i + \mu f'(P_2)\left[\frac{\sum_{i=1}^{3} P_i f'(P_i)}{\sum_{i=1}^{3} f'(P_i)} - P_2\right] = 0 \qquad (22)$$

$$\text{and } 2P_3 - 2/3\sum_{i=1}^{3} P_i + \mu f'(P_3)\left[\frac{\sum_{i=1}^{3} P_i f'(P_i)}{\sum_{i=1}^{3} f'(P_i)} - P_3\right] = 0 \qquad (23)$$

when the minimum is achieved at (P_1,P_2,P_3) for any given (P_1,P_2,P_3). Since (21), (22), (23) must hold for any given (P_1,P_2,P_3), we can choose P_1, P_2, P_3 such that $P_2 + P_3 = 2P_1$ and $P_2 \neq P_1$, $P_3 \neq P_1$ which reduces (21) to

$$\mu f'(P_1)\left[\frac{\sum_{i=1}^{3} P_i f'(P_i)}{\sum_{i=1}^{3} f'(P_i)} - P_1\right] = 0$$

$$\Rightarrow \sum_{i=1}^{3} P_i f'(P_i) = P_1\sum_{i=1}^{3} f'(P_i)$$

since $\mu \neq 0$ and $f'(P_1) \neq 0$. (If $f'(P_1) = 0$ then $f(P_1)$ = constant, $\forall P_1$ since we can arbitrarily choose P_1.

Therefore, $(P_2 - P_1)f'(P_2) = -(P_3 - P_1)f'(P_3)$

$$\Rightarrow (P_2 - P_1)f'(P_2) = (P_2 - P_1)f'(P_3)$$

$$\Rightarrow (P_2 - P_1)(f'(P_2) - f'(P_3)) = 0$$

$$\Rightarrow f'(P_2) = f'(P_3), \text{ since } P_2 \neq P_1$$

Now, since we can arbitrarily choose P_1 and P_2, it follows that $f'(P) = c$ for some cosntant c, which implies f(p) is linear, i.e., $f(P) = K + cP$ for some constants K and c. Substituting this functional

form into requirement condition, eq. (20), we have

$$\sum_{i=1}^{3} P_i(cP_i - \frac{c}{3}\sum_{i=1}^{3} P_i) \leq 0$$

$$\Rightarrow c \cdot (\sum_{i=1}^{3} P_i^2 - \frac{1}{3}(\sum_{i=1}^{3} P_i)^2) \leq 0$$

$$\Rightarrow c < 0,$$

Since $\sum_{i=1}^{3} P_i^2 - \frac{1}{3}(\sum_{i=1}^{3} P_i)^2 = 3\,\mathrm{var}(P) > 0$ whenever P_1, P_2 and P_3 are not all the same and f(P) is nontrivial (i.e., $c \neq 0$). Thus we have shown that to satisfy Friedman's conjecture, f(P) must be linear with negative slope. Now, by Farrell [2], Kemp [4], or Telser [7], we know this functional form satisfies Friedman's conjecture.[5] Hence, the proof is complete.

Q.E.D.

Actually, we can easily extend the proof of Theorem 3 to a T-period model ($T \geq 3$), and establish the following theorem:

Theorem 4

For a T-period model with $T \geq 3$, the only nontrivial continuously differentiable non-speculative excess demand functional form which can satisfy Friedman's conjecture is linear with negative slope.

Hence, Farrell's two results hold.

Lag-Responsive Non-speculative Excess Demand

Although Friedman [3, p. 269] claimed there shouldn't be a lag response in non-speculative excess demand functions, it's still worth studying to what extent the above results generalize to this case. Before proceeding with this approach, note that Friedman seemed to argue that if an individual's demand was a function of past prices, then he should be classified as a speculator. Therefore, Friedman's position was that if a function involves past prices, then it can never qualify as a non- speculative excess demand function.

Despite this, Schimmler generalized Farrell's approach to consider interdependent demand situations. First, let $P^S = [P_1^S, P_2^S, \cdots, P_T^S]'$, $P^W = [P_1^W, P_2^W, \cdots, P_T^W]'$, $Q^S = [Q_1^S, Q_2^S, \cdots, Q_T^S]'$, $Q^W = [Q_1^W, Q_2^W, \cdots, Q_T^W]'$, $S = Q^S - Q^W$, all (T X 1) vectors. Schimmler assumed that the non-speculative excess demand function has the following property:

$$P^S - P^W = H(S) \tag{24}$$

where H is a mapping from $\mathbb{R}^T$ to $\mathbb{R}^T$. Under (24), the Schimmler showed that $H^*(S) = H(S) - \frac{H(S)\cdot U}{T}U = b(S)\cdot S$, where b(S) is a real-valued function and $U = [1,1,1,\ldots,1]'$ is a (T X 1) vector, is necessary and sufficient for Friedman's conjecture to be true.

Since Schimmler's assumption (eq. (24)) is only a generalized version of Farrell's assumption (eq. (5)), then by employing similar procedures, we can prove once again that only linear mappings are consistent with eq. (24).

Theorem 5

Let P^S, P^W, Q^S, Q^W be (T X 1) vectors satisfying $Q^S = f(P^S)$, $Q^W = f(P^W)$. Within the class of continuously differentiable mappings, the only mapping $h(\cdot)$ which can satisfy $P^S - P^W = H(Q^S - Q^W)$ is linear. Also, $f(\cdot)$ is linear.

[Proof]

Since $Q^S = f(P^S)$, $Q^W = f(Q^W)$, $P^S - P^W = H(f(P^S) - f(P^W))$. Taking the Jacobian[6] on both sides with respect to P^S, we have

$$I = J(H(Q^S - Q^W)) \cdot J(f(P^S)), \quad \forall P^S, P^W \tag{25}$$

Where I is the (T X T) identity matrix.

Similarly, taking the Jacobian on both sides with respect to P^W,

$$-I = J(H(Q^S - Q^W)) \cdot [-J(f(P^W))], \quad \forall P^S, P^W \tag{26}$$

Now, since

$$\begin{aligned} T = \text{rank } I &= \text{rank}[J(H(Q^S - Q^W)) \cdot J(f(P^S))] \\ &\leq \text{rank}[J(H(Q^S - Q^W))] \text{ , rank}[J(f(P^S))] \leq T \\ &\Rightarrow \text{rank}[J(H(Q^S - Q^W))] = \text{rank}[J(f(P^S))] = T \\ &\Rightarrow [J(H(Q^S - Q^W))]^{-1} \text{ and } [J(f(P^S))]^{-1} \text{ exists,} \end{aligned}$$

Therefore $J(f(P^S)) = [J(H(Q^S - Q^W))]^{-1} = J(f(P^W))$, $\forall P^S, P^W$

$$\Rightarrow J[f(P)] = A \text{ for some constant (T X T) matrix, } \forall P.$$

$$\Rightarrow J[H(Q^S - Q^W)] = A^{-1}, \ \forall Q^S, Q^W \Rightarrow H \text{ is linear.}$$

Q.E.D.

Again, Theorem 5 shows that Schimmler's approach is partially correct, i.e., he only proved that even if non-speculative excess demand is linear, and interdependent, then we still need other restrctions in order to satisfy Friedman's conjecture. This also means we have to reconsider interdependent excess demand situations.[7]

Theorem 6

Any continuously differentiable non-speculative excess demand function $f(P_t,P_{t-1})$ with $f_1(P_t,P_{t-1}) < 0$, $f_2(P_t,P_{t-1}) \leq 0$ will satisfy Friedman's conjecture in the two-period model.

[Proof]

Let P_0 be the exogeneously determined price at 0 period, and let P_1, P_2 be the prices in period 1 and 2 under the speculation sequence $\{S_1,S_2\}$. Also, let q_1, q_2 be the prices in periods 1 and 2 when there is no speculation. Therefore,

$$Q_1^W = f(q_1,P_0),\ Q_2^W = f(q_2,q_1),$$
$$Q_1^S = f(P_1,P_0),\ Q_2^S = f(P_2,P_1)$$
$$\Rightarrow S_1 = Q_1^S - Q_1^W = f(P_1,P_0) - f(q_1,P_0), \text{ and}$$
$$S_2 = Q_2^S - Q_2^W = f(P_2,P_1) - f(q_2,q_1).$$

Hence, the minimization problem we're considering now is

$$\underset{\{q_1,q_2\}}{\text{Min}}\ V(q_1,q_2) = (q_1^2 + q_2^2) - \tfrac{1}{2}(q_1 + q_2)^2$$

$$\text{Subject To: } f(P_1,P_0) - f(q_1,P_0) + f(P_2,P_1) - f(q_2,q_1) = 0 \quad (27)$$

$$P_1[f(P_1,P_0) - f(q_1,P_0)] + P_2[f(P_2,P_1) - f(q_2,q_1)] \geq 0 \quad (28)$$

By the same arguments as in the proof of Theorem 2, we have two cases:

Case A: If both (27) and (28) are binding, then

$$[f(P_1,P_0) - f(q_1^*,P_0)](P_1 - P_2) = 0$$
$$\Rightarrow f(P_1,P_0) = f(q_1^*,P_0) \Rightarrow P_1 = q_1^*, \text{ if } P_1 \neq P_2$$

substituting into (27), we have

$$f(P_2,P_1) - f(q_2^*,P_1) = 0 \Rightarrow P_2 = q_2^*.$$

If $P_1 = P_2$, then it can easily be shown that

$$q_1^* = q_2^* = P_1 = P_2$$

Case B: If (28) is not binding, then we have $q_1^* = q_2^* = q^*$, and $[f(P_1,P_0) - f(q^*,P_0)](P_1 - P_2) \geq 0$.

(i) If $P_1 > P_2$, then $f(P_1,P_0) \geq f(q^*,P_0)$ which implies $f(P_2,P_1) - f(q^*,q^*) \leq 0$ and $P_1 \leq q^*$.

Since $f_1 < 0$, $f_2 \leq 0$, we must have $P_2 \geq q^* \Rightarrow P_2 \geq P_1$, contradicting to $P_1 > P_2$.

(ii) If $P_1 < P_2$, then $f(P_1,P_0) \leq f(q^*,P_0)$ which implies $f(P_2,P_1) - f(q^*,q^*) \geq 0$ and $P_1 \geq q^*$.

By $f_1 < 0$, $f_2 \leq 0$, we have $P_2 \leq q^* \Rightarrow P_1 \geq P_2$, a contradiction.

Thus far, we have shown that the optimum point must be (P_1,P_2) for any given (P_1,P_2). To make sure V achieves a minimum, we'll check the local properties of V at (P_1,P_2) while still satisfying (27) and (28) as follows:

(i) If $P_1 > P_2$, then to satisfy (24) and (25), we must have

$$(f(P_1,P_0) - f(q_1,P_0))(P_1 - P_2) \geq 0$$

which implies $f(P_1,P_0) \geq f(q_1,P_0) \Rightarrow P_1 \leq q_1$. Now by (24), we have $f(P_2,P_1) \leq f(q_2,q_1) \Rightarrow P_2 \geq q_2$. Hence, $V(P_1,P_2) \leq V(q_1,q_2)$.

(ii) If $P_1 < P_2$, then $(f(P_1,P_0) - f(q_1,P_0))(P_1 - P_2) \geq 0$

$\Rightarrow f(P_1,P_0) \leq f(q_1,P_0) \Rightarrow P_1 \geq q_1$.

By (24), we have $f(P_2,P_1) \geq f(q_2,q_1)$

$\Rightarrow P_2 \leq q_2$, hence $V(P_1,P_2) \leq V(q_1 q_2)$.

(iii) If $P_1 = P_2$, then (P_1,P_2) is the only feasible point satisfying (27) and (28).

Therefore, (P_1,P_2) achieves a minimum of $V(q_1,q_2)$ subject to constraints (27) and (28), which implies Friedman's conjecture is satisfied.

Q.E.D.

Theorem 7

For a three-period model, within the class of twice continuously differentiable functions, the only (nontrivial) non-speculative excess demand functional form $f(P_t,P_{t-1},P_{t-2})$, which can satisfy Friedman's conjecture is linear with $f_1 < 0$, $f_2 = f_3 = 0$.

[Proof]

Let P_{-1} and P_0 be exogeneously determined prices in periods -1 and 0, respectively, and let P_1, P_2 and P_3 be prices in periods 1, 2 and 3 associated with speculative sequence $\{S_1,S_2,S_3\}$. Also, let q_1, q_2, and q_3 be prices in periods 1, 2 and 3 when there is no speculation. Hence, we have:

$$\begin{aligned}
Q_1^W &= f(q_1,P_0,P_{-1}), \quad Q_1^S = f(P_1,P_0,P_{-1}) \\
Q_2^W &= f(q_2,q_1,P_0), \quad Q_2^S = f(P_2,P_1,P_0) \\
Q_3^W &= f(q_3,q_2,q_1), \quad Q_3^S = f(P_3,P_2,P_1) \\
\Rightarrow S_1 &= Q_1^S - Q_1^W = f(P_1,P_0,P_{-1}) - f(q_1,P_0,P_{-1}) \\
S_2 &= Q_2^S - Q_2^W = f(P_2,P_1,P_0) - f(q_2,q_1,P_0) \\
S_3 &= Q_3^S - Q_3^W = f(P_3,P_2,P_1) - f(q_3,q_2,q_1)
\end{aligned}$$

Now, the minimization problem under consideration is

$$\underset{\{q_1,q_2,q_3\}}{\text{Min}} \sum_{i=1}^{3} q_i^2 - \frac{1}{3}\left(\sum_{i=1}^{3} q_i\right)^2$$

Subject To:

$$f(P_1,P_0,P_{-1}) - f(q_1,P_0,P_{-1}) + f(P_2,P_1,P_0) - f(q_2,q_1,P_0) + f(P_3,P_2,P_1) - f(q_3,q_2,q_1) = 0 \quad (29)$$

and

$$P_1[f(P_1,P_0,P_{-1}) - f(q_1,P_0,P_{-1})] + P_2[f(P_2,P_1,P_0) - f(q_2,q_1,P_0)] + P_3[f(P_3,P_2,P_1) - f(q_3,q_2,q_1)] \geq 0 \quad (30)$$

Forming the Lagrangian, we can derive the first-order conditions as following:

$$\frac{\partial \mathbb{L}}{\partial q_1} = 2q_1 - \frac{2}{3}\left(\sum_{i=1}^{3} q_i\right) + \lambda[f_1(q_1,P_0,P_{-1}) + f_2(q_2,q_1,P_0) + f_3(q_3,q_2,q_1)] + \mu[-f_1(q_1,P_0,P_{-1})P_1 - f_2(q_2,q_1,P_0)P_2 - f_3(q_3,q_2,q_1)P_3] = 0 \quad (31)$$

$$\frac{\partial \mathbb{L}}{\partial q_2} = 2q_2 - \frac{2}{3}\left(\sum_{i=1}^{3} q_i\right) + \lambda[f_1(q_2,q_1,P_0) + f_2(q_3,q_2,q_1)] + \mu[-f_1(q_2,q_1,P_0)P_2 - f_2(q_3,q_2,q_1)P_3] = 0 \quad (32)$$

$$\frac{\partial \mathbb{L}}{\partial q_3} = 2q_3 - \frac{2}{3}(\sum_{i=1}^{3} q_i) + \lambda f_1(q_3,q_2,q_1) - \mu f_1(q_3,q_2,q_1)P_3 = 0 \qquad (33)$$

$$\frac{\partial \mathbb{L}}{\partial \lambda} = f(P_1,P_0,P_{-1}) - f(q_1,P_0,P_{-1}) + f(P_2,P_1,P_0) - f(q_2,q_1,P_0) + f(P_3,P_2,P_1) - f(q_3,q_2,q_1) = 0$$

$$\mu \cdot \frac{\partial \mathbb{L}}{\partial \mu} = 0, \qquad \mu \geq 0, \qquad \frac{\partial \mathbb{L}}{\partial \mu} \geq 0$$

Now, consider only the binding case, i.e., $\lambda \neq 0$, $\mu \neq 0$. Then to satisfy Friedman's conjecture, we require (P_1,P_2,P_3) to satisfy eqs. (29) to (33). To simplify the notation, let

$$f_i^{(1)} \equiv f_i(P_1,P_0,P_{-1}),\ f_i^{(2)} = f_i(P_2,P_1,P_0),\ f_i^{(3)} = f_i(P_3,P_2,P_1),\ i = 1,2,3.$$

Hence, we must have

$$2P_1 - \frac{2}{3}(\sum_{i=1}^{3} P_i) + \lambda[f_1^{(1)} + f_2^{(2)} + f_3^{(3)}] - \mu[f_1^{(1)}P_1 + f_2^{(2)}P_2 + f_3^{(3)}P_3] = 0 \qquad (34)$$

$$2P_2 - \frac{2}{3}(\sum_{i=1}^{3} P_i) + \lambda[f_1^{(2)} + f_2^{(3)}] - \mu[f_1^{(2)}P_2 + f_2^{(3)}P_3] = 0 \qquad (35)$$

$$2P_3 - \frac{2}{3}(\sum_{i=1}^{3} P_i) + \lambda f_1^{(3)} - \mu f_1^{(3)}P_3 = 0 \qquad (36)$$

summing up (34), (35), (36),

$$\lambda = \frac{f_1^{(1)}P_1 + (f_1^{(2)} + f_2^{(2)})P_2 + (f_1^{(3)} + f_2^{(3)} + f_3^{(3)})P_3}{f_1^{(1)} + f_1^{(2)} + f_2^{(2)} + f_1^{(3)} + f_2^{(3)} + f_3^{(3)}} \cdot \mu \qquad (37)$$

Since for any given P_1, P_2 and P_3, Eqs. (34) - (37) always hold, we can choose P_1, P_2, P_3 such that $P_1 + P_2 = 2P_3$ and $P_1 \neq P_3$, $P_2 \neq P_3$. Thus, from (36),

$$\lambda f_1^{(3)} - \mu f_1^{(3)}P_3 = 0 \Rightarrow f_1^{(3)}(\lambda - \mu P_3) = 0 \Rightarrow \lambda = \mu P_3,$$

since f is nontrivial. Substituting this into (37),

$$\mu \cdot \{\frac{f_1^{(1)}P_1 + (f_1^{(2)} + f_2^{(2)})P_2 + (f_1^{(3)} + f_2^{(3)} + f_3^{(3)})P_3}{f_1^{(1)} + f_1^{(2)} + f_2^{(2)} + f_1^{(3)} + f_2^{(3)} + f_3^{(3)}} - P_3\} = 0$$

$$\Rightarrow f_1^{(1)}P_1 + (f_1^{(2)} + f_2^{(2)})P_2 - (f_1^{(1)} + f_1^{(2)} + f_2^{(2)})P_3 = 0, \text{ since } \mu \neq 0$$

$$\Rightarrow (f_1^{(2)} + f_2^{(2)} - f_1^{(1)})P_2 + (f_1^{(1)} - f_1^{(2)} - f_2^{(2)})P_3 = 0 \text{ (using } P_1 = 2P_3 - P_2)$$

$$\Rightarrow (f_1^{(2)} + f_2^{(2)} - f_1^{(1)})(P_2 - P_3) = 0$$

$$\Rightarrow f_1^{(2)} + f_2^{(2)} - f_1^{(1)} = 0, \text{ since } P_2 \neq P_3$$

i.e., $f_1(P_2,P_1,P_0) - f_1(P_1,P_0,P_{-1}) = -f_2(P_2,P_1,P_0)$ when $P_1 + P_2 = 2P_3$, and $P_1 \neq P_2$. Now, since we can arbitrarily change P_3, also by twice continuous differentiability of f, we have

$$f_1(P_2,P_1,P_0) - f_1(P_1,P_0,P_{-1}) = -f_2(P_2,P_1,P_0), \quad \forall P_2,P_1,P_0,P_{-1}. \qquad (38)$$

Note that P_{-1} is arbitrarily given, and only $f_1(P_1,P_0,P_{-1})$ involves this term, implies $f_{13}(P_1,P_0,P_{-1}) = 0$, $\forall\ P_1,P_0,P_{-1}$

$$\Rightarrow f_{13}(P_t,P_{t-1},P_{t-2}) = 0, \quad \forall\ P_t,P_{t-1},P_{t-2} \qquad (39)$$

Similarly, if we fix P_1,P_0,P_{-1} and change P_2, we'll have

$$f_{11}(P_2,P_1,P_0) + f_{21}(P_2,P_1,P_0) = 0, \quad \forall\ P_2,P_1,P_0$$

$$\Rightarrow f_{11}(P_t,P_{t-1},P_{t-2}) + f_{21}(P_t,P_{t-1},P_{t-2}) = 0, \quad \forall\ P_t,P_{t-1},P_{t-2} \qquad (40)$$

which is a partial differential equation. The solution of (40) is

$$f_1(P_t,P_{t-1},P_{t-2}) = a + bP_t - bP_{t-1} \qquad (41)$$

for some constants a and b (Note: $f_{13} = 0$ by (39)).

Substituting (41) into (38),

$$a + bP_2 - bP_1 - (a + bP_1 - bP_0) = -f_2(P_2,P_1,P_0)$$

$$\Rightarrow -f_2(P_2,P_1,P_0) = b(P_2 - P_1) + b(P_0 - P_1) \quad \forall\, P_2,P_1,P_0$$

$$\Rightarrow f_2(P_t,P_{t-1},P_{t-2}) = -bP_t + 2bP_{t-1} - bP_{t-2}, \quad \forall\, P_t,\ P_{t-1},\ P_{t-2}$$

Now, choose P_1,P_2,P_3 such that $P_1 + P_3 = 2P_2$, $P_1 \neq P_2$, $P_3 \neq P_2$, then (35) reduces to

$$\lambda(f_1^{(2)} + f_2^{(3)}) - \mu(f_1^{(2)}P_2 + f_2^{(3)}P_3) = 0$$

Substituting (37),

$$\mu \cdot \left[\frac{f_1^{(1)}P_1 + (f_1^{(2)} + f_2^{(2)})P_2 + (f_1^{(3)} + f_2^{(3)} + f_3^{(3)})P_3}{f_1^{(1)} + f_1^{(2)} + f_2^{(2)} + f_1^{(3)} + f_2^{(3)} + f_3^{(3)}} \cdot (f_1^{(2)} + f_2^{(3)}) - f_1^{(2)}P_2 - f_2^{(3)}P_3\right] = 0$$

After algebraic operations, we have

$$(f_1^{(1)}f_1^{(2)} + 2f_1^{(1)}f_2^{(3)} + f_2^{(2)}f_2^{(3)} - f_1^{(3)}f_1^{(2)} - f_3^{(3)}f_1^{(2)})(P_2 - P_3) = 0. \qquad (42)$$

Since $f_2^{(3)} = f_2(P_3,P_2,P_1) = -bP_3 + 2bP_2 - bP_1 = 0$, (42) reduces to

$$(f_1^{(1)}f_1^{(2)} - f_1^{(3)}f_1^{(2)} - f_3^{(3)}f_1^{(2)})(P_2 - P_3) = 0$$

$$\Rightarrow f_1^{(2)}(f_1^{(1)} - f_1^{(3)} - f_3^{(3)})(P_2 - P_3) = 0$$

$$\Rightarrow f_1^{(1)} - f_1^{(3)} = f_3^{(3)}, \text{ since } f_1^{(2)} \neq 0,\ P_2 \neq P_3$$

$$\Rightarrow f_3(P_3,P_2,P_1) = (a + bP_1 - bP_0) - (a + bP_3 - bP_2)$$

$$= -bP_3 + bP_2 + bP_1 - bP_0, \text{ when } P_1 + P_3 = 2P_2,\ P_1 \neq P_3$$

Hence, by arbitrarily changing P_0, we must have

$$\partial f_3(P_3,P_2,P_1)/\partial P_0 = -b = 0.$$

which implies $f_1(P_t,P_{t-1},P_{t-2}) \equiv a$ (43)

and $f_2(P_t,P_{t-1},P_{t-2}) \equiv 0$ (44)

$\Rightarrow f_3^{(3)} = 0$ when $2P_2 = P_1 + P_3$ and $P_1 \neq P_3$, i.e.,

$$f_3(P_3,P_2,2P_2 - P_3) = 0, \quad \forall\, P_2,P_3 \qquad (45)$$

But, $f_{31}(P_t,P_{t-1},P_{t-2}) = f_{13}(P_t,P_{t-1},P_{t-2}) \equiv 0$, and

$f_{32}(P_t,P_{t-1},P_{t-2}) = f_{23}(P_t,P_{t-1},P_{t-2}) \equiv 0$, hence

$f_3(P_t,P_{t-1},P_{t-2}) = g(P_{t-2})$ for some function $g(\cdot)$,

and $f_3(P_3,P_2,2P_2 - P_3) = g(2P_2 - P_3)$.

Since we can arbitrarily choose P_2, P_3, therefore $g(2P_2 - P_3) = 0$, $\forall\, P_2,P_3$ implies $g(P_{t-2}) \equiv 0$, and

$$f_3(P_t,P_{t-1},P_{t-2}) \equiv 0 \qquad (46)$$

Combining (43), (44) and (46), we have

$f(P_t,P_{t-1},P_{t-2}) = K + aP_t$ for some constants K and a.

Next, consider the nonbinding case (i.e., $\lambda = \mu = 0$), and follow the same arguments as in proof of Theorem 4, $a < 0$, which completed the proof.

Q.E.D.

Theorem 6 shows that, as in the independent excess demand case, suppose we only ocnsider two-period models, a large class of continuously differentiable nospeculative excess demand functions will satisfy Friedman's conjecture, even though they involve past prices. However, Theorem 7 states that for Friedman's conjecture to be true in three- period model, we can never have a lag structure, if we assume a nonspeculative excess demand functional form that is independent of time. This implies Friedman's classification of speculators cannot be relaxed, otherwise his conjecture will in general be invalidated.[8]

Further Discussions

Although Theorem 7 seems to be a little suprise, yet it was already implied by Schimmler's conclusion as long as we can prove linear excess demand is necessary for Friedman's conjecture to be true. (Recall that, instead of doing this, Schimmler assumed eq. (24), which is equivalent to assumption of linear excess demand). To show this, first we state Theorem 8, which is also a property derived from Schimmler's conclusions.

Theorem 8

If H(S) is a real-valued differentiable vector function which satisfies:

(i) $P^S - P^W = H(S)$, $\forall$ S, and

(ii) $H^*(S) = H(S) - \frac{H(S)'\mu}{T}\mu = b(S)'S$, $\forall$ S for some real-valued function $b(\cdot)$, where $\mu' = [1,1,\cdots]$,

Then $b(S) = b_0$, $\forall$ S, for some constant b_0.

[Proof]

First, from Theorem 5, we know H(S) satisfies (i) which implies H(S) is linear , i.e., there exists a (T X T) matrix A such that $H(S) = A'S$, where T is the number of periods under considerations. Therefore,

$$H^*(S) = H(S) - \frac{H(S)'\mu}{T}\mu$$

is also a linear function which can be expressed as $A^{*\prime}S$. Now, by condition (ii),

$$A^{*\prime}S = b(S)'S$$

$$\Rightarrow [A^* - b(S)I]'S = 0 \tag{47}$$

To have a nontrivial solution for the above linear homogeneous T-equation system in T variables, we must have

$$\det[A^* - b(S)'I] = 0, \ \forall S$$

Therefore, b(S) is determined by A, independent of S, i.e., $b(S) = b_0$, $\forall$ S, for some constant b_0.

Q.E.D.

Now, assume

$$P^S - P^W = H(S) = A'S = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

then we have $a_{12} = a_{13} = a_{23} = 0$, since the changes in prices shouldn't be affected by future speculations. Also, by the time-independent excess demand assumption, $a_{11} = a_{22} = a_{33}$.[9] From Theorem 8, $b(S) = b_0$, $\forall$ S. So,

$$H^*(S) = b(S)\cdot S$$

$$\Rightarrow a_{11}S_1 - \frac{(a_{11}+a_{21}+a_{31})S_1 + (a_{22}+a_{32})S_2 + a_{33}S_3}{3} = b_0S_1 \tag{48}$$

$$a_{21}S_1 + a_{22}S_2 - \frac{(a_{11}+a_{21}+a_{31})S_1 + (a_{22}+a_{32})S_2 + a_{33}S_3}{3} = b_0S_2 \tag{49}$$

$$a_{31}S_1 + a_{32}S_2 + a_{33}S_3 - \frac{(a_{11}+a_{21}+a_{31})S_1 + (a_{22}+a_{32})S_2 + a_{33}S_3}{3} = b_0S_3 \tag{50}$$

Substituting $S_1 + S_2 + S_3 = 0$ and by requiring nontrivial solutions, we have

$$\begin{cases} a_{21} - a_{11} + b_0 = 0 \\ a_{31} - a_{11} + 2b_0 - a_{33} = 0 \\ a_{22} = b_0 \\ a_{32} - a_{33} + a_{22} = 0 \end{cases} \Rightarrow \begin{cases} a_{11} = a_{22} = a_{33} = b_0 \\ a_{21} = a_{31} = a_{32} = 0 \end{cases} . \tag{51}$$

Since Schimmler also showed $b_0 < 0$,[10] we have the same conclusion as Theorem 8, only that instead of proving, he assumed linearity is a necessary condition.

In fact, Theorem 7 can be generalized to a T-period model with $T \geq 3$ as follows:[11]

<u>Theorem 9</u>

For a T-period model with $T \geq 3$, within the class of twice continuously differentiable functions, the only (nontrivial) non-speculative excess demand functional form $Q_t = f(P_t, P_{t-1}, \cdots, P_{t-(T-1)})$ which can satisfy Friedman's conjecture is linear with $f_1 < 0$, $f_2 = f_3 = \cdots = f_{t-(T-1)} = 0$.

Now, we really come to a deadend. That is, if non-speculative excess demand involves a non-degenerate lag structure, then Friedman's conjecture is always false. However, this contradicts Friedman's claim that this kind of functional form can't be used to represent non-speculative demand. There seems no way to resolve this problem. These conclusions only applied to T-period model, when $T \geq 3$. When T=2, there is a large class of functions which can satisfy Friedman's conjecture even if a lag structure exists.[12]

Footnotes

* I am indebted to James Quirk for helpful discussions and editings, also to Richard McKelvey and Jennifer Reinganum for comments on earlier drafts. All errors, of course, remain mine.

1. This idea was originated by Telser [7, p. 295].

2. Though there are some problems associated with using the variance as a measure of price stability [7, p. 296], yet it seems to be widely accepted [2][4].

3. Farrell's formula is correct, if we fix the initial non-speculative price unchanged. Nonetheless, in this case, $h(\cdot)$ is not only a function of S_t, also of P_t^W, and hence his proof is still unjustified.

4. By "nontrivial," we meant the non-speculative excess demand $Q_t =$

$$\frac{\partial f(P_t, P_{t-1}, \cdots)}{\partial P_t} \neq 0, \ \forall P_t$$

5. To be consistent with our approach, we provide the following proof of T-period model. The problem under consideration is

$$\underset{S}{\text{Min}}\ q'[I - e(e'e)^{-1}e']q$$
$$\text{subject to:}\ \ P'\cdot S \geq 0,\ e'\cdot S = 0$$

where $q = [q_1, q_2, \cdots, q_T]'$, $P = [P_1, P_2, \cdots, P_T]'$, $e = [1, 1, \ldots, 1]'$, $S = [S_1, S_2, \cdots, S_T]'$, I is T X T identity matrix.

Now, since demand is linear with negative slope α,

$$P'\cdot S \geq 0 \ \Rightarrow\ P'\cdot(\alpha P - \alpha q) \geq 0 \ \Rightarrow\ P'\cdot(P - q) \leq 0 \ \Rightarrow\ P'\cdot P \leq P'\cdot q$$

also, $e'\cdot S = 0 \ \Rightarrow\ e'\cdot(\alpha P - \alpha q) = 0 \ \Rightarrow\ e'\cdot P = e'\cdot q$

Assume $q = P + \delta$, i.e., δ is the deviation vector, then

$$q'[I - e(e'e)^{-1}e']q = q'q - q'e(e'e)^{-1}e'q$$
$$= q'q - P'e(e'e)^{-1}e'P = (P + \delta)'(P + \delta) - P'e(e'e)^{-1}e'P$$
$$= P'P + \delta'P + P'\delta + \delta'\delta - P'e(e'e)^{-1}e'P$$

Since $P'\cdot q \geq P'\cdot P \ \Rightarrow\ P'\cdot(q - P) \geq 0 \ \Rightarrow\ P'\cdot\delta \geq 0$, and $\delta'\cdot P \geq 0$, therefore

$$q'[I - e(e'e)^{-1}e']q = P'[I - e(e'e)^{-1}e']P + 2P'\cdot\delta + \delta'\delta$$
$$\geq P'[I - e(e'e)^{-1}e']P$$

which implies P is the solution of this minimization problem, and Friedman's conjecture is true under linear demand cases.

6. The Jacobian is defined as the following: Let $f(y) \equiv f(y_1, y_2, \cdots, y_n) \equiv (f_1(y_1, y_2, \cdots, y_n), f_2(y_1, y_2, \cdots, y_n), \cdots, f_m(y_1, y_2, \cdots, y_n))$ be a real vector function mapping from $\mathbb{R}^n$ to $\mathbb{R}^m$, then the Jacobian of $f(y)$ is

$$J[f(y)] = \begin{bmatrix} \frac{\partial f_1}{\partial y_1} & \frac{\partial f_2}{\partial y_1} & \cdots & \frac{\partial f_m}{\partial y_1} \\ \frac{\partial f_1}{\partial y_2} & \frac{\partial f_2}{\partial y_2} & \cdots & \frac{\partial f_m}{\partial y_2} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \frac{\partial f_1}{\partial y_n} & \frac{\partial f_2}{\partial y_n} & \cdots & \frac{\partial f_m}{\partial y_n} \end{bmatrix}$$

7. There are two important points worth noting about Theorem 6 and 7:

(a) Though we assume $Q_t = f(P_t, P_{t-1})$ in two- period models, $Q_t = f(P_t, P_{t-1}, P_{t-2})$ in three-period model, actually we can add more lags since they're irrevelant in the proofs.

(b) In this paper, we only consider time-independent lag-repsonsive speculative excess demand. Therefore, Theorem 6 and 7 only apply to this case.

8. An example of an inconsistency between Friedman's conjecture and lag-responsive non-speculative demand will be:

Let $Q_t = 20 - 0.5P_t + 0.3P_{t-1}$ and assume $P_0 = 4$. Now, if we have $P_1^W = 5.2$, $P_2^W = 5.72$, $P_3^W = 7.032$, which implies $Q_1^W = 18.6$, $Q_2^W = 18.7$, $Q_3^W = 18.2$. And then, pick up $S_1 = 0.1$, $S_2 = -0.21$, $S_3 = 0.11$, we have $Q_1^S = 18.7$, $Q_2^S = 18.49$, $Q_3^S = 18.31$ and $P_1^S = 5$, $P_2^S = 6.02$, $P_3^S = 7.02$.

Hence,

(a) $\sum_{i=1}^{S_i} = 0.1 - 0.21 + 0.11 = 0$, i.e., $\{S_1, S_2, S_3\}$ is a

complete speculation sequence

(b) $\pi = \sum_{i=1}^{3} P_i^S S_i = 0.00492 > 0$

(c) $C = [\sum_{i=1}^{3} (P_i^W)^2 - \frac{1}{3}(\sum_{i=1}^{3} P_i^W)^2] - [\sum_{i=1}^{3} (P_i^S)^2 - \frac{1}{3}(\sum_{i=1}^{3} P_i^S)^2]$

$= 1.78266 - 1.98442 < 0$

which shows this particular profitable speculation destabilizes prices.

9. Originally, Schimmler didn't assume time-independent functional form, therefore solving the equation system (51), we have

$$A = \begin{bmatrix} b_0 + \varepsilon & 0 & 0 \\ \varepsilon & b_0 & 0 \\ \varepsilon + \delta & \delta & b_0 + \delta \end{bmatrix}$$

where ε, δ are arbitrary numbers, and $b_0 < 0$. Here, we could have lag structure, but it must change over time.

10. In Schimmler's paper, $b(S) > 0$. But we explained S as speculative sales, while he explained it as speculative buys. Hence, in our case, it becomes $b(S) < 0 \Rightarrow b_0 < 0$.

11. One way to think about this is, assuming $q_{t-3} = P_{t-3}, q_{t-4} = P_{t-4}, \ldots, q_{t-T+1} = P_{t-T+1}$, for given $(P_t, P_{t-1}, P_{t-2}, \ldots, P_{t-T+1})$. Then, the minimization problem for T-period models is exactly the same as three-period models which, by Theorem 7, implies no lag structure. Now, picking up any three consecutive periods and letting the other time periods τ satisfy $q_\tau = P_\tau$, we have the conclusion as above, i.e., no lag structure. This derives Theorem 9.

12. Note that, this paper along with Farrell [2], Schimmler [5], are all ex post analyses.

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