DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA 91125

PERFECTLY COMPETITIVE MARKETS AS THE LIMITS OF COURNOT MARKETS

William Novshek
California Institute of Technology
Stanford University

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ABSTRACT: We consider a perfectly competitive, partial equilibrium market for a single homogeneous good with a (bounded) continuum of infinitesimal firms. Cost functions are essentially unrestricted and are allowed to vary smoothly across firms. We introduce a sequence (net) of Cournot markets (each with a finite number of firms) which converge smoothly to the perfectly competitive limit in terms of both the inverse demand functions and the distribution of firm technologies and show that all markets sufficiently far along the sequence have a Cournot equilibrium and all the Cournot equilibria converge to the perfectly competitive equilibrium of the limit market.

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1. Introduction

A growing literature has been examining the noncooperative foundations of perfect competition by comparing the limits of noncooperative equilibria of a sequence of markets (or economies) and the perfectly competitive equilibria of the limit market (or economy). See for example Dasgupta and Ushio [2], Fraysse and Moreaux [3], Hart [4], Mas-Colell [5], Novshek [6], Novshek and Sonnenschein [8,9], and Ushio [10,11]. Here we examine this problem in the context of a single, partial equilibrium market for a homogeneous good. We consider a limit market, $M(0)$, with a bounded continuum of infinitesimal firms. Since individual infinitesimal firms cannot affect aggregate output or market price, firms must act as price takers, and the proper notion of equilibrium is the usual perfectly competitive equilibrium for the market. We also consider markets $M(\alpha)$, $\alpha \in (0,1]$, in which firms are not infinitesimal. We assume firms correctly perceive their influence on price, and use pure strategy Nash equilibrium in outputs (or Cournot equilibrium) as the equilibrium concept in these markets. We show that if the demand sectors of $M(\alpha)$ converge smoothly to the demand sector of $M(0)$, and the production sectors of $M(\alpha)$ converge smoothly to the production sector of $M(0)$ in terms of the distribution of firm types, then (1) for all small α, $M(\alpha)$ has a Cournot equilibrium and (2) the Cournot equilibria of $M(\alpha)$ converge to the perfectly competitive equilibrium of $M(0)$ as α converges to zero. We allow firms to differ within and between markets, and allow average cost curves to have any shape.

The results are a substantial generalization of Novshek [6] in which all firms were identical within markets and rescaled versions of each other between markets, with average cost U-shaped (or decreasing). Other than standard assumptions on cost and inverse demand functions, our only assumption needed for eventual existence of Cournot equilibrium and convergence of the Cournot equilibria to the perfectly competitive equilibrium of the limit market is that the markets $M(\alpha)$ converge smoothly to the limit market $M(0)$.

The existence proof follows from showing that for small α we can apply an existence theorem for n-firm Cournot equilibria which does not require identical firms or nondecreasing marginal cost (see Novshek [7]). That theorem shows that if each reaction correspondence r_i is nonincreasing in the sense that $Y' > Y$ implies $\max r_i(Y') \leq \min r_i(Y)$ (as is the case when each firm's marginal revenue is decreasing in the aggregate output of other firms) then a Cournot equilibrium exists. Bamon and Fraysse [1] independently proved a similar (though weaker) result in the form of a fixed point theorem and used it to show existence of Cournot equilibrium in a market in which demand is sufficiently large (due to replication of the consumer sector) relative to the capacity constraint of firms and a constant determined by the derivatives of the inverse demand function.

The convergence result follows from the smooth convergence of $M(\alpha)$ to $M(0)$. If the limiting aggregate output is not the perfectly competitive equilibrium of $M(0)$ then for α sufficiently small some firm could improve its profit by changing output, and the purported Cournot equilibrium would not in fact be an equilibrium.

The paper is organized as follows. In Section 2 we introduce the model and basic assumptions. Section 3 contains the results and proofs. Section 4 contains some remarks exploring the nature of the assumptions and how they can and cannot be weakened.

2. The Model

For each element of $[0,1]$ we define a different market. $M(0)$ will be a perfectly competitive market with a continuum of infinitesimal firms. For each $\alpha \in (0,1]$, $M(\alpha)$ will be an imperfectly competitive market with a finite number of noninfinitesimal firms. The inverse demand function in $M(0)$ is $P(\cdot,0)$ and the inverse demand function in $M(\alpha)$ is $P(\cdot,\alpha)$. We assume both that standard conditions hold for each inverse demand function and that the inverse demand function for $M(\alpha)$ converges smoothly to the inverse demand function for $M(0)$:

(A1) $P : \mathbb{R}_+ \times [0,1] \to \mathbb{R}_+$ is twice continuously differentiable; $P(Y,0)$ converges to zero as Y converges to ∞; and $P_1 < 0$ whenever $P > 0$, where subscripts on P denote partial derivatives.

In order to define the production sectors of each market so that the production sector for $M(\alpha)$ converges smoothly to the production sector for $M(0)$ we introduce:

(A2) $T : [0,1]^3 \to \mathbb{R}_+$ is continuously differentiable; and $T(x,y,z) + x(1-x)T_1(x,y,z) \geq 0$ for all $(x,y,z) \in [0,1]^3$, where subscripts on T denote partial derivatives.

The production sector for $M(0)$ consists of a continuum of infinitesimal firms with output dependent on expenditure: if firm $\beta \in [0,1]$ spends $c \geq 0$ then its output is $y(c,0,\beta) := cT(c/(c+1),0,\beta)$. If $c(\beta)$ is the expenditure of firm β, $\beta \in [0,1]$, then aggregate expenditure is $\int_0^1 c(\beta)d\beta$ and aggregate output is $\int_0^1 y(c(\beta),0,\beta)d\beta$. (The integrals are with respect to Lebesgue measure.) Note that output is nondecreasing in expenditure:

$$\frac{\partial y}{\partial c}(c,0,\beta) = T(c/(c+1),0,\beta) + [c/(c+1)][1 - c/(c+1)]T_1(c/(c+1),0,\beta) \geq 0.$$

Set up costs are allowed since $y(c,0,\beta) = 0$ for $c > 0$ is possible, and the shape of the derived average cost curve is essentially unrestricted.

For $\alpha \in (0,1]$ the production sector for $M(\alpha)$ consists of a finite number of noninfinitesimal firms with output dependent on expenditure. Let $[1/\alpha]$ be the greatest integer less than or equal to $1/\alpha$. $M(\alpha)$ has $[1/\alpha]$ firms and if firm $j \in \{1,2,...,[1/\alpha]\}$ spends $c \geq 0$ then its output is $y(c,\alpha,j\alpha) = cT(c/(c+\alpha),\alpha,j\alpha)$. Aggregate expenditure (aggregate output) is found by summing the expenditures (outputs) over the $[1/\alpha]$ firms. As for $M(0)$, by A2 $\frac{\partial y}{\partial c}(c,\alpha,j\alpha) \geq 0$, and the cost functions are essentially unrestricted.

To see the relationship between the production sectors of the $M(\alpha)$ and $M(0)$ note that if $T(\cdot,\alpha,\cdot) \equiv T(\cdot,0,\cdot)$ and $y = cT(c/(c+1),0,\beta)$ then $\alpha y = \alpha cT(\alpha c/(\alpha c+\alpha),\alpha,\beta)$. This implies that the average cost curve for firm β in $M(\alpha)$ is a rescaling of the average cost curve for β in $M(0)$ exactly as in Novshek [6]: the average cost in $M(\alpha)$ for y is the same as the average cost in $M(0)$ for y/α. By assumption the "normalized" cost functions of firms in $M(\alpha)$ converge smoothly to the cost functions of the corresponding firms in $M(0)$.

We define T on the closed set $[0,1]^3$ to guarantee that the aggregate production possibilities for $M(\alpha)$ converge to those for $M(0)$ (see Remark 1 Section 4). The aggregate production set in $M(\alpha)$,

$$Y(\alpha) = \sum_{j=1}^{[1/\alpha]} \{(-c,y) \in (-\mathbb{R}_+) \times \mathbb{R}_+ \mid y \leq y(c,\alpha,j\alpha)\},$$

converges to the aggregate production set in $M(0)$,

$$Y(0) = \int_0^1 \{(-c,y) \in (-\mathbb{R}_+) \times \mathbb{R}_+ \mid y \leq y(c,0,\beta)\} d\beta$$

$$= \{(-c,y) \in (-\mathbb{R}_+) \times \mathbb{R}_+ \mid (-c,y) = \int_0^1 (-c(\beta),y(\beta))d\beta \text{ for some } c(\beta),$$

$$\beta \in [0,1] \text{ where } y(\beta) \leq y(c(\beta),0,\beta) \text{ for all } \beta\},$$

in the following sense. For any $z \in Y(0)$ and sequence $\{\alpha_k\}$ converging to zero, there exists a sequence $\{z_k\}$ converging to z such that $z_k \in Y(\alpha_k)$ for all k. Also, for any sequences $\{\alpha_k\}$ converging to zero and $\{x_k\}$ converging to x such that $x_k \in Y(\alpha_k)$ for all k, x is a limit point of $Y(0)$. Both of these convergence results follow using

standard arguments concerning the approximation of integrals by finite sums since T and its derivatives are uniformly continuous.

If we define the cost of producing output y for firm β in market $M(\alpha)$ as $c(y,\alpha,\beta) := \min\{c \in \mathbb{R}_+ \mid y(c,\alpha,\beta) \geq y\}$ for outputs that are feasible then we can define a Cournot equilibrium for $M(\alpha)$.

<u>Definition</u>: A set of feasible nonnegative outputs $\{y_1,y_2,\ldots,y_{[1/\alpha]}\}$ is a Cournot equilibrium for $M(\alpha)$ if for all $i \in \{1,2,\ldots,[1/\alpha]\}$

$$P(\sum_{j=1}^{[1/\alpha]} y_j,\alpha)y_i - c(y_i,\alpha,i\alpha) \geq P(\sum_{j=1}^{[1/\alpha]} y_j - y_i + y,\alpha)y - c(y,\alpha,i\alpha) \text{ for}$$

all $y \geq 0$ which are feasible for firm i. The set of Cournot equilibrium aggregate outputs for $M(\alpha)$ is defined as

$$C(\alpha) := \{\sum_{j=1}^{[1/\alpha]} y_j \mid \{y_1,y_2,\ldots,y_{[1/\alpha]}\} \text{ is a Cournot equilibrium for } M(\alpha)\}.$$

We must define the "perfectly competitive equilibrium" of $M(0)$ with a little care since the usual "supply equals demand" definition may not work. For example, if $T(x,0,\beta) = x$ then all firms are identical and have strictly decreasing average cost, $(1/2) + (1/2)\sqrt{1 + 4/y}$. Aggregate supply in $M(0)$ is zero at any price less than <u>or equal to</u> one, but infinite at any price strictly above one. In this case we define the "perfectly competitive equilibrium" price as the price at which supply "crosses" demand. Let $S(p)$ be the "supply correspondence" defined as $S(p) := \{y \in \mathbb{R}_+ \cup \{\infty\} \mid$ there exist $c \in \mathbb{R}_+ \cup \{\infty\}$ and a sequence $\{(-c_k,y_k)\} \subset Y(0)$ which converges to $(-c,y)$ such that $\sup \{(1,p) \cdot z \mid z \in Y(0)\} = \lim_k (1,p) \cdot (-c_k,y_k)\}$.

<u>Definition</u>: (p^*,Y^*) is the "perfectly competitive equilibrium" for $M(0)$ if

(i) $P(Y^*,0) = p^*$ and

(ii) $p^* = \sup \{p \in \mathbb{R}_+ \mid S(p) \subset [0,Y^*]\}$.

This is the usual "supply equals demand" definition when that works. In other cases it extends the usual definition so that a "perfectly competitive equilibrium" always exists.

3. <u>Theorem and Proof</u>

We are now ready to state our main results: For α sufficiently small $M(\alpha)$ has a Cournot equilibrium, and the aggregate Cournot equilibrium outputs for $M(\alpha)$, $C(\alpha)$, converge to the "perfectly competitive equilibrium" output Y^*. By the continuity of P and the fact that the aggregate production sets converge, this implies that Cournot equilibria of $M(\alpha)$ exist and are approximately price taking equilibria when α is sufficiently small. This provides further support for the "Folk Theorem" that when firms are small relative to the market then the outcome is approximately competitive.

<u>Theorem</u>: Let $M(\alpha)$, $\alpha \in (0,1]$ and $M(0)$ be the markets defined by P and T and assume A1 and A2 hold. Let (p^*,Y^*) be the "perfectly competitive equilibrium" for $M(0)$. Then for all $\varepsilon > 0$ there exists an $\alpha' > 0$ such that for all $\alpha \in (0,\alpha']$,

(i) $C(\alpha) \neq \emptyset$ i.e., $M(\alpha)$ has a Cournot equilibrium, and,

(ii) $C(\alpha) \subset [Y^*-\varepsilon, Y^*+\varepsilon]$.

<u>Proof</u>: Pick $\varepsilon > 0$. We start with some basic observations. Let $t = \max \{T(x,\alpha,\beta) \mid (x,\alpha,\beta) \in [0,1]^3\}$. By A2, $t < \infty$. If $t = 0$ then no firm is productive, $C(\alpha) = \{0\}$ for all α, and the results of the Theorem hold since $(p^*,Y^*) = (P(0,0), 0)$. Henceforth assume $t > 0$. For feasible outputs y, the average cost of producing y for firm β in $M(\alpha)$ is $c(y,\alpha,\beta)/y = 1/T(c(y,\alpha,\beta)/(c(y,\alpha,\beta)+\alpha),\alpha,\beta) \geq 1/t$. If $P(0,0) < 1/t$ then for α sufficiently small no firm could ever be active without incurring losses, and again the results of the Theorem hold with $C(\alpha) = \{0\}$ for all small α. Assume $P(0,0) \geq 1/t$ for the remainder of the proof. If $1/T(1,0,\beta) < p$ for some β then a positive measure of firms in $M(0)$ can produce arbitrarily large outputs at average cost less than p, so the "perfectly competitive equilibrium" price p^* must be less than p. Finally, by A1 there exist $a \in (0,1]$, $\delta \in (0,\infty)$, $\varepsilon' \in (0,\varepsilon]$, and $Y' \in (Y^*,\infty)$ such that for all $\alpha \in (0,a]$, $y \in [0,\varepsilon']$, and $Y \in [0,Y']$, both $P(Y',\alpha) < 1/t$ and $P_1(Y+y,\alpha) + yP_{11}(Y+y,\alpha) < -\delta$. For all i, if Y is the aggregate output of other firms and y is the output of firm i then the firm's marginal revenue is decreasing in both y and Y in this region.

We now turn to the proof of part (ii) of the Theorem. For all α sufficiently small $p' := P(Y^*,0)/2 + P(Y^*+\varepsilon, 0)/2 > P(Y^*+3\varepsilon/4, \alpha)$, and in any Cournot equilibrium $(y_1,\ldots,y_{[1/\alpha]})$ for $M(\alpha)$, if $\sum_{j\neq i} y_j \geq Y^* + 3\varepsilon/4$ then firm i faces marginal revenue which is everywhere less than p', and firm i's Cournot equilibrium action is not larger than its optimal response to a fixed price p'. For any output $y \geq \varepsilon/4$ the corresponding expenditure is $c \geq \varepsilon/4t$ and average cost is $1/T(c/(c+\alpha),\alpha,i\alpha)$. For all β, $1/T(1,0,\beta) \geq p^* > p'$ so for α sufficiently small no firm could profitably produce $y \geq \varepsilon/4$ in any Cournot equilibrium with aggregate output $Y^* + \varepsilon$ or larger. Thus for all i, if $\sum_{j=1}^{[1/\alpha]} y_j > Y^* + \varepsilon$ then $\sum_{j\neq i} y_j \geq Y^* + 3\varepsilon/4$ and firm i's action is no larger than its competitive response to the fixed price p'. But the aggregate production set for $M(\alpha)$ converges to the aggregate production set for $M(0)$, so for small α the aggregate competitive supply for $M(\alpha)$ at price $p' < p^*$ is less than or equal to the aggregate competitive supply for $M(0)$ at price $p'/2 + p^*/2$, which is less than $Y^* + \varepsilon$. Thus $C(\alpha) \subseteq [0,Y^*+\varepsilon]$ for all small α.

To complete the proof of part (ii) pick $\varepsilon'' \in (0,\varepsilon']$ and $a' \in (0,a]$ such that for all $\varepsilon \in [0,\varepsilon'']$ and $\alpha \in (0,a']$

$$p := P(Y^*,0)/2 + P(Y^*-\varepsilon'',0)/2 < P(Y^*-3\varepsilon/4,\alpha) + (\varepsilon/4)P_1(Y^*-3\varepsilon/4,\alpha).$$

For $\alpha \in (0,a']$ and $\varepsilon \in (0,\varepsilon'']$, if $(y_1,\ldots,y_{[1/\alpha]})$ is a Cournot equilibrium for $M(\alpha)$ and $\sum_{j\neq i} y_j \leq Y^* - \varepsilon$ then firm i faces marginal revenue which exceeds p for all actions less than or equal to $\varepsilon/4$. Thus y_i must be no less than the competitive supply for a firm with technology identical to firm i's except that output cannot exceed $\varepsilon/4$, facing fixed price p. The sum of these restricted production sets converges to the aggregate production set for $M(0)$ as α converges to zero (this follows just as the convergence result for the unrestricted production sets), so for sufficiently small α the aggregate (restricted) competitive supply at price p ($> p^*$) is no less than the aggregate (restricted) competitive supply at $p^*/2 + p/2$ which exceeds

$Y^* - \varepsilon$. Thus $C(\alpha) \subseteq [Y^*-\varepsilon,\infty)$ for all small α. This completes the proof of part (ii).

We now turn to the proof of part (i). By our choice of ε', a, and Y', for all $\alpha \in (0,a]$ and $\varepsilon \in (0,\varepsilon']$, if we restrict the firms in $M(\alpha)$ to outputs $y \leq \varepsilon$ then each firm's marginal revenue is declining in both its own output, y, and the aggregate output of other firms, Y, as long as $Y \leq Y'$. But $P(Y',\alpha) < 1/t$ so no firm could ever profitably produce if other firms produced $Y > Y'$. Thus, with these restricted production sets we are able to apply the existence theorem of Novshek [7] to show the existence of a Cournot equilibrium. It remains to show that the restriction $y \leq \varepsilon$ can be removed.

As noted in the proof of part (ii), the sum of the restricted production sets converges to the aggregate production set of $M(0)$ as α converges to zero, so an argument similar to the proof of part (ii) shows that the aggregate production in the Cournot equilibrium is close to Y^* for α sufficiently small. For any $\eta > 0$, for all β, for all $y > \varepsilon$, either y is not a feasible output for β in $M(\alpha)$ or, letting $c = c(y,\alpha,\beta) > \varepsilon/t$, $1/\{T(c/(c+\alpha),\alpha,\beta) + (\alpha c/(c+\alpha)^2)T_1(c/(c+\alpha),\alpha,\beta)\} > p^* - \eta$ for all sufficiently small α (since $1/T(1,0,\beta) \geq P^*$ for all β). Thus for all $\eta > 0$, for all sufficiently small α all firms have marginal cost exceeding $p^* - \eta$ for all outputs $y > \varepsilon$. All elements of these firms' reaction correspondences (with unrestricted output) are no bigger than the larger of ε and the optimal action for a firm with constant marginal and average cost $p^* - \eta$.

Pick $\varepsilon \in (0,\varepsilon']$, $\eta \in (0,-\varepsilon P_1(Y^*,0)/3]$, and $a' \in (0,a]$ such that for all $\alpha \in (0,\alpha']$, when firms are restricted to $y \leq \varepsilon$ Cournot equilibrium exists for $M(\alpha)$ and aggregate output lies in $[Y^* + \eta/P_1(Y^*,0),\ Y^* - \eta/P_1(Y^*,0)]$. If $(y_1,\ldots,y_{[1/\alpha]})$ is such a Cournot equilibrium then for all i, $\sum_{j\neq i} y_j \geq Y^* + \eta/P_1(Y^*,0) - \varepsilon$ and for sufficiently small ε, for all sufficiently small α the optimal response by a firm with constant marginal and average cost $p^* - \eta$ is less than $(Y(\eta,\alpha) - (Y^* + \eta/P_1(Y^*,0) - \varepsilon))/2 + \varepsilon/12$ where $Y(\eta,\alpha)$ is defined by $P(Y(\eta,\alpha),\alpha) = p^* - \eta$. (This follows from the uniform convergence of $P(\cdot,\alpha)$ and $P_1(\cdot,\alpha)$ to $P(\cdot,0)$ and $P_1(\cdot,0)$, where $P(\cdot,0)$ and $P_1(\cdot,0)$ are arbitrarily close to linear functions on $[Y^*-2\varepsilon, Y^*+2\varepsilon]$ for ε sufficiently small.) As α converges to zero $Y(\eta,\alpha)$ converges to $Y^*-\eta/P_1(Y^*,0)$ and $(Y(\eta,\alpha) - (Y^*+\eta/P_1(Y^*,0) - \varepsilon))/2 + \varepsilon/12$ converges to $-\eta/P_1(Y^*,0) + 7\varepsilon/12 < \varepsilon/3 + 7\varepsilon/12 < \varepsilon$. Thus for sufficiently small α no firm prefers to switch to an output exceeding ε, and the Cournot equilibria with restricted outputs are also Cournot equilibria with unrestricted outputs.

Q.E.D.

4. Remarks

(4.1) The cost functions for the firms were introduced in a rather unorthodox manner for a partial equilibrium market. This is because we need as a basic assumption that the markets $M(\alpha)$ converge to the perfectly competitive market $M(0)$. To see this we will consider an example in which cost functions are introduced directly. Though the

cost functions are seemingly well behaved, the markets $M(\alpha)$ do not converge to $M(0)$.

Consider the cost function $c(y,0,\beta) = y + (\sqrt{1-\beta})y^2$, and for all $\alpha \in (0,1]$, $c(y,\alpha,\beta) = y + (\sqrt{1-\beta})y^2/\alpha$. This cost function has the rescaling property for average cost curves: for all β, for all $\alpha > 0$, for all $y > 0$, $c(y,\alpha,\beta)/y = c(y/\alpha,0,\beta)/(y/\alpha)$. The competitive supply in $M(0)$ is $\int_0^1 [(p-1)/(2\sqrt{1-\beta})]\,d\beta = p-1$ for $p \geq 1$ and zero otherwise. The competitive supply for $M(\alpha)$ does not converge to this function. For any integer n, when $\alpha = 1/n$ $M(\alpha)$ has a firm ($\beta = [1/\alpha]\alpha = 1$) with constant average cost at level one. Competitive supply in $M(1/n)$ is $\mathbb{R}_+$ at $p = 1$ and unbounded at any $p > 1$.

For $P(Y,\alpha) := 3 - Y$ there is a unique Cournot equilibrium for each $\alpha \in (0,1]$, with aggregate output

$$Q(\alpha) = \{\sum_{j=1}^{[1/\alpha]} 2\alpha/(\alpha + 2\sqrt{1-j\alpha})\} \;/\; \{1 + \sum_{j=1}^{[1/\alpha]} \alpha/(\alpha + 2\sqrt{1-j\alpha})\}.$$

The perfectly competitive equilibrium for $M(0)$ is $(p^*,Y^*) = (2,1)$ while the limit points of $Q(\alpha)$ as α converges to zero are all outputs in $[1,4/3]$. The convergence result fails because the markets $M(\alpha)$ do not converge to market $M(0)$.

The problem is that in $M(0)$ the constant returns to scale technology has zero mass (only $\beta = 1$ has it) with very small mass for firms close to it in technological terms (for $y > 0$, $\frac{\partial}{\partial\beta}(c(y,0,\beta)/y)$ is unbounded as β approaches 1) while in $M(1/n)$, the constant returns to scale technology is available to one firm, and is unaffected by α.

The proportion of firms with the constant returns to scale technology does converge to zero but the size of the firm relative to the market does not. For all $\alpha > 0$ the single firm could meet market demand at price $p = 1$ without incurring losses. With Lebesgue measure in M(0) the single firm is irrelevant for aggregate output. Note that if the rescaling property for average cost curves held and the maximum possible output of firms was uniformly bounded in M(0) then this problem would not arise (feasible outputs for firms in $M(\alpha)$ would be uniformly bounded by α times the bound for M(0) by the rescaling property).

The problem arises at arbitrarily large outputs. By defining T on $[0,1]^3$ the properties of average cost "at infinity" are determined by $T(1,\alpha,\beta)$. The smoothness assumption on T guarantees that $M(\alpha)$ converges to M(0) in terms of the production sector.

(4.2) The assumption that T be continuously differentiable ruled out production with avoidable set up costs and constant marginal cost. However, this case could be easily treated. Any firm's output with average cost greater than $\max_{\alpha \in [0,1]} P(0,\alpha)$ would never be chosen, so cost functions of this type could be modified in the irrelevant region to satisfy the smoothness assumption on T. This would in no way affect the results.

(4.3) The convergence result (part (ii) of the Theorem) can be improved in the following ways. If firms have U-shaped average cost then the bounds in (ii) can be sharpened to $[Y^*-K\alpha, Y^*+K\alpha]$ for

sufficiently small α, where K is a constant determined by the minimum efficient scale of the marginal firm in M(0) and the derivatives of P and T evaluated at points corresponding to the perfectly competitive equilibrium of M(0).

It is well known that when firms have identical "fixed cost plus constant marginal cost" technologies and the rescaling property holds for average cost curves then the bound for small α can be sharpened to $[Y^*-K\sqrt{\alpha}, Y^*+K\sqrt{\alpha}]$ where K is a constant (see Dasgupta and Ushio [2], Fraysse and Moreaux [3], and Ushio [10,11]). This result can be extended to the case in which firms are not identical.

(4.4) Can the results be extended to allow "free entry?" By this we mean is it possible to set up the model so that for small α there are equilibria at which some firms are inactive. In some circumstances this can be accomplished by starting with a sufficient mass of firms in M(0) (that is, allowing $\beta \in [0,B]$ for B sufficiently large). In one case it is not possible.

First consider the cases with identical firms and the rescaling property for average cost curves. For the case of strictly positive set up cost and constant marginal cost, the number of active firms in $M(\alpha)$ is on the order of $1/\sqrt{\alpha}$ if a countable infinity of firms is available (see for example Dasgupta and Ushio [2]). Since $(1/\sqrt{\alpha})/[1/\alpha]$ converges to zero as α converges to zero, for α sufficiently small the $[1/\alpha]$ firms available in $M(\alpha)$ are already more than enough to have some firms inactive in equilibrium. For the case of U-shaped average cost with minimum average cost p^*, attained at output $y^* > 0$ (in M(1), and with curvature of the average cost curves at y^*) and Y^* the output satisfying $P(Y^*,0) = p^*$, when the mass of available firms in M(0) exceeds Y^*/y^*, for sufficiently small α the equilibria of $M(\alpha)$ will have inactive firms. Finally, for the case of no set up costs and nondecreasing marginal cost, for any α either all firms or no firms will be active in equilibrium. Since average cost is nondecreasing, if any firm is earning strictly positive profit then any inactive firm could produce a small output at a positive profit. If any firm produces positive output and earns zero profit it could produce less, thus increasing price and profit. Thus it is never possible to extend this case to have both active and inactive firms in equilibrium. (With a countable infinity of available firms there is no equilibrium in this case unless the monopoly output is zero.)

Now consider the case in which firms within M(0) differ, and assume the differences include differences in minimum (or infimum) of average cost, $MAC(\alpha,\beta)$, with $\frac{\partial}{\partial\beta}MAC(0,\beta)$ bounded away from zero. Then for a sufficiently large mass of available firms in M(0) there will be a marginal firm, the "least efficient" active firm, and a pool of inactive firms in the perfectly competitive equilibrium. For α sufficiently small Cournot equilibria exist, and by the convergence result we know that some firms must be inactive in equilibrium (approximately those with minimum average cost exceeding p^*). Thus Cournot equilibria with both active and inactive firms can exist for small α in all cases when firms differ in terms of minimum average cost.

This discussion shows that in all cases we have considered except the case of identical firms with no set up costs and nondecreasing marginal cost, we can extend the Theorem to the case of an unbounded number of firms: an unbounded mass $\beta \in [0,\infty)$ in M(0) and a countable infinity of firms $j \in \{1,2,3,...\}$ in $M(\alpha)$ for all $\alpha \in (0,1]$.

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