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GENERAL EQUILIBRIUM WITH FREE ENTRY: A SYNTHETIC APPROACH TO THE THEORY OF PERFECT COMPETITION

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ABSTRACT

The purpose of this essay is to explain a new theory of perfect competition that synthesizes the Arrow-Debreu-McKenzie and Marshallian theories and summarizes the recent work of a large number of researchers. The synthetic theory provides a logically precise general equilibrium framework that can be used both for posi analysis as well as to form a basis for the classical theorems of welfare economics; in these aspects it follows the ADM theory. However, as in the Marshallian theory, there is free-entry in the form of an unbounded pool of firms which have access to the existing technology. Also as in the Marshallian theory, the average cost curves available to each firm are U-shaped.

In contrast to both the Marshallian and the ADM theories, we do not take price taking as a hypothesis. We use the term "perfect competition" to describe a situation in which firms are arbitrarily small relative to the markets in which they are involved. The firms in our model correctly perceive the effect of the amount that they place on the market on prices and they act to maximize profit wi this in mind. "Perfectly competitive equilibria" are defined as the limit points of equilibria as firms become small relative to the market, and following Cournot, we observe that as firms become small their ability to influence price disappears.

A new condition is identified which is important for the viability of competitive markets. Loosely speaking, this condition requires that prices provide the proper entry signals for firms. If one thinks of each firm as associated with the use of an unpriced and nondivisible resource, sometimes referred to as entrepreneurship, then in equilibrium the returns to that factor must fall with entry and rise with exit. Not only the stability theorems of the synthetic theory but also the existence theorem reject the application of the competitive model to a regime in which entry drives up (and exit reduces) the profit of similar firms.

GENERAL EQUILIBRIUM WITH FREE ENTRY: A SYNTHETIC APPROACH TO THE THEORY OF PERFECT COMPETITION

I. INTRODUCTION

The fundamental axiom of perfect competition is price taking behavior; however, within this framework two strikingly different theories are advanced. These are referred to as the Marshallian Theory and the Arrow-Debreu-McKenzie Theory. The Marshallian Theory is emphasized at the undergraduate level. It conjures up images of blackboards filled with partial equilibrium diagrams, and it is what Chicagoans refer to when they speak of "bread and butter" economics. The Arrow-Debreu-McKenzie Theory, hereafter referred to as the ADM theory, is primarily reserved for our more advanced students.[1] It is the dominant framework of "highbrow theorists," and its most important application is to the classical theorems of welfare economics. During the first half of this century, the Marshallian Theory was unquestionably the leading theory of value. This probably remains true today. However, a growing number of papers in the applied areas; for example, finance, international trade, and monetary theory adopt a variant of the ADM framework.

One must acknowledge that the Marshallian and ADM theories have many striking and essential differences, for example:

(i) In the ADM theory a finite number of firms are specified in advance, while in the Marshallian theory there is normally a pool of firms, which have free access to the existing technology, and are ready to enter when the conditions become right.

(ii) In the ADM theory the technology of each firm is convex, while in the Marshallian theory the average cost curve of each firm is U-shaped.

(iii) In the ADM theory price-taking is assumed independent of the number of firms, while in Marshallian theory price taking is assumed only for an environment in which efficient scale is small relative to demand, and, as a consequence, there are many firms.

(iv) ADM theory is general equilibrium, and looks toward the classical theorems of welfare economics while most of Marshallian analysis ignores intermarket effects.

(v) Finally, the ADM theory is static, and the major attempt to introduce a dynamics via a tatonnement has not been very successful. In contrast, the Marshallian analysis has an essentially dynamic aspect, and this aspect requires that a process of entry is at rest in the equilibrium.

The purpose of this essay is to explain a new theory of perfect competition; one that synthesizes the ADM and Marshallian theories and may be viewed as summarizing the recent work of a large number of researchers; for example, Hart [1979], Novshek [1980], Novshek and Sonnenschein [1978], and others. (See the symposium issue [1980] and the references therein.) The synthetic theory provides a logically precise general equilibrium framework that can be used both for positive analysis as well as to form a basis for the classical theorems of welfare economics; in these aspects it follows the ADM theory. However, as in the Marshallian theory, there is free-entry in the form of an unbounded pool of firms which have access to the existing technology. Also as in the Marshallian theory, the average cost curves available to each firm are U-shaped.

In contrast to both the Marshallian and the ADM theories, we do not take price taking as a hypothesis. Rather, we integrate into our analysis the leading classical explanation for price taking behavior. We use the term "perfect competition" to describe a situation in which firms are arbitrarily small relative to the markets in which they are involved. The firms in our model correctly perceive the effect of the amount that they place on the market on prices and they act to maximize profit with this in mind. "Perfectly competitive equilibria" are defined as the limit points of equilibria as firms become small relative to the market, and following Cournot, we observe that as firms become small their ability to influence price disappears.[2]

A new condition is identified which is important for the viability of competitive markets. Loosely speaking, this condition requires that prices provide the proper entry signals for firms, and is a consequence of the essentially dynamic aspect of Marshallian analysis. If one thinks of each firm as associated with the use of an unpriced and nondivisible resource, sometimes referred to as entrepreneurship, then in equilibrium the returns to that factor must fall with entry and rise with exit. Not only the stability theorems of the synthetic theory but also the existence theorem reject the application of the competitive model to a regime in which entry drives up (and exit reduces) the profit of similar firms.

II. THE ADM AND MARSHALLIAN MODELS

It is useful to begin by presenting stylized versions of the ADM and Marshallian models. We will use these simple versions to explain more carefully the relation between the ADM and Marshallian theories and to list the features that are desired in our synthesis.

A. A Stylized Arrow-Debreu-McKenzie Model

Our development follows T. Koopmans' classic exposition of a Robinson-Crusoe economy [1957]. Robinson produces food using his labor, and to simplify matters we assume that production takes place according to constant returns to scale. Employing a convention of the Arrow-Debreu-McKenzie theory, labor input is denoted as a negative quantity and food output is denoted as a positive quantity.

The technology is summarized in Figure 1, where by a choice of units, one unit of labor input yields one unit of food output. Robinson has 24 hours of labor to offer; this represents his entire initial endowment of resources. When his labor is added to the technology set one obtains the set T of possible aggregate supplies.

[Figure 1 around here]

Robinson's preferences are indicated by indifference curves, which connect equally preferred combinations of food and retained labor. In the economy of Figure 1, there is a unique "best attainable point," which we have denoted by x.

A price system is a non-zero vector that lies in the first quadrant; it defines the value of each bundle (ℓ,f) in the usual manner. Given the price system (1,1), the vector $x = (\overline{\ell},\overline{f})$ maximizes the value of supply and this value is 24. Observe that $(\overline{\ell},\overline{f})$ is not the unique maximizer, since each point on the northeast boundary of T has the same value. If the value of the supply action is distributed to Robinson, then at prices (1,1) he will be able to afford bundles in the first orthant that satisfy the budget inequality $p_\ell \ell + p_f f \leqq 24$. Robinson's utility maximizing demand action is then $(\overline{\ell},\overline{f})$, and so at prices (1,1) there is a profit maximizing supply and a utility maximizing demand which coincide.
It is quite apparent that this is the unique equilibrium for the ADM model. If the technology set is interpreted as that available to a competitive firm, and if the economy is interpreted as a private ownership economy in which Robinson owns both the firm and the single scarce input labor, then one has the following description of the equilibrium: At prices (1,1) the firm takes these prices as given and maximizes profit by purchasing $24-\overline{\ell}$ units of labor and producing $24-\overline{\ell}$ units of food. It pays $24-\overline{\ell}$ for the labor and sells the food for $24-\overline{\ell}$, making zero profit. Robinson receives no dividends from his ownership of the firm; however, as a holder of labor resource he receives offers for all 24 units and

thus derives an income of 24. These offers come from the firm ($24-\overline{\ell}$ units) and from himself viewed as a consumer ($\overline{\ell}$ units). Finally, as a consumer, Robinson uses his 24 units of income to purchase $\overline{\ell}$ units of his own labor (that is, he buys leisure) and $\overline{f} = 24 - \overline{\ell}$ units of food. Both markets clear and all accounts balance.

This is general equilibrium because the interaction between markets is emphasized; for example, the wage determines not only the supply of labor, but also the demand for food through its effect on both the relative price of leisure and food and the value of Robinson's initial endowment of labor. The model was developed with the classical theorems of welfare economics in mind, and it has been skillfully employed to determine precise conditions under which a) every equilibrium is a Pareto optimum, and b) every Pareto optimum is an equilibrium after a suitable redistribution of ownership.

We emphasize that the ADM model does not concern itself with the plausibility of price-taking behavior. The model offers descriptions of perfect competition for situations in which bilateral monopoly (one consumer and one producer) or single agent maximization (Robinson) should apply.

B. A Stylized Marshallian Model

We begin with a familiar textbook figure (Figure 2) with average cost, marginal cost, and demand labelled AC, MC, and FF respectively. All firms are identical and the number of firms is fixed at 5. The aggregate supply is zero up to price p^* (= minimum AC); at p^* supply is the indicated five point set, and above p^* supply is the horizontal sum of the marginal cost curves.

[Figure 2 around here]

Observe that if the technology represented by AC is freely available to additional firms, then the aggregate supply in the figure does not describe a

situation in which firms take prices as given and maximize profit. This is because at any price above p^* it is possible for any firm in the market to make a positive profit, and so all firms should be active. Clearly, with free entry, an exact price-taking equilibrium will exist only in the unlikely event that the value of inverse demand at price p^* is an integral multiple of the minimum average cost quantity. In Marshallian analysis this observation is minimized, and equilibrium is taken to represent a situation in which each active firm is maximizing profit ($p = MC$) and the number of firms is right in the sense that there is not much incentive for firms to enter or leave. It is the latter aspect that gives Marshallian analysis a dynamic flavor.

Marshall applies his model of perfect competition when efficient scale (minimum average cost output) is small relative to demand. This is not only the case in which there is especially strong justification for the price-taking assumption, but in addition, it is the case in which the horizontal gaps in the value of supply at price p^* become small relative to demand. When efficient scale is small relative to demand, one is able to simultaneously have active firms maximizing profit and the number of active firms chosen so that the incentive for inactive firms to enter is very small. The price in the market will be a bit above p^*. Alternatively, one can consider an approximate equilibrium at price p^*, in which a finite number of firms are active and maximizing profit by offering the efficient scale output to the market, and demand very nearly matches supply. This is illustrated in Figure 3a.

[Figure 3a and 3b around here]

In Figure 3b the efficient scale of each firm is the infinitesimal quantity M and there is an unbounded mass of available firms. Let D be demand at prices p^*, then an exact equilibrium is achieved by having a mass $\frac{D}{M}$ of firms active, each producing at efficient scale M. Clearly Figure 3b represents a natural limit of markets of the type considered in Figure 3a as efficient scale becomes small relative to demand. In the equilibrium of Figure 3b there is a continuum of active firms.

C. The Beginnings of the Synthesis

Let us first place the Marshallian model in the framework of the ADM theory. This is done to demonstrate that the framework of the ADM theory is sufficiently rich so as to enable us to capture the Marshallian ideas described above. The U-shaped average cost leads to the firm technology represented in the second quadrant of Figure 4. By summing this technology a countable infinity

[Figure 4 around here]

of times and using the initial endowment, the feasible set of aggregate supplies takes the form of the set denoted by T. If the indifference curves are as indicated in the first quadrant of Figure 4, then there will be no price-taking equilibrium. To see this observe that the only price systems that lead to a positive output of food at a profit maximum are of the form (p,p). (Without loss of generality we will assume $p = 1$.) But at prices $(1,1)$ demand is z, while profit maximizing supply must take on one of the values $\{(24,0), (24-1,1), (24-2,2)\ldots\}$. In particular, observe that the "best attainable point" x is not an equilibrium of the system.

It is clear that by limiting the number of firms to five (as in the Marshallian example), we can find a price system $(1,1+\varepsilon)$, at which supply equals demand and there is not much incentive for the excluded firms to enter. Furthermore, the associated allocation is close to the "best point" x. This is illustrated in Figure 5.

[Figure 5 around here]

Alternatively, at prices $(1,1)$, we could consider an approximate price-taking equilibrium in which a finite number of firms are active and maximizing profit by offering the efficient scale output to the market, and demand very nearly matches supply. This is essentially illustrated in Figure 4, when the gap between s_5 and z is small. Once again, observe that demand is close to the "best point" x.

In Figure 6 the efficient scale of each firm is an infinitesimal quantity and there is an unbounded mass of available firms. As in the Marshallian Figure 3b, an exact equilibrium at prices $(1,1)$ is achieved when one properly chooses the mass of active firms and has them each produce at efficient scale. Clearly Figure 6 represents a natural limit of markets of the type considered in Figure 4. Observe the similarity with the Arrow-Debreu-McKenzie Figure 1. Also observe that the equilibrium and the "best point" coincide.

[Figure 6 around here]

D. An Outline of the Approach

The synthetic theory begins with the above ideas and develops them into a well-specified model. As in the Marshallian theory, we assume that there is free entry and that firms have nonconvex technologies. A perfectly competitive economy is a sequence of economies $\{\mathcal{E}(\alpha_k)\}$ in which firms become arbitrarily small relative to the markets in which they are involved. The limit of the sequence is denoted by $\mathcal{E}(0)$, and in the limit economy firms are infinitesimal. In each economy of the sequence $\{\mathcal{E}(\alpha_k)\}$ firms are assumed to correctly perceive the (typically non-negligible) effect of the amount that they place on the market on prices, and they act to maximize profit. We have seen above that non-convexities in the production sets of firms lead to generic non-existence of price-taking equilibrium. Fortunately, when efficient scale is small relative to demand, it turns out that Cournot equilibrium with entry frequently exists for the Marshallian model. An equilibrium for the perfectly competitive sequence $\{\mathcal{E}(\alpha_k)\}$ is naturally defined as the limit of Cournot equilibria of $\{\mathcal{E}(\alpha_k)\}$. In other words, perfectly competitive equilibrium is defined as the limit of Cournot equilibria with entry as firms become small relative to the market.

As one might expect, it is possible to characterize the perfectly competitive equilibria of $\{\mathcal{E}(\alpha_k)\}$ in terms of its limit economy $\mathcal{E}(0)$. The main theorems of this essay concern this characterization and show, in particular, that the actions of firms in perfectly competitive equilibrium are "as if" they were price takers in $\mathcal{E}(0)$. On the other hand, these theorems also show that there are price-taking equilibria of $\mathcal{E}(0)$ which are not the limit of Cournot equilibria with entry, and thus not perfectly competitive equilibria.

III. THE PARTIAL EQUILIBRIUM SYNTHESIS

In this section we develop the notion of a perfectly competitive sequence of partial equilibrium markets. This is a sequence of Marshallian markets in which there is a single homogeneous good and firms decrease in size relative to the market. As was suggested above, we define a perfectly competitive equilibrium as the limit of Cournot quantity setting equilibria with entry of the markets in the sequence.

Let us begin by returning to the Marshallian specification as in Figure 2. In order to simplify the analysis we assume some special structure for the cost function $C(y)$,

(C) $C(y) = 0$, if $y = 0$, and

$C(y) = C_o + V(y)$ if $y > 0$,

where $C_o > 0$, and for all $y \geqq 0$, $V' > 0$, $V'' \geqq 0$. Average cost is minimized uniquely at $y = 1$.

For the inverse demand function $F(y)$, we assume

(F) $F \in C^2([o,\infty))$; that is, F is twice continuously differentiable and $F(y) = C(1)$ implies $F'(y) \neq 0$. These are regularity conditions.

[Figure 7 around here]

(C,F) specifies the basic Marshallian market. We interpret there to be a countable infinity of firms with access to the cost function C .

A perfectly competitive sequence of markets $\{M(\alpha_k)\}$ is a sequence of markets in which the size of firms changes relative to the market. An α size firm corresponding to C is a firm with cost function $C_\alpha(y) = \alpha C(y/\alpha)$. For each $\alpha > 0$, C, F, one considers a market with a countable infinity of firms with technology C_α facing market inverse demand F; this market is denoted by $M(\alpha)$. As $\alpha \to 0$, firms become smaller relative to the market (for an α size firm average cost is $AC_\alpha(y) \equiv AC_1(y/\alpha)$ and an α size firm attains minimum average cost uniquely at output α), and the aggregate production possibilities in the market converge to the constant returns to scale case diagrammed in Figures 3b and 6. Given the cost function C and the inverse demand function F , a competitive sequence is defined by any $\{\alpha_k\}_{k=1}^{\infty} \subset (0,1]$ such that $\alpha_k \to 0$. The limit market is denoted by $M(0)$.

In the Cournot theory quantities are the strategic variables. Cournot equilibrium requires that the quantity actions of firms maximize profit given the quantity actions of all other firms. In particular, this implies that in an equilibrium no firm makes negative profit, since it could leave the market and make zero profit. Similarly, equilibrium requires that no firm can enter the market and make a positive profit. Stated precisely, a Cournot equilibrium with entry for the market $M(\alpha)$ is an integer n and a set of positive outputs $(y_1, y_2, \ldots, y_n)$ such that:

(a) $\{y_1, y_2, \ldots, y_n\}$ is an n-firm Cournot equilibrium (without entry); that is, for all $i=1,2,\ldots,n$, $F(\sum_{j\neq i} y_j + y_i)y_i - C_\alpha(y_i) \geqq F(\sum_{j\neq i} y_j + y)y - C_\alpha(y)$ for all $y \geqq 0$, and

(b) entry is not profitable; that is, $F(\sum_{j=1}^{n} y_j + y)y - C_\alpha(y) \leqq 0$ for all $y \geqq 0$.

Observe that Cournot equilibrium with entry is an "exact" equilibrium notion. In equilibrium, firms maximize profit and supply equals demand.

Finally, we define the equilibria of the perfectly competitive sequence $\{M(\alpha_k)\}$ as the limit of $\Sigma y_i(\alpha_k)$ where $(y_1(\alpha_k), \ldots, y_n(\alpha_k))$ is a Cournot equilibrium with entry of the market $M(\alpha_k)$ These are called perfectly competitive equilibria.

Theorem 1 characterizes the perfectly competitive equilibria for a sequence $\{M(\alpha_k)\}$. It shows that they can be identified by the conditions "demand price equals minimum per unit cost" and "demand slopes downward" in the limit market $M(0)$. The number of active firms is endogenously determined by free entry and exit. Average cost curves are U-shaped (and so firm technology is not convex). Price-taking behavior is not a primitive of the theory; rather, the ability of a firm to effect price becomes arbitrarily small. Equilibrium is exact, with both production equals demand and all firms maximizing profit.

<u>Theorem 1</u> (Novshek):[3] Given the cost function C satisfying (C), the inverse demand function F satisfying (F), and the perfectly competitive sequence $\{M(\alpha_k)\}$, the following conditions are equivalent:

(1a) y^* is a perfectly competitive equilibrium for $\{M(\alpha_k)\}$, and

(1b) $F(y^*) = C(1)$ and $F'(y^*) < 0$.

For the case of partial equilibrium, the preceding result describes precisely our approach to perfect competition. Perfectly competitive equilibria are defined as the limit points of Cournot equilibria of the Marshallian markets $\{M(\alpha_k)\}$.

Novshek's characterization theorem establishes that these perfectly competitive equilibria are the quantities in $M(0)$ that (a) equate the inverse demand price of consumers and minimum average cost, and (b) satisfy the condition that demand is downward sloping. For the case of globally downward sloping demand, the perfectly competitive equilibria of the sequence $\{M(\alpha_k)\}$ are identified with the unique Walrasian equilibrium in the derived constant returns to scale market $M(0)$. But if inverse demand is not globally downward sloping, there may be Walrasian equilibria of the derived (constant returns to scale in the aggregate) economy $\mathcal{E}(0)$ that are not by our definition competitive equilibria. This is due to the requirement in our analysis, that for equilibrium the process of entry must be at rest.

In Figure 7, the points y^* and y^{**} are equilibria of the perfectly competitive sequence $\{M(\alpha_k)\}$, but $\hat{y}$ is not, despite the fact that the demand price of consumers and minimum average cost coincide at $\hat{y}$. We argue that $\hat{y}$ is not an equilibrium because an infinitesimal firm in $M(0)$ can enter and make a positive profit. To make this precise we observe first that in any Cournot equilibrium for the market $M(\alpha)$ all firms must make non-negative profit so aggregate output must lie in $[0,y^*]$ or $[\hat{y}\ y^{**}]$. Second, inactive firms must not be able to profit from switching to minimum average cost production, α, so aggregate Cournot equilibrium output plus α must lie in $[y^*,\hat{y}]$ or $[y^{**},\infty)$. Thus the aggregate output in a Cournot equilibrium must lie in either the interval $[y^*-\alpha,y^*]$ or $[y^{**}-\alpha, y^{**}]$. For small α neither interval is near $\hat{y}$; in fact this is how one proves 1a implies 1b. The hard part of the theorem is to show that for α sufficiently small, if $F(y^*) = C(1)$ and $F'(y^*) < 0$, then $M(\alpha)$ has a Cournot equilibrium with entry with aggregate output in $[y^*-\alpha,y^*]$.

The above argument makes it clear why the equilibrium concept we put forth leads to a different theory than does the notion of Walrasian equilibrium in the limit economy. In the Walrasian theory firms are assumed to take prices as given, and in perfect competition we justify this as an approximation. The approximation applies when firms are small relative to the market, and thus have little influence in most any specification of strategic variables. In our analysis, the ability of each firm to affect price becomes small; however, each firm will normally have some effect, and if the entry of a firm will drive up price and make entry profitable, then firms will understand this effect and enter. Because we model perfect competition as a situation in which small firms correctly perceive their influence, equilibrium requires that no firm can drive up prices by entering.

IV. THE GENERAL EQUILIBRIUM MODEL, PRELIMINARIES

In this section we present the general equilibrium model that provides the basis for our concept of perfectly competitive equilibrium.

We consider a perfectly competitive sequence of economies $\{\mathcal{E}(\alpha)\}$, $\alpha \varepsilon (0,1]$.[4] As α converges to 0 firms become arbitrarily small relative to the economy, and in the limit economy, $\mathcal{E}(0)$, firms are infinitesimal. For $\alpha \varepsilon (0,1]$, each economy $\mathcal{E}(\alpha)$ has a countable infinity of firms available. This provides our notion of free entry: in any equilibrium for $\mathcal{E}(\alpha)$ only a finite number of firms can be active, so additional inactive firms are always available to test the profitability of entry. The process of entry is at rest if none of the inactive firms could gain positive profit by being active.

In each economy $\mathcal{E}(\alpha)$, firms are the only strategic agents and pure strategy Nash equilibrium in quantities is the solution concept. As in Marshall, we treat consumers as a competitive, price-taking sector to focus on the role of firms and entry. In the Cournot tradition, quantity setting provides a tractable basis for our analysis and avoids the obvious problem of nonexistence which arises when using price setting with nonconvex technologies. Pure strategies are used because of their natural appeal.[5] When firms consider a quantity action (a vector of input and output levels) they evaluate the corresponding profit using an "inverse demand" function F. To each vector of quantity actions of all firms, y, the "inverse demand" function associates a price vector, p, such that the competitive consumer sector's excess demand given prices, p, and the income generated by the consumers' dividend payments (their fraction of the profit or loss for each firm j, $p \cdot y_j$) exactly matches the aggregate quantity action of the firms. Hence the payoff for the j^{th} firm is a well defined function of its own strategy, y_j, and the strategies of other firms, $y_{)j(}$.[6] For any vector of strategies played by the firms, all markets clear, though some firms may be making losses, and in general, firms are not profit maximizing. The firms are in Cournot equilibrium if: (1) each firm's action is feasible (in its production set); and (2) each firm is maximizing profit given F and the actions of other firms. Because of nonconvex technologies, only a finite number of firms have nonzero actions in equilibrium. Note that the entry process is at rest in an equilibrium: additional, inactive firms are available but cannot earn positive profit from entry. Thus we have a notion of Cournot equilibrium with free entry for each economy $\mathcal{E}(\alpha)$, $\alpha\epsilon(0,1]$.

For each economy $\mathcal{E}(\alpha)$, let $C(\alpha,F)$ be the set of aggregate firm actions corresponding to Cournot equilibria of $\mathcal{E}(\alpha)$ relative to "inverse demand" F. The perfectly competitive equilibria of the sequence of economies $\{\mathcal{E}(\alpha)\}$, $\alpha\,\epsilon\,(0,1]$ are defined to be the limit points of robust sequences of Cournot equilibria[7] of the $\mathcal{E}(\alpha)$ economies; i.e., (p^*,y^*) is a perfectly competitive equilibrium of the sequence $\{\mathcal{E}(\alpha)\}$ if there exists (1) an "inverse demand" selection F which is continuously differentiable in a neighborhood of y^* and satisfies $F(y^*) = p^*$, (2) a strictly decreasing sequence $\{\alpha_r\}_{r=1}^{\infty} \subset (0,1]$ which converges to zero and (3) for each $\alpha\;\epsilon\;\bigcup_{r=1}^{\infty}[\alpha_{2r},\alpha_{2r-1}]$ an equilibrium output for $\mathcal{E}(\alpha)$, $y(\alpha)\;\epsilon\;C(\alpha,F)$ such that $y(\alpha)$ converges to y^* as α converges to zero (α varying over $\bigcup_{r=1}^{\infty}[\alpha_{2r},\alpha_{2r-1}]$ only). Since $\alpha_{2r} < \alpha_{2r-1}$ each interval $[\alpha_{2r},\alpha_{2r-1}]$ is nondegenerate and the perfectly competitive equilibria of the sequence are limit points of truly robust sequences of Cournot equilibria.

This framework allows us to analyze a logically precise general equilibrium model with nonconvex technologies and endogenous determination of the number of active firms. Despite the nonconvex technologies, the equilibria are exact rather than approximate equilibria. Price taking behavior has not been assumed; rather, when α converges to zero the ability of a firm to affect price

becomes arbitrarily small. At any perfectly competitive equilibrium of the sequence, equilibrium production maximizes profit relative to the equilibrium prices. Firms' actions are as if firms were price takers (Theorem 2). The standard welfare theorems still hold in this framework: every perfectly competitive equilibrium of the sequence is Pareto optimal (Theorem 4) and every Pareto optimal allocation for the sequence can be supported as a perfectly competitive equilibrium of the sequence (Theorem 5). However, since the process of entry must be at rest in an equilibrium, prices, as determined by the "inverse demand" functions F, must give correct entry signals. This introduces a new condition which we will call DSD, for the existence of a perfectly competitive equilibrium of the sequence. Every perfectly competitive equilibrium of the sequence satisfies weak DSD (Theorem 2). This condition is not part of the Arrow-Debreu-McKenzie model and there are perfectly competitive sequences of (private ownership) economies with no perfectly competitive equilibrium in our sense but such that the limit economy, $\mathcal{E}(0)$, has an Arrow-Debreu-McKenzie equilibrium (see Section V). An allocation in $\mathcal{E}(0)$ which satisfies the standard Arrow-Debreu-McKenzie conditions for equilibrium and also satisfies strong DSD is a perfectly competitive equilibrium for the sequence (Theorem 3). If the consumer sector of $\mathcal{E}(0)$ acts as a single consumer then the sequence, $\{\mathcal{E}(\alpha)\}$, has an equilibrium (Corollary 1).

V. A GENERAL EQUILIBRIUM EXAMPLE IN WHICH PRICES GIVE THE WRONG ENTRY SIGNALS

The characterization results for partial equilibrium in Section III depended on a condition of downward sloping demand. The results for general equilibrium depend on an analogous condition, called DSD. Prices determined by the "inverse demand" selection F must give proper entry signals: at a point satisfying the ADM equilibrium conditions, additional entry must lead to new prices at which the entrants make losses. Since input and output prices change, the requirement is that the net effect of all the price changes leads to a loss for the entrant.

Walras had something similar to DSD in mind (Walras [1874-7], p. 255):

> ...under free competition, if the selling price of a product exceeds the cost of the productive services for certain firms and a profit results, entrepreneurs will flow towards this branch of production or expand their output, so that the quantity of the product [on the market] will increase, its price will fall, and the difference between price and cost will be reduced; and, if [on the contrary], the cost of the productive services exceeds the selling price for certain firms, so that a loss results, entrepreneurs will leave this branch of production or curtail their output, so that the quantity of the product [on the market] will decrease, its price will rise and the difference between price and cost will again be reduced....

The natural question to ask now is whether or not the presence of a perfectly competitive equilibrium for the sequence of economies $\{\mathcal{E}(\alpha)\}$ can be normally expected. We show, in the example below, that simple economies $\mathcal{E}(0)$ exist (two consumers with homothetic preferences, and constant returns to scale) for which there is an ADM equilibrium but no perfectly competitive equilibrium of the sequence $\{\mathcal{E}(\alpha)\}$ which converges to $\mathcal{E}(0)$. Because DSD does not hold, for all sufficiently small α, no economy $\mathcal{E}(\alpha)$ will have a noncooperative equilibrium in pure quantity strategies that is close to the ADM equilibrium. (Recall the reason why $\hat{y}$ was not an equilibrium in Figure 7.) The relevance of the DSD condition should to some extent be measured by the plausibility that an economy will rest at such an ADM equilibrium.

Consider an economy with two consumers or two types of consumers (A and B), two consumption commodities (leisure and food), and a process which converts

one (small) unit of labor into one (small) unit of output (food). The use of a process requires a resource (e.g., land), all of which is owned by B (in an amount which does not effectively constrain the economy). We idealize the fact that the process is small in scale by defining the aggregate output as $(-\mu,\mu)$, when a mass μ of processes now called firms, are active. Aggregate output $(-\mu,\mu)$ means that μ (big) units of labor are supplied and converted into μ (big) units of food. Finally, for any output vector $(-\mu,\mu)$ the price of food relative to labor is determined by the condition that the sum of demands of A (taking into account the value of his initial holding of labor) and B (taking into account the value of his holding of labor and the profit from the firms which he owns) equals the supply action $(-\mu,\mu)$.

Since labor can be converted into food on a one-to-one basis, ADM equilibrium requires that the price of food in terms of labor be one. At this price profits are zero, and the aggregate demand of consumers is met by the activity of an appropriate mass of small firms. All firms make zero profit, whether or not they are active. Now the punch line! This example, to be given an analytic form below, has the following disturbing feature at the unique ADM equilibrium. When firms are treated as infinitesimal (rather than as points), the entry of a firm will result in a lower price for labor relative to food, and yield profit for the entering firm. The additional labor which is required is obtained by lowering the wage (A's demand for both leisure and food falls, as his income (the value of his labor) falls), and the additional food produced is demanded by B, whose income is augmented by the profits of the firms which he owns. Agent B demands more leisure and more food. Thus, from both the viewpoint of the preceding quotation (Walras), and the viewpoint of our concept of equilibrium of the perfectly competitive sequence, the single ADM equilibrium cannot be expected to be an equilibrium state under competition! (For a subsequent purpose we include a parameter θ, which defines the fraction of each firm which is owned by A.)

Example: In $\mathcal{E}(0)$ there are two commodities, two consumers, and one industry composed of a continuum of infinitesimal firms. Preferences and technology are shown in Figure 8. Agent A owns fraction θ of each firm and agent B owns fraction $1-\theta$. Each consumer also holds an endowment of one unit of labor, the numeraire good, which he can trade for food at price p. The unique ADM equilibrium for this economy, $\mathcal{E}(0)$, has prices (1,1), consumption vectors (1/2,1/2) and (1/3,2/3), and aggregate production (-7/6,7/6). In order to check whether DSD holds we first determine equilibrium prices in an exchange economy with endowments $((1-\theta\mu,\theta\mu), (1-(1-\theta)\mu,(1-\theta)\mu)$, where μ, the measure of active firms producing (-1,1), is near the measure of active firms in ADM equilibrium, 7/6. (This follows a standard procedure for determining the "inverse demand" function faced by an oligopolist; see, for example, Novshek and Sonnenschein [1978], and takes into account the profits A and B receive from the firms they own, and the subsequent influence on demand.) Consumer A's optimal net trade in this exchange economy is:

$$t_A(p,\mu) = \left(\frac{2p\theta\mu-p}{1+p}, \frac{1-2\theta\mu}{1+p}\right)$$

[Figure 8 around here]

and consumer B's optimal net trade is:

$$t_B(p,\mu) = \left(\frac{3p(1-\theta)\mu-2p}{1+2p}, \frac{2-3(1-\theta)\mu}{1+p}\right).$$

The exchange economy equilibrium price $p(\mu)$ satisfies $t_A(p(\mu),\mu) + t_B(p(\mu),\mu) = (0,0)$, so

$$p(\mu) = \frac{3\mu-\theta\mu-3}{4-\theta\mu-3\mu} \quad \text{and} \quad \left.\frac{dp}{d\mu}\right|_{\mu=7/6} = \frac{3-7\theta}{[4-(3+\theta)(7/6)]^2}.$$

For $\theta < 3/7$ DSD fails: output price increases as the measure of active firms increases (near the perfectly competitive measure, 7/6.)[8] (See Figure 9.) The ADM equilibrium is not the limit of pure strategy Cournot equilibria from the sequence of economies $\{\mathcal{E}(\alpha)\}$, $\alpha \in (0,1]$ with non-infinitesimal firms which converge to the limit economy $\mathcal{E}(0)$. In $\mathcal{E}(\alpha)$ each active firm uses α units of labor to produce α units of food. If the number of active firms is k and $k\alpha \geq 7/6$, then entry will be profitable; alternatively, if $k\alpha < 7/6$, then firms will be making negative profits and should exit. Thus the ADM equilibrium is not a perfectly competitive equilibrium of the sequence. However, if ownership shares are reallocated so that $\theta > 3/7$, then DSD holds (see Figure 9), and the ADM equilibrium is the limit of Cournot equilibria from the sequence of economies $\{\mathcal{E}(\alpha)\}$, $\alpha \in (0,1]$; that is, it is a perfectly competitive equilibrium of the sequence. For the original specification of ownership there exists no perfectly competitive equilibrium of the sequence. At each non-negative profit state the entry of a firm will lead to a positive profit.

[Figure 9 around here]

VI. THE GENERAL EQUILIBRIUM SYNTHESIS

A. Existence and Characterization of Equilibrium

Our basic general equilibrium model is, with the exception of production sets (and our sequence of economies approach), the standard Arrow-Debreu-McKenzie model with standard assumptions. For expository purposes, we make no effort here to use the most general assumptions possible. There are ℓ commodities $k = 1,2,\ldots,\ell$ and m consumers $i = 1,2,\ldots,m$. Each consumer i has (1) preferences $\succsim_i$ defined over a consumption set $X_i \subset R^\ell$, (2) an initial endowment $w_i \in R^\ell$, and (3) for each t, an ownership share, $\theta_{it} \in [0,1]$ of the firm t. To simplify notation we will assume that the ownership share is independent of the firm, ($\theta_{it} = \theta_i$ for all t) and that the consumer sector is the same in the limit economy $\mathcal{E}(0)$ and in every $\mathcal{E}(\alpha)$, $\alpha \in (0,1]$. The assumptions on the consumer sector $(X_i, \succsim_i, w_i, \theta_i)_{i=1}^m$ are standard (see; e.g., Debreu [1959]):

A1: (i) for each $i=1,\ldots,m$, $X_i \subset R^\ell$ is nonempty, closed, convex, and bounded below,

(ii) for each $i=1,\ldots,m$, $\succsim_i$ is a complete, convex preorder on X_i which is continuous,

(iii) for each $i=1,\ldots,m$, there is no satiation consumption in X_i,

(iv) for each $i=1,\ldots,m$, $w_i \in$ interior X_i,

(v) for each $i=1,\ldots,m$, $\theta_i \in [0,1]$, and

(vi) $\sum_{i=1}^m \theta_i = 1$.

To treat nonconvex technologies and free entry at the firm level our assumptions on the producer sector differ from the standard assumptions in two important ways. First, individual firm production sets are the union of the

origin and a compact strictly convex set which is bounded away from the origin.

[Figure 10 around here]

This assumption provides a simple production set analog of U-shaped average cost in partial equilibrium. The origin is included in the production set to guarantee free exit -- in the long run a firm can always avoid losses by shutting down completely. Second, each economy has an infinity of potential firms; a countable infinity of firms in each economy $\mathcal{E}(\alpha)$, and a continuum of firms in the limit economy $\mathcal{E}(0)$. In the Cournot equilibria of $\mathcal{E}(\alpha)$ only a finite number of firms will be active so there will always be additional firms available to check the profitability of entry.

To generate a model with aggregate constant returns to scale production in the limit economy, let $Y_1, Y_2, \ldots, Y_n$ be nonempty, compact, strictly convex subsets of R^ℓ with $Y_j \cap R^\ell_+ = \emptyset$ for $j=1,2,\ldots,n$ and, for each j, let $C(Y_j)$ be the smallest closed, convex cone with vertex 0 containing Y_j.

[Figure 11 around here]

We generate a sequence of economies $\{\mathcal{E}(\alpha)\}$, $\alpha \in (0,1]$, converging to a limit economy $\mathcal{E}(0)$ with production sets $C(Y_1), C(Y_2),\ldots,C(Y_n)$ for "industries" $1,2,\ldots,n$ as follows: for each $\alpha \in (0,1]$ let the production sector of $\mathcal{E}(\alpha)$ consist of, for each j, a countable infinity of firms with identical production sets $\alpha Y_j \cup \{0\}$; let the production sector of $\mathcal{E}(0)$ consist of, for each j, a continuum $[0,\infty)$ of firms with production set $Y_j \cup \{0\}$. The aggregate production set in $\mathcal{E}(\alpha)$ for "industry" j is $\sum_{t=1}^{\infty} (\alpha Y_j \cup \{0\})$.[9] Similarly, the aggregate production set in $\mathcal{E}(0)$ for "industry" j is $\int_0^\infty (Y_j \cup \{0\})dt = C(Y_j)$.[10]

For every $\alpha \in (0,1]$, $\sum_{t=1}^{\infty} (\alpha Y_j \cup \{0\}) \subset C(Y_j)$. For any $M < \infty$ the set of actions in $C(Y_j)$ with norm less than M, which are not in $\sum_{t=1}^{\infty} (\alpha Y_j \cup \{0\})$, becomes arbitrarily small as α converges to zero. Thus the "industry" production sets in $\mathcal{E}(\alpha)$ converge to the "industry" production sets in $\mathcal{E}(0)$. Since all economies have the same consumer sector, the economies $\mathcal{E}(\alpha)$ converge to the limit economy $\mathcal{E}(0)$.

The simplifying assumption that each consumer's ownership share is identical across firms, implies that each consumer's wealth depends only on prices p, and aggregate production y, and not on the arrangement of production among firms. Thus the excess demand D of the consumer sector (the sum of individual consumer's gross demands, minus the sum of resources owned as initial endowments by the consumer sector) is a function only of prices and aggregate production, $D = D(p,y)$. An "inverse demand" selection $F(y)$ is a function from aggregate outputs y, to prices $F(y)$, which clear markets given the action of firms: $D(F(y),y) = y$.[11]

An equilibrium for $\mathcal{E}(\alpha)$ is a Cournot equilibrium with profits evaluated using a selection F:

Definition: Given an inverse demand selection F, $\{y_{jt}\}_{j=1,t=1}^{n,\infty}$ is a Cournot equilibrium for $\mathcal{E}(\alpha)$ relative to F if

(i) $y_{jt} \in \alpha Y_j \cup \{0\}$ for all $t=1,2,\ldots$ for all $j=1,2,\ldots,n$, and

(ii) $y_{jt} \cdot F(\sum_{k=1}^{n} \sum_{s=1}^{\infty} y_{ks}) \geq y'_{jt} \cdot F(\sum_{k=1}^{n} \sum_{s=1}^{\infty} y_{ks} - y_{jt} + y'_{jt})$
for all $y'_{jt} \in \alpha Y_j \cup \{0\}$, for all $t=1,2,\ldots$
for all $j=1,2,\ldots,n$.

The first condition for equilibrium requires individual feasibility. The second condition requires that each firm be profit maximizing given the selection F and the actions of the other firms; i.e., the equilibrium is a Nash equilibrium in quantities (or a Cournot equilibrium). Let $C(\alpha,F)$ be the set of equilibrium "industry" outputs for $\mathcal{E}(\alpha)$ relative to F:

$$C(\alpha,F) = \{ (\sum_{t=1}^{\infty} y_{1t},\ldots, \sum_{t=1}^{\infty} y_{nt}) \mid \{y_{jt}\}_{j=1,t=1}^{n,\infty} \text{ is a Cournot equilibrium for } \mathcal{E}(\alpha) \text{ relative to } F\}.$$

When the cones $C(Y_1), C(Y_2),\ldots,C(Y_n)$ are positively semi-independent (see Debreu [1959] p. 22), and the consumer sector satisfies assumption A1, a standard argument shows that $C(\alpha,F)$ lies in some compact cube (otherwise some points would lead to allocations in which some consumers would be outside their consumption sets, and F would not be defined). By our assumptions on the Y_j, a countable infinity of active firms with production set $\alpha Y_j \cup \{0\}$ would require at least one input at an unbounded level. Therefore in an equilibrium for $\mathcal{E}(\alpha)$ at most a finite number of firms can be active in any industry.

A perfectly competitive equilibrium for the sequence $\{\mathcal{E}(\alpha)\}$ is defined as a limit point of a robust sequence of Cournot equilibria for the economies $\mathcal{E}(\alpha)$:

Definition: (p^*,y^*) is a perfectly competitive equilibrium for the sequence $\{\mathcal{E}(\alpha)\}_{\alpha \in (0,1]}$, if there exists

(i) an inverse demand selection F which is continuously differentiable in a neighborhood of y^* and satisfies $F(y^*) = p^*$;

(ii) a strictly decreasing sequence $\{\alpha_r\}_{r=1}^{\infty} \subset (0,1]$ which converges to zero; and

(iii) for each $\alpha \in \bigcup_{r=1}^{\infty} [\alpha_{2r}, \alpha_{2r-1}]$ an equilibrium "industry" output vector for $\mathcal{E}(\alpha)$ relative to F, $y(\alpha) \in C(\alpha, F)$, such that $y(\alpha)$ converges to y^* as α converges to zero (varying over $\bigcup_{r=1}^{\infty} [\alpha_{2r}, \alpha_{2r-1}]$ only).

The same "inverse demand" selection F, is used throughout the definition. The requirement that F be continuous in a neighborhood of y^* rules out "boundary equilibria" (see K. Roberts [1980]) which we take to be an artifact of the requirement that inverse demand is correctly perceived, even at discontinuities. Under some conditions (as in the partial equilibrium model in section III), it is possible to find $y(\alpha) \in C(\alpha, F)$ for all small α such that $y(\alpha)$ converges to y^*. In other situations gaps repeatedly occur in the set of α values for which an equilibrium near y^* exists for $\mathcal{E}(\alpha)$. However, the definition demands a robust sequence of α values, with <u>nondegenerate intervals</u> of existence, not an ordinary sequence with a countable infinity of points of existence. Thus, the definition requires something much stronger than the coincidence that "demand at the competitive price is an integral multiple of the efficient output for firms."

What are the properties of equilibria of the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$? How are these equilibria related to the ADM (price taking) equilibria of the limit economy $\mathcal{E}(0)$? The answers are analogous to the partial equilibrium results of Theorem 1: the equilibria of the perfectly competitive sequence are those ADM equilibria of the limit economy which satisfy an additional condition, DSD, which is related to downward sloping demand in the partial equilibrium case.

Consider an equilibrium of the perfectly competitive sequence, (p^*, y^*), and the corresponding "inverse demand" function, F. In $\mathcal{E}(\alpha)$, a firm in

industry j has production set $\alpha Y_j \cup \{0\}$, and the "inverse demand" function is continuously differentiable in a neighborhood of y^*. Though price taking behavior has not been assumed, when α converges to zero a firm's ability to affect price becomes arbitrarily small. Even the entry or exit of firms has a small effect on price. Thus each "industry" output in equilibrium, y_j^*, must maximize profit over the industry production set, $C(Y_j)$, relative to prices p^*, and must satisfy $p^* \cdot y_j^* = 0$. (If $p^* \cdot y_j^* > 0$ ($<$ 0) inactive (active) firms could not have been profit maximizing in the Cournot equilibrium of $\mathcal{E}(\alpha)$ for small α. If $p^* \cdot y_j = 0$ but $p^* \cdot y > 0$ for some $y \in C(Y_j)$ then inactive firms could not have been profit maximizing in $\mathcal{E}(\alpha)$.) By an interchangeability lemma (see Koopmans [1957] p. 13) any decomposition of y^* into feasible actions for individual firms in $\mathcal{E}(0)$ is such that <u>all</u> firms' actions must be profit maximizing over their production sets relative to prices p^*. Thus the actions of firms are <u>as if</u> they are price takers in equilibrium, and an equilibrium of the perfectly competitive sequence is an ADM equilibrium of the limit economy $\mathcal{E}(0)$.

The example of Section V clearly demonstrates the difference between the <u>result</u> we obtain, "as if" price taking, and the <u>assumption</u> of price taking behavior. The equilibria of the perfectly competitive sequence must satisfy an additional condition, (weak) DSD. As discussed in Section IV, the DSD condition is necessary for the process of entry to be at rest. As α converges to zero, in the sequence of economies $\{\mathcal{E}(\alpha)\}$, firms become arbitrarily small relative to the market, and thus have arbitrarily small impact on prices. Therefore, any $y_j \in Y_j$ with $p^* \cdot y_j < 0$ would not (for α sufficiently small) be able to change price enough to make a nonnegative profit when introduced as $\alpha y_j \in \alpha Y_j$. Since $\max p^* \cdot Y_j = \max p^* \cdot C(Y_j) = 0$, no actions αy_j in αY_j with $p^* \cdot y_j > 0$ are available. The expression $\alpha^2 y_j'[\frac{\partial F}{\partial y}(\sum_{j=1}^{n} y_j^*)]y_j$ is (approximately) the profit differential (above the profit made by an active firm) available to an inactive firm by switching to $\alpha y_j \in \alpha Y_j$ with $p^* \cdot y_j = 0$ in the $\mathcal{E}(\alpha)$ equilibrium. When weak DSD fails the profit differential is positive, and inactive firms could not be profit maximizing for α sufficiently small. Thus the process of entry could not be at rest in $\mathcal{E}(\alpha)$, and $(y_1^*, \ldots, y_n^*)$ could not be a limit point of <u>any</u> sequence of equilibria of $\mathcal{E}(\alpha)$ relative to F.

These ideas are used to prove the following result:

<u>Theorem 2</u>: Let $(X_i, \geq_i, w_i, \theta_i)_{i=1}^m$ be a consumer sector satisfying assumption A1 and let $Y_1, Y_2, \ldots, Y_n$ be nonempty, compact, strictly convex subsets of R^ℓ with $Y_j \cap R_+^\ell = \emptyset$ and $C(Y_1), C(Y_2), \ldots, C(Y_n)$ positively semi-independent. Let $\{\mathcal{E}(\alpha)\}$, $\alpha \in (0,1]$ and $\mathcal{E}(0)$ be the sequence of economies generated by $(X_i, \geq_i, w_i, \theta_i)_{i=1}^m$ and $Y_1, Y_2, \ldots, Y_n$. Let $(p^*, y_1^*, y_2^*, \ldots, y_n^*)$ be a perfectly competitive equilibrium of the sequence $\{\mathcal{E}(\alpha)\}$, and let F be the corresponding "inverse demand" selection. Then (1) for each j, $p^* \cdot y_j^* = \max p^* \cdot C(Y_j) = 0$, and (2) for each j, for each $y_j \in Y_j$ with $p^* \cdot y_j = 0$, $y_j' [\frac{\partial F}{\partial y} (\sum_{j=1}^n y_j^*)] y_j \leq 0$ (weak DSD).

The first result implies that an equilibrium of $\mathcal{E}(0)$ is a price taking equilibrium. By an interchangeability lemma (see Koopmans [1957]) any decomposition of y^* into feasible actions for individual firms is such that <u>all</u> firms must be profit maximizing given the fixed vector of prices p^*.

The second result is that at equilibrium (weak) DSD must hold.

Theorem 2 shows that the equilibria of the perfectly competitive sequence are ADM equilibria of $\mathcal{E}(0)$ and satisfy (weak) DSD. To complete the analogy to the partial equilibrium results of Theorem 1, we next state a theorem which shows that the ADM equilibria of $\mathcal{E}(0)$ which satisfy (strong) DSD are equilibria of the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$.

<u>Theorem 3</u>: Let $(X_i, \geq_i, w_i, \theta_i)_{i=1}^m$ be a consumer sector satisfying assumption A1 and let $Y_1, Y_2, \ldots, Y_n$ be nonempty, compact, strictly convex subsets of R^ℓ with $Y_j \cap R_+^\ell = \emptyset$ and $C(Y_1), C(Y_2), \ldots, C(Y_n)$ positively semi-independent. Let $\{\mathcal{E}(\alpha)\}$, $\alpha \in (0,1]$, and $\mathcal{E}(0)$ be the sequence of economies generated by $(X_i, \geq_i, w_i, \theta_i)_{i=1}^m$, and $Y_1, Y_2, \ldots, Y_n$. Given $(p^*, y_1^*, y_2^*, \ldots, y_n^*)$ and an "inverse demand" selection F, if

(1) $p^* = F(\sum_{j=1}^n y_j^*)$ and F is twice continuously differentiable in a neighborhood of $\sum_{j=1}^n y_j^*$,

(2) for each $j = 1, 2, \ldots, n$, $y_j^* \in C(Y_j)$,

(3) for each $j=1,2,\ldots,n$, $p^* \cdot y_j^* = \max p^* \cdot C(Y_j)$,

(4) for each $j=1,2,\ldots,n$, for all $y_j \in Y_j$ with $p^* \cdot y_j = 0$,

$y_j' [\frac{\partial F}{\partial y} (\sum_{j=1}^n y_j^*)] y_j < 0$ (strong DSD), and

(5) a regularity condition holds,[12]

then $(p^*, y_1^*, \ldots, y_n^*)$ is an equilibrium of the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$.

Condition (1) of the theorem requires that markets clear at prices p^* when firms produce $\sum_{j=1}^n y_j^*$ in aggregate, and that F selects these prices and is well behaved locally. The second condition requires that each "industry" production be feasible. The third condition requires that each "industry" production be profit maximizing at prices p^*. Conditions (1)-(3) imply that $(p^*, y_1^*, \ldots, y_n^*)$ is an ADM (price-taking) equilibrium. The fourth condition requires that (strong) DSD holds at $(p^*, y_1^*, \ldots, y_n^*)$ for the selection F: the prices determined by F give proper entry signals so that the process of entry could be at rest at $(p^*, y_1^*, \ldots, y_n^*)$. Along with the regularity condition (5), Theorem 3 states that these conditions imply that $(p^*, y_1^*, \ldots, y_n^*)$ is the limit of a robust sequence of Cournot equilibria of $\mathcal{E}(\alpha)$ relative to section F. Thus an ADM equilibrium of $\mathcal{E}(0)$ which satisfies (strong) DSD is an equilibrium of the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$. The proof of this result is fairly long and technical. See Novshek and Sonnenschein [1978].

Theorem 2 showed that (weak) DSD was a necessary condition for an equilibrium of the perfectly competitive sequence. Theorem 3 shows that (strong) DSD (along with the standard conditions for an ADM equilibrium and a regularity condition) is a sufficient condition for an equilibrium of the perfectly competitive sequence. As the example in Section V demonstrates, there are perfectly competitive sequences of economies with no equilibrium, but for which the ADM equilibrium conditions (and the regularity condition) hold at some point. Thus the (strong) DSD condition is required as part of the set of sufficient conditions.

While Theorem 3 gives conditions under which a particular point will be an equilibrium of the perfectly competitive sequence, it does not specify conditions on the perfectly competitive sequence or the limit economy $\mathcal{E}(0)$ to guarantee that the sequence has <u>some</u> equilibrium. The following corollary does provide sufficient conditions on the limit economy $\mathcal{E}(0)$ to guarantee existence of an equilibrium for the perfectly competitive sequence.

<u>Corollary 1</u>: Let $(X_i, \gtrsim_i, w_i, \theta_i)_{i=1}^m$ be a consumer sector satisfying assumption A1 and let $Y_1, Y_2, \ldots, Y_n$ be nonempty, compact, strictly convex subsets of R^ℓ with $Y_j \cap R_+^\ell = \emptyset$ and $C(Y_1), C(Y_2), \ldots, C(Y_n)$ positively semi-independent. Let $\{\mathcal{E}(\alpha)\}$, $\alpha \in (0,1]$ and $\mathcal{E}(0)$ be the sequence of economies generated by $(X_i, \gtrsim_i, w_i, \theta_i)_{i=1}^m$ and $Y_1, Y_2, \ldots, Y_n$. If the consumer sector of $\mathcal{E}(0)$ acts as a single consumer with differentiably convex preferences[13] (and the regularity condition holds everywhere), then the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$ has an equilibrium and it is unique.

The corollary follows from Theorem 3 and three observations: (a) The assumptions of Corollary 1 are sufficient to guarantee the existence of a unique point (p^*, y^*) satisfying conditions (1) - (3) of Theorem 3 (the ADM equilibrium conditions). (b) A single consumer with differentiably convex preferences generates a unique selection F. When aggregate production is y and the consumer's initial endowment is w, the consumer must demand $y + w$ at prices $F(y)$ and wealth $F(y)\cdot(y+w)$. Thus $F(y)$ is the unique (after normalization) supporting price for the bundle $y+w$. (c) F satisfies (strong) DSD at y^*. If prices are normalized so that the first commodity is the numeraire, then, using Walras' law, the properties of F can be determined by examining the properties of the excess demand for the last $\ell-1$ commodities. Computation of $y' [\frac{\partial F}{\partial y} (\sum_{j=1}^n y_j^*)] y$ for any nonzero y with $p^* \cdot y = 0$ shows that (strong) DSD follows from the negative definiteness of the consumer's $(\ell-1)\times(\ell-1)$ substitution matrix (the substitution matrix with first row and column deleted).

Conditions under which the consumer sector of an economy acts as a single consumer are well known (e.g., see Shafer and Sonnenschein [1982]).

B. The Classical Theorems of Welfare Economics

We now turn to the two classical theorems of welfare economics in the context of free entry and firms which are arbitrarily small relative to the economy. First we must define a Pareto optimal allocation for the sequence $\{\mathcal{E}(\alpha)\}$. For any $\alpha \in (0,1]$ a vector $(x_1^*, \ldots, x_m^*, y_1^*, \ldots, y_n^*)$ is a feasible allocation in $\mathcal{E}(\alpha)$ if x_i^* is in the consumption set for consumer i for $i=1,2,\dots,m$, y_j^* is in the production set for "industry" j $(\sum_{t=1}^{\infty}(\alpha Y_j \cup \{0\}))$ for $j=1,2,\dots,n$, and resources balance, $\sum_{i=1}^{m} x_i^* - \sum_{j=1}^{n} y_j^* = w$ where w is the aggregate endowment of the economy. A feasible allocation in $\mathcal{E}(\alpha)$ is Pareto optimal in $\mathcal{E}(\alpha)$ if there does not exist a second feasible allocation at which all consumers are at least as well off as at the first allocation and at least one consumer is strictly better off. We define the Pareto optimal allocations of the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$ to be the limit points of Pareto optimal allocations for the $\mathcal{E}(\alpha)$. That is, t is a Pareto optimal allocation for the sequence if there exists sequences $\{\alpha_k\}$ and $\{t_k\}$ such that α_k converges to zero, t_k converges to t, and t_k is a Pareto optimal allocation of $\mathcal{E}(\alpha_k)$ for each k.

Because of the way in which the sequence $\{\mathcal{E}(\alpha)\}$ converges to the limit economy $\mathcal{E}(0)$, it is easy to see that every Pareto optimal allocation for the sequence is also a Pareto optimal allocation for $\mathcal{E}(0)$. Also, every Pareto optimal allocation of $\mathcal{E}(0)$ is the limit of Pareto optimal allocations for the $\mathcal{E}(\alpha)$, and thus is a Pareto optimal allocation for the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$.[14]

Under what conditions are equilibria of the sequence Pareto optimal allocations? Under what conditions can Pareto optimal allocations be supported as equilibria of the sequence? An answer to the first question was produced by Oliver Hart [1979]. He showed that for a differentiated products general equilibrium model, limit points of the Cournot equilibria of prelimit economies must be Pareto optimal allocations. In our model the equilibria of the perfectly competitive sequence satisfy all the conditions of an ADM equilibrium. As long as the conditions of the ADM First Welfare Theorem are satisfied, equilibria

of the sequence are ADM equilibria of the limit economy, which are Pareto optimal. Thus, with weaker assumptions than A1 (for example, see Debreu [1959] p. 94), we get:

<u>Theorem 4:</u> Equilibria of the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$ are Pareto optimal.

The Second Welfare Theorem takes on an additional subtlety in our context. By the ADM Second Welfare Theorem, under suitable assumptions, every Pareto optimal allocation can be supported as an ADM equilibrium. However, not every ADM equilibrium of the limit economy $\mathcal{E}(0)$ is an equilibrium of the perfectly competitive sequence $\mathcal{E}(\alpha)$. We must be able to redistribute initial endowments and ownership shares (as was possible in the example of Section III by setting $\theta > \frac{3}{7}$) so that prices give correct entry signals (DSD holds). Otherwise, the process of entry might never be at rest near the Pareto optimal allocation, and therefore that allocation would not be a limit point of a robust sequence of Cournot equilibria of $\mathcal{E}(\alpha)$ (that is, an equilibrium of the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$). We next show that the ADM assumptions for their Second Welfare Theorem, plus assumptions on consumers' preferences which guarantee that each consumer has a twice continuously differentiable demand function (because we work in a differentiable framework) with a negative definite substitution effects matrix,[15] are enough to allow us to redistribute ownership shares and endowments so that (strong) DSD holds. Then, with the addition of the regularity condition, the Pareto optimal allocation for the sequence can be supported as an equilibrium of the sequence; i.e., as a limit point of a robust sequence of Cournot equilibria of $\mathcal{E}(\alpha)$, with suitable redistribution of initial endowments and ownership shares.

Our initial specification of the consumer sector does not specify individual initial endowments w_i nor ownership shares θ_i. It only specifies the aggregate endowment $w = \sum_{i=1}^{m} w_i$, since that is all that is relevant for the Pareto optimal allocation. The theorem establishes that, under suitable conditions, it is always possible to redistribute initial endowments and ownership shares so that for the new sequence of private ownership economies, $\mathcal{E}(\alpha)$, the Pareto optimal allocation for the sequence is the limit point of a robust sequence of Cournot equilibria of $\mathcal{E}(\alpha)$; i.e., an equilibrium allocation for the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$.

<u>Theorem 5</u>: Let $((X_i,\geq_i)_{i=1}^{m}, w)$ be a consumer sector satisfying

(i) for $i=1,2,\ldots,m$, X_i is convex, and

(ii) for $i=1,2,\ldots,m$, $\geq_i$ have no satiation point and generate a twice continuously differentiable demand function with a negative definite substitution-terms matrix.[15]

Let $Y_1,Y_2,\ldots,Y_n$ be nonempty, compact, strictly convex subsets of R^ℓ with $Y_j \cap R_+^\ell = \emptyset$, $C(Y_1), C(Y_2),\ldots,C(Y_n)$ positively semi-independent and $(-R_+^\ell) \subset \sum_{j=1}^{n} C(Y_j)$.[16] Let $\{\mathcal{E}(\alpha)\}$, $\alpha \in (0,1]$ and $\mathcal{E}(0)$ be the sequence of economies generated by $((X_i,\geq_i)_{i=1}^{m},w)$ and $Y_1,Y_2,\ldots,Y_n$. Let $(x_1^*,\ldots,x_m^*,y_1^*,\ldots,y_n^*)$ be a Pareto optimal allocation for the sequence such that

(iii) for $i=1,2,\ldots,m$, $x_i^* \in$ interior X_i.

Then if the regularity condition holds, there exists a vector of prices, p^*, initial endowments w_i^*, $i=1,2,\ldots, m$ with $\sum_{i=1}^{m} w_i^* = w$, and nonnegative ownership shares θ_i^*, $i=1,2,\ldots,m$ (with $\sum_{i=1}^{m} \theta_i^* = 1$) such that (p^*,y^*) is an equilibrium for the

sequence $\{\mathcal{E}(\alpha)\}$ (when endowments are w_i^*, $i=1,\ldots,m$ and ownership shares are θ_i^*, $i=1,\ldots,m$), and for each $i=1,2,\ldots,m$, x_i^* is the corresponding demand for consumer i (that is, x_i^* maximizes $\geq_i$ on $\{x\epsilon X_i \mid p^*\cdot x \leq p^*\cdot w_i + \theta_i \sum_{j=1}^{n} p^*\cdot y_j^*\}$).

The Pareto optimal allocation for the sequence is also a Pareto optimal allocation for the limit economy $\mathcal{E}(0)$, so by the standard Second Welfare Theorem there are prices, p^*, and an assignment of endowments and ownership shares such that the Pareto optimal allocation is supported as an ADM equilibrium of $\mathcal{E}(0)$. It remains to show that the assignment can be made so that (strong) DSD holds. If we assign endowments and ownership shares so that $w_i + \theta_i \sum_{j=1}^{n} y_j^* = x_i^*$ for each i, then (strong) DSD holds. (The argument is similar to the argument that a single consumer generates an "inverse demand" satisfying (strong) DSD used for Corollary 1. In this case a sum of m negative definite matrices appears instead of the single negative definite matrix in the single consumer case.) Application of Theorem 3 completes the proof of Theorem 5.

Theorem 5 shows that, under suitable conditions, every Pareto optimal allocation for the sequence $\{\mathcal{E}(\alpha)\}$ can be supported as an equilibrium of the sequence. Endowments and ownership shares are redistributed not only to make incomes "right" but also to make prices give correct entry signals (DSD must hold).

It is important to observe that Theorem 5 is not as positive as the corresponding ADM result (see; e.g., Debreu [1959], p. 95). In that theory all of the required redistribution can be achieved by means of a single commodity, which may be thought of as "government transfer." Here this may not be possible. Leo Hurwicz [1959] refers to the property that every optimum is an equilibrium (after redistribution) as "unbiasedness." Our analysis suggests a bias of the competitive mechanism that goes beyond what can be corrected by "government transfer." This bias can be corrected, but the correction in general requires more than the redistribution of a single commodity. From the viewpoint presented here, the competitive mechanism has the bias that it can only seek out those optima that, with the means of transfer at hand, give the necessary entry signals.

VII. DYNAMICS

Marshall and Walras looked past the tatonnement to a long-run equilibrium in which all factors are free to vary, and they "flow towards that branch of production" in which there are profits to be realized. In the short run certain factors are immobile, and the returns to these factors are not necessarily equalized. Prices clear markets at each moment, but these prices reflect only the relative scarcity of factors that are instantaneously variable. Walras described the process by which these quickly adjusting prices are reached by a tatonnement.

In each $\mathcal{E}(\alpha)$ firms correctly perceive the prices, $F(y)$, that will prevail given any aggregate production y. Thus our analysis has always included an implicit perceived adjustment of prices to clear markets, a tatonnement for the exchange economy generated by any fixed production vector y.[17] For our model of dynamics we assume that the adjustment of prices to clear markets is instantaneous relative to the speed at which firms are able to change production levels.

Following the standard Marshallian framework, the next consideration is the quantity adjustments made by a fixed number of active firms to get to a short run equilibrium. In $\mathcal{E}(\alpha)$ we assume a continuous Cournot dynamics; that is, each active firm changes its output continuously, recognizing its own effect on price but myopically assuming all other firms' outputs are fixed. In the limit economy $\mathcal{E}(0)$, each firm is infinitesimal and this dynamics is equivalent to that generated by firms' continuous adjustment viewing price as fixed. In both $\mathcal{E}(0)$ and $\mathcal{E}(\alpha)$ the firms are myopic since all firms are adjusting and and price is changing over time. Exit is not allowed in the short run, so a firm in "industry" j must produce an output from αY_j. If F is continuously differentiable and α is small, then the firm has little effect on price. Thus the incentives and behavior of the firm in the short run dynamic for $\mathcal{E}(\alpha)$ "converge to" that in the short run dynamic for $\mathcal{E}(0)$.

The bridge between the short run and the long run is built in the following manner. For concreteness, let us refer to the factor that is fixed in the short run as entrepreneurship. After each tatonnement we assume that entrepreneurship flows in the direction of increasing profit; that is, the number of active firms changes based on the profit in each industry. Following such a period of factor movement, a new short run equilibrium is determined with associated prices, and these new prices lead to new incentives for factors to move, and so on. From this point of view the tatonnement takes place with great speed relative to the short run quantity adjustment of firms, which takes place with great speed relative to the entry and exit adjustment of the entrepreneurial factor. As in the short run adjustment, in $\mathcal{E}(\alpha)$ firms enter or leave recognizing their own effect on price, but viewing other firms' actions as fixed, so again, in $\mathcal{E}(0)$ this entry dynamic is equivalent to that generated by viewing price as fixed. In both the short and long run, firms' behavior is myopic. The entry-exit decision is a choice of production set αY_j or $\{0\}$. If F is continuously differentiable and α is small, then the firm's decision has little effect on price. Thus the incentives and behavior of the firm in the long run dynamic for $\mathcal{E}(\alpha)$ "converge to" that in the long run dynamic for $\mathcal{E}(0)$.

The DSD condition suggests a dynamic theory of the movement towards the long-run equilibrium, in which the realignment of the entrepreneurial factor plays a significant role. Since DSD is a necessary condition for equilibrium of the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$, no infinitesimal firm in $\mathcal{E}(0)$ can enter with positive profit at an equilibrium. But out of a long-run equilibrium we can conceive of entry and exit that is proportional to the returns to the entrepreneurial factor. This leads to consideration of the stability of the equilibrium introduced in the previous sections. It is related to the question of whether returns to the homogeneous entrepreneurial factors have a tendency to be equalized and whether myopic profit seeking behavior moves an economy toward a Pareto optimum. In addition, it is relevant to the viability of a planning procedure in which production units are centrally added in those sectors which are the most profitable.

For simplicity we will discuss the dynamics in terms of the limit economy $\mathcal{E}(0)$. We have already argued that when α is small, both the short and long run dynamics in $\mathcal{E}(\alpha)$ will be similar to the dynamics in $\mathcal{E}(0)$. Our three stages of dynamics are:

(1) instantaneous adjustment of prices to clear markets given any aggregate production y;

(2) output adjustment by a fixed number (mass) of firms, each viewing price as fixed at each instant, to reach a short run equilibrium; and

(3) entry and exit at a rate proportional to the (firm) profit levels in each industry to reach a long run equilibrium.

A. Partial Equilibrium and Simple General Equilibrium Dynamics

The dynamics we have in mind are conveniently illustrated by means of the partial equilibrium market M(0) that was used in Section III. We add to the hypotheses from that section the condition that F is non-increasing and there exists a unique y^* such that $F(y^*) = C(1)$. In this case there is a unique equilibrium.

The dynamics are introduced as follows. For each aggregate output y, market clearing prices are given by $F(y)$, and the usual tatonnement adjustment is stable since F is nonincreasing. In the short run an active firm is forced to produce a positive quantity whether or not the positive, profit maximizing action leads to positive profit. Thus each active firm increases output whenever the current price exceeds the marginal cost at the current output level, and decreases output when current marginal cost exceeds price. This adjustment is stable since F is nonincreasing and all firms have nondecreasing marginal cost. For each mass of active firms μ, the short run equilibrium price associated with μ, $p(\mu)$, is determined as follows:

1. Supply, $S(p(\mu))$, is the integral of the profit maximizing actions of the μ active firms at price $p(\mu)$, and

2. $F(S(p(\mu))) = p(\mu)$; that is, the inverse demand of supply at $p(\mu)$ is $p(\mu)$ (or short run supply equals demand at $p(\mu)$).

Let $\pi(\mu)$ denote the profit of each active firm when there are μ active firms in short run equilibrium. Then it is clear that the long run equilibrium is globally stable under the long run adjustment process

$$\frac{d\mu}{dt} = \pi(\mu) .$$

A word of interpretation is in order. Suppose that we start with the initial mass of active firms $\mu(0)$. The prices $p(\mu(0))$ are short run equilibrium prices and provide signals for entry. Over time the mass of active firms continuously changes and there is a continuum of short run equilibria during this adjustment. When the mass of active firms is $\mu(t)$, the short run equilibrium prices are $p(\mu(t))$. The mass of active firms adjusts towards the final equilibrium at which profits are zero and each firm produces at efficient scale.

Before beginning the general equilibrium analysis, it is useful to explore what these ideas mean for the general equilibrium example considered in Section V. For $\theta > \frac{3}{7}$ there is a unique equilibrium for $\{\mathcal{E}(\alpha)\}$. It is the unique ADM equilibrium of $\mathcal{E}(0)$ and DSD is satisfied. For any initial mass of active firms that leaves consumers in their consumption sets when each active firm produces at the efficient point (-1,1), the dynamics defined by $\frac{d\mu}{dt} = \pi(\mu)$ converge to the unique equilibrium $\mu^* = \frac{7}{6}$. On the other hand, if $\theta < \frac{3}{7}$, then the unique ADM equilibrium remains $\mu^* = \frac{7}{6}$, but this allocation is not even locally stable.[18]

The example at hand is useful for presenting a planning interpretation. Suppose that the central authority provides licenses for opening or closing facilities, but aside from this allows prices to be flexible. In the example

one starts with a certain mass of licensed facilities. If $\theta > \frac{3}{7}$, then the procedure which has facilities added when profits are positive, and facilities closed when there are losses, converges to the unique Pareto optimal allocation.

B. General Equilibrium Dynamics

Our general equilibrium existence theorem applies to a sequence of economies $\{\mathcal{E}(\alpha)\}$ in which the demand sector acts as a single consumer (Corollary 1). We now apply our entry dynamics to this case to test whether the existence and stability theories can be tied together as they are in Marshall's theory. Is the entry dynamics stable or does it lead to a form of market failure in which the economy fails to approach the unique equilibrium? It is clear that the dynamics can only be discussed at aggregate productions y, which lead to feasible allocations for consumers (otherwise $F(y)$ is not defined). For the single consumer case this means we can only discuss dynamics at aggregate outputs y, which, when added to the consumer's initial endowment w, are feasible consumptions for the consumer; that is $y + w \in X$. (To avoid further problems with the boundary of the consumption set, X, we assume that the boundary of X forms a single indifference curve for the consumer or consumer sector.) Is it the case that, starting from any such aggregate production, the entry dynamics leads to the redistribution of entrepreneurial activity, entry, and exit which converges to the unique equilibrium of $\{\mathcal{E}(\alpha)\}$ guaranteed by Corollary 1? Theorem 6 shows the answer is yes, the unique equilibrium of $\{\mathcal{E}(\alpha)\}$ is "globally" stable under the entry dynamics. When the consumer sector acts as a single consumer, prices give the proper entry signals so that the entry dynamics leads to a Pareto optimal allocation.

The general equilibrium dynamics are perfectly analogous to the partial equilibrium dynamics. For any aggregate y, market clearing prices are given by $F(y)$. In the short run the fixed mass of active firms adjust their productions myopically till a short run equilibrium is attained. The resulting short run equilibrium prices provide the incentives for long run entry and exit decisions by entrepreneurs. The rate of entry in any "industry" is proportional to (firm) profit levels in that industry at that time.

We begin by defining a short run, perfectly competitive equilibrium (without free entry or exit) for the economy with a single consumer $(X,\succeq,w,1)$ and measure η_j of active infinitesimal firms with production set Y_j in "industry" j. We assume $Y_1,\ldots,Y_n$ are nonempty, compact, strictly convex subsets of R^ℓ with $Y_j \cap R^\ell_+ = \emptyset$ and $C(Y_1), C(Y_2),\ldots,C(Y_n)$ positively semi-independent, and $(X,\succeq,w,1)$ satisfies assumption A1 with differentiably convex preferences as in Corollary 1.

<u>Definition</u>: $(p, y_1,\ldots,y_n)$ is a short run competitive equilibrium with measures of active firms $(\eta_1,\ldots,\eta_n)$ if

(1) for each j, $y_j \in \eta_j Y_j$,

(2) for each j, y_j maximizes $p\cdot y$ on $\eta_j Y_j$, and

(3) $w + \sum_{j=1}^n y_j \in X$ maximizes $\succeq$ on $\{x \in X \mid p\cdot x \leq p\cdot(w + \sum_{j=1}^n y_j)\}$.

The first condition requires that the aggregate action y_j be feasible for "industry" j when mass η_j of firms are active. The second condition requires that all active firms be profit maximizing given prices p and subject to the constraint of remaining active. Since firms cannot shut down $(0 \notin y_j)$

they may be making losses. The third condition requires that the consumer sector demand at prices p and income $p\cdot(w + \sum_{j=1}^n y_j)$ exactly match the available supply, $w + \sum_{j=1}^n y_j$.

Before discussing the stability of the long run entry adjustment process, it is natural to examine the stability of both the tatonnement and the short run adjustment process. It is well known that for any fixed aggregate production y, the tatonnement for this single consumer economy is stable, converging to $F(y)$. Whenever a short run equilibrium exists, the price and aggregate output, summed over "industries", $\sum_{j=1}^n y_j$, must be a Pareto optimal allocation for the short run economy with the single consumer and convex production sets $\eta_1 Y_1,\ldots,\eta_n Y_n$. Strict convexity of preferences leads to a unique Pareto optimal allocation and unique supporting prices. The stability of the short run adjustment process with a fixed mass of firms follows from the observation that when firms adjust their outputs to increase profit at the current prices, they move aggregate output in a direction that increases the consumer's utility (since the current prices are supporting prices for the consumer's choice of the current bundle). Thus the short run adjustment process moves to the Pareto optimal allocation for the short run economy with a fixed mass of active firms, and this limit is the short run competitive equilibrium.

The long run entry dynamics operates as follows: for the vector of masses of active firms at time t, $(\eta_1(t),\ldots,\eta_n(t))$, let $(p(t), y_1(t),\ldots, y_n(t))$ be the unique, stable, short run equilibrium. We assume firms enter (leave) industry j in proportion to the current profits (losses) of individual firms in industry j: for each j, $\dot{\eta}_j = \frac{1}{\eta_j} p(t) \cdot y_j(t)$.[19]

Theorem 6: Let $(X_i, \succsim_i, w_i, \theta_i)_{i=1}^m$ be a consumer sector satisfying assumption A1 and let $Y_1, Y_2, \ldots, Y_n$ be nonempty, compact, strictly convex subsets of R^ℓ with $Y_j \cap R_+^\ell = \emptyset$ and $C(Y_1), C(Y_2), \ldots, C(Y_n)$ positively semi-independent. Let $\{\mathcal{E}(\alpha)\}$, $\alpha \in (0,1]$ and $\mathcal{E}(0)$ be the sequence of economies generated by $(X_i, \succsim_i, w_i, \theta_i)_{i=1}^m$ and $Y_1, Y_2, \ldots, Y_n$. If the consumer sector of $\mathcal{E}(0)$ acts as a single consumer with differentiably convex preferences,[20] whose consumption set boundary is a single indifference curve (and the regularity condition holds everywhere), then the unique equilibrium of $\mathcal{E}(\alpha)$ is "globally" stable for the entry dynamic.[21]

The proof of the stability of the long run entry dynamics follows from the observation that entry (or exit) takes place if and only if the change in the mass of active firms leads to new short run "industry" production sets, $\eta_1 Y_1, \ldots, \eta_n Y_n$, such that $\sum_{j=1}^{n} n_j Y_j$ contains points in the upper contour set for the consumer. (This does not depend on the relative speeds of adjustment in the different "industries.") Thus the new short run equilibrium leads to a higher utility level for the consumer, and the entry/exit adjustment continues until the consumer reaches the Pareto optimal allocation for $\mathcal{E}(0)$, which is the unique equilibrium of $\{\mathcal{E}(\alpha)\}$.

For economies with multiple industries and multiple consumers who do not act as a single consumer, the DSD conditions are necessary but, by themselves not sufficient for stability of equilibrium under the entry dynamic, not even for local stability. With multiple consumers, additional conditions on the interactions among "industries" must be satisfied for (local) stability. The DSD conditions are then something like the Hicksian stability conditions, which check only one market at a time. We also note that with multiple "industries," entry in "industry" 1 may lead to increased profit in "industry" 2, etc. Thus the dynamic path may have a decrease in the measure of firms in industry 2 followed by an increase in the measure of active firms, and so on. This emphasizes the myopia of firm decision making. Even with this myopia, convergence is guaranteed when the demand sector acts as if it were a single consumer.

Let us summarize the dynamics. Before this section we were concerned with static analysis. Time was integrated into the analysis in the spirit of Debreu's Theory of Value, with all commodities dated and all markets opening only once for the purpose of deciding on "who was to deliver what to whom and on what date." In this section the analysis is dynamic and temporary equilibrium in spirit. Markets open for today's exchange. The prices determined in those markets determine profits, which are the basis for factor movements and a different distribution of firms tomorrow. Because of the new distribution of factors that are immobile in the short run, tomorrow's prices are different from today's and so are profits, etc. In our temporary equilibrium analysis we assume that firms are myopic; they do not consider how prices will change over time. Even with such myopia, for the leading case in which prices provide the correct entry signals and so equilibrium exists, namely the case in which the demand sector behaves as a single agent, equilibrium is globally stable. This of course provides a strong link between the existence and stability theorem, and we take this link to be very much in the spirit of both Walrasian and Marshallian analysis.

VIII. DECREASING RETURNS TO SCALE IN THE AGGREGATE [22]

Our analysis has been carried out for the case of constant returns to scale in the aggregate. We now consider the case of decreasing returns to scale in the aggregate, caused by variation in the "quality" of some resource, such as land, as more and more land is used. The standard analysis for the partial equilibrium case is quite familiar. The "high quality" land is more productive (or has lower average cost of production) so that, in equilibrium, a rent is paid for the high quality land. The marginal land, the "lowest quality" land being used in equilibrium, is paid no rent. The aggregate supply curve is upward sloping, since land of lower and lower quality must be brought into use and high quality land must be used more intensively (see Figure 12). What happens when these variations in efficiency are introduced in our framework? For concreteness let us assume that different firms have different efficiencies of production

[Figure 12 around here]

(minimum average costs in partial equilibrium). The first issue is merely a technical one: it becomes more complicated notationally to set up the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$ and to define a notion of convergence to the limit economy $\mathcal{E}(0)$.

The second issue is the definition of DSD. The idea remains the same, but the form of the condition must change since the additional firms that might consider entry are less efficient than the firms already active in equilibrium. The DSD condition requires that these less efficient firms could not profit from entry; it does not require that additional firms just as efficient as the marginal firm could not profit from entry. A perfectly competitive equilibrium of the

perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$ is defined as a limit point of a robust sequence of Cournot equilibria with entry of $\mathcal{E}(\alpha)$.

In this decreasing returns to scale in the aggregate framework, with the revised definition of DSD, the results of our previous analysis are unchanged: (weak) DSD is a necessary condition for a price-allocation pair to be a perfectly competitive equilibrium of $\{\mathcal{E}(\alpha)\}$; (strong) DSD, the ADM equilibrium conditions (and a regularity condition) are sufficient conditions for a price-allocation pair to be a perfectly competitive equilibrium of $\{\mathcal{E}(\alpha)\}$ (see Novshek and Sonnenschein [1983]); if the consumer sector of $\mathcal{E}(0)$ acts as a single consumer, then $\{\mathcal{E}(\alpha)\}$ has a unique equilibrium and it is "globally" stable for all entry dynamics;[23] perfectly competitive equilibria are Pareto optimal; and every Pareto optimal allocation of $\{\mathcal{E}(\alpha)\}$ can, after a suitable redistribution of initial endowments and ownership shares, be supported as an equilibrium of $\{\mathcal{E}(\alpha)\}$.

As a final comment we note the effect of allowing mixed strategy equilibria in $\mathcal{E}(\alpha)$. For the constant-returns-to-scale-in-the-aggregate case, there exists an equilibrium of $\mathcal{E}(\alpha)$ for <u>all</u> sufficiently small α if and only if there exists a robust sequence of pure strategy equilibria. Thus mixed strategies in $\mathcal{E}(\alpha)$ only serve to "fill out" the sequence of equilibria, when it already exists for pure strategies. For the decreasing-returns-to-scale-in-the-aggregate case, the results are quite different. Mas-Colell [1983] has shown that (mixed strategy) equilibrium exists in $\mathcal{E}(\alpha)$ for all small α , regardless of the satisfaction of the DSD condition. With mixed strategies it is always possible to have a sufficiently large number of marginal firms using mixed strategies (which include action 0 with positive probability) so that the most efficient inactive firm is too inefficient to enter profitably.

IX. CONCLUSION

Taken together the preceding results demonstrate that there is less tension than one might have thought between the rather institutional Marshallian model, with free entry and U-shaped average costs, and the formal ADM theory with its insistence on a general existence theorem and price-taking as a primitive. Recent advances in the theory of imperfect competition and mathematical economics make it possible to build a synthesis between the Marshallian and ADM models that incorporates the important aspects ofboth, and provides a rigorous foundation for the theory of perfect competition. Let us review the accomplishment.

We have presented a formal model of perfect competition with free entry, and U-shaped average costs. Price-taking is not a hypothesis of the theory, yet the actions of firms in perfectly competitive equilibrium are as if they cannot affect price. This is because perfect competition is defined by means of a sequence of economies $\{\mathcal{E}(\alpha)\}$ in which firms become arbitrarily small relative to the market.

Consistent with the demands of the formalist school of economic theory, we have presented general conditions (Corollary 1) which guarantee that a perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$ has at least one equilibrium. These conditions require that the consumer sector of the economy generates demand of the class that is generated by a single consumer. With this hypothesis, the entry (exit) of firms will reduce (increase) profit, and this condition is necessary for the existence of Cournot equilibrium with free entry in the economies $\mathcal{E}(\alpha)$. The perfectly competitive equilibrium of the perfectly competitive sequence $\{\mathcal{E}(\alpha)\}$ is defined as the limit of these Cournot equilibria.

Theorems 2 and 3 identify the perfectly competitive equilibria of the sequence $\{\mathcal{E}(\alpha)\}$ with the set of ADM equilibria of the limit economy $\mathcal{E}(0)$ which satisfy DSD (weak DSD). These theorems (and the example of Section V) tell us that the notion of perfectly competitive equilibrium presented here is more demanding than the notion of ADM equilibrium, and indicate precisely which ADM equilibria are in our sense perfectly competitive.

Theorems 4 and 5 concern the classical theorems ofwelfare economics in the context of U-shaped average costs and free entry. Perfectly competitive equilibria are optima; however, the theorem that optima are perfectly competitive equilibria requires more than the usual caveat "relative to some redistribution of purchasing power." In our analysis, it is generally required that several commodities be redistributed so that prices will provide the appropriate entry signals. In contrast with the standard analysis, the welfare theorems presented here do not assume that agents are price-takers, and they do not apply for the case in which "one buyer faces one seller." The hypotheses of the theorems allow firms to recognize their market power; however, they also require that firms be arbitrarily small relative to the market.

Section VII is devoted to a dynamic theory of entry with Marshallian features. The theory complements the usual tatonnement dynamics. Prices adjust quickly (instantaneously); however, the quantity adjustment of firms and the entry and exit of firms take place more slowly. Firms are myopic and do not consider the fact that prices will change with changes in the quantity actions of active firms and the entry of new firms. The main theorem shows that for the case in which the consumer sector of the economy generates demand of the class that is generated by a single consumer, the unique perfectly competitive equilibrium is globally stable.

Finally, we have shown how this new model of perfect competition can be adjusted to include the case of decreasing returns to scale in the aggregate, caused by variation in the quality of the resource that defines the production set of firms.

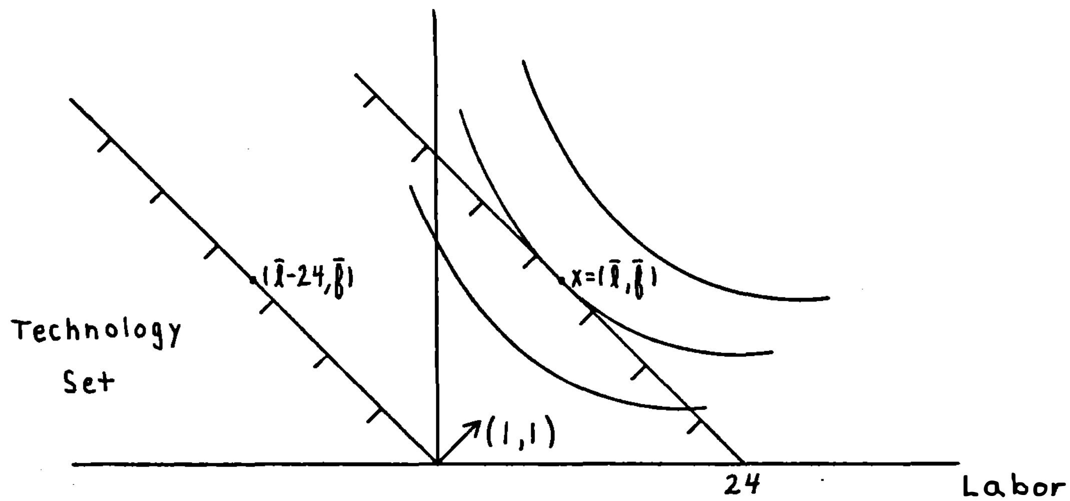


Figure 1

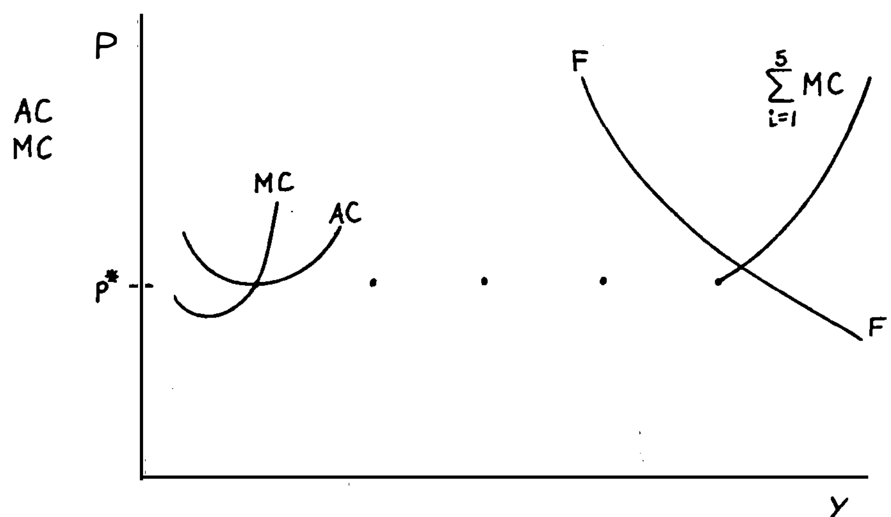


Figure 2

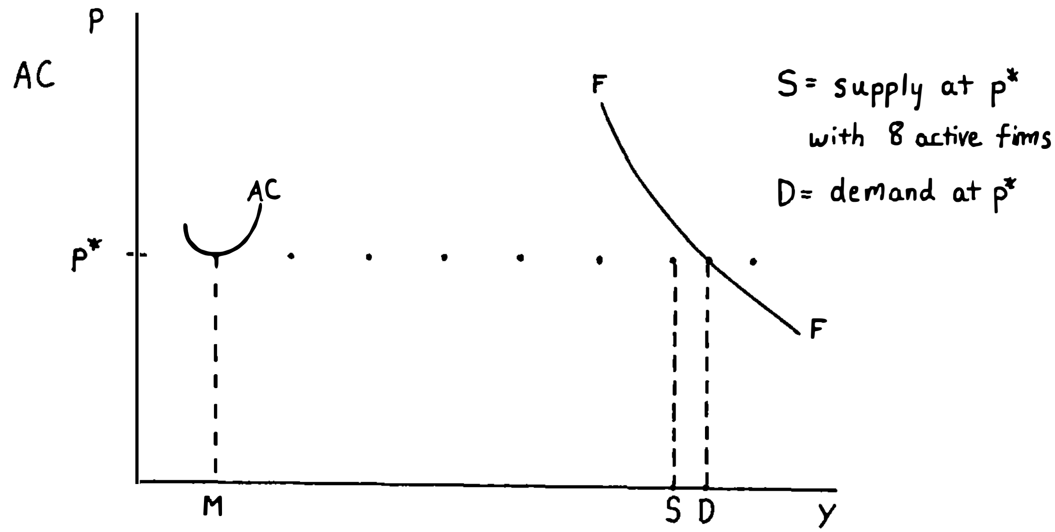


Figure 3a

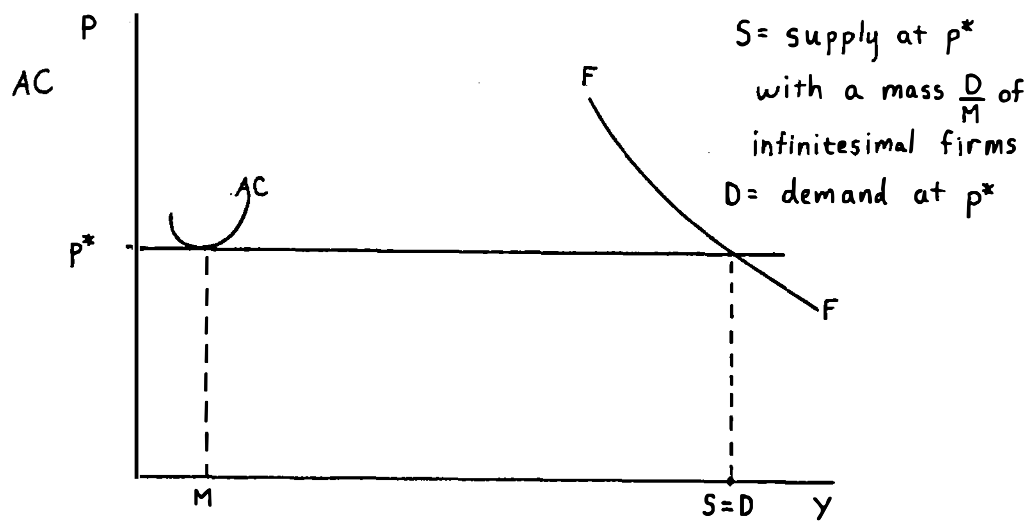


Figure 3b

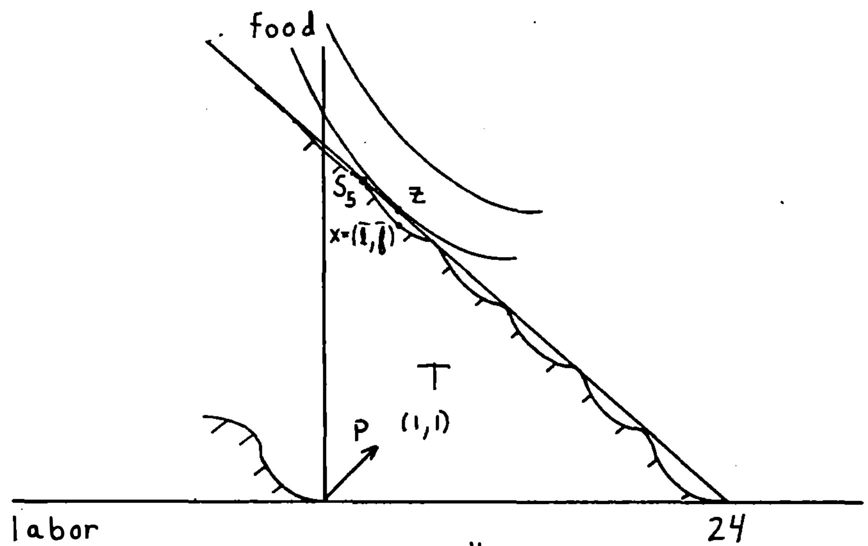


Figure 4

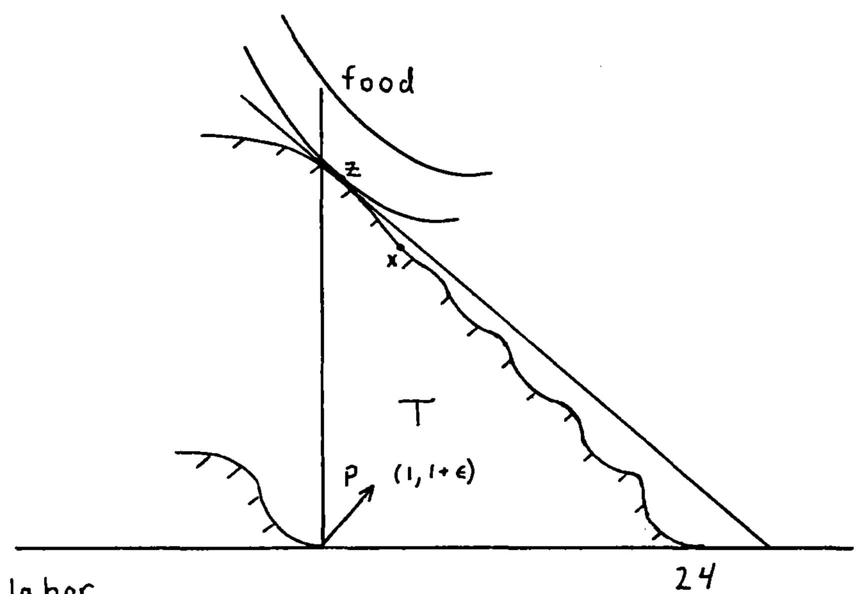


Figure 5

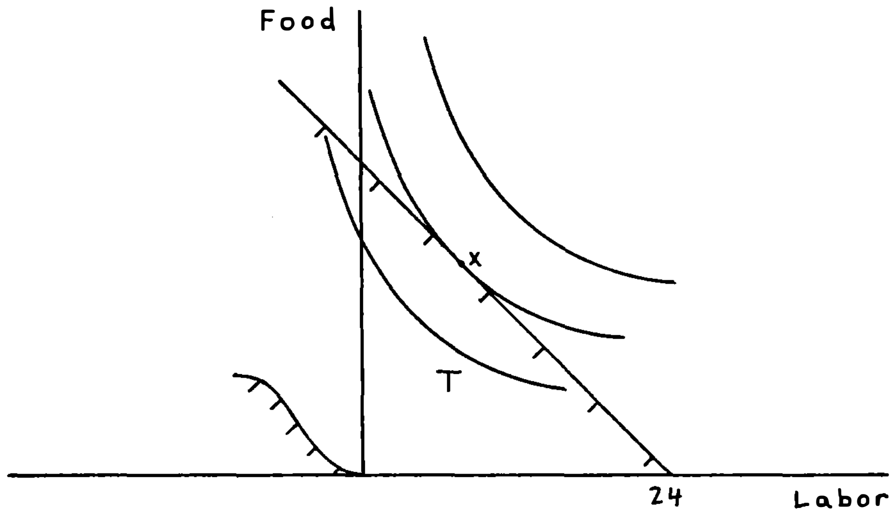


Figure 6

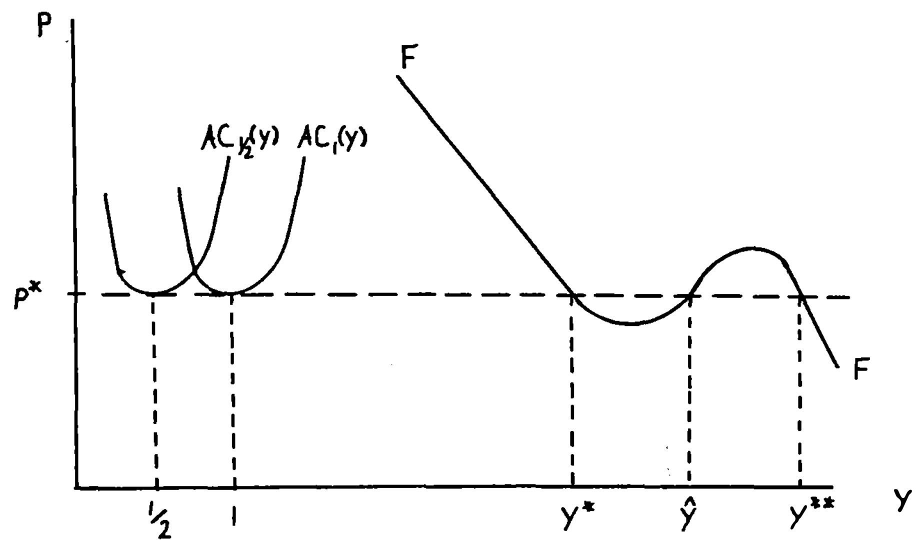
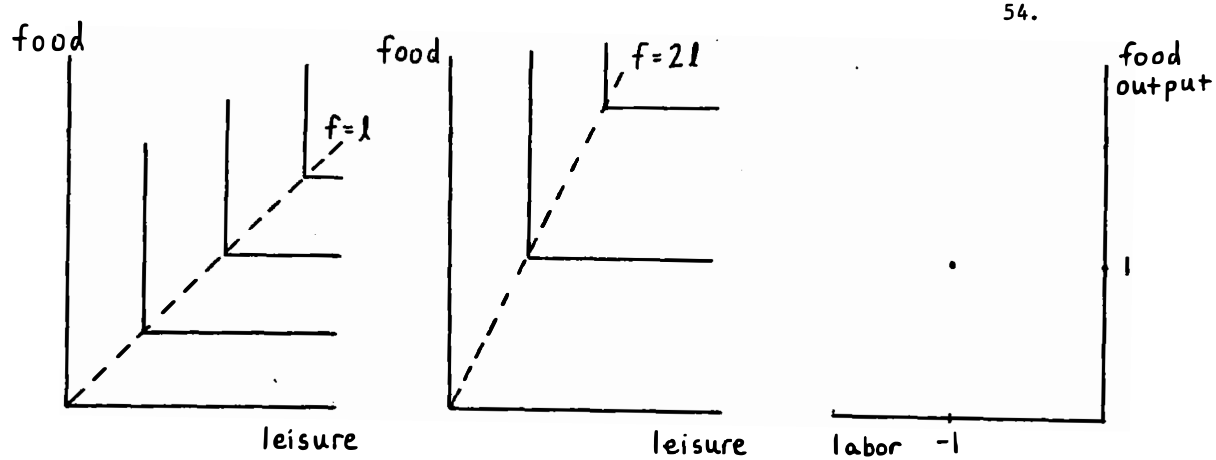


Figure 7

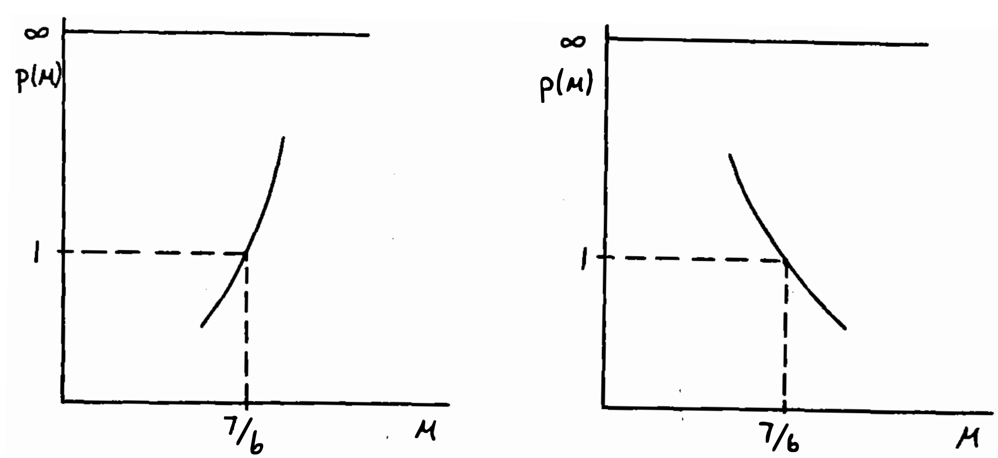


indifference curves for A

indifference curves for B

technology for a single infinitesimal firm

Figure 8



"inverse demand" correspondence for $\theta < 3/7$

"inverse demand" correspondence for $\theta > 3/7$

Figure 9

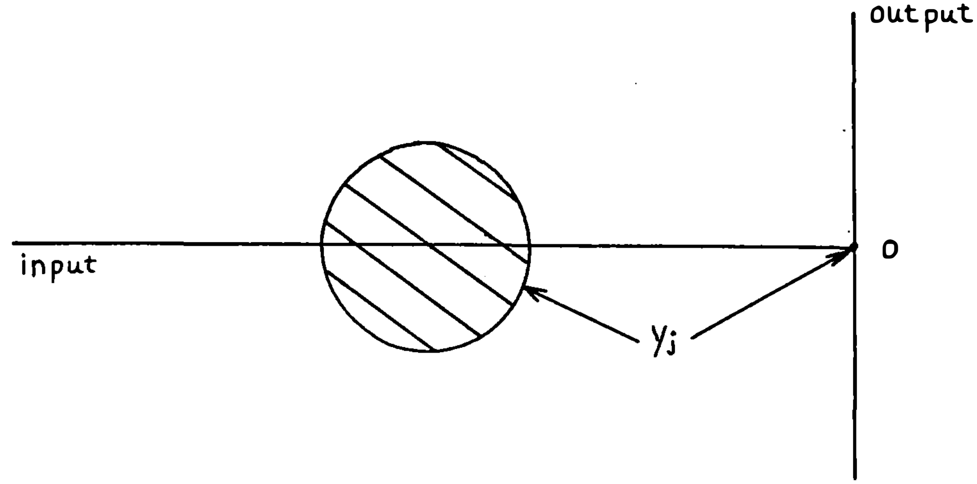


Figure 10

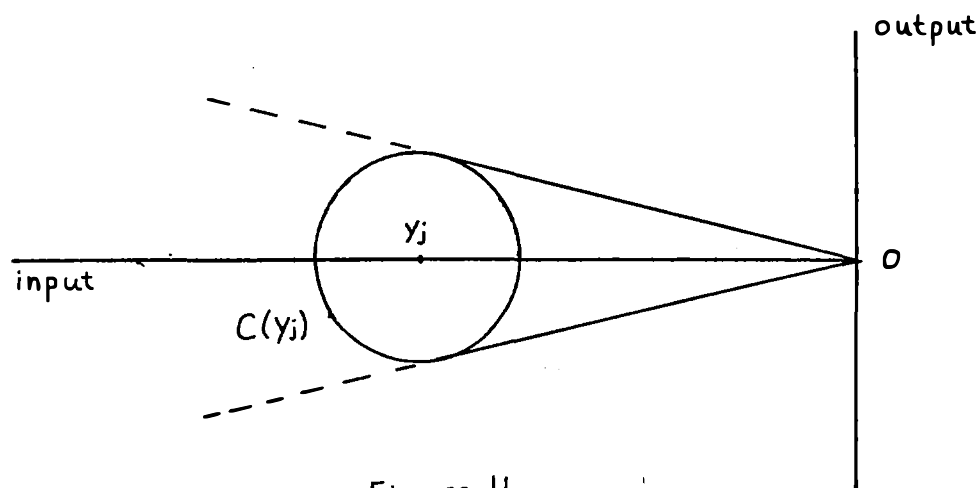
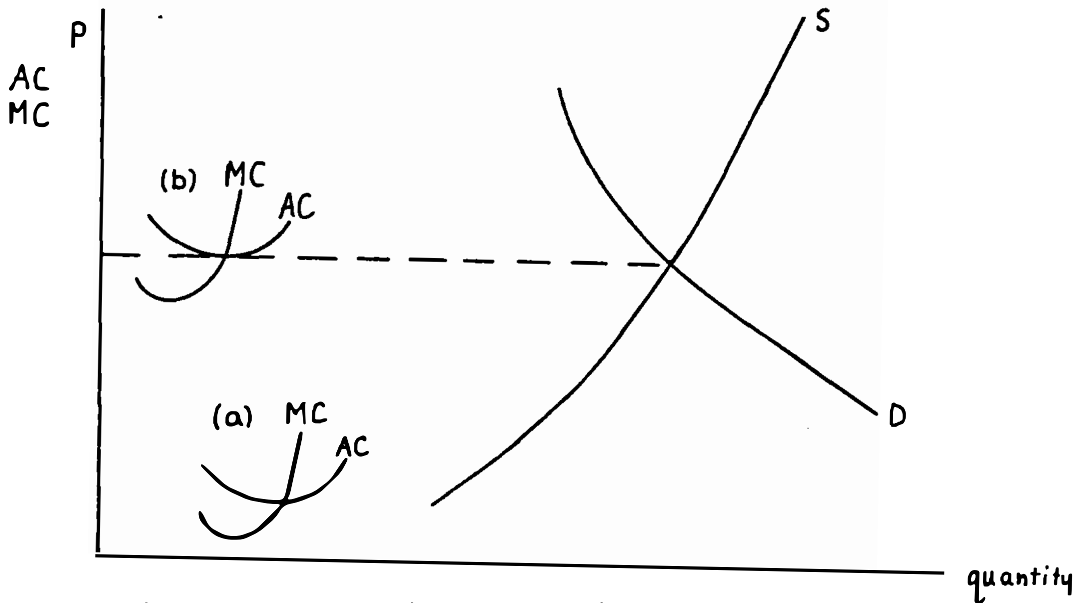


Figure 11

Figure 12



(a) costs of production on high quality land
(exclusive of rent)

(b) costs of production on marginal quality land
(exclusive of rent)

FOOTNOTES

*We gratefully acknowledge the support of the National Science Foundation and our universities.

[1]Frequently we will be concerned with the interpretation of the ADM model rather than the model itself. When we speak of the interpretation we will follow Arrow and Debreu and in particular have Debreu's Theory of Value [1959] in mind. McKenzie's interpretation of the model is different, and is in fact more in the spirit of the synthesis we advance here.

[2]Because price taking behavior is explained along the lines of the Cournot theory, what we offer might be better termed a Cournot-Marshall-ADM synthesis. It is also important to acknowledge that small efficient scale and free entry, while sufficient to guarantee price taking behavior in the limit, may not be necessary (see for example Spence [1983]). The extent to which free entry alone is sufficient (for the price taking conclusion) is a much debated issue, and not one on which our analysis sheds light. Suffice it to say that without the assumption of small efficient scale the case for price taking behavior becomes less compelling.

[3]See Novshek [1980], p. 473-486.

[4]Notice we have changed from a usual sequence index $\mathcal{E}(\alpha_k)$, k=1,2... to a continuous index $\mathcal{E}(\alpha)$, $\alpha \in (0,1]$. Thus we really have a net rather than a sequence.

[5]The results for mixed strategies are discussed in Section VIII.

[6]In general equilibrium several price vectors may clear markets for a single vector of strategies. Then F is a selection from the "inverse demand" correspondence. We assume all firms evaluate their profit using the same selection F, i.e. they agree on the prices that will arise in any situation.

[7]Robust sequences of Bertrand equilibria cannot be found.

[8]Labor has been chosen as numeraire in this example to make it similar to the partial equilibrium analysis of Section III. The price vector is $F(\mu) = (1,p(\mu))$. For μ near 7/6 there are two other exchange equilibria, with prices (1,0) and (0,1), but they do not satisfy the continuity and value condition $(F(7/6) = (1,1))$ imposed on F.

[9]This is an abuse of terminology since "industries" that are different (according to our definition) may use the same inputs to produce the same outputs. Here an "industry" is just the set of firms with identical technologies.

[10]Given action $y_{jt} \in Y_j \cup \{0\}$ for each $t \in [0,\infty)$, the aggregate production in "industry" j is the (Lebesgue) integral of the individual productions, $\int_0^\infty y_{jt}dt$. The integral $\int_0^\infty (Y_j \cup \{0\})dt$ is just the set of all possible aggregate productions $\int_0^\infty y_{jt}dt$ where $y_{jt} \in Y_j \cup \{0\}$ for all $t \in [0,\infty)$.

[11]Even after prices have been normalized, there may be several price vectors which clear markets given y. We assume one of the price vectors is selected by F.

[12]The regularity condition is similar to the condition in optimization problems that the second derivative is nonzero. It holds generically (i.e. except for a negligible set of cases). See Novshek and Sonnenschein [1978] for details of the regularity condition.

[13]By differentiably convex preferences we mean preferences which generate a twice continuously differentiable demand function.

[14]For every Pareto optimal allocation for $\mathcal{E}(0)$, say t, there exist sequences $\{\alpha_k\}$ and $\{s_k\}$ such that α_k converges to zero, s_k converges to t, and s_k is a feasible allocation for $\mathcal{E}(\alpha_k)$ for each k. For each $\mathcal{E}(\alpha_k)$ pick a Pareto optimal allocation, t_k, at which all consumers are at least as well off as at s_k, and take a convergent subsequence of the $\{t_k\}$. If t_0 is the limit of the subsequence, then every consumer is at least as well off at t_0 as at t (each consumer weakly prefers t_k to s_k and t_k converges to t_0 while s_k converges to t). Since t is Pareto optimal for $\mathcal{E}(0)$ and t_0 is feasible in $\mathcal{E}(0)$, strict convexity of preferences implies all consumers have the same allocation in t_0 as in t, and the combined aggregate production of all "industries" is the same in both allocations. Thus the Pareto optimal allocations for the sequence of perfectly competitive economies $\{\mathcal{E}(\alpha)\}$ are precisely the Pareto optimal allocations of the limit economy $\mathcal{E}(0)$ (with production aggregated across "industries").

[15]This is the $(\ell-1)\times(\ell-1)$ matrix with one row and column deleted.

[16]Observe that the aggregate production set is not assumed to be convex. Convexity holds only in the limit economy $\mathcal{E}(0)$, and there it is a consequence of infinitesimal efficient scale.

[17]In each Cournot equilibrium, F(y) is the perceived price vector that will result from production y. Since no firm actually changes output at an equilibrium, no post change tatonnement is required. The analysis could have been carried out with a subjective F(y), but we required that F(y) actually clear markets.

[18]It is not locally stable for either the first stage tatonnement or for the third stage entry dynamic using $p(\mu)$ given in Section V. See Svensson [1983].

[19]If $\eta_j = 0$ we assume $\dot{\eta}_j = \max p(t)\cdot(Y_j \cup \{0\})$.

[20]Recall that by differentiably convex preferences we mean preferences which generate a twice continuously differentiable demand function.

[21]"Globally," means starting at any aggregate output at which prices are defined.

[22]Increasing returns to scale, due to externalities that are internal to an industry but external to firms (see; for example, Novshek and Sonnenschein [1980]), can also be included in the analysis.

[23]A more restrictive dynamics is considered in which the active firms in any industry are all "more efficient" than any inactive firm, at every time t.

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