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A TWO-STAGE MODEL OF RESEARCH AND DEVELOPMENT WITH ENDOGENOUS SECOND-MOVER ADVANTAGES

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ABSTRACT

This paper describes a simple two-stage model of research and development, in which the "winner" of the research stage has the option of moving first in the development stage. Some peculiar results emerge: in equilibrium, the leader in the development stage invests less than each follower, and is consequently least likely to collect the patent. Moreover, the leader receives a lower expected payoff than each of the followers. Thus there are endogenous second-mover advantages. Using a game of timing (in which the identity of the Stackelberg leader is determined) to link the two stages, we find that firms face quite different incentives in the research stage. Although the leader invests less than each follower in the research stage as well, the leader enjoys higher expected revenue from the complete (two-stage) game than does each follower. The equilibrium is inefficient because there is a lag between the time at which research is completed and the time at which development is begun, and because aggregate investment is inefficiently (asymmetrically) distributed across firms.

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I. INTRODUCTION

It is commonplace in informal discussion to distinguish two stages in the process of perfecting a new product or technology -- "research" and "development." "Research" is frequently described as a relatively risky activity, whose results are generally unappropriable, while "development" consists of refining the results of research into a commercial, often patentable, product. We propose to develop a model of research and development which embodies the characterization above. We intend to analyze the possibilities for commitment generated by early research success or precedence with the original idea. For example, suppose that development can commence only upon successful completion of the research stage. Suppose in addition that research findings rapidly become common knowledge. Then the firm which succeeds first at research does not gain a perceptible "lead" on its rivals -- it possesses only one advantage -- it can "move first" in the development stage. That is, the winner in the research stage becomes (if it so chooses) a Stackelberg leader in the development stage. One of the more surprising results of this analysis is that the winner in the research stage <u>will not</u> choose to move first in the development stage. Although it prefers to be a leader than a Nash player, it would most prefer the role of <u>follower</u>. This implies that the first successful researcher may decide to delay rather than to commit itself. Thus in order for the first successful researcher to prefer to move first in the subsequent development stage, it is necessary for early research success to confer a considerable advantage in the development stage (either by providing a substantial lead over rivals or by increasing the likelihood of development success).

The fact that each firm prefers to move "second" in the development stage necessitates the examination of an intervening game which determines the identity of the first mover (for simplicity, we lump all followers together). Once equilibrium behavior is characterized at this level, we can consider the preceding (research) stage, again with the possibility that one firm, having first conceived the idea, may have "moved first" in the research stage. We find that the first mover in the research stage still invests less than each follower; however, the first mover receives a higher expected payoff than each follower.

This equilibrium is inefficient for two reasons. First, there is no merit in the leader/follower structure. Coordinated investment would imply immediate, simultaneous investment. Instead the equilibrium is characterized by the firm with the initial idea moving first (rather than revealing the idea); in the second stage, firms hesitate rather than beginning development immediately. Thus while new projects are researched promptly, they are not developed promptly, which is inefficient from either a joint profit maximizing or a welfare maximizing perspective. Second, there is no merit in the use of different investment levels by the leader and the followers. Given the symmetry of the problem as posed, any particular expected date of completion is achieved at least cost by having all firms invest at equal rates.

II. THE DEVELOPMENT STAGE

In typical dynamic programming style, we consider the last stage first. In particular, suppose that firm 1 is a Stackelberg leader in the development stage, while firms 2,3,...,n+1 are followers. Suppose that firm 1 has chosen to invest at the rate x_1, implying a development intensity of $h_1 = h(x_1)$, where h(.) is twice differentiable with $h'(.) > 0$ and $h''(.) < 0$. The development process is assumed to be stochastic with the probability of success by firm i by time t equal to $1 - e^{-h_i t}$, where $h_i = h(x_i)$ is the development intensity selected by firm i, at a cost of x_i dollars per unit time. This is a sort of "putty-clay" model of development -- the firm selects a scale at which to run its project, which it must continue to maintain unless it succeeds or gives up entirely.[1]

1. In this regard we follow Lee and Wilde (1980) rather than Loury (1979) and Dasgupta and Stiglitz (1980). Once selected, the costs x_i per unit time are fixed but not sunk. The constancy of the investment rate is assumed by Lee and Wilde (1980). In Reinganum (1982), it was shown that, in a purely Nash equilibrium context, allowing the investment rate to depend on time and a relevant state variable adds no generality when the hazard function h(.) depends only on current investment; in a stationary environment, a constant rate of investment will be an equilibrium strategy. This need not be true in the Stackelberg case; we specifically require constant investment by making this putty-clay assumption about the technology of the inventive process.

Successful development rewards the first successful firm with a patent of value P. Assuming that firm 1 has selected x_1, firms 2,3,...,n+1 play a simultaneous-move game in which each acts as a Nash player.

Definition 1. A strategy for firm j is a decision rule $x_j : [0,\infty) \rightarrow [0,\infty)$. The expression $x_j(x_1)$ represents the investment rate for firm j, conditional on that chosen by the leader, x_1.

Definition 2. The payoff to firm j which results from the development game is

$$V_j(x_j,a_j) = \int_0^\infty e^{-rt} e^{-(h(x_j) + a_j)t} [h(x_j)P - x_j]dt - K, \tag{1}$$

where $a_j = \sum_{i \neq j} h(x_i)$ and K is a fixed, nonrecoverable cost associated with development activity.

That is, firm i collects the patent at t with probability density $h(x_j)e^{-(h(x_j) + a_j)t}$ -- this is the probability that j succeeds at t and no other firm has yet done so. Firm j pays the development cost x_j so long as no firm has yet succeeded -- with probability $e^{-(h(x_j) + a_j)t}$. Integrating equation (1) above yields

$$V_j(x_j,a_j) = [h(x_j)P - x_j]/[r + h(x_j) + a_j] - K. \tag{2}$$

Suppose that there exist values of x_j such that $h(x_j)P - x_j > 0$. Thus there is at least a possibility of positive profit for firm j.

<u>Definition 3.</u> A <u>best response function</u> for firm j is a decision rule $\phi_j : [0,\infty) \rightarrow [0,\infty)$, where $\phi_j(a_j)$ represents the value of x_j which maximizes $V_j(x_j,a_j)$. That is, for each a_j, $V_j(\phi_j(a_j),a_j) \geq V_j(x_j,a_j)$ for all $x_j \in [0,\infty)$.

<u>Definition 4.</u> A <u>Nash equilibrium</u> for the followers, given x_1, is an n-tuple of strategies $(\mu_2(x_1),\ldots,\mu_{n+1}(x_1))$ such that

$$\mu_j(x_1) = \phi_j(h(x_1) + \sum_{i\neq j,1} h(\mu_i(x_1)))$$

for all $j = 2,3,\ldots,n+1$.

Consider the first- and second-order necessary conditions which characterize a unique interior best response for firm j.

$$\begin{aligned}\partial V_j(\phi_j,a_j)/\partial x_j &\propto (r + h(\phi_j) + a_j)(h'(\phi_j)P - 1) \\ &\quad - (h(\phi_j)P - \phi_j)h'(\phi_j) = 0,\end{aligned} \tag{3}$$

which can be simplified to yield

$$V_j(\phi_j,a_j) + K = (h'(\phi_j)P - 1)/h'(\phi_j). \tag{4}$$

Since there exist values of x_j which guarantee that $h(x_j)P - x_j > 0$, it follows that $V_j(\phi_j,a_j) + K > 0$. Thus $h'(\phi_j)P - 1 > 0$.

<u>Remark 1.</u> $h'(\phi_j)P - 1 > 0$ for all $a_j \in [0,\infty)$.

An alternative simplification of equation (3) yields

$$h'(\phi_j)(rP + a_jP + \phi_j) - (r + h(\phi_j) + a_j) = 0. \tag{5}$$

The second-order necessary condition

$$\partial^2 V_j(\phi_j,a_j)/\partial x_j^2 = h''(\phi_j)(rP + a_jP + \phi_j)/(r + h(\phi_j) + a_j)^2 \leq 0 \tag{6}$$

holds with strict inequality since $h''(.) < 0$.

<u>Lemma 1.</u> $\phi_j(a) \equiv \phi_i(a) \equiv \phi(a)$ for all $j,i \neq 1$. In addition, $\phi'(a) > 0$. That is, the form of the best response function is the same for all the followers, and an increase in aggregate rival investment stimulates an increase in the best response of firm j.

<u>Proof.</u> The first claim is apparent from the first-order condition (3) which implicitly defines the best response function. By the implicit function theorem,

$$\phi'(a) = -(h'(\phi)P - 1)/h''(\phi)(rP + aP + \phi) > 0$$

by Remark 1.

Q.E.D.

If a Nash equilibrium exists, then

$$\mu_j(x_1) = \phi(h(x_1) + \sum_{i\neq 1,j} h(\mu_i(x_1))).$$

<u>Lemma 2.</u> $\mu_j(x_1) \equiv \mu_i(x_1) \equiv \mu(x_1)$ for all $j,i \neq 1$. That is, the form of the equilibrium decision rules is the same for all followers.

<u>Proof.</u> Suppose $\mu_j(x_1) > \mu_i(x_1)$. Then $a_j(x_1) < a_i(x_1)$, where

$a_j(x_1) = h(x_1) + \sum_{k\neq 1,j} h(\mu_k(x_1))$ and $a_i(x_1) = h(x_1) + \sum_{k\neq 1,i} h(\mu_k(x_1))$.

Then

$$\mu_j(x_1) = \phi(a_j(x_1)) < \phi(a_i(x_1)) = \mu_i(x_1),$$

which is a contradiction. Thus $\mu_j(x_1) \equiv \mu_i(x_1) \equiv \mu(x_1)$ for all $j,i \neq 1$.

Q.E.D.

In what follows, we make use of the following modification of the Lee and Wilde (1980) stability condition for an n-firm Nash equilibrium.

Assumption 1. Suppose that for all x_1,

$$1 - \phi'(a(x_1))(n - 1)h'(\mu(x_1)) > 0,$$

where $a(x_1) = h(x_1) + (n - 1)h(\mu(x_1))$.

The content of Assumption 1 is that if all other Nash equilibrium players except firm j increase their investment by a fixed amount, then firm j's best response is to increase its own investment by less than that amount. To see this, suppose that every other (follower) firm increases its investment by $d\mu$; x_1 is regarded as fixed throughout. Then rewrite the expression above as $d\phi < da/(n - 1)h'(\mu)$. Since $a = h(x_1) + (n - 1)h(\mu)$, $da = (n - 1)h'(\mu)d\mu$. Substituting this for da above yields $d\phi < d\mu$.

Lemma 3. Under Assumption 1, $\mu'(x_1) > 0$. Thus the greater is the investment rate of the leader firm, the greater is individual and aggregate follower investment.

Proof.

$$\mu'(x_1) = \phi'(a(x_1))(h'(x_1) + (n - 1)h'(\mu)\mu'(x_1))$$

so

$$\mu'(x_1) = \phi'(a(x_1))h'(x_1)/(1 - \phi'(a(x_1))(n - 1)h'(\mu(x_1))) > 0$$

by Assumption 1 and Lemma 1.

Q.E.D.

This characterizes the Nash equilibrium among the followers. Each follower firm invests more the greater is the aggregate investment by others (i.e., $\phi'(.) > 0$). Moreover, in equilibrium, each firm invests more the greater is the investment by the Stackelberg leader (i.e., $\mu'(.) > 0$).

Now consider the behavior of the leader. Firm 1 recognizes that its followers will each invest $\mu(x_1)$ if it invests x_1.

Definition 5. A strategy for firm 1 is a number $x_1 \in [0,\infty)$. Firm 1's payoff is

$$\begin{aligned} V_1(x_1) &= \int_0^\infty e^{-rt} e^{-(nh(\mu(x_1)) + h(x_1))t}(h(x_1)P - x_1)dt - K \\ &= (h(x_1)P - x_1)/(r + h(x_1) + nh(\mu(x_1))) - K. \end{aligned} \tag{7}$$

Firm 1 wishes to maximize its payoff, taking into account the dependence of its rivals' investment decisions upon its own strategy. In general, there is no need for this problem to be particularly well-behaved; however, we will proceed under the assumption that the solution can be characterized as an interior stationary point. The first-order necessary condition for an interior maximum is

$$V_1'(x_1^*) \propto (r + h(x_1^*) + nh(\mu(x_1^*)))(h'(x_1^*)P - 1) - (h(x_1^*)P - x_1^*)(h'(x_1^*) + nh'(\mu(x_1^*))\mu'(x_1^*)) = 0. \tag{8}$$

Simplification of equation (8) yields

$$V_1(x_1^*) + K = (h'(x_1^*)P - 1)/(h'(x_1^*) + nh'(\mu(x_1^*))\mu'(x_1^*)). \tag{9}$$

Again, firm 1 can guarantee that $V_1(x_1) + K > 0$, so the Remark below follows.

<u>Remark 2.</u> $h'(x_1^*)P - 1 > 0$.

Alternatively, equation (8) can be simplified to obtain

$$h'(x_1^*)(rP + nh'(\mu(x_1^*))P + x_1^*) - (r + h(x_1^*) + nh(\mu(x_1^*))) - (h(x_1^*)P - x_1^*)(nh'(\mu(x_1^*))\mu'(x_1^*)) = 0. \tag{10}$$

The second-order necessary condition for an interior maximum, that $V_1''(x_1^*) \leq 0$, is rather complicated, and will be assumed to hold with strict inequality.

<u>Proposition 1.</u> $x_1^* < x_j^* = \mu(x_1^*)$ for all $j \neq 1$. That is, the first mover invests strictly less than each follower.

<u>Proof.</u> Let

$$g(x) = h'(x_1^*)(rP + (n-1)h(\mu(x_1^*))P + h(x)P + x_1^*) - h'(x)(rP + (n-1)h(\mu(x_1^*))P + h(x_1^*)P + x). \tag{11}$$

Note that $g(x_1^*) = 0$, while $g(\mu(x_1^*)) = (h(x_1^*)P - x_1^*)nh'(\mu(x_1^*))\mu'(x_1^*)$ by equation (10). Since there exist values of x such that $h(x)P - x > 0$, it follows that $h(x_1^*)P - x_1^* > 0$. Thus $g(\mu(x_1^*)) > 0$. Moreover,

$$g'(x) = h'(x)(h'(x_1^*)P - 1) - h''(x)(rP + (n-1)h(\mu(x_1^*))P + h(x_1^*)P + x_1^*) \tag{12}$$

which is strictly positive by Remark 2 and the fact that $h''(.) < 0$. Thus $x_1^* < x_j^* \equiv \mu(x_1^*)$.

Q.E.D.

Thus the firm which moves first in the development phase is <u>least</u> likely to succeed first in the development phase, and is hence <u>least</u> likely to collect the ultimate reward.

An even more surprising result is that, under the assumptions of this model, the firm which moves first has a lower expected profit than every firm which moves later.

<u>Proposition 2.</u> Let V^L and V^F denote the equilibrium payoffs to the leader and each follower, respectively. Then $V^L < V^F$.

<u>Proof.</u>

$$V^L \equiv (h(x_1^*)P - x_1^*)/(r + h(x_1^*) + nh(\mu(x_1^*))) - K$$

while

$$V^F \equiv (h(\mu(x_1^*))P - \mu(x_1^*))/(r + h(x_1^*) + nh(\mu(x_1^*))) - K.$$

Then $V^L < V^F$ if and only if $h(x_1^*)P - x_1^* < h(\mu(x_1^*))P - \mu(x_1^*)$. Since $h'(x)P - 1 > 0$ at both x_1^* and at $\mu(x_1^*)$, and since $h''(x) < 0$, it follows that $h'(x)P - 1 > 0$ for all x between x_1^* and $\mu(x_1^*)$. Since $x_1^* < \mu(x_1^*)$ and $h(x)P - x$ is strictly increasing on $[x_1^*,\mu(x_1^*)]$, it follows that $h(x_1^*)P - x_1^* < h(\mu(x_1^*))P - \mu(x_1^*)$. Thus $V^L < V^F$.

Q.E.D.

This peculiar result is due to the fact that the first mover provides the followers with a public good -- by holding down its own development intensity, it reduces each follower's Nash rivals' intensities as well. This is quite the opposite result from the usual quantity-setting oligopoly equilibrium in which a Stackelberg leader is better off than the followers. They differ because an increase in aggregate output decreases the best response for each firm. Thus the incentives work in the opposite direction from those encountered in this model.[2] It should be noted that the results described above are likely to be sensitive to the specification of R and D costs. For instance, in the models of Loury (1979) and Dasgupta and Stiglitz (1980), costs are lump-sum rather than in flow terms. Under this assumption, best response functions are decreasing at equilibrium; since a crucial element of the preceding analysis is the fact that best response functions are increasing, we might suspect different results under this alternative specification. However, the lump-sum cost formulation implies an even more extreme version of commitment;

once the level of investment is selected, all R and D costs are henceforth sunk.

Using an assumption which is a slight extension of Assumption 1, we can compare equilibrium investment in the leader/follower framework with symmetric equilibrium investment in a simultaneous-move framework. Let x^N represent individual investment in a symmetric (n+1)-firm Nash equilibrium.

<u>Assumption 1'.</u> Suppose that for all x_1,

$$1 - \phi'(a(x_1))[h'(x_1) + (n-1)h'(\mu(x_1))] > 0,$$

where $a(x_1) = h(x_1) + (n - 1)h(\mu(x_1))$.

Assumption 1' can be interpreted much like Assumption 1: in this case, if <u>all</u> other firms except firm j (not just followers) increase their investments by the same amount, then firm j's best response is to increase its own investment by less than that amount. To formalize this notion, rewrite the condition above as

2. In a labor market model with adverse selection, Weiss and Guasch (1980) derive the result that it is preferable to enter a labor market later rather than earlier. This is because less able workers accept the lower offers made by earlier firms, and thus a better average quality of worker remains for later firms. The result of the present paper is not due to adverse selection. Similarly, Hendricks (1982) and Sadanand (1982) also discover circumstances in which firms may prefer to move later rather than earlier; this is due to the fact that common uncertainty is resolved by the first mover (Hendricks), or is resolved between moves (Sadanand). Since ours is a deterministic model, the result is unrelated to the resolution of any form of uncertainty. This result is, however, similar to the equilibrium consequences of strategic pricing in differentiated-products oligopolies, and to the equilibrium outcomes in spatial models of electoral competition.

$$d\phi < da/[h'(x_1) + (n-1)h'(\mu(x_1))].$$

Now if all other players except firm j increase their investments by the same amount dx, then $da = [h'(x_1) + (n-1)h'(\mu(x_1))]dx$. Substituting this expression for da above yields $d\phi < dx$.

Proposition 3. Under Assumption 1', $x_1^* < x^N$ and $\mu(x_1^*) < x^N$. That is, each firm invests less in the leader/follower framework than in a symmetric (n+1)-firm Nash equilibrium.

Proof. At a symmetric equilibrium, it must be that $x^N = \mu(x^N)$. That is, firm 1 plays the same strategy as every other firm. From Proposition 1 we know that $x_1^* < \mu(x_1^*)$. Define the function $\alpha(x_1) = x_1 - \mu(x_1)$. Then $\alpha(x_1^*) < 0$, $\alpha(x^N) = 0$, and

$$\begin{aligned}\alpha'(x_1) &= 1 - \mu'(x_1)\\ &= 1 - \phi'(a(x_a))h'(x_1)/(1 - \phi'(a(x_1))(n-1)h'(\mu(x_1)))\\ &= \Big[1 - \phi'(a(x_1))[h'(x_1) + (n-1)h'(\mu(x_1))]\Big]/\Big[1 - \phi'(a(x_1)(n-1)h'(\mu(x_1))\Big].\end{aligned}$$

Both numerator and denominator are strictly positive by Assumption 1'. Thus $x_1^* < x^N$. Moreover, since $\mu'(x_1) > 0$, $\mu(x_1^*) < \mu(x^N)$.

Q.E.D.

We know from previous work (see e.g., Lee and Wilde (1980), Theorem 2) that firms invest at inefficiently high rates in a Nash equilibrium. Thus the leader/follower structure may actually be welfare-improving relative to the symmetric (n+1)-firm Nash equilibrium.

III. THE TIMING GAME

The first successful researcher has the option to be a Stackelberg leader in the subsequent development stage. While this option is preferable to being just another Nash player, it is not preferable to being a follower. Let V^N represent the Nash payoff in an n+1-firm simultaneous-move game. Then $V^F > V^L > V^N$. Thus the first successful researcher may elect to forego the option of going first, in the hope that some other firm will do so.

The fact that $V^F > V^L > V^N$ necessitates the analysis of a game which intervenes between the research and develpment stages; in this game, the timing of play is determined. That is, whether there is a leader/follower structure or a simultaneous-move structure.

Notice that there is no point in delaying investment after the first mover has moved (assuming all followers move simultaneously). Thus all followers move simultaneously immediately after the first mover selects its investment. Thus the timing game essentially ends with the first move.[3]

It is easy to show that there is no pure strategy Nash equilibrium in this game. For instance, there is no symmetric equilibrium, because suppose that everyone else plays "move at $\hat{t}$ if no one else has yet moved (and choose the n+1-firm Nash equilibrium

3. Guasch and Weiss (1980) examine a two-firm timing game which is formally equivalent to this one (if we assume two firms rather than an arbitrary finite number). Thus the analysis of this section, though independently done, is only marginally more general than their work. The results are included to provide continuity and intuition for how the research and development stages are linked together by this intervening game of timing.

investment level)." Then firm i's payoff if it moves at t_i is

$$V^L e^{-rt_i} \quad \text{if } t_i < \hat{t}$$

$$V^N e^{-rt_i} \quad \text{if } t_i = \hat{t}$$

$$V^F e^{-rt_i} \quad \text{if } t_i > \hat{t}$$

Either $t_i = \hat{t} + \varepsilon$ or $t_i = \hat{t} - \varepsilon$ would be preferred to $t_i = \hat{t}$ for sufficiently small ε, since $V^F > V^N$ and $V^L > V^N$. Thus there can be no symmetric equilibrium in pure strategies.

Now suppose that there is an asymmetric equilibrium, with firm i the leader. Let t_{-i} be the time selected by everyone else. Then $t_i < t_{-i}$, say $t_{-i} = t_i + \varepsilon$. Firm i's equilibrium payoff is $V^L e^{-rt_i}$. But since $V^F > V^L$, $t_i = t_{-i} + \varepsilon'$ is strictly preferred to $t_i = t_{-i} - \varepsilon$ for sufficiently small ε'. Thus $t_i < t_{-i}$ can't be part of an asymmetric Nash equilibrium. Since this argument works for all i, there can be no asymmetric pure strategy equilibrium, and hence no pure strategy equilibrium at all.

There is, however, a unique symmetric Nash equilibrium in the space of continuously differentiable mixed strategies. To see this, let $1 - F_i(t)$ be the probability that i has not moved by t. This is the planned distribution; once another firm actually moves, the followers all follow instantly. Thus the payoff to firm i when i uses F_i and the aggregate probability that none of the rivals has moved by t is $1 - G(t)$, is

$$U_i(F_i,G) = \int_0^\infty e^{-rt}[V^L f_i(t)(1 - G(t)) + V^F g(t)(1 - F_i(t))]dt \qquad (13)$$

where $F_i(0) = 0$, $f_i(t) \geq 0$, $\lim_{t \to \infty} F_i(t) \leq 1$, and, similarly, $G(0) = 0$, $g(t) \geq 0$, and $\lim_{t \to \infty} G(t) \leq 1$. That is, firm i selects its hazard rate, $f_i(t)/(1 - F_i(t))$, which is its conditional probability density of moving at t, given that it has not moved by t. The probability that it moves first at t is then this hazard rate times the probability that it has not yet moved, times the probability that no other firm has yet moved. This product is $[f_i(t)/(1 - F_i(t))](1 - F_i(t))(1 - G(t))$, which simplifies to $f_i(t)(1 - G(t))$, as in the first term of equation (13) above. The second term represents the probability that some other firm moves first at date t, in which case firm i becomes a follower in the development stage. Using the calculus of variations, firm i chooses $F_i(t)$ subject to the contraints on it, so as to maximize U_i, taking G(t) as given. The Euler equation for this problem is

$$-g(t)e^{-rt}V^F = d[e^{-rt}V^L(1 - G(t))]/dt \qquad (14)$$

or, performing the differentiation,

$$g(t)/(1 - G(t)) = rV^L/(V^F - V^L). \qquad (15)$$

This ordinary differential equation has the unique solution

$$G(t) = 1 - \exp\{-[rV^L/(V^F - V^L)]t\} \qquad (16)$$

through the point $G(0) = 0$. If we assume a symmetric equilibrium (i.e., $F_i(t) \equiv F^*(t)$ for all i), then

$$1 - G^*(t) = (1 - F^*(t))^n = \exp\{-[rV^L/(V^F - V^L)]t\}.$$

Proposition 4. In a symmetric continuously differentiable mixed-strategy equilibrium, each firm will randomize using an exponential distribution over the date of its move.

$$F^*(t) = 1 - \exp\{-[rV^L/n(V^F - V^L)]t\}. \tag{17}$$

It will follow this plan unless another firm moves first. If this occurs, it follows immediately. Thus the distribution of the date of the first move is exponential with parameter $(n+1)rV^L/n(V^F - V^L)$. The expected date of the first move is the reciprocal

$$E[\min_i\{t_i\}] = n(V^F - V^L)/(n+1)rV^L.$$

This waiting game eventually ends, and a first mover is thus endogenously determined. Since the game is symmetric, all firms *ex ante* expect a payoff of $\bar{V} \equiv U_i(F^*,G^*)$.

IV. THE RESEARCH PHASE

The payoff to successful research then is $\bar{V}$. Moreover, this is received *regardless* of who succeeds first. This creates quite different incentives relative to the those of the development stage. Essentially the intervening "waiting game" smooths out any asymmetries

in the subsequent payoffs. Suppose the technology for producing likelihood of success at research is also exponential. Let $\lambda_j = \lambda(y_j)$ be firm j's "research intensity," sustained at a cost of y_j dollars per unit time. Then the probability that firm j successfully completes research by time t is $1 - e^{-\lambda(y_j)t}$. Suppose that $\lambda(.)$ is nonnegative and twice differentiable with $\lambda'(.) > 0$ and $\lambda''(.) < 0$. Again there may be a "first mover," a firm which first conceives of the idea, but which anticipates imitation or further entry into the research phase.

Then the payoff to each follower firm is

$$W_j(y_j,b_j) = \int_0^\infty e^{-rt}e^{-(\lambda(y_j) + b_j)t}[(\lambda(y_j) + b_j)\bar{V} - y_j]dt - C \tag{18}$$

where $b_j = \sum_{i\neq j}\lambda(y_i)$, and C is a fixed, nonrecoverable research cost. Integrating equation (18) yields

$$W_j(y_j,b_j) = [(\lambda(y_j) + b_j)\bar{V} - y_j]/(r + \lambda(y_j) + b_j) - C. \tag{19}$$

The definitions of a strategy, a best response and a Nash equilibrium for the followers, given y_1, are analogous to those of Section II. Let $y_j(y_1)$ denote a strategy for firm j, $\varphi_j(b_j)$ j's best response function, and $(\gamma_j(y_1))_{j=2}^{n+1}$ a Nash equilibrium for the followers, given y_1.

The first-order necessary condition for an interior best response for firm j is

$$\partial W_j(\varphi_j,b_j)/\partial y_j \propto (r + \lambda(\varphi_j) + b_j)(\lambda'(\varphi_j)\bar{V} - 1)$$
$$- [(\lambda(\varphi_j) + b_j)\bar{V} - \varphi_j]\lambda'(\varphi_j) = 0 \qquad (20)$$

which can be simplified to yield

$$W_j(\varphi_j,b_j) + C = (\lambda'(\varphi_j)\bar{V} - 1)/\lambda'(\varphi_j). \qquad (21)$$

Again, since there exist values of y_j which ensure that $\lambda(y_j)\bar{V} - y_j > 0$, it follows that $W_j(\varphi_j,b_j) + C > 0$. Then we have the following remark.

Remark 3. $\lambda'(\varphi_j)\bar{V} - 1 > 0$.

Alternatively, we could rewrite (20) as

$$\lambda'(\varphi_j)(r\bar{V} + \varphi_j) - (r + \lambda(\varphi_j) + b_j) = 0. \qquad (22)$$

The second-order necessary condition

$$\partial^2 W_j(\varphi_j,b_j)/\partial y_j^2 = \lambda''(\varphi_j)(r\bar{V} + \varphi_j)/(r + \lambda(\varphi_j) + b_j)^2 \leq 0$$

holds with strict inequality since $\lambda''(.) < 0$.

Lemma 4. $\varphi_j(.) \equiv \varphi_i(.) \equiv \varphi(.)$ for all $i,j\neq 1$, and $\varphi'(.) < 0$. That is, the best response function for the research phase is the same for all followers, and an increase in aggregate rival research results in a decrease in the best response of a given firm.

Proof. The first claim is apparent from equation (20), which

implicitly defines $\varphi_j(.)$. The second claim follows by the implicit function theorem. Differentiating (20), we can solve for

$$\varphi'(b_j) = \bar{V}/\lambda''(\varphi)(r\bar{V} + \varphi) < 0.$$

Q.E.D.

This is precisely the opposite result from the development stage, in which $\phi'(a) > 0$. Now firms are indifferent about who succeeds first; they only prefer that success occur earlier rather than later. Thus the greater the investment undertaken by everyone else, the less firm j is willing to invest.

As before, in a Nash equilibrium

$$\gamma_j(y_1) = \varphi(\lambda(y_1) + \sum_{i\neq j,1}\lambda(\gamma_i(y_1))).$$

Assumption 2. Suppose $\lambda''(\gamma)(\gamma + r\bar{V}) + \lambda'(\gamma) < 0$ for all $\gamma \in [0,\infty)$.

Lemma 5. If Assumption 2 holds, then $\gamma_j(y_1) \equiv \gamma_i(y_1) \equiv \gamma(y_1)$ for all $i,j\neq 1$. That is, the Nash equilibrium is symmetric among the follower firms.

Proof. Let

$$g(\gamma) = \lambda'(\gamma)(r\bar{V} + \gamma) - \lambda'(\gamma_j(y_1))(r\bar{V} + \gamma_j(y_1)).$$

By equation (22), $g(\gamma_i(y_1)) = 0$, while $g(\gamma_j(y_1)) = 0$ by inspection. Since $g'(\gamma) = \lambda''(\gamma)(r\bar{V} + \gamma) + \lambda'(\gamma) < 0$, it follows that $\gamma_j(y_1) = \gamma_i(y_1) = \gamma(y_1)$ for all $i,j \neq 1$.

Q.E.D.

Lemma 6. $\gamma'(y_1) < 0$ for all y_1. That is, the greater is the leader's investment, the smaller is that of each follower.

Proof. Recall that

$$\gamma(y_1) = \varphi(\lambda(y_1) + (n-1)\lambda(\gamma(y_1))).$$

Thus

$$\gamma'(y_1) = \varphi'(b(y_1))\lambda'(y_1)/(1 - \varphi'(b(y_1))(n-1)\lambda'(\gamma)) < 0$$

since $\varphi'(.) < 0$.

Q.E.D.

Again, this is quite opposite to the results obtained in the development stage. To summarize the research stage among the followers: each follower firm invests the amount $\gamma(y_1)$ on research, this amount being smaller the greater is y_1.

Now consider the first mover's problem. As before the leader takes account of its impact upon the followers' subsequent decisions.

$$W_1(y_1) = \int_0^\infty e^{-rt} e^{-(\lambda(y_1) + n\lambda(\gamma(y_1))t} [\lambda(y_1)\bar{V} + n\lambda(\gamma(y_1)\bar{V} - y_1]dt - C$$
$$= [\lambda(y_1)\bar{V} + n\lambda(\gamma(y_1))\bar{V} - y_1]/(r + \lambda(y_1) + n\lambda(\gamma(y_1))) - C. \quad (23)$$

A necessary condition for an interior maxiumum of W_1 is

$$W_1'(y_1^*) \propto (r + \lambda(y_1^*) + n\lambda(\gamma(y_1^*))[\lambda'(y_1^*)\bar{V} + n\lambda'(\gamma)\gamma'(y_1^*)\bar{V} - 1]$$

$$- [\lambda(y_1^*)\bar{V} + n\lambda(\gamma(y_1^*)\bar{V} - y_1^*](\lambda'(y_1^*) + n\lambda'(\gamma)\gamma'(y_1^*)) = 0. \quad (24)$$

We can simplify equation (24) to obtain

$$W_1(y_1^*) + C = [\lambda'(y_1^*)\bar{V} + n\lambda'(\gamma)\gamma'(y_1^*)\bar{V} - 1]/(\lambda'(y_1^*) + n\lambda'(\gamma)\gamma'(y_1^*))$$
$$= \bar{V} - 1/(\lambda'(y_1^*) + n\lambda'(\gamma)\gamma'(y_1^*)).$$

Remark 4. Notice that since $W_1 + C < \bar{V}$, it is necessary that $\lambda'(y_1^*) + n\lambda'(\gamma)\gamma'(y_1^*) > 0$.

Equation (24) may also be simplified to yield

$$\lambda'(y_1^*)(r\bar{V} + y_1^*) + n\lambda'(\gamma)\gamma'(y_1^*)(r\bar{V} + y_1^*)$$
$$- (r + \lambda(y_1^*) + n\lambda(\gamma(y_1^*))) = 0. \quad (25)$$

Proposition 5. If Assumption 2 holds, then $y_1^* < \gamma(y_1^*)$. That is, the leader invests less than each follower.

Proof. Let

$$g(\gamma) = \lambda'(\gamma)(r\bar{V} + \gamma) - \lambda'(y_1^*)(r\bar{V} + y_1^*).$$

Inspection indicates that $g(y_1^*) = 0$, while $g(\gamma(y_1^*)) = n\lambda'(\gamma)\gamma'(y_1^*)(r\bar{V} + y_1^*) < 0$. Moreover, $g'(\gamma) = \lambda''(\gamma)(r\bar{V} + \gamma) + \lambda'(\gamma) < 0$, under Assumption 2. Therefore, $y_1^* < \gamma(y_1^*)$.

Q.E.D.

Thus the leader still invests less than each follower. However, in the two-stage game, the leader is better off than each follower.

<u>Proposition 6.</u> If Assumption 2 holds, then $W^L > W^F$.

<u>Proof.</u> Using the definitions of the individual firm payoffs, it is clear that $W^L > W^F$ so long as $y_1^* < \gamma(y_1^*)$. Proposition 5 says that this is true, if Assumption 2 holds.

Q.E.D.

Again we can compare equilibrium investment in the leader/follower framework with that of an (n+1)-firm Nash equilibrium. Let y^N denote this symmetric equilibrium rate of investment.

<u>Proposition 7.</u> $y_1^* < y^N$ and $\gamma(y_1^*) > y^N$. That is, the leader invests less than it would in a Nash equilibrium, while each follower invests more than it would in a Nash equilibrium.

<u>Proof.</u> At a symmetric (n+1)-firm Nash equilibrium, $y^N = \gamma(y^N)$. From Proposition 5, $y_1^* < \gamma(y_1^*)$. Let $\beta(y_1) = y_1 - \gamma(y_1)$. Then $\beta(y_1^*) < 0$, $\beta(y^N) = 0$, and $\beta'(y_1) = 1 - \gamma'(y_1) > 0$ since $\gamma'(y_1) < 0$ by Lemma 6. Thus $y_1^* < y^N$. Since $\gamma'(y_1) < 0$, $\gamma(y_1^*) > \gamma(y^N) = y^N$.

Q.E.D.

Again the analysis of the research stage differs from that of the development stage. In the development stage both leader and followers invested less than the symmetric Nash equilibrium investment rate, while in the research stage the leader invests less and the followers more than they would in a symmetric Nash equilibrium.

V. CONCLUSIONS

We have developed a simple two-stage model of research and development, in which the "winner" of the research stage has the option of moving first in the development stage. We have found that an increase in the rate of aggregate expenditure of rivals induces each firm to increase its (best response) rate of investment. We discovered the somewhat surprising result that, in equilibrium, the leader in the development stage invests less than each follower, and is consequently least likely to collect the patent. Moreover, the leader receives a lower expected payoff than each of the followers. Thus there are endogenous second-mover advantages.[4]

Using a game of timing (in which the identity of the Stackelberg leader is determined) to link the two stages, we have found that firms face quite different incentives in the research stage. In this stage, an increase in the rate of aggregate expenditure of rivals induces each firm to decrease its (best response) rate of investment. Although the leader invests less than each follower in the research stage as well, the leader enjoys higher expected revenue from the

4. We also noted the likely sensitivity of this result to at least two of our assumptions. First, if the R and D costs are of the lump-sum variety, it may be that committing a large investment makes follower firms invest less (because best response functions are decreasing at equilibrium with lump-sum costs). Second, we assumed that the first successful researcher is no more efficient at R and D and enjoys essentially no head start on development. It seems clear that advantages of sufficient magnitude in either of these directions will make it preferable to lead than to follow.

complete (two-stage) game than does each follower.

The equilibrium described above is manifestly inefficient from the perspective of joint profit maximizing (or welfare maximizing, if the patent value is regarded as the social value of the innovation). First, there is no merit in the leader/follower structure. If firms are coordinating their investment, they may as well invest simultaneously. In the second stage, noncooperative firms hesitate rather than beginning development immediately. This hesitation is inefficient and would be eliminated by cooperative firms. Thus the noncooperative leader/follower structure means that new projects are researched promptly, but they are not developed as promptly as one would desire. Second, there is no virtue in the use of different investment levels by the leader and the followers. The time till completion is determined solely by the aggregate hazard rate; given the symmetry of the problem as posed, any particular aggregate hazard rate is achieved at least cost by having all firms invest at equal rates.

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