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THE ECONOMICS OF INCOME TAXATION: COMPLIANCE IN A PRINCIPAL-AGENT FRAMEWORK

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ABSTRACT

Previous analyses have modeled income tax evasion as a "portfolio problem," deriving the optimal consumption of the "risky asset" (unreported income) under the assumption of a fixed probability of detection. The purpose of this paper is to examine some of these issues in tax compliance starting from a different set of assumptions. In particular, we compare alternative audit policies to the standard random audit policy. We focus on an "audit cutoff" policy, in which an agent triggers an audit if his or her reported income is "too low," and is not audited if reported income is "sufficiently high."

This paper establishes two major results. First, random audit rules are weakly dominated by audit cutoff rules. It can be shown, given lump-sum taxes and fines, that these audit cutoff rules are the least-cost policies which induce truthful reporting of income. Second, the dominance of audit cutoff rules over random audit rules holds for "lump-sum" as well as proportional taxation — in fact, the equilibrium consequences of the two are equivalent.

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1. INTRODUCTION

One of the ubiquitous features of modern systems of income taxation is their essentially voluntary nature. While an individual taxpayer always faces some chance of being audited by the Internal Revenue Service, to take the U.S. as an example, most individuals pay a sum which depends upon the income they choose to report, which is not necessarily the same as their actual income. The purpose of this paper is to analyze income tax compliance (or evasion) starting from a set of assumptions which is somewhat different from the usual ones.

There are three topic areas in the economics literature to which this work is related. The first is optimal taxation. There is a large literature on this subject; a recent survey can be found in Sandmo (1976). A second related topic area is the work on "crime and punishment" (e.g., Becker (1968), Stigler (1970) and Becker and Stigler (1974)), and the third is the literature on tax evasion. While the literature on tax evasion is closely related to that on crime and punishment, the two are distinguished partly by the absence of a tax rate in the work on crime and punishment. Research in these last two areas typically treats noncompliance as a decision under uncertainty where the individual faces a given probability of detection and conviction, and given tax and penalty functions (e.g., Allingham and Sandmo (1972), Srinivasan (1973), and Yitzhaki (1974)). Having solved the problem of optimal under-reporting (in this framework), some authors have investigated endogenous labor supply decisions (e.g., Andersen (1977), Baldry (1979) and Pencavel (1979)), and the extent to which individuals shift their labor supply from primary markets in which wage income is reported, to secondary or "underground" markets in which wage income goes unreported (e.g., Isachsen and Strom (1980), Cowell (1981) and Sandmo (1981)). Others have investigated random audits (or random taxes), noting that they may have beneficial incentive effects when labor supply decisions are endogenous and individuals are risk averse (e.g., Stiglitz (1976) and Weiss (1976)). Finally, there has been some treatment of the determination of the optimal probability of detection and/or the penalty for evasion, usually under the assumption that the objective of the government is the maximization of some measure of social welfare (e.g., Kolm (1973), Singh (1973), Fishburn (1979), Kemp and Ng (1979), Polinsky and Shavell (1979), Christiansen (1980), and Landsberger and Meilijson (1982)).

The most complete analysis of the tax evasion problem under a random audit/linear tax schedule formulation can be found in Sandmo (1981). In the Sandmo model, individuals may supply labor in a primary market or in a secondary, or "underground" market, thereby evading taxes. The individual, in effect, faces a kind of "portfolio problem" with respect to the allocation of his or her labor. The government, on the other hand, must choose a probability of detection, a fine for evasion, a proportional tax rate and possibly some lump-sum transfers so as to maximize a social welfare function, subject to a required revenue constraint. While this analysis is quite comprehensive, it is also quite complicated, and few unambiguous qualitative results emerge.

As mentioned earlier, the purpose of this paper is to examine some of these issues in income tax compliance starting from a different set of assumptions than those used in the extant literature. In particular, we will compare <u>alternative</u> audit policies to the standard random audit policy. Accordingly, we will use a highly simplified framework which yields sharp results suitable for comparison across audit policies.

First, we will assume that income is a random variable. Second, income is observed by the taxpayer costlessly, but can only be observed by the government if an audit cost is paid. Third, we will assume, at least initially, that the objective of the tax-collecting agency is to maximize expected revenue net of audit costs. Other objective functions, such as minimizing the cost of raising some given level of expected revenue, will be discussed later in the paper.

The idea here is that the principal (the tax-collecting agency) desires to maximize net revenue but cannot observe the true income of the agent (the taxpayer) unless an "audit" is performed. The principal asks the agent to report his or her income and then

makes a decision whether or not to audit (this decision can be independent of reported income or probabilistic). The agent responds to this system so as to maximize his or her own well-being.

Besides setting an audit policy, there is also a question of the optimal form and level of taxation. We initially assume that taxation is lump-sum; that is, the agent is asked to pay a tax of T dollars if that is feasible; otherwise the taxpayer pays all of his or her income. Proportional taxation is considered later in the paper.

Finally, there is the question of what fine should be imposed for noncompliance, or in our model, under-reporting of income. This is also taken to be a choice variable of the principal.

The outline of the paper is as follows. Section 2 will set up the general model using a principal-agent framework in which the probability of an audit is a function of reported income. Section 3 considers the standard case in which audits are random, occuring with a probability which is independent of the reported level of income. In this case the principal sets a lump-sum tax, a probability of audit and a fine for under-reporting. Taxpayers respond by reporting a level of income, based upon their true income, which maximizes net expected income (i.e., they are assumed to be risk-neutral). In this situation the size of the fine is irrelevant to the optimal tax and audit policy and to the form of the optimal reporting strategy for the taxpayer (so long as the fine is positive). Moreover, either the optimal audit probability is one and the tax is infinite, or the optimal audit probability is zero and the tax is irrelevant.

Section 4 analyzes another simple, but quite different, form of audit policy. The principal still sets a lump-sum tax and a fine for under-reporting income, but now sets a cutoff level on reported income such that taxpayers who report income less than the cutoff are audited with probability one and taxpayers who report income greater than or equal to the cutoff are never audited. In this situation the optimal cutoff will equal the optimal tax and the fine will again be irrelevant to the form of the actors' optimal strategies. However, three outcomes are now possible; the optimal tax/cutoff is zero or it is infinite (corresponding to the two cases possible with a random audit policy) or it takes on some finite value determined by the cost of audits and the distribution of income. Thus the audit cutoff policy weakly dominates the random audit policy for any given distribution, and strictly dominates it for some distributions.

The audit cutoff rule analyzed in Section 4 induces truthful reporting of income. In Section 5 we show that, given lump-sum taxes and fines, the only audit policies which induce truthful reporting are those which require the principal to audit with probability one any agent who reports income less than the tax. The audit cutoff policy of Section 4 is shown to be the audit policy which induces truthful reporting at least cost to the principal and thus is optimal within the class of audit policies which induce truthful reporting.

Section 6 presents some simple examples in which the audit cutoff policy strictly dominates the random audit policy, and

considers some comparative static effects. Sections 7 and 8 reconsider the preceding analysis under proportional taxes. It is shown that, in this simple model, there is complete equivalence between the equilibrium results under lump-sum and proportional taxation. Finally, Section 9 will summarize our results and outline several variations and extensions of the basic model.

2. THE GENERAL CONTROL MODEL

The formal assumptions outlined in the Introduction are as follows.

Assumption 1. Income is distributed via $G(I)$, where $g(I) = G'(I) > 0$ for all $I \in [0,\infty)$.

Assumption 2. The taxpayer observes I costlessly, the principal observes I only if it audits the taxpayer, at a per audit cost of $c \geq 0$.

Assumption 3. Taxation is lump-sum; that is, the principal asks the agent to pay $\min\{T,I\}$.

Assumption 4. If an audit is performed and the agent has under-reported, he or she is assessed a fine of F dollars in addition to paying T; i.e., he or she pays $\min\{T + F,I\}$. No fine is ever paid if income is reported truthfully.

Assumption 5. The principal sets T,F and an audit probability function $p(\mathcal{I})$, based on reported income $\mathcal{I}$, so as to maximize expected revenue net of audit costs.

Assumption 6. The agent sets $\delta(I)$ so as to maximize expected net income.

Let $R(I,x)$ be expected net revenue to the principal from an individual with income I who reports x. Then

$$R(I,x) = \min\{T,x\}[1 - p(x)] \tag{1}$$

$$+ \begin{cases} [\min\{I,T + F\} - c]p(x) & \text{if } x < I \\ [\min\{I,T\} - c]p(x) & \text{if } x \geq I \end{cases}$$

Equation (1) allows for the possibility of over-reporting; i.e., for $x > I$. Although it turns out to make no difference in the optimal policies, we rule this out since it makes the analysis which follows less tedious.

Assumption 7. Let $\delta(I)$ be the reported income for an individual with true income I. Then $\delta(I) \leq I$.

Let $r(x,I)$ be expected net income to the agent if income I is observed and x is reported. Given (1), if $x < I$, then

$$r(x,I) = \begin{cases} [1 - p(x)][I - \min\{x,T\}] + p(x)[I - T - F] & \text{if } I \geq T + F \\ [1 - p(x)][I - \min\{x,T\}] & \text{if } I < T + F \end{cases} \tag{2}$$

If an individual under-reports and is not audited, then he or she pays only the minimum of reported income or the tax; if audited, the

individual must pay the fine F as well, whenever that is possible. If $x \geq I$, then A7 implies $x = I$, so

$$r(I,I) = \begin{cases} [1 - p(I)][I - T] + p(I)[I - T] & \text{if } I \geq T \\ 0 & \text{if } I < T \end{cases} \tag{3}$$

If the individual reports his or her income truthfully, then no fine is ever assessed. Define $\delta(I)$ to be the optimal report for an individual with income I; that is,

$$\delta(I) = \operatorname*{argmax}_{0 \leq x \leq I} r(x,I),$$

where $r(x,I)$ is given by (2) for $x < I$ and by (3) for $x = I$. Given $\delta(I)$, the principal selects T, F and $p(.)$ so as to maximize the expected value of $R(I,\delta(I))$. This is a well-defined control problem, but it does not appear to be a very tractable one -- the solution to the general problem is not readily discernible. Consequently, we will consider two special (but interesting) cases; we will subsequently show that the second, despite its simplicity, is the least-cost policy of the form $(p(.),T,F)$ which induces truthful reporting of income.

3. RANDOM AUDITS

In this section we consider the optimal strategy for the principal when $p(.)$ is independent of the level of reported income. As mentioned earlier, this is the random audit formulation used in all of the previous literature on this subject.

Assumption 8. The audit probability function p(.) is independent of x.

Denote the probability of an audit by p. Then from (2) and (3), if $x < I$,

$$r_x(x,I) = \begin{cases} -(1-p) & \text{if } x < T \\ 0 & \text{if } x \geq T \end{cases} \tag{4}$$

and if $x = I$,

$$r_x(x,I) = 0. \tag{5}$$

The implications of (4) and (5) are that if an agent under-reports his or her income, $d(I) = 0$ is optimal. Thus $d(I)$ is either 0 (given under-reporting) or I (given truthful reporting). Comparing these choices using (2) and (3) yields the following lemma.[1]

Lemma 1. Under Assumptions 1-8, for any given p,T and F, the agent's optimal reporting rule is

$$d(I) = \begin{cases} \begin{cases} 0 & \text{if } p < T/(T+F) \\ I & \text{if } p \geq T/(T+F) \end{cases} & \text{for } I \geq T+F \\ \begin{cases} 0 & \text{if } p < T/I \\ I & \text{if } p \geq T/I \end{cases} & \text{for } T \leq I < T+F \\ 0 & \text{for } I < T \end{cases}$$

Proof. If $I \geq T + F$, $r(0,I) = I - p(T + F)$ and $r(I,I) = I-T$. Thus $d(I) = 0$ if and only if $p < T/(T + F)$.

If $T \leq I < T + F$, $r(0,I) = (1-p)I$ and $r(I,I) = I-T$. Thus $d(I) = 0$ if and only if $p < T/I$.

Finally, if $I < T$, $r(0,I) = (1-p)I$ and $r(I,I) = 0$. Thus $d(I) = 0$ for all $I < T$. Q.E.D.

Remark 1. If $p < T/(T + F)$ and $I < T + F$, then $p < T/I$. Thus if $p < T/(T + F)$, then the agent will always report no income. However, if $p \geq T/(T + F)$, an agent with income less than T/p will evade (i.e., report income equal to zero) while those with incomes greater than or equal to T/p will report truthfully.

Using Lemma 1 and Remark 1, we can write the principal's expected revenue as

$$ER(I,d(I)) = \begin{cases} -cp + \int_0^{T+F} pIdG(I) + \int_{T+F}^{\infty} p(T+F)dG(I) & \text{if } p < T/(T+F) \\ -cp + \int_0^{T/p} pIdG(I) + \int_{T/p}^{\infty} TdG(I) & \text{if } p \geq T/(T+F) \end{cases} \tag{6}$$

Notice that ER(.) is continuous at $p = T/(T + F)$. We are concerned with maximizing (6) with respect to T,p and F. Consider T first. Using subscripts to denote partial derivatives, we have

$$ER_T(I,d(I)) = \begin{cases} p[1 - G(T+F)] & \text{if } p < T/(T+F) \\ [1 - G(T/p)] & \text{if } p \geq T/(T+F) \end{cases} \tag{7}$$

Using (7) we have immediately that if $p > 0$, then the optimal tax is $T^* = \infty$. If $p = 0$, then T is irrelevant.

Next consider the choice of p. Here

$$ER_p = \begin{cases} -c + \int_0^{T+F} IdG(I) + \int_{T+F}^{\infty} (T + F)dG(I) & \text{if } p < T/(T + F) \\ -c + \int_0^{T/p} IdG(I) & \text{if } p \geq T/(T + F) \end{cases} \tag{8}$$

When $T = \infty$, equation (8) becomes

$$ER_p = -c + \int_0^{\infty} IdG(I), \tag{9}$$

since $T/(T + F) \to 1$ as $T \to \infty$. In this case, $p^* > 0$ if and only if (9) is positive, and then $p^* = 1$. Otherwise, $p^* = 0$ and T is irrelevant. In either case F is irrelevant to the determination of the optimal (p^*,T^*) combination, so long as F is nonnegative. In particular,

$$ER_F = \begin{cases} \int_{T+F}^{\infty} pdG(I) & \text{if } p < T/(T + F) \\ 0 & \text{if } p \geq T/(T + F) \end{cases} \tag{10}$$

If $p^* = 0$, then F is clearly irrelevant. If $p^* = 1$, then $T^* = \infty$ and F is again irrelevant.[2]

Thus we have the following theorem.

<u>Theorem 1</u>. Under Assumptions 1-8, F is irrelevant. Either $p^* = 0$ and T is also irrelevant or $p^* = 1$ and $T^* = \infty$. Moreover, $p^* = 0$ if and only if average income is no greater than the audit cost c, and $p^* = 1$ if and only if average income exceeds the audit cost c. That is,

$$p^* = 0 \text{ if and only if } \int_0^{\infty} IdG(I) \leq c$$

and

$$p^* = 1 \text{ if and only if } \int_0^{\infty} IdG(I) > c.$$

4. AUDIT CUTOFFS

In this section, we modify Assumption 8 in the following way.

<u>Assumption 9</u>. The audit probability function must take the form

$$p(x) = \begin{cases} 1 & \text{if } x < i \\ 0 & \text{if } x \geq i \end{cases}$$

where $i \in [0,\infty)$.

In other words, the principal always audits the agent if income less than i is reported. Otherwise an audit never takes place. This audit policy has the feature that an agent can "trigger" an audit by reporting income which is "too low." In this case, revenue to the principal (net of audit costs) from an agent with income I who reports income x is

$$R(I,x) = \begin{cases} \min\{x,T\} & \text{if } i \leq x \leq I \\ \min\{I,T\} - c & \text{if } i > x = I \\ \min\{I,T + F\} - c & \text{if } x < \min\{i,I\} \end{cases} \tag{11}$$

The logic behind (11) is as follows. If the agent reports an income greater than the cutoff, no audit takes place and either the reported income or the tax is paid, whichever is smaller. If the agent reports truthfully, but less than the cutoff, an audit takes place but no fine is imposed. Finally, if the agent reports less than the cutoff but lies, the tax plus the fine is imposed, whenever that is possible.

Similarly, the agent's residual income is

$$r(x,I) = \begin{cases} I - \min\{x,T\} & \text{if } i \leq x \leq I \\ I - \min\{T,I\} & \text{if } i > x = I \\ I - \min\{T + F,I\} & \text{if } x < \min\{i,I\} \end{cases} \tag{12}$$

Following as in Section 3, we have an analogous result to Lemma 1.

Lemma 2. Under Assumptions 1-7 and 9, for any given i,T and F,

$$\phi(I) = \begin{cases} \begin{cases} \{I,0\} & \text{if } I < i \\ i & \text{if } I \geq i \end{cases} & \text{for } i < T \\ \\ \begin{cases} \{I,0\} & \text{if } I < T \\ \{i,I\} & \text{if } I \geq T \end{cases} & \text{for } i \geq T \end{cases}$$

Proof. The level of reported income only affects r(x,I) on the first branch of (12), and for that case $r_x \leq 0$. Hence $\phi(I) = i$ on the first branch. On the second branch we set $\phi(I) = I$ and on the third $\phi(I) = 0$. Hence (12) implies

$$\max_x r(x,I) = \begin{cases} I - \min\{i,T\} & & \text{for } x \geq i \\ \\ \begin{cases} I - T & \text{if } I \geq T \\ 0 & \text{if } I < T \end{cases} & & \text{for } i > x = I \\ \\ \begin{cases} I - T - F & \text{if } I \geq T + F \\ 0 & \text{if } I < T + F \end{cases} & & \text{for } x < \min\{i,I\} \end{cases} \tag{13}$$

To determine optimal reports, we need only compare reports of i, I and 0. It turns out that there are but two relevant cases.

Case 1. $i < T$.

When $I < i$ in this case, a report of i is inadmissible by Assumption 7. A report of I implies $r(I,I) = 0$ as does a report of 0, $r(0,I) = 0$. Hence $\phi(I) \in \{0,I\}$.

When $i \leq I < T$, $r(i,I) = I-i$, $r(I,I) = 0$ and $r(0,I) = 0$. Thus $\phi(I) = i$.

For $T \leq I < T + F$, $r(i,I) = I-i$, $r(I,I) = I-T$, and $r(0,I) = 0$. Hence again $\phi(I) = i$.

Finally, when $I \geq T + F$, $r(i,I) = I-i$, $r(I,I) = T$ and $r(0,I) = I-T-F$. Hence $\phi(I) = i$.

To summarize this case, when $I < i$, either 0 or I is reported, an audit occurs regardless, and the agent keeps nothing. If $I \geq i$, the agent always reports i, pays i, and is never audited. The fine F is irrelevant.

Case 2. $i \geq T$.

If $I < T$, a report of i is inadmissible. Furthermore, $r(I,I) = 0 = r(0,I)$. Hence $\phi(I) \in \{0,I\}$.

If $T \leq I < i$, i is still inadmissible as a report. But $r(I,I) = I-T$ and $r(0,I) = \max\{I-T-F,0\}$. Hence $\phi(I) = I$.

If $I \geq i$, then $r(i,I) = I-T$, $r(I,I) = I-T$, and $r(0,I) = \max\{0,I-T-F\}$. Hence $\phi(I) \in \{i,I\}$.

Thus if $i \geq T$, the agent reports I or 0 when $I < T$ and reports i or I when $I \geq T$ (whenever i is admissible). Q.E.D.

Assuming the agent is truthful whenever he or she and the principal are otherwise indifferent (see footnote 1), Lemma 2 implies

$$\delta(I) = \begin{cases} \begin{cases} I & \text{if } I < i \\ i & \text{if } I \geq i \end{cases} & \text{for } i < T \\ \\ I & \text{for } i \geq T \end{cases} \tag{14}$$

It is interesting to note that under the audit cutoff rule, there may be universal truthful reporting; when there is some evasion, it is the relatively high income individuals who evade. This is in direct contrast to the results in the random audit case; in that case, there might be universal evasion; when there is some compliance, it is the relatively high income agents who comply.

From (14), then,

$$ER(I,\delta(I)) = \begin{cases} \int_0^i (I-c)dG(I) + \int_i^\infty idG(I) & \text{if } i < T \\ \\ \int_0^T (I-c)dG(I) + \int_T^i (T-c)dG(I) + \int_i^\infty TdG(I) & \text{if } i \geq T \end{cases} \tag{15}$$

It is clear from (14) and (15) that F is irrelevant to both the optimal reporting rule of the agent and the optimal policy of the principal. Furthermore,

$$ER_i(I,\delta(I)) = \begin{cases} -cg(i) + 1 - G(i) & \text{if } i < T \\ \\ -cg(i) & \text{if } i > T \end{cases} \tag{16}$$

Now $ER(I,\delta(I))$ is continuous at $i = T$. It is decreasing in i for

$i > T$, so clearly $i \leq T$. However, in this case only the cutoff i matters; T is irrelevant. Thus we can set $i = T$ and use either branch of (15) to solve for T^*. Substituting $i = T$ and differentiating with respect to T implies

$$ER_T(I,\delta(I)) = -cg(T^*) + 1 - G(T^*). \tag{17}$$

This yields the basic result for audit cutoff policies.

<u>Theorem 2</u>. Under Assumptions 1-7 and 9, F is irrelevant so long as it is nonnegative. Furthermore, $i^* = T^*$ always. Beyond this, three cases are possible.

(1) $i^* = T^* = 0$

(2) $i^* = T^* = \infty$

(3) $i^* = T^* = \hat{T}$, where $\hat{T}$ solves

$$-cg(\hat{T}) + 1 - G(\hat{T}) = 0.$$

The optimal tax/audit policy includes three possibilities: audit no one; audit everyone and take all their income; or audit those with reported income less than $\hat{T}$ and take all their money, while taking $\hat{T}$ from each agent who reports income greater than this amount.

<u>Remark 2</u>. Note that for $\hat{T}$ to be a maximum, we must have

$$-cg'(\hat{T}) + g(\hat{T}) < 0. \tag{18}$$

Denoting $h(I) = g(I)/[1 - G(I)]$ as the hazard rate, $\hat{T}$ is defined by

$h(\hat{T}) = 1/c$ and (18) is equivalent to $h'(\hat{T}) > 0$; i.e., the hazard rate must be increasing at $\hat{T}$. Actually, we can say more than this. Suppose $h'(I) > 0$ for all I. Then there is at most one interior critical point and it is a local maximum of ER. If this value exists it must also provide a global maximum of ER. For otherwise there would exist an interior local minimum as well, which is impossible. Thus if $h'(I) > 0$ for all I and there exists $\hat{T} \in (0,\infty)$ such that $h(\hat{T}) = 1/c$, then $\hat{T}$ is optimal.

Cases (1) and (2) of Theorem 2 correspond to the two possibilities under random audits -- either audit no one or audit everyone and take whatever income they have. The third possibility is the interesting one; it requires the principal to audit those with incomes less than some finite positive amount. Taxpayers with incomes below this level are always audited and those with incomes above this level are never audited. Taxpayers with incomes above the cutoff are indifferent about reporting their true incomes or the cutoff level; the principal is also indifferent between these two reports, so one could observe some evasion among these higher income individuals. Notice that only truthful taxpayers are audited. Thus no fines are ever collected and audit costs are, in a sense, wasted. This highlights the nature of the audit as an incentive device; actual audit costs are the price for inducing those with higher incomes to report at least T^*.

Since the audit cutoff policy can generate the optimal random audit outcomes, but also admits an interior solution, it weakly

dominates the random audit policy. In addition to raising at least as much revenue as the random audit policy, the audit cutoff policy has several other desirable features. Stiglitz (1976) and Weiss (1976) have suggested that randomness in the tax or audit rates may have beneficial incentive effects when labor supply decisions are endogenous and individuals are risk averse. When viewed from an ex ante perspective (i.e., before income is realized), the audit cutoff policy looks like a random audit policy with $p = G(T^*)$. Thus if labor supply decisions are made before the realization of (say) a random wage rate, then whatever benefits might be derived from using a random audit policy still apply when one uses an audit cutoff policy.

There has also been much discussion of the desirability of horizontal equity in a tax system (e.g., Stiglitz (1976) and Rosen (1978)). While a random audit policy is horizontally equitable in an ex ante sense (i.e., before anyone is audited, they face an identical probability of audit), it is not horizontally equitable ex post; that is, some individuals with the same income are audited, while others are not. The audit cutoff policy, however, is horizontally equitable both ex ante and ex post. That is, before income is determined, each individual faces a probability of $G(T^*)$ of being audited; after income is realized, all those with the same income make the same report and suffer the same consequences. Thus there is no question of treating identical people differently.

5. INDUCIBILITY OF TRUTHFUL REPORTING

<u>Definition</u>. A given policy $(p(.),T,F)$ <u>induces truthful reporting</u> if $I \varepsilon \delta(I)$ for that policy. That is, $(p(.),T,F)$ induces truthful reporting if I is a best report for the agent.

Note that truthful reporting need not be the only best report. The audit cutoff policy of Section 4

$$p(x) = \begin{cases} 1 & \text{if } x < T \\ 0 & \text{if } x \geq T \end{cases} \tag{19}$$

induces truthful reporting, but agents with income greater than T also have $\delta(I) = T$ as a best report.

<u>Lemma 3</u>. A necessary and sufficient condition for a policy $(p(.),T,F)$ to induce truthful reporting is that $p(x) \equiv 1$ for all $x < T$.

<u>Proof</u>. Using equations (2) and (3), we see that for $I < T$, $r(I,I) \geq r(x,I)$ for all $x < I$ (note that Assumption 7 rules out $x > I$) if and only if

$$0 \geq (1 - p(x))(I - \min\{x,T\})$$

for all $x < I < T$. This inequality holds if and only if $p(x) \equiv 1$ for all $x < T$.

For $T \leq I < T + F$, $r(I,I) \geq r(x,I)$ for all $x < I$ if and only if

$$I - T \geq (1 - p(x))(I - \min\{x,T\}).$$

Note that for $x < T$, $p(x) \equiv 1$, so this inequality is satisfied for all $x < T$. For $x \geq T$, with $x < I < T + F$, $\min\{x,T\} = T$, so the above inequality reduces to

$$I - T \geq (1 - p(x))(I - T).$$

This is satisfied for any nonnegative $p(x)$.

Finally, for $I \geq T + F$, $r(I,I) \geq r(x,I)$ for all $x < I$ if and only if

$$I - T \geq (1 - p(x))(I - \min\{x,T\}) + p(x)(I - T - F).$$

For $x < T$, $p(x) \equiv 1$ and $I - T \geq I - T - F$ for all nonnegative F. For $x \geq T$, $\min\{x,T\} = T$, so the inequality reduces to

$$I - T \geq (1 - p(x))(I - T) + p(x)(I - T - F)$$

which is true for all nonnegative $p(x)$ and F. Q.E.D.

<u>Theorem 3</u>. The audit policy given in equation (19) is the least-cost policy of the form $(p(.),T,F)$ which induces truthful reporting.

<u>Proof</u>. By Lemma 3, the only restriction imposed on $(p(.),T,F)$ by the requirement of truthful reporting is that $p(x) \equiv 1$ for all $x < T$, $p(x) \geq 0$ for all x, and $F \geq 0$. Any function $p(x)$ for $x \geq T$ will induce truthful reporting and $p(x) \equiv 0$ does so at least audit cost to the principal. Q.E.D.

We have also examined audit policies which combine the aspects of random audits and audit cutoffs. They are called mixed policies and are of the form

$$p(x) = \begin{cases} p_1 & \text{if } x < i \\ p_2 & \text{if } x \geq i \end{cases}$$

where $0 \leq p_j \leq 1$ for $j = 1,2$ and $i \in [0,\infty)$.

Mixed policies allow for audit probabilities which may increase or decrease with reported income.[3] Although the proof is too tedious to include here, it can be shown that the optimal mixed policy reduces to the optimal audit cutoff policy of Section 4. Thus there is reason to believe that the audit cutoff rule is actually the solution to the general control problem as well. The validity of this conjecture is now under investigation.

<u>Conjecture</u>. The solution to the general control problem presented in Section 2 is the optimal audit cutoff rule of Theorem 2.

<u>Remark 3</u>. It can be shown that, in any solution to the general control problem, $p(x) \equiv 0$ for all $x \geq T$. To see this, note that for agents with incomes $I \geq T$ (these are the only ones who can report $x \geq T$ by Assumption 7), truthful reporting dominates any report $x \in [T,I)$. That is, $r(I,I) = I - T$, while $r(x,I) = (1 - p(x))(I - T) + p(x)(I - \min\{I,T+F\})$ for all $x \in [T,I)$. So long as $p(x) \geq 0$ (> 0), it is at least as good (better) to report I as to report $x \in [T,I)$. Thus any report $x \geq T$ will be truthful and it will not pay to audit agents reporting $x \geq T$ with positive

probability. Therefore $p(x) \equiv 0$ for all $x \geq T$. All that remains to be proved is the optimality of $p(x) \equiv 1$ for all $x < T$.[4]

6. EXAMPLES AND COMPARATIVE STATICS

Recall that for any given distribution of income $G(.)$, the audit cutoff rule weakly dominates the random audit policy. However, for some distributions, this dominance may be strict. In particular, we know that this is true whenever a positive solution T^* exists for the equation

$$h(T^*) = 1/c \tag{20}$$

and the hazard rate $h(.)$ is strictly increasing. Thus the class of distributions with increasing hazard rate are of particular interest. Members of this class include the uniform distribution, truncated normal distributions, the Weibull distribution (with parameter $\alpha > 1$), the gamma distribution (with parameter $\beta > 1$), and distributions with linear hazard rates.[5]

Whenever equation (20) has an interior solution and the hazard rate is increasing, differentiation implies that $\partial T^*/\partial c < 0$. That is, an increase in audit costs lowers the optimal tax T^* (and the audit cutoff i^*). Consequently, fewer agents are audited and less revenue is collected: maximized expected revenue is

$$ER = \int_0^{T^*} (I-c)\,dG(I) + \int_{T^*}^{\infty} T^* dG(I).$$

By the envelope theorem,

$$\partial ER/\partial c = - \int_0^{T^*} dG(I) < 0$$

so maximized expected revenue declines.

We frequently think of taxing different classes of agents differently. For instance, there may be other observable characteristics of individuals which are correlated with income (e.g., education or location of residence). Then we could think of G(.) as being parametrized by ρ, where ρ yields some information about the likelihood of a type-ρ agent having income level I. Denote this dependence by $G(.;\rho)$. To determine the comparative static effects of ρ on $T^* = T^*(\rho)$, we would need to know the effect of a change in ρ upon $h(.;\rho) = g(.;\rho)/[1 - G(.;\rho)]$. Since this seems beyond intuition, we now compute T^* for some examples and consider several parametric variations of interest.

For the uniform distribution on [a,b], $G(I) = I/M$, where $M = a-b$. Solving for T^* yields $T^* = M - c$, which is interior so long as $M > c$. The mean of I is $\mu = (a+b)/2$, while the variance is $\sigma^2 = (b-a)^2/12$. One might consider increasing the mean of the distribution, holding the variance constant. Since $\sigma^2 = M^2/12$, we must hold M fixed. Consequenty, $T^* = M - c$ implies that T^* is unaffected by a variance-preserving increase in the mean. On the other hand, a mean-preserving increase in the variance would increase M, while holding a+b constant. Thus $(a+b)/2 \equiv \mu$ implies $b = 2\mu-a$, or $M = b-a = 2(\mu-a)$. So an increase in M holding μ fixed is equivalent to a decrease in a. Since $T^* = 2(\mu-a) - c$, $\partial T^*/\partial a < 0$. Thus a decrease in a (corresponding to an increase in σ^2) holding μ fixed,

results in an increase in T^*. Agents facing uniform distributions with greater variance (and the same mean) should be required to pay higher taxes.

Another interesting parameter change is related to stochastic dominance. We say that $G(.;M_1)$ dominates $G(.;M_2)$ if $G(I;M_1) < G(I;M_2)$ for all I; that is, income is stochastically greater under M_1 than under M_2. For the uniform distribution, $G(.;M_1)$ dominates $G(.;M_2)$ if $M_1 > M_2$. Since $\partial T^*/\partial M > 0$, $T^*(M_1) > T^*(M_2)$. Thus agents facing stochastically better uniform distributions of income should be taxed more (and should face a harsher audit cutoff policy).

For distributions with linear hazard rate, $G(I) = 1 - e^{-(\alpha I + \theta I^2/2)}$, for $I \in [0,\infty)$ and $\alpha, \theta > 0$. Solving for T^* yields $T^* = (1 - c\alpha)/c\theta$, which is interior for $1 > c\alpha$. Since

$$\partial G/\partial \alpha = Ie^{-(\alpha I + \theta I^2/2)}$$

and

$$\partial G/\partial \theta = (1/2)I^2 e^{-(\alpha I + \theta I^2/2)},$$

an increase in either α or θ results in a stochastically dominated distribution. Since $\partial T^*/\partial \alpha = -1/\theta < 0$, and $\partial T^*/\partial \theta = -(1 - c\alpha)/c\theta^2 < 0$, again we should raise the optimal tax/cutoff T^* for distributions with stochastically higher income (i.e., those with lower values of α and/or θ).

7. PROPORTIONAL TAXATION WITH RANDOM AUDITS

In this section, we consider proportional, as opposed to lump-sum taxation, in conjunction with a random audit policy. Let $t \in [0,1]$ be the constant average and marginal tax rate, assessed on all reported income. In addition, randomly selected individuals are subjected to an audit. If they are discovered to be under-reporting, a fine of F is assessed.

<u>Assumption 3'</u>. Taxation is proportional; that is, the principal asks the agent to pay tI.

<u>Assumption 4'</u>. If an audit is performed and the agent has underpaid, he or she is assessed a fine of F dollars in addition to paying tI; i.e., he or she pays $\min\{tI+F, I\}$. No fine is ever paid if income is reported truthfully.

<u>Assumption 5'</u>. The principal sets t,F and an audit probability function $p(\partial)$, based on reported income ∂, so as to maximize expected revenue net of audit costs.

Maintaining Assumption 8 ($p(x) \equiv p$) implies that revenue to the principal from a taxpayer with income I who reports x is

$$R(I,x) = t\partial(1-p) + \begin{cases} \min\{I, tI+F\} - c & \text{if } \partial < I \\ tI & \text{if } \partial \geq I \end{cases} \qquad (21)$$

Residual income to the taxpayer with income I who reports income x is

$$r(x,I) = \begin{cases} (1-p)(I-t\partial) + p(I-tI-F) & \text{if } I \geq tI+F \text{ and } \partial < I \\ (1-p)(I-t\partial) & \text{if } I < tI+F \text{ and } \partial < I \\ (1-p)(I-t\partial) + p(I-tI) & \text{if } \partial \geq I \end{cases} \qquad (22)$$

Notice that all three branches of (22) are decreasing in ∂, so for the first two branches the optimal report is $x = 0$, while the optimal report in the last branch is $x = I$. Thus one need only compare the reports $x = 0$ and $x = I$. Substituting these reports into (22) and comparing the payoff under truthful reporting to the payoff under a report of zero yields the following optimal reporting rule $\partial(I)$.

<u>Lemma 4</u>. Under Assumptions 1-2,3'-5', and 6-8, the optimal reporting rule for the taxpayer is

$$\partial(I) = \begin{cases} \begin{cases} 0 & \text{if } p < t \\ I & \text{if } p \geq t \end{cases} & \text{for } I < F/(1-t) \\ \\ \begin{cases} 0 & \text{if } I > pF/t(1-p) < F/(1-t) \\ I & \text{if } I \leq pF/t(1-p) \end{cases} & \text{for } I \geq F/(1-t) \end{cases}$$

<u>Proof</u>. For values of income $I \geq tI+F$ (i.e., $I \geq F/(1-t)$), a taxpayer should evade ($\partial = 0$) if and only if $(1-p)I + p(I-tI-F) > I(1-t)$; that is, if and only if $I > pF/t(1-p)$.

For values of income $I < tI+F$ (i.e., $I < F/(1-t)$), evasion is optimal if and only if $(1-p)I > (1-t)I$; that is, if and only if $p < t$. Q.E.D.

Notice that if $p < t$ and $I \geq F/(1-t)$, then $I > pF/t(1-p)$.

Thus if $p < t$ everyone evades, while if $p \geq t$, lower-income individuals are truthful while higher income individuals evade. This is the opposite of our finding for the lump-sum case, in which under a random audit policy it was the lower-income agents who evaded.

Expected revenue to the tax authority can be computed using $d(I)$. There are two relevant cases, but expected revenue is continuous across the point $p = t$.

$$ER(I, d(I)) = \begin{cases} \int_0^{F/(1-t)} p(I-c)\,dG(I) + \int_{F/(1-t)}^{\infty} p(It+F-c)\,dG(I) & \text{if } p < t \\ \int_0^{pF/t(1-p)} (tI-pc)\,dG(I) + \int_{pF/t(1-p)}^{\infty} p(tI+F-c)\,dG(I) & \text{if } p \geq t \end{cases} \tag{23}$$

The principal maximizes this expression by a choice of t,p and F. First consider the choice of the tax rate t.

$$ER_t(I, d(I)) = \begin{cases} \int_{F/(1-t)}^{\infty} pI\,dG(I) & \text{if } p < t \\ \int_0^{pF/t(1-p)} I\,dG(I) + \int_{pF/t(1-p)}^{\infty} pI\,dG(I) & \text{if } p \geq t \end{cases} \tag{24}$$

Since ER is increasing in t if $p > 0$, $t^* = 1$ for $p > 0$. If $p = 0$, then t is irrelevant. We may use either branch of (23) evaluated at $t^* = 1$ to solve for the optimal value of p. However, given $t^* = 1$,

$$ER_p = \int_0^{\infty} p(I-c)\,dG(I). \tag{25}$$

Thus we have the following theorem.

<u>Theorem 4</u>. Under Assumptions 1-2,3'-5',and 6-8, the fine F is irrelevant. Either $p^* = 0$ and t is also irrelevant, or $p^* = 1$ and $t^* = 1$. Moreover, $p^* = 0$ if and only if mean income is no greater than the cost per audit, while $p^* = 1$ if and only if mean income exceeds the cost per audit. That is,

$$p^* = 0 \text{ if and only if } \int_0^{\infty} I\,dG(I) \leq c$$

and

$$p^* = 1 \text{ if and only if } \int_0^{\infty} I\,dG(I) > c.$$

Notice that these results are <u>exactly the same as in the lump-sum formulation.</u> That is, if mean income exceeds the cost per audit, the optimal policy is to audit everyone and take all they have; if mean income is less than the cost per audit, the optimal policy is to audit no one, and collect no revenue.

8. PROPORTIONAL TAXATION WITH AUDIT CUTOFF

In this section, we consider proportional taxation under the simple audit cutoff rule. Thus we invoke Assumptions 1-2,3'-5', 6-7 and 9. Under this set of assumptions, revenue to the principal from a randomly selected taxpayer is

$$R(I,x) = \begin{cases} tx & \text{for } x \geq i \\ tI - c & \text{for } I \leq x < i \\ \min\{tI+F, I\} - c & \text{for } x < i \text{ and } d < I \end{cases} \tag{26}$$

Residual income to an agent with income I who reports income x is

$$r(x,I) = \begin{cases} I-tx & \text{for } x \geq i \\ I-tI & \text{for } I \leq x < i \\ \begin{cases} I-tI-F & \text{if } I \geq F/(1-t) \\ 0 & \text{if } I < F/(1-t) \end{cases} & \text{for } x < I \text{ and } x < i \end{cases} \tag{27}$$

Now there are three candidates for an optimal report; x = i, x = I and x = 0, for the three branches of (27), respectively. We need to compare these using (27) to determine the optimal report $\phi(I)$.

Lemma 5. Under Assumptions 1-2,3'-5',6-7 and 9, for any given i,t and F,

$$\phi(I) = \begin{cases} I & \text{if } I \leq i \\ i & \text{if } I > i \end{cases}$$

Proof. There are two relevant cases.

Case 1. $i \leq F/(1-t)$.

For $I \leq i$, the report i is inadmissible by Assumption 7 (again, the results are unchanged if over-reporting is allowed, but several additional sub-cases arise). A truthful report yields residual income r(I,I) = I(1-t), while under-reporting yields r(0,I) = 0. To see this, note that $I \leq i$ implies $I \leq F/(1-t)$ or $I < tI+F$, so the audited evader can pay at most I (rather than tI+F). Thus for $I < i$, $\phi(I) = I$.

For $i < I$, a report of i implies residual income r(i,I) = I-ti, a truthful report implies income r(I,I) = I(1-t), and evasion implies

$$r(0,I) = \begin{cases} 0 & \text{if } I \leq F/(1-t) \\ I(1-t)-F & \text{if } I \geq F/(1-t) \end{cases}$$

Thus the optimal report for $I > i$ is $\phi(I) = i$.

Case 2. $i > F/(1-t)$.

Again there are several ranges of income to be considered, but so long as $I \leq i$, $\phi(I) = I$, and for $I > i$, $\phi(I) = i$. Q.E.D.

Notice that evasion is practiced by those agents with relatively high incomes.[6] This coincides with the results under a random audit policy, in which evasion occurs in the ranks of the higher-income taxpayers. This is to be contrasted with the lump-sum results, in which the random audit policy resulted in evasion by the relatively low-income taxpayers, while under the audit cutoff policy relatively high-income taxpayers were indifferent between truthful reporting and evasion.

Expected revenue to the tax authority, net of auditing costs, under the optimal reporting rule given above, is

$$ER(I,\phi(I)) = \int_0^i (tI-c)dG(I) + \int_i^\infty ti dG(I). \tag{28}$$

The principal wishes to maximize this expression by a choice of i and t (F is clearly irrelevant). Differentiating with respect to t yields

$$ER_t(I,\delta(I)) = \int_0^i IdG(I) + i(1 - G(i)) \tag{29}$$

which is strictly positive if $i > 0$. Thus either $i > 0$ and $t^* = 1$, or $i = 0$, in which case t is irrelevant. In any event, $t^* = 1$ is optimal. Evaluating (30) at $t^* = 1$ and differentiating with respect to i yields

$$ER_i(I,\delta(I)) = -cg(i) + 1 - G(i). \tag{30}$$

Theorem 5. Under Assumptions 1-2,3'-5',6-7 and 9, F is irrelevant so long as it is nonnegative. Furthermore, $t^* = 1$. Beyond this, three cases are possible.

(1) $i^* = 0$

(2) $i^* = \infty$

(3) $i^* = \hat{i}$, where $\hat{i}$ solves

$$-cg(\hat{i}) + 1 - G(\hat{i}) = 0.$$

Notice that the value of $\hat{i}$ in Theorem 5 is the same as the value of $\hat{T}$ in Theorem 2. Thus the results of Theorem 5 are identical to those of Theorem 2. That is, the principal may audit no one ($i^* = 0$); audit everyone and take all their income ($i^* = \infty$); or audit those with reported income below $\hat{i}$ and take all their income, while taking $\hat{i}$ from each agent who reports at least $\hat{i}$. The same equivalence exists between Theorems 4 and 1 in the random audit case. Thus, despite the fact that there may be some differences in the best reporting rule for taxpayers, there is essentially complete

equivalence between the equilibrium consequences of lump-sum and proportional taxation.

9. CONCLUSIONS, VARIATIONS AND EXTENSIONS

This paper has established two major results and offered one conjecture in the context of a simple model of income tax compliance. First, random audit rules are weakly dominated by audit cutoff rules. It can be shown, given lump-sum taxes and fines, that these audit cutoff rules are the least-cost policies which induce truthful reporting of income. Second, the dominance of audit cutoff rules over random audit rules holds for "lump-sum" as well as proportional taxation -- in fact, the equilibrium consequences of the two are equivalent. In addition, we conjectured that an audit cutoff rule is, in fact, the solution to the general problem outlined in Section 2.

These results were established under fairly strong assumptions, however. In particular, we assumed risk-neutrality for all agents and that the principal desired to maximize net expected revenue. In spite of its simple nature, the model developed in this paper yielded interesting results not found in the previous literature, and several extensions seem worth pursuing.

An important extension would be to consider risk averse agents. It is easy to show that the problem under the audit cutoff rule is unchanged; agents face no real uncertainty, and (as long as utility is increasing in income) they will behave precisely as described in Section 4. However, the risk averse agent will respond differently to the random audit policy. This analysis leaves open the possibility that a policy of random audits may be more effective than an audit cutoff policy when taxpayers are risk averse.[7]

Obviously, it is important to consider alternative objective functions; for example, the tax authority may desire to minimize the cost of raising a specified amount of revenue. One might also consider the more traditional objective of maximizing a utilitarian criterion.

There is a presumption in the tax policy literature that some taxpayers are always honest, even when they could increase their expected net income by under-reporting. To capture this formally, we could assume some fraction of taxpayers, say α, always report honestly. The issue is how this affects the principal's optimal strategy; in particular, what happens as α decreases? One might speculate that in the lump-sum case, the presence of an "honest" group will drive a wedge between i^* and T^* when audit cutoffs are used. Beyond this, however, it is not clear what might happen.

Instead of knowing the function p(.), taxpayers might only know $\bar{p} = E[p(d)]$, particularly if they don't know the distribution of income, G(.). In this case, the issue is whether it is in the interest of the principal to reveal p(.) or whether net revenue can be increased by exploiting taxpayer ignorance of the audit rule.

In a move toward realism, one might reformulate the model so that the principal sees true income I (via W-2 forms, for example) but the taxpayer can claim deductions, say $\delta(I)$. The audit rule could

then be based on the ratio of deductions to income.

Alternative formulations of the fine might be considered; for instance, the fine might be proportional to the unreported income (e.g., Allingham and Sandmo (1972) and Srinivasan (1973)) or to evaded taxes (e.g., Yitzhaki (1974)).

Finally, as a formal matter, it might be of interest to compare bonuses for truthful reporting to fines for under-reporting. Other variations are possible, but it is clear from the suggestions above that much interesting work remains.

FOOTNOTES

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1. Throughout this paper we will assume that whenever an agent is indifferent between two or more actions, he or she will take that action which is most preferred by the principal. This is a standard assumption in the principal-agent literature.

2. Having noted that one can "trade off" penalties for probability of detection in attempting to increase compliance, several authors have remarked that if it is costly to detect and convict offenders, but there is no cost to imposing a penalty, then the "optimal" solution is a very small probability of detection and a very large penalty (e.g., Becker (1968), Stern (1978), Kemp and Ng (1979), Polinsky and Shavell (1979) and Kolm (1973)). These studies have typically taken the tax as exogenous, or have focused on crime and punishment, in which there is no analog to the tax, and have taken as objective the maximization of a social welfare function.

3. Of the three policies examined in this paper, the mixed audit policy most closely approximates the "true" audit policy. "As a part of its Taxpayer Compliance Measurement Program, the IRS

periodically conducts random audits of the tax-paying population. From information obtained there on characteristics of tax evaders and other underreporters, they devise formulas designed to identify likely candidates for audits... About all that is known generally is that individual tax returns are separated into audit classes -- based on income and type of income -- for analysis, and occasionally the average probabilities for each class have been made public. Still, the probability for any tax return in a given class is a function of its reported items (Clotfelter (1982))." Thus while individuals do face the possibility of being audited in a purely random fashion, they also face a "formula," and consequently may "trigger" an audit through their reporting behavior.

4. In papers on the general principal-agent problem, there is frequent reference to the Revelation Principle. In our context, this would say: given any tax/audit/fine policy in some feasible set, there exists another feasible policy which induces truthful reporting and leaves both principal and agent at least as well off as the originial policy. Thus without loss of generality, one can restrict attention to policies which induce truthful reporting. The key to using this principle is defining an appropriate feasible set; not just any set will do. In particular, we have been unable to prove that the Revelation Principle applies within our restricted domain of policies of the form (p(.),T,F). (Note that if we could show this, we would be done, because of Lemma 3

and Theorem 3). The revelation principal does apply in the slightly larger set of policies of the form $(p(x),\tau(x),f(x))$, where the audit policy is $p(x)$, the tax policy is $\min\{\tau(x),T\}$ and the fine policy is

$$\begin{cases} \min\{I,T\} & \text{if } f(x) = I \\ \min\{I,T+F\} & \text{if } f(x) < I \end{cases}$$

If the agent optimally reports $d(I)$ in response to this policy, then the policy $(\tilde{p}(x),\tilde{\tau}(x),\tilde{f}(x)) = (p(d(x)),\tau(d(x)),f(d(x)))$ gives both the principal and the agent the same payoffs and induces truthful reporting. However, now one needs to characterize <u>three</u> functions rather than one. We have made little headway on this problem.

5. The Weibull (with $\alpha = 1$) and the gamma (with $\beta = 1$) coincide with the exponential distribution, which has constant hazard rate. Common distributions with decreasing hazard rates are the lognormal distribution, the Weibull (with $\alpha < 1$), the gamma (with $\beta < 1$) and the Pareto distribution.

6. It is interesting to note that, if income can be arbitrarily large, then no policy of the form $(p(.),t,F)$ can induce truthful reporting with proportional taxation unless $p(x) \equiv 1$ for all x. To see this, note that residual income with proportional taxation is

$$r(x,I) = (1 - p(x))(I - tx) + p(x)(I - \min\{tI + F,I\}$$

for $x < I$, and

$$r(I,) = I(1 - t).$$

For $tI + F \leq I$ (i.e., $I \geq F/(1-t)$), truthful reporting is optimal if and only if

$$I(1 - t) \geq (1 - p(x))(I - tx) + p(x)(I - tI - F)$$

for all $x < I$ and all $I \geq F/(1 - t)$. Simplifying implies

$$(1 - p(x))t(I - x) \leq p(x)F$$

for all $x < I$ and for all $I \geq F/(1-t)$. But since I can be arbitrarily large, this requires $p(x) \equiv 1$ for all x. If $p(x) \equiv 1$ for all x, then those with incomes $I < F/(1-t)$ will also tell the truth, so there is truthful reporting by all agents.

7. In some ways, our problem of choosing an optimal audit policy is similar to Townsend's (1979) problem of optimal contracts with costly state verification. In that model, agent 1's endowment is certain, while agent 2's endowment is random. Motivated by risk-sharing considerations, they agree to a contract to transfer endowment from one to the other for certain realizations of the random endowment, which is observed only by agent 2. The decision to ask for verification and the responsibility for the attendant costs lies with agent 2, the informed agent. Townsend examines deterministic verification schemes, but concludes from an example that these can be dominated by a stochastic verification scheme.

By contrast, in our problem the audit policy is imposed on the agent by the principal; there is no mutuality of interest. The agent reports an income realization, and the principal is responsible for the decision to audit and the accompanying costs.

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