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SIMULATION OF THE DEMAND FOR ELECTRICITY  
UNDER ALTERNATIVE RATE STRUCTURES

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## ABSTRACT

This paper reviews the theory of price specification and considers the comparative static analysis of demand subject to alternative rate schedules. An econometric analysis of the 1975 Washington Center for Metropolitan Studies survey resolves four empirical issues related to the estimation of the demand for electricity: (1) measured average price and measured marginal price are statistically endogenous so that least squares techniques are not appropriate for the determination of price and income elasticities, (2) while the rate structure premium (RSP) has established theoretical merit its statistical contribution is negligible, (3) consumer behavior in the demand for electricity follows the marginal price rather than the average price specification, and (4) estimates of price responsiveness are not statistically different using the tail-end price rather than the true marginal price.

We demonstrate a practical way of making probabilistic comparisons between alternative rate schedules which is applied in several examples to illustrate the prevalence of block switching. The methodology is easily applied to inverted tariff schedules even when structural parameters have been determined from a cross-section of individuals who face declining block rates.

## SIMULATION OF THE DEMAND FOR ELECTRICITY

### UNDER ALTERNATIVE RATE STRUCTURES<sup>1</sup>

#### I. INTRODUCTION

Recent studies in the demand for electricity have raised again the question of price specification. The early work of Houthakker (1951a) discussed demand subject to a quantity dependent rate structure as compared to the classical situation of parametrically given prices. Taylor (1975), in his survey of the electricity demand literature, reviews the rate structure problem and indicates a simple procedure which converts the complex optimization problem of the consumer to the standard case of a linear budget constraint set in marginal prices. Modifications to the Taylor procedure were noted by Berndt (1978) and Nordin (1976).

A behavioral question is whether consumers can detect prevailing marginal rates in the presence of automatic appliances and billing cycle variations. An alternative hypothesis suggests that consumers respond to a summarizing statistic for the quantity dependent rate structure such as average price.

This paper reviews the theory of price specification and considers the comparative static analysis of demand under alternative rate structures. We investigate the statistical endogeneity of prices whose construction depends on the observed

in 1975 by the Washington Center for Metropolitan Studies (WCMS).<sup>2</sup>

We consider the problem of determining demand response subject to non-marginal changes in the underlying budget set. An important illustration attempts to predict the probability of block switching when consumers are shifted from declining to inverted block rate structures.

Section II considers the theory of price specification while Section III presents empirical estimates of price responsiveness. Section IV presents the simulation of demand under alternative rate schedules and Section V provides a brief summary and some conclusions.

#### II. SPECIFICATION OF PRICE: THEORY

##### 1. Quantity Dependent Rate Structure

We begin by reviewing the general quantity dependent outlay or expenditure function. Let  $B(Q)$  be the total expenditure on electricity when an amount  $Q$  is consumed. The rate structure premium,  $B(Q) - B'(Q)Q$ , is an adjustment to income such that consumers choose quantity level  $Q$  at constant marginal price  $B'(Q)$ .

If  $V(P, Y)$  is the indirect utility at prices  $p$  and income level  $Y$  then the consumer's optimal choice of quantity subject to the expenditure function  $B(Q)$  solves the problem:

$$\text{MAX}_Q V[B'(Q), Y - [B(Q) - B'(Q)Q]].$$

The first-order condition implies that optimal  $Q$  is given as

the solution to Roy's identity:

$$Q = - \frac{V_P[B'(Q), Y - (B(Q) - B'(Q)Q)]}{V_Y[B'(Q), Y - (B(Q) - B'(Q)Q)]}$$

$$= D[B'(Q), Y - (B(Q) - B'(Q)Q)]$$

where  $D[P, Y]$  is the Marshallian demand curve. Thus  $Q$  is not generally given as a reduced form in terms of prices, income, and other parameters of the uncompensated demand function. Furthermore the usual monotonicity properties of the demand function are not sufficient to imply unique optimal consumption levels.

The case of the declining block rate structure is somewhat more structured and permits us to derive a simple relation among quantity, average price, marginal price, and the rate structure premium. A declining block rate schedule implies an expenditure function which increases in linear segments, the slope of each succeeding segment being smaller than the one preceding it.

Suppose:

$$B = C \quad \text{for } 0 \leq Q \leq X_1$$

$$B = C + \pi_1(Q - X_1) \quad \text{for } X_1 < Q \leq X_2$$

$$B = C + \sum_{j=1}^{r-1} (X_{j+1} - X_j)\pi_j + \pi_r(Q - X_r) \quad \text{for } X_r < Q \leq X_{r+1}, 1 < r \leq n$$

where  $X_i$  denote the lower block boundaries and where we have set  $X_{n+1} = \infty$ . The constant  $C$  is the connect charge and  $\pi_j$  is the price of electricity in block  $j$ . Suppose measured consumption,  $Q^*$ , lies in the

$r$ th block so that  $X_r < Q^* \leq X_{r+1}$  and total expenditure,  $B^*$ , is  $C + \sum_{j=1}^{r-1} (X_{j+1} - X_j)\pi_j + \pi_r(Q^* - X_r)$ . We then define the measured average price as  $B^*/Q^*$ , the measured marginal price as  $\pi_r$ , and the rate structure premium (RSP) as the difference between total expenditure and the cost of purchasing the quantity  $Q^*$  at the marginal rate  $\pi_r$ :  $RSP = B^* - \pi_r Q^*$ . Dividing by quantity we obtain the simple relation average price = marginal price +  $RSP/Q^*$ . Taylor (1975) shows that the rate structure premium is an adjustment to income such that consumers choose quantity  $q^*$  at price  $\pi_r$  and income level  $Y - RSP$ .

## 2. Comparative Static Analysis of Demand Subject to a Declining Block Rate Structure

We now consider the comparative static analysis of demand subject to a declining block rate structure. Let  $U[q, Z]$  denote the utility derived from the consumption of electricity  $q$  and a Hicksian or numeraire commodity  $Z$ . We assume a two-tier tariff for electricity with the price of electricity  $\pi$  given by

$$(1) \quad \pi = \begin{cases} \pi_1 & \text{for } 0 \leq q \leq X \\ \pi_2 & \text{for } X < q \end{cases} \quad \text{with } \pi_1 > \pi_2$$

Normalizing the price of the numeraire commodity to equal one the budget constraint satisfies:

$$(2) \quad \begin{aligned} \pi_1 q + Z &\leq y && \text{for } q \leq X \\ \pi_1 X + (q - X)\pi_2 + Z &\leq y && \text{for } X < q \end{aligned}$$

where  $y$  denotes income.

[Insert Diagram 1 and 2 here]

We illustrate the declining tariff in Diagram 1 and the corresponding budget set in Diagram 2.

Denote by  $D[\pi, y; \beta]$  the Marshallian or uncompensated demand for electricity where  $\beta$  is a vector of behavioral parameters and let  $\pi^*$  denote the price at which demand equals the lower block boundary, i.e.,  $D[\pi^*, y; \beta] = X$ . Let  $q_1$  denote demand along the segment with slope  $\pi_1$  and let  $q_2$  denote demand along the segment with slope  $\pi_2$ . Demand along the first budget segment satisfies

$$(4) \quad q_1 = D[\pi_1, y; \beta] \text{ for } (\pi_2, \pi_1) \in S_1$$

while demand in the second segment satisfies

$$(5) \quad q_2 = D[\pi_2, y - (\pi_1 - \pi_2)X; \beta] \text{ for } (\pi_2, \pi_1) \in S_2$$

We have defined the sets  $S_1$  and  $S_2$  to indicate which price pairs  $(\pi_2, \pi_1)$  imply optimal first and second segment demand. We show below that  $S_1$  and  $S_2$  constitute a proper partition of all prices which correspond to declining two-part tariffs. Note that the term  $(\pi_1 - \pi_2)X$  is the rate structure premium adjustment for demand in the marginal or tail-end block. We now derive certain results concerning local price response.

Lemma 1: Suppose the uncompensated demand for electricity is decreasing in price and increasing in income. Then:

- 1a)  $\partial q_1 / \partial \pi_1 < 0$  for  $(\pi_2, \pi_1) \in S_1$
- 1b)  $\partial q_2 / \partial \pi_1 < 0$  for  $(\pi_2, \pi_1) \in S_2$
- 1c)  $\partial q_2 / \partial \pi_2 < 0$  and  $(\pi_2, \pi_1) \in S_2$

Proof Lemma 1:

1a) By assumption demand is downward sloping.

1b)  $\partial q_2 / \partial \pi_1 = (D_y)(-X) < 0$  since we have assumed that electricity is a normal good.

1c)  $\partial q_2 / \partial \pi_2 = D_\pi + D_y X \leq D_\pi + D_y q_2$  since  $X \leq q_2$ . But  $D_\pi + D_y q_2$  equals the partial derivative with respect to price of the Hicksian or compensated demand function (by Slutsky's relation) and is thus negative.

Remarks: For  $\pi_1 \geq \pi^*$ ,  $q_1 \leq X$  by Lemma 1a. For  $\pi_1 < \pi^*$ ,  $q_1 > X$  so that optimal demand falls outside the range in which  $\pi_1$  is the prevailing price. Furthermore Lemma 1c implies that for  $\pi_2 < \pi^*$ ,  $q_2 > X$ . The pattern of prices in which  $\pi_2 < \pi^* \leq \pi_1$  implies that  $q_1$  and  $q_2$  are each feasible.

Let  $V(\pi, y)$  be the indirect utility function corresponding to the problem  $\text{Max } U[q, Z]$  subject to  $\pi q + Z \leq y$ . For  $\pi_2 < \pi^* \leq \pi_1$ , the

budget segment with price  $\pi_1$  is optimal when

$V(\pi_1, y) > V(\pi_2, y - (\pi_1 - \pi_2)X)$ . It is clear that combinations of  $\pi_1$  and  $\pi_2$  exist which satisfy  $\pi_2 < \pi^* \leq \pi_1$  and imply equal indirect utility so that demand for electricity is multi-valued. For the set

of prices which imply equal indirect utility a trade-off exists where an increase in  $\pi_1$  may be compensated by a decrease in  $\pi_2$ . We have the following result:

**Lemma 2:** Let  $S = \{(\pi_2, \pi_1) | V(\pi_1, y) = V(\pi_2, y - (\pi_1 - \pi_2)X) \text{ and } \pi_2 < \pi^* \leq \pi_1\}$ . Then  $\partial\pi_1/\partial\pi_2 < 0$  for  $(\pi_2, \pi_1) \in S$  and for  $V_y(\pi_1, y) < V_y(\pi_2, y - (\pi_1 - \pi_2)X)$ .

**Proof Lemma 2:** Let  $V_{y_1} = V_y[\pi_1, y]$  and  $V_{y_2} = V_y[\pi_2, y - (\pi_1 - \pi_2)X]$ .

For  $(\pi_2, \pi_1) \in S$ ,

$$(\partial\pi_1/\partial\pi_2) \cdot V_{\pi_1} = V_{\pi_2} + V_{y_2}((-X)((\partial\pi_1/\partial\pi_2) - 1)). \text{ Then}$$

$$(\partial\pi_1/\partial\pi_2)(V_{\pi_1} + V_{y_2}X) = V_{\pi_2} + V_{y_2}X \text{ which implies}$$

$$(\partial\pi_1/\partial\pi_2) = (V_{\pi_2} + V_{y_2}X)/(V_{\pi_1} + V_{y_2}X)$$

$$= (X - q_2)/(X - q_1(V_{y_1}/V_{y_2})) < 0 \text{ for } q_1 < X \text{ and } q_2 > X.$$

Q.E.D.

To complete the static analysis we need the following result which indicates the direction of change in indirect utility from changes in price.

**Lemma 3:** Let  $V_1 = V[\pi_1, y]$  and  $V_2 = V[\pi_2, y - (\pi_1 - \pi_2)X]$ .

For  $\pi_2 < \pi^* \leq \pi_1$ :

$$3a) \quad \partial V_1/\partial\pi_1 < 0$$

$$3b) \quad \partial V_2/\partial\pi_1 < 0$$

$$3c) \quad \partial V_2/\partial\pi_2 < 0$$

$$3d) \quad \partial(V_2 - V_1)/\partial\pi_1 < 0 \text{ when } V_{y_1} < V_{y_2}.$$

**Proof Lemma 3:**

$$3a) \quad \partial V_1/\partial\pi_1 = V_{\pi}(\pi_1, y) < 0 \text{ (monotonicity property of indirect utility function).}$$

$$3b) \quad \partial V_2/\partial\pi_1 = V_{y_2}(-X) < 0$$

$$3c) \quad \partial V_2/\partial\pi_2 = V_{\pi} + V_{y_2}X \leq V_{\pi} + V_{y_2}q_2 < 0 \text{ as } X \leq q_2.$$

$$3d) \quad \partial(V_2 - V_1)/\partial\pi_1 = -[V_{\pi_1} + V_{y_2}X] \\ = -V_{y_2}[X - q_1(V_{y_1}/V_{y_2})] < 0$$

as  $V_{y_1}/V_{y_2} < 1$  and  $q_1 < X$ .

Q.E.D.

We now collect the results in the following theorem.

**Theorem 1:** (Two-Tier Declining Block Rate Comparative Statics)

Let  $\pi^*$  be defined by  $D[\pi^*, y; \beta] \equiv X$ . Define the functions  $\pi_1^*(\pi_2)$  and  $\pi_2^*(\pi_1)$  by

$$V(\pi_1^*, y) = V(\pi_2, y - (\pi_1^* - \pi_2)X) \text{ and}$$

$$V(\pi_1, y) = V(\pi_2^*, y - (\pi_1 - \pi_2^*)X) \text{ respectively.}$$

Then equilibrium occurs in the first segment for:

$$S_1 = \{(\pi_2, \pi_1) | \pi^* \leq \pi_1 \text{ and } \pi_2^*(\pi_1) \leq \pi_2 \leq \pi_1\}$$

and equilibrium occurs in the second segment for:

$$S_2 = \{(\pi_2, \pi_1) | 0 \leq \pi_2 \leq \pi^* \text{ and } \pi_2 \leq \pi_1 \leq \pi_1^*(\pi_2)\}.$$

[Insert Diagram 3 here]

**Proof Theorem 1:** The shaded region above the diagonal line in Diagram 3 represents the set of feasible declining block rate structures. The curve with declining slope which intersects the  $(\pi^*, \pi^*)$  point is the set S of Lemma 2. Suppose we begin at a point on the curve S and increase  $\pi_2$  while leaving  $\pi_1$  unchanged. Since we are in a region in which both budget segments are feasible, Lemma 3c implies that the increase in  $\pi_2$  decreases the utility  $V_2$ . As we began at a point of equal utility and  $V_2$  has decreased while  $V_1$  remains constant it must be the case that budget segment one is preferred to budget segment two.

Similarly consider a decrease in  $\pi_1$  leaving  $\pi_2$  constant. In this case, Lemma 3d applies so that  $V_2 - V_1 > 0$  and budget segment two becomes optimal. In the southwest quadrant above the 45° degree line, demand occurs in the second budget segment since optimal demand for prices  $\pi_1 < \pi^*$  exceeds the block boundary X. The other quadrants are similarly derived using the results of Lemma 1 and Lemma 3.

Q.E.D.

**Remarks:** Note that price pairs below the diagonal imply increasing or non-decreasing block rate schedules which correspond to convex budget sets. The triangular area in the southwest quadrant below the diagonal implies optimal demand in the second budget segment while the area below the diagonal in the northeast quadrant implies demand in the first budget segment. The southeast quadrant which

includes the boundary  $\pi_1 = \pi^*$  but excludes the boundary  $\pi_2 = \pi^*$  implies optimal demand at the block boundary X. We further note that the set S of equal utility points has measure zero in the price space of Diagram 3.

We now use Diagram 3 to answer simple comparative static problems. Suppose for example that we increase the lower block boundary. Diagram 4 illustrates that the partition moves to an intersection with the 45° line at the point  $(\pi^{*'}, \pi^{*'})$  with  $\pi^{*'} < \pi^*$  as  $X' > X$ . If equilibrium had initially occurred at point A, the discontinuous change in lower block boundary from X to X' would now imply that point A corresponds to optimal demand in budget segment one versus the initial equilibrium in budget segment two.

[Insert Diagram 4 here]

Finally we note that our comparative static analysis applies to the more general case of multiple tier declining block rate schedules where we interpret  $\pi_2$  as the marginal rate and  $\pi_1$  as the intramarginal average price, i.e., the average price up to but not including the marginal block.

### III. SPECIFICATION OF PRICE: EMPIRICAL RESULTS

We now address the issue of price specification with an econometric analysis of the 1975 survey of 1502 households carried out by the Washington Center for Metropolitan Studies (WCMS) for the Federal Energy Administration. Individual household locations (identified at the level of primary sampling units) permitted matching

of actual rate schedules used in 1975 to each household. The use of disaggregated data is necessary to avoid the confounding effects of misspecification due to aggregation bias or due to approximation of the rate data.

We resolve four empirical issues related to the estimation of the demand for electricity: (1) measured average price and measured marginal price are statistically endogenous so that least squares techniques are not appropriate for the determination of price elasticities, (2) while the rate structure premium adjustment has established theoretical merit its statistical contribution is negligible, (3) consumer behavior in the demand for electricity follows the marginal price specification rather than the average price specification, and (4) estimates of price responsiveness are not statistically different using the tail-end price rather than the true marginal rate.

#### 1. Endogeneity of Measured Prices

The general proposition is that explanatory variables which utilize the observed consumption level introduce correlation between those variables and the error term. To illustrate the direction of least squares estimation bias write the demand for electricity equation as  $Q = \beta p + Z\delta + \varepsilon$  where  $p$  is the measured marginal price with coefficient  $\beta$ ,  $Z$  is a vector of socioeconomic variables with coefficient vector  $\delta$  and  $\varepsilon$  is the equation error. For simplicity assume that  $p$  is uncorrelated with  $Z$  so that  $\hat{\beta}_{LS} = \beta + p' \varepsilon / p' p$ . An unobserved increase in electricity consumption induces a decrease in

price so that we expect an a priori negative correlation between  $p$  and  $\varepsilon$ . The formula for  $\hat{\beta}_{LS}$  shows that least squares over estimates in absolute magnitude the price-response coefficient  $\beta$ .<sup>3</sup>

McFadden (1978) and Hausman et al. (1979) have demonstrated that an instrumental variable estimation technique provides consistent estimates of the electricity demand equation where instruments are constructed utilizing predicted rather than actual consumption to determine measured prices. In forming predicted consumption levels all endogenous variables are purged from the set of explanatory variables. One must insure that the instruments so constructed are not exact linear combinations of the exogenous variables included in the demand for electricity equation. This is usually not a problem given the non-linearity of the rate schedule and given the existence of other prices which are exogenous. The tail-end block price, for example, will be used in exactly this role.

To establish empirical verification of the hypothesis of endogeneity of measured price we apply the specification test due to Wu (1973) and recently discussed in Hausman (1978). The methodology consists of isolating a group of explanatory variables whose endogeneity is under test. Using the result that the least squares estimator has zero asymptotic covariance with its difference from the instrumental variable estimator, we are able to form a simple statistic which is asymptotically chi-squared under the null hypothesis of statistical exogeneity for the test group.

To illustrate the test write the demand for electricity in



schematic form as  $Q = X\beta + Z\gamma + \varepsilon$  where  $X$  is a  $k$ -vector of price and income terms under various specifications and  $Z$  is a group of assumed exogenous variables. The variables in  $X$  are presumed to be suspect of endogeneity. The test statistic is then:

$$T = (\hat{\beta}_{IV} - \hat{\beta}_{LS})' [V[\hat{\beta}_{IV}] - V[\hat{\beta}_{LS}]]^{-1} (\hat{\beta}_{IV} - \hat{\beta}_{LS}) \sim \chi^2(k)$$

where  $\hat{V}$  is the estimated variance covariance matrix and  $k$  is the number of coefficients in  $\beta$ .

The dependent variable in each estimated equation is monthly consumption of kilowatt hours of electricity used by the family in 1975. The socioeconomic variables include appliance ownership dummies for the electric dishwasher, electric washing machine, food freezer, electric range, color television, black and white television, electric clothes dryer, and central air conditioner. To capture the effects of climate, the annual number of cooling degree days (the number of days in which the daily average temperature was greater than 65°) and this number multiplied by respectively the central air conditioner dummy and the number of room air conditioners were included, as well as scale variables for the number of rooms, the number of persons, and the number of room air conditioners.<sup>4</sup>

Price terms included the average price, measured marginal price and the tail-end block rate. These rates are used below in various combinations and are taken from the rate schedules prevailing in the winter of 1975.

[Insert Table 1 and 2 here]

In Table 1 we present the mean values of all variables. To demonstrate the bias induced by least squares under the marginal price specification we compare the least squares and instrumental variable estimates of the equation:  $Q = \alpha(\text{measured marginal price}) + Z\delta + \varepsilon$ . For brevity we report the coefficient estimates on the variables: measured marginal price, income, electric water heat and electric space heat in Table 2. At sample means the price elasticity implied by least squares is  $-0.266$  while the instrumental variable estimates imply a price elasticity of  $-0.159$ . The direction of the bias agrees with our a priori expectation that least squares will overestimate in magnitude the price sensitivity coefficient.

Taylor (1975) reports both short-run and long-run price and income elasticities. Of nine estimates of residential elasticities two used marginal price. Each of the studies by Houthakker (1951a, 1951b) reports short-run elasticities of approximately  $-0.90$ .<sup>5</sup> Both our least squares and instrumental estimates are well below this estimate in magnitude but are entirely consistent with other estimates of electricity demand price elasticity using an average price specification.<sup>6</sup>

The Hausman statistic for the endogeneity test of measured marginal price is computed to be 34.18. This well exceeds the critical value for a Chi-squared test of any size given the single degree of freedom. We note that the respective income elasticities for least squares and instrumental variables are 0.118 and 0.109. Both

estimates are consistent with those obtained in previous studies.

If the same test is repeated using measured average price in place of measured marginal price we find price elasticities for least squares and instrumental variables of respectively -0.437 and -0.416. Note that the direction of bias is the same as that obtained with measured marginal price—a general increase in price sensitivity magnitude. Income elasticities were robustly estimated at 0.120 and 0.104 for the two procedures. The Chi-squared statistic was computed in this case to be 118.2 which well exceeds the critical value of 3.84 for a 5 percent test. Parameter estimates for the average price specification are reported in Table 3.

[Insert Table 3 here]

In summary we remark that previous studies in the demand for electricity have undoubtedly been subject to the bias illustrated above. The bias has been demonstrated to be statistically significant for the two most common specifications of price and is qualitatively impressive on the order of 67 percent.<sup>7</sup>

## 2. Rate Structure Premium Adjustment

From Table 1 we see that the mean value of rate structure premium is \$3.12 compared to the mean value of income of \$1321/month. The negligible value of RSP as compared to INCOME implies that the difference (INCOME - RSP) could not be distinguished from general measurement error in the definition of monthly income. In Table 4 we present instrumental variable estimates of the electricity demand

equation using the marginal price specification and income adjusted by the rate structure premium.

[Insert Table 4 here]

Comparison of the estimates in Table 4 with estimates given in Table 2 for instrumental variables demonstrates the qualitative similarity. Based on these results we do not advocate the rate structure premium correction to income in the WCMS data for 1978. This confirms the findings of Hausman et al. (1979) for insignificance of the RSP adjustment.

## 3. Average versus Marginal Price

Estimation in demand for electricity studies has followed the predominant usage of either marginal or average price. A simple observation will allow us to nest both the marginal and average price specification in a more general model. We have demonstrated above that the difference between measured average price and measured marginal price is the rate structure premium divided by measured consumption. Hence an unrestricted specification of marginal and average prices has the form:  $Q = (\text{measured marginal price})\alpha_0 + (\text{Rate structure premium/Quantity})\alpha_1 + Z\delta + \epsilon$ . Clearly when  $\alpha_0$  equals  $\alpha_1$  we have the average price specification. When  $\alpha_1 = 0$  we have the marginal price specification.

[Insert Table 5 here]

Ordinary least squares and instrumental variable estimates for the unrestricted model are presented in Table 5. For brevity we report only the coefficient estimates of measured marginal price, rate structure premium/quantity, income, WHE, and SHE. The Hausman statistic of 83.8 with the two degrees of freedom confirms the endogeneity of the explanatory variables measured marginal price and rate structure premium/quantity.

Using the instrumental variables estimates in Table 5 we compute a Wald test of the hypothesis that the coefficients of measured marginal rate and rate structure premium/Q are equal. The test statistic which compares the difference in the estimated coefficients has a value of 7.09 and is distributed chi-squared with one degree of freedom (the number of imposed restrictions). We thus reject the average price specification at the 1 percent critical level. Furthermore the individual t-statistics for the coefficients of measured marginal price and RSP/Q confirm the marginal price specification as the former coefficient is significant while the latter is insignificant at the 5 percent level.<sup>8</sup> It is interesting to note that inspection of the least squares estimates would lead one to choose the average price specification over the marginal price specification. Given the differential in sum of squared residuals for the measured marginal price and average price specifications (using the consistent estimates in Tables 2 and 3 respectively) it is likely that a non-nested test (see Pesaran (1974) for example) would also discriminate between the two models. We are thus led to conclude that

consumer behavior in the demand for electricity follows the marginal price specification rather than the average price specification.

#### 4. Measurement Error in Marginal Price

We now consider the impact of the measurement-error misspecification which results from the use of the tail-end rate in place of the measured marginal rate. In Table 6 we reproduce the least squares regression results for this specification. Note that least squares estimation provides consistent parameter estimates since the tail-end price is by definition exogenous. The use of the tail-end rate in place of the measured marginal rate introduces measurement error in the price variable. However it is not appropriate to apply the usual measurement error bias formulae since price is expected to reveal significant correlation with the other explanatory variables and since the difference between the two measures of price is not a mean zero random disturbance.

[Insert Table 6 here]

Comparing the estimate of the tail-end price coefficient in Table 6 with the consistent estimate of the measured marginal price coefficient in Table 2, we see that relative to the standard error the difference is not significant. ( $t = ((-6006.) - (-6828.))/1837. = 0.45$ ). This result is confirmed through the inspection of the variables WMPE75 and RATE; the correlation coefficient between the two variables is 0.87 and the mean difference is approximately one-third of a standard deviation. While there is no specific suggestion that

the rate schedules in the WCMS data are flat, these estimates suggest that many individuals are close to the tail-end of the rate schedule so that measured marginal rates are well approximated by the tail-end price.<sup>9</sup>

#### IV. SIMULATION OF DEMAND WITH ALTERNATIVE RATE STRUCTURES

We now consider the problem of determining demand response subject to non-marginal changes in the underlying budget constraint set. In particular we attempt to estimate changes in expected demand, revenue, etc. when consumers are shifted from declining block to inverted block rate structures. The inverted block structure has gained increasing prominence in recent years in an attempt to guarantee the poor a level of electricity consumption which would maintain a minimum standard of living in the face of rising electricity prices.

In this section we follow recent empirical studies which indicate the importance of population taste variation. In the labor supply context, for example, small R-squareds are symptomatic of the inadequacies of explaining observed hours of work under the assumption of a representative individual. This source of variability in electricity demand is expected to be particularly important when considering alternative rate structures aimed at benefiting the low income segment of the population.

We follow the approach of Burtless and Hausman (1978) and Hausman (1978) and allow the income parameter to be randomly

distributed across individuals. The objective of their studies was the determination of labor supply with non-linear net wages. Hall (1973) had demonstrated the usefulness of the net wage approach and defined the concept of virtual income which is identical to our definition of income less rate structure premium.

To compare budget segments, an indirect utility function,  $V[p, y; \beta]$ , in prices, income, an unknown income parameter  $\beta$ , is found which is consistent with the assumed demand specification. Each value of  $\beta$  between zero and  $+\infty$  gives rise to an associated global optimum level of electricity demand given a particular rate structure. The probability that desired demand lies in a given range is thus equivalent to the probability of an associated range in  $\beta$ . Actual demand is assumed to differ from desired demand by an additive disturbance  $\eta$ .

In the non-convex budget case consisting of two segments, Burtless and Hausman (1978) demonstrate that desired demand will fall in the steeper budget segment for  $0 \leq \beta \leq \beta^*$  and will fall in the marginal segment for  $\beta^* \leq \beta$ . The parameter  $\beta^*$  denotes a point in the parameter space of equal indirect utility for both segments  $V[p_1, y_1; \beta^*] = V[p_2, y_2; \beta^*]$ . If  $F(\eta, \beta)$  denotes the cumulative distribution function for  $\eta$  and  $\beta$  and  $q_j$  denotes desired demand in the  $j^{\text{th}}$  segment ( $j = 1$  or  $2$ ) then the likelihood of observed demand  $q$  is

$$\text{Prob} = \int_0^{\beta^*} \int_{\eta} (q - q_1) dF[\eta, \beta] + \int_{\beta^*}^{+\infty} \int_{\eta} (q - q_2) dF[\eta, \beta]$$

The full-information maximum likelihood solution to convex and non-convex budget set estimation is implementable provided simple distributional assumptions are made concerning  $\eta$  and  $\beta$ . If more than one or two parameters are assumed to vary in the population, the requirement of evaluating multiple integrals over nonrectangular regions implies too complex a problem for maximum likelihood.

As an alternative to the maximum likelihood solution we consider consistent estimation of moments of the random parameter distribution. Write the demand equation as:

$$(6) \quad y = \sum_{j=1}^J \alpha_j X_j + X^* \beta^* + \varepsilon$$

where  $\alpha_j$  is the random coefficient of variable  $X_j$ ,  $\beta^*$  is a  $L \times 1$  column vector of non-random but unknown parameters for the variables  $X^*$ , and  $\varepsilon$  is an additive disturbance. We assume that  $\alpha_j - \bar{\alpha}_j = v_j$  where  $E[v_j] = 0$  and  $E[v_j^2] = \sigma_j^2$  and that  $E[\varepsilon] = 0$ ,  $E[\varepsilon^2] = \sigma^2$ . Under the maintained assumption that the  $X_j$  and  $X^*$  are uncorrelated asymptotically with the disturbances  $\varepsilon$  and  $v_j$ , we could proceed to estimate the parameters  $\bar{\alpha}_j$ ,  $\beta^*$ , and  $\sigma_j^2$ ,  $\sigma^2$  using the methods of Hildreth and Hauck (1968). The Hildreth-Hauck result is not applicable in the presence of stochastic regressors which we have demonstrated to be an important consideration in the specification of demand subject to a non-linear rate schedule.

Fortunately, a simple alternative to the Hildreth-Hauck method exists which does guarantee consistency in the presence of stochastic regressors. We present the results in Theorem 2.

**Theorem 2:** Consistent Estimation of the Random Parameter Model with Stochastic and Non-Stochastic Regressors.

Assume that  $v_j$  and  $\varepsilon$  are uncorrelated,  $v_j$  and  $X_j$  are uncorrelated,  $X^*$  non-stochastic, and  $\text{plim}(\frac{1}{T} X_j' \varepsilon) \neq 0$ .

(i) Equation (6) implies  $y = \sum_j \bar{\alpha}_j X_j + X^* \beta^* + v$  where  $v = \varepsilon + \sum_j v_j X_j$

$$E(v) = 0, \quad E[v^2] = \sigma^2 + \sum_j \sigma_j^2 X_j^2$$

(ii) Instrumental variables yields consistent estimates of the parameters  $\bar{\alpha}_j$  and  $\beta^*$ .

(iii) Consistent estimates of the variances  $\sigma^2$  and  $\sigma_j^2$  are obtained in an auxiliary regression of the squared residuals from instrumental variables on a constant term and the variables  $X_j^2$ .

**Proof of Theorem 2:** See Appendix.

**Remark:** While consistency of the estimates of  $\sigma_j^2$  and  $\sigma^2$  is guaranteed, standard errors given in the auxiliary regression procedure are incorrect. We implement this procedure for the specification given in Table 2 of section III:

$$(7) \quad Q = \alpha(\text{measured marginal rate}) + \beta(\text{income}) + Z\delta + \varepsilon \quad \text{where } \beta = \bar{\beta} + \varepsilon_\beta$$

[Insert Table 7 here]

Results of the auxiliary regression of squared fitted residuals are presented in Table 7. The estimates in Table 7 provide

an indirect method of decomposing the total variance into the variance of  $\varepsilon$  and the variance of  $(\varepsilon_\beta \cdot \text{Income})$ . From Table 7 we have:

$$(8) \quad \hat{\varepsilon}_t^2 = (0.8447E+15)1 + (0.1762E-1)(\text{Income}_t)^2 + \hat{\xi}_t^2$$

where  $\hat{\varepsilon}_t$  is the instrumental variable estimated residual and  $\hat{\xi}_t$  denotes the fitted residual in the auxiliary regression. Adjusting for degrees of freedom, equation (8) implies:

$$(9) \quad \frac{1}{(744-19)} \sum_t \hat{\varepsilon}_t^2 = (0.8447E+15)(744)/(744-19) + (0.1762E-1)(744)/(744-19)$$

$$\frac{1}{(744)} \sum_t (\text{Income}_t)^2 + 0 = \text{var}(\varepsilon) + \text{var}[\varepsilon_\beta \text{Income}].$$

Since the standard error of the regression in (7) is (355.6), and income has population mean (1322.0) and variance (.4508E+6) we find  $\text{var}[\varepsilon] = (.866837E+5)$  and  $\text{var}[\varepsilon_\beta \text{Income}] = (.39753E+5)$ . Over 30 percent of total variance is thus accounted for by randomness in the income taste parameter. Hausman (1981), by way of contrast, reports that virtually all unexplained variation in labor supply may be attributed to taste variation.

For the purpose of making explicit probability statements we assume that  $\beta$  is log-normally distributed so that  $\log \beta$  is a normal random variable with mean  $\mu$  and variance  $\lambda^2$ . The log-normal distribution has the desirable feature that  $\beta$  has positive support. This is consistent with the assumption that income is a normal good. Furthermore, the translation between the observed moments  $(\bar{\beta}, \sigma_\beta^2)$  and

the moments  $(\mu, \lambda^2)$  is accomplished in a simple calculation. An alternative assumption of the truncated normal distribution for  $\beta$  would not allow this simple translation and would be equally arbitrary.

Recall that the log-normal distribution for  $\beta$  implies  $\bar{\beta} = E(\beta) = \exp[\mu + \frac{1}{2}\lambda^2]$  and  $\sigma_\beta^2 = \text{var}(\beta) = (\exp(\lambda^2)-1)\exp(2\mu + \lambda^2)$ . From these we find:

$$(10) \quad \lambda^2 = \ln[\sigma_\beta^2 / \bar{\beta}^2 + 1] \text{ and}$$

$$(11) \quad \mu = \ln[(\bar{\beta}^2 / (\sigma_\beta^2 + \bar{\beta}^2))^{.5}]$$

From Table 2,  $\bar{\beta} = 0.0757$  and from Table 7,  $\sigma_\beta^2 = 0.01762$  so that we estimate  $\mu = -3.2834$  and  $\lambda^2 = 1.4048$ . The median of distribution for  $\beta$  occurs at  $\exp[\mu] = 0.0375$  which indicates the direction of skewness relative to the mean of 0.0757. A simple result shows that approximately 71 percent of  $\beta$  distribution lies below the mean value.

Having determined the  $\beta$  distribution we may now make probabilistic comparisons between alternative rate structures.

The results of Dubin and McFadden (1982) demonstrate that the indirect utility function

$$(12) \quad V[p_i, y_i] = e^{-\beta p_i} [y_i + (\alpha/\beta)p_i + \alpha/\beta^2 + Z\delta/\beta]$$

is consistent with the demand equation

$$(13) \quad X_i = -V_{p_i} / V_{y_i} = \beta y_i + \alpha p_i + Z\delta$$

The coefficients and explanatory variables in (13) are chosen to be consistent with the specification of desired demand given in (7). Price  $p_i$  and income  $y_i$  are defined below to follow actual budget segments. Using (12) and (13) we note that

$$(14) \quad V[p_i, y_i] = e^{-\beta p_i [X_i/\beta + \alpha/\beta^2]}$$

The distribution of the random variable  $V[p_i, y_i]$  induced by distribution of  $\beta$  is not readily calculated in closed-form given the complicated form of equation (14). We approximate (14) by a Taylor's series expansion of  $\beta$  around its observed mean  $\bar{\beta}$ :

$$(15) \quad V[p_i, y_i; \beta] - V[p_i, y_i; \bar{\beta}] = \frac{\partial V[p_i, y_i; \beta]}{\partial \beta} (\beta - \bar{\beta})$$

where the slope  $\partial V/\partial \beta$  is given by

$$(16) \quad \begin{aligned} \partial V[p_i, y_i; \beta]/\partial \beta &= e^{-\beta p_i [(y_i - X_i)/\beta^2 - 2\alpha\beta^{-3}]} \\ &+ e^{-\beta p_i [X_i/\beta + \alpha/\beta^2]} (-p_i) \\ &= (e^{-\beta p_i/\beta^3}) [\beta^2 (y_i - p_i X_i) - \beta (X_i + p_i \alpha) - 2\alpha] \end{aligned}$$

We now consider two probability calculations. In the first case consumers are assumed to choose between two segments of a declining two-part tariff. In the second case we compare inverted rate with declining block structures. The analysis and empirical results focus on the two-part tariff structure considered in Section II above.

#### CASE I. Declining Two-Part Tariffs

Recall that the declining two-part tariff assumes a price  $\pi_1$  for consumption up to and including an amount  $X$  and price  $\pi_2$  for any additional consumption. We want to find the probability that  $V_1$  is greater than  $V_2$  while taking into account the possibility that certain ranges of prices make either budget segment infeasible. Given the assumption of a declining tariff we note that  $\pi_2 < \pi_1$  so that:

$$(17) \quad \begin{aligned} \text{Prob}[V_1 \geq V_2] &= \text{Prob}[V_1 \geq V_2 | \pi_2 < \pi^* \leq \pi_1] * \text{Prob}[\pi_2 < \pi^* \leq \pi_1] \\ &+ \text{Prob}[V_1 \geq V_2 | \pi^* \leq \pi_2] * \text{Prob}[\pi^* \leq \pi_2] \\ &+ \text{Prob}[V_1 \geq V_2 | \pi^* > \pi_1] * \text{Prob}[\pi^* > \pi_1] \end{aligned}$$

where  $V_i = V[p_i, y_i; \beta]$  is defined in (15) with  $p_1 = \pi_1$ ,  $y_1 = y$ ,  $p_2 = \pi_2$ , and  $y_2 = y - (\pi_1 - \pi_2)X$ . (Here  $y$  without the subscript denotes total income.) The price  $\pi^*$  defined by  $D[\pi^*, y; \beta] = X$  is stochastic given the unknown distribution of  $\beta$ . To evaluate (17) we note that the first term is simply the joint probability  $\text{Prob}[V_1 \geq V_2 \text{ and } \pi_2 < \pi^* \leq \pi_1]$ . In the second term, the condition  $\pi^* \leq \pi_2$  implies  $V_1 \geq V_2$  with certainty since the second segment is not feasible. Similarly the third term is zero as  $V_1$  may not be selected when  $\pi^*$  is strictly larger than  $\pi_1$ . Thus (17) implies:

$$(18) \quad \text{Prob}[V_1 \geq V_2] = \text{Prob}[V_1 \geq V_2 \text{ and } \pi_2 < \pi^* \leq \pi_1] + \text{Prob}[\pi^* \leq \pi_2]$$

Let  $m_i = \partial V[p_i, y_i; \bar{\beta}]/\partial \beta$  and  $u_i = V[p_i, y_i; \bar{\beta}]$  so that the first term in (18) may be rewritten (using equation (15)) as:

$$\begin{aligned}
(19) \quad & \text{Prob}[V_1 \geq V_2 \text{ and } \pi_2 < \pi^* \leq \pi_1] \\
& = \text{Prob}[(m_2 - m_1)(\beta - \bar{\beta}) \leq (u_1 - u_2) \text{ and } \pi_2 < \frac{X - \beta y - Z\delta}{\alpha} \leq \pi_1] \\
& = \text{Prob}[(m_2 - m_1)(\beta - \bar{\beta}) \leq (u_1 - u_2) \text{ and } \frac{X - \hat{X}_2}{y} < \beta - \bar{\beta} \leq \frac{X - \hat{X}_1}{y}]
\end{aligned}$$

where we have defined  $\hat{X}_1 = \alpha p_i + \bar{\beta} y_i + Z\delta$ . We similarly find that the second term in (18) may be written as:

$$(20) \quad \text{Prob}[\pi^* \leq \pi_2] = \text{Prob}\left[\frac{X - \beta y - Z\delta}{\alpha} \leq \pi_2\right] = \text{Prob}\left[\beta - \bar{\beta} \leq \frac{X - \hat{X}_2}{y}\right].$$

When  $m_2 - m_1 > 0$ , (18) implies:

$$(21) \quad \text{Prob}[V_1 \geq V_2] = \text{Prob}\left[\frac{X - \hat{X}_2}{y} \leq \beta \leq \min\left(\frac{X - \hat{X}_1}{y}, \frac{u_1 - u_2}{m_2 - m_1}\right)\right].$$

When  $m_2 - m_1 < 0$ , (18) implies:

$$(22) \quad \text{Prob}[V_1 \geq V_2] = \text{Prob}\left[\max\left(\frac{u_1 - u_2}{m_2 - m_1}, \frac{X - \hat{X}_2}{y}\right) \leq \beta \leq \frac{X - \hat{X}_1}{y}\right]$$

When  $m_2 = m_1$  we note that  $V_1 \geq V_2$  as  $u_1 \geq u_2$ . Thus:

$$(23) \quad \text{Prob}[V_1 \geq V_2] = \begin{cases} 1 & \text{if } u_1 \geq u_2 \\ 0 & \text{otherwise} \end{cases}$$

As an application we consider three empirical examples. In the first example we assume that consumers choose between their observed block and an artificial block consisting of a flat rate set at the intra-marginal average price. Since the observed block is

feasible by definition and highly correlated with the desired block, we expect that the probability of block switching to be very low. To illustrate this example we examine the rate structure faced by individuals living in Boston, Massachusetts in 1975.

[Insert Figure 1A and 1b here]

Figure 1a plots the expenditure or outlay schedule while Figure 1b illustrates the declining block rate schedule. In our sub-sample of the WCMS data ten households are located in this primary sampling unit. In addition to the tariff prices, a fixed charge of \$1.43 is applied to the bill which appears in the non-zero intercept of Figure 1a. The symbol R in Figure 1a denotes a rate boundary; the symbol A denotes an observation. Table 8 lists the observed demands and corresponding expenditure for households in Boston, Massachusetts.



TABLE 8

	<u>Quantity</u>	<u>Expenditure</u>
1.	473	20.43
2.	334	15.29
3.	192	9.82
4.	300	14.04
5.	246	12.03
6.	298	13.96
7.	373	16.73
8.	526	22.38
9.	263	12.67
10.	374	16.77

Consider household #1 with consumption in the tail-end block and marginal price \$0.03693/KWH. The lower block boundary for this household is 250 KWH and the intra-marginal average price is \$0.04876 since  $(\$0.04876)(250)$  is the total cost of 250 KWH. In the notation of Section II we set  $\pi_1 = 0.04876$ ,  $\pi_2 = 0.03693$ ,  $y_1 = \text{Income}$ ,  $y_2 = (\text{Income} - (\pi_1 - \pi_2)X)$ , with  $X = 250$ . The population average value for  $\text{Prob}[V_1 \geq V_2]$  is 0.1688 with a standard deviation of 0.4025. In doing the calculation we note that  $\text{Prob}[V_1 \geq V_2]$  was evaluated by (21) in 1 of 744 cases, by (22) in 216 of 744 cases, and by (18) in the remaining 527 of 744 cases. In 79 of 744 cases, the calculated value of  $u_1$  was greater than  $u_2$ . We thus have found strong evidence that the observed block is the predicted optimal block.

As a second example consider the effect of a uniform increase of 30 percent in the observed lower block boundary,  $X$ , for each household. In this case we expect that a greater number of households will find that the first budget segment yields greater utility. If we repeat the probability calculation we find a population average value of 0.3382 for  $\text{Prob}[V_1 \geq V_2]$  with standard deviation of 0.5518. The 30 percent increase in lower block boundary therefore induces a 16.94 percent increase in the likelihood that individual will select the intra-marginal price over the observed marginal rate. While it is true that individual predicted probabilities will differ from the sample average values, we feel that the magnitude of the difference in population means is large enough to indicate a systematic behavioral shift.

As a final example we assume consumers choose between their observed block and the block immediately preceding. The probability calculation must insure that feasibility is respected at all times. For household #1, we would set  $\pi_1 = 0.04093$ ,  $\pi_2 = 0.03693$ ,  $y_1 = \text{Income}$ ,  $y_2 = (\text{Income} - (\pi_1 - \pi_2)X)$  with  $X = 250$ . The population average value for  $\text{Prob}[V_1 \geq V_2]$  is 0.1723 with standard deviation 0.4395 confirming the supposition that the observed block is not far from the desired block.

#### CASE II. Inverted Two-Part Tariff and Flat Rates

To compare declining block rates with inverted rates it would be useful to examine a time-series of individuals who are observed in

declining block and an inverted block regimes. Unfortunately, the 1975 WCMS survey includes only households facing declining block rate structures. Nevertheless, it is both possible and desirable to use the 1975 cross-sectional data to derive structural estimates of taste parameters which enable hypothetical probability comparisons.

We have selected primary sampling unit #111 in the WCMS survey which comprises Boston, Massachusetts for such a calculation. From Table 8, we note that mean consumption is 337.90 KWH per month with a standard deviation of 102.49 KWH. We choose a two-part inverted tariff with a single block boundary selected at approximately one-standard deviation below the mean value. The prices in the two-tariffs are selected so that the initial block has a rate approximately equal to the prevailing tail-end rate under declining block rates, and so that average consumption reproduces the average expenditure level of \$15.41 per month. Figure 2 presents the two-part inverted tariff schedule chosen for our calculations.

Corresponding to the linear approximation of equation (10) we define

$$(24) \quad W_i = z_i + f_i(\beta - \bar{\beta}), \quad i = 1, 2 \quad \text{where} \\ z_i = V[p_i, y_i; \bar{\beta}] \quad \text{and} \quad f_i = \partial V[p_i, y_i; \bar{\beta}] / \partial \beta.$$

(We use the symbols  $z_i$  and  $f_i$  as distinguished from  $u_i$  and  $m_i$  to denote the inverted tariff calculation.) The analysis of Section II indicates that  $W_1 > W_2$  as  $\pi_2 > \pi^*$  whenever  $\pi_2 \geq \pi_1$ ; the condition for an inverted two-part tariff.

Consider the probability of each segment having maximal utility when compared to the base case  $V_1 = u_1 + m_1(\beta - \bar{\beta})$ . The first budget segment will have maximal utility when  $\{W_2 > V_1 \text{ and } W_1 > W_2\}$ . The second budget segment in the inverted tariff will have maximal utility when  $\{W_2 > V_1 \text{ and } W_2 > W_1\}$ . The base case  $V_1$  has maximal utility in the complement of the union of the two above sets. We have:

$$(25) \quad \text{Prob}[W_1 > V_1 \text{ and } W_1 > W_2] \\ = \text{Prob}[(f_1 - m_1)(\beta - \bar{\beta}) \geq -(z_1 - u_1) \text{ and } (\beta - \bar{\beta}) \leq \frac{X - X_2}{y}]$$

where  $X_j = \alpha p_j + \bar{\beta} y_j + Z\delta$  and  $p_j$ ,  $y_j$ , and  $X$  are defined for the two-part inverted tariff. Similarly,

$$(26) \quad \text{Prob}[W_2 > V_1 \text{ and } W_2 > W_1] \\ = \text{Prob}[(f_2 - m_1)(\beta - \bar{\beta}) \geq -(z_2 - u_1) \text{ and } (\beta - \bar{\beta}) \geq \frac{X - X_2}{y}]$$

We have made these calculations for the ten households in Boston, Massachusetts using the hypothetical inverted two-part tariff of Figure 2 and observed marginal rates for the base case. The mean value of the probability in (25) is 0.70 with standard deviation 0.46. The mean value of the probability in (21) is 0.125 with standard deviation 0.183. The probability in the base case has mean value 0.1747 with standard deviation 0.517.

Finally we compute the probability that  $W_1$  is greater than  $W_2$  in a choice between the two segments. In this situation we have

$$(27) \quad \text{Prob}[W_1 > W_2] = \text{Prob}[\pi_2 > \pi^*] = \text{Prob}[(\beta - \bar{\beta}) \leq \frac{X - X_2}{Y}]$$

For the sample of ten households we find a mean value of 0.01897. The probabilities in (25), (26), and (27) are sufficient to characterize the population shares of desired demands and could be used to calculate expected demand, deadweight loss, revenue or other policy variables.

#### V. SUMMARY AND CONCLUSIONS

This paper has reviewed the theory and estimation of price specification in the demand for electricity. We have seen that the price space which parameterizes a quantity dependent rate schedule may be partitioned in such a way as to uniquely determine demand in each price tier. Furthermore it is possible to use the price partition to conduct gross and marginal comparative static analysis. This approach might be successfully applied to an empirical mapping of the marginal revenue function facing a given utility.

We have further explored the empirical specification of price in a class of electricity demand models and have demonstrated that (1) measured average price and measured marginal price are statistically endogenous, (2) the statistical contribution of the rate structure premium adjustment is negligible, (3) consumer behavior follows the marginal rather than the average price specification, and (4) estimated price elasticities are not significantly different using the tail-end price in place of the measured marginal rate.

Finally, we have demonstrated a practical way of comparing alternative rate structures in their impact on demand response. The methodology was applied in a comparison of a prevailing rate structure with a hypothetical inverted structure in order to determine the individual tier probability shares.

TABLE 1

<u>VARIABLE NAME</u> <sup>a</sup>	<u>DESCRIPTION</u>	<u>MEAN</u>
AKWH75	monthly consumption of electricity in 1795	916.5
RATE	measured marginal price in 1975	.02427
AVPRICE	measured average price in 1975	.03128
WMPE75	winter tail-end block price for electricity in 1975	.02138
INCOME	monthly income of household head	1322
RSP	measured rate structure premium	5.151
WHE	electric water heat dummy	0.2728
SHE	electric space heat dummy	0.1411
ROOMS	number of rooms in household	6.078
PERSONS	number of persons in household	3.550
CAC	central air-conditioning dummy	0.2890
CDDCAC	(annual cooling degree days) * (CAC)	463.7
RACNUM	number of room air-conditioners	.4382
CDDRACNUM	(annual cooling degrees days) * (RACNUM)	642.3
AUTOWSH	automatic washing machine dummy	0.8898
AUTODSH	automatic dishwasher dummy	0.4921
FOODFRZ	food freezer dummy	0.5323
ELECRNGE	electric range dummy	0.6411
ECLTHDR	electric clothes dryer dummy	0.4990
BWTV	black and white television dummy	0.5806
CLRTV	color television dummy	0.7446

<sup>a</sup>A subsample of the original 1502 observations was selected so that all price and income data were positive and so that complete information was available for each individual.

TABLE 2

<u>VARIABLE</u> <sup>a</sup>	<u>LS ESTIMATES</u>	<u>IV ESTIMATES</u>
Measured Marginal Price	-10050. (-5.909) <sup>b</sup>	-6006. (-3.269)
Income	.08169 (3.330)	.07570 (3.071)
WHE	405.6 (10.22)	404.5 (10.15)
SHE	694.8 (14.08)	714.9 (14.40)
R <sup>2</sup>	.7074	.7051
Number of Observations	744	744
Sum of Squared Residuals	.9094E+8	.9166E+8
Standard Error of Regression	354.2	355.6

<sup>a</sup>In Tables 2-6 coefficient estimates are not reported for the variables: PERSONS, BWTV, ROOMS, RMCLCAC, CDDCAC, CAC, RACNUM, CDDRACNUM, FOODFRZ, ELECRNGE, CLRTV, ECLTHDR, AUTODSH, AUTOWSH, and the intercept. The dependent variable is AKWH75.

<sup>b</sup>t-statistics presented in parentheses.

TABLE 3

<u>VARIABLE</u>	<u>LS ESTIMATES</u>	<u>IV ESTIMATES</u>
Average Price	-12810. (-8.731)	-4266. (-2.563)
Income	.08304 (3.484)	.07221 (2.959)
WHE	388.8 (10.05)	398.1 (10.06)
SHE	669.2 (13.90)	719.6 (14.56)
R <sup>2</sup>	.7225	.7095
Number of Observations	744	744
Sum of Squared Residuals	.8626E+8	.9029E+8
Standard Error of Regression	344.9	352.9

TABLE 4

<u>VARIABLE</u>	<u>IV ESTIMATES</u>
Measured Marginal Price	-6006. (-3.269)
NETINC	.7560E-01 (3.067)
WHE	404.5 (10.15)
SHE	715.0 (14.40)
R <sup>2</sup>	.7050
Number of Observations	744
Sum of Squared Residuals	.9167E+8
Standard Error of Regression	355.6

TABLE 5

<u>VARIABLE</u>	<u>LS ESTIMATES</u>	<u>IV ESTIMATES</u>
Measured Marginal Rate	-10130. (-6.158)	-6430. (-3.352)
Rate Structure Premium/Q	-22410 (-7.236)	10040. (1.777)
Income	.07702 (3.248)	.07846 (3.068)
WHE	374.9 (9.717)	418.4 (9.961)
SHE	673.6 (14.10)	722.1 (14.00)
R <sup>2</sup>	.7271	.6840
Number of Observations	744	744
Sum of Squared Residuals	.8481E+8	.9823E+8
Standard Error of Regression	342.3	368.3

TABLE 6

<u>VARIABLE</u>	<u>LS ESTIMATES</u>
WMPE75	-6828. (-3.644)
Income	.08299 (3.277)
WHE	414.1 (10.26)
SHE	721.7 (14.51)
R <sup>2</sup>	.6988
Number of Observations	744
Sum of Squared Residuals	.9361E+8
Standard Error of Regression	359.3

TABLE 7

## AUXILIARY REGRESSION OF SQUARED FITTED RESIDUALS

ORDINARY LEAST SQUARES

<u>VARIABLE</u>	<u>ESTIMATED COEFFICIENT</u> <u>(Standard Error)</u>
Constant	0.8447E+15 (0.2827E+5)*
(Income) <sup>2</sup>	0.1762E-1 (0.9540E-2)*
R-Squared	0.458E-2
Sum of Squared Residuals	0.1984E+15
Number of Observations	744

\*Standard errors are not corrected.

FIGURE 1a

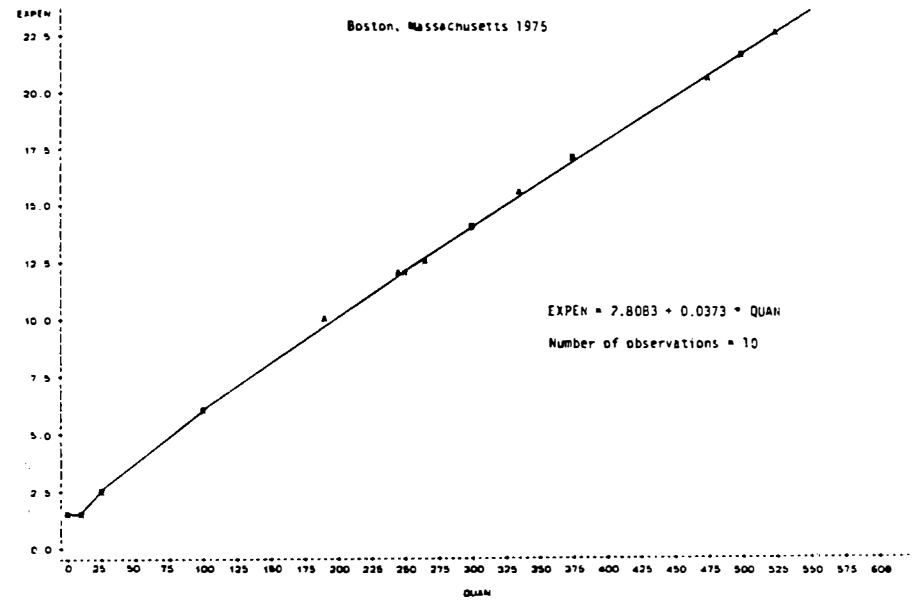


FIGURE 1b

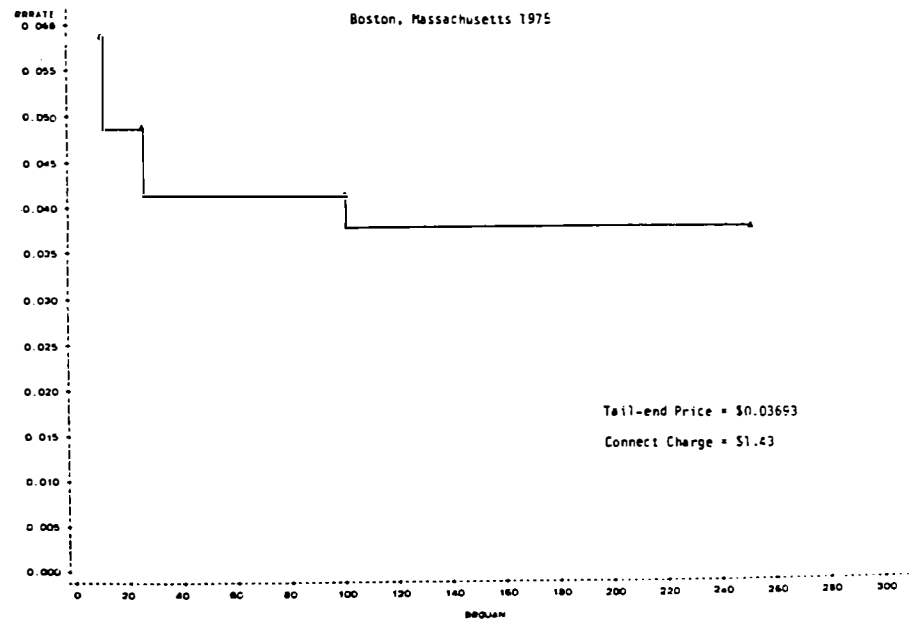


FIGURE 2

HYPOTHETICAL INVERTED RATE SCHEDULE FOR BOSTON, MA.

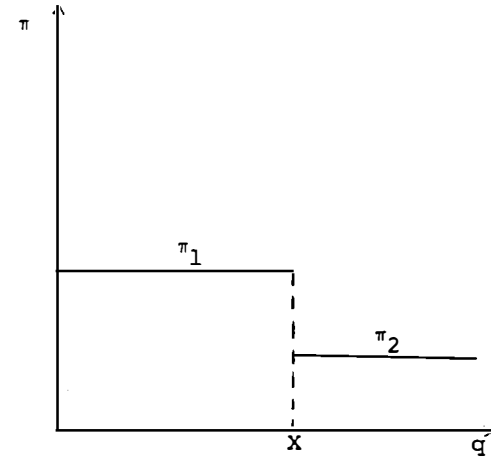
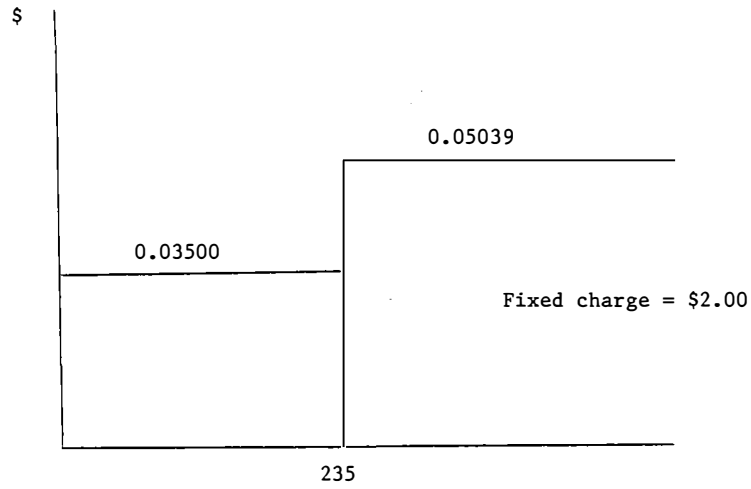


Diagram 1

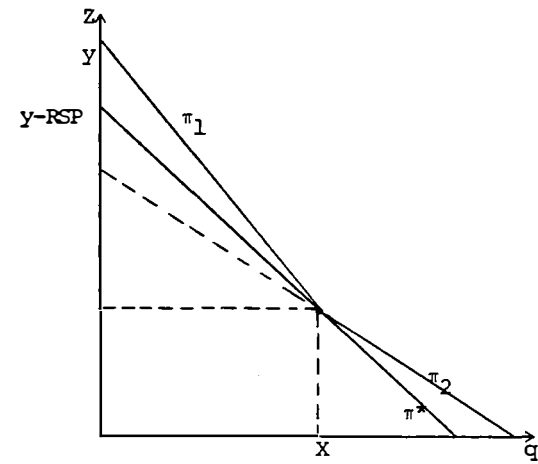


Diagram 2



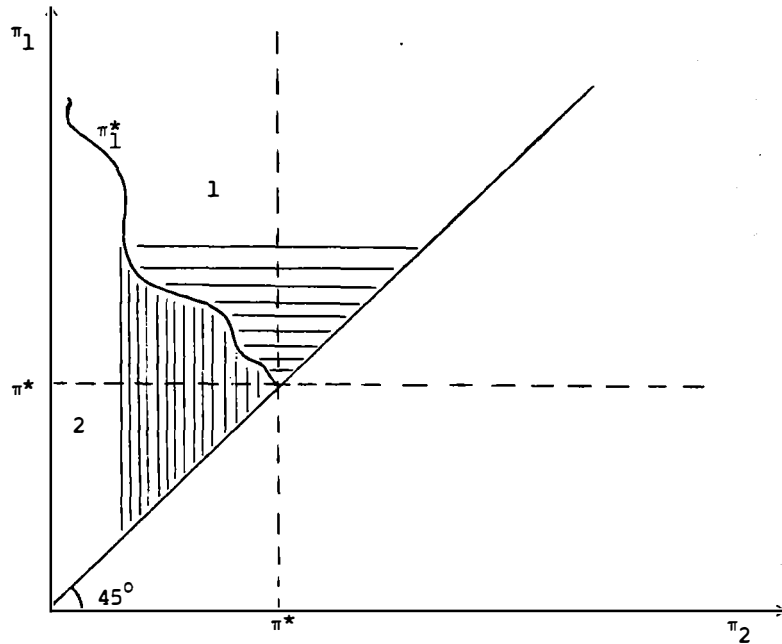


Diagram 3

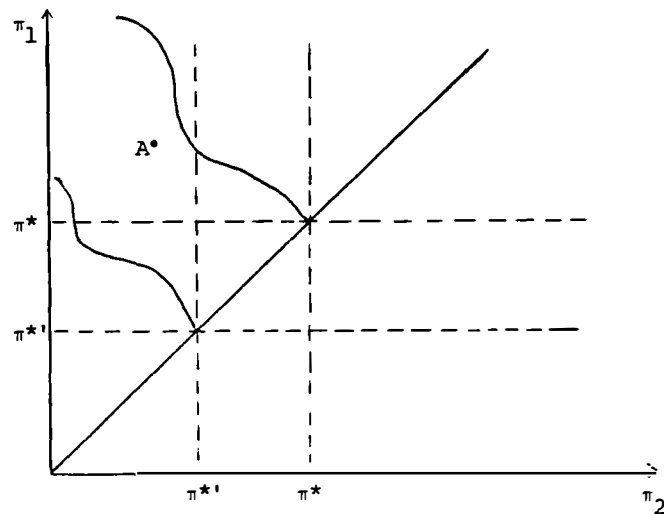


Diagram 4

FOOTNOTES

1. The author wishes to thank Ernst Berndt, Daniel McFadden, Franklin Fisher, and Jerry Hausman for helpful comments. This paper revises and extends a chapter from the author's M.I.T. Ph.D thesis, "Economic Theory and Estimation of the Demand for Consumer Durable Goods and their Utilization: Appliance Choice and the Demand for Electricity," Massachusetts Institute of Technology. An earlier version of this paper circulated under the title, "Rate Structure and Price Specification in the Demand for Electricity."
2. A source of bias not discussed in this paper arises from the endogeneity of appliance ownership dummies. Generally, unobserved factors which influence the choice of a durable will also influence its use. For a complete discussion of this problem see Dubin and McFadden (1979) who find evidence that this leads to under estimates (in magnitude) of the true price effects.
3. This result is further true when  $p$  is correlated with  $Z$ . However, it is not in general possible to determine the magnitude of the bias when several explanatory variables are correlated with the error term.
4. Issues of specification are considered in Dubin (1982).
5. The rate schedule in Houthakker's study consisted of a connect

charge and a fixed marginal price. The marginal price elasticity estimated by Houthakker is not tainted by simultaneity bias.

6. Studies by Acton, Mitchell, and Mowill (1976) and Taylor, Blattenberger, and Verleger (1977), find short-run price elasticities from  $-.08$  to  $-.35$  with endogenous marginal price specifications.
7. The bias for the average price specification is not as large at approximately 5 percent.
8. We have rejected the null hypothesis that demand for electricity follows the average price specification. This, of course, is not equivalent to accepting the marginal price specification. However, given the sign change on the coefficient of  $(RSP/Q)$  and its standard error we cannot reject the marginal price specification.
9. Using the results of Section II, we have seen that  $RSP = X(\pi_1 - \pi_2)$ . As average price less marginal price equals  $RSP/Q$  we have average price  $= \pi_2 + (X/Q)(\pi_1 - \pi_2)$ . For  $Q \gg X$ , average price and marginal price will be approximately equal. Thus individuals with demand much larger than the tail-end block will have approximately equal marginal, average, and tail-end prices.
10. For further details concerning life-line rates the reader may consult Berg and Roth (1976).

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