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ASYMMETRIC ARBITRAGE AND NORMAL BACKWARDATION

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ABSTRACT

This paper provides a theoretical explanation for the existence of backwardation on the futures markets, based on Houthakker's work dealing with asymmetry of arbitrage on such markets. The central assumption of the paper is that cash and futures prices tend to be more highly correlated at low than at high cash prices. This assumption reflects the asymmetry in arbitrage opportunities in futures markets; in particular, at the maturity date of a futures contract, the futures price cannot exceed the cash price of any grade-location combination deliverable under the futures contract. The main result of the paper is a proposition that asserts that with identical long and short hedgers, with the same wheat commitments on both sides of the market, and with utility functions exhibiting constant or decreasing absolute risk aversion, if the probability density function over cash and futures prices is sufficiently concentrated at low cash prices, then the resulting market equilibrium will exhibit backwardation, that is, the current future price is a downward biased estimator of the future futures price as well as being a downward biased estimator of the future cash price.

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INTRODUCTION

The central role played by contingent claims markets in the optimal allocation of resources under uncertainty is a dominant theme in the theoretical literature of modern welfare economics. In contrast, the operational characteristics of their real-world counterparts, futures markets, are less well-understood. One of the intriguing and controversial issues concerning futures markets is that of the existence of backwardation in the pattern of futures prices. While ultimately, the presence or absence of backwardation is an empirical matter, the occurrence of backwardation as a theoretical matter remains to be completely developed. This paper attempts to clarify the theory of backwardation as applied to a commodity such as wheat. We begin by briefly reviewing the backwardation literature and the way in which the term backwardation itself has evolved over time.

The notion of "normal backwardation"¹ first appears in a letter written by Keynes (1923) but his best known comment appears in the Treatise on Money (1930).

If the supply and demand are balanced, the spot price must exceed the forward price by the amount which the producer (of a commodity) is ready to sacrifice in order to hedge himself i.e., to avoid the risk of price fluctuations during the production period. Thus in normal conditions, the spot price exceeds the forward price, i.e., there is backwardation (vol. II, p. 143).

Keynes offered no theoretical explanation for the premise that only producers hedge, as opposed to both producers and purchasers. In Value and Capital, Hicks provides such an explanation, relying on substitution possibilities available to purchasers but not to producers.

. . . technical conditions give the entrepreneur a much freer hand about the acquisition of inputs (which are largely needed to start new processes) than about the completion of outputs. . . . Thus, while there is likely to be some desire to hedge planned purchases, it tends to be less insistent than the desire to hedge planned sales (p. 137).

According to the Keynes-Hicks formulation, producers dominate hedging as short hedgers (long in the spot market, short in the futures market) and pay the amount of the backwardation (spot price greater than forward price) to avoid price risks.

The next important development in backwardation involves the Kaldor-Dow exchange. Kaldor (1939), accepting the Keynes-Hicks formulation, assumes all hedgers are short hedgers and derives backwardation as an equilibrium condition on the futures market. Commenting on Kaldor's work, Dow (1940) emphasizes that both long and short hedgers are present in commodity markets but that short hedgers should dominate. Dow regards backwardation as the normal state of affairs, primarily because existing stocks must be held by someone.

Hence, they pose inevitable risks that are available to be hedged. In contrast, long hedging arises only when purchasers of a good for future delivery decide to engage in such commitments prior to the delivery date. In his rejoinder, Kaldor (1940) admits that long hedging could dominate.

. . . in cases where the technical uncertainties associated with production are much greater in the stages of production prior to the stage where the futures market is situated than in subsequent stages (p. 197).

The idea here is that hedges engaged in by an early stage producer to avoid price risks can expose the producer to quantity risks due to uncertainties concerning production. Hence, such technical uncertainties would tend to discourage short hedging. The notion of quantity risk also is emphasized by Hirshleifer (1975).

We should also note that the Kaldor-Dow exchange introduces a concept of backwardation differing from the Keynes-Hicks notion in important respects. For Kaldor, backwardation means that the futures price lies below the expected spot price as of the maturity date of the futures contract. In another matter of central importance to the backwardation debate, Dow draws a clear distinction between futures and forward contracts.

Houthakker (1959) introduces another notion of backwardation, namely, backwardation exists when the futures price is a downward biased estimator of its price at maturity of the futures contract, as well as being a downward biased estimator of the spot price at that maturity date. Thus if backwardation exists in a market, on average

the futures price should rise over time. In a later paper, Houthakker (1968) rejects Hicks' notion of differential substitution possibilities between producers and purchasers as the argument for backwardation. Houthakker's alternative explanation rests on the asymmetry of arbitrage situations confronting short and long hedgers. Short hedgers have a limited risk because the futures price cannot exceed the spot price by more than carrying charges, but long hedgers have no such protection.

As a result of this asymmetry (in arbitraging), short hedgers have a limited risk, while long hedgers have an unlimited risk, of adverse changes in the basis (p. 196).

The limited risk enjoyed by short hedgers encourages such hedging relative to long hedging. Houthakker also notes that short hedging increases as the basis (the difference between the futures and the spot prices) increases, and as inventories rise, since large inventories are associated with large values of the correlation coefficient between spot and futures prices. Thus short hedging tends to peak several months after harvest, during the period of low cash prices (since large inventories occur at low cash prices as a result of burgeoning supply), and reaches a trough around harvest time. Long hedging is not so seasonal.²

We believe the most convincing argument for backwardation rests on Houthakker's claim that asymmetric arbitrage leads to an imbalance of hedging, with short hedgers dominating long hedgers.³ The resulting market equilibrium, (\bar{p}_0^f, \bar{N}) in Figure 1, is one in which the

current futures price, p_0^f , is depressed below its expected market value at time of maturity, $E(p_1^f)$, where curve D is the sum of demand for futures contracts by speculators and long hedgers while curve S_S represents the supply of futures contracts by short hedgers.

We attempt to spell out in some detail certain conditions under which asymmetric arbitrage leads to backwardation. We do this in a simplified two-period model of short and long hedging. Admittedly, some of the richness of the Houthakker approach is lost. However, by constructing a formal model of the hedging process, it is possible to identify the critical importance of Houthakker's assumption that cash and futures prices tend to be more highly correlated at low than at high cash prices.

We show that under this assumption with short and long hedgers having the same commodity commitments and identical utility functions characterized by constant or decreasing absolute risk aversion, the resulting market equilibrium exhibits backwardation. In contrast, when perfect hedges are possible, backwardation does not occur.

SPOT, FORWARD, AND FUTURES MARKETS

Hedging and speculation occur within the structure of existing commodity markets. Spot (immediate delivery) and forward delivery markets for a commodity like wheat exist at every facility capable of storage.⁴ In principle, these "cash markets" could exist at every farm producing wheat as well. In contrast, the futures market is one central market, as illustrated by the market for Chicago wheat

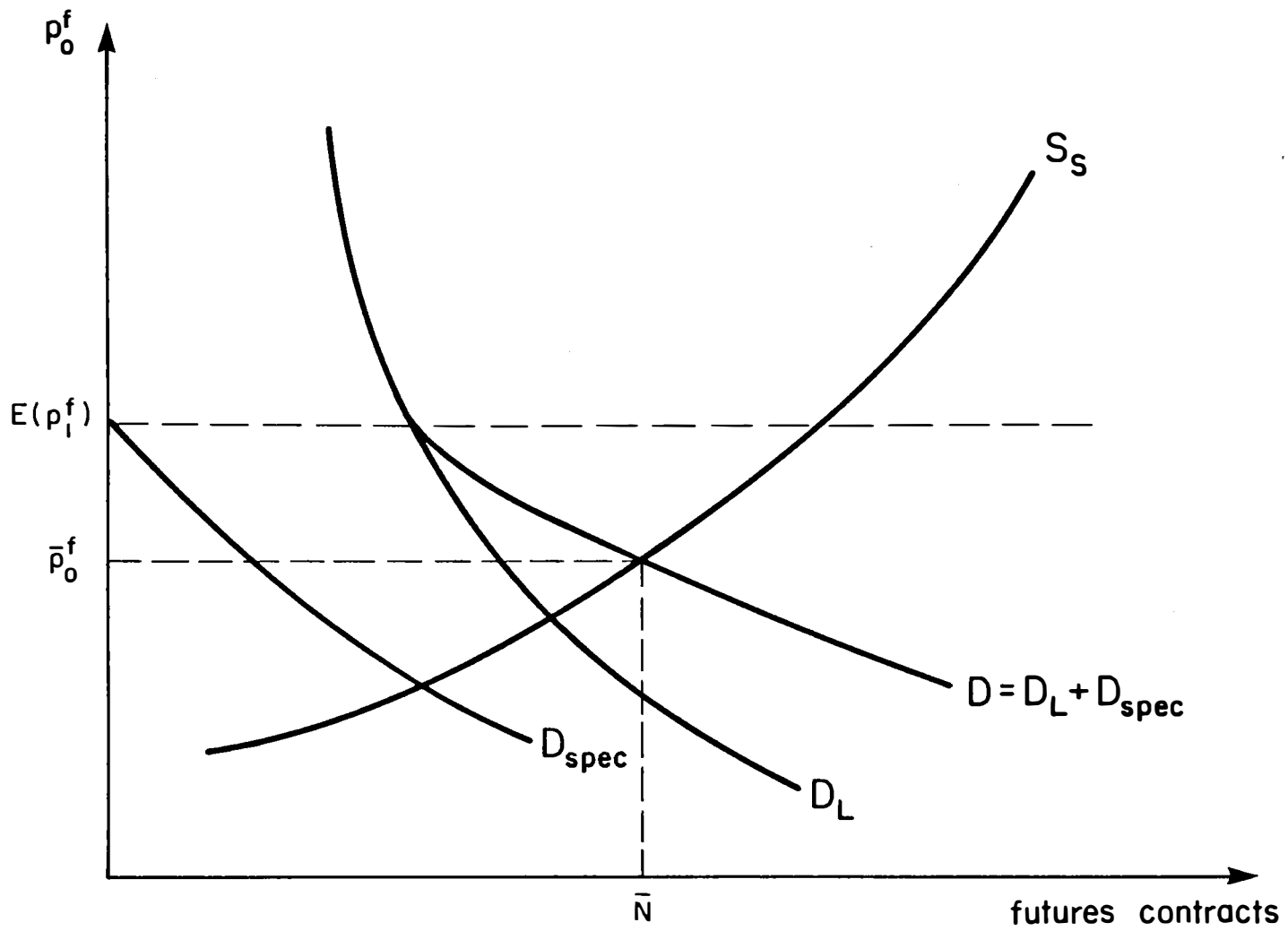


FIGURE I. Backwardation Equilibrium

futures. The crucial difference between a futures contract and cash contracts is that the futures contract provides flexibility with respect to delivery terms to the seller of the contract. The seller has the choice of the date during the delivery month to make delivery, he has the choice of the grade to deliver (subject to contractual premiums or penalties for nonstandard grades), and he has the choice of the delivery location, from among a list of locations specified in the contract. This flexibility of delivery terms is an essential ingredient in avoiding problems of thinness or cornering of markets, but it also creates problems of uncertainty as to delivery terms for the buyer of a futures contract. For this reason, delivery rarely takes place under futures contracts; instead, cash or forward contracts are typically used when actual transfer of a commodity is contemplated.

In a world of competitive markets operating under perfect certainty, the time paths of spot, forward, and futures prices would be simple to describe. In the inter-harvest period, the spot price of wheat must rise each month by the cost of storage (including interest, insurance, and warehousing costs), with the price of wheat falling when the new harvest comes in, assuming no carryover. At any point in time, the quoted forward and futures prices would be simply the corresponding spot prices that would prevail at the dates in the future when these forward or futures contracts would mature. (See Samuelson (1957)).

When uncertainty is introduced into the picture, things are

not at all so straightforward. However, certain basic arbitrage relationships among markets can be identified. As a matter of notation, let L denote the set of locations where markets exist and/or where delivery of wheat can take place, and let $L^* \subseteq L$ denote those locations specified for delivery under the futures contract. Let I denote the set of grades of wheat, and let $I^* \subseteq I$ denote those grades deliverable at no penalty or premium under the futures contract.⁵ Let p_{ijt}^c denote the cash price of wheat of grade i at time t , for delivery at location j . Let p_{st}^f denote the price at time t of a futures contract that matures at time s . To keep things manageable, we ignore the flexibility as to delivery date in the futures contract. Let r_{jkt} denote the cost of shipping a bushel of wheat between locations j and k , at time t ; and let $w_{j(uv)t}$ denote the public warehousing charge (including interest and insurance) at location j , for storing a bushel of wheat between times u and v , the charge being quoted at time t .

Then we can identify the following arbitrage relations:

$$\text{cash to cash: } |p_{ijt}^c - p_{ikt}^c| \leq r_{jkt}; \quad (1)$$

cash prices for the same grade i at any two locations j and k at time t cannot differ by more than the transportation cost between the two locations.

$$\text{futures to futures: } p_{st}^f - p_{ut}^f \leq w_{j(us)t} \text{ for } j \in L^*, s > u; \quad (2)$$

the price at time t of a futures contract maturing at time s cannot exceed the price at time t of a futures contract maturing at time u by

more than the cost at time t of warehousing a unit of the commodity between times u and s , at a location specified for delivery under the futures contract.

$$\text{futures to cash: } p_{st}^f - p_{ijt}^c \leq w_j(ts)t \quad \text{for } j \in L^*, \quad (3)$$

$s > t$ and $i \in I^*$; the price at time t of a futures contract maturing at time s cannot exceed the cash price at time t , of a grade i and a location j eligible for delivery under the futures contract, by more than the cost at time t of warehousing a unit of the commodity between times t and s at location j .

$$\text{futures to cash: } p_{tt}^f = \min_{i,j} p_{ijt}^c \quad \text{for } i \in I^*, j \in L^*; \quad (4)$$

the price at time t of a futures contract maturing at time t is the minimum of the cash prices at time t taken over the grades and locations eligible for delivery under the futures contract.

A few comments are in order concerning arbitrage relations (1)-(4). Relation (1) permits a considerable amount of leeway in the spatial structure of prices. Arbitrage reduces but does not eliminate the added uncertainties facing producers, consumers, and merchants located at points that are not delivery points under the futures contract. The specialized knowledge possessed by country elevator operators as to how the local cash prices might be expected to vary relative to the cash prices at a terminal elevator delivery location might explain their readiness to "speculate on the basis."

Relations (2) and (3) are intertemporal arbitrage relations

that are asymmetric, that is, they operate in one direction only. These relations specify a maximum premium of a distant futures price over a near futures or a cash price, but no corresponding arbitrage operation is available to determine a maximum discount for the distant futures price relative to the cash price or the price of a near futures. Thus, in principle, the cash price at t can exceed the price at t of a futures maturing at s ($s \geq t$) by any amount. Extreme cases of discounts of futures under cash prices arise in the case of squeezes or corners.

Relation (4) reflects the incentives incorporated into a futures contract. Since the seller of a futures contract is free to choose the grade and location of delivery (from among the sets I^* and L^*), the terminal value of the futures contract is equal to the market value of the least cost grade-location combination eligible for delivery under the futures contract. It is clear from relation (4) that the case of a "perfect hedge," where the cash and futures prices always are equal at the maturity of the futures contract, is the special case in which there is just one grade and one location eligible for delivery under the futures contract. To put it another way, the case of a "perfect hedge" arises when the futures contract is in fact simply a forward contract.

In the more interesting case of a range of eligible grades and locations, relation (4) can be used to say something about the joint probability distribution over a cash price and the futures price, as of the maturity date of the futures contract. Let p_0 be the cash

price at time t of some grade-location combination eligible for delivery under the futures contract ($p_0 = p_{ijt}^c$) and let $h(p_0)$ be the probability density function over p_0 . Let y be the price at time t of a futures contract maturing at time t ($y = p_{tt}^f$). Suppose that there are n other grade-location combinations eligible for delivery under the futures contract, with prices p_i , $i = 1, \dots, n$ and that all eligible grade-location combinations are perfect substitutes for good 0 at ratios α_i , $i = 1, \dots, n$. Heroically, assume that the α 's are all independent and identically distributed random variables, each independent of p_0 . Let $f(\alpha_i)$ denote the probability density function over α_i with F denoting the cumulative distribution function. The market clearing equilibrium is given by $p_i = \alpha_i p_0$, $i = 1, \dots, n$. Using the arbitrage relation (4), we find the following joint density $g(p_0, y)$ where $y = \min_i p_i$, $i = 0, \dots, n$:

$$g(p_0, y) = \begin{cases} 0 & \text{for } y > p_0 \\ h(p_0)(1 - F(1))^n & \text{for } y = p_0 \\ h(p_0) \frac{nf(y/p_0)}{p_0} [1 - F(y/p_0)]^{n-1} & \text{for } y < p_0 \end{cases} \quad (5)$$

Now admittedly the assumption of independence of the α_i 's is not at all realistic. In fact, as Houthakker and others have argued persuasively, it is the presumed high correlation among the cash prices that makes the futures contract a valuable tool for hedging. On the other hand, what the joint density (5) shows is that even when we assume independence among the cash prices, still the futures price

is correlated with any such cash price through the arbitrage relation (4). Moreover, (5) also shows that the joint density over the cash price for a grade-location combination deliverable under the futures contract and the futures price includes a spike of strictly positive probability relating to the case where the cash price equals the futures price. This spike has to be incorporated into our formulation of the decision problems facing hedgers in the market.

One other feature of the joint density (5) should be noted. Cash and futures prices are more highly correlated at low cash prices than at high cash prices in the sense that for any constant value of y/p_0 , the conditional density of y given p_0 , (g/h) increases as p_0 decreases. This result extends empirically to the case of dependence among cash prices since cash prices tend to be most highly correlated with one another and with the futures price when inventories are high, as Houthakker has pointed out. Obviously, inventories are high just after the harvest with cash and futures prices at their lowest levels. Since this high cash-futures price correlation at low values of the cash price figures large in the backwardation argument, it deserves a little more elaboration. Suppose there are three grades of wheat deliverable under the futures contract, grade X, grade Y, and grade Z. X is the only grade that can be used to produce output A, Y is the only grade usable in the production of output B, while all grades are more or less perfect substitutes in the production of output C (say, a cheap grade of flour). In Figure 2, W_A , W_B , and W_C denote bushels of wheat used to produce goods A, B, and C, respectively. Curves D_A ,

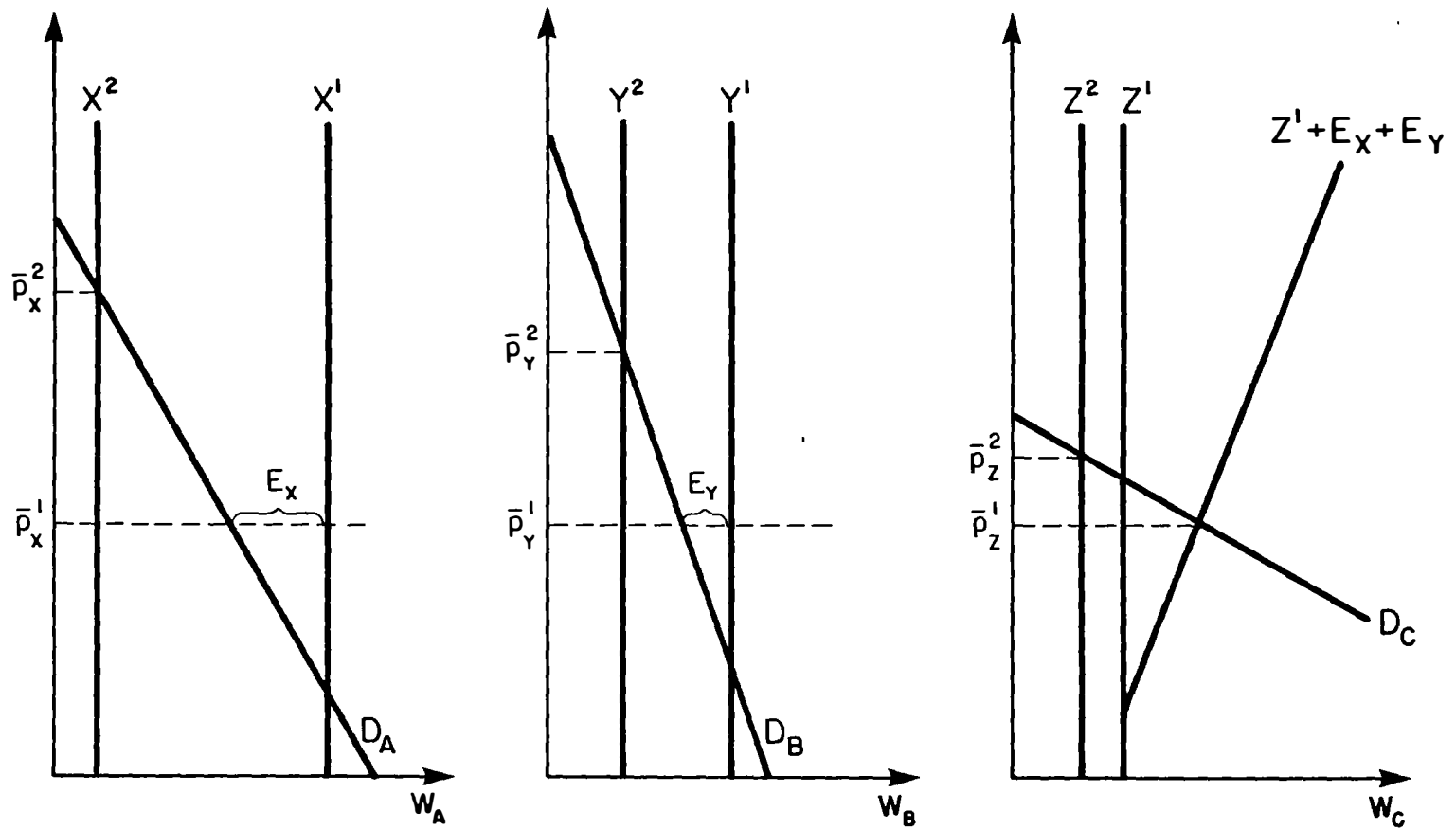


FIGURE 2. Inventories and Price Patterns

D_B , and D_C are input demands for wheat in the production of each good.

Early in the crop year (just after harvest), denoted with time subscript "1," stocks of all grades are abundant. In particular, the amounts of X and Y available are large enough that more X is available than is demanded in producing A at the market clearing price for C and similarly for Y with respect to good B. Excess supplies, E_X and E_Y , of grades X and Y are added to the supply of Z, to be used in producing good C. Thus at time 1, all grades sell for the same price which is determined by their common marginal productivity in producing good C. According to arbitrage condition (4), the price of a futures contract maturing at time 1 is equal to the cash price at time 1 of any of the three grades of wheat, $\frac{-1}{P_X} = \frac{-1}{P_Y} = \frac{-1}{P_Z}$.

Late in the crop year, time 2, stocks of all grades are low. When there is no longer any excess supply of grades X and Y in producing A and B at the market clearing price for C, then prices of the three grades move away from each other. In the particular case represented by Figure 2, arbitrage condition (4) dictates the price of a futures contract maturing at time 2 to be $\frac{-2}{P_Z}$. The extent of the correlation between this futures price and the cash prices of grades X and Y depends upon the inventory levels of the two grades and the location and slope of demand curves D_A and D_B . With uncertainty incorporated into the picture (say as to the locations of the demand curves D_A, D_B, D_C), then it is apparent that cash prices for X and Y tend to be more highly correlated with the futures price at low cash prices (high inventories) than at high cash prices.

We use the implications of (4) to examine separately the decision problems of short and long hedgers in a simplified two-period setting. Within this two-period setting, asymmetry of arbitrage reduces to the implications of (4), so that some of the aspects of Houthakker's arguments are lost. However the basics of that argument are preserved. Following the stylized facts in the hedging literature, we take short hedgers to be elevator operators while long hedgers are millers.⁷

SHORT AND LONG HEDGERS

We undertake our simplified two-period analysis in the context of a specific commodity market, say wheat. There are two participants—the elevator operator and the miller. The elevator operator deals in a deliverable grade of wheat and is located at an eligible delivery point under a futures contract. The elevator operator buys spot wheat and stores it until he decides to sell it. To the extent that he hedges, the elevator operator is a short hedger selling futures contracts to offset his long position in the cash market.

The operation of long hedging by millers has been described in detail by Working (1953). Millers make bids on flour contracts with flour users such as bakeries, in a more or less competitive environment. A successful bid in effect commits the miller to a forward contract to deliver flour to the bakery at a fixed price. For large flour contracts, wheat requirements are difficult to satisfy

through immediate purchases in the cash markets, because of thinness of such markets. As a consequence, the miller buys wheat futures at the time of a successful bid for a flour contract. As wheat is purchased over time in the cash markets to meet milling requirements, each cash purchase is offset by a corresponding sale of futures, so that gradually over time the initial long hedge is terminated. It was in describing this behavior that Working (1953) introduced a new view of hedging, namely,

the hedge in futures should not be regarded as an offset to the forward sale of flour, but as a temporary substitute for a purchase of spot wheat. Indeed, any hedging purchase or sale of futures is a temporary substitute for a 'cash' purchase or sale, regardless of the reason for choosing to hedge (p. 131).

The following assumptions are made in modeling the behavior of the elevator operator and the miller. At time 0 (today), there is just one futures contract being traded, maturing at time 1 (tomorrow). p_0^f is the price of the futures contract today and p_1^f is its price tomorrow. Similarly, p_0^c and p_1^c denote the cash prices today and tomorrow. We take the grade-location combination to be the same for the miller and the elevator operator. Each firm has an equity of M dollars. Furthermore, the miller has the same wheat requirement as the elevator operator, W bushels, and both participants are risk-averse expected-utility maximizers with identical joint p.d.f.s over cash and futures prices.

Looking first at the elevator operator, we assume that all assets of the operator are in the form of stocks of wheat. With w_h

denoting hedged stocks and w_u denoting unhedged stocks, $w_h + w_u = W$. In addition, there is an out-of-pocket cost, k dollars per unit, for warehousing and insurance per period.

Let A_0 and A_1 denote asset values for our firms at times 0 and 1, respectively. We have the following accounting identities for the elevator operator (short hedger, S):

$$A_0^S = M = p_0^c W \quad (6)$$

$$A_1^S = p_1^c W + (p_0^f - p_1^f) w_h - kW. \quad (7)$$

In expression (7), A_1^S is the value of assets net of warehousing and insurance costs, after futures contracts sold at time 0 are cancelled by purchases at time 1. Note that if $A_1^S < 0$, then the elevator operator defaults on his futures contracts and/or his commitments for insurance and other services. We will ignore this possibility in what follows.

The activities of the miller are more complicated. Our simplified analysis does some violence to the process of gradual release of the long hedge as described by Working. We assume that at time 0, the miller undertakes a flour commitment involving a wheat requirement, W , with forward delivery on the flour contract at time 1. The bid price on the flour contract is based on p_0^c , the cash price of wheat at time 0. In addition, at the same time (time 0), the miller institutes a hedge against a portion (up to 100%) of the wheat commitments by buying futures contracts that mature at time 1. For

modeling convenience, purchases of cash wheat are telescoped into a single purchase at time 1 in the cash market, following which the wheat is immediately converted into flour. The long hedge is terminated at the time of the cash wheat purchase through an offsetting sale of futures.

Let α denote the markup over milling cost by the miller. Then the net revenue from the milling operations is given by $(p_0^c + \alpha - p_1^c)W$. However, since p_1^c is uncertain at time 0, there is no guarantee that the miller will have enough cash to purchase the W bushels of wheat needed to complete the flour contract. That is, the miller might default on his forward contract to the bakery. Hence, initial and final asset values for the miller (long hedger, L) are given by

$$L_0^L = M \quad (8)$$

$$L_1^L = \begin{cases} B & \text{if } B \geq 0 \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

where

$$B = (p_1^f - p_0^f)w_h + (p_0^c + \alpha - p_1^c)W + M. \quad (10)$$

Note that B is simply the sum of profits from the miller's futures market activity, profits from milling, and the initial value of assets, assuming that W bushels of wheat are throughput. If $B < 0$, then the miller defaults on his flour contract to the bakery. Through the rest of this paper we will assume that there is no risk of default

by the miller on his flour contract, that is, $B \geq 0$ for (p_1^f, p_1^c) pairs occurring with positive probability weights.⁹

HEDGING AND BACKWARDATION

We consider the equity constraint to be binding and look first at the case of a perfect hedge, that is, the case in which $p_1^f = p_1^c$ is an identity for all (p_1^f, p_1^c) . Let $h(p_1^c)$ denote the density over p_1^c in this case. The utility function u for a short or long hedger is monotone increasing and strictly concave. The maximization problems for the short and long hedger are, respectively,

$$\begin{aligned} \max_{w_h, w_h} \quad & EU^S = \int_0^\infty u(A_1^S) h(p_1^c) dp_1^c \\ \text{s.t.} \quad & p_0^c W = M \end{aligned}$$

and

$$\begin{aligned} \max_{w_h, w_u} \quad & EU^L = \int_0^\infty u(A_1^L) h(p_1^c) dp_1^c \\ \text{s.t.} \quad & p_0^c W = M. \end{aligned}$$

Since the expressions involved are tedious, we carry through the remaining decision calculus for the short hedger only.¹⁰

With W fixed, the problem of the short hedger can be restated as that of maximizing EU^S with respect to w_h , $0 \leq w_h \leq W$. In this formulation, $\frac{dEU^S}{dw_h} = \frac{\partial EU^S}{\partial w_h} - \frac{\partial EU^S}{\partial w_u}$, where¹¹

$$\frac{\partial EU^S}{\partial w_h} = \int_0^\infty u'(A_1^S) [p_0^f - k] h(p_1^c) dp_1^c \quad (11)$$

$$\frac{\partial EU^S}{\partial w_u} = \int_0^\infty u'(A_1^S) [p_1^c - k] h(p_1^c) dp_1^c \quad (12)$$

Thus we have

$$\frac{dEU^S}{dw_h} = \int_0^\infty u'(A_1^S) [p_0^f - p_1^c] h(p_1^c) dp_1^c$$

Integrating this expression by parts yields

$$\begin{aligned} \int_0^\infty u'(A_1^S) [p_0^f - p_1^c] h(p_1^c) dp_1^c &= u'(A_1^S) \int_0^{p_1^c} [p_0^f - x] h(x) dx \Big|_0^\infty \\ &- \int_0^\infty \left\{ \int_0^{p_1^c} [p_0^f - x] h(x) dx \right\} u''(A_1^S) W dp_1^c. \end{aligned} \quad (13)$$

Additionally, if there is no backwardation in the market ($p_0^f \geq Ep_1^c (= Ep_1^f)$), then (13) becomes

$$\begin{aligned} \int_0^\infty u'(A_1^S) [p_0^f - p_1^c] h(p_1^c) dp_1^c &\geq \\ u'(A_1^S) \int_0^{p_1^c} [Ep_1^c - x] h(x) dx \Big|_0^\infty & \\ - \int_0^\infty \left\{ \int_0^{p_1^c} [Ep_1^c - x] h(x) dx \right\} u''(A_1^S) W dp_1^c &> 0. \end{aligned} \quad (13')$$

With $\frac{dEU^S}{dw_h} > 0$, clearly the optimal solution is to set $w_h = W$. This

leads into the following.

Proposition 1: Assume a risk-averse elevator operator facing a strictly binding equity constraint, M . Suppose that a perfect hedge is possible and that there is no backwardation in the market ($p_0^f \geq Ep_1^f$). Then the operator will hedge all stocks; only if there is backwardation in the market will any of his stocks be left unhedged.

By the same argument, we have the following for long hedgers.

Proposition 2: Assume a risk-averse miller facing a strictly binding equity constraint. Suppose that a perfect hedge is possible and that no contango exists, that is, $p_0^f \leq Ep_1^f$. The miller will hedge all stocks; only if there is a contango in the market will any of his commitments be left unhedged.

These propositions can be viewed as applications of Jensen's Inequality. With all stocks hedged, and no backwardation (no contango), the elevator operator (miller) achieves an income (with certainty) whose utility exceeds the expected utility of income under any mixed (some stocks hedged, some unhedged) portfolio. With a strictly concave utility function, the option of hedging all stocks has a higher expected utility than that associated with any mixed portfolio.

This leads into the following result concerning backwardation as an equilibrium condition in the futures market.

Proposition 3: Assume that perfect hedges are possible to both elevator operators and millers, that there is no default risk, and that wheat commitments of millers equal stocks of wheat held by elevator operators. Then a market equilibrium can be sustained without speculative activity, with $p_0^f = E(p_1^f)$.

With $p_0^f = E(p_1^f)$, both millers and elevator operators hedge all their commitments, and with speculators with the same pdf over prices that hedgers have, the futures market clears.

Propositions 1 and 2 do not hold when a perfect hedge is not possible, that is, when the futures contract is not simply a forward contract. We formalize this as follows.

Proposition 4: Assume a short (long) hedger, facing a binding equity constraint. Then with only imperfect hedges possible, the absence of backwardation (of a contango) is not sufficient to guarantee that all commitments will be hedged, for arbitrary concave utility functions.

Proof: We outline the proof for the case of a short hedger. The argument is identical for the case of a long hedger.

When a perfect hedge is not possible, the elevator operator finds that even with all of his stocks hedged, he is still faced with a nondegenerate pdf over income. The decision problem in this case becomes (recall that $p_1^f \leq p_1^c$ by (4))

$$\max_{w_h} EU^S = \int_0^\infty \int_0^{p_1^c} u(A_1^S) g(\cdot) dp_1^f dp_1^c + \int_0^\infty u(A_1^S) h(p_1^c) dp_1^c$$

subject to $p_0^c W = M$.

Taking the equity constraint as fixed and binding, we have

$$\frac{dEU^S}{dw_h} = \int_0^\infty \int_0^{p_1^c} u'(A_1^S) (p_0^f - p_1^f) g(\cdot) dp_1^f dp_1^c + \int_0^\infty u'(A_1^S) (p_0^f - p_1^f) h(p_1^c) dp_1^c.$$

If the optimal w_h occurs at W , then $\frac{dEU^S(w_h = W)}{dw_h} \geq 0$.

In contrast to the "perfect hedge" case, in general we cannot prove that in the absence of backwardation all of the operator's inventory will be hedged.

In fact, it is well known (see Quirk and Saposnik (1962) and Hadar and Russell (1969)) that a definite ordering of preferences over probability distributions holding for all monotone (or monotone concave) measurable utility functions occurs only with stochastically dominated distributions. In particular, $EU(f) \geq EU(g)$ for all monotone concave utility functions u iff $\int_{-\infty}^x F(t) dt \leq \int_{-\infty}^x G(t) dt$ for all x , where F and G are the cdf's associated with f and g respectively.

In our context, we have

$$P(t) \equiv \Pr\{A_1 \leq t\} = \int_0^\infty \int_{\gamma(p_1^c, t)}^{p_1^c} g(p_1^f, p_1^c) dp_1^f dp_1^c + \int_0^\beta g(p_1^c, p_1^c) dp_1^c.$$

$$\text{where } \beta(t) = \frac{t + kW - M - P_0^f w_h}{w_u}, \text{ and}$$

$$\gamma(P_1^c, t) = (P_1^c - k) \frac{W}{w_h} + P_0^f - \frac{t - M}{w_h}$$

Differentiating $P(t)$ with respect to w_h :

$$\frac{\partial F(t)}{\partial w_h} = - \int_0^\infty \frac{\partial \gamma}{\partial w_h} g(\gamma, P_1^c) dP_1^c + \frac{\partial \beta}{\partial w_h} g(\beta, \beta),$$

$$\text{where } \frac{\partial \gamma}{\partial w_h} = \frac{t - M - (P_1^c - k)W}{w_h^2},$$

$$\frac{\partial \beta}{\partial w_h} = \frac{1}{w_u} \{t + kW - M - P_0^f - P_0^f w_h\}$$

Since $\frac{\partial P(t)}{\partial w_h}$ and $\int_{-\infty}^t \frac{\partial P(x)}{\partial w_h} dx$

are ambiguous in sign, depending on t and the functional form of g , stochastic dominance of either the first or second degree cannot be established, and hence Proposition 1 does not extend to the general case. Similarly, Proposition 2 is generally limited to the case of a perfect hedge.

Intuitively, for the short hedger, the reason for this failure is as follows. The terminal value of the elevator operator's assets can be written as

$$A_1^S = P_1^c w_u + (P_1^c + P_0^f - P_1^f) w_h - kW$$

with

$$EA_1^S = w_u E P_1^c + (E P_1^c + P_0^f - E P_1^f) w_h - kW,$$

and

$$\sigma_{A_1}^2 S = W^2 \sigma_c^2 + w_h^2 \sigma_f^2 - 2W w_h \sigma_{P_1^f c}$$

so that for fixed W ,

$$\frac{\partial EA_1^S}{\partial w_h} = E P_1^c + P_0^f - E P_1^f$$

$$\frac{\partial \sigma_{A_1}^2 S}{\partial w_h} = 2(w_h \sigma_f^2 - W \sigma_{P_1^f c}).$$

The nonbackwardation condition $P_0^f \geq E P_1^f$ insures that $\frac{\partial EA_1^S}{\partial w_h} \geq 0$,

but increasing the share of inventory hedged increases the variance of A_1 at the same time it increases the expected value of A_1 , as w_h approaches W . For a risk averse operator, this tradeoff can lead to a diversified portfolio involving some hedged and some unhedged inventory.

While Propositions 1 and 2 do not extend to the case where only imperfect hedges are possible, still it is possible to say something about backwardation in the imperfect hedge case, given that most of the probability weight is concentrated in the range of low values of P_1^c . In particular, when the utility function exhibits constant or decreasing absolute risk aversion (which implies u' is strictly convex), then backwardation can be established. The following proposition holds.

Proposition 5: Assume that millers and elevator operators are identical, with the wheat commitments of millers equaling the stocks of wheat held by operators. The utility function common to millers and elevator operators exhibits constant or decreasing absolute risk aversion. Let w_h^S, w_h^L denote the amounts of wheat hedged by short (S) and long (L) hedgers. Then for $g(p_1^f, p_1^c)$ sufficiently concentrated about low values of $p_1^c, w_h^S \geq w_h^L$ if $p_0^f \geq Ep_1^f$, with $w_h^S > w_h^L$ if $0 < w_h^S < W$.

Proof: With $W^S = W^L = W$ and with $w_h^S = w_h^L = w_h, A_1^S = M + \pi$ and $A_1^L = M - \pi$ where

$$\pi = (p_1^c - p_0^c)W + (p_0^f - p_1^f)w_h - kW,$$

assuming k , the per bushel storage cost facing the elevator operator, equals α , the per bushel markup of the miller. With an interior maximum for w_h^S , we have

$$\begin{aligned} \frac{\partial EU^S}{\partial w_h} &= \int_0^\infty \int_0^{p_1^c} u'(M + \pi)(p_0^f - p_1^f)g(\cdot) dp_1^f dp_1^c \\ &+ \int_0^\infty u'(M + \pi)(p_0^f - p_1^c)h(p_1^c) dp_1^c = 0 \end{aligned}$$

while

$$\begin{aligned} \frac{\partial EU^L}{\partial w_h} &= \int_0^\infty \int_0^{p_1^c} u'(M - \pi)(p_1^f - p_0^f)g(\cdot) dp_1^f dp_1^c \\ &+ \int_0^\infty u'(M - \pi)(p_1^c - p_0^f)h(p_1^c) dp_1^c \end{aligned}$$

Hence,

$$\frac{\partial EU^L}{\partial w_h} - \frac{\partial EU^S}{\partial w_h} = \frac{\partial EU^L}{\partial w_h} \text{ is given by}$$

$$\begin{aligned} &\int_0^\infty \int_0^{p_1^c} [u'(M - \pi) + u'(M + \pi)](p_1^f - p_0^f)g(\cdot) dp_1^f dp_1^c \\ &+ \int_0^\infty [u'(M - \pi) + u'(M + \pi)](p_1^c - p_0^f)h(p_1^c) dp_1^c. \end{aligned}$$

Integrating the first integral by parts yields

$$\begin{aligned} \frac{\partial EU^L}{\partial w_h} &= \int_0^\infty \{ [u'(M - \pi) + u'(M + \pi)] \int_0^{p_1^c} (p_1^f - p_0^f)g(\cdot) dp_1^f \\ &- w_h \int_0^{p_1^c} \int_0^{p_1^f} (x - p_0^f)g(\cdot) dx [u''(M - \pi) - u''(M + \pi)] dp_1^f \} dp_1^c \\ &+ \int_0^\infty [u'(M - \pi) + u'(M + \pi)](p_1^c - p_0^f)h(p_1^c) dp_1^c. \end{aligned}$$

Or, rewriting,

$$\begin{aligned} \frac{\partial EU^L}{\partial w_h} &= \int_0^\infty [u'(M - \pi) + u'(M + \pi)] \{ (p_1^c - p_0^f)h(p_1^c) + \int_0^{p_1^c} (p_1^f - p_0^f)g(\cdot) dp_1^f \} dp_1^c \\ &- w_h \int_0^\infty \int_0^{p_1^c} \int_0^{p_1^f} (x - p_0^f)g(\cdot) dx [u''(M - \pi) - u''(M + \pi)] dp_1^f dp_1^c. \end{aligned}$$

Integrating again inside the first integral, by parts, and combining terms we obtain

$$\begin{aligned} \frac{\partial EU^L}{\partial w_h} &= (Ep_1^f - p_0^f) \lim_{p_1^c \rightarrow \infty} [u'(M - \pi) + u'(M + \pi)] \\ &+ w_h \int_0^\infty \int_0^{p_1^c} [u''(M - \pi) - u''(M + \pi)] \{ [(x - p_0^f)h(x) dx \end{aligned}$$

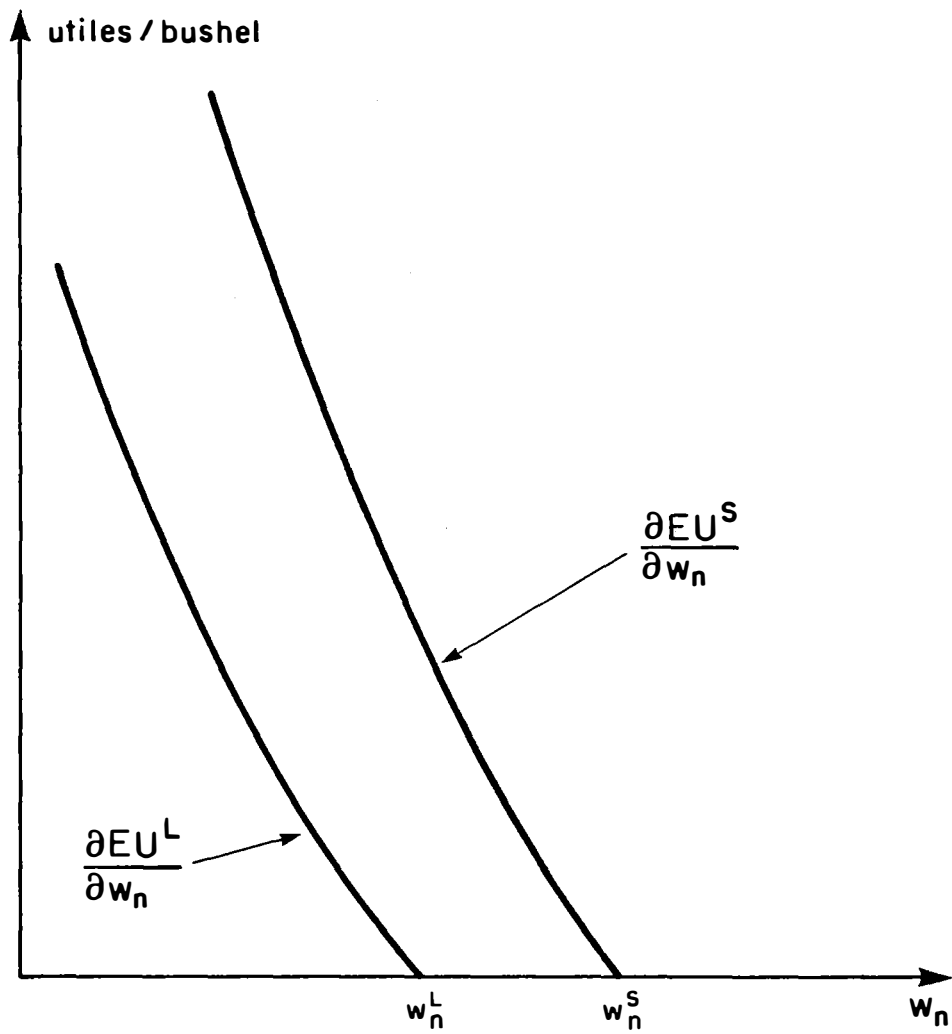


FIGURE 3. Excess of Short over Long Hedging

$$+ \int_0^{p_1^f} (y - p_0^f) g(y, p_1^c) dy dp_1^f dp_1^c. \quad (14)$$

Assume that $Ep_1^f \leq p_0^f$, i.e. no backwardation. Then the first term on the RHS of (14) is non-positive, since $u' > 0$. In the second term, the expression inside the curved brackets is negative at low values of p_1^f and p_1^c . (Recall that $p_1^f = p_1^c$ is associated with the p.d.f. $h(p_1^c)$).

Moreover $u'' < 0$ while constant or decreasing absolute risk aversion implies $u''' > 0$, which in turn implies that at low values of p_1^c , $u''(M - \pi) > u''(M + \pi)$. Hence, for g sufficiently concentrated around low values of (p_1^f, p_1^c) , the second integral is strictly negative for $w_u > 0$.

Thus we have that $p_0^f \geq Ep_1^f$ implies that $\frac{\partial Eu^L}{\partial w_h^L} \leq 0$, evaluated at $w_h^L = w_h^S$. It follows that $p_0^f \geq Ep_1^f$ implies $w_h^S \geq w_h^L$; further, if $0 < w_h^S < W$, then $w_h^S > w_h^L$. (See Figure 3).¹²

Corollary. Under the conditions of Proposition 5, with speculators having the same pdf, $g(p_1^f, p_1^c)$ as hedgers, with $0 < w_h^S < W$, and with the demands of speculators for futures contracts of less than infinite elasticity, then the equilibrium price in the futures market exhibits backwardation, that is $p_0^f < Ep_1^f$, with speculators sufficiently long in futures to absorb the excess of short over long hedging.

The intuitive reason for backwardation under the conditions of Proposition 5 is this. With cash and futures prices more highly correlated at low than at high cash prices, the futures contract

offers a more effective hedging protection to an individual who wants to avoid the risk of low cash prices than it does to one worried about high cash prices. This means that short hedgers have stronger incentives to hedge than long hedgers do, given the same pdf's, utility functions, and wheat commitments, and assuming $p_0^f = Ep_1^f$. Backwardation emerges as a result of these market forces. This also helps explain the seasonality of backwardation, since the concentration of the density $g(\cdot)$ at low values of p_1^c is more pronounced early in the crop year than later as inventories are depleted.

CONCLUSION

Consider a situation in which elevator operators as a group have W bushels of wheat to hedge, and millers have flour commitments involving W bushels as well. Suppose that the futures price p_0^f satisfies $p_0^f = Ep_1^f$. In the general case of diverse probability beliefs and attitudes toward risk on the part of millers and elevator operators, we have no definite conclusions to report. Depending on their probability beliefs and their utility functions, there might be an excess of long hedging or of short hedging, or the two might be balanced. However, when there is no default risk and a perfect hedge is possible, by Proposition 1, short hedgers prefer to hedge all of their inventories when $p_0^f \geq Ep_1^f$ and by Proposition 2 long hedgers prefer to hedge all of their inventories when $p_0^f \leq Ep_1^f$. With identical probability beliefs, the knife-edge case of $p_0^f = Ep_1^f$ is

consistent with market clearing with all commitments hedged on both sides of the market.

In the perfect hedge case, the explanation for "normal backwardation" must rest on arguments other than asymmetry in arbitrage, for example, the technical uncertainties cited by Kaldor, the substitution possibilities argument of Hicks, or differences in probability beliefs on the part of short and long hedgers and speculators.

But once we look to the case where hedges are not perfect, then the Houthakker argument that cash and futures prices tend to be more highly correlated at low than at high cash prices, comes to the fore. Assuming no default risk, Proposition 5 asserts that with identical individuals on the short and long hedging sides of the market, with the same commitments available to hedge, then when $p_0^f = E p_1^f$, and with the density $g(\cdot)$ sufficiently concentrated at low values of p_1^c , the volume of short hedging will exceed the volume of long hedging, given constant or decreasing absolute risk aversion.¹³ Market equilibrium will exhibit backwardation, and speculators will on average earn profits by going long in the futures market.

This paper has explored the underpinnings of the argument that asymmetric arbitrage provides an explanation for the existence of backwardation in futures markets. A high correlation between cash and futures prices at low cash prices is at the heart of this argument. Clearly the arguments advanced by Hicks, Kaldor, Dow, Working and others also have relevance in explaining the presence of backwardation

and/or the size of the premium earned by long speculators. An important caveat with respect to the current paper is that it ignores the problem of default risk, which is a potentially important omission since it is precisely at low cash prices that default is most probable for short hedgers. Beyond clearing up default problems, it would be of interest to see what empirical tests might distinguish among these various backwardation hypotheses.

FOOTNOTES

1. The "abnormal" situation in which backwardation might not occur is one in which there are "redundant liquid stocks" of a good; such redundant stocks can lead to a "contango" on the market (futures price greater than the spot price).
2. One of the most interesting exchanges concerning backwardation is that between Telser (1958, 1960) and Cootner (1960). Telser's model of the commodity markets is one in which competitive pressures are assumed to reduce speculative profits to zero, so that the market clearing futures price equals the expected future spot price. Cootner points out that speculative profits are deferred in time, so that the purchaser of a futures contract has an opportunity cost equal to the market rate of return (adjusted for risk) on the speculator's investment in margin, a cost ignored in Telser's formulation. Cootner also objects to Telser's empirical findings that speculators don't make money on the average, by pointing out that Telser's assumption that the futures price should rise over the entire inter-harvest period ignores the seasonal pattern of short and long hedging. In fact, based on an analysis of Telser's data on wheat, Cootner notes that:

If we assume that speculators go short before the harvest, continue that short position until net hedging by large traders reaches a peak, and then go long, they consistently make a profit (p. 402).

In his reply, Telser (1960) reexamines the pattern of hedging, showing that short hedging peaks on average just after the harvest and reaches a trough just before the harvest, with net (short minus long) hedging being positive all year long, this pattern holding for both wheat and cotton. Moreover, Telser shifts to the normal backwardation camp:

. . . it is fair to conclude that, on the basis of powerful statistical tests, the weight of the evidence lends some support to Keynes and Hicks (p. 415).

3. We have identified the literature leading to Houthakker's (1959, 1968) backwardation hypothesis. To be sure, other views exist. Holbrook Working (1948) introduces the notion of storage supply as a function of the basis; the amount of storage offered increases with market carrying charges as measured by the basis. See also Samuelson (1957). According to Working and his supply curve for storage, when backwardation in the Keynes-Hicks sense (spot price exceeds futures price) exists, it reflects an "inverse carrying charge" due to the convenience yield of inventories held by processors of a commodity. Later, Working (1962) argues that "A significant tendency for futures prices to rise during the life of each future is not uniformly present in futures markets, and when it exists, it is to be attributed chiefly to lack of balance in the market" (p. 432). Some time before, Blau (1945) felt that the case for backwardation had not been proven. In particular, Blau notes that speculative demands and supplies can overwhelm market forces exerted by hedgers, a

point made earlier by Kaldor. In a related view, Working (1961) comments as follows: "The concept that risk-bearing commands a reward applied at several related points in the theory of futures markets, has served poorly to account for observed phenomena of the markets. One of its applications led J. M. Keynes to advance the concept of 'normal backwardation,' but the observed tendencies toward backwardation vary widely according to circumstances, as shown by R. W. Gray, in a manner not reasonably explainable on the basis of differences in risk premium" (p. 163). Houthakker's 1959 studies indicate that speculators earn a risk premium from hedgers: ". . . the risk premium accruing to long futures speculators with 'general skill' before deducting commissions and other expenses, is of the order of 8% per year on the value of their holdings; Keynes, himself, on the basis of unspecified data, had put it at 10%" (p. 154). With a 10 percent margin requirement (plus 10 percent more to cover margin calls), an 8 percent annual rate of increase in the futures price amounts to a 40 percent return on a speculator's investment, a point duly noted by Houthakker.

Other recent work in the rational expectations framework has focused attention on the role of information in futures markets, with differential information as between insider-speculators and uninformed hedgers being a possible explanation for backwardation. However, the rationale for such differential information situations is not easily established, and certain

fundamental attributes of futures markets (as contrasted, say, with forward markets) are ignored in this recent literature. For example, Danthine (1978) outlines a rational expectations model of a commodity market in which informed speculators participate. In the Danthine model, the only hedgers are short hedgers (producers of the good) and the futures contract is identical to a forward contract. Given informed speculators and uninformed hedgers, it turns out that ". . . the futures price is not an unbiased estimator of the future spot price" (p. 91). In fact, the futures price is a downward biased estimator of the future spot price, i.e., there is backwardation in the Danthine model, which is a kind of intellectual grandchild of the Kaldor (1939) model. In two other related studies, Baesel and Grant (1982) derive backwardation in the futures market in a "perfect hedge" setting when producers, rather than processors, hedge through the futures market while Stevenson and Bear (1970) perform a statistical analysis of the corn and soybean markets, rejecting the random walk hypothesis, and providing some evidence for backwardation.

4. Forward contracts can be quite detailed. For example, the standard forward contract employed by a potato processor in dealing with potato farmers lays down rules concerning such items as the kinds of pesticides to be used, fumigation, the temperature at which the potatoes are to be stored, and like matters. Most of the contract deals with rewards and penalties

for quality differentials, these taking up almost 5 of the 10 single spaced typed pages of the contract. We wish to thank Lowell Bassett for making this contract available to us.

5. To simplify notation in what follows, we ignore grade premiums and penalties written into futures contracts.
6. To verify that $g(p_0, y)$ is a density, note that $g(p_0, y) \geq 0$ for all (p_0, y) . Further $\int_0^\infty \int_0^{p_0} g(p_0, y) dp_0 dy = \int_0^\infty h(p_0) (1 - F(1))^n dp_0 + \int_0^\infty h(p_0) \left\{ \int_0^{p_0} \frac{f(y/p_0)}{p_0} [1 - F(y/p_0)]^{n-1} dy \right\} dp_0$. The term inside the curved brackets is $1 - (1 - F(1))^n$, hence $\int_0^\infty \int_0^{p_0} g(p_0, y) dp_0 dy = 1$.
7. In a recent USDA survey (see Heifner, et al [1977]), hedging was done almost exclusively by these agents. Farmers rarely trade in the futures markets, but instead shift price risks by forward selling. Presumably this reflects the technical risks (e.g., weather) facing farmers, as in Kaldor's (1939) comments on hedging.
8. Both (7) and (9) ignore the opportunity costs associated with margin requirements on futures contracts. See Cootner (1960).
9. Hedging can have the effect of limiting the risks facing market participants. One example of such risks is the possibility of default on debt contracts. In this paper, the model is structured purely upon equity financing and any such loan

contract default is assumed away implicitly. In a forthcoming paper, the analysis is extended to the context of debt financing and the effect of additional risk of loan contract default upon short hedgers, long hedgers, and debt suppliers. Our analysis thus far has revealed that the presence of loan default risk lessens the incentives for short and long hedging because unfavorable price outcomes for hedgers lead to losses by lenders, while favorable price outcomes increase payoffs to borrowers; hedgers are encouraged to be more "risk taking" in the presence of default risk. This result leads us to shift our attention to the incentives facing lenders in the forthcoming paper.

10. As an aid to the interested, note that for the long hedger:

$$\frac{\partial EU^L}{\partial w_h} = (p_0^c - p_0^f) \int_0^b u'(A_1^L) h(p_1^c) dp_1^c$$

$$\frac{\partial EU^L}{\partial w_u} = \int_0^b u'(A_1^L) (p_0^c - p_1^c) h(p_1^c) dp_1^c.$$

11. Recall that in this case of a perfect hedge, $p_1^f \equiv p_1^c$ so that $A_1^S = p_1^c w_u + p_0^f w_h - kW$.
12. As is clear from the proof of Proposition 5, $u''' > 0$ is a necessary condition to establish backwardation. In particular, if u is quadratic so that expected utility depends on the first and second moments of the pdf over A_1 , then with $p_0^f = E p_1^f$, $w_h^S = w_h^L$, as is easily verified. On the other hand, concave quadratic utility functions violate monotonicity.

13. The only exception to this is the corner case in which short and long hedgers alike wish to hedge their entire stocks when $\frac{f}{p_0} = E p_1^f$. As contrasted with the situation in the perfect hedge case, this can arise only under restrictive assumptions concerning the utility function u and the joint pdf $g(\cdot)$, as follows from Proposition 4.

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