Supporting Information for "The influence of meridional ice transport on Europa's ocean stratification and heat content"

Peiyun Zhu,[1] **Georgy E. Manucharyan,**[2] **Andrew F. Thompson,**[2] **Jason C. Goodman,** [3] **and**
Steven D. Vance [4]

[1] Department of Earth and Environmental Sciences, University of Michigan, Ann Arbor, Michigan, USA.
[2] Department of Environmental Science and Engineering, California Institute of Technology, Pasadena, California, USA.
[3] Department of Physics, Wheaton College, Norton, Massachusetts, USA
[4] Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, USA.

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Introduction

Text S1: Derivation of d_{min}.

Here we describe derivation details of the analytical solution of the minimum freshwater layer depth d_{min}. The salinity balance of the freshwater layer and parameterization of entrainment rate c are given by

$$(cu^* + \frac{\kappa}{d})\Delta S = (S_0 - \Delta S)\frac{\rho_i}{\rho}F_h, \tag{1}$$

$$c = 1.5Ri^{-3/2}, \qquad Ri = \frac{dg\beta\Delta S}{u^{*2},}, \qquad \Delta S = S_0 - S^e. \tag{2}$$

Using (2), we can express c with the following relationship:

$$c = 1.5(\frac{dg\beta\Delta S}{u^{*2}})^{-2/3}. \tag{3}$$

Corresponding author: Peiyun Zhu, `zhpeiyun@umich.edu`

Assuming $cu^* \gg \frac{\kappa}{d}$, and thus neglecting the κ term, substituting (3) into (1) results in a cubic equation for $(\Delta S)^{1/2}$:

$$(\Delta S)^{3/2} - S_0(\Delta S)^{1/2} + A = 0, \qquad \text{where} \qquad A = \frac{3u^{*4}}{2\frac{\rho_i}{\rho} F_h \, (dg\beta)^{3/2}}. \tag{4}$$

This cubic equation (4) always has one negative root, which is not a physical solution because we require $\Delta S > 0$. The two other roots are either both positive real roots or complex numbers. The transition between real and complex roots occurs when the two real roots are identical. This occurs when the derivative of (4) with respect to ΔS is equal to zero, i.e. when $\Delta S = S_0/3$. Substituting ΔS in (4) leads to

$$d_{\min} = \frac{0.84u^{*8/3}}{(\frac{\rho_i}{\rho} F_h)^{2/3} g\beta S_0}. \tag{5}$$

Any value of d smaller than $d_{\min}$ is not a solution because (4) does not have real roots. Physically, only the smaller of the two real roots can be a solution for ΔS. The root with a larger magnitude gives a value of Ri that suppresses c and breaks the assumption that turbulent mixing dominates over diffusion at the base of the freshwater layer, i.e. $cu^* \gg \frac{\kappa}{d}$.

Text S2: Other parameterization for turbulence at the freshwater layer base

The parameterization for turbulent entrainment (see equations in (2)) applies to a turbulence induced by the vertical shear of horizontal velocities at the interface between the freshwater layer and the deep ocean, assuming a significant circulation in the latter. We estimate the entrainment rate at the ice-ocean interface on Europa, c_{ice}, to be 10^{-3} in our study because calculations based on a formula from *Jenkins* [1991] suggests c_{ice} to be o(10^{-4}), and *McPhee et al.* [1999] suggests $c_{\text{ice}} = 0.0058$ based on sea ice observations in the Arctic Ocean. $c_{\text{ice}} = 10^{-3}$ is within the uncertainty range for this parameter based on terrestrial studies.

However, turbulence may also arise from other mechanisms, such as plumes rising from the seafloor and impinging the interface. Entrainment rates of plume-induced turbulence can be parameterized by an interfacial Froude number $F_r = w_i/\sqrt{b_i \frac{g\Delta\rho}{\rho_0}}$ [*Bains*, 1975] where w_i is the characteristic center-line vertical velocity of the plume, b_i is the local plume radius, $\Delta\rho$ is the density difference across the interface and ρ_0 is a reference density. Entrainment rate is given by $c = AF_r^n$, coefficient A and exponent n varying with different ranges of F_r [*Shrinivas and Hunt*, 2014]. Theoretical models [*Shrinivas and Hunt*, 2014] and laboratory experiments [*Bains*, 1975; *Kumagai*, 1984] support n ranging from 2 to 3.5 for $F_r < 1.5$.

As F_r is equivalent to $Ri^{-1/2}$, these values of n show no significant deviation from our parameterization of c in (2). The range of Ri in our results is 100 to 1000, corresponding to $F_r < 0.1$. The coefficient A varies by less than one order of magnitude under different experimental conditions [*Kit et al.*, 1980], but this variation can be incorporated into the uncertainty of u^*.

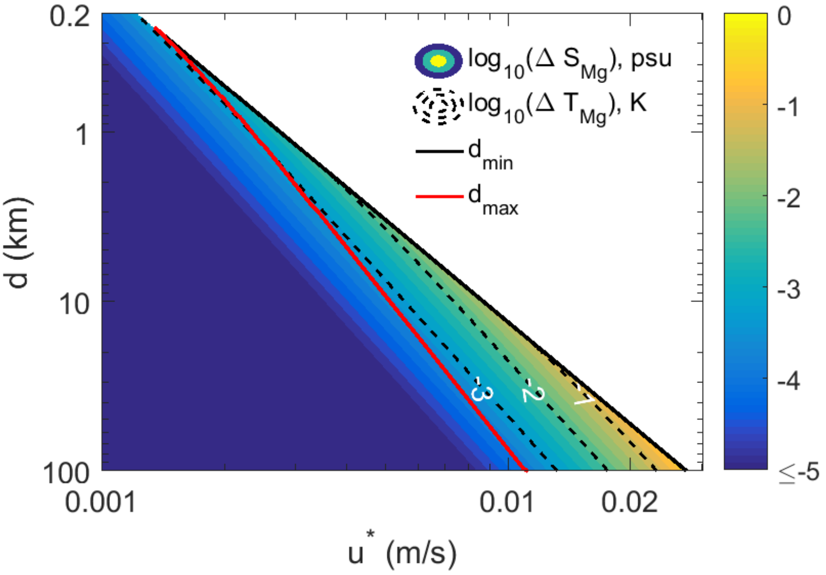

Figure S1: Solutions of freshwater layer salinity balance for a magnesium sulfate ($MgSO_4$) ocean. Salinity contrast ΔS (color-filled contours) and temperature contrast ΔT (dashed contours) between the deep ocean and the freshwater layer for a $MgSO_4$ ocean at an average salinity of 100 psu, $F_h = 1.76 \times 10^{-11}$ m s^{-1} and $F_b = 0.01$ W m^{-2}. The black and red contours indicate d_{min} and d_{max} respectively. All ΔT and ΔS values are in $\log_{10}$ space; u^* and d axes are logarithmic.

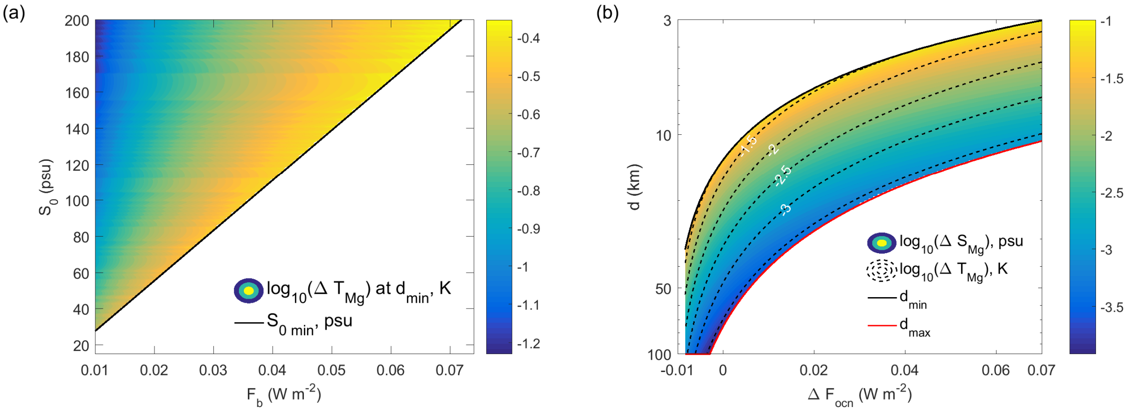

Figure S2: Sensitivity of temperature and salinity contrast at the freshwater layer base to F_b, S_0 and F_h (via ΔF_{ocn}) for a magnesium sulfate ocean (a) Temperature contrast ΔT between the freshwater layer and the deep ocean corresponding to $d_{\min}$, at $u^* = 0.01$ m s^{-1} and $F_h = 1.76 \times 10^{-11}$ m s^{-1}, as a function of F_b and S_0 for a $MgSO_4$ ocean. The black contour indicates the minimum permissible salinity. ΔT is plotted in $\log_{10}$ space. (b) Range of freshwater layer depth d, bounded by $d_{\min}$ and $d_{\max}$ (black and red contours respectively), temperature contrast ΔT (dashed lines) and salinity contrast ΔS (colors) as a function of ΔF_{ocn} (W m^{-2}), for a $MgSO_4$ ocean at $S_0 = 100$ psu, $u^* = 0.01$ m s^{-1} and $F_b = 0.01$ W m^{-2}.