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A DYNAMIC MODEL OF TARGETING IN R AND D CONTRACTS

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ABSTRACT

This paper extends our earlier work on dynamic models of R and D contracts to a case in which the firm must specify in each period of the contractual horizon a research "target" which will govern payoffs in the next period. Targets may be "safe" or "risky." By definition, the former are less than the firm's existing stock of knowledge while the latter exceed it. We show that the firm is more likely to do research the longer is the contractual horizon (given a suitably high discount rate), the lower are research costs, and the higher is the level of sponsor knowledge. Such parameter changes also imply it is more likely to set a risky target. We also establish a number of results relating changes in parameters to the optimal level of safe and risky targets. Finally, we analyze the intertemporal relationship between the targeting decision and incentives to do research.

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I. INTRODUCTION

In a recent article in The Bell Journal, we analysed the behavior of a single firm engaged in R and D for a "sponsor" who was assumed to be interested in reducing the cost of some technology (Balbien and Wilde, 1982). The firm earned a reward in each period of a multi-period contractual relationship which was a function of the current state of sponsor knowledge and the new state of sponsor knowledge created by the firm's research as reported that period. However, it is often the case that a research firm's payoff is based, at least in part, on some preset "target" level of performance (see e.g. Murrell, 1979 and Weitzman, 1980). As we noted in our earlier paper, the methodology developed therein can be used to study this more realistic, but more complex, case. It is our purpose in this paper to do so.

In our earlier model, we assumed the sponsor was interested in lowering the unit cost of some technology. The index of performance was thus taken to be costs--a lower index of performance was preferred by the sponsor. This generated some nonintuitive notation so in this model we assume research performance is represented in such a way that a higher index of performance is preferred by the sponsor. The sponsor again contracts with a single firm to engage in research over a number of periods. At the beginning of each period the firm inherits a performance target (a fixed level of the index) that it hopes to meet by the end of the period. This target might represent, for example, a reduction in expected cost for some manufacturing process or the increased potency of an anti-cancer drug. The firm selects a level of research effort, conducts research, observes the (random) output of the research process, and makes a report to the sponsor. If the current target is achieved a new target is selected for the next period. The reported level of performance (necessarily greater than or equal to the old target) becomes the new state of sponsor knowledge for the next period and provides a baseline for measuring further advances.

We make a number of simplifying assumptions concerning the nature of the research process and the set of admissable contracts. The research process is modeled as random draws from a probability distribution defined over a range of performance levels. The firm pays a fixed cost, c, and gets one draw from this distribution per period. It does, however, have the option of not doing research at all if it so desires. The distribution is meant to represent the firm's expectations (possibly subjective) about research potential. In principle, placing no particular structure on expectations is ideal (Balbien and Wilde, 1982). However, in this case there are two problems with taking a general approach. Interviews with R and D engineers suggest they seldom have a strong notion of the shape of the distribution of research potential, but feel confident about its upper and lower bounds.[1] This implies a uniform distribution might be the most appropriate assumption. Second, certain technical problems arise, further complicating the model, if no structure is placed on expectations. These relate to concavity of the various value-functions associated with the dynamic programming problem which characterizes the firm's optimal strategy. But even with uniform expectations over research potential we will have to deal with some nonconcavities. That sharp results can be obtained in spite of this suggests some weakening of the uniform expectations is possible, and we comment on this in the conclusion. For notational convenience we normalize the distribution describing expectations over research potential to be uniform over [0,1].

Several assumptions will also be made about the nature of the research contract. First it is assumed that in any given period the firm's reward depends on fulfillment of the current performance target, X. If that goal is met, then the reward is a function of the target and the current state of sponsor knowledge, a level of performance, R. If the target is not met, the firm earns nothing in the current period and its contractual relationship with the sponsor is terminated. The firm earns no additional bonus for reporting progress beyond the target. One could presumably relax these assumptions to include bonuses and penalities of the sort considered

by Bonin (1976) and Weitzman (1976).

More formally, we let the instantaneous reward for fulfilling the current target be $W(X,R)$ where X is the target and R is current sponsor knowledge. It is natural to assume that $W_X > 0$ and $W_R > 0$. We also assume $W_{XX} < 0$ and $W_{RR} < 0$. Finally, there is the cross partial W_{XR}. In our earlier paper we assumed that $W_{XR} > 0$ (see Balbien and Wilde, 1982, pp. 109-10 for a discussion of this), but here we only need $W_{XR} \geq 0$. This assumption implies the marginal return to setting higher performance targets does not fall as sponsor knowledge increases.[2]

Our results focus on the firm's incentives to do research and the nature of optimal targets. We define a target as risky if it exceeds the firm's stock of knowledge and safe if it does not. In general we have the following conclusions. Regarding the firm's research strategy, we show the firm is more likely to do research the longer is the sequence of potential contracts (given an appropriately high discount rate), the lower are research costs, and the higher is the level of sponsor knowledge. Such parameter changes also imply it is more likely to set a risky target. Thus, anything which induces the firm to do more research also induces it to set risky targets more often. Moreover, given that the firm sets a unique risky target, this target is higher (i.e. less likely to be met) the lower are research costs and the higher is the state of sponsor knowledge. However, the risky target is independent of the level of firm knowledge. Given that the firm sets a unique safe target, it reveals more of what it knows the higher are research costs, the level of sponsor knowledge, or the level of firm knowledge. Finally, we analyze the intertemporal relationship between the targeting decision and incentives to do research. In particular, we show that if there exist levels of private knowledge such that the firm desires to do research when it faces a safe target, then in setting a safe target optimally, it will also intend to do research. In other words, in choosing a target for any period, if there exist levels of private knowledge and safe targets for which it will want to do research, then it will always do research the next period, whether it sets a safe or a risky target. However, if no safe target could ever induce the firm to do research, then it may or may not do research, depending on whether the optimal target is risky or safe, respectively.

II. THE GENERAL MODEL

Let $V_t(\sigma,R,X)$ be discounted expected profits from pursuing an optimal research and targeting plan when there are t periods remaining in the firm's planning horizon. The time index t represents the number of contracts the firm believes are available to it if all future targets are met. The firm's level of privately held knowledge at the beginning of the research period is represented by σ. Again, R is the level of sponsor knowledge and X is the target for the t'th period. In describing the firm's optimal strategy (in this case a choice of whether or not to do research in the current period and a choice of next period's target), it is useful to distinguish between

two cases. In the first σ is less than X; the level of privately held knowledge yields a performance level which falls short of the currently active target (presumably set by the firm in the previous period). In this situation the firm must either conduct research or forfeit both the current reward and future contract opportunities on this particular project. Thus for $\sigma < X$,

$$V_t(\sigma,R,X) = \max \begin{cases} -c + [W(X,R) + \underset{Z \geq X}{E}\ \beta \underset{1 \geq x \geq X}{\max} V_{t-1}(Z,X,x)](1-X) \\ 0 \end{cases} \tag{1}$$

The logic of (1) is as follows. If the firm does research it incurs a cost c. If the outcome of its research, a random variable denoted by Z, is less than X (the currently active target) the firm gets no reward and its contract with the sponsor is not renewed. If $Z \geq X$ then it earns a reward for meeting the current target, $W(X,R)$, and gets to sign a new contract which specifies a new target, x. Of course x is set to maximize $V_{t-1}(Z,X,x)$. Note that the new target is set after the random variable representing research output is observed. Therefore the expectation, E, of maximized discounted profits when t-1 contracts remain, is evaluated conditional upon the outcome of the random performance variable, Z, being greater than or equal to the currently active target, X. Current and expected future returns are multiplied by $1 - X = \text{Prob}\{Z \geq X\}$ since this represents the likelihood of meeting the current target. Note also that if $Z \geq X$ the firm has the choice of setting the new target at a level of performance either above, equal to, or below its level of private knowledge. The firm will be said to set a "risky target" if $x > \max\{\sigma, Z\}$, and a "safe target" if $x \leq \max\{\sigma, Z\}$.

For $\sigma \geq X$, the firm can fulfill the current target by drawing upon its technology inventory; i.e., research is not compulsory. Thus, for $\sigma \geq X$,

$$V_t(\sigma,R,X) = \max \begin{cases} -c + W(X,R) + E\beta \max\limits_{1>x\geq X} V_{t-1}(\max\{\sigma,Z\},X,x) \\ W(X,R) + \beta \max\limits_{1\geq x\geq X} V_{t-1}(\sigma,X,x) \end{cases} \tag{2}$$

The first term in (2) again reflects discounted expected profits when the firm conducts research and then, depending on the results of that research, decides whether to set a risky or safe target. The second term reflects expected profits when the firm does not conduct new research in the current period and merely meets the current target "out of inventory." Nevertheless, even in this case either a risky target or a safe target may be set for the next contract, depending on σ.

In both cases, the relevant discount rate is $\beta \in (0,1)$. Equations (1) and (2) hold for $t \geq 1$. For $t = 0$ we define $V_0(\sigma,R,X) = 0$ for all σ,R,X. If $\sigma > R$ there might be some profit to the firm from selling the residual information to other private parties, but it is assumed that penalities for such action are so severe as to eliminate this possibility.

One final assumption important to a firm's targeting strategy concerns whether the sponsor will renew the firm's contract if the research firm sets a "safe target" equal to the level of sponsor

knowledge, i.e. the firm sets $X = R$. Under one scenario, a sponsor might require that a firm demonstrate some minimal improvement in sponsor knowledge as a condition for contract renewal. Such a policy would encourage the setting of risky targets when a firm exhausted its inventory of knowledge, but might lead to premature cancellation of research projects if targets are not achieved. An alternative policy, implicit in equations (1) and (2) permits contract renewal when the firm sets $X = R$. However $W(R,R)$ is still assumed to be equal to 0.

The formal analysis of this model focuses on two aspects of a firm's research strategy over a sequence of contracts: (i) a firm's choice of research effort in the current period as determined by the firm's level of private knowledge at the beginning of the research period, the level of sponsor known performance at the beginning of the research period, the currently active target, the cost of research, and the number of remaining contracts in which the firm expects to participate; and (ii) the decision to set a safe target versus a risky target for the next research period as determined by the level of privately known performance at the end of the research period, the level of sponsor knowledge at the end of the research period, research costs, and again the length of the firm's planning horizon. The analysis proceeds recursively working backwards from the end of the horizon, i.e. beginning with $t = 1$.

III. THE ONE-PERIOD PROBLEM

As is apparent from section 2, the firm's problem is quite different when it inherits a risky target as compared to when it inherits a safe target. We consider the former first.

3a. Risky Initial Targets

In this case $\sigma < X$. Thus the firm must simply decide whether or not to do research, and

$$V_1(\sigma,R,X) = \max\begin{cases} -c + W(X,R)(1-X) \\ 0 \end{cases} \tag{3}$$

The firm conducts research if and only if $W(X,R)(1-X) > c$. Otherwise it willingly accepts zero profits in the final period.

Lemma 1: $W(X,R)(1-X)$ is concave in X for $X \geq R$.

The proof of this Lemma, and all subsequent results, are found in the appendix. Given Lemma 1, we can easily characterize the set of targets for which the firm does research. Define

$$S_1(c,R) = \{X \mid -c + W(X,R)(1-X) < 0\},$$

and let $\bar{S}_1(c,R)$ be the complement of $S_1(c,R)$ in $[0,1]$.

Figure 1 illustrates $S_1(c,R)$ and $\bar{S}_1(c,R)$. It also illustrates that no research is conducted if X is near 0 or near 1. In the latter case the likelihood of meeting the current target is so low that it doesn't pay to try. In the former the likelihood of meeting the target is high but the payoff is low.[3]

[Figure 1 about here]

Proposition 1: (a) $S_1(c_1,R) \subset S_1(c_2,R)$ for $c_1 < c_2$,

(b) $S_1(c,R_1) \subset S_1(c,R_2)$ for $R_1 < R_2$.

This result shows that when a firm faces a risky target in the final research period, the worse the state of sponsor knowledge (i.e. the lower is R) the more likely the firm is to conduct research. This follows from the assumption that the reward to the firm for fulfillment of the current target increases as the difference between sponsor knowledge and the current target increases (i.e., $W_R(X,R) < 0$). Raising R reduces the profitability of conducting research without affecting the likelihood of achieving the target. As expected, Proposition 1 also implies that the lower the cost of research, the more likely the firm is to conduct research. As can be seen in Figure 1, increasing the cost of research decreases the profitability of conducting research for all levels of the performance target.

Given the definition of $S_1(c,R)$ we can rewrite (3) as

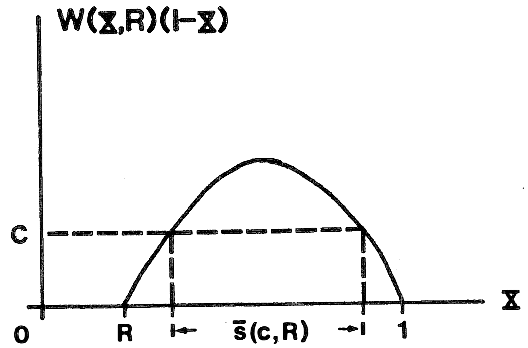
$$V_1(\sigma,R,X) = \begin{cases} -c + W(X,R)(1-X) & \text{if } X \in \bar{S}_1 \\ 0 & \text{if } X \in S_1 \end{cases} \tag{4}$$

3b. Safe Initial Targets

All that remains in the one period problem is the case of $\sigma \geq X$. Since $V_0(\sigma,R,x)$ is defined to be zero, for $\sigma \geq X$

FIGURE 1

Definition of $\bar{S}(c,R)$



$$V_1(\sigma,R,X) = W(X,R). \tag{5}$$

No research is conducted and at least a level of performance X is delivered to the sponsor.

IV. THE TWO-PERIOD PROBLEM

The two period problem is richer than its one period analogue since the firm now sets an optimal target for the last period's research after deciding whether to conduct research during the second to the last period.

From (1) and (2) we have for $X > \sigma$

$$V_2(\sigma,R,X) = \max\begin{cases} -c + [W(X,R) + \underset{Z \geq X}{E}\ \beta \max\limits_{X \leq x \leq 1} V_1(Z,X,x)](1-X) \\ 0 \end{cases} \tag{6}$$

and for $X \leq \sigma$

$$V_2(\sigma,R,X) = \max\begin{cases} -c + W(X,R) + E\ \beta \max\limits_{X \leq x \leq 1} V_1(\max\{\sigma,Z\},X,x) \\ W(X,R) + \max\limits_{X \leq x \leq 1} \beta\ V_1(\sigma,X,x). \end{cases} \tag{7}$$

Recall that the target for the final period is set after observing the current period's research output. The analysis of the two period problem begins by taking that output as given and examining the target setting decision of the firm as a function of it. Thus we need to know more about $V_1(\sigma,R,X)$ as X ranges over [R,1].

4a. Optimal Targeting

Since $\sigma \geq R$ or the firm could not have survived the current period, it has a choice of setting a risky target for the final period ($X > \sigma$) or a safe target ($X \leq \sigma$). In the latter case it reveals all it knows since $W_X > 0$ and $V_0 = 0$. That is, $X = \sigma$ if any safe target is set.

If a risky target is set ($X > \sigma$) then the firm must do research in the final period. Thus we define

$$r_2 = \operatorname*{argmax}_{R\leq X\leq 1} V_1(\sigma,R,X) = \operatorname*{argmax}_{R\leq X\leq 1} W(X,R)(1-X) - c. \tag{8}$$

If $r_2 > \sigma$ then the firm's optimal risky target is r_2 and it compares this targeting strategy to setting $X = \sigma$. If $r_2 < \sigma$, though, the firm will want to set risky targets arbitrarily close to σ (since $W(X,R)(1-X) - c$ is concave). In this case setting a safe target of σ will dominate any risky target due to the following result.

Lemma 2: $W(\sigma,R) > \lim_{X\to\sigma} W(X,R)(1-X) - c$

Given Lemma 2 we need only compare $W(\sigma,R)$ to $W(r_2,R)(1-r_2) - c$. Define x_2^* as the optimal target in the two period problem. Then

$$x_2^* = \begin{cases} \sigma & \text{if } W(\sigma,X) \geq W(r_2,R)(1-r_2) - c \\ r_2 & \text{if } W(\sigma,X) < W(r_2,R)(1-r_2) - c \end{cases} \tag{9}$$

Whether x_2^* equals σ or r_2 clearly depends on σ. Hence we define

$$\bar{\sigma}_2 = \begin{cases} R & \text{if } c > W(r_2,R)(1-r_2) - W(\sigma,R) \text{ for all } \sigma \geq R \\ \{\sigma \mid c = W(r_2,R)(1-r_2) - W(\sigma,R)\} & \text{otherwise} \end{cases} \tag{10}$$

Proposition 2: $\bar{\sigma}_2$ is unique and well-defined. Furthermore, if $\bar{\sigma}_2 > R$ then

(i) $d\bar{\sigma}_2/dc < 0$

(ii) $d\bar{\sigma}_2/dR > 0$

(iii) $r_2 > \bar{\sigma}_2 > R$

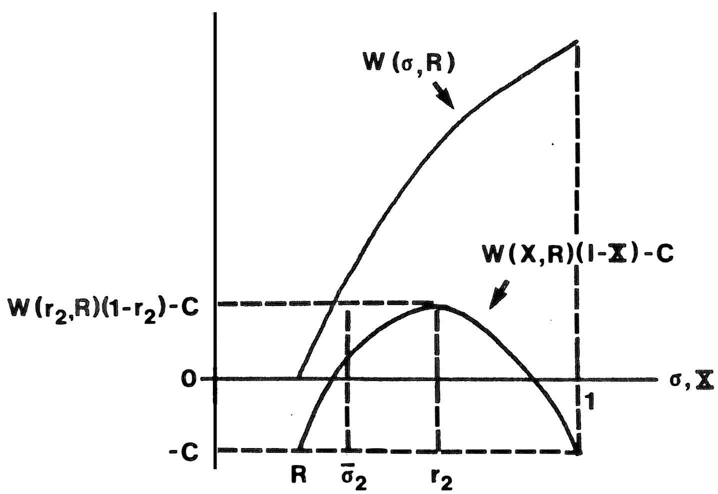
Figure 2 illustrates $\bar{\sigma}_2$. For $\sigma < \bar{\sigma}_2$, $W(r_2,R)(1-r_2) - c > W(\sigma,R)$ so $x_2^* = r_2$. For $\sigma \geq \bar{\sigma}_2$, $W(r_2,R)(1-r_2) - c \leq W(\sigma,R)$ so $x_2^* = \sigma$.

[Figure 2 about here]

That $\bar{\sigma}_2$ is uniquely defined by c and R obtains because the optimal risky target, r_2, is independent of the firm's level of private knowledge, σ, while $W(\sigma,R)$ is increasing in σ. Proposition 2 also shows that the lower the cost of research and the better the state of sponsor knowledge, the more likely the firm is to set a risky target for the final research period. The first of these is obvious since a decision to set a risky target implies that research is compulsory in the final period, and a larger value of c makes this research more costly. The second obtains because an increase in sponsor knowledge hurts the firm less under a risky strategy than a safe strategy, hence at the margin it is more willing to engage in risky targeting when R increases. Finally, we have related results on dr_2/dR and dr_2/dc.

FIGURE 2

Definition of $\bar{\sigma}_2$



Proposition 3: If $r_2 \varepsilon (\sigma,1)$ then $dr_2/dR > 0$ and $dr_2/dc = 0$.[4]

As the level of sponsor knowledge increases, the optimal risky target for the final period also increases. Thus, as R increases, not only is the firm more likely to set a risky target, but it sets one which has less probability of being met. Research costs have no effect on r_2 since the firm will have to do research in any case when it sets a risky target and behavior in later periods is irrelevant since only one period remains. This result will not generalize to $t > 2$.

Finally, we can show that whenever setting a risky target dominates setting a safe target, the firm will never set the risky target in such a way so as to want to drop out of the contractual relationship in the last period.

Corollary 1: If $\sigma < \bar{\sigma}_2$ and $\bar{S}_1 \neq 0$ then $r_2 \varepsilon \bar{S}_1(c,R)$.

This result is immediate and is stated without proof. It follows because r_2 is the maximum of $W(X,R)(1 - X)$ on $[R,1]$.

Using $\bar{\sigma}_2$ we can rewrite $V_1(\sigma,R,x_2^*)$ in a more useful form:

$$V_1(\sigma,R,x_2^*) = \begin{cases} W(r_2,R)(1 - r_2) - c & \text{if } \sigma < \bar{\sigma}_2 \\ W(\sigma,R) & \text{if } \sigma \geq \bar{\sigma}_2 \end{cases} \tag{11}$$

These results allow us to analyze the two-period problem.

4b. Risky Initial Targets

Using $\bar{\sigma}_2$ we can rewrite (6) as

$$V_2(\sigma,R,X) = \max\begin{cases} -c + W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2} V_1(z,X,r_2)dz + \beta\int_{\bar{\sigma}_2}^1 V_1(z,X,z)dz \\ 0 \end{cases} \tag{12}$$

where $\bar{\sigma}_2 = \bar{\sigma}_2(X)$ and $r_2 = r_2(X)$; that is, the current target X becomes the level of sponsor knowledge in the final period.

As in the one-period problem, we first characterize the set of targets for which the firm does research. Define

$$S_2(c,R) = \{X | c > W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2} V_1(z,X,r_2)dz + \beta\int_{\bar{\sigma}_2}^1 V_1(z,X,z)dz\},$$

and let $\bar{S}_2(c,R)$ be the complement of $S_2(c,R)$ in [0,1].

<u>Proposition 4</u>: (a) $S_2(c_1,R) \subset S_2(c_2,R)$ for $c_1 < c_2$

(b) $S_2(c,R_1) \subset S_2(c,R_2)$ for $R_1 < R_2$

(c) $S_2(c,R) \subset S_1(c,R)$

The firm is more likely to do research when it faces a risky target the lower is c or R. Furthermore, it is more likely to do so when two research periods remain as compared to one. One more rewrite of (4), and thus (12), is possible:

$$V_2(\sigma,R,X) = \begin{cases} -c + W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2} V_1(z,X,r_2)dz + \beta\int_{\bar{\sigma}_2}^1 V_1(z,X,z)dz & \text{if } x \in \bar{S}_2 \\ 0 & \text{if } x \in S_2 \end{cases} \tag{13}$$

4c. Safe Initial Targets

In this case $\sigma \geq X$, but we can't say whether σ exceeds $\bar{\sigma}_2$ or not. Thus (7) becomes

$$V_2(\sigma,X,R) = \max\begin{cases} -c + W(X,R) + \beta\int_0^{\sigma} V_1(\sigma,X,x_2^*)dz + \beta\int_{\sigma}^1 V_1(z,X,x_2^*)dz \\ W(X,R) + \beta V_1(\sigma,X,x_2^*) \end{cases} \tag{14}$$

Here

$$x_2^*(y) = \begin{cases} y & \text{if } y \geq \bar{\sigma}_2 \\ r_2 & \text{if } y < \bar{\sigma}_2 \end{cases}$$

where $y \in \{\sigma,Z\}$. The issue is whether or not to do research. The first term in (14) exceeds the second if and only if

$$c < \beta\int_{\sigma}^1 [V_1(z,X,x_2^*) - V_1(\sigma,X,x_2^*)]dz. \tag{15}$$

Hence we define a "reservation level" of private knowledge, σ_2^*, by

$$\begin{cases} \sigma_2^* = 0 \quad \text{if } c \geq \beta\int_{\sigma}^1 [V_1(z,X,x_2^*) - V_1(\sigma,X,x_2^*)]dz \text{ for all } \sigma \\ c = \beta\int_{\sigma_2^*}^1 [V_1(z,X,x_2^*) - V_1(\sigma_2^*,X,x_2^*)dz \quad \text{otherwise} \end{cases} \tag{16}$$

<u>Proposition 5</u>: σ_2^* is unique and well-defined. Furthermore, if $\sigma_2^* > 0$ then

(a) $\sigma_2^* > \bar{\sigma}_2$

(b) $d\sigma_2^*/dc < 0$

(c) $d\sigma_2^*/dX \geq 0$.

If $\sigma < \sigma_2^*$ then the firm conducts research. If $\sigma \geq \sigma_2^*$ then it does not. If $\sigma_2^* > 0$ then it is also greater than $\bar{\sigma}_2$, the critical level of private knowledge that makes the firm indifferent between setting a risky versus a safe target at the end of the second to the last research period. This implies that when the firm enters the second to the last research period with a level of private knowledge below the cutoff point for setting a safe target, it always conducts research hoping to avoid the need to set a risky performance target for its last contract, unless <u>all</u> safe targets imply no research (i.e. $\sigma_2^* = 0$). Hence if $\sigma_2^* > 0$, the equation which defines it reduces to

$$c = \beta\int_{\sigma_2^*}^{1}[W(z,X) - W(\sigma_2^*,X)]dz. \tag{17}$$

Proposition 5 also shows an increase in sponsor knowledge or a decrease in research costs will increase the likelihood the firm conducts research. Finally, the definition of σ_2^* allows one more rewrite of (7) and hence (16):

$$V_2(\sigma,R,X) = \begin{cases} -c + W(X,R) + \beta\int_0^{\sigma} V_1(\sigma,X,x_2^*)dz + \beta\int_{\sigma}^{1} V_1(z,X,x_2^*)dz & \text{if } \sigma < \sigma_2^* \\ W(X,R) + \beta V_1(\sigma,X,x_2^*) \quad \text{if } \sigma \geq \sigma_2^*. \end{cases} \tag{18}$$

A brief summary of the two-period problem will be useful before turning to the three-period problem. The firm enters the beginning of the second to the last period with a stock of knowledge in inventory, a current target and a given level of sponsor knowledge. If the current target is risky it first must decide whether to do research or drop out of the contractual arrangement altogether. If it

conducts research and meets the current target it must then choose a target for the final period. Whether that target is risky or not depends on a critical level of private knowledge, $\bar{\sigma}_2$. If the current target is safe, it must again decide whether to do research, but in this case if it does not, it is not forced to drop out. Whether it does research depends on a reservation value of private knowledge, σ_2^*. However, if $\sigma_2^* > 0$ then $\sigma_2^* > \bar{\sigma}_2$, so that the firm always does research when it sets a safe target unless all safe targets imply no research (i.e. $\sigma_2^* = 0$). On the other hand, if its stock of private knowledge is such that it knows it will want to set a risky target in the final period, then it necessarily conducts research during the second to the last period. Hence, if there exist levels of private knowledge and safe targets for which it will want to do research in the last period, then the firm will always do research in the last period. Otherwise, whether or not it does research is determined by whether or not the optimal is risky, respectively.

5. THE THREE-PERIOD PROBLEM

Now that the firm's research strategy in the two-period problem has been fully characterized, it is possible to consider the firm's selection of targets and decision to conduct research when three or more periods remain in the planning horizon. This problem is more complicated than the two-period analogue because when $t \geq 3$ the firm's best safe target at the end of the period is not necessarily to reveal everything it knows. Instead the firm may have an incentive to temporarily withhold some of its private knowledge. This knowledge inventory can then be depleted over the remaining research periods so as to maximize the net present value of profits. A conservative strategy of sequentially setting higher safe performance targets also insures the firm against the risk of losing profits on remaining contracts if it encounters a bad draw from the distribution over research potential.

When $t = 3$ and $\sigma < X$ equation (1) gives

$$V_3(\sigma,R,X) = \begin{cases} -c + [W(X,R) + \underset{Z>X}{E}\ \beta \underset{X\leq x\leq 1}{\max} V_2(Z,X,x)](1-X) \\ 0 \end{cases} \tag{19}$$

and when $\sigma \geq X$ equation (2) implies

$$V_3(\sigma,R,X) = \max \begin{cases} -c + W(X,R) + \underset{X\leq x\leq 1}{E}\ \beta\, V_2(\max\{\sigma,Z\},X,x) \\ W(X,R) + \underset{X\leq x\leq 1}{\max}\ \beta\ V_2(\sigma,X,x) \end{cases} \tag{20}$$

Analogous to the two-period problem, we begin by taking the output of the third to the last research period as given, and examine the target setting decision of the firm as a function of it. Once the targeting decision is fully characterized one can again back up to the beginning of the third to the last research period and embed the solution to the two-period targeting problem in the three-period setting.

5a. Optimal Targeting

The firm's objective at the end of the third to the last research period is to choose a target for the next contract which maximizes $V_2(\sigma,R,X)$, where σ may be knowledge held in inventory or the outcome of research conducted in the third to the last research period. Again, X is allowed to range over $[R,1]$. But $\sigma \geq R$ as well. Suppose $\sigma > R$. Then $X \in [R,\sigma]$ is called a safe target and $X \in (\sigma,1]$ is called a risky target. The two cases need to be considered separately.

Consider safe targeting strategies. Define

$$s_3 = \underset{R\leq X\leq \sigma}{\operatorname{argmax}}\ V_2(\sigma,R,X). \tag{21}$$

As yet we know nothing about V_2. In particular, it may not be concave in X! Thus s_3 may not be unique and may not be interior to $[R,\sigma]$. Furthermore, properties of V_2 when $\sigma \geq X$ depend on where σ lies relative to $\bar{\sigma}_2$ and σ_2^*. But when $\sigma \geq X$, we do know that

$$V_2(\sigma,R,X) = \begin{cases} -c + W(X,R) + \beta\int_0^{\sigma} V_1(\sigma,X,x_2^*)dz - \beta\int_{\sigma}^{1} V_1(z,X,x_2^*)dz & \text{if } \sigma < \sigma_2^* \\ W(X,R) + \beta V_1(\sigma,X,x_2^*) \quad \text{if } \sigma \geq \sigma_2^* \end{cases} \tag{22}$$

CASE 1: $\sigma < \bar{\sigma}_2 < \sigma_2^*$

In this case $\sigma < \sigma_2^*$ so, from (22),

$$V_2(\sigma,R,X) = -c + W(X,R) + \beta\int_0^\sigma V_1(\sigma,X,r_2)dz$$

$$+ \beta\int_\sigma^{\bar{\sigma}_2} V_1(z,X,r_2)dz + \beta\int_{\bar{\sigma}_2}^1 V_1(z,X,z)dz. \quad (23)$$

But r_2 is independent of the level of private knowledge, so for $z \le \bar{\sigma}_2$,

$$V_1(\sigma,X,r_2) = -c + W(r_2,X)(1 - r_2) = V_1(z,X,r_2).$$

For $z \ge \bar{\sigma}_2$,

$$V_1(z,X,z) = W(z,X).$$

Thus (23) becomes

$$V_2(\sigma,R,X) = -c + W(X,R) + \beta\int_0^{\bar{\sigma}_2} [-c + W(r_2,X)(1-r_2)]dz + \beta\int_{\bar{\sigma}_2}^1 W(z,X)dz. \quad (24)$$

Since $\bar{\sigma}_2$ and r_2 depend on X, $V_2(\sigma,R,X)$ is independent of σ for $\sigma < \sigma_2^*$! Thus if $\sigma < \sigma_2^*$, the set of safe targets which maximize V_2 is independent of σ, as is the maximized value.

<u>CASE 2:</u> $\bar{\sigma}_2 \le \sigma < \sigma_2^*$

In this case V_2 becomes, again from the first branch of (22),

$$V_2(\sigma,R,X) = -c + W(X,R) + \beta\int_0^{\bar{\sigma}_2} V_1(\sigma,X,\sigma)dz + \beta\int_{\bar{\sigma}_2}^{\sigma} V_1(\sigma,X,\sigma)dz + \beta\int_\sigma^1 V_1(z,X,z)dz,$$

or

$$V_2(\sigma,R,X) = -c + W(X,R) + \beta\int_0^\sigma W(\sigma,X)dz + \beta\int_\sigma^1 W(z,X)dz. \quad (25)$$

Taking the derivative of (25) with respect to σ we have

$$\partial V_2/\partial\sigma = \beta\int_0^\sigma W_X(\sigma,X)dz = \beta(1-\sigma)W_X(\sigma,X) > 0. \quad (26)$$

Thus the set of safe targets which maximize V_2 depends on σ and the maximized value of V_2 increases with σ at rate $\beta(1-\sigma)W_X(\sigma,s_3)$ for any one of them.

<u>CASE 3:</u> $\bar{\sigma}_2 < \sigma_2^* < \sigma$.

In this case, from the second branch of (22),

$$V_2(\sigma,R,X) = W(X,R) + \beta V_1(\sigma,X,\sigma)$$
$$= W(X,R) + \beta W(\sigma,X). \quad (27)$$

Hence

$$\partial V_2/\partial\sigma = \beta W_X(\sigma,X) > 0. \quad (28)$$

Again, the set of safe targets which maximizes V_2 depends on σ and the maximized value of V_2 increases with σ at rate $\beta W_X(\sigma,s_3)$ for any one of them.

Using these results we can graph max $V_2(\sigma,R,X)$ for $X \in [R,\sigma]$ as in figure 3.

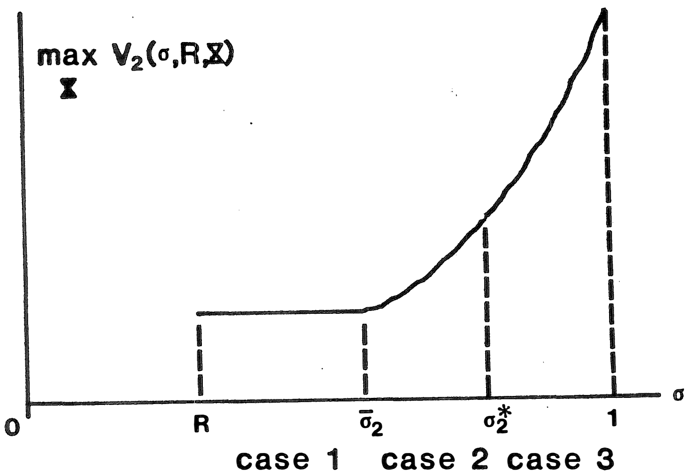
[Figure 3 about here]

Comparing (28) to (26) implies the slope of max $V_2(\sigma,R,X)$ on $[\sigma_2^*,1]$ is positive and greather than that on $[\bar{\sigma}_2,\sigma_2^*]$ since $W_{XR} \ge 0$.

Consider next risky targeting strategies. This applies to $X > \sigma$ so some care must be taken as X approaches σ. When $X > \sigma$ we

FIGURE 3

Form of max $V_2(\sigma,R,X)$ for $X \varepsilon [R,\sigma]$



know

$$V_2(\sigma,R,X) = \begin{cases} -c + W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2} V_1(z,X,r_2)dz + \beta\int_{\bar{\sigma}_2}^1 V_1(z,X,z)dz & \text{if } X \varepsilon \bar{S}_2 \\ 0 \quad \text{if } X \varepsilon S_2 \end{cases} \tag{29}$$

For $x \varepsilon \bar{S}_2$, this can be rewritten as

$$V_2(\sigma,R,X) = -c + W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2} [-c + W(r_2,X)(1-r_2)]dz + \beta\int_{\bar{\sigma}_2}^1 W(z,X)dz \tag{30}$$

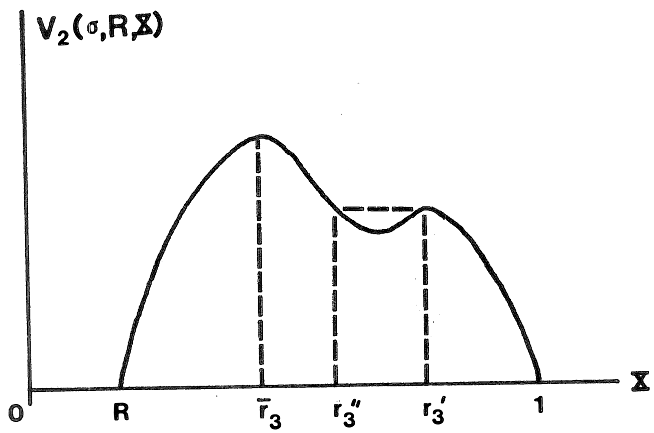
which is clearly independent of σ. Consider the form of $V_2(\sigma,R,X)$ for $X \varepsilon [R,1]$. For now we ignore whether $X \varepsilon S_2$ or $X \varepsilon \bar{S}_2$, and assume the firm always conducts research when it faces a risky target. Figure 4 illustrates one possibility for $V_2(\sigma,R,X)$ as a function of X. Note it has a unique global maximum at $\bar{r}_3$ and a local maximum at r_3'.

[Figure 4 about here]

If $\sigma < \bar{r}_3$ then $\bar{r}_3$ is the optimal risky target. If $\bar{r}_3 \leq \sigma < r_3''$ then the optimal risky target is not well-defined, if $r_3'' \leq \sigma < r_3'$, then the optimal risky target is r_3' and if $r_3' \leq \sigma \leq 1$, then the optimal risky target is again not well-defined. Of concern here are the ranges $[\bar{r}_3,r_3'')$ and $[r_3',1]$. If σ falls in these ranges, the optimal risky target is arbitrarily close to σ. But as soon as σ is reached, the firm has a safe target, and V_2 is given by (22), not (29). It turns out that in these cases, a safe target of σ dominates all risky

FIGURE 4

Example of $V_2(\sigma,R,X)$ when $1 \geq X \geq R$ and $X > \sigma$



targets, as the next Lemma shows. Here V_2^s and V_2^r refer to the value functions as defined by (22) and (29), respectively, for safe and risky targets.

<u>Lemma 3</u>: $\lim_{X \to \sigma} V_2^r(\sigma,R,X) < V_2^s(\sigma,R,\sigma)$.

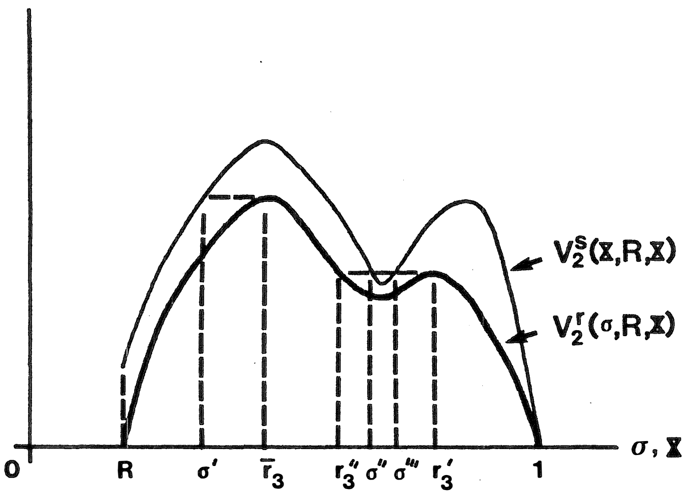
Lemma 3 is really quite intuitive. It simply implies that if the firm sets a risky target of X when it knows only σ, then it would be better off if it actually knew X and could set a safe target. In Figure 5, $V_2^s(X,R,X)$ is plotted above $V_2^r(\sigma,R,X)$. In that figure,

[Figure 5 about here]

suppose $\sigma \leq \sigma'$. Then the optimal risky target is $\bar{r}_3$ since $V_2^s(\sigma,R,\sigma) \leq V_2^r(\sigma,R,\bar{r}_3)$. But when $\sigma \varepsilon (\sigma', \bar{r}_3)$, $V_2^s(\sigma,R,\sigma) > V_2^r(\sigma,R,\bar{r}_3)$; that is, when $\sigma \varepsilon (\sigma', \bar{r}_3)$, it is possible to set a safe target of σ, and this dominates the optimal risky target of $\bar{r}_3$. When $\sigma \varepsilon [\bar{r}_3, r_3'')$, the optimal risky target is not well-defined, but $V_2^s(\sigma,R,\sigma) > V_2^r(\sigma,R,X)$ for all $X > \sigma$, so again some safe target is preferred (possibly less than σ). On $[r_3'', \sigma'']$, an optimal risky target exists (r_3'), but $V_2^s(\sigma,X,\sigma)$ again dominates it. On (σ'', σ'''), $V_2^r(\sigma,R,r_3') > V_2^s(\sigma,R,\sigma)$ so the risky target of r_3' is both well-defined and preferred to a safe target of σ. Finally, on $[\sigma''', 1]$, the safe target of σ dominates all risky strategies, whether an optimal risky target is well-defined or not. Figure 6 illustrates the form of the maximized value of $V_2^r(\sigma,R,X)$ with respect to $X \geq \sigma$ as a function of σ for the situation illustrated in Figure 5. The heavy line represents max $V_2^r(\sigma,R,X)$ where it is well-defined and the lighter

FIGURE 5

Example of safe versus risky targeting; safe is optimal for $\sigma \in [\sigma', \sigma'']$ and $\sigma \in [\sigma''', 1]$. Optimal risky target is not well-defined on $[\bar{r}_3, r_3'']$ and $[r_3', 1]$.



line represent $V_2^s(\sigma,R,\sigma)$ on regions where it dominates all risky strategies.

[Figure 6 about here]

Figures 6 and 5 can be combined to compare risky versus safe strategies as a function of σ. This is done in Figure 7. Notice that

[Figure 7 about here]

max V_2^s can never cut max V_2^r to the right of σ' since max $V_2^s(\sigma,R,X)$ on $X \leq \sigma$ must dominate $V_2^s(\sigma,R,\sigma)$. Thus there exists a unique value of σ such that the firm is indifferent between a risky target and a safe target unless safe targets are preferred for all σ. We define this value as $\bar{\sigma}_3$. Figure 7 also illustrates that $\bar{r}_3 \geq \bar{\sigma}_3 > \bar{\sigma}_2$! Therefore we have shown the following. Define

$$\begin{cases} \bar{\sigma}_3 = R & \text{if } \max V_2^s(\sigma,R,X) \geq \max V_2^r(\sigma,R,X) \quad \text{for all } \sigma \geq R \\ \{\sigma \mid \max V_2^s(\sigma,R,X) = \max V_2^r(\sigma,R,X)\} & \text{otherwise} \end{cases} \tag{31}$$

Proposition 6: $\bar{\sigma}_3$ is unique and well-defined. Furthermore, if $\bar{\sigma}_3 > R$, then

(i) $d\bar{\sigma}_3/dc < 0$

(ii) $d\bar{\sigma}_3/dR > 0$

(iii) $\bar{r}_3 > \bar{\sigma}_3 > \bar{\sigma}_2$

where $\bar{r}_3 = \min\{X \mid X = \underset{\sigma < X \leq 1}{\text{argmax}}\ V_2(\sigma,R,X)\}$.

Up to this point we have said nothing about the uniqueness of s_3 or $r_3 \equiv \text{argmax } V_2(\sigma,R,X)$ for $\sigma < X \leq 1$. No simple set of conditions guarantee either is unique or that r_3 is even well-defined. Nevertheless, $\bar{\sigma}_3$ is unique and well-defined. If $\sigma < \bar{\sigma}_3$ then the firm

FIGURE 6

Form of $\max_X V_2^r(\sigma,R,X)$ for $X > \sigma$ and relation to $V_2^s(\sigma,X,\sigma)$

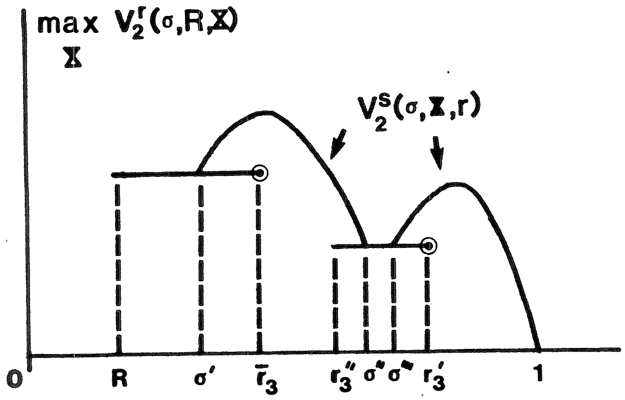
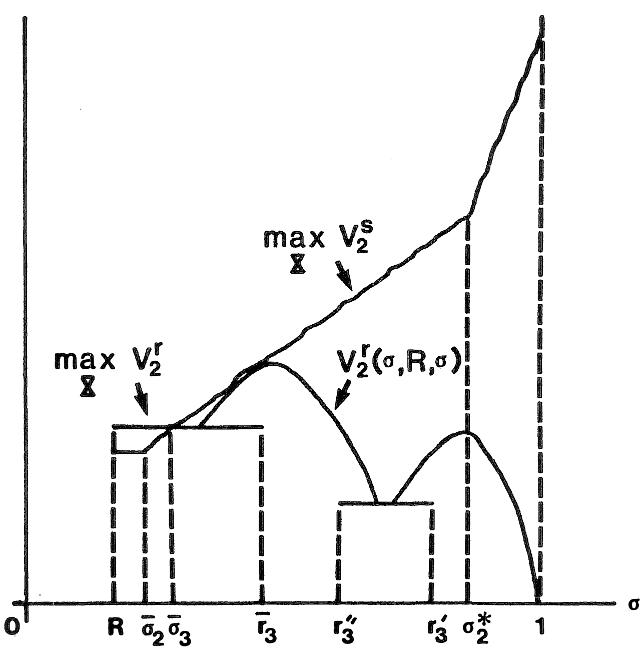


FIGURE 7

Definition of $\bar{\sigma}_3$



sets a risky target for the second to the last period and if $\sigma \geq \bar{\sigma}_3$ it sets a safe target for the second to the last period. Note that if $\sigma < \bar{\sigma}_3$, then r_3 must necessarily exist! We can state the following Corollary to Proposition 6 even though the premise may not hold for r_3.

<u>Corollary 2</u>: s_3 is unique and well-defined for $\sigma \geq \bar{\sigma}_2$. If $s_3 \in (0,\sigma)$ and $r_3 \in (\sigma,1)$ then

(i) $dr_3/dc \leq 0 \leq ds_3/dc$

(ii) $ds_3/dR > 0$ and $dr_3/dR > 0$

(iii) $ds_3/d\sigma \geq 0 = dr_3/d\sigma$.

The comparative statics given in Proposition 6 and its Corollary can be summarized as follows. As the cost of research <u>increases</u>, the firm is <u>less</u> likely to set a risky target. If it does, it generally sets a <u>less</u> risky one. If it does not, it reveals <u>more</u> of what it knows in a safe target. If sponsor knowledge <u>increases</u>, the firm is <u>more</u> likely to set a risky target and, if it does, the target will be <u>riskier</u>. If it does not set a risky target it reveals <u>more</u> of what it knows. Finally, as the prviate knowledge of the firm <u>increases</u> it is <u>more</u> likely to set a <u>safe</u> target. It's optimal safe target also <u>increases</u>, so that it reveals some of this additional knowledge, but its optimal risky target is unaffected. Interestingly, Corollary 2 also shows s_3 is uniquely defined on any relevant range of σ; that is, if $\sigma < \bar{\sigma}_3$ then the firm sets a risky target; it only sets a safe target if $\sigma \geq \bar{\sigma}_3$ and in this case s_3 is unique.

Before considering the firm's optimal research strategy in the three-period problem, a decision which is made prior to the selection of an optimal target for the two-period problem, two further issues are of interest. The first concerns our assumption, made at the beginning of the analysis of risky targeting, that the firm always conducts research in the two-period problem if it faces a risky target. We know that $X \varepsilon S_2$ if and only if $c > V_2^r(\sigma,R,X)$ when $\sigma < X$. Referring to the example of V_2^r used in the above discussion, Figure 8 illustrates S_2 and $\bar{S}_2$.

[Figure 8 about here]

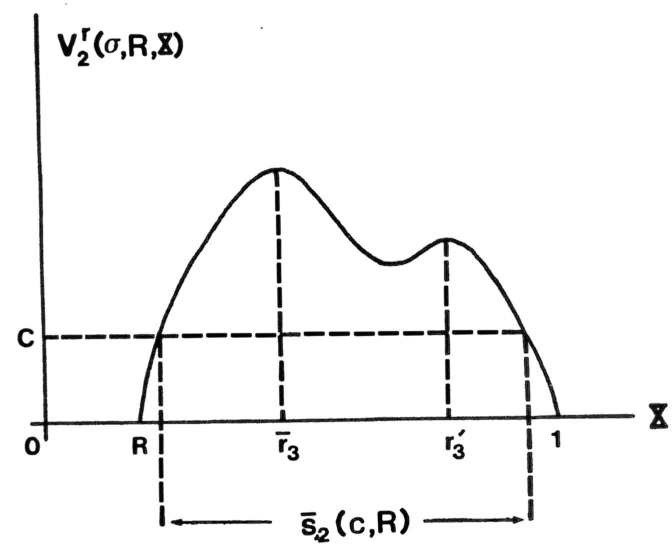
It is clear from Figure 8 that if V_2^r is not concave, $\bar{S}_2$ will not necessarily be a connected set. However, $\min\{r_3 | r_3 = \underset{R \leq X \leq 1}{\text{argmax}}\ V_2^r(\sigma,R,X)\} \equiv \bar{r}_3 \ \varepsilon\ \bar{S}_2$ as long as $\bar{S}_2 \neq \emptyset$. As shown above, $\bar{r}_3$ is the crucial risky target (see Proposition 6). If the firm sets a risky target, it will be $\bar{r}_3$, and $\bar{r}_3 \ \varepsilon\ \bar{S}_2$. Thus it will never set a risky target at the end of the third to the last period in such a way that it will want to drop out of the contractual relationship at the beginning of the second to the last period. This result is stated without proof in Corollary 3.

<u>Corollary 3</u>: If $\sigma < \bar{\sigma}_3$ and $\bar{S}_2 \neq \emptyset$, then $\bar{r}_3 \ \varepsilon\ \bar{S}_2$.

The second issue related to the target setting decision at the end of the third to the last research period concerns conditions under which the firm sets a safe target which reveals all of its current knowledge, i.e. conditions under which $s_3 = \sigma$, and r_3 is either not

FIGURE 8

Definition of S_2(c,R



well-defined or is dominated by a safe strategy. To analyze, this possiblity, note that $s_3 = \sigma$ if and only if

$$\left.\frac{\partial V_2^s(\sigma,X,R)}{\partial X}\right|_{X=\sigma} > 0$$

But an examination of this derivative in the three cases relevant to safe targeting reveals no apparent systematic behavior.[6]

5b. Risky Initial Targets

Using $\bar{\sigma}_3$ we can write (19) as

$$V_3(\sigma,R,X) = \max\begin{cases} -c + W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_3} V_2(z,X,r_3)dz + \beta\int_{\bar{\sigma}_3}^1 V_2(z,X,s_3)dz \\ 0 \end{cases} \quad (32)$$

where $\bar{\sigma}_3 = \bar{\sigma}_3(X), r_3 = r_3(X)$ and $s_3 = s_3(z,X)$. Here r_3 can be any risky target which maximizes $V_2^r(z,X,r)$ on $r \varepsilon (\sigma,1]$.[7] Again, the initial target, X, for the three-period problem becomes the level of sponsor knowledge in the two-period problem. Define

$$S_3(c,R) = \{X|c > W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_3} V_2(z,X,r_3)dz + \beta\int_{\bar{\sigma}_3}^1 V_2(z,X,s_3)dz\}$$

and let $\bar{S}_3(c,R)$ be the complement of $S_3(c,R)$ in [0,1]. The following result follows in an analogous fashion to Proposition 4 and is stated without proof.

Proposition 7:

(a) $S_3(c_1,R) \subset S_3(c_2,R)$ for $c_1 < c_2$

(b) $S_3(c,R_1) \subset S_3(c,R_2)$ for $R_1 < R_2$

(c) $S_3(c,R) \subset S_2(c,R)$.

Again, lower costs and a lower level of sponsor knowledge encourage the firm to conduct research when it faces a risky target. Similarly, it also finds a longer sequence of potential research "sub-contracts" an encouragement to do research when the current target is risky.

Analogous to the two-period problem, (32) becomes

$$V_3(\sigma,R,X) = \begin{cases} -c + W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_3} V_2(z,X,r_3)dz + \beta\int_{\bar{\sigma}_3}^1 V_2(z,X,s_3)dz & \text{if } X \varepsilon \bar{S}_3 \\ 0 & \text{if } X \varepsilon S_3 \end{cases} \quad (33)$$

5c: Safe Initial Targets

As in the two-period problem, we know $\sigma \geq X$ in this case but whether σ exceeds $\bar{\sigma}_3$ or not is unknown. Thus (20) becomes

$$V_3(\sigma,X,R) = \max\begin{cases} -c + W(X,R) + \beta\int_0^{\sigma} V_2[\sigma,X,x_3^*(\sigma)]dz + \beta\int_{\sigma}^1 V_2[z,X,x_3^*(z)]dz \\ W(X,R) + \beta V_2[\sigma,X,x_3^*(\sigma)]. \end{cases} \quad (34)$$

where

$$x_3^*(y,X) = \begin{cases} r_3(X) & \text{if } y < \bar{\sigma}_3(X) \\ s_3(y,X) & \text{if } y \geq \bar{\sigma}_3(X). \end{cases}$$

and $y \varepsilon \{\sigma,z\}$. The issue here is again whether or not to do research. The first term in (34) exceeds the second if and only if

$$c < \beta\int_{\sigma}^{1}\{V_2[z,X,x_3^*(z)] - V_2[\sigma,X,x_3^*(\sigma)]\}dz. \tag{35}$$

A reservation level of private knowledge for the three-period problem, σ_3^*, can thus be defined by

$$\begin{cases} \sigma_3^* = 0 \quad \text{if } c \geq \beta\int_{\sigma}^{1}\{V_2[z,X,x_3^*(z)] - V_2[\sigma,X,x_3^*(\sigma)]\}dz \quad \text{for all } \sigma \\ c = \int_{\sigma_3^*}^{1}\{V_2[z,X,x_3^*(z)] - V_2[\sigma_2^*,X,x_3^*(\sigma_3^*)]\}dz \quad \text{otherwise.} \end{cases} \tag{36}$$

<u>Proposition 8</u>: σ_3^* is unique and well-defined. Furthermore, if $\sigma_3^* > 0$ then

(a) $\sigma_3^* > \bar{\sigma}_3$

(b) $d\sigma_3^*/dc < 0$

(c) $d\sigma_3^*/dX \geq 0$

If $\sigma < \sigma_3^*$ then the firm conducts research. If $\sigma \geq \sigma_3^*$ then it does not. If $\sigma_3^* > 0$ then it is also greater than $\bar{\sigma}_3$, the critical level of private knowledge that makes the firm indifferent between setting a risky versus a safe target at the end of the third to the last research period. As in the two-period problem, if the firm enters the three-period problem with a level of private knowledge below the cutoff point for setting a safe target in the subsequent research period, it always conducts research, hoping to avoid the need to set a risky target in that period. Thus, if $t \geq 2$, and $\sigma_3^* > 0$, the firm <u>always</u> conducts research if it doesn't pay to drop out. If $\sigma_3^* = 0$,

whether or not it does research depends entirely on whether or not it sets a risky target. Again, increases in research costs or decreases in sponsor knowledge reduce the likelihood of doing research. Finally, the definition of σ_3^* allows (33) to be written as

$$V_3(\sigma,X,R) = \begin{cases} -c + W(X,R) + \beta\int_0^{\sigma}V_2[\sigma,X,x_3^*(\sigma)]dz + \beta\int_{\sigma}^{1}V_2[z,X,x_3^*(z)]dz \quad \text{if} \quad \sigma < \sigma_3^* \\ W(X,R) + \beta V_2[\sigma,X,x_3^*(\sigma)] \quad \text{if} \quad \sigma \geq \sigma_3^* \end{cases}$$

One final result is of interest when $t = 3$. This concerns the relation of σ_3^* to σ_2^*. Proposition 9 shows $\sigma_3^* > \sigma_2^*$ when the discount rate is sufficiently high so that the longer the contractual horizon, the more likely the firm is to do research when it faces a safe target. Otherwise, $\sigma_3^* \leq \sigma_2^*$.

<u>Proposition 9</u>: There exists $\hat{\beta}_3 \varepsilon (0,1)$ such that $\sigma_3^* \gtrless \sigma_2^*$ if $\beta \lessgtr \hat{\beta}_3$.

VI. SUMMARY AND CONCLUSION

The formal analysis this paper has dealt only with contractual horizons of $t = 1$, 2, and 3. It is tedious but relatively straightforward to generalize the results to $t \geq 4$. We will not do so. The unique feature of the model from a technical point of view is

that in spite of nonconcavities in the value functions for $t \geq 2$, a number of strong results are possible. In particular, we have shown that an increase in the number of potential contracts increases the likelihood that the firm will set a risky target and makes it more likely to do research if it faces a safe target (the latter depending on a relatively high discount rate). In fact, any parametric change which increases the likelihood that the firm will do research will also increase the likelihood that it sets a risky target. This is an unintuitive result, especially for increases in the number of potential contracts. To see why it holds, suppose the firm has a level of private knowledge at which it is just indifferent between a safe and risky target. Let the number of remaining contracts increase by one. If the firm now sets a safe target it will also want to do research in the next period. But if this research then increases its stock of knowledge it cannot capitalize on it until another research period passes. Hence, at the margin, it is willing to incur some risk in order to capture part of these potential gains one period earlier. This argument holds as long as there exist safe targets for which research is desirable.

Of course our results were all obtained under the assumption that expectations over research potential are uniform. While this is a moderately strong assumption, the results may well hold for other distributions. This is likely to be the case because even with the uniform distribution, the value functions for horizons greater than or equal to two are not concave in targets. That we were able to derive our results in spite of this suggests a cautious optimism regarding generalizations to nonuniform distributions.

It is also possible to analyze other assumptions regarding the payoff function W(X,R). An assumption that $W_{XR} < 0$ will not change the basic qualitative features of the model, but will reverse some of

the comparative statics results and leave others ambiguous. However, if $W_{RR} > 0$, the firm's optimal strategy can be quite different. In particular, if W_{RR} is large enough (and positive), the firm pursues a kind of "bang-bang" policy -- it either reveals none of what it knows or everything. This also holds for our earlier analysis (Balbien and Wilde, 1982).

The more interesting generalizations, though, would introduce some degree of sophistication on the sponsor side of the problem. It is these we intend to pursue in future research. It should be pointed out that the main issue from the sponsor's point of view is not misrepresentation of results per se (either through under-reporting safe targets or over-reporting risky targets). The sponsor is interested in some overall measure of project benefits, and misrepresentation may be optimal if it is associated with positive incentives to do research. Our partial equilibrium results, both here and in our earlier paper, suggest this might well be the case.

APPENDIX

Proof of Lemma 1: Define $f(x) = W(X,R)(1 - X)$. Then

$$f'(X) = W_X(1 - X) - W,$$
$$f''(X) = W_{XX}(1 - X) - 2W_X.$$

Hence $f''(X) < 0$ since $W_{XX} < 0$ and $W_X > 0$. Q.E.D.

Proof of Lemma 2: It is trivial that

$$\lim_{X \to \sigma} W(X,R)(1-X)-c = W(\sigma,R)(1-\sigma)-c < W(\sigma,R)$$

since $c > 0$ and $0 \leq \sigma \leq 1$. Q.E.D.

Proof of Proposition 1: The result is trivial for changes in c. For changes in R, note $\partial f(X)/\partial R = W_R(X,R)(1-X) < 0$, where f(x) is defined as in Lemma 1. Since increases in R decrease f(X) for all X, they enlarge the set of targets for which research does not pay (see Figure 1). Q.E.D.

Proof of Proposition 2: The proof of this proposition is obvious from Figure 2. The optimal risky target, r_2, is independent of σ, but $W(\sigma,R)$ is strictly increasing in σ. Hence if $\bar{\sigma}_2 > R$, then it is unique and well-defined and strictly less than r_2. Taking the total derivative of $c = W(r_2,R)(1-r_2) - W(\bar{\sigma}_2,R)$ with respect to $\bar{\sigma}_2$ and c gives

$$d\bar{\sigma}_2/dc = -1/W_X(\bar{\sigma}_2,R) < 0$$

Similarly $0 = W_R(r_2,R)(1 - r_2)dR - W_X(\bar{\sigma}_2,R)d\bar{\sigma}_2$. Hence $W_{XR} \geq 0$ and $r_2 > \bar{\sigma}_2$ imply $0 = W_X(\bar{\sigma}_2,R)d\bar{\sigma}_2 + W_R(\bar{\sigma}_2,R)dR$ or

$$\frac{d\bar{\sigma}_2}{dR} = \frac{-W_R(r_2,R)(1-r_2) - W_R(\bar{\sigma}_2,R)}{W_X(\bar{\sigma}_2,R)} > 0.$$

Proof of Proposition 3: By definition,

$$W_X(r_2,R)(1-r_2) - W(r_2,R) = 0. \tag{A1}$$

Hence

$$dr_2/dR = [W_R(r_2,R) - W_{XR}(r_2,R)(1-r_2)]/[W_{XX}(r_2,R)(1-r_2) - 2W_X(r_2,R)].$$

But $W_{XR} \geq 0$. Hence $dr_2/dR > 0$. Clearly $dr_2/dc = 0$. Q.E.D.

Proof of Proposition 4: Define

$$f_2(X) = W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2} V_1(z,X,r_2)dz + \beta\int_{\bar{\sigma}_2}^1 V_1(z,X,z)dz.$$

Part (a) follows trivially since f_2 is independent of c. To see part (b) note that $\partial f_2(X)/\partial R = W_R(X,R)(1-X) < 0$. Finally, part (c) follows since $f_2(X) > f_1(X)$, where f_1 is defined in Lemma 1. Q.E.D.

Proof of Proposition 5: Define

$$H_2(\sigma,X) = \beta\int_\sigma^1 [V_1(z,X,x_2^*) - V_1(\sigma,X,x_2^*)]dz \tag{A2}$$

where $x_2^* = x_2^*(z,X)$ and $x_2^* = x_2^*(\sigma,X)$, respectively.

But

$$x_2^*(y,X) = r_2(X) \quad \text{if} \quad y \le \bar{\sigma}_2(X)$$

where $y \varepsilon \{z,\sigma\}$. Hence (A2) becomes

$$H_2(\sigma,X) = \begin{cases} \beta\int_\sigma^{\bar{\sigma}_2} [V_1(z,X,r_2) - V_1(\sigma,X,r_2)] + \beta\int_{\bar{\sigma}_2}^1 [V_1(z,X,z)-V_1(\sigma,X,r_2)]dz & \text{if } \sigma < \bar{\sigma}_2 \\ \beta\int_\sigma^1 [V_1(z,X,z)-V_1(\sigma,X,\sigma)]dz & \text{if } \sigma \ge \bar{\sigma}_2 \end{cases}$$

But $V_1(z,X,r_2) = -c + W(r_2,X)(1-r_2)$ and $V_1(z,X,z) = W(z,X)$. Hence

$$H_2(\sigma,X) = \begin{cases} \beta\int_{\bar{\sigma}_2}^1 [W(z,X)-W(r_2,X)(1-r_2) + c]dz & \text{if } \sigma < \bar{\sigma}_2 \\ \beta\int_\sigma^1 [W(z,X)-W(\sigma,X)]dz & \text{if } \sigma \ge \bar{\sigma}_2 \end{cases} \tag{A3}$$

Differentiating (A3) gives

$$\partial H_2/\partial\sigma = \begin{cases} 0 & \text{if } \sigma < \bar{\sigma}_2 \\ -\beta(1-\sigma)W_X(\sigma,X) & \text{if } \sigma \ge \bar{\sigma}_2 \end{cases} \tag{A4}$$

Furthermore,

$$\beta\int_{\bar{\sigma}_2}^1 [W(z,X)-W(r_2,X)(1-r_2) + c]dz = \beta\int_{\bar{\sigma}_2}^1 [W(z,X)-W(\bar{\sigma}_2,X)]dz$$

by definition of $\bar{\sigma}_2$. Also, $H_2(1,X) = 0$. Thus we have Figure A1.

[Figure A1 about here]

Clearly $\bar{\sigma}_2 < \sigma_2^* < 1$ for $0 < c < \beta\int_{\bar{\sigma}_2}^1 [W(z,X)-W(\bar{\sigma}_2,X)]dz$. To sign $d\sigma_2^*/dc$ and $d\sigma_2^*/dX$ take the total derivatives of

$$c = \beta\int_{\sigma_2^*}^1 [V_1(z,X,x_2^*) - V_1(\sigma_2^*,X,x_2^*)]dz,$$

which reduces to

$$c = \beta\int_{\sigma_2^*}^1 [W(z,X) - W(\sigma_2^*,X)]dz$$

since $\sigma_2^* > \bar{\sigma}_2$. This gives

$$d\sigma_2^*/dc = -1/\beta(1-\sigma_2^*)W_X(\sigma_2^*,X) < 0$$

$$d\sigma_2^*/dX = \int_{\sigma_2^*}^1 [W_R(z,X) - W_R(\sigma_2^*,X)]dz/\int_{\sigma_2^*}^1 W_X(\sigma_2^*,X)dz \ge 0,$$

the latter since $W_{RX} \ge 0$. Q.E.D.

Proof of Lemma 3: Since the form of V_2^s depends on where σ lies relative to $\bar{\sigma}_2$ and σ_2^* we again need to consider three cases.

CASE 1: $\sigma < \bar{\sigma}_2 < \sigma_2^*$

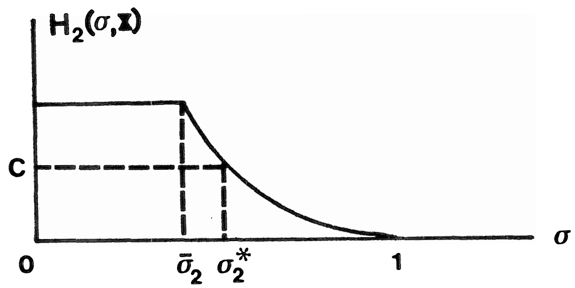
From (24) we have

$$V_2^s(\sigma,R,\sigma) = -c + W(\sigma,R) + \beta\int_0^{\bar{\sigma}_2} [-c + W(r_2,\sigma)(1-r_2)]dz + \beta\int_{\bar{\sigma}_2}^1 W(z,\sigma)dz. \tag{A5}$$

But from (30)

FIGURE A1

Definition of σ_2^*



$$\lim_{X\to\sigma} V_2^r(\sigma,R,X) = -c + W(\sigma,R)(1-\sigma) + \beta\int_{\sigma}^{\bar{\sigma}_2} [-c + W(r_2,\sigma)(1-\sigma_2)]dz + \beta\int_{\bar{\sigma}_2}^{1} W(z,\sigma)dz. \quad (A6)$$

Clearly (A6) is less than (A5).

<u>CASE 2</u>: $\bar{\sigma}_2 \leq \sigma < \sigma_2^*$

From (25),

$$V_2^s(\sigma,R,\sigma) = -c + W(\sigma,R) + \beta\int_0^{\sigma} W(\sigma,\sigma)dz + \beta\int_{\sigma}^{1} W(z,\sigma)dz$$
$$= -c + W(\sigma,R) + \beta\int_{\sigma}^{1} W(z,\sigma)dz. \quad (A7)$$

But $\sigma \geq \bar{\sigma}_2$ so (A6) reduces to

$$\lim_{X\to\sigma} V_2^r(\sigma,R,X) = -c + W(\sigma,R)(1-\sigma) + \beta\int_{\sigma}^{1} W(z,\sigma)dz. \quad (A8)$$

Clearly (A7) is greater than (A8).

<u>CASE 3</u>: $\bar{\sigma}_2 < \sigma_2^* \leq \sigma$

Here (27) gives

$$V_2^s(\sigma,X,\sigma) = W(\sigma,R). \quad (A9)$$

Thus, using (A8) we need to show

$$W(\sigma,R) > -c + W(\sigma,R)(1-\sigma) + \beta\int_{\sigma}^{1} W(z,\sigma)dz$$

or

$$c > \beta\int_{\sigma}^{1} W(z,\sigma)dz - W(\sigma,R)\sigma. \quad (A10)$$

But $\sigma \geq \sigma_2^*$ implies

$$c > \beta\int_{\sigma}^{1} [W(z,X) - W(\sigma,X)]dz.$$

Since we are letting $X \to \sigma$, this implies $c > \beta\int_\sigma^1 W(z,\sigma)dz$, which implies (A10). Q.E.D.

Proof of Proposition 6: That $\bar{\sigma}_3$ is unique and well-defined follows from the text. That $\bar{\sigma}_3 < \bar{r}_3$ where $\bar{r}_3$ is the minimum of the set of optimal risky target follows from the fact that max V_2^s is increasing for $\sigma > \bar{\sigma}_2$. The partial derivatives can be signed by examining the following derivatives.

For safe targets we need to consider $\partial V_2^s/\partial c$ and $\partial V_2^s/\partial R$ on ranges of $\sigma \in (\bar{\sigma}_2,\sigma_2^*)$ and $\sigma \in (\sigma_2^*,1)$, cases 2 and 3 given in equations (25) and (27) in the text. Hence

$$\partial V_2^s(\sigma,R,X)/\partial c = \begin{cases} -1 & \text{for } \sigma \in (\bar{\sigma}_2,\sigma_2^*) \\ 0 & \text{for } \sigma \in (\sigma_2^*,1] \end{cases} \tag{A11}$$

$$\partial V_2^s(\sigma,R,X)/\partial R = W_R(X,R) \qquad \text{for } \sigma \in (\bar{\sigma}_2,1]. \tag{A12}$$

For risky targets we need only consider equation (30), or

$$V_2^r(\sigma,R,X) = -c + W(X,R)(1-X) + \beta\int_X^{\bar{\sigma}_2}[-c + W(r_2,X)(1-r_2)]dz + \beta\int_{\bar{\sigma}_2}^1 W(z,X)dz$$

Hence

$$\partial V_2^r(\sigma,R,X)/\partial R = W_R(X,R)(1 - X) \tag{A13}$$

and

$$\partial V_2^r(\sigma,R,X)/\partial c = -1 + \beta\int_X^{\bar{\sigma}_2}[-1 + W_X(r_2,X)(1-r_2)\frac{dr_2}{dc} - W(r_2,X)\frac{dr_2}{dc}]dz$$

$$+ \beta[-c + W(r_2,X)(1-r_2)]\frac{d\bar{\sigma}_2}{dc} - \beta W(\bar{\sigma}_2,X)\frac{d\bar{\sigma}_2}{dc}. \tag{A14}$$

But (A1) implies $W_X(r_2,X)(1 - r_2) - W(r_2,X) = 0$ and the definition of $\bar{\sigma}_2$ implies $-c + W(r_2,X)(1 - r_2) = W(\bar{\sigma}_2,X)$. Hence (A14) is simply

$$\partial V_2^r(\sigma,R,X)/\partial c = -1 + \beta\int_X^{\bar{\sigma}_2}(-1)dz = -[1 + \beta(\bar{\sigma}_2 - X)]. \tag{A15}$$

Comparing (A15) to (A11) we have for all X such that they are defined,

$$|\partial V_2^s(\sigma,R,X)/\partial c| \le |\partial V_2^r(\sigma,R,X)/\partial c|$$

and

$$|\partial V_2(\sigma,R,X)/\partial R| \ge |\partial V_2^r(\sigma,R,X)/\partial R|.$$

Hence $d\bar{\sigma}_3/dc > 0$ and $d\bar{\sigma}_3/dR > 0$. These same results can be obtained by totally differentiating $V_2^s(\sigma,R,s_3) = V_2^r(\sigma,R,r_3)$ using some selection for s_3, say $\hat{s}_3 = \max\{s_3 | s_3 = \underset{1\le X\le\sigma}{\operatorname{argmax}} V_2(\sigma,R,X)\}$. Q.E.D.

Proof of Corollary 2: In general s_3 is defined by $\partial V_2^s(\sigma,R,s_3)/\partial X = 0$ and r_3 by $\partial V_2^r(\sigma,R,r_3)/\partial X = 0$. Consider first case 1 in which V_2^s is given by (24). Here s_3 is given by

$$W_X(s_3,R) + \beta\int_0^{\bar{\sigma}_2} W_R(r_2,s_3)(1-r_2)dz + \beta\int_{\bar{\sigma}_2}^1 W_X(z,s_3)dz = 0. \tag{A16}$$

Hence

$$-[\partial^2 V_2^s/\partial X^2]ds_3 = \{\beta\int_0^{\bar{\sigma}_2}[W_{RX}(r_2,s_3)(1-r) - W_R(r_2,s_3)](\partial r_2/\partial c)dz + [W_R(r_2,s_3)(1-r_2)$$

$$+ [W_R(r_2,s_3)(1-r_2) - W(\bar{\sigma}_2,s_3)](\partial\bar{\sigma}_2/\partial c)\}dc$$

But $\partial r_2/\partial c = 0$, and $\partial^2 V_2^s/\partial X^2 < 0$ by the necessary second order condition. Also, $d\bar{\sigma}_2/dc < 0$. Hence

$$ds_3/dc = -[W_R(r_2,s_3)(1-r_2) - W(\bar{\sigma}_2,s_3)](\partial\bar{\sigma}_2/\partial c/(\partial^2 V_2^s/\partial X^2) > 0.$$

In cases 2 and 3 (equations 25 and 27), s_3 is clearly independent of c. Furthermore, taking $\partial^2 V_2^s/\partial X^2$ in (25) and (27) shows V_2^s is concave for $\sigma \geq \bar{\sigma}_2$. In these cases s_3 is uniquely defined.

For r_3 we use equation (30). Differentiating with respect to X gives

$$W_X(r_3,R)(1-r_3) - W(r_3,R) + \beta\int_{r_3}^{\bar{\sigma}_2} W_R(r_2,r_3)(1-r_2)dz + \beta\int_{\bar{\sigma}_2}^{1} W_R(z,r_3)dz$$

$$-\beta[-c + W(r_2,r_3)(1-r_2)] = 0 \qquad \text{(A17)}$$

as the equation which defines r_3. Hence

$$-(\partial^2 V_2^r/\partial X^2)dr_3 = \{\beta[W_R(r_2,r_3)(1-r_2) - W_R(\bar{\sigma}_2,r_3)](\partial\bar{\sigma}_2/\partial c)\}dc,$$

or

$$dr_3/dc = -\beta[W_R(r_2,r_3)(1-r_2) - W_R(\bar{\sigma}_2,r_3)](\partial\bar{\sigma}_2/\partial c)/(\partial^2 V_2^r/\partial X^2).$$

But we know from Proposition 2 that $r_2 > \bar{\sigma}_2$. Hence

$W_R(\bar{\sigma}_2,r_3) \leq W_R(r_2,r_3)(1-r_2) < 0$. This implies $dr_3/dc \leq 0$.

For changes in R, we have from (17) that

$$-(\partial^2 V_2^r/\partial X^2)dr_3 = [W_{XR}(r_3,R)(1-r_3) - W_R(r_3,R)]dR$$

or

$$dr_3/dR = -[W_{XR}(r_3,R)(1-r_3) - W_R(r_3,R)]/(\partial^2 V_2^r/\partial X^2) > 0.$$

For s_3, (A16) gives, in case 1,

$$-(\partial^2 V_2^s/\partial X^2)ds_3 = W_{XR}(s_3,R)dR$$

or

$$ds_3/dR = -W_{XR}(s_3,R)/(\partial^2 V_2^s/\partial X^2) > 0 \qquad \text{(A18)}$$

In cases 2 and 3 (equations (25) and (27), we get results identical to (A18).

For changes in σ we already know r_3 is independent of σ. For s_3, it is apparent (A16) is independent of σ. In case 2, (26) implies

$$ds_3/d\sigma = -(\partial^2 V_2^s/\partial\sigma\partial X)/(\partial^2 V_2^s/\partial X^2)$$

$$= -\beta(1-\sigma)W_{XR}(\sigma,X)/(\partial sup[2V_2^s/\partial X^2) > 0,$$

and in case 3, (28) implies

$$ds_3/d\sigma = -\beta W_{XR}(\sigma,s_3)/(\partial^2 V_2^s/\partial X^2) > 0. \qquad \text{Q.E.D.}$$

<u>Proof of Proposition 8</u>: Define

$$H_3(\sigma,X) = \beta\int_\sigma^1 [V_2[z,X,x_2(z)] - V_2[\sigma,X,x_2(\sigma)]dz$$

where x_2^* is defined as in the text following equation (33). Using those definitions,

$$H_3(\sigma,X) = \begin{cases} \beta\int_\sigma^{\bar\sigma_3} [V_2(z,X,r_3)-V_2(\sigma,X,r_3)]dz + \int_{\bar\sigma_3}^1 \{V_2[z,X,s_3(z)]-V_2(\sigma,X,r_3)\}dz & \text{if } \sigma < \bar\sigma_3 \\ \beta\int_\sigma^1 \{V_2[z,X,s_3(z)]-V_2[\sigma,X,s_3(\sigma)]\}dz & \text{if } \sigma \geq \bar\sigma_3 \end{cases}$$

where r_3 is any optimal risky target (e.g. $\bar r_3$) and s_3 is the unique safe target.

But r_3 is independent of σ for $\sigma < \bar\sigma_3$ (and for $z < \bar\sigma_3$). Hence

$$\partial H_3(\sigma,X)/\partial\sigma = \begin{cases} 0 & \text{if } \sigma < \bar\sigma_3 \\ -\beta\int_\sigma^1 [\frac{\partial V_2}{\partial\sigma}(\sigma,X,s_3) + \frac{\partial V_2}{\partial X}(\sigma,X,s_3)\frac{ds_3}{d\sigma}]dz & \text{if } \sigma \geq \bar\sigma_3. \end{cases}$$

Now if $s_3(\sigma) < 0$ then $\partial V_2/\partial X = 0$ evaluated at (σ,X,s_3). If $s_3(\sigma) = \sigma$, then $ds_3/d\sigma = 0$. Hence

$$\partial H_3(\sigma,X)/\partial\sigma = \begin{cases} 0 & \text{if } \sigma < \bar\sigma_3 \\ -\beta\int_\sigma^1 \frac{\partial V_2}{\partial\sigma}(\sigma,X,s_3)dz & \text{if } \sigma \geq \bar\sigma_3 \end{cases} \tag{A19}$$

But $\partial V_2/\partial\sigma > 0$ evaluated at (σ,X,s_3) when $\sigma \geq \bar\sigma_3$. Moreover,

$H_3(1,X) = 0$. Thus H_3 has the form illustrated in Figure A2.

[Figure A2 about here]

Thus σ_3^* is unique and well-defined. Furthermore, $\sigma_3^* > \bar\sigma_3$. That $d\sigma_3^*/dc < 0$ and $d\sigma_3^*/dX \geq 0$ follow from differentiating $c = H_3(\sigma,X;c)$ in the usual fashion. Q.E.D.

<u>Proof of Proposition 9</u>: We know from Proposition 6 that $\bar\sigma_3 > \bar\sigma_2$. Moreover, $H_2(1,X) = 0 = H_3(1,X)$> Hence if we can show $|\partial H_2(\sigma,X)/\partial\sigma| \lesseqgtr |\partial H_3(\sigma,X)/\partial\sigma|$ on $\sigma \in [\sigma_2^*,1]$ as $\beta \gtreqless \hat\beta_3$ we are done. (See Figure A3)

[Figure A3 about here]

For $\sigma > \bar\sigma_3 > \bar\sigma_2$, from (A4) we have

$$\partial H_2(\sigma,X)/\partial\sigma = -\beta(1 - \sigma)W_X(\sigma,X),$$

and from (A19)

$$\partial H_3(\sigma,X)/\partial\sigma = -\beta\int_\sigma^1 \frac{\partial V_2}{\partial\sigma}(\sigma,X,s_3)dz.$$

But $s_3 > X$ since X is the new state of sponsor knowledge. Furthermore s_3 is a safe target and on $\sigma > \sigma_2$,

$$\partial H_3(\sigma,X)/\partial\sigma = -\beta^2(1-\sigma)W_X(\sigma,s_3).$$

Thus $|\partial H_2(\sigma,X)/\partial\sigma| < |\partial H_3(\sigma,X)/\partial\sigma|$ for $\sigma \in [\bar\sigma_3,1]$ if and only if

$$\beta^2(1-\sigma)W_X(\sigma,s_3) > \beta(1-\sigma)W_X(\sigma,X),$$

or

FIGURE A2

Definition of σ_3^*

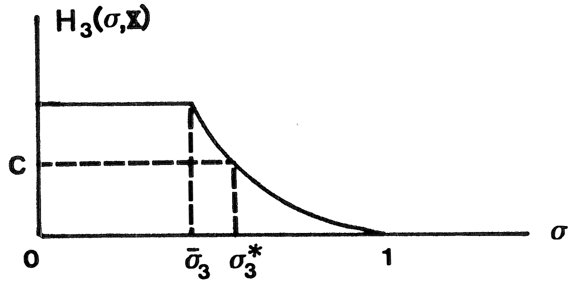
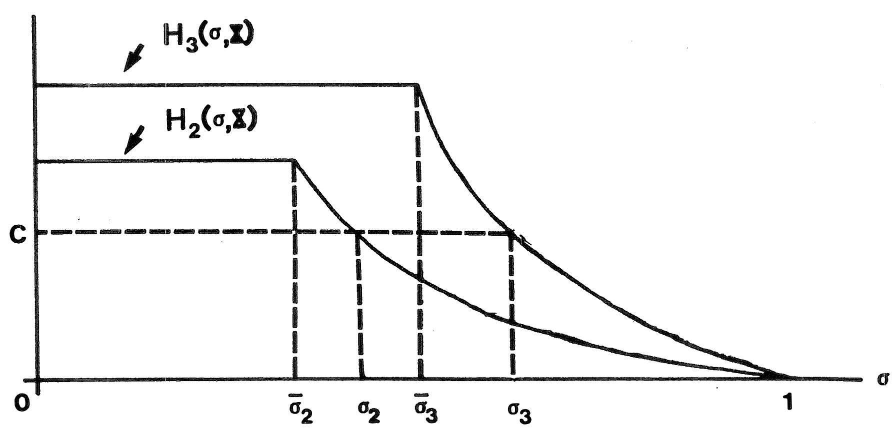


FIGURE A3

Relation $\sigma_2^* < \sigma_3^*$



$$\beta W_X(\sigma,s_3) > W_X(\sigma,X).$$

But $s_3 > X$ implies $W_X(\sigma,s_3) > W_X(\sigma,X)$. Thus if β is sufficiently close to one we have

$$\beta W_X(\sigma,s_3) > W_X(\sigma,X),$$

and $\sigma_3^* > \sigma_2^*$. If $\beta = \hat{\beta}_3$ where

$$\hat{\beta}_3 = W_X(\sigma,s_3)$$

$\sigma_3^* = \sigma_2^*$. Otherwise, if $\beta < \hat{\beta}_3$, $\sigma_3^* < \sigma_2^*$. Q.E.D.

FOOTNOTES

1. In Balbien (1981) cost estimates for parabolic dish power systems as elicited from Jet Propulsion Laboratory R and D engineers are typically found to have concave or convex distributions. However, later follow up interviews dealing with physical performance characteristics (based on engineering efficiencies) yielded estimates which were almost always uniform. These latter estimates were reported in an unpublished JPL internal memo.

2. The crucial assumption is, surprisingly, $W_{RR} < 0$. See the discussion on page 110 of Balbien and Wilde (1982).

3. The set of targets for which the firm does research has a simple structure in the one period problem. The general definition thus seems specious. However, as the length of the horizon increases, the value functions are not necessarily quasi-concave. Thus $\bar{S}_t(c,R)$ may not be connected when $t > 1$. We start with a general notation in order to be consistent throughout the paper.

4. Conditions which guarantee r_2 is an interior maximum are similar to those stated in Balbien and Wilde (1982). See footnote 8, page 112. Again, Lemma 2 is an obvious result. We state it formally in order to maintain consistency with more complicated analogues when $t > 1$. See footnote 2.

5. Conditions which guarantee s_3 and r_3 are interior maxima are again similar to those stated in Balbien and Wilde (1982, footnote 8, page 112).

6. Once the firm reaches a situation where no safe target ever induces it to do research, it will never again do research. In this case it still may not wish to reveal all it knows immediately. However, prior to this point it may reveal all it knows in a particular period, do research the next period, and withhold some of its new knowledge.

7. There is no real problem with nonuniqueness of r_3 from the firm's perspective--all yield identical expected payoffs. Formally some selection is required but as the discussion associated with Proposition 6 showed, $\bar{r}_3 = \min\{r_3\}$ is the natural one.

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