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CONSUMER SURPLUS UNDER UNCERTAINTY:
AN APPLICATION TO DAM-RESERVOIR PROJECTS

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ABSTRACT

The use of cost-benefit analysis to evaluate the net welfare
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many years. One of the interesting aspects of such cost-benefit
studies is that water projects involve an uncertain flow of costs and
benefits, arising from the stochastic nature of streamflows. Hence a
basic problem for the cost-benefit analyst is that of incorporating
this uncertainty into his measures of costs and benefits. In the
present paper, we examine the problem of computing an appropriate
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1. INTRODUCTION

The use of cost-benefit analysis to evaluate the net welfare
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many years. One of the interesting aspects of such cost-benefit
studies is that water projects involve an uncertain flow of costs and
benefits, arising from the stochastic nature of streamflows. Hence a
basic problem for the cost-benefit analyst is that of incorporating
this uncertainty into his measures of costs and benefits. In the
present paper, we examine the problem of computing an appropriate
consumer surplus measure to evaluate water project benefits under
uncertainty. Detailed treatment is given to the case in which a
complete set of contingent claim markets exist in the economy, as well
as to the case of a spot market economy. Generally speaking, the spot
market economy model appears to be a closer approximation to reality
than the complete contingent claim market model. One of the
interesting aspects of the spot market model is that there is an
unobserved component of consumer benefits in a spot market economy
that limits the applicability of the usual consumer surplus measure.

2. A CONTINGENT CLAIM WATER ECONOMY

To keep the notational clutter to a minimum, we consider the
case of a world in which there are two consumer goods, y and z, where
y denotes the consumption of water and z is a non-water good (say, a
composite commodity) which does not involve the use of water as an
intermediate product or input in production. The only uncertainty in
the first model we consider is uncertainty as to streamflows. Let π_{jt}
denote the probability of streamflow j in period t. There are assumed
to be J possible streamflow levels in each period, and T periods. We
take the probability distribution over streamflows to be "objective,"
since historical streamflows are tabled for most major rivers and
their tributaries; and we assume that water users are aware of these
data and make use of them in their decision making. Again, mainly for
notational convenience, we take the probability distribution over
streamflows to be stationary with respect to time, and intertemporal
independence is assumed as well. Thus the joint probability of the
sequence of streamflows $(x_{j_1}, x_{j_2}, \ldots, x_{j_t})$ is given by $\pi_{j_1} \cdot \pi_{j_2} \cdot \ldots \pi_{j_t}$.

As a matter of notation, let

$$x(j_1 \to j_t) = (x_{j_1}, \ldots, x_{j_t}).$$

We first examine the cost-benefit problem in the context of an
Arrow-Debrew [1] type world in which there is a complete set of
contingent claim markets all of which open at time 0. Consumers make
binding commitments to consumption over the T-period horizon through
purchases and sales of contingent claims to the goods y and z at time

0; all market transactions occur in the period 0 contingent claim markets. Randomness enters the picture because of stochastic streamflows; there is no randomness emanating from the consumption side, or from the supply side with respect to commodity z.

Let $y^i_{j_1 \cdots j_t}$ denote the number of contingent claim contracts acquired by consumer i, satisfying the property that each claim pays one unit of good y in period t if the streamflow sequence $x(j_1 \to j_t)$ occurs. Similarly, let $p_{j_1 \cdots j_t}$ denote the price of such a contingent claim contract. $z^i_{j_1 \cdots j_t}$ is the number of contingent claims to z acquired by i, satisfying the property that each claim pays one unit of good z in period t if the streamflow $x(j_1 \to j_t)$ occurs, with $q_{j_1 \cdots j_t}$ being the price of such a contract. Sellers of these contracts are the firms producing goods y and z. Note also that if the streamflow sequence $x(j_1 \to j_t)$ occurs, then consumption by consumer i in period t is the vector $(y^i_{j_1 \cdots j_t}, z^i_{j_1 \cdots j_t})$, because of the absence of spot markets in the periods following period 0; that is, purchases of contingent claim contracts are binding commitments to consumption.

To simplify notation, let Ω_t denote the index set over streamflows $x(j_1 \to j_t)$ commencing at time 1 and terminating at time t. Let $\Omega = \overset{T}{\underset{t=1}{U}} \Omega_t$ denote the set of all index sets over streamflow sequences, ranging from one to T-period sequences. Then Ω_t is also the index set over consumptions in period t, and over prices (p,q) of contingent claim contracts to t^{th} period consumption.

Given the usual separability assumptions concerning preferences, the decision problem for consumer i reduces to the following:

$$\max_{(y^i, z^i)} \quad EU^i = \sum_{t=1}^{T} \beta_i^t \sum_{r \varepsilon \Omega_t} \pi_r u_t^i(y_r^i, z_r^i) \qquad (1)$$

subject to

$$W^i(T) = \sum_{t=1}^{T} \delta^t \sum_{r \varepsilon \Omega_t} (M_r^i - p_r y_r^i - q_r z_r^i) = 0$$

where (y^i, z^i) is the matrix of contingent claim purchases by consumer i, β_i is consumer i's personal discount rate, $\delta = (\frac{1}{1+i})$ where i is the market rate of interest, M^i is income for consumer i, and $W^i(T)$ is consumer i's wealth at time T. In this formulation, we assume a perfect capital market with a T-period horizon.

This gives rise to the usual first order conditions, assuming a regular interior maximum, that is,

$$\frac{\partial EU^i}{\partial y_r^i} - \lambda \frac{\partial W^i(T)}{\partial y_r^i} = 0$$

$$\frac{\partial EU^i}{\partial z_r^i} - \lambda \frac{\partial W^i(T)}{\partial z_r^i} = 0 \qquad (2)$$

$$W^i(T) = 0$$

Given the structure of markets outlined above, we are interested in constructing a measure, in the form of consumers' surplus, of the benefits associated with controlling a river. In

particular, we are interested in a measure of the benefits from a change from a completely uncontrolled river to one with a dam-reservoir complex controlling flows, with a reservoir storage capacity of S^* acre-feet of water, where S^* is some given constant. To finesse the well known aggregation problems plaguing consumers' surplus, we will instead calculate consumer's surplus for one individual, say consumer i. Then the relevant measure of benefits for the consumer is presumably the amount of discounted income that consumer i is willing to give up to be indifferent between facing a controlled river with S^* units of storage capacity, and facing an ucontrolled river. Thus we are dealing with the compensated demand curve(s) for y^i, along which EU^i is a constant. In the usual partial equilibrium approach, we assume that the prices $q_{(\cdot)}$ of the non-water commodity z are unaffected by changes in the water storage capacity levels.

To simply the notation, we will drop the sub (super) script identifying the individual where there is no problem of ambiguity. Let DPV denote the discounted present value of income for consumer i. Then DPV is given by

$$ DPV = \sum_{t=1}^{T} \delta^t \sum_{s \varepsilon \Omega_t} M_s. \tag{3} $$

Since $W(T) = 0$ is a binding constraint that holds for all values of S, then, given a change dS in the reservoir storage level, $dW(T) = 0$. Thus we have from (1) and (3):

$$ dDPV = (-) \sum_{r \varepsilon \Omega} \left[\frac{\partial W(T)}{\partial p_r} + \sum_{s \varepsilon \Omega} \left(\frac{\partial W(T)}{\partial y_s} \frac{\partial y_s}{\partial p_r} + \frac{\partial W(T)}{\partial z_s} \frac{\partial z_s}{\partial p_r} \right) \right] (\frac{dp_r}{dS}) dS \tag{4} $$

Along a compensated demand curve, dEU=0, thus

$$ dEU = \sum_{r \varepsilon \Omega} \left[\sum_{s \varepsilon \Omega} \frac{\partial EU}{\partial y_s} \frac{\partial y_s}{\partial p_r} + \frac{\partial EU}{\partial z_s} \frac{\partial z_s}{\partial p_r} \right] (\frac{dp_r}{dS}) dS = 0 \tag{5} $$

By the first order conditions (2),

$$ \frac{\partial EU}{\partial y_s} = \lambda \frac{\partial W(T)}{\partial y_s}, \quad \frac{\partial EU}{\partial z_s} = \lambda \frac{\partial W(T)}{\partial z_s} \tag{6} $$

Given non-satiation, $\lambda > 0$, and thus (4) reduces to

$$ dDPV = - \sum_{r \varepsilon \Omega} (\frac{\partial W(T)}{\partial p_r}) (\frac{dp_r}{dS}) dS $$

$$ = \sum_{t=1}^{t} \delta^t \sum_{r \varepsilon \Omega_t} y_r (\frac{dp_r}{dS}) dS \tag{7} $$

Thus the consumer surplus measure relevant for the comparison between an uncontrolled river and one with a reservoir capacity of S^* units is given by

$$ CS = (-) \sum_{t=1}^{T} \delta^t \sum_{r \varepsilon \Omega_t} \int_0^{S^*} y_r (\frac{dp_r}{dS}) dS \tag{8} $$

In the special case where y_r depends only on p_r and where S affects only one price p_r, then by a change of variables we end up with the usual consumer's surplus area computed from the compensated demand curve for y_r, but clearly this is a case with no practical significance for the evaluation of the typical water project. In any case, it is clear that the consumer's surplus measure calculated in (8) is of precisely the same form as the corresponding consumer's

surplus measure for the certainty case. The major difference is that the contingent claim version incorporates the alternative possible streamflows indexed by the set Ω. As Rogerson (2) has noted in a different context, this correspondence reflects in part at least the fact that uncertainty is assumed to enter the picture in our model only from the supply side of the market.

Turning to the supply side of the market, assume that a public agency is charged with the task of operating the dam-reservoir complex, using the rule of maximizing expected profits taking prices (of contingent claims) as parameters. In analyzing the decision making process of the agency, timing of inflows and releases is of importance. We assume that in any period, inflows and releases occur at the end of the period, just before the reservoir level is measured. The tth period begins at time $t-1$ and ends at time t.

Let $S_{j_1 \cdots j_t}$ be the amount of water stored in the reservoir at time t, given the sequence of streamflows $x(j_1 \to j_t)$ and given an optimal (expected profit maximizing) release policy. $y^*_{j_1 \cdots j_t}$ is the optimal level of releases in period t, given the streamflow sequence $x(j_1 \to j_t)$, and $R_{j_1 \cdots j_t}$ is the amount of water available for release in the tth period, being measured near the end of that period, after inflows are known but before releases have occurred.

Then we have the following accounting identities:

$$S_r = (1 - \alpha)S_s + x_{j_t} - y^*_r, \quad s \; \varepsilon \; \Omega_{t-1}, r = (s,j_t) \tag{9}$$

where α is the percentage loss in reservoir stocks due to evaporation;

$$R_r = (1 - \alpha)S_s + x_{j_t}, \quad s \; \varepsilon \; \Omega_{t-1}, \; r = (s,j_t) \tag{10}$$

so that

$$R_r = (1 - \alpha)[R_s - y^*_s] + x_{j_t}, \quad s \; \varepsilon \; \Omega_{t-1}, \; r = (s,j_t). \tag{11}$$

The dam-reservoir operator then chooses a sequence of releases to maximize expected profits. Let $r = (j_1,\ldots,j_t) \; \varepsilon \; \Omega_t$, and let $x(r) \equiv x(j_1 \to j_t)$. Then at time t, when the streamflow sequence $x(r)$ is already known to have occurred, the operator chooses the release y^*_r to maximize $V(R_r)$, where

$$V(R_r) = \delta^t p_r y^*_r + \delta^{t+1} \sum_{j_{t+1}=1}^{J} \pi_{j_{t+1}} \max_{0 \leq R_s - y_s \leq S^*} V[(1 - \alpha)(R_r - y^*_r) + x_{j_{t+1}}] \tag{12}$$

with $s = (r, j_{t+1}) \; \varepsilon \; \Omega_{t+1}$.

$V(R_r)$ is to be maximized subject to the constraint

$$0 \leq R_r - y^*_r \leq S^* \tag{13}$$

Thus at time T, with R_w acre-feet of water available for release ($w = (j_1,\ldots,j_T) \; \varepsilon \; \Omega_T$), y_w is chosen to max $\delta^T p_w y^*_w$ subject to $0 \leq R_w - y_w \leq S^*$.

Given the linear increasing objective function together with the linear constraints, clearly the optimal policy involves setting

$y_w^* = R_w$.

At T-1, with R_v acre-feet of water available (v = $(j_1,...,j_{T-1}) \varepsilon \Omega_{T-1}$), y_v^* is chosen to

$$\max V(R_v) = \max \delta^{T-1} p_v y_v^* + \delta^T \sum_{j_T=1}^{J} \pi_{j_T} p_w[(1-\alpha)(R_v - y_v^*) + x_{j_T}]$$

subject to $0 \le R_v - y_v^* \le S^*$, where w = $(v, j_T) \varepsilon \Omega_T$.

Thus $\dfrac{\partial V}{\partial y_v^*} = \delta^{T-1} \left[p_v - \delta(1-\alpha) \sum_{j_T=1}^{J} \pi_{j_T} p_w \right]$

The term inside the square brackets is the difference between the value at time T-1 of a unit of water released at T-1, and the discounted expected value at time T-1 of a unit of water stored at time T-1 for release at time T. The "own interest rate" $\gamma = \dfrac{\alpha}{1-\alpha}$ is included as a discount factor, with $\delta(1-\alpha) = (\dfrac{1}{1+i})(\dfrac{1}{1+\gamma})$.

When the term inside the square brackets is positive, the optimal release $y_v^* = R_v$, the stock of water available. When the term is negative, $y_v^* = \max \{0, R_v - S^*\}$, that is, as much water as possible is stored. When the term inside the square brackets is zero, then any feasible release is optimal.

In the general case of an arbitrary index t and streamflow sequence indexed by $r \varepsilon \Omega_t$, the relevant criterion for the dam-reservoir operator is the expression

$$\left[p_r - \max_s \sum_{j_{t+1}=1}^{J} \cdots \sum_{j_{t+s}=1}^{J} \delta^s (1-\alpha)^s \pi_{j_{t+1}} \cdots \pi_{j_{t+s}} p_{w_s} \right] \quad (14)$$

where s = 1,...T - t and $w_s = (r, j_{t+1},...,j_{t+s}) \varepsilon \Omega_{t+s}$.

If the expression (14) is positive, then it pays (in discounted expected profit terms) to release as much water as possible at time t; if it is negative, then there is a time in the future such that water released at that time has a higher discounted expected value today than water released today, so as much water as possible should be held in storage. If the expression is zero, any feasible release is optimal.

Using (14) to regulate releases determines the supply of water to be made available at any point in time for any given sequence of streamflows and hence this determines the supply of contingent claims to water offered by the dam-reservoir operator as a function of the set of prices $p_{(\cdot)}$ for contingent claims, and the storage capacity S^*.

Equilibrium in the market for contingent claims occurs when there is simultaneous clearing of all such markets, so that the price vector p satisfies

$$\sum_{i=1}^{I} y_s^i(p;q) = y_s^*(p,S^*) \text{ for } s \varepsilon \Omega. \quad (15)$$

Hence we have

$$\sum_{r\varepsilon\Omega} \left[\sum_{i=1}^{I} \frac{\partial y_s^i}{\partial p_r} - \frac{\partial y_s}{\partial p_r} \right] (\frac{dp_r}{dS}) = \frac{\partial y_r^*}{\partial S} \quad s \varepsilon \Omega, \quad (16)$$

with all magnitudes evaluated at the equilibrium prices p, q.

Solving this system of $(\frac{J}{J-1})(\frac{T}{T-1})$ simultaneous equations in the same number of variables for each value of S, $0 \le S \le S^*$ would (in

principle!) enable us to determine the values of the terms $(\frac{dp_r}{dS})$ appearing in the expression (8) for consumer's surplus, and hence to complete the formal calculation of that magnitude. We will not attempt that task here. It should be noted, however, that in the calculations to obtain $(\frac{dp_r}{dS})$, $\frac{\partial y_s^i}{\partial p_r}$ is to be interpreted as a movement along the compensated demand curve for y_s^i.

Under the appropriate assumptions, when total (consumer plus producer) surplus is maximized, then an _ex ante_ Pareto optimum is achieved, that is, it is not possible to make one consumer better off in terms of expected utility without making some other consumer worse off. This assumes, of course, that aggregation problems relative to consumers' surplus are successfully resolved. It should also be pointed out that expected utility calculations involve only the probabilities associated with streamflows, since this is the only source of randomness in our model of a water economy with a complete set of contingent claim markets. We next turn to the other extreme, an economy with no contingent claim markets, but with spot markets open in each period.

3. A SPOT MARKET WATER ECONOMY

In this section, we consider an economy with no contingent claim or futures markets, but instead one with a complete set of spot markets, opening at each of the T dates in the planning horizon. From the point of view of a typical consumer in this economy, there are two

sources of uncertainty, that is, uncertainty as to streamflows at various dates in time, and uncertainty as to the prices that will prevail in future spot markets. To minimize notational clutter, we will assume that both kinds of uncertainty get incorporated into the consumer's subjective probability distribution over prices in future spot markets. That is, each consumer has a subjective pdf over the spot prices that will prevail at each point in time, which pdf reflects the underlying uncertainty as to stream flows and the behavior of other consumers. Once again we assume a two good (y,z) economy, with (p,q) denoting the prices of the goods. The reservoir storage capacity S then shows up as a parameter of the consumer's pdf over prices $f(p_1,\ldots,p_T;q_1,\ldots,q_T;S)$.

Once again we are interested in constructing a measure of the benefits to a consumer from the construction of a dam-reservoir complex with capacity S^* to control streamflows. The decision problem for the consumer over a T-priced horizons is to choose consumption vectors $y = (y_1,\ldots,y_T)$ and $z = (z_1,\ldots,z_T)$ to maximize

$$EU = \sum_{t=1}^{T} \beta^t \int \cdots \int U_t(y_t,z_t) f_t(p_t,\ldots,p_T;q_t,\ldots,p_T;S)dpdq \qquad (17)$$

subject to

$$W(T) = \sum_{t=1}^{T} (M_t - p_t y_t - q_t z_t)/(1 + i)^t = 0. \qquad (18)$$

In this formulation, the time stream of income M^i is known with certainty, as is the (constant) one period interest rate i. We

assume a perfect capital market and no default risk. Let

$$DPV = \sum_{t=1}^{T} M_t/(1+i)^t \qquad (19)$$

denote the discounted present value of income, and let

$$M_t^* = DPV - \sum_{s=1}^{t-1}(p_s y_s + q_s z_s)/(1 + i)^s \qquad t = 1,\ldots T \qquad (20)$$

denote the net amount of DPV available to be spent from time t on. Furthermore, we make the (heroic) assumption that default never occurs, i.e., M_t^* is always positive. This avoids the problem of introducing a complicated budget constraint in which consumption is at a subsistence level if default ever occurs. Allowing for default would not change the basic qualitative conclusions of our approach.

The consumer presumably adopts a dynamic programming approach to solving his decision problem. We illustrate for the case of a two-period horizon.

In the case where T = 2, then at time 2, when p_2, q_2 and M_2^* are known the consumer chooses y_2 and z_2 to

$$\text{maximize } U_2(y_2,z_2) \text{ subject to } M_2^* = p_2 y_2 + q_2 z_2 \qquad (21)$$

Solving this problem, we obtain the demand functions

$$y_2 = y_2(p_2,q_2,M_2^*)$$

$$\qquad (22)$$

$$z_2 = z_2(p_2,q_2,M_2^*),$$

Reverting to time 1, the consumer chooses y_1 and z_1 to maximize

$$\begin{aligned} EU = &\ \beta U_1(y_1,z_1) \\ &+ \beta^2 \iint U_2(y_2(p_2,q_2,M_2^*),z_2(p_2,q_2,M_2^*))f(p_2,q_2,S)dp_2 dq_2 \qquad (23) \end{aligned}$$

First order conditions characterizing an (unconstrained) maximum are given by

$$\frac{\partial EU}{\partial y_1} = \beta\frac{\partial U_1}{\partial y_1} + \beta^2\frac{\partial EU_2}{\partial y_1} = 0$$

$$\qquad (24)$$

$$\frac{\partial EU}{\partial z_1} = \beta\frac{\partial U_1}{\partial z_1} + \beta^2\frac{\partial EU_2}{\partial z_1} = 0$$

Given this framework, how much is the consumer willing to pay (in terms of DPV) for an increase dS in the storage capacity of the reservoir? Since the consumer is to be indifferent between a situation with a storage capacity of S and one with a storage capacity of S + dS, thus we have

$$dEU = 0, \qquad (25)$$

where $EU = EU(p_1,q_1,y_1,z_1,DPV,S)$.

Hence, we have

$$dEU = \left\{ \frac{\partial EU}{\partial y_1}\frac{dy_1}{dS} + \frac{\partial EU}{\partial z_1}\frac{dz_1}{dS} + \frac{\partial EU}{\partial q_1}\frac{dq_1}{dS} + \frac{\partial EU}{\partial p_1}\frac{dp_1}{dS} + \frac{\partial EU}{\partial S} \right\} dS + \frac{\partial EU}{\partial DPV}dDPV = 0$$

$$\qquad (26)$$

From the first order conditions, $\frac{\partial EU}{\partial y_1} = \frac{\partial EU}{\partial z_1} = 0$. By the usual

partial equilibrium assumption, $\frac{dq_1}{dS} = 0$. Also note that

$$\frac{\partial EU}{\partial p_1} = \beta^2 \frac{\partial EU_2}{\partial M_2^*}(-y_1), \text{ while } \frac{\partial EU}{\partial DPV} = \beta^2 \frac{\partial EU_2}{\partial M_2^*},$$

where

$$\frac{\partial EU_2}{\partial M_2^*} = \iint \left[\frac{\partial U_2}{\partial y_2} \cdot \frac{\partial y_2}{\partial M_2^*} + \frac{\partial U_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial M_2^*} \right] f(p_2 q_2, S) dp_2 dq_2.$$

Hence we obtain

$$dDPV = y_1 \left(\frac{dp_1}{dS}\right) dS - [(\partial EU/\partial S)/(\partial EU_2/\partial M_2^*)] dS \qquad (27)$$

where

$$\frac{\partial EU}{\partial S} = \int_0^\infty \int_0^\infty U_2(y_2, z_2) \frac{\partial f}{\partial S}(P_2, q_2, S) dp_2 dq_2 \equiv \frac{\partial EU_2}{\partial S}.$$

Given a change from an uncontrolled river (S = 0) to a river with a dam-reservoir complex with reservoir storage capacity of S^* units, the relevant consumer surplus measure in a spot market economy is given by

$$CS = |DPV(S = S^*) - DPV(S = 0)| = -\int_0^{S^*} y_1 \left(\frac{dp_1}{dS}\right) dS + \int_0^{S^*} \left(\frac{\partial EU_2/\partial S}{\partial EU_2/\partial M_2^*}\right) dS \qquad (28)$$

Two things should be noted about this measure of consumer surplus as contrasted to that for the case of the contingent claim

economy. First, only the first period compensated demand curve appears in the CS measure; benefits in the second period are already incorporated into the first period demand curve through the dynamic programming algorithm. Second, the CS measure includes a term that is nonobservable, representing the benefits that accrue to the consumer from the change in S, when p_1, y_1 and z_1 remain unchanged. A less variable time stream of releases increases expected utility of second period consumption, even when first period consumptions (and prices) are unaffected. Since $\partial EU_2/\partial S \geq 0$ and $\partial EU_2/\partial M_2^* > 0$, using the first integral on the right in (28) to measure the benefits from the dam-reservoir complex _understates_ the value of such benefits. Since the market (compensated) demand curve does not reveal the information contained in the second term of (28), there is a need for supplementary work for the cost-benefit analyst, designed to elicit information as to the properties of the underlying EU function.

The extension to the T-period case is straightforward. Define $H_t(y_t, z_t, M_t^*)$ by

$$H_t(y_t, z_t, M_t^*) = U_t(y_t, z_t) + \beta E \left[\max_{y_{t+1}, z_{t+1}} H_{t+1}(y_{t+1}, z_{t+1}, M_{t+1}^*) \right] \qquad (29)$$

where

$$M_{t+1}^* = M_t^* - (p_t y_t + q_t z_t)/(1 + i)^t.$$

Then at any time t, t = 1,...,T, the consumer chooses y_t, z_t to maximize H_t. By an inductive argument, it can be shown that this

leads to the derivation of the demand functions

$$y_t = y_t(p_t, q_t, M_t^*, S)$$

$$(30)$$

$$z_t = z_t(p_t, q_t, M_t^*, S)$$

$t = 2, \ldots, T.$

Hence, at time 1, the consumer chooses y_1 and z_1 to maximize EU^1 where EU is given by

$$EU^1 = \beta U(y_1, z_1) + \sum_{t=2}^{T} \beta^t EU_t(y_t(p_t, q_t, M_t^*, S), z_t(p_t, q_t, M_t^*, S))$$

$$(31)$$

First order conditions are given by

$$\frac{\partial EU}{\partial y_1} = \beta \frac{\partial U}{\partial y_1} + p_1 \sum_{t=2}^{T} \beta^t \frac{\partial EU_t}{\partial M_t^*} = 0$$

$$(32)$$

$$\frac{\partial EU}{\partial z_1} = \beta \frac{\partial U}{\partial z_1} + q_1 \sum_{t=2}^{T} \beta^t \frac{\partial EU_t}{\partial M_t^*} = 0$$

Consider now the derivation of a consumer surplus measure. $EU = EU(y_1, z_1, p_1, q_1, DPV, S)$ so that under the partial equilibrium assumption, again we have

$$dDPV = -\left[\frac{(\partial EU/\partial p_1)}{(\partial EU/\partial DPV)} \left(\frac{dp_1}{dS}\right) - \frac{(\partial EU/\partial S)}{(\partial EU/\partial DPV)} \right] dS,$$

$$(33)$$

where

$$\frac{\partial EU}{\partial p_1} = -y_1 \sum_{t=2}^{T} \beta^t \frac{\partial EU_t}{\partial M_t^*}, \text{ and}$$

$$\frac{\partial EU}{\partial DPV} = \sum_{t=2}^{T} \beta^t \frac{\partial EU_t}{\partial M_t^*}.$$

Hence the consumer surplus measure for consumer i, for the evaluation of the benefits from increasing S from 0 to S^* is again given by

$$CS = |DPV(S = S^*) - DPV(S = 0)| = \qquad (34)$$

$$- \int_0^{S^*} y_1^i \left(\frac{dp_1}{dS}\right) dS + \int_0^{S^*} \frac{(\partial EU^i/\partial S)}{(\partial EU^i/\partial DPV)} dS.$$

While the expression for $\partial EU^i/\partial S$ is considerably more complicated in this case than in the 2-period case, still $\partial EU^i/\partial S \geq 0$, so that if the area under the compensated y_1^i demand curve is used as a measure of consumer surplus, this understates the value of the benefits derived from the dam-reservoir project.

On the supply side, the introduction of uncertainty as to future spot market prices does not raise any new conceptual problems, since the decision problem for the dam-reservoir operator is one of dynamic programming even when price uncertainty is absent. Given an expected income maximizing operator, the effect of introducing price uncertainty is simply that of replacing the known prices p_r of contingent claims associated with the streamflow sequence x(r), with the expected value Ep_r of the uncertain spot market price, in the operator's criterion function. That is, the rule for the operator is based on the expression

$$\left[p_r - \max_s \sum_{j_{t+1}=1}^{J} \cdots \sum_{j_{t+s}=1}^{J} \delta^s (1-a)^s \pi_{j_{t+1}} \cdots \pi_{j_{t+s}} E p_{w_s} \right] \quad (35)$$

where $s = 1, \ldots, T - t$ and $w_s = (r, j_{t+1}, \ldots, j_{t+s}) \in \Omega_{t+s}$.

If the term inside the brackets is positive, the operator should empty the reservoir at time t, releasing as much water as he can; if it is negative, then as much water as possible is stored; and if the expression is zero, any feasible release is consistent with a maximum of expected profits.

Equilibrium in the spot market for water in period t, given that the streamflow sequence x(r) has occurred, is given by the condition

$$\sum_{i=1}^{I} y_t^i(p_t, q_t, M_t^*, S) = y_r^*(p_r, q_r, S) \quad (36)$$

where y_t^i is the demand for water by consumer i in period t, and y_r is releases by the dam-reservoir operator in period t, given the streamflow sequence x(r), $r \in \Omega_t$.

Once again, in principle, the system represented by (36) could be solved to obtain the appropriate equilibrium spot prices and in particular, a rational expectations equilibrium might be of interest to explore.

4. EXAMPLE

To illustrate, consider the case where $U_t = y_t z_t$, t = 1, 2, with $f(p_2, q_2, S) = \frac{S+1}{S+2}$, $1 \leq q_2 \leq 2$, $1 \leq p_2 \leq \frac{2S+3}{S+1}$.

Then it is easy to verify that

$$y_2 = \frac{M_2^*}{2p_2}, \quad z_2 = \frac{M_2^*}{2q_2}$$

so that

$$EU = y_1 z_1 + \beta \int_1^2 \int_1^{2S+3} \left(\frac{1}{S+1}\right) \frac{S+1}{S+2} \frac{(DPV - p_1 y_1 - q_1 z_1)^2}{4p_2 q_2} \, dp_2 \, dq_2$$

Hence

$$EU = y_1 z_1 + K(DPV - p_1 y_1 - q_1 z_1)^2$$

where

$$K = \frac{B}{4} \frac{S+1}{S+2} \ln\frac{(2S+3)}{S+1} \ln 2$$

Thus $y_1 = \frac{Kq_1(DPV)}{1 + 2p_1 q_1}$ and $z_1 = \frac{Kp_1(DPV)}{1 + 2p_1 q_1}$ are the ordinary demand curves, and the compensated demand function for y_1, y_1^c, with $EU = \overline{EU}$, is given by

$$y_1^c = \frac{KDPVp_1 + \sqrt{K^2 p_1^2 DPV^2 - (KDPV^2 - \overline{EU})(4Kp_1^2 + p_1/q_1)}}{4Kp_1^2 + p_1/q_1}.$$

Note that

$$\frac{\partial EU_2}{\partial S} = \frac{(M_2^*)^2}{4} \frac{(\ln 2)}{(S+2)^2} \left[\ln \frac{2S+3}{S+1} - \frac{1}{(2S+3)} \right] > 0$$

while $\frac{\partial EU_2}{\partial M^*} = \frac{2EU_2}{M^*} > 0.$

5. SUMMARY AND COMMENTS

We have examined the construction of consumer surplus measures appropriate to the evaluation of a water project in two different economic environments--an economy containing a complete set of contingent claim markets (and with no spot markets); and an economy with a complete set of spot markets (but no contingent claim markets). In the Arrow-Debreu type world, the consumer surplus measure that applies under uncertainty as to streamflows is of the same form as the measure that would apply under certainty, being the sum of the discounted consumer surplus areas for each period over the horizon. The only difference between the certainty and uncertainty cases is that under uncertainty there is a consumer surplus area for each possible state of the world (sequence of streamflows) for each period.

Things are quite different in the case of a spot market economy. In the first place, only the consumer surplus area for first period compensated demand appears in the consumer surplus measure; the dynamic programming approach implies that all of the benefits over the T-period horizon are captured in the first period compensated demand function. But of more interest is the fact that the "correct" measure of consumer surplus includes an unobserved term, measuring the benefits accruing to the consumer from the change in expected utility due to a shift in the pdf over future spot prices, given unchanged first period prices and consumptions.

The unobserved term in the CS measure raises some basic questions about the welfare implications of a CS measure designed to

express the benefits accruing from a project such as a dam-reservoir complex, when the effect (in part) of the project is to alter the probability distributions of consumers over future spot prices. This is related to what is involved in the notion of an _ex ante_ Pareto optimum. The idea is this. _Ex ante_ optimality refers to any feasible allocation that maximizes the expected utility of one consumer for given levels of expected utility for other consumers; alternatively, at an _ex ante_ optimum, we can't make one consumer better off in terms of expected utility without making some other consumer worse off in terms of expected utility. But this is to hold for _given_ subjective probability distributions over payoffs. If pdfs change, then any resulting allocation is simply non comparable (in the _ex ante_ sense) with allocations holding before the change in pdfs.

The CS measure in (34) measures the benefits to an individual resulting from a parameter change that alters his subjective pdf over future price vectors. While the CS measure in (34) is the measure relevant for decision making wish respect to a proposed dam-reservoir project, it is irrelevant to policy decisions linked to _ex ante_ optimality or to the attainment of _ex ante_ Pareto superior allocations, because of the effect of the dam-reservoir project on the pdf's of consumers.

What is needed in the case of the spot market economy is a different notion of a Pareto ranking, one defined over vectors $(f^1, \ldots, f^I)$ of subjective pdfs. In this ranking, one vector is Pareto superior to another if the vector of expected utilities $(EU^1, \ldots, EU^I)$

associated with the first vector of pdf's is weakly larger (component by component) than the vector of expected utilities associated with the second vector of pdf's. A Pareto optimum is achieved with a feasible vector of pdf's if no other feasible vector of pdf's is Pareto superior to it. This is a completely different kind of Pareto ranking than the ex ante ranking, and presumably it is this difference that in part at least accounts for the quite different specifications of CS as between (8) and (34), including the unobserved component in (34).

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