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Double-Trace Flows and the Swampland

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Abstract

We explore the idea that large N, non-supersymmetric conformal field theories with a parametrically large gap to higher spin single-trace operators may be obtained as infrared fixed points of relevant double-trace deformations of superconformal field theories. After recalling the AdS interpretation and some potential pathologies of such flows, we introduce a concrete example that appears to avoid them: the ABJM theory at finite k, deformed by $\int \mathcal{O}^2$, where $\mathcal{O}$ is the superconformal primary in the stress-tensor multiplet. We address its relation to recent conjectures based on weak gravity bounds, and discuss the prospects for a wider class of similarly viable flows. Next, we proceed to analyze the spectrum and correlation functions of the putative IR CFT, to leading non-trivial order in $1/N$. This includes analytic computations of the change under double-trace flow of connected four-point functions of ABJM superconformal primaries; and of the IR anomalous dimensions of infinite classes of double-trace composite operators. These would be the first analytic results for anomalous dimensions of finite-spin composite operators in any large N CFT_3 with an Einstein gravity dual.

Contents

1 Introduction

The paradigmatic example of AdS/CFT [1–3] is a duality between two theories: a theory of AdS quantum gravity whose low-energy limit includes a weakly coupled Einstein gravity subsector; and a CFT with many degrees of freedom ("large N"), and a sparse spectrum of local, single-trace operators of spin no greater than two. The absence of parametrically light single-trace operators of higher spin is often phrased as a holographic gap condition, $\Delta_{\text{gap}} \gg 1$, where Δ_{gap} is the dimension of the lightest spin-four single-trace operator [4]. As a matter of consistency, these conformal field theories must, among other properties, be unitary, crossing-symmetric and causal; must they also be supersymmetric?

There are recent conjectures in the affirmative, motivated in part by an absence of explicit constructions. It was argued in [5, 6] that weak gravity bounds on the tension of charged objects in quantum gravity may only be saturated when a BPS condition is obeyed; combined with known instabilities of non-BPS branes in AdS spacetimes, [5, 6] argued that, indeed, sparse non-supersymmetric CFTs with large N and large gap do not exist. This connection arises because the weak gravity bounds are meant to hold in theories whose low-energy gravitational sector is described by general relativity, a feature which is automatically implied by the CFT spectral condition $\Delta_{\text{gap}} \gg 1$ [7–9].

In this paper, we explore a general approach to the construction of large N, large gap, non-supersymmetric CFTs, and investigate specific examples that avoid some well-known pathologies that are both perturbative and non-perturbative in N. The main idea is to impose supersymmetry-breaking boundary conditions in AdS. On the CFT side, we perturb a UV superconformal field theory by a relevant, supersymmetry-breaking, double-trace deformation, which flows to an IR CFT that still obeys the gap condition. The construction is on firmest footing when the UV SCFT has no symmetry-preserving exactly marginal operators. While not all such flows may lead to a viable IR fixed point, we present a large class of examples in three spacetime dimensions that appear to be especially promising.[1] This construction may be viewed as a potential counterexample to the CFT conjecture of [5, 6], but is consistent with their sharpened version of the weak gravity bounds as applied to flat space quantum gravity.

In Section 2, we introduce the basic idea of a supersymmetry-breaking double-trace flow from a large N, large gap SCFT; make a connection to the conjectures of [5, 6]; list some pathologies that a consistent construction must avoid; and discuss the most basic constraints on such flows from superconformal representation theory in various spacetime dimensions.

In Section 3, we introduce a specific class of examples: namely, the ABJM theories [17] at finite k deformed by $\int \mathcal{O}^2$, where $\mathcal{O}$ is the superconformal primary in the stress-tensor multiplet. We argue for their viability at large but finite N, and address a potential issue on the moduli space of vacua. We also discuss a preliminary proposal for using double-trace deformations of more general three-dimensional CFTs to generate other non-supersymmetric theories with a large gap.

In Section 4, having made a concrete proposal for a non-supersymmetric, large gap CFT, we perform some quantitative computations of its observables. We show that certain classes of correlation functions vanish in the IR at *leading* order in $1/N$, and very briefly discuss the implications for thermal physics. We then study the change under RG flow of an infinite class of four-point functions of ABJM superconformal primaries $\mathcal{O}_p$. This calculation generalizes the approach of [18]. By performing the conformal block decomposition of those results, we extract, in Section 4.3, the leading-order change in the anomalous dimensions of spinning double-trace operators $\mathcal{O}_p \partial_{\mu_1} \dots \partial_{\mu_\ell} \mathcal{O}_p$. Moreover, using the non-renormalization

[1]There is a long history of attempts at consistent AdS/CFT constructions without supersymmetry. Some are cited in [5]. Those with a direct connection to M2-branes include [10–16].

properties of UV-protected supermultiplets, we can extract the anomalous dimension of several classes of double-trace operators *at the IR fixed point*, not only their change under RG flow. The main results may be found in (4.34), (4.38)–(4.40), and (4.42). These would be, to our knowledge, the first analytic computations of anomalous dimensions of finite-spin double-trace operators in any large N, three-dimensional CFT with an Einstein gravity dual, supersymmetric or otherwise. Our technique for analytically deriving these anomalous dimensions applies to any IR fixed point obtained by double-trace flow from a CFT with a greater number of supersymmetries, including, in particular, IR SCFTs that preserve a fraction of the UV supersymmetry.

In Section 5, we conclude with some comments on future work. Appendices A–B include some technical details needed for Section 4, while Appendix C gives a brief historical recollection of the attempt to construct non-supersymmetric, large gap CFTs by orbifolding $\mathcal{N} = 4$ super-Yang-Mills.

Before proceeding, it is worth being more precise about our definitions, and our interpretation of the conjectures of [5,6] that motivated this work. In general, a CFT with a weakly coupled gravity dual belongs to a sequence of CFTs with central charge C_T parameterized by N. The sequence exists above some critical value of N and admits a $N \to \infty$ limit with a finite spectral density and operator product expansion. The prototypical holographic CFT, whose gravity sector is pure general relativity, obeys the further constraint $\Delta_{\rm gap} \gg 1$, and the "sparseness" condition of polynomially-bounded growth of low-spin operators. It is this class of CFTs whose existence we are addressing. It should be emphasized that the question of whether holographic CFTs can have a parametrically large gap ceases to make sense at small enough N: these theories always obey a parametric hierarchy $\Delta_{\rm gap} \lesssim \sqrt{C_T}$. (This was recently proven in CFT in [19].) In string theory terms, $M_{string} \lesssim M_{Planck}$. For finite N and $\Delta_{\rm gap}$, UV-finiteness demands corrections to the gravitational action beyond general relativity, at which point it is unclear what becomes of the conjecture of [5,6] then.

2 Basic Idea

Let us first quickly recall the definition of a double-trace flow. Consider a large N CFT$_d$ which contains a scalar conformal primary O of conformal dimension $\Delta < d/2$. If we deform the action by

$$\delta S_{\rm CFT} = g \int d^d x\, O^2 \ , \tag{2.1}$$

this triggers a flow to an IR CFT, in which $\Delta \to d - \Delta + \ldots$ to leading order in $1/N$. If O is not a singlet under global symmetries, O^2 is to be understood as the singlet in the operator product. The generating functional for connected correlators of the IR CFT is given by the Legendre transform of the UV functional with respect to a source for O [20]. At infinite N, such flows always exist. Besides the change $\Delta \to d - \Delta$, the spectrum of the planar dilatation operator is identical in the UV and IR. At the non-planar level, conformal dimensions and

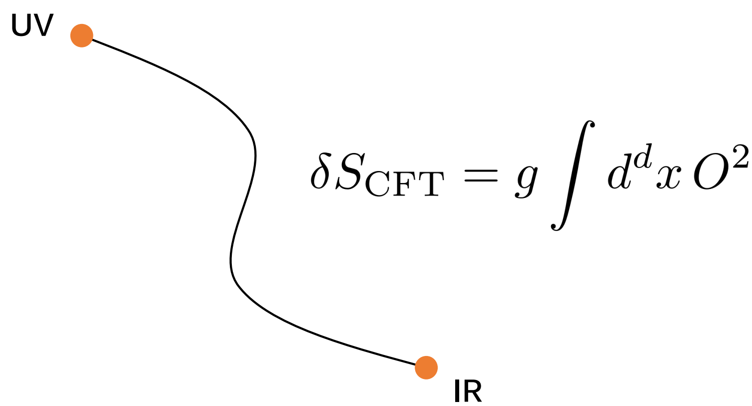

Figure 1: A cartoon of a general double-trace flow. In this paper, we take the UV theory to be a superconformal field theory with large N and a large gap, and O to be a superconformal primary.

OPE coefficients are modified, both for single-trace and multi-trace operators. The change in the double-trace spectrum was recently analyzed in [18], and we will reprise those results later in this paper.

In the holographic context [20–27], in which O is dual to a scalar field ϕ of mass squared $m^2 = \Delta(\Delta - d)$ in AdS units, there are two choices of normalizable boundary conditions when $-d^2/4 \leq m^2 \leq -d^2/4 + 1$:

$$\begin{aligned} \textbf{Standard:} \quad & \Delta_+ = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2} \\ \textbf{Alternate:} \quad & \Delta_- = d - \Delta_+ \end{aligned} \tag{2.2}$$

Each of these corresponds to a unitary conformal dimension at one end of the RG flow triggered by (2.1). The flow is not visible in the bulk as a soliton interpolating between two AdS vacua with a macroscopic difference in curvatures, due to $1/N$ suppression.

We now consider this deformation in a top-down setting. Consider a SUSY AdS$_{d+1} \times \mathcal{M}$ compactification of string or M-theory. In the limit in which the bulk theory is a weakly coupled supergravity and $\mathcal{M}$ is of AdS size, as in Freund-Rubin compactifications, the dual SCFT has a parametrically large gap to single-trace operators of spin greater than two, and the light spectrum is sparse.[2] If the Kaluza-Klein spectrum on $\mathcal{M}$ contains scalar fields with masses in the aforementioned range, SUSY may require alternate quantization. This happens, for instance, for bottom components of chiral multiplets of unit R-charge in 4d $\mathcal{N} = 1$ SCFTs (where $\Delta = 3/2$), or of flavor current multiplets in 3d $\mathcal{N} = 2$ SCFTs (where $\Delta = 1$). In such cases, we may deform the dual SCFT as in (2.1), where O is the conformal primary operator dual to such a scalar field. Such a double-trace deformation will generically break all SUSY in the IR. Thus, by choosing the SUSY-breaking Δ_+ boundary condition in

[2]In $d = 2$, a less restrictive and more explicit definition of sparseness, $\rho(\Delta) \lesssim e^{2\pi\Delta}$ for $\Delta - \Delta_{\rm vac} \leq c/12$, is sufficient to capture many aspects of holographic universality [28].

AdS, we are studying a non-SUSY, large N fixed point with $\Delta_{\rm gap} \gg 1$.

Let us make contact with [5,6]. Their proposed sharpening of the weak gravity conjecture states that in a consistent theory of quantum gravity, the weak gravity bounds on charge-to-mass ratios may only be saturated by BPS objects. AdS spacetimes suffer from a brane nucleation instability for non-BPS branes with a sufficiently large charge-to-tension ratio [29,30], which destroys the near-horizon region of a stack of branes in asymptotically (locally) flat space; [5, 6] take this as evidence that non-SUSY CFTs with large N and a large gap do not exist. But the special class of non-SUSY CFTs studied herein – obtained by SUSY-breaking boundary conditions in AdS geometries built from BPS branes – may nevertheless be consistent with the refined bound on charged objects in flat space. In the brane picture – that is, before taking the near-horizon limit – there is no boundary, and hence no notion of SUSY-breaking boundary condition.[3] We return to this point in Section 3.

What can go wrong?

Obviously, the question is how robust this construction is away from infinite N. But some pathologies are absent by design. At infinite N the CFT has a unitary spectrum of operator dimensions; in the bulk, the classical theory is perturbatively stable. Moreover, non-perturbative instabilities arising from geometry or topology of $\mathcal{M}$ are also absent.

There are, still, various potential pitfalls that may render the IR CFT ill-defined, including **i)** complex operator dimensions in the $1/N$ expansion; **ii)** an unstable vacuum; **iii)** the breaking of conformal symmetry via nonzero beta functions for marginal or nearly-marginal gauge-invariant operators, which can be generated for single-trace or multi-trace operators. If the CFT has a conformal manifold, the offending pathology – and its bulk dual – may change as a function of marginal couplings (as in e.g. [32, 33]).

On the bulk side, one may likewise find **i)** bulk tachyons that violate the Breitenlohner-Freedman bound [34]. This can occur at tree- or loop-level, but the most dangerous situation is when a tree-level scalar mass sits at the bound, $m^2_{BF} = -d^2/4$: in the absence of SUSY, quantum corrections may push $m^2 < m^2_{BF}$; **ii)** the development of a runaway direction in the effective potential for probe branes in AdS; **iii)** non-perturbative instabilities due to the geometry and/or topology of $\mathcal{M}$; **iv)** non-perturbative instabilities in AdS, such as the potential for brane nucleation of [29, 30].

In which spacetime dimensions?

With these issues in mind, let us briefly comment on the viability of this construction in various space-time dimensions, before moving on to the case of $d = 3$, our main interest.

[3] The sharpened weak gravity bounds are implied by [5, 6] to hold in both flat space and AdS quantum gravity. The natural argument for the latter, assuming the former, is that AdS geometries in string/M-theory are canonically constructed by bringing BPS branes together and zooming into the near-horizon region of the backreacted geometry. But logically speaking, these seem to be distinct claims. We thank H. Ooguri for a discussion on this issue. See also [31] for a discussion of the weak gravity bounds in AdS.

In $d = 5, 6$, superconformal representation theory prohibits the existence of (unitary) scalar conformal primaries in the range $\frac{d-2}{2} < \Delta < \frac{d}{2}$ (see e.g. [35]).[4] In $d = 4$, the same is true for maximal $\mathcal{N} = 4$ SUSY, but $\mathcal{N} < 4$ theories can (and do) contain operators with $1 < \Delta < 2$. In $d = 2, 3$, the superconformal algebra admits unitary representations containing scalar primaries of $\Delta < d/2$ for any amount of SUSY.

In $d = 4$ SCFTs, a ubiquitous class of operators with $\Delta < 2$ are $\mathcal{N} = 1$ chiral primaries with unit $\mathfrak{u}(1)_R$-charge, which have $\Delta = 3/2$. However, finding viable examples of SUSY-breaking double-trace flows to stable IR fixed points in $d = 4$ seems challenging. The bottom component of a four-dimensional $\mathcal{N} = 1$ current multiplet is a $\Delta = 2$ superconformal primary, $\mathcal{O}$. The dual scalar field in AdS sits exactly at the BF bound. One can also form double-trace operators $\mathcal{O}^2$ that are classically marginal global symmetry singlets; such operators are typically marginally relevant, and may develop nonzero beta functions along a conformal manifold. The latter issue also applies to composites made from fermions in $\mathcal{N} = 1$ chiral and anti-chiral multiplets, which have $\Delta = 2$. An explicit example of a large N, large gap CFT where these problems would arise is the Klebanov-Witten theory, dual to type IIB string theory on $\mathrm{AdS}_5 \times T^{1,1}$ [36]. The theory contains an operator of unit R-charge, $\mathcal{O}_1$, that lives in the $(2, 2)$ of an $\mathfrak{su}(2)_A \times \mathfrak{su}(2)_B$ global symmetry. This is the only operator of $\Delta < 2$. Thus, if we turn on the deformation

$$\delta S_{KW} = g \int (\mathcal{O}_1)_{ab} (\overline{\mathcal{O}}_1)^{ab} \ , \tag{2.3}$$

by choosing the Δ_+ boundary condition on the dual bulk scalar field, the CFT will naively flow to a non-SUSY fixed point. This was briefly considered in [24, 37, 38]. But for the reasons given above, the putative IR fixed point is unlikely to exist. (The Klebanov-Witten theory also has an exactly marginal coupling, which can acquire a nonzero beta function in the $1/N$ expansion after double-trace flow.)

On the other hand, the representation theory of $\mathfrak{osp}(\mathcal{N}|4)$, the $d = 3$ superconformal algebra, is much more favorable, and we focus on this case below.

3 A SUSY-Breaking Double-Trace Flow from ABJM

We now turn to our main proposal: that a non-SUSY double-trace flow from the ABJM theory leads to a stable IR fixed point.

Let us quickly review the salient aspects of the ABJM theory that we will need. (For more detailed reviews, see e.g. [39–41].) The ABJM theory [17] is a parity-invariant, $U(N)_k \times U(N)_{-k}$ Chern-Simons theory coupled to bifundamental matter. For $k = 1, 2$, the theory has $\mathcal{N} = 8$ SUSY, with $\mathfrak{so}(8)$ R-symmetry, whereas for $k > 2$ it has $\mathcal{N} = 6$ SUSY, with $\mathfrak{su}(4)_R \times \mathfrak{u}(1) \subset \mathfrak{so}(8)$ R-symmetry. The nature of the large N limit depends on whether

[4] This eliminates one approach to constructing the elusive, and perhaps non-existent, interacting non-SUSY $d > 4$ CFT.

k is held finite. At finite k, the holographic dual is 11d supergravity on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$, supported by four-form flux. At large k, one may take a ‘t Hooft limit, $k \to \infty, N \to \infty$, with $\lambda = N/k$ fixed, where, up to numerical prefactors, $\alpha' \sqrt{\lambda} \sim 1$. Upon increasing k while keeping $\lambda \gg 1$, the bulk dual remains 11d supergravity up to $k^5 \sim N$, where it crosses over to type IIA supergravity on $\mathrm{AdS}_4 \times \mathbb{CP}^3$. Eventually, for large enough k, stringy effects become parametrically important. It is useful to view the $S^7/\mathbb{Z}_k$ as a circle fibered over $\mathbb{CP}^3$ with length $2\pi L_{\mathrm{AdS}}/k$; this essentially corresponds to the $\mathfrak{u}(1)$ symmetry of the CFT. The “central charge” C_T of the ABJM theory is

$$C_T \approx \frac{64}{3\pi}\sqrt{2k}N^{3/2} + \mathcal{O}(\sqrt{N}) \tag{3.1}$$

where C_T is defined by the stress tensor two-point function as

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = C_T \frac{\mathcal{I}_{\mu\nu\rho\sigma}(x)}{x^6} \tag{3.2}$$

with a fixed tensor structure $\mathcal{I}_{\mu\nu\rho\sigma}(x)$ defined in [42].

For any value of k, the bottom component of the stress tensor multiplet is a $\Delta = 1$ scalar, which we call $\mathcal{O}$. For $k = 1, 2$, $\mathcal{O}$ resides in the $\mathbf{35_c}$ of the $\mathfrak{so}(8)$ R-symmetry,[5] with Dynkin labels [0020]; we may represent it as $\mathcal{O}_{IJ}$, the symmetric, traceless rank-two tensor of $\mathfrak{so}(8)$, where $I, J = 1 \ldots 8$. In terms of the fundamental scalars X_I in the ABJM Lagrangian,

$$\mathcal{O}_{IJ} = \mathrm{tr}(X_I X_J - \frac{\delta_{IJ}}{8} X^2) \tag{3.3}$$

For $k > 2$, $\mathcal{O}$ resides in the $\mathbf{15}$ of the $\mathfrak{su}(4)$ R-symmetry, with $\mathfrak{su}(4)$ Dynkin labels [101]; we may represent it as $\mathcal{O}^a_b$. In either case, from these components, one may form many double-trace operators, preserving varying degrees of R-symmetry. A natural choice is to preserve the full R-symmetry, but break all SUSY. Thus, we propose to consider the RG flow away from the ABJM theory triggered by the R-symmetry singlet $\mathcal{O}^2$, with all R-symmetry indices contracted:

$$\delta S_{ABJM} = g \int d^3x\, \mathcal{O}^2 \tag{3.4}$$

where

$$\begin{aligned} \mathcal{O}^2 &= \mathcal{O}_{IJ}\mathcal{O}^{IJ} \quad \text{for} \quad k = 1, 2 \\ \mathcal{O}^2 &= \mathcal{O}^a_b \mathcal{O}^b_a \quad \text{for} \quad k > 2\ . \end{aligned} \tag{3.5}$$

This relevant deformation triggers a flow to an IR fixed point where $\mathcal{O}$ has dimension $\Delta_{\mathcal{O}} = 2 + O(1/N)$. In the dual AdS theory, this corresponds to choosing Δ_+ boundary conditions for all components of the $m^2 = -2$ scalar field dual to $\mathcal{O}$. Our proposal is that such IR fixed points may be viable CFTs for finite k. More precisely, this statement is on firmest footing

[5] We have made a choice of $\mathfrak{so}(8)$ triality frame, following conventions of [43]. See e.g. Table 1.

for $k \neq 1,3$, as we explain below.

Let us lay out some arguments for this. First, as mentioned earlier, the $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ geometry admits no tunneling solutions that can be ascribed to the transverse manifold, because it descends from a SUSY compactification. Second, at finite k the ABJM theories form a discrete set, with no exactly marginal gauge coupling. This is one virtue of $\mathrm{AdS}_4 \times \mathcal{M}_7$ compactifications in general as compared to $\mathrm{AdS}_5 \times \mathcal{M}_5$ compactifications.

Another appealing property of the finite k ABJM theories subject to the R-symmetry-preserving double-trace deformation (3.4) is their especially sparse spectrum of light operators: all symmetry-preserving gauge-invariant operators in the IR are irrelevant. More precisely, the IR fixed point has the following two properties, to all orders in $1/N$:

A There are no global singlet, parity-preserving relevant operators.

B There are no global singlet, parity-preserving marginal operators for $k \neq 1,3$.

B implies that there are no $\Delta = 3/2$ scalar operators, which would be dual to BF bound-saturating scalars. (If there were, we could use them to form a classically marginal singlet double-trace operator.) Together these imply the complete attractiveness of the double-trace RG flow and the absence of conformal symmetry breaking.

To show **A** and **B**, we examine the spectrum of gauge-invariant operators of the ABJM theory. The spectrum at large N may be obtained by starting from the KK reduction on S^7 [44, 45], and restricting to $\mathbb{Z}_k$-invariant modes. See [46] for a summary. These are the modes with $\mathfrak{u}(1)$ charge 0 mod k.[6] The scalar spectrum on S^7 is comprised of a tower of KK modes in $\mathfrak{so}(8)$ representations $[00p0]$, where $p = 2, 3, \ldots$. The dual operators are single-trace, superconformal primaries which we call $\mathcal{O}_p$, with $\Delta_p = p/2$. Each of these is a bottom component of a 1/2-BPS superconformal multiplet, namely, the $B_1[0]^{(0,0,p,0)}_{p/2}$ multiplets in the notation of [35]. The $p = 2$ multiplet is the stress tensor multiplet, which contains $\mathcal{O}_2 \equiv \mathcal{O}$; we list its content in Table 1. Under the branching $\mathfrak{so}(8) \to \mathfrak{su}(4) \times \mathfrak{u}(1)$, the representation $[00p0]$ yields $\mathfrak{u}(1)$ charges $\pm(p - 2n)$ with $n = 0, 1, \ldots \lfloor p/2 \rfloor$.

We now look to construct relevant and marginal scalar operators at the IR fixed point that are singlets under all global symmetries. We have broken SUSY completely, but kept the full R-symmetry. The theory is also parity-invariant. So we are looking for parity-even R-singlets. As described above, all single-trace scalars are charged under the R-symmetry, so only multi-trace operators can be R-singlets. In the UV, before the RG flow, the only operators that can possibly be used to form marginal or relevant multi-trace scalar singlets have $\Delta = 1, 3/2, 2$. Taking $k \neq 1,3$, which removes the potentially unitarity-violating $\mathcal{O}_3$

[6] For instance, the fact that $\mathcal{O}$ lives in the adjoint of $\mathfrak{su}(4)_R$ in the $k > 2$ ABJM theories may be easily understood via the projection onto $\mathbb{Z}_k$-invariant states, together with the branching of the 35_c of $\mathfrak{so}(8)$ into $\mathfrak{su}(4) \times \mathfrak{u}(1)$ representations,

$$\mathbf{35_c} \to \mathbf{15}_0 + \mathbf{10}_2 + \mathbf{10}_{-2} \tag{3.6}$$

where the subscript labels the $\mathfrak{u}(1)$ charge. Henceforth we will show only $\mathfrak{so}(8)$ Dynkin data $[abcd]$ explicitly, leaving the branching into $\mathfrak{su}(4) \times \mathfrak{u}(1)$ for the $\mathcal{N} = 6$ theories implicit.

Operator	Δ	ℓ	$\mathfrak{so}(8)$
$\mathcal{O}$	1	0	$\mathbf{35}_c = [0020]$
$Q\mathcal{O}$	3/2	1/2	$\mathbf{56}_v = [0011]$
$Q^2\mathcal{O}$	2	0	$\mathbf{35}_s = [0002]$
$Q^2\mathcal{O}$	2	1	$\mathbf{28} = [0100]$
$Q^3\mathcal{O}$	5/2	3/2	$\mathbf{8}_v = [1000]$
$Q^4\mathcal{O}$	3	2	$\mathbf{1} = [0000]$

Table 1: The conformal primaries of the stress-tensor multiplet in a 3d, $\mathcal{N} = 8$ SCFT, together with their $\mathfrak{so}(3,2) \times \mathfrak{so}(8)$ quantum numbers and their positions in the multiplet. This is the $B_1[0]_1^{(0,0,2,0)}$ multiplet in the notation of of [35]. The superconformal primary is $\mathcal{O}$. In a theory with $\mathcal{N} = 6$ SUSY, the first three columns are identical.

from the spectrum, the list of such operators is short:

- $\mathcal{O}$, with $\Delta = 1$ in the $\mathbf{35_c}$, and its superconformal descendants with $\Delta \leq 2$. These include spin-1/2 fermions ψ, with $\Delta = 3/2$ in the $\mathbf{56_v}$, and scalars, with $\Delta = 2$ in the $\mathbf{35_s}$ – see Table 1.
- $\mathcal{O}_4$, with $\Delta = 2$ in the $\mathbf{294_c} = [0040]$.

Due to $\mathfrak{so}(8)$ selection rules, the only relevant singlet comprised of these constituents is $\mathcal{O}^2$, our deforming operator. In the UV, there is also a nearly marginal triple-trace operator obtained from $\mathcal{O}^3$ (projecting onto the singlet part). However, the RG flow (3.4) takes $\Delta \to 2 + O(N^{-3/2})$, for *all* components of $\mathcal{O}$. This immediately implies that for generic k, there are no relevant singlets in the IR.[7] Likewise, the only candidate (nearly-)marginal singlets in the IR are the two-fermion operators, $:\psi\psi:$, projected onto the singlet. However, this operator is parity odd.[8] Thus, we have shown Properties **A** and **B**. These imply that the RG flow (3.4) leads to a stable fixed point, and the IR CFT admits no symmetry-preserving relevant flows.

Having established the absence of relevant and marginal global singlet operators, let us however note that such operators would not have been expected to spoil the existence of the IR fixed point anyway. Given some set of operators $\mathcal{O}_i$ with $\Delta_i - d = \epsilon_i \ll 1$

[7] We do not expect that the presence of the nearly marginal triple-trace operator $\mathcal{O}^3$ in the UV renders the RG flow (3.4) invalid. A similar situation arises in the standard Wilson-Fisher fixed point in the 3d $O(N)$ model: there is a nearly marginal "triple-trace" operator $(\phi^i\phi^i)^3$ in the UV, but this does not affect the RG flow triggered by the relevant operator $(\phi^i\phi^i)^2$. See also [47] for a similar example with SUSY in the UV.

[8] This can be seen by noting that in any CFT with a weakly coupled AdS dual, the $1/N$ expansion may be viewed as an expansion around generalized free fields. The parity of a given operator is discrete, so we may determine it at infinite N. It is known that, in the theory of generalized free spin-1/2 fermions, the scalar operator $:\psi\psi:$ with dimension $2\Delta_\psi$ is parity odd. This can be seen, for instance, by decomposing the four-point function of identical, generalized free spin-1/2 fermions.

and corresponding couplings g^i, ordinary conformal perturbation theory admits both trivial ($g^i = 0$) and non-trivial ($g^i \sim \epsilon^i$) fixed points. For either sign of ϵ^i, the trivial fixed point is guaranteed to exist. This should be contrasted with the case of non-SUSY orbifolds of $\mathcal{N} = 4$ SYM in the 't Hooft limit, in which, due to the exactly marginal 't Hooft coupling, nearly-marginal global singlets develop nonzero beta functions to any order in perturbation theory [48–50]. Indeed, our restriction to finite k avoids a potential problem in the 't Hooft limit of the ABJM theory: the marginal coupling λ may acquire a nonzero beta function in the $1/N$ expansion. Note, though, that even when there are flat directions in a theory with nearly-marginal singlets, the addition of a *relevant* deformation, like $\int \mathcal{O}^2$, may still lead to a stable IR fixed point. This happens in e.g. [36, 51, 52]; while these examples retain some SUSY in the IR, perhaps the same also happens in the 't Hooft regime of the ABJM theories after our SUSY-breaking double-trace deformation.

3.1 A general prescription for 3d CFT

There are obvious variations of the above, including extension to the $\mathcal{N} = 6$ $U(N)_k \times U(M)_{-k}$ ABJ theories [53], or the introduction of double-trace deformations that preserve only a subgroup of $\mathfrak{so}(8)$. More interesting are generalizations beyond ABJ(M). Consider an $\text{AdS}_4 \times \mathcal{M}_7$ solution of 11d supergravity, where $\mathcal{M}_7$ is a Sasaki-Einstein manifold. The dual $\mathcal{N} = 2$ SCFT is, like ABJM, isolated. A preliminary proposal for generating stable, IR non-SUSY CFTs is to change the AdS boundary conditions for *all* single-trace scalar operators with $\Delta < 3/2$. That is, for every such operator $\mathcal{O}_i$ with $\Delta_i < 3/2$, deform the UV SCFT action by

$$\delta S_{CFT} = \sum_{\{\mathcal{O}_i | \Delta_i < \frac{3}{2}\}} g^i \int d^3x \, \mathcal{O}_i^2 \tag{3.7}$$

for some couplings g^i. Following our ABJM discussion, the question of IR stability boils down (modulo issues we have not exorcised yet) to the question of whether there are any single-trace scalar operators with $\Delta = 3/2$.

Examples abound. In $d = 3$, the bottom component of $\mathcal{N} = 2$ flavor multiplets is a $\Delta = 1$ scalar. This includes the R-current multiplet, so every $\mathcal{N} = 2$ SCFT admits such a double-trace flow. ($\mathcal{N} = 1$ SCFTs have no R-symmetry, nor do $\mathcal{N} = 1$ flavor multiplets contain scalars; so $\mathcal{N} = 1$ SCFTs may, but need not, contain a $\Delta < 3/2$ scalar with which to flow.) A well-studied $\mathcal{N} = 2$ example is the 11d supergravity compactification on $\text{AdS}_4 \times M^{111}$ [54, 55], dual to a quiver gauge theory with three nodes [56]. This manifold has isometry group $G = \mathfrak{su}(3) \times \mathfrak{su}(2) \times \mathfrak{u}(1)_R \times \mathfrak{u}(1)_B$, where the $\mathfrak{u}(1)_B$ is a baryonic or "Betti" vector multiplet. From the analysis of the KK spectrum in [54] (see also Table 5.2 of [13]), one can check that the CFT contains no $\Delta = 3/2$ single-trace scalar operators.[9] It would be

[9] Any manifold $\mathcal{M}_7$ with topologically non-trivial two-cycles has b_2 Betti multiplets. The dual CFT has a global symmetry group $G = G' \times \mathfrak{u}(1)^{b_2}$. Members of Betti multiplets are G'-singlets. The top component of a Betti multiplet is a $\Delta = 2$ scalar. Thus, in CFT_3's with Betti multiplets, the putative IR fixed

worthwhile to examine this proposal further.

3.2 Comments

Connection to other proposals

The double-trace technique considered in this paper is "milder" than other SUSY-breaking constructions that break SUSY in the bulk, not just by boundary conditions. In Appendix C, we briefly recall the story of one of the most well-studied – and ultimately unsatisfactory – non-SUSY constructions, namely, the type IIB orbifolds of the form $\mathrm{AdS}_5 \times S^5/\Gamma$, dual to non-SUSY orbifolds of 4d $\mathcal{N} = 4$ SYM.

In [16], a morally similar construction was suggested for the $k = 1$ ABJM theory, in which the $\mathfrak{so}(8)$ R-symmetry is gauged and augmented by a Chern-Simons term. This may be implemented by a double-trace deformation $\delta S_{\mathrm{CFT}} = \int d^3x J_\mu J^\mu$, where J_μ is the R-symmetry current, which is induced holographically through a mixed boundary condition on the bulk gauge field [57]. Though the IR fixed point may indeed exist, it has more potentially problematic operators whose dynamics may destabilize the theory and/or drive it to non-unitarity (such as the triple-trace singlet $\mathcal{O}^3$ discussed in [16]); it also would have the relevant singlets $\mathcal{O}^2$, thus making the RG flow less stable. Our proposal is simpler, and eliminates these operators, as described above.

Moduli space of vacua

A potential issue with this class of theories is the stability of the moduli space of vacua. In the ABJM theory on $\mathbb{R}^3$, the effective potential on the moduli space vanishes. In the IR, in the absence of SUSY, these flat directions will presumably be lifted at finite N; in principle, the origin of moduli space could cease to be a minimum, or runaway instabilities could develop in the $1/N$ expansion.[10] Definitively understanding the fate of the moduli space appears to be highly involved, but let us make some observations.[11]

Intuitively, the deformation (3.4) appears to lift the moduli space. With $g > 0$, it is a stabilizing quartic potential for all ABJM scalars, that grows as one flows toward the IR. It is nevertheless possible that, in the deep IR, the minimum is pushed away from the origin, or worse, by higher order effects in $1/N$. (If there is indeed a minimum away from the origin, this would be an interesting non-SUSY, non-conformal field theory to study.)

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point obtained after double-trace flow has (at least) b_2 relevant, single-trace, parity-even G-singlet scalars. Accordingly, while the IR fixed point may be stable, it is thus not a "dead-end" CFT.

[10] See [32, 58, 59] for discussions of similar instabilities in the non-SUSY $\mathcal{N} = 4$ SYM orbifold context. In [50], it was shown that in fixed lines of 4d CFTs with adjoint matter, Coleman-Weinberg instabilities exist if and only if conformal symmetry is broken via nonzero beta functions. If this equivalence extends to the present case (though we know no reason this would be so), our previous arguments about the spectrum imply an absence of moduli space instabilities for the ABJM double-trace deformation.

[11] We thank O. Aharony for raising this issue and for valuable discussions about it.

At finite k, the CFTs are inherently strongly coupled, so one must resort to a bulk M-theory computation. The essential question of whether a nucleation instability [29,30] occurs boils down to whether the effective potential for probe M2-branes in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$, with the modified boundary condition for ϕ (dual to $\mathcal{O}$), is attractive or repulsive. (A related approach would be to compute the force between two probe M2-branes using the $\Delta = 2$ boundary condition for ϕ.) The only contributions to the potential that differ from the SUSY case must involve ϕ propagators. At leading order in $1/N$, one can compute the "self-energy" correction due to the emission and re-absorption of ϕ from the brane, given the $\Delta = 2$ boundary condition. This should yield the leading order effect of the field theory potential (3.4) in the IR. (A related calculation was performed in [60].) We expect that this leaves flat the $\langle \mathcal{O} \rangle = 0$ subspace of the moduli space, so to determine whether this is lifted requires a higher-loop bulk computation. Unfortunately, this is no longer an AdS_4 supergravity computation, as it involves all of the scalar KK modes ϕ_p, dual to $\mathcal{O}_p$. For instance, the emission/re-absorption of ϕ_p from the brane receives a loop correction, because all ϕ_p couple to ϕ through loops. A vev for $\mathcal{O}_p$ would also seem to receive linear contributions from one-loop tadpole diagrams, where a ϕ loop attaches to a ϕ_p propagator attached to the probe; however, we note the encouraging feature that all cubic couplings $\phi_p \phi^2$ vanish.[12] There are likely other effects to consider as well; we leave a systematic exploration for future work.

4 Spectrum and Operator Products of the IR CFT

The IR CFTs described here have a rigid structure despite the lack of SUSY. The spectrum of local operators is integer- or half-integer spaced (depending on the parity of k), to leading order in $1/N$. Their operator products obey strict selection rules imposed by $\mathfrak{so}(8)$ global symmetry. The three-sphere free energy, $F = -\log Z_{S^3}$, can also be written to several subleading orders in $1/N$ [61]:

$$F_{IR} = F_{ABJM} - \frac{\zeta(3)}{8\pi^2} + \mathcal{O}(N^{-3/2}) \tag{4.1}$$

F_{ABJM} is known exactly from the ABJM matrix model [62]. In particular, it includes terms of order $N^{\pm 1/2}$ and $\log N$ that are identical in the IR CFT: higher loop effects start at $\mathcal{O}(N^{-3/2})$. It is somewhat remarkable that these subleading terms representing quantum effects in M-theory are robust to SUSY-breaking boundary conditions.

Below we derive some new results on the CFT data at the putative IR fixed points.

In Section 4.1, we show that even the *leading* large-N contribution to certain single-trace correlators flows to zero in the IR, to leading order in $1/N$. In particular, this is true for

[12]For $p > 4$, this is just $\mathfrak{so}(8)$ group theory (see (4.15)); for $p = 2, 4$, while the coupling is allowed by group theory, it actually vanishes. We show this in Section 4.1.

"extremal" n-point correlators, and for the non-extremal three-point function $\langle\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\rangle$.[13]

In Section 4.2, we compute the change in the connected four-point functions $\langle\mathcal{O}_p\mathcal{O}_p\mathcal{O}_p\mathcal{O}_p\rangle$, with $p\neq 2$, between UV and IR. From this we extract, in Section 4.3, the leading-order change in the conformal dimensions of double-trace operators $\mathcal{O}_p\partial^{2n}\partial_{\mu_1}\dots\partial_{\mu_\ell}\mathcal{O}_p$. However, the real power of this calculation comes in considering the consequences of $\mathfrak{so}(8)$ global symmetry. There exist several families of double-trace operators, one for each $\mathfrak{so}(8)$ representation appearing in the product $\mathcal{O}_p\otimes\mathcal{O}_p$. In the UV, some of these operators reside in SUSY-protected multiplets, and thus have vanishing anomalous dimensions to all orders in $1/N$. But in the IR, absent SUSY, these multiplets are no longer protected. Therefore, for these $\mathfrak{so}(8)$ representations, the *change* in the anomalous dimension between UV and IR *equals* the IR anomalous dimension! Thus, our computation allows us to read off some analytical double-trace spectral data about the IR CFT.

4.1 Extremal correlators vanish after double-trace flow

An extremal correlator is defined by the condition

$$\left\langle\prod_{i=1}^{n}\mathcal{O}_i(x_i)\right\rangle\,,\quad\text{where}\quad\Delta_1=\sum_{i=2}^{n}\Delta_i\tag{4.2}$$

These were studied mainly in the $\mathcal{N}=4$ SYM context in [63], and then in the ABJM context in [64]. We now demonstrate two simple vanishing conditions of extremal correlators under double-trace flow, and of the non-extremal correlator $\langle\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\rangle$.

4.1.1 Three-point functions

First, for completely general double-trace flows, an extremal three-point function that involves $\mathcal{O}$, the operator that triggers the flow, becomes zero in the IR to leading order in $1/N$:

$$\textbf{General double-trace flows:}\quad\langle\mathcal{O}_i\mathcal{O}_j\mathcal{O}\rangle_{UV}\neq 0\quad\xrightarrow{\int\mathcal{O}^2}\quad\langle\mathcal{O}_i\mathcal{O}_j\mathcal{O}\rangle_{IR}=0\tag{4.3}$$

for $\Delta_i=\Delta_j+\Delta_{\mathcal{O}}$. For the flow from ABJM, this implies that an infinite set of three-point functions vanishes at leading order in the IR: for all superconformal primaries $\mathcal{O}_p$ for any integer p in the ABJM spectrum,

$$\textbf{ABJM:}\quad\langle\mathcal{O}_{p+2}\mathcal{O}_p\mathcal{O}_2\rangle=\frac{8(p+1)}{\pi}\sqrt{\frac{2(p+3)}{(p+2)C_T}}\quad\xrightarrow{\int\mathcal{O}_2^2}\quad\langle\mathcal{O}_{p+2}\mathcal{O}_p\mathcal{O}_2\rangle_{IR}=0\tag{4.4}$$

[13]In this Section, we revert to using $\mathcal{O}_2$ to label the superconformal primary in the ABJM stress tensor multiplet.

where the UV result may be read off from [65] (we have used a unit normalization of the two-point functions $\langle \mathcal{O}_p \mathcal{O}_p \rangle$).

It is straightforward to prove (4.3). For simplicity, consider the three-point extremal correlators $\langle \Phi \mathcal{O} \mathcal{O} \rangle$, where $\Delta_\Phi = 2\Delta_{\mathcal{O}}$ in the UV. In the IR, we trade $\mathcal{O}$ for its Hubbard-Stratanovich field σ [38] inside correlation functions. In Appendix A, we show that the IR OPE coefficient, $C^{IR}_{\Phi\sigma\sigma}$, is

$$C^{IR}_{\Phi\sigma\sigma} = \frac{C^{UV}_{\Phi\mathcal{O}\mathcal{O}}}{\pi^d C^2_{\mathcal{O}\mathcal{O}}} \frac{\Gamma(\frac{d}{2} + \frac{\Delta_\Phi}{2} - \Delta_{\mathcal{O}})\Gamma^2(\Delta_{\mathcal{O}})}{\Gamma(\Delta_{\mathcal{O}} + \frac{\Delta_\Phi}{2} - \frac{d}{2})\Gamma^2(\frac{d}{2} - \Delta_{\mathcal{O}})} \frac{\Gamma(d - \frac{\Delta_\Phi}{2} - \Delta_{\mathcal{O}})}{\Gamma(\Delta_{\mathcal{O}} - \frac{\Delta_\Phi}{2})} \tag{4.5}$$

where $C_{\mathcal{O}\mathcal{O}}$ is the norm of $\mathcal{O}$. The denominator of the last factor implies that the UV-extremal correlator vanishes in the IR: $C^{IR}_{\Phi\sigma\sigma} = 0$. The analogous calculation was done for $\langle \Phi \Psi \mathcal{O} \rangle$ where $\Delta_\Phi = \Delta_\Psi + \Delta_{\mathcal{O}}$ in [18], which, being extremal, can also be seen to vanish in the IR.

This can be understood holographically using well-known facts about extremal correlators [63]. The bulk fields participating in extremal CFT three-point functions have a vanishing bulk cubic coupling, regardless of the boundary condition. In the UV, multiplying this zero by the infinity from the AdS integral gives a finite result (a more formal treatment involves subtle boundary terms). But in the IR – that is, after changing quantization of the bulk field dual to $\mathcal{O}$ – the AdS integral does not produce an infinity because the correlator is no longer extremal. Thus, one gets zero in the IR. This can also be understood yet another way, by thinking about operator mixing in the identification of bulk fields with CFT operators. A nonzero CFT extremal correlator $\langle \Phi \mathcal{O} \mathcal{O} \rangle$ is only consistent with a vanishing bulk coupling if the bulk field Φ_{bulk} is dual not only to Φ, but to a linear combination

$$\Phi_{\text{bulk}} \ := \ \Phi + \frac{c}{\sqrt{C_T}} \mathcal{O}^2 \tag{4.6}$$

for some c such that the three-point function of bulk modes, $\Phi_{\text{bulk}} \mathcal{O}_{\text{bulk}} \mathcal{O}_{\text{bulk}}$, vanishes. In the IR, $\Delta_{\mathcal{O}} \to d - \Delta_{\mathcal{O}}$, and this operator mixing is not allowed: thus, the holographic identification is $\Phi_{\text{bulk}} := \Phi$, and the CFT correlator vanishes.

Self-coupling of $\mathcal{O}_2$

Let us also point out that the three-point function of $\mathcal{O}_2$ vanishes in the IR: again assuming unit normalization of $\mathcal{O}_2$,

$$\textbf{ABJM:} \quad \langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle = \frac{128}{C_T} \quad \xrightarrow{\int \mathcal{O}_2^2} \quad \langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle_{IR} = 0 \tag{4.7}$$

This correlator is not extremal, but shares the feature that the bulk cubic vertex for ϕ_2 vanishes; the nonzero result in the UV is due to a compensating factor $\Gamma\left(\frac{\Delta_1+\Delta_2+\Delta_3-d}{2}\right)$ in the AdS three-point scalar integrals [66]. (See [67] for a proper treatment of boundary terms in $\mathcal{N} = 8$ supergravity that yields the correct result.) In the IR where $\Delta \to 2 + \ldots$, this gamma function becomes finite, so the CFT three-point function vanishes. The analogous statement is true for the three-point function of ϕ^2 in the large N critical $O(N)$ model [68–70].

Application: Thermal one-point functions

The fact that $\langle \mathcal{O}_4 \mathcal{O}_2 \mathcal{O}_2 \rangle_{IR} = \langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle_{IR} = 0$ after flowing from ABJM modifies the leading large-N behavior of thermal one-point functions in the IR. Consider first the one-point function of $\mathcal{O}_4$, defined on $S^1_\beta \times S^2$ as

$$\langle \mathcal{O}_4 \rangle_{S^1_\beta \times S^2} = \mathrm{Tr}_{\mathcal{H}}(\mathcal{O}_4 e^{-\beta H}) \tag{4.8}$$

where $\mathcal{H}$ is the local operator Hilbert space. The leading low-temperature asymptotics are determined by the dimension of the lightest operator to which $\mathcal{O}_4$ couples linearly. Thanks to the result above, the thermal one-point function of $\mathcal{O}_4$ has different behavior in UV and IR:

$$\begin{aligned} \text{UV}: \quad & \langle \mathcal{O}_4 \rangle_{S^1_\beta \times S^2} \approx \langle \mathcal{O}_2 \mathcal{O}_4 \mathcal{O}_2 \rangle e^{-\beta} + \ldots \\ \text{IR}: \quad & \langle \mathcal{O}_4 \rangle_{S^1_\beta \times S^2} \approx \langle \mathcal{O}_4 \mathcal{O}_4 \mathcal{O}_4 \rangle e^{-2\beta} + \ldots \end{aligned} \tag{4.9}$$

The leading term in the IR comes from the cubic self-coupling of $\mathcal{O}_4$ because neither of the other IR operators with $\Delta \leq 2$ – in particular, the $\Delta = 3/2$ fermion and $\Delta = 2$ scalar in the stress tensor multiplet – produce a $\mathbf{294_c}$ in their $\mathfrak{so}(8)$ tensor product [71]. Moreover, in the IR, all multi-trace operators made of $\mathcal{O}_2$ do not contribute to $\langle \mathcal{O}_4 \rangle_{S^1_\beta \times S^2}$ at leading order in $1/N$: for these operators, the leading order contribution comes from (generalized) free field Wick contractions,

$$\langle [\underbrace{\mathcal{O}_2 \ldots \mathcal{O}_2}_{n}]\, \mathcal{O}_4\, [\underbrace{\mathcal{O}_2 \ldots \mathcal{O}_2}_{n}] \rangle_{IR} \approx \langle \mathcal{O}_2 \mathcal{O}_2 \rangle^{n-1} \langle \mathcal{O}_2 \mathcal{O}_4 \mathcal{O}_2 \rangle_{IR} + \ldots\ , \tag{4.10}$$

which always leaves a three-point factor $\langle \mathcal{O}_2 \mathcal{O}_4 \mathcal{O}_2 \rangle_{IR} = 0$.

Similar statements are true for the thermal one-point function of $\mathcal{O}_2$. For instance, at small β,

$$\begin{aligned} \text{UV}: \quad & \langle \mathcal{O}_2 \rangle_{S^1_\beta \times S^2} \approx \langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle e^{-\beta} + \ldots \\ \text{IR}: \quad & \langle \mathcal{O}_2 \rangle_{S^1_\beta \times S^2} \approx \langle \psi \mathcal{O}_2 \psi \rangle e^{-\frac{3\beta}{2}} + \ldots \end{aligned} \tag{4.11}$$

where ψ is the spin-1/2 fermionic operator in the $\mathbf{56}_v$ of $\mathfrak{so}(8)$ (see Table 1).

4.1.2 n-point functions

Next, in the double-trace flow from ABJM, any n-point extremal correlator involving at least one $\mathcal{O}_2$ also vanishes in the IR to leading non-trivial order in $1/N$:

$$\textbf{ABJM:}\quad \Big\langle \prod_{i=1}^{n-1} \mathcal{O}_{p_i}\mathcal{O}_2 \Big\rangle \neq 0 \quad \xrightarrow{\int \mathcal{O}_2^2} \quad \Big\langle \prod_{i=1}^{n-1} \mathcal{O}_{p_i}\mathcal{O}_2 \Big\rangle_{IR} = 0 \tag{4.12}$$

where $p_1 = 2 + \sum_{i=2}^{n-1} p_i$. Although it is not directly to the question of whether tree-level extremal n-point correlators vanish after double-trace flow, we note for completeness that in ABJM (indeed, in maximally-SUSY CFTs in $3 \leq d \leq 6$ [72]), extremal correlators of chiral primaries exhibit the factorized form

$$\Big\langle \prod_{i=1}^{n} \mathcal{O}_{p_i} \Big\rangle_{ABJM} = \prod_{i=2}^{n} \langle \mathcal{O}_{p_i}\mathcal{O}_{p_i} \rangle \tag{4.13}$$

where $p_1 = \sum_{i=2}^{n} p_i$ [64]. The mechanism can again be viewed as coming from the admixture of $\mathcal{O}_{p_1}$ with the $(n-1)$-trace operator $[\mathcal{O}_{p_2} \dots \mathcal{O}_{p_n}]$. Upon flowing to the IR, this vanishes.

The proof of (4.12) adapts the arguments of [63] to this setting. In fact, this was already done in [64]. For simplicity, we consider the $\mathcal{N} = 8$ ABJM theories, so we study the four-point function

$$\langle \mathcal{O}_{p_1}\mathcal{O}_{p_2}\mathcal{O}_{p_3}\mathcal{O}_{p_4} \rangle \ , \quad \text{where} \quad p_1 = p_2 + p_3 + p_4 \tag{4.14}$$

For the double-trace application, we take (say) $p_4 = 2$. Using the $\mathfrak{so}(8)$ tensor product (e.g. [72])

$$[00p_1 0] \otimes [00p_2 0] = \bigoplus_{k=0}^{p_2} \bigoplus_{j=0}^{p_2-k} [0, j, p_1 + p_2 - 2k - 2j, 0] \tag{4.15}$$

and likewise for $[00p_3 0] \otimes [00p_4 0]$, one sees that the only $\mathfrak{so}(8)$ representation in both tensor products is $[00(p_3 + p_4)0]$. The logic of [63] applies verbatim, and all bulk diagrams contributing to these correlators involve at least one vanishing bulk vertex. As explained earlier, it follows that the IR correlator vanishes.

4.2 Four-point functions

In [18], the change in connected four-point functions $\langle \Phi\Phi\Phi\Phi \rangle$ was computed under general double-trace flows $\int \mathcal{O}^2$, for single-trace operators Φ that couple to $\mathcal{O}$. This was used to extract the change in the spectrum of double-trace operators $\Phi\partial^{2n}\partial_{\mu_1} \dots \partial_{\mu_\ell}\Phi$ in the IR, as well as their OPE coefficients. We can generalize this result to the present case, in which we take Φ to be $\mathcal{O}_p$, the superconformal primaries of ABJM

4.2.1 Setup

The spectrum of superconformal primary operators of the ABJM theory, for any k, includes the infinite tower of 1/2-BPS chiral primaries $\mathcal{O}_p$, where $p = 2, 4, \ldots$, living in the $[00p0]$ representations of $\mathfrak{so}(8)$ or its branching into $\mathfrak{su}(4) \times \mathfrak{u}(1)$. These operators have conformal dimension $\Delta = p/2$. For concreteness, in the remainder of this section we specialize to $k = 1, 2$, and hence an $\mathfrak{so}(8)$ global symmetry, though the results are easily generalized.

We may form $\mathfrak{so}(8)$ invariants by contracting their indices with the null vectors Y^I,

$$\mathcal{O}_p = \mathcal{O}_{I_1 \ldots I_p} Y^{I_1} \cdots Y^{I_p} \tag{4.16}$$

We will consider four-point functions $\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_p \mathcal{O}_p \rangle$ for $p > 2$, and compute their change under the RG flow triggered by (3.4): that is, under the deformation (3.4), we compute the quantity

$$\langle \mathcal{O}_p(x_1; Y_1) \mathcal{O}_p(x_2; Y_2) \mathcal{O}_p(x_3; Y_3) \mathcal{O}_p(x_4; Y_4) \rangle_{IR} - \langle \mathcal{O}_p(x_1; Y_1) \mathcal{O}_p(x_2; Y_2) \mathcal{O}_p(x_3; Y_3) \mathcal{O}_p(x_4; Y_4) \rangle_{UV} \tag{4.17}$$

to leading order. This is controlled, roughly speaking, by the order g term in $\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_p \mathcal{O}_p e^{-g \int \mathcal{O}_2^2} \rangle$. Due to SUSY Ward identities relating $\mathcal{O}_2$ to $T_{\mu\nu}$, $\mathcal{O}_2$ couples universally to all operators, so this difference is guaranteed to be nonzero for all p. The rest of the calculation is an extension of $\mathfrak{so}(8)$ group theory to the results of [18].

The functional form of two- and three-point functions of $\mathcal{O}_p$ are determined by $\mathfrak{so}(3,2) \times \mathfrak{so}(8)$ symmetry. Let us introduce the following UV correlators,

$$\begin{aligned} \langle \mathcal{O}_p(x_1; Y_1) \mathcal{O}_p(x_2; Y_2) \rangle &= C_{pp} \frac{(Y_1 \cdot Y_2)^p}{x_{12}^p} \\ \langle \mathcal{O}_p(x_1; Y_1) \mathcal{O}_p(x_2; Y_2) \mathcal{O}_2(x_3; Y_3) \rangle &= C_{pp2} \frac{(Y_1 \cdot Y_2)^{p-1} (Y_2 \cdot Y_3)(Y_3 \cdot Y_1)}{x_{12}^{p-1} x_{23} x_{31}} \end{aligned} \tag{4.18}$$

for some constants C_{pp}, C_{pp2}, where $x_{12} \equiv |x_1 - x_2|$. We may form the normalization-independent ratio,

$$a_{pp2}^{UV} = \frac{C_{pp2}^2}{C_{pp}^2 C_{22}} \tag{4.19}$$

This may be computed in various ways (see e.g. [65]) to be

$$a_{pp2}^{UV} = \frac{32 p^2}{C_T} \tag{4.20}$$

Symmetry allows us to write the four-point function at either fixed point in the form

$$\langle \mathcal{O}_p(x_1; Y_1) \mathcal{O}_p(x_2; Y_2) \mathcal{O}_p(x_3; Y_3) \mathcal{O}_p(x_4; Y_4) \rangle = C_{pp}^2 \left(\frac{Y_1 \cdot Y_2 Y_3 \cdot Y_4}{x_{12} x_{34}} \right)^p \mathcal{F}_p(u, v; \sigma, \tau) \tag{4.21}$$

where we introduced the internal cross-ratios

$$\sigma = \frac{Y_1 \cdot Y_3 Y_2 \cdot Y_4}{Y_1 \cdot Y_2 Y_3 \cdot Y_4} \ , \quad \tau = \frac{Y_1 \cdot Y_4 Y_2 \cdot Y_3}{Y_1 \cdot Y_2 Y_3 \cdot Y_4} \tag{4.22}$$

$\mathcal{F}_p(u, v; \sigma, \tau)$ has an expansion in the $\mathfrak{so}(8)$ representations appearing in the tensor product $\mathcal{R}_p \otimes \mathcal{R}_p$, where we sometimes employ the shorthand

$$\mathcal{R}_p \equiv [00p0] \tag{4.23}$$

The list of such representations is

$$\begin{aligned} \mathcal{R}_p \otimes \mathcal{R}_p &= \bigoplus_{a=0}^{p} \bigoplus_{b=0}^{a} [0(a-b)(2b)0] \\ &\equiv \bigoplus_{a=0}^{p} \bigoplus_{b=0}^{a} (ab) \end{aligned} \tag{4.24}$$

Representations in the symmetric product have $a + b =$ even, whereas those in the anti-symmetric product have $a + b =$ odd. The contribution of each representation to $\delta\mathcal{F}_p$ is encoded in a harmonic polynomial of $\mathfrak{so}(8)$ which depends on both σ and τ [73, 74]. These polynomials $Y_{ab}(\sigma, \tau)$, associated to the representation (ab), obey an orthogonality condition

$$\iint Y_{ab}(\sigma, \tau) Y_{cd}(\sigma, \tau) \propto \delta_{ac}\delta_{bd} \tag{4.25}$$

with

$$\iint \equiv \int_0^{(1-\sqrt{\tau})^2} d\sigma \int_0^1 d\tau \ \left[(\sigma - 1)^2 + \tau(\tau - 2\sigma - 2)\right]^{3/2} \tag{4.26}$$

Hence the four-point function enjoys the decomposition

$$\mathcal{F}_p(u, v; \sigma, \tau) = \sum_{a,b} Y_{ab}(\sigma, \tau) f_{ab}(u, v) \ , \quad \text{where} \quad \sum_{a,b} \equiv \sum_{a=0}^{p} \sum_{b=0}^{a} \tag{4.27}$$

Crossing symmetry of $\mathcal{F}_p(u, v; \sigma, \tau)$ acts in all four variables. The algorithm for constructing these polynomials, as well as the first several explicit polynomials, can be found in [73, 74]. Of particular importance for what follows are the polynomials associated to $(00) = [0000]$, the singlet, and $(11) = [0020]$, in which $\mathcal{O}_2$ lives:

$$\begin{aligned} Y_{00}(\sigma, \tau) &= 1 \\ Y_{11}(\sigma, \tau) &= \sigma + \tau - \frac{1}{4} \end{aligned} \tag{4.28}$$

In what follows, we will compute $\delta\mathcal{F}_p(u, v; \sigma, \tau)$, the difference between IR and UV connected correlators:

$$\delta\mathcal{F}_p(u,v;\sigma,\tau) \equiv \mathcal{F}_p(u,v;\sigma,\tau)\big|_{\rm IR} - \mathcal{F}_p(u,v;\sigma,\tau)\big|_{\rm UV} \tag{4.29}$$

4.2.2 Change in four-point function

As explained in [18], $\delta\mathcal{F}_p$ is given by a sum of three terms, each of which computes the change in the contribution of $\mathcal{O}_2$ in a given channel. Each contribution carries the R-symmetry polynomial $Y_{11}(\sigma,\tau)$ in its respective channel. This is the only $\mathfrak{so}(8)$ representation that appears in a given channel, because we are taking the difference of the four-point functions at the two fixed points. Combining this global symmetry structure with the explicit result of [18], we find

$$\begin{aligned}\delta\mathcal{F}_p(u,v;\sigma,\tau) = -\frac{16p^2}{\pi^{5/2}C_T}\Big[&Y_{11}(\sigma,\tau)u\bar{D}_{1,1,\frac{1}{2},\frac{1}{2}}(u,v)\\ &+\sigma^p\,Y_{11}(\sigma^{-1},\tau\sigma^{-1})u^{\frac{p}{2}}\bar{D}_{1,\frac{1}{2},1,\frac{1}{2}}(u,v)\\ &+\tau^p\,Y_{11}(\sigma\tau^{-1},\tau^{-1})\left(\frac{u}{v}\right)^{\frac{p}{2}}v\bar{D}_{\frac{1}{2},1,1,\frac{1}{2}}(u,v)\Big]\end{aligned} \tag{4.30}$$

where the $\bar{D}$-function is defined by the integral

$$\int d^dy\prod_{i=1}^4\frac{\Gamma(\Delta_i)}{(x_i-y)^{2\Delta_i}} \overset{\sum\Delta_i=d}{=} \pi^{\frac{d}{2}}\,\frac{x_{14}^{d-2\Delta_1-2\Delta_4}x_{34}^{d-2\Delta_3-2\Delta_4}}{x_{13}^{d-2\Delta_4}x_{24}^{2\Delta_2}}\bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}(u,v) \tag{4.31}$$

and we have used the OPE coefficients (4.19). Each line of (4.30) represents a different OPE channel. In writing this, we have used the obvious transformation properties of σ and τ under permutation of the indices, together with (4.21).[14]

Eq. (4.30) is the complete result. It is useful to project $\delta\mathcal{F}_p$ into a single OPE channel – say, the $12\to34$ channel – by putting it in the form (4.27). This makes it straightforward to extract anomalous dimensions for the double-trace operators $[\mathcal{O}_p\mathcal{O}_p]$. To do so, we project (4.30) onto each representation (ab). Let us define a normalized projection operator $\mathcal{P}_{ab|cd}(p)$, that projects Y_{cd} polynomials in the t-channel onto Y_{ab} polynomials in the s-channel:

$$\mathcal{P}_{ab|cd}(p) \equiv \frac{1}{\mathcal{N}_{ab}}\iint \sigma^p\,Y_{ab}(\sigma,\tau)\,Y_{cd}(\sigma^{-1},\tau\sigma^{-1}) \tag{4.32}$$

where $\mathcal{N}_{ab}$ is the norm,

$$\mathcal{N}_{ab} = \iint Y_{ab}(\sigma,\tau)^2 \tag{4.33}$$

[14] The first line of (4.30) is $Y_{11}(\sigma,\tau)/2$ times the result one would obtain for the same double-trace flow without global symmetries (likewise for the other two channels). The factor of 1/2 comes from contracting the vectors $Y_{1,2,3,4}$ with the tensor structure $\delta_{IK}\delta_{JL}+\delta_{IL}\delta_{JK}-\delta_{IJ}\delta_{KL}/4$ that appears in the two-point function of the Hubbard-Stratanovich field for $\mathcal{O}_2$, and using the normalization (4.28). See [18] for details.

The u-channel projection is identical up to a $(-1)^{a+b}$. Applied to (4.30), this projector acts on the (σ,τ)-dependent parts of the second two lines, with $(cd)=(11)$: one finds

$$\begin{aligned}\delta\mathcal{F}_p(u,v;\sigma,\tau) = -\frac{16p^2}{\pi^{5/2}C_T}\Bigg\{&Y_{11}(\sigma,\tau)u\bar{D}_{1,1,\frac{1}{2},\frac{1}{2}}(u,v)\\ +\sum_{a,b}Y_{ab}(\sigma,\tau)\mathcal{P}_{ab|11}(p)&\left[u^{\frac{p}{2}}\bar{D}_{1,\frac{1}{2},1,\frac{1}{2}}(u,v)+(-1)^{a+b}\left(\frac{u}{v}\right)^{\frac{p}{2}}v\bar{D}_{\frac{1}{2},1,1,\frac{1}{2}}(u,v)\right]\Bigg\}\end{aligned} \tag{4.34}$$

The factor of $(-1)^{a+b}$ indicates whether the (ab) representation appears in the symmetric $(+)$ or anti-symmetric $(-)$ product $\mathcal{R}_p\otimes\mathcal{R}_p$.

4.3 Double-trace anomalous dimensions

The first line of (4.34) represents the exchange of $\mathcal{O}_2$, while the second line of (4.34) represents the exchange of double-trace operators of the schematic form

$$[\mathcal{O}_p\mathcal{O}_p]^{(ab)}_{n,\ell} \simeq \mathcal{O}_p\partial^{2n}\partial_{\mu_1}\ldots\partial_{\mu_\ell}\mathcal{O}_p\Big|_{(ab)\,\subset\,\mathcal{R}_p\otimes\mathcal{R}_p} \tag{4.35}$$

The notation indicates that there exist several families of such operators, one for each $\mathfrak{so}(8)$ representation appearing in the product $\mathcal{R}_p\otimes\mathcal{R}_p$. We denote their total conformal dimension as $\Delta^{(ab)}_{n,\ell}(p)$, and introduce an anomalous dimension

$$\gamma^{(ab)}_{n,\ell}(p) \equiv \Delta^{(ab)}_{n,\ell}(p) - (2\Delta_p + 2n + \ell) \tag{4.36}$$

The $\gamma^{(ab)}_{n,\ell}(p)$ have a $1/C_T$ expansion; we will henceforth take $\gamma^{(ab)}_{n,\ell}(p)$ to be the leading term, of order $1/C_T$, dual to tree-level contributions to the binding energy in AdS. We focus on the leading-twist operators, with $n=0$, and introduce the shorthand $\gamma_\ell\equiv\gamma_{0,\ell}$.

By decomposing the second line of (4.34) into double-trace conformal blocks and working in the $1/C_T$ expansion, we can extract the flow of anomalous dimensions from UV to IR. Define

$$\delta\gamma^{(ab)}_\ell(p) \equiv \gamma^{(ab)}_\ell(p)\big|_{\rm IR} - \gamma^{(ab)}_\ell(p)\big|_{\rm UV}\,. \tag{4.37}$$

In Appendix B, we carry out the remaining steps in the calculation. The result for even p is

$$\boxed{\delta\gamma^{(ab)}_\ell(p) = \delta\gamma^{(ab)}_0(p)\sum_{k=0}^{\frac{p-4}{2}}\frac{c_k(p)}{\ell+\frac{p}{2}+k}} \tag{4.38}$$

where

$$c_k(p) = \frac{(p-2)\left(2-\frac{p}{2}\right)_k\left(\frac{p-1}{2}\right)_k}{(p-3)\left(\frac{5}{2}-\frac{p}{2}\right)_k\left(\frac{p}{2}\right)_k} \tag{4.39}$$

and

$$\delta\gamma_0^{(ab)}(p) = \frac{64p^2}{\pi^2 C_T}\frac{\mathcal{P}_{ab|11}(p)}{\mathcal{P}_{ab|00}(p)} \tag{4.40}$$

where ℓ is even/odd if $a+b$ is even/odd. (One can easily check that the sum on the RHS of (4.38) equals unity for $\ell = 0$.) In Appendix B, we also give the result for odd p, and the explicit functions $\mathcal{P}_{ab|00}(p)$ and $\mathcal{P}_{ab|11}(p)$ for $a, b \leq 4$ and $a+b = \text{even}$. Note that the ratio of $\delta\gamma_\ell^{(ab)}$ for two different spins is completely independent of the $\mathfrak{so}(8)$ representation (ab).

Let us make some comments on signs. By inspection, $c_k(p) > 0$ for all $p \geq 4$, so the sign of $\delta\gamma_\ell^{(ab)}(p)$ is given by the sign of the ratio of projectors. In general, these ratios need not be sign-definite: whereas $\mathcal{P}_{ab|00}(p) > 0$ due to unitarity of mean field theory (see (B.10)), there is no unitarity constraint on $\mathcal{P}_{ab|cd}(p)$ for $(cd) \neq (00)$. Explicit calculation using the projectors in (B.6)-(B.7) does in fact produce both signs for different representations at fixed p.[15] For instance, in the case $p = 4$, for representations appearing in the symmetric product $[\mathcal{R}_4 \otimes \mathcal{R}_4]_{\text{sym}}$, one finds

$$\begin{aligned} &\delta\gamma_0^{(ab)}(p=4) > 0 \ \ \forall \ \ (ab) \in \{(00),(11),(20),(22),(31),(33),(44)\} \\ &\delta\gamma_0^{(ab)}(p=4) < 0 \ \ \forall \ \ (ab) \in \{(40),(42)\} \end{aligned} \tag{4.41}$$

This pattern appears to generalize to $p \neq 4$: the only symmetric representations for which $\delta\gamma_\ell^{(ab)} < 0$ are those with $a = p, b < p$. It would be nice to prove this.

4.3.1 IR dimensions for UV-protected operators

In the product $\mathcal{R}_p \otimes \mathcal{R}_p$, the operators living in representations (ab) with $a = p-1, p$ are protected by SUSY in the UV; all others are unprotected [74]. For this subset of protected representations, our double-trace data is especially interesting: since $\gamma_{n,\ell}^{(ab)}(p)\big|_{\text{UV}} = 0$, the change in anomalous dimension under RG flow equals the IR anomalous dimension.

There are further constraints from $\mathfrak{osp}(8|4)$ representation theory on which composites are protected. In Table 2 we show the relation between internal and spacetime quantum numbers for the protected $\mathfrak{so}(8)$ multiplets in the (ab) representations. Because $\Delta_{n,\ell}^{(ab)}(p) = p + 2n + \ell$ for protected representations, Table 2 implies the following:

- When $a = p$, the protected superconformal primary double-trace operators lie in the B series of BPS representations, with $\ell = 0$.
- When $a = p-1$, the protected superconformal primary double-trace operators lie in the A series of BPS representations, with $b - \ell = \text{even}$.

[15] Both signs are consistent with lightcone bootstrap constraints on large spin anomalous dimensions, due to the non-trivial global symmetry representations involved. See e.g Section 2 of [75] for similar examples of charged correlators, there studied in the $\ell \gg 1$ limit, where an intricate pattern of signs was found.

Class	$\tau_{\mathcal{O}}$	$\ell_{\mathcal{O}}$
A	$a+1$	≥ 0
B	a	0

Table 2: The two classes of BPS multiplets of $\mathfrak{so}(8)$, specified to the $(ab) \equiv [0(a-b)(2b)0]$ representations, along with the twist $\tau_{\mathcal{O}} = \Delta_{\mathcal{O}} - \ell_{\mathcal{O}}$ and Lorentz spin $\ell_{\mathcal{O}}$ of the superconformal primary $\mathcal{O}$. See e.g. [35] or Table 2 of [43]] for further refinement.

- Among superconformal primaries, only the $n = 0$ operators are protected.[16]

Thus, equations (4.38)-(4.40) give analytic formulas for the leading order anomalous dimensions of infinite classes of double-trace operators at the non-SUSY IR fixed point: for the values of (n, ℓ) noted above,

$$\boxed{\delta\gamma^{(ab)}_{n,\ell}(p) = \gamma^{(ab)}_{n,\ell}(p)\Big|_{\rm IR} \quad \text{when} \quad a = p-1 \text{ or } p\,.} \tag{4.42}$$

For example, let us provide the explicit IR dimensions of the infinite class of scalar double trace primaries in the symmetric traceless representation $(pp) = [00(2p)0]$, with $n = 0$:

$$[\mathcal{O}_p\mathcal{O}_p]^{(pp)}_{0,0} \equiv \,:\!\mathcal{O}_p\mathcal{O}_p\!:\Big|_{(pp)} \tag{4.43}$$

In the UV, these operators are superconformal primaries living in a 1/2-BPS B series multiplet, with vanishing anomalous dimension. In the IR, the anomalous dimension $\gamma^{(pp)}_0(p)$, as defined in (4.36), is given in (4.40). By inspection of the projectors through $p = 18$, we find that for this representation, the ratio of projectors appearing in (4.40) is actually p-independent:

$$\frac{\mathcal{P}_{pp|11}(p)}{\mathcal{P}_{pp|00}(p)} = \frac{3}{4}\,. \tag{4.44}$$

This leads to a particularly simple result for the leading-order IR anomalous dimension of (4.43),

$$\gamma^{(pp)}_0(p)\Big|_{\rm IR} = \frac{48p^2}{\pi^2 C_T} \tag{4.45}$$

As another example, let us also, using (B.6)-(B.7), give the explicit IR anomalous dimensions of the leading-twist $p = 4$ spinning double-trace operators,

$$[\mathcal{O}_4\mathcal{O}_4]^{(ab)}_{0,\ell} \equiv \,:\!\mathcal{O}_4\partial_{\mu_1}\ldots\partial_{\mu_\ell}\mathcal{O}_4\!:\Big|_{(ab)}\,, \tag{4.46}$$

[16] These operators have conformal primary descendants, which are also UV-protected; these can be easily enumerated by expanding the supermultiplet operator content. Such conformal primaries may have $0 \leq n \leq 3$, depending on how many supercharges generate the full multiplet. For $n > 0$, there is mixing among double-trace operators of $n' \leq n$ and identical spins.

in the symmetric ($3b$) and ($4b$) representations:

$$\begin{aligned}
\gamma_\ell^{(31)}(4)\Big|_{\rm IR} &= \frac{3328}{3\pi^2 C_T}\frac{1}{\ell+2} \\
\gamma_\ell^{(33)}(4)\Big|_{\rm IR} &= \frac{1536}{\pi^2 C_T}\frac{1}{\ell+2} \\
\gamma_0^{(40)}(4)\Big|_{\rm IR} &= -\frac{512}{\pi^2 C_T} \\
\gamma_0^{(42)}(4)\Big|_{\rm IR} &= -\frac{128}{\pi^2 C_T} \\
\gamma_0^{(44)}(4)\Big|_{\rm IR} &= \frac{768}{\pi^2 C_T}
\end{aligned} \tag{4.47}$$

where $\ell \in 2\mathbb{Z}_{\geq 0}$.

These results for $\gamma_\ell^{(ab)}(p)$ at the IR fixed point are the first analytic computations of anomalous dimensions of finite-spin double-trace operators in any large N CFT$_3$ with an Einstein gravity dual. The only previously known data at finite spin, either analytic or numeric, is a numerical bootstrap estimate in $\mathcal{N} = 8$ ABJM for the $\mathfrak{so}(8)$-singlet operators $[\mathcal{O}_2\mathcal{O}_2]_{0,\ell}$ for $\ell = 0, 2$ [43]. This technique could also be applied to derive anomalous dimensions in IR CFTs obtained by double-trace RG flows that *preserve* a fraction of the UV SUSY: again, certain UV-protected double-trace operators become unprotected in the IR, as determined by the branching rules of the UV superalgebra. The above results are also, to our knowledge, the first calculations of γ_ℓ at finite spin for double-trace operators in non-trivial global symmetry representations, in any large N, large gap CFT in any d. (Large spin results have been obtained using the lightcone bootstrap [75–79].) The interpretation of CFT anomalous dimensions as AdS binding energies has been discussed elsewhere [80–82].

5 Final comments

The proposal we have made, and the specific example involving ABJM, provide a way to construct non-SUSY CFTs with large N and a large gap that appear to obey all necessary CFT consistency conditions. Of course, it is paramount to understand if there is nevertheless an obstruction. It would be enlightening, though challenging, to fully determine the fate of the moduli space of vacua after the RG flow. If instabilities do develop, the conjecture of [5,6] will have passed a novel test; if they do not, a plausible modification of the conjecture is that *all* non-SUSY CFTs with a large gap are obtained by SUSY-breaking RG flows. This is still a radical statement that, if true, would be fascinating from the CFT perspective: in the absence of SUSY, the typical large N, large gap CFT is believed to be complicated, with a highly disordered set of irrational operator dimensions and OPE coefficients and a host of possible sporadic phenomena. On the other hand, CFTs constructed via double-trace flow are highly ordered.

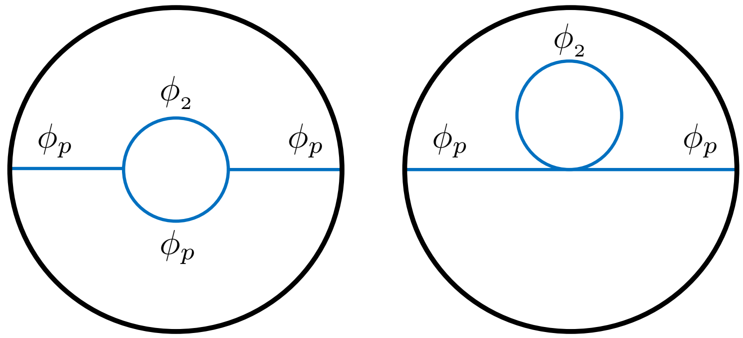

Figure 2: The bulk diagrams needed to determine the leading anomalous dimensions of the single-trace operators $\mathcal{O}_p$ after the ABJM double-trace flow by $\int \mathcal{O}_2^2$.

We have only computed a handful of gauge-invariant observables of the putative IR fixed point obtained by flowing from ABJM, but it is worth exploring its properties further. For instance, one would like to compute the leading-order shift of the single-trace spectrum in the IR, where $\mathcal{O}_p$ may acquire anomalous dimensions. We may do so by an AdS computation of the one-loop correction to the propagators of KK modes ϕ_p on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. The relevant bubble and tadpole diagrams – see Figure 2 – can be computed as explained e.g. in [83]. More precisely, to compute the IR dimensions of $\mathcal{O}_p$, it would be sufficient to compute differences of such diagrams with $\Delta = 1$ and $\Delta = 2$ boundary conditions on ϕ_2. A missing ingredient are the quartic couplings $\phi_2^2\phi_p^2$, which have never been computed. These would also allow computation of the four-point functions in the large N ABJM theory itself, which remains an outstanding problem.[17]

An intriguing question, independent of the concerns of this paper, is whether the RG flow (3.4) survives all the way down to small values of N. Can one reach an analog of the Wilson-Fisher model, endowed with $\mathfrak{so}(8)$ global symmetry, by RG flow from ABJM? Away from large N, the notion of "double-trace flow" is meaningless, but (3.4) can be understood as a fancier version of the typical ϕ^4 deformation, in analogy with the usual construction of the Wilson-Fisher fixed point via RG flow from the free scalar theory. Such a CFT, if it exists, may (but need not) be a non-SUSY Chern-Simons-bifundamental matter theory. Recent studies of non-SUSY Chern-Simons-matter theories have revealed a rich landscape of fixed points and dualities (e.g. [85–89]); it would be interesting to ask whether this landscape accommodates the $\mathfrak{so}(8)$ Wilson-Fisher-type theory described above. One promising approach to this problem may be to ask the conformal bootstrap whether such a theory is allowed to exist, for instance, by generalizing the analysis of [43] to include $\mathfrak{so}(8)$ global symmetry but not SUSY.[18]

[17] In fact, note that the calculation of differences of loop diagrams with $\Delta = 2$ and $\Delta = 1$ boundary conditions on ϕ_2 can be mapped, following [26, 84], to a conformal perturbation theory computation on the CFT side. This includes a contribution proportional to $\int dz_1 dz_2 G_\sigma(z_1, z_2)\langle \mathcal{O}_p(x_1)\mathcal{O}_p(x_2)\mathcal{O}(z_1)\mathcal{O}(z_2)\rangle_{UV}$, which entails knowing the four-point function in the ABJM theory in the UV.

[18] A preliminary problem is to understand how many relevant singlet operators there are in the ABJM theories at some finite N and k. At large but finite N, the answer is two: the finite N continuations of

One might also try to construct SUSY-breaking double-trace flows from large-gap SCFTs in $d = 2$. It behooves us to look for more M-theory examples. A canonical one is M-theory on $\text{AdS}_3 \times S^2 \times CY_3$, whose dual is the MSW CFT with $\mathcal{N} = (0, 4)$ SUSY [90]. This theory remains poorly understood, but the BPS spectrum is known [91–93], and contains no $\Delta < 1$ operators. It would be worthwhile to seek other examples, particularly given the paucity of explicit constructions of large N CFTs in $d = 2$ with sparse spectra.

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A Extremal three-point functions under double-trace flow

Consider the extremal three-point function

$$\langle \Phi(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle = \frac{C_{\Phi\mathcal{O}\mathcal{O}}}{x_{12}^{\Delta_\Phi} x_{13}^{\Delta_\Phi}} \ , \quad \text{where} \ \ \Delta_\Phi = 2\Delta_{\mathcal{O}} \tag{A.1}$$

We now perturb the CFT by $\int \mathcal{O}^2$. To leading order in $1/N$, in the IR we take $\mathcal{O} \to \sigma$ inside correlation functions, whereupon we must compute the "triangle diagram" with two σ legs; see Figure 3. σ has a two-point function [38, 94]

$$\langle \sigma(x_1)\sigma(x_2)\rangle = \frac{(\frac{d}{2} - \Delta_{\mathcal{O}}) \sin\left((\frac{d}{2} - \Delta_{\mathcal{O}})\pi\right) \Gamma\left(d - \Delta_{\mathcal{O}}\right) \Gamma\left(\Delta_{\mathcal{O}}\right)}{\pi^{d+1} C_{\mathcal{O}\mathcal{O}} x_{12}^{2(d-\Delta_{\mathcal{O}})}} \equiv \frac{C_\sigma}{x_{12}^{2(d-\Delta_{\mathcal{O}})}} \tag{A.2}$$

the double- and triple-trace operators $[\mathcal{O}_2\mathcal{O}_2]$ and $[\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2]$ projected onto the R-symmetry singlet. As we decrease N further, singlet single-trace operators such as $\text{Tr}(X_I X_I)$, the ABJM analog of the Konishi operator, re-enter the spectrum. For $k = 1, 2$, it should be possible to extend the $\mathcal{N} = 8$ numerical bootstrap methods of [43] to determine the number of relevant operators.

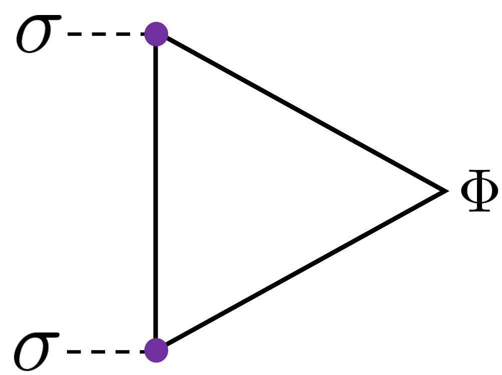

Figure 3: The triangle diagram determines the three-point coupling $\langle \Phi\sigma\sigma\rangle$ in the IR, to which the UV coupling $\langle \Phi\mathcal{O}\mathcal{O}\rangle$ flows. The purple points are integrated over.

where $x_{ij} \equiv |x_i - x_j|$, and

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \frac{C_{\mathcal{O}\mathcal{O}}}{x_{12}^{2\Delta_{\mathcal{O}}}} \tag{A.3}$$

The necessary integral is

$$\begin{aligned}\langle \Phi(x_1)\sigma(x_2)\sigma(x_3)\rangle_{IR} &= \int d^dx_4 \int d^dx_5 \frac{C_\sigma}{x_{24}^{2(d-\Delta_{\mathcal{O}})}} \frac{C_\sigma}{x_{35}^{2(d-\Delta_{\mathcal{O}})}} \langle \Phi(x_1)\mathcal{O}(x_4)\mathcal{O}(x_5)\rangle_{UV} + \dots \\ &= \int d^dx_4 \int d^dx_5 \frac{C_\sigma}{x_{24}^{2(d-\Delta_{\mathcal{O}})}} \frac{C_\sigma}{x_{35}^{2(d-\Delta_{\mathcal{O}})}} \frac{C^{UV}_{\Phi\mathcal{O}\mathcal{O}}}{x_{14}^{\Delta_\Phi} x_{15}^{\Delta_\Phi}} + \dots\end{aligned} \tag{A.4}$$

where $\dots$ denotes higher orders in $1/N$, and we have used conformal symmetry to go from the first to the second line. Two applications of the conformal integral

$$\int \frac{d^dx_4}{x_{14}^{2\Delta_1} x_{24}^{2\Delta_2} x_{34}^{2\Delta_3}} \overset{\sum \Delta_i = d}{=} \frac{\pi^{\frac{d}{2}} a(\Delta_1)a(\Delta_2)a(\Delta_3)}{x_{12}^{d-2\Delta_3} x_{23}^{d-2\Delta_1} x_{31}^{d-2\Delta_2}}\,, \tag{A.5}$$

where

$$a(\Delta_i) \equiv \frac{\Gamma(d/2 - \Delta_i)}{\Gamma(\Delta_i)}\,, \tag{A.6}$$

lead to the result (4.5) quoted in the text. One concludes that UV-extremal correlators involving $\mathcal{O}$ vanish in the IR. Note that the reverse is also true: if the correlator is not extremal in the UV, but becomes extremal in the IR – that is, if $\Delta_\Phi = 2(d - \Delta_{\mathcal{O}})$ – the numerator of the last factor in (4.5) blows up; this gives a finite result only if $C^{UV}_{\Phi\mathcal{O}\mathcal{O}} = 0$.

B Double-trace computations and $\mathfrak{so}(8)$ group theory

B.1 $\mathfrak{so}(8)$ projectors

In (4.32), we introduced the projector $\mathcal{P}_{ab|cd}(p)$, whose definition we recall here:

$$\mathcal{P}_{ab|cd}(p) \equiv \frac{1}{\mathcal{N}_{ab}} \iint \sigma^p \, Y_{ab}(\sigma,\tau) Y_{cd}(\sigma^{-1}, \tau\sigma^{-1}) \tag{B.1}$$

where

$$\iint \equiv \int_0^{(1-\sqrt{\tau})^2} d\sigma \int_0^1 d\tau \, \left[(\sigma-1)^2 + \tau(\tau - 2\sigma - 2)\right]^{3/2} \tag{B.2}$$

and

$$\mathcal{N}_{ab} \equiv \iint Y_{ab}(\sigma,\tau)^2 \tag{B.3}$$

This projects a t-channel exchange in the (cd) representation of $\mathfrak{so}(8)$ of the correlator $\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_p \mathcal{O}_p \rangle$, where $\Delta_p = p/2$, onto the s-channel. The identical formula, up to an overall $(-1)^{a+b}$, holds for projection of a u-channel term onto the s-channel. A nice exposition of the polynomials $Y_{ab}(\sigma,\tau)$, and a list of those with $a \leq 3$, is given in Appendix B of [73]. For what follows, we will also need

$$\begin{aligned}
Y_{40}(\sigma,\tau) &= \left(\sigma^4 - 4\sigma^3\tau + 6\sigma^2\tau^2 - 4\sigma\tau^3 + \tau^4\right) - \frac{6}{5}\left(\sigma^3 - \sigma^2\tau - \sigma\tau^2 + \tau^3\right) \\
&\quad + \frac{3}{110}\left(17\sigma^2 - 12\sigma\tau + 17\tau^2\right) - \frac{3}{55}(\sigma+\tau) + \frac{1}{330} \\
Y_{42}(\sigma,\tau) &= \left(\sigma^4 + 2\sigma^3\tau - 6\sigma^2\tau^2 + 2\sigma\tau^3 + \tau^4\right) + \frac{1}{4}\left(-5\sigma^3 - 3\sigma^2\tau - 3\sigma\tau^2 - 5\tau^3\right) \\
&\quad + \frac{3}{44}\left(7\sigma^2 + 6\sigma\tau + 7\tau^2\right) + -\frac{3}{44}(\sigma+\tau) + \frac{1}{308} \\
Y_{44}(\sigma,\tau) &= \left(\sigma^4 + 16\sigma^3\tau + 36\sigma^2\tau^2 + 16\sigma\tau^3 + \tau^4\right) - \frac{8}{5}\left(\sigma^3 + 9\sigma^2\tau + 9\sigma\tau^2 + \tau^3\right) \\
&\quad + \frac{4}{5}\left(\sigma^2 + 4\sigma\tau + \tau^2\right) + -\frac{2}{15}(\sigma+\tau) + \frac{1}{210}
\end{aligned} \tag{B.4}$$

which can be derived from [73].

The projector with $(cd) = (00)$ is relevant for the conformal block decomposition of $\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_p \mathcal{O}_p \rangle$ in mean field theory, which is a sum over channels of identity exchange:

$$\begin{aligned}
\mathcal{F}_p^{MFT}(u,v;\sigma,\tau) &= 1 + (\sigma u)^p + \left(\tau \frac{u}{v}\right)^p \\
&= 1 + \sum_{a,b} Y_{ab}(\sigma,\tau) \mathcal{P}_{ab|00}(p) \left[u^p + (-1)^{a+b}\left(\frac{u}{v}\right)^p\right]
\end{aligned} \tag{B.5}$$

The first several low-lying representations (ab) in the symmetric product $[\mathcal{R}_p \otimes \mathcal{R}_p]_{\rm sym}$ are

$$
\begin{aligned}
\mathcal{P}_{00|00}(p) &= \frac{360}{(p+1)(p+2)(p+3)^2(p+4)(p+5)} \\
\mathcal{P}_{11|00}(p) &= \frac{16800p}{(p+1)(p+3)^2(p+4)^2(p+5)(p+6)} \\
\mathcal{P}_{20|00}(p) &= \frac{151200(p-1)p}{(p+1)(p+2)(p+3)^2(p+4)(p+5)(p+6)(p+7)} \\
\mathcal{P}_{22|00}(p) &= \frac{264600(p-1)p}{(p+3)^2(p+4)^2(p+5)^2(p+6)(p+7)} \\
\mathcal{P}_{31|00}(p) &= \frac{4191264(p-2)(p-1)p}{(p+1)(p+3)^2(p+4)^2(p+5)(p+6)(p+7)(p+8)} \\
\mathcal{P}_{33|00}(p) &= \frac{2794176(p-2)(p-1)p^2}{(p+3)^2(p+4)^2(p+5)^2(p+6)^2(p+7)(p+8)} \\
\mathcal{P}_{40|00}(p) &= \frac{15135120(p-3)(p-2)(p-1)p}{(p+1)(p+2)(p+3)^2(p+4)(p+5)(p+6)(p+7)(p+8)(p+9)} \\
\mathcal{P}_{42|00}(p) &= \frac{51891840(p-3)(p-2)(p-1)p}{(p+3)^2(p+4)^2(p+5)^2(p+6)(p+7)(p+8)(p+9)} \\
\mathcal{P}_{44|00}(p) &= \frac{23783760(p-3)(p-2)(p-1)^2p^2}{(p+3)^2(p+4)^2(p+5)^2(p+6)^2(p+7)^2(p+8)(p+9)}
\end{aligned}
\tag{B.6}
$$

The projector with $(cd) = (11)$ was needed for the conformal block decomposition of the change in $\langle \mathcal{O}_p\mathcal{O}_p\mathcal{O}_p\mathcal{O}_p\rangle$ after double-trace flow triggered by $\int \mathcal{O}_2^2$, where $\mathcal{O}_2$ sits in the (11) representation. The first several low-lying representations (ab) in the symmetric product

$[\mathcal{R}_p \otimes \mathcal{R}_p]_{\text{sym}}$ are

$$\begin{aligned}
\mathcal{P}_{00|11}(p) &= \frac{270(p+6)}{p(p+1)(p+2)^2(p+3)^2(p+5)} \\
\mathcal{P}_{11|11}(p) &= \frac{4200\left(3p^4+36p^3+100p^2-48p-64\right)}{p(p+1)(p+2)^2(p+3)^2(p+4)^2(p+5)(p+6)} \\
\mathcal{P}_{20|11}(p) &= \frac{37800(p-1)\left(3p^2+18p-56\right)}{p(p+1)(p+2)^2(p+3)^2(p+5)(p+6)(p+7)} \\
\mathcal{P}_{22|11}(p) &= \frac{66150(p-1)\left(3p^4+36p^3+52p^2-336p+320\right)}{p(p+2)^2(p+3)^2(p+4)^2(p+5)^2(p+6)(p+7)} \\
\mathcal{P}_{31|11}(p) &= \frac{1047816(p-2)(p-1)\left(3p^4+36p^3+28p^2-480p-352\right)}{p(p+1)(p+2)^2(p+3)^2(p+4)^2(p+5)(p+6)(p+7)(p+8)} \\
\mathcal{P}_{33|11}(p) &= \frac{2095632(p-2)(p-1)\left(p^4+12p^3-4p^2-240p+576\right)}{(p+2)^2(p+3)^2(p+4)^2(p+5)^2(p+6)^2(p+7)(p+8)} \\
\mathcal{P}_{40|11}(p) &= \frac{11351340(p-3)(p-2)(p-1)\left(p^2+6p-48\right)}{p(p+1)(p+2)^2(p+3)^2(p+5)(p+6)(p+7)(p+8)(p+9)} \\
\mathcal{P}_{42|11}(p) &= \frac{38918880(p-3)(p-2)(p-1)\left(p^4+12p^3-12p^2-288p+224\right)}{p(p+2)^2(p+3)^2(p+4)^2(p+5)^2(p+6)(p+7)(p+8)(p+9)} \\
\mathcal{P}_{44|11}(p) &= \frac{5945940(p-3)(p-2)(p-1)^2\left(3p^4+36p^3-92p^2-1200p+4928\right)}{(p+2)^2(p+3)^2(p+4)^2(p+5)^2(p+6)^2(p+7)^2(p+8)(p+9)}
\end{aligned} \tag{B.7}$$

For both $(cd) = (00)$ and (11), one check on these functions are the zeroes at $p = 1, 2, 3$: these reflect, correctly, the absence of the (ab) representations in the product $\mathcal{R}_p \otimes \mathcal{R}_p$ for $p < a$. Note the universal behavior of these projectors at large p, where $\sim 1/p^6$; in particular, their ratio goes to a constant.

The above data are sufficient, using (4.38)-(4.40), to compute $\delta\gamma_\ell^{(ab)}(p)$, the change under RG flow of anomalous dimensions of $[\mathcal{O}_p\mathcal{O}_p]_{0,\ell}^{(ab)}$, for all representations appearing in the symmetric product $[\mathcal{R}_p \otimes \mathcal{R}_p]_{\text{sym}}$ with $a \leq 4$. For $p = 4$, this is the full set of symmetric representations; moreover, for $p = 4$ and $a = 3, 4$, these operators are protected in the UV, so $\delta\gamma_\ell^{(ab)}(4)$ equals the IR anomalous dimensions, as explained in Section 4.3.1.

B.2 Computing $\delta\gamma_\ell^{(ab)}(p)$

The starting point for this calculation is (4.34). The strategy is to decompose it into conformal blocks, picking off the terms that contain the anomalous dimensions and applying the results of [18].

The second line of (4.34) contains the double-trace exchanges, which have twist $\tau = p+2n$ at infinite N. This can be inferred from the leading power of u: at $u \ll 1$, a conformal family whose primary has twist τ contributes terms of order $u^{\tau/2}$ times positive integer powers. Each power of the anomalous dimension comes with a power of $\log u$. Putting these facts together, we must solve the equations

$$
\begin{aligned}
-\frac{16p^2}{\pi^{5/2}C_T}\sum_{a,b} Y_{ab}(\sigma,\tau)\mathcal{P}_{ab|11}(p)&\left[u^{\frac{p}{2}}\bar{D}_{1,\frac{1}{2},1,\frac{1}{2}}(u,v)+(-1)^{a+b}\left(\frac{u}{v}\right)^{\frac{p}{2}} v\bar{D}_{\frac{1}{2},1,1,\frac{1}{2}}(u,v)\right]_{\log u} \\
&=\sum_{a,b} Y_{ab}(\sigma,\tau)\sum_{n=0}^{\infty} u^{\frac{p}{2}+n}\sum_{\ell=0}^{\infty}\frac{1}{2}\widetilde{a}^{(ab)}_{n,\ell}(p)\delta\gamma^{(ab)}_{n,\ell}(p)\, g_{p+2n,\ell}(u,v)
\end{aligned}
\tag{B.8}
$$

We have employed the notation $\widetilde{a}^{(ab)}_{n,\ell}(p)$ for the squared OPE coefficients of MFT, and written the conformal block as

$$G_{\tau,\ell}(u,v) = u^{\tau/2} g_{\tau,\ell}(u,v) \tag{B.9}$$

Let us first compute the MFT OPE coefficients $\widetilde{a}^{(ab)}_{n,\ell}(p)$. It follows from (B.5) that they are simply those of ordinary MFT in $d=3$, in the absence of any global symmetry – call them $a^{(0)}_{n,\ell}(p)$ – times the $\mathcal{P}_{ab|00}(p)$ factors:

$$\widetilde{a}^{(ab)}_{n,\ell} = (1+(-1)^{\ell+a+b})\mathcal{P}_{ab|00}(p)\, a^{(0)}_{n,\ell}(p) \tag{B.10}$$

where [95]

$$a^{(0)}_{n,\ell}(p) = \frac{\left(\frac{p-1}{2}\right)_n^2\left(\frac{p}{2}\right)_{\ell+n}^2}{\ell!n!\left(\ell+\frac{3}{2}\right)_n (p-2+n)_n(p+2n+\ell-1)_\ell\left(p+n+\ell-\frac{3}{2}\right)_n} \tag{B.11}$$

For (ab) in the symmetric/anti-symmetric product of $\mathcal{R}_p\otimes\mathcal{R}_p$, only even/odd ℓ double-trace operators are exchanged.

Having computed the MFT result, we return to (B.8). We focus on the leading twist operators, with $n=0$, henceforth, and use the shorthand $\gamma_\ell\equiv\gamma_{0,\ell}$ and $a_{0,\ell}=a_\ell$. This allows us to utilize the lightcone limit,

$$u\ll 1\ ,\quad v \text{ fixed}\ . \tag{B.12}$$

In this limit,

$$g_{p,\ell}(u\ll 1,v)\approx u^{\frac{p}{2}} g^{coll}_\ell(v) \tag{B.13}$$

where

$$g^{coll}_\ell(v)\equiv {}_2F_1\left(\frac{p}{2}+\ell,\frac{p}{2}+\ell,p+2\ell;1-v\right) \tag{B.14}$$

is the colinear, or lightcone, block. Therefore, for every (ab), we must solve

$$
\begin{aligned}
-\frac{16p^2}{\pi^{5/2}C_T}\frac{\mathcal{P}_{ab|11}(p)}{\mathcal{P}_{ab|00}(p)}&\left[\bar{D}_{1,\frac{1}{2},1,\frac{1}{2}}(u,v)+(-1)^{a+b}\left(\frac{1}{v}\right)^{\frac{p}{2}} v\bar{D}_{\frac{1}{2},1,1,\frac{1}{2}}(u,v)\right]_{\log u} \\
&=\sum_{\ell=0}^{\infty}\frac{1}{2}(1+(-1)^{\ell+a+b})a^{(0)}_\ell\delta\gamma^{(ab)}_\ell(p) g^{coll}_\ell(v)
\end{aligned}
\tag{B.15}
$$

This is essentially identical to the same problem in the case where $\mathcal{O}_p$ is uncharged under global symmetry – in particular, it is clear that $\delta\gamma_\ell^{(ab)}(p)$ is simply the uncharged result, multiplied by the ratio of projectors. The uncharged problem was solved in [18], using the fact that $g_\ell^{coll}(v)$ obeys a simple orthogonality condition. The result is

$$\delta\gamma_\ell^{(ab)}(p) = \left(\frac{64p^2}{\pi^2 C_T}\frac{\mathcal{P}_{ab|11}(p)}{\mathcal{P}_{ab|00}(p)}\right) {}_4F_3\left(\begin{matrix} -\ell, 1, \frac{1}{2}, p+\ell-1 \\ \frac{3}{2}, \frac{p}{2}, \frac{p}{2} \end{matrix}\Big| 1\right) \tag{B.16}$$

where ℓ is even/odd if $a+b$ is even/odd. This result is valid for any $p \in \mathbb{Z}_{>0}$. Moreover, it turns out that for $p \in 2\mathbb{Z}$ and $\ell \in \mathbb{Z}_{\geq 0}$, this function simplifies tremendously:

$${}_4F_3\left(\begin{matrix} -\ell, 1, \frac{1}{2}, p+\ell-1 \\ \frac{3}{2}, \frac{p}{2}, \frac{p}{2} \end{matrix}\Big| 1\right) = \sum_{k=0}^{\frac{p-4}{2}} \frac{c_k}{\ell + \frac{p}{2} + k} \quad (p = 4, 6, 8, \ldots) \tag{B.17}$$

where

$$c_k(p) = \frac{(p-2)\left(2-\frac{p}{2}\right)_k \left(\frac{p-1}{2}\right)_k}{(p-3)\left(\frac{5}{2}-\frac{p}{2}\right)_k \left(\frac{p}{2}\right)_k} \tag{B.18}$$

This is the result (4.38)–(4.39) quoted in the main text.

C Contrast with SUSY-breaking $\mathcal{N} = 4$ SYM orbifolds

For context, we give a brief historical overview of one of the most well-studied non-SUSY constructions, namely, the $\mathrm{AdS}_5 \times S^5/\Gamma$ orbifolds, dual to orbifolds of 4d $\mathcal{N} = 4$ SYM [96, 97]. Non-freely-acting orbifolds $\Gamma = \mathbb{Z}_k$ have AdS tachyons; explicit CFT calculations at weak coupling revealed the existence of an unstable effective potential [32, 58], and, later, nonzero beta functions for double-trace operators comprised of twisted sector single-trace operators [48, 49]. The AdS tachyons were conjectured to be the strongly coupled avatars of these weak coupling phenomena [32]. The freely-acting orbifolds have no tachyons in AdS, but do suffer from a non-perturbative instability [33]; though an initial weak coupling calculation [32] revealed no apparent instability of the effective potential, these theories were nevertheless shown to break conformality [48, 49]. The field theory picture was tied together by [50], who showed that along any fixed line of $d = 4$ CFTs with adjoint fields, and at any value of the marginal coupling, the CFT develops nonzero beta functions if and only if the perturbative vacuum at the origin of moduli space is unstable. No such examples survived further scrutiny. The orbifold studies were extended in [13] to $\mathrm{AdS}_4 \times S^7/\Gamma$, and to the "skew-whiffed" $\mathrm{AdS}_4 \times S^7$ background [10], where global singlet marginal operators were found in both cases; this is likely to imply a breaking of conformality, but deserves further study.

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