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ELECTIONS WITH LIMITED INFORMATION: A FULFILLED
EXPECTATIONS MODEL USING CONTEMPORANEOUS POLL
AND ENDORSEMENT DATA AS INFORMATION SOURCES

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ABSTRACT

This paper is one of several papers in which we develop and test models of 2 candidate elections under extremely decentralized and incomplete information conditions. We assume candidates do not know voter utility functions, and that most voters do not observe the policy positions adopted by the candidates. We assume that uninformed actors (voters and candidates alike) have "beliefs" about parameters of which they are uninformed, and that they attempt to inform these beliefs on the basis of readily observable variables endogenous to the system. Specifically, in this paper, we assume that uninformed actors inform their beliefs, and hence condition their behavior, on the basis of contemporaneous poll and (binary) endorsement data. An equilibrium is defined to be a set of strategies, together with a set of beliefs, such that all actors are maximizing expected utility subject to their beliefs, and such that no actor wants to revise his beliefs conditional on the information he does observe. This paper develops the above model only for the case of a one dimensional policy space with symmetric single peaked preferences.

When the electorate is modeled as being infinite, with the cumulative density of ideal points for both informed and uninformed voters being invertible, we show that regardless of the number of informed voters, in an equilibrium, the candidates behave exactly as if all voters have complete information. They respond to the preferences of the uninformed as well as the informed voters, ending

up at the median ideal point of the entire electorate. Further, we show that regardless of candidate behavior, if voters are in equilibrium, their votes will extract all available information, in the sense that all voters, informed and uninformed alike, will vote as if they had perfect information about candidate positions. Finally, we give a dynamic for convergence of voting behavior, which shows that the model implies a "bandwagon" effect, with the speed of convergence depending on the ratio of the density of informed to uninformed voters at the true candidate midpoint.

In addition to the theoretical results, we run some experiments to test the implications of the model. The experiments show a moderate degree of support for the model.

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I. INTRODUCTION

In the last 30 years or so, considerable effort has been expended at attempting to develop a formal theory of political systems and processes based on the economic paradigm of rational choice. Labeled variously positive political theory, public choice, or social choice, this effort encompasses a broad area of study, including spatial election models, coalition processes, voting rules and agenda manipulation. (See Riker and Ordeshook [1972] for a review of the early work in this area and Shepsle [1979], Kramer [1977], McKelvey et al [1978] for a sampling of the recent directions of this literature). Generally, however, the models and theories that form the component parts of this effort are subject to a common and compelling criticism—they assume that political actors such as voters, candidates, legislators, etc., possess a level of knowledge of other voters' preferences, candidate positions and the like that empirical investigation does not support. Thus, in these models, it is supposed typically that candidates adopt well defined positions on all issues and that voters know these positions and the issues (at least up to

some well defined probability measure)—despite the well documented empirical fact that voters oftentimes do not even know the names of the candidates, much less the policies they espouse (cf Berelson et al [1954], Almond and Verba [1963], Converse [1975], and for an up to date review of this literature, Kinder and Sears [1982]). The effort of this paper can be thought of as an attempt to bring the informational assumptions of such models more in line with what we know empirically.

This paper is one in a series of papers in which we study election processes under limited and decentralized information conditions. (See also McKelvey and Ordeshook [1983a, 1983b, 1983c]). Specifically, in this and other papers, we develop models of policy formation in two candidate elections where most voters have little or no information about the policies or platforms adopted by the candidates, and where candidates have little or no information about the voter preference functions.

The key to understanding and modeling systems in which participants have limited access to information seems to us to be related to the ideas that have recently been applied successfully to similar situations in economics. When voters do not possess the perfect information assumed in earlier models, and when it is costly to obtain this information relative to the presumed expected benefits, we assume that voters take cues from other sources, endogenous in the system, that are easily observable and which they believe may convey useful information. Such sources may be other voters, interest

groups, historical behavior of the candidates, or poll results. Regardless of the source, we assume voters will condition their choices on such "low cost" data. Candidates, too, may condition their actions on such data. Thus, there are variables, endogenous in the system, which carry information to the uninformed participants. When actors condition their behavior on the information from these endogenous sources, this, in itself will change the observed values of some of the endogenous variables. The system is in equilibrium only if all participants are acting optimally given available information, and further if the information generated when participants act in such a fashion does not change--i.e. it is stable, and consistent with this optimization behavior of all participants.

Several questions of considerable theoretical interest can be addressed by models of this sort. The most important and interesting of these questions is the extent to which the equilibria of systems with limited information correspond to the equilibria of systems with full information. That is, do the policy outcomes correspond to the outcomes that would prevail were all participants to have full information? As we show, in this and other papers, such a correspondence can frequently be established.

The model developed in this paper assumes the information source for uninformed voters is poll data and interest group endorsements. Our model is a model of a single election, so no historical information is available. There are two classes of participants: voters and candidates. However, the voters are further

partitioned into informed and uninformed voters. All voters have single peaked preferences over a one dimensional issue space, X . Strategies available to candidates are to adopt positions in the policy space, and strategies available to voters are to vote for one candidate or the other. The candidates do not know voter utility functions, and the uninformed voters do not know the candidate positions. The only source of information for the uninformed agents is "interest group endorsement" information and the results of a "Gallup poll" of all voters. In addition, uninformed voters know where their ideal point is in the distribution of total ideal points--i.e. they know how liberal or conservative they are with respect to the remaining population.

In this model one might expect that the informed voters, by virtue of their better information, would have a disproportionate impact on the final outcome. On the contrary, the informed voters by acting on their superior information, end up revealing it, (through the poll and endorsement data) to others, so that in equilibrium, we obtain the result described above: The equilibrium extracts all choice relevant information so that all voters vote as if they had full information. The policies adopted by the candidate in equilibrium reflect the preferences of the uninformed as well as the informed voters.

Our approach in this and companion papers parallels the development of rational expectations models in economics. (cf, Muth [1961], Lucas [1972], Radner [1972, 1979] and Grossman [1978, 1982]

for development and references to some of this extensive literature). In that literature, the actors are buyers and sellers, and the information that is of concern is the future state of the world—a state that effects the future market value of the commodity being traded. The question addressed by the rational expectations literature is what will happen to the market price of a commodity when only a few specialized participants, called insiders, have information regarding the future state of the world. In those models, agents are able to condition their choices, and derive information from endogenous variables such as the price or historical market data. The principal result to emerge is called the efficient markets hypothesis, which asserts that the market will behave as if everyone had information, since the relevant information about the state of the world is itself conveyed to the other participants through the price.

This paper shows how the rational expectations view can be applied to models of political processes. The correspondence between the models we develop and the rational expectations models in economics is that in our models interest groups or informed voters perform the same function as the insiders in the rational expectation market models: their choices provide signals to the other participants (voters and candidates alike) that convey information about the relevant properties of the election system. Namely, they convey some sketchy, but useful information about the relative positions of the candidates. Further, an election or poll outcome serves the same role as the price in the market models. Just as the

price elicits demand information in market models, so the election outcome conveys information to the candidates about the preferences of the voters. In the model developed here, the true distribution of voter preferences is, to the candidates, the unknown state of the world, so the price conveys information about the true state of the world. Only when no participant wants to change his behavior given the information that is being revealed by the behavior of the interest groups and the outcome of the election can the system be in equilibrium. This is the type of equilibrium we search for in these models.

The organization of the paper is as follows: Section 2 develops the basic model and defines the equilibrium notion used here. Section 3 presents the main results, which prove existence of and derive properties of equilibria to our model. Section 4 specifies a dynamic "tatonnement" process, corresponding to a series of successive polls, which converges to a voter equilibrium for any fixed candidate positions. Finally, in addition to the theoretical work, we design and run some experiments intended to test certain implications of the model. Section 5 reports on these experimental results.

2. The Formal Development

We are given a measure space $\Omega = (\Omega, \mathcal{F}, \mu)$ of voters, a set $K = \{1, 2\}$ of candidates, and a closed convex set $X \subseteq \mathbb{R}$ of alternatives. Voters in Ω are partitioned into two classes, I and U , which are referred to as informed and uninformed voters, respectively.

There is a function $y^*: \Omega \rightarrow \mathbb{R}$ which assigns to each voter, α , a characteristic, $y^*(\alpha)$, referred to as voter α 's ideal point. We write $y_\alpha^* = y^*(\alpha)$. All voters then have utility functions $u_\alpha: X \rightarrow \mathbb{R}$ which are assumed to be of the form

$$u_\alpha(x) = u(x, y_\alpha^*) = -|x - y_\alpha^*| \quad (2.1)$$

for all $x \in X$.

We now define a game, with players consisting of the voters and candidates. The strategy sets are

$$\text{Voter } \alpha: B_\alpha = K_0 = K \cup \{0\} \quad (2.2)$$

$$\text{Candidate } k: S_k = X.$$

We let $\underline{B}$ denote the set of measurable functions from Ω into K_0 .

Elements of $\underline{B}$ are denoted b , with $b_\alpha = b(\alpha) \in B_\alpha$ representing the choice of strategy by $\alpha \in \Omega$. Similarly, let $\underline{S}$ denote the set of functions from K to X . Elements of $\underline{S}$ are denoted s , with

$s_k = s(k) \in S_k$ representing the choice of strategy by candidate k . We call b_α voter α 's ballot, with $b_\alpha = k$ representing a vote for candidate k if $k \in K$, and an abstention if $k = 0$. We call s_k candidate k 's policy position. For any $b \in \underline{B}$, $\alpha \in \Omega$, and $b'_\alpha \in B_\alpha$, we use the notation b/b'_α to denote the ballot which results when we replace b_α by b'_α in b . Similarly, for $s \in \underline{S}$, $s_k \in S_k$, the notation s/s'_k denotes the strategy pair resulting when we replace s_k by s'_k in s .

A choice of strategies $(s, b) \in \underline{S} \times \underline{B}$ by all players yields a vote, in any measurable $C \subseteq \Omega$ of

$$v_k^C(b) = \mu(\{\alpha \in C | b_\alpha = k\}) \quad (2.3)$$

for each $k \in K_0$. Thus for $k \in K$, $v_k^C(b)$ represents the vote in C for candidate k , and for $k = 0$, it represents the total abstentions in C .

We write $v_k(b) = v_k^\Omega(b)$ for the total vote for k , and $v_k^{-\alpha}(b) = v_k^{\Omega - \{\alpha\}}(b)$ for the vote of all voters except α .

The election outcome is then

$$k(b) = \begin{cases} k & \text{if } v_k(b) > v_{\bar{k}}(b), \text{ and } k \in K \\ 0 & \text{otherwise} \end{cases} \quad (2.4)$$

I.e., the candidate with the greatest number of votes wins unless there is a tie. If there is a tie--i.e., $k(b) = 0$, we assume below that a coin is flipped).

We pick $\delta \in \mathbb{R}$ with $\delta \geq 0$. The payoff functions,

$M_\alpha: \underline{S} \times \underline{B} \rightarrow \mathbb{R}$ and $M_k: \underline{S} \times \underline{B} \rightarrow \mathbb{R}$, to voters and candidates, are now defined by:

$$M_\alpha(s, b) = u_\alpha(s_{k(b)}) + \delta u_\alpha(s_{b_\alpha}) \quad (2.5)$$

and

$$M_k(s, b) = \begin{cases} 1 & \text{if } k(b) = k \\ -1 & \text{if } k(b) = \bar{k} \\ 0 & \text{otherwise} \end{cases} \quad (2.6)$$

for all $\alpha \in \Omega$, $k \in K$, and $(s, b) \in \underline{S} \times \underline{B}$. We use the convention that

$$u_\alpha(s_0) = \frac{1}{2}u_\alpha(s_1) + \frac{1}{2}u_\alpha(s_2).$$

So the payoff to the voter consists of the utility he receives for the policy position of the winning candidate plus some small increment of utility of the candidate he votes for. The candidates

care only about winning. The reason for the second term in (2.5) is to deal with the non atomic case, where $\mu(\{a\}) = 0$. In order to insure that, in equilibrium, such voters have positive incentives to vote for their preferred candidate, the second term must be added. For the finite voter case, it is immaterial if we set $\delta = 0$ or $\delta > 0$.

In addition to the above, rather standard, development, we introduce some structure to allow us to model the imperfect information. Specifically, each player has beliefs over certain parameters of the above model which he does not observe. We define $\underline{Y} \subseteq \underline{Y}^0 = \{y|y: \Omega \rightarrow \mathbb{R} \text{ such that } y \text{ is } \underline{F} \text{ measurable}\}$ to be a set of possible assignments of ideal points to voters. For any measurable set $C \subseteq \Omega$, $x \in \mathbb{R}$ and $y \in \underline{Y}$ we use the notation

$$\begin{aligned} F_C(x|y) &= \mu(\{a \in C | y_a \leq x\}) \\ G_C(x|y) &= \mu(\{a \in C | y_a \geq x\}) \\ E_C(x|y) &= \mu(\{a \in C | y_a = x\}) \end{aligned} \quad (2.7)$$

Thus, $F_C(x|y)$ is the cumulative density function of y in C , while $G_C(x|y)$ is the "reverse" cumulative density function. We write $F_C(x|y^*) = F_C(x)$, and $F_\Omega(x|y) = F(x|y)$ so $F(x) = F_\Omega(x|y^*)$ is the cumulative density function of the true ideal points, y_a^* . Also, we write $F^{-a}(x)$ for $F_{\Omega-\{a\}}(x)$. Similar notation is used for G and E .

We now define the belief spaces for voters and candidates by

$$\text{Voter } a: \Lambda_a = \tilde{\underline{S}} \quad (2.8)$$

$$\text{Candidate } k: \Gamma_k = \tilde{\underline{Y}}$$

Here, $\tilde{\underline{S}}$ denotes the set of probability measures over the Borel sets of $\underline{S}$, and $\tilde{\underline{Y}}$ denotes the set of probability measures over some σ algebra of $\underline{Y}$. (We assume sets of the form $\{y \in \underline{Y} | F_C(x|y) \geq t\}$ and $\{y \in \underline{Y} | G_C(x|y) \geq t\}$ are $\underline{Y}$ measurable for all $x, t \in \mathbb{R}, C \in \underline{F}$.) Thus, voters have beliefs about candidate policy positions, while candidates have beliefs about voter ideal points. A belief is simply a probability measure over the relevant space.

We let Δ be the set of functions from Ω into $\tilde{\underline{S}}$, and Γ denote the set of functions from K into $\tilde{\underline{Y}}$. Elements of Δ are denoted λ , with $\lambda_a = \lambda(a) \in \Lambda_a$ denoting the belief of $a \in N$. Elements of Γ are denoted γ , with $\gamma_k = \gamma(k) \in \Gamma_k$ denoting the belief by $k \in K$. We will routinely use shorthand notation of the form $\lambda_a(s_k < s_k^-)$ to represent $\lambda_a(\{s \in \underline{S} | s_k < s_k^-\})$, $\gamma_k(F(x|y) < t)$ to represent $\gamma_k(\{y \in \underline{Y} | F(x|y) < t\})$, etc.

For any $s \in \underline{S}$, we define the endorsement

$$e(s) = \begin{cases} k & \text{if } s_k < s_k^-, \text{ for } k \in K \\ 0 & \text{otherwise} \end{cases} \quad (2.9)$$

and the anti endorsement:

$$\bar{e}(s) = \begin{cases} \bar{k} & \text{if } s_k < s_k^- \text{ for } k \in K \\ 0 & \text{otherwise.} \end{cases} \quad (2.10)$$

So the endorsed candidate is the candidate whose policy position is to

the left, while the anti endorsement represents the candidate to the right. Also, we let $q = (\frac{1}{2}, \frac{1}{2})$, so for any $s \in \underline{S}$, the candidate midpoint between s_1 and s_2 is $q \cdot s = \frac{s_1 + s_2}{2}$.

For any $s \in \underline{S}$, and $y \in \underline{Y}$, we define the predicted ballot $\hat{b}(s, y) \in \underline{B}$ by

$$\hat{b}_a(s, y) = \begin{cases} k & \text{if } u(s_k, y_a) > u(s_{-k}, y_a) \text{ for } k \in K \\ 0 & \text{otherwise.} \end{cases} \quad (2.11)$$

With this notation, we can now define the equilibrium definitions used here.

Definition 2.1 Let $s^* \in \underline{S}$. A voter equilibrium, conditional on s^* , is a profile $(b^*, \lambda^*) \in \underline{B} \times \underline{\Delta}$ satisfying

$$(a) \quad \forall \alpha \in \Omega, \quad \forall b \in \underline{B}, \quad b_\alpha^* \in \arg \max_{b'_\alpha \in B_\alpha} E_{\lambda_\alpha^*} [M_\alpha(s^*, b/b'_\alpha)]$$

$$(b) \quad \forall \alpha \in I, \quad \lambda_\alpha^*(\{s^*\}) = 1$$

$$(c) \quad \forall \alpha \in U, \text{ if } e \in K, \text{ then}$$

$$(i) \quad \lambda_\alpha^*(s_e < s_{-e}) = 1$$

$$(ii) \quad v_e^{-\alpha}(b^*) > F^{-\alpha}(y_a^*) \Rightarrow \lambda_\alpha^*(q \cdot s > y_a^*) = 1$$

$$(iii) \quad v_e^{-\alpha}(b^*) > G^{-\alpha}(y_a^*) \Rightarrow \lambda_\alpha^*(q \cdot s < y_a^*) = 1$$

where $e = e(s^*)$, and $\bar{e} = \bar{e}(s^*)$

Definition 2.2 A full equilibrium is a candidate profile

$(s^*, \gamma^*) \in \underline{S} \times \underline{\Gamma}$, together with a voter profile $(b^*, \gamma^*) \in \underline{B} \times \underline{\Delta}$, such that (b^*, λ^*) is a voter equilibrium, conditional on s^* , and (s^*, γ^*)

satisfies, $\forall k \in K$

$$(a) \quad s_k^* \in \arg \max_{s_k \in S_k} E_{\gamma_k^*} [M_k(s^*/s_k, \hat{b}(s^*/s_k, y))]$$

$$(b) \quad \text{If } e \in K, \quad \gamma_k^*(F(q \cdot s^* | y) \geq v_e^*) = 1, \text{ and}$$

$$\gamma_k^*(G(q \cdot s^* | y) \geq v_{\bar{e}}^*) = 1$$

where $v_e^* = v_e(s^*)$, and $v_{\bar{e}}^* = v_{\bar{e}}(s^*)$.

Thus, in full equilibrium, each player chooses a strategy which maximizes his payoffs subject to his beliefs (i.e., voters satisfy 2.1(a), candidates satisfy 2.2(a)). Further, the beliefs of each player must be consistent with the data he observes (i.e., informed voters satisfy 2.1(b), uninformed voters satisfy 2.1(c), and candidates satisfy 2.2(b)).

For players to be able to behave according to the above equilibrium definitions, some implicit assumptions are made about what contemporaneous information each player observes, and about what each player knows about the structure of the model. Before discussing the equilibrium conditions, we specify these information assumptions.

The contemporaneous data each player observes is:

$$I: s^*$$

$$U: e(s^*), v_k(s^*)$$

$$K: s^*, v_k(s^*)$$

Thus, informed voters observe the actual candidate positions. The uninformed voters, on the other hand, only observe the contemporaneous

endorsement and vote (or poll) totals. The candidates observe candidate positions and contemporaneous poll data.

Also, all players are assumed to have some knowledge of the basic structure of the model: Specifically, they know that all voters have utility functions of the form (2.1), although they do not know other voters' ideal points. Further, each voter, a , is assumed to know the relative position of his own ideal point in the issue space: Thus, voter a knows $F(y_a^*)$ and $G(y_a^*)$ - the number of voters to his left and right. Note that we do not require that he know $F(x)$ or $G(x)$ for $x \neq y_a^*$. Finally, when there is no reason to conclude otherwise, each player assumes that all other players are fully informed and rational. As we will see, in equilibrium, this assumption is justified.

We now discuss each of the conditions in the equilibrium definitions, in turn.

Voter Strategies: Definition 2.1(a)

This condition requires each voter to pick a strategy which maximizes his expected payoff subject to his beliefs—regardless of the behavior of other voters. Thus, he must pick a dominant strategy to the game whose payoff is $E_{\lambda_a^*}[M_a(s, b)]$. Fortunately, since the choice is discrete, and the payoff $M_a(s, b)$ is "positively responsive," a strategy of choosing b_a to maximize $E_{\lambda_a^*}[u_a(s, b_a)]$ achieves this, as is shown in the next section.

Informed voter beliefs: Definition 2.1(b)

Informed voters each observe the true candidate positions, s^* . Their beliefs about the candidate positions must be consistent with this information. Hence, their beliefs must be degenerate point masses centered on s^* .

Uninformed voter beliefs: Definition 2.1(c)

Uninformed voters observe the endorsement, $e(s^*)$, and the vote $v_k(b^*)$, for each $k \in K_0$.

Condition (i) insures that voter a 's belief is consistent with the endorsement. I.e., he assigns zero probability to situations where the endorsed candidate is to the right of the other candidate.

Conditions (ii) and (iii) bring the voter's beliefs in agreement with the observed vote outcome. Voter a knows that, excluding his own vote, all votes for the endorsed candidate must come from voters with ideal points at or to the left of the candidate

midpoint, $q^*s^* = \frac{s_1^* + s_2^*}{2}$, wherever that might be. This follows from his assumption that all voters who are voting are informed and rational. Hence condition (ii) requires that if he observes the vote for the endorsed candidate to be greater than the number of voters he knows to have ideal points at or to his left, then he must infer that the candidate midpoint is to his right. Similarly, condition (iii) requires that if more voters are voting for the unendorsed candidate than voter a knows to be at or to his right, then he must infer that the candidate midpoint is to his left.

Conditions (ii) and (iii) are the key conditions driving the results in the subsequent sections. These conditions formalize the notion of how it is possible for uninformed voters to gather useful information from contemporaneous voting data. A very loose idea of the type of inference being required of the uninformed voter in condition (ii) can be captured in the statement "Well if that many voters are voting for candidate e, he can't be too liberal."

Figure 1 illustrates the situation for the case when Ω is infinite and F is invertible. In this case we have $\mu(\{a\}) = 0$ for all a , so $v_e^{-a} = v_e$, $F^{-a} = F$, etc. Also $G(x) = \mu(\Omega) - F(x)$ for all x . Here, voters in group A (with $y_\alpha^* < F^{-1}(v_e)$) must infer that the candidate midpoint is to the right of their ideal point. Voters in group B (with $y_\alpha^* > G^{-1}(v_-)$, or $y_\alpha^* > F^{-1}(v_e + v_0)$) must infer that the candidate midpoint is to the left of their ideal points. The remaining voters, those in group C, can make no inference.

Candidate Strategies: Definition 2.2(a).

This condition requires that each candidate chooses a policy position to maximize his expected payoff, based on his belief of the voter ideal points. Note that each candidate knows the position of the other candidate, but does not know the true voter ideal points y_α^* ; hence his belief γ_k^* over $\underline{Y}$. Further, each candidate, in choosing his optimal strategy, assumes that all voters are informed and rational. I.e., given ideal points $y \in \underline{Y}$ for each voter, and candidate positions $s \in \underline{S}$, the candidate assumes that voters vote according to $\hat{b}(s, y)$.

Figure 1

The results in section 3 show that if the voters are in equilibrium, then they satisfy this condition.

Candidate beliefs: Definition 2.2(b)

These conditions require that candidate beliefs about voter characteristics be consistent with the observed vote. The candidates observe s^* as well as $v_k(s^*)$. Hence they know the candidate midpoint $q \cdot s^* = \frac{s_1^* + s_2^*}{2}$. Since all voters who are voting are assumed to be voting rationally, it must be that all voters voting for $e(s^*)$ have ideal points at or to the left of $q \cdot s^*$, while all those voting for $\bar{e}(s^*)$ must have ideal points at or to the right of $q \cdot s^*$. Hence for any $y \in Y$ in the support set of γ_k^* , it must be that $F(q \cdot s^* | y) \geq v_e^*$ and $G(q \cdot s^* | y) \leq v_{\bar{e}}^*$.

In concluding this section, we relate our equilibrium definition to other types of equilibria for games with incomplete information. Our equilibrium is similar to Harsanyi's Bayesian equilibrium (see Harsanyi [1968]) in that each player has beliefs about the unknown characteristics of other players. However, we differ significantly from the Bayesian equilibrium framework in that we allow for revision of beliefs based on observed strategy choices of other players. The beliefs are part of the equilibrium in our setup, unlike in Bayesian equilibria, where they are datum. In this respect our equilibrium resembles more closely the "rational expectations" or "fulfilled expectations" equilibria. Like these equilibria (see eg.,

Krepps [1977] or Grossman [1981]). Our players optimize subject to beliefs, and then beliefs cannot be countermanded by observed data. However our equilibrium also differs somewhat from these equilibria. In a fulfilled expectations equilibrium, for example, it is assumed that the function relating states of the world to observed data (prices) is not known a priori, but is determined as part of the equilibrium definition. Here, on the other hand, we assume the correspondence relating states of the world (i.e., candidate positions and voter ideal points) to observed data (i.e., endorsement and vote totals) is known. In the context of voting models, we believe this assumption makes more sense than the assumption that uninformed voters must learn the voting correspondence.

3. Results

This section states and proves some properties of equilibria to the above model. Lemmas 1 and 2 deal with voter equilibria, while Theorems 1 and 2 deal with full equilibria.

Lemma 1 Assume Ω is finite, with $|\Omega| = n$, and with $\mu(C) = |C|$ for all $C \subseteq \Omega$. Let $s^* \in \underline{S}$ satisfy $s_1^* \neq s_2^*$. Then if $(b^*, \lambda^*) \in \underline{B} \times \underline{\Delta}$ is a voter equilibrium conditional on s^* , it must satisfy

(a) For all $\alpha \in I$,

$$\begin{aligned} y_\alpha^* < q \cdot s^* &\Rightarrow b_\alpha^* = e(s^*) \\ y_\alpha^* > q \cdot s^* &\Rightarrow b_\alpha^* = \bar{e}(s^*) \end{aligned}$$

(b) $\exists x_L, x_R \in X$, satisfying

$$F_I(x_R) \leq F_I(q^*s^*) + T - 1$$

$$G_I(x_L) \leq G_I(q^*s^*) + T - 1$$

where $T = \max_{x \in X} E(x)$, such that for all $\alpha \in U$,

$$y_\alpha^* < x_L \Rightarrow b_\alpha^* = e(s^*)$$

$$y_\alpha^* > x_R \Rightarrow b_\alpha^* = \bar{e}(s^*)$$

Proof: From Definition 2.1(a) and (2.5), it follows that, $\forall \alpha \in \Omega$,

$$b_\alpha^* \in \arg \max_{b_\alpha \in B_\alpha} E_{\lambda_\alpha^*}[u_\alpha(s_{k(b)}) + \delta u_\alpha(s_{b_\alpha})]. \quad (3.1)$$

But, from the definition of $k(b)$, it follows that by voting for candidate k , voter α can never cause candidate k to do worse than he would if α voted for $\bar{k}$ or abstained. Further, by finiteness of Ω , for some b , voter α is pivotal. Hence, regardless of whether $\delta > 0$ or $\delta = 0$, (3.1) can be rewritten as, $\forall \alpha \in \Omega$,

$$b_\alpha^* \in \arg \max_{b_\alpha \in B_\alpha} E_{\lambda_\alpha^*}[u_\alpha(s_{b_\alpha})] \quad (3.2)$$

For $\alpha \in I$, from Definition 2.1(b), $\lambda_\alpha(\{s^*\}) = 1$, so, for

$\alpha \in I$,

$$b_\alpha^* \in \arg \max_{b_\alpha \in B_\alpha} [u_\alpha(s_{b_\alpha}^*)] = \arg \min_{k \in K} [|s_k^* - y_\alpha^*|] \quad (3.3)$$

But $y_\alpha^* < q^*s^* \Rightarrow |s_e^* - y_\alpha^*| < |s_{\bar{e}}^* - y_\alpha^*|$, where $e = e(s^*)$ and

$\bar{e} = \bar{e}(s^*)$. So

$$y_\alpha^* < q^*s^* \Rightarrow b_\alpha^* = e(s^*) \quad (3.4)$$

Similarly,

$$y_\alpha^* > q^*s^* \Rightarrow b_\alpha^* = \bar{e}(s^*), \quad (3.5)$$

which proves (a). For part (b), we first show, for all $\alpha \in U$, that

$$F(y_\alpha^*) \leq v_e^* \Rightarrow b_\alpha^* = e \quad (3.6)$$

$$G(y_\alpha^*) \leq v_{\bar{e}}^* \Rightarrow b_\alpha^* = \bar{e},$$

where $v_e^* = v_e(b^*)$, and $v_{\bar{e}}^* = v_{\bar{e}}(b^*)$. To see this, suppose $F(y_\alpha^*) \leq v_e^*$ and $b_\alpha^* \neq e$. Then $F^{-\alpha}(y_\alpha^*) = F(y_\alpha^*) - 1$, and $v_{\bar{e}}^{-\alpha}(b^*) = v_{\bar{e}}^*$. So for all $\alpha \in U$, using (ii) of Definition 2.1(c),

$$\begin{aligned} F(y_\alpha^*) \leq v_e^* &\Rightarrow F^{-\alpha}(y_\alpha^*) < v_{\bar{e}}^{-\alpha}(b^*) \\ &\Rightarrow \lambda_\alpha^*(q^*s^* > y_\alpha^*) = 1. \end{aligned} \quad (3.7)$$

But now, for a set of s of λ_α^* measure 1, we have $q^*s^* > y_\alpha^*$ and (from (i) of Definition 2.1(c)), $s_e < s_{\bar{e}}$. For such s , $u_\alpha(s_e) > u_\alpha(s_{\bar{e}})$, so $\{e\} = \arg \max_{b_\alpha \in B_\alpha} [u_\alpha(s_{b_\alpha})]$. Hence

$$\{e\} = \arg \max_{b_\alpha \in B_\alpha} E_{\lambda_\alpha^*}[u_\alpha(s_{b_\alpha})] \quad (3.8)$$

So, from (3.2), it follows that $b_\alpha^* = e$, a contradiction. It follows that $F(y_\alpha^*) \leq v_e^* \Rightarrow b_\alpha^* = e$. The second inequality of (3.6) follows in an analogous fashion. Now, we set

$$\begin{aligned} x_L &= \sup \{x \in X | F(x) \leq v_e^*\} \\ x_R &= \inf \{x \in X | G(x) \leq v_{\bar{e}}^*\} \end{aligned} \quad (3.9)$$

then, using (3.6), for all $\alpha \in U$,

$$\begin{aligned} y_a^* < x_L &\Rightarrow F(y_a^*) \leq v_e^* \Rightarrow b_a = e \\ y_a^* > x_R &\Rightarrow G(y_a^*) \leq v_e^* \Rightarrow b_a = \bar{e} \end{aligned} \quad (3.10)$$

So x_L and x_R satisfy the last inequalities in part (b) of the lemma.

We must only show the first inequalities in (b). Using (3.10)

together with (a) of the Lemma, we get

$$\begin{aligned} v_0^* + v_e^* &\leq F_I(q's^*) + F_U(x_R) \\ v_0^* + v_e^* &\leq G_I(q's^*) + G_U(x_L), \end{aligned} \quad (3.11)$$

where $v_0^* = v_0(b^*)$. (For example, the contrapositive of the second equation in (a) and the second equation in (3.10) yields the first equation of (3.11)). Adding and subtracting $F_I(x_R)$ to the first equation of (3.11), using $F_I(x_R) + F_U(x_R) = F(x_R)$, and rearranging, we get

$$v_0^* + v_e^* - F(x_R) \leq F_I(q's^*) - F_I(x_R) \quad (3.12)$$

But, from the definition of x_R , and since G is a lower semi continuous, monotone decreasing step function, we have $G(x_R) > v_e^*$. So $G(x_R) \geq v_e^* + 1$. Hence, $n - F(x_R) + E(x_R) \geq n - v_e^* - v_0^* + 1$, which implies

$$v_0^* + v_e^* - F(x_R) \geq 1 - E(x_R) \geq 1 - T \quad (3.13)$$

Now (3.12) together with (3.13) yield $1 - T \leq F_I(q's^*) - F_I(x_R)$, or

$$F_I(x_R) \leq F_I(q's^*) + T - 1 \quad (3.14)$$

A similar argument yields

$$G_I(x_L) \leq G_I(q's^*) + T - 1 \quad (3.15)$$

which completes the proof of the lemma.

Q.E.D.

The above Lemma establishes that in equilibrium, all voters (informed and uninformed voters alike) with the exception of a few uninformed voters with ideal points in the neighborhood of the candidate midpoint vote as if they had correct information about candidate positions. In the special case when $T = \sup_{x \in X} E(x) = 1$, we get the equations in (b) of the lemma become:

$$\begin{aligned} F_I(x_L) &\geq F_I(q's^*) \\ G_I(x_R) &\geq G_I(q's^*) \end{aligned} \quad (3.16)$$

Figure 2 illustrates the interpretation of this result.

Clearly, as the number of informed voters around $q's^*$ becomes denser and denser, the uninformed voters' voting will also become more and more informed.

It is of considerable interest to determine the limiting behavior of the model as the number of voters becomes large. To do this, we consider the infinite voter model. Here, we can formalize the above notion of the denseness of the informed voters through the invertibility of the cumulative density function of ideal points. We get the following analogue of Lemma 1.

Figure 2

Illustration of Theorem 1 where $T=1$. All voters with $y_{\alpha} < x_L$ vote for candidate 1. Those with $y_{\alpha} \geq x_L$ vote for candidate 2.

Lemma 2 Assume Ω is infinite, $\mu(I) > 0$, F_I and F are invertible and $\delta > 0$. Let $s^* \in \underline{S}$ satisfy $s_1^* \neq s_2^*$, and let $(b^*, \lambda^*) \in \underline{B} \times \underline{\Delta}$ be a voter equilibrium conditional on s^* . Then it must satisfy, for all $\alpha \in \Omega$,

$$\begin{aligned} y_{\alpha}^* < q \cdot s^* &\Rightarrow b_{\alpha}^* = e(s^*) \\ y_{\alpha}^* > q \cdot s^* &\Rightarrow b_{\alpha}^* = \bar{e}(s^*) \end{aligned}$$

Or, equivalently, for all $\alpha \in \Omega$, all $k \in K$,

$$u_{\alpha}(s_k^*) > u_{\alpha}(s_{-k}^*) \Rightarrow b_{\alpha}^* = k$$

Proof Invertibility of F implies $E_I(x) = E_U(x) = 0$ for all $x \in X$, which further implies that $\mu(\{a\}) = 0$ for all $a \in \Omega$. Thus for all $\alpha \in \Omega$, and $b'_{\alpha} \in B_{\alpha}$, we have $k(b/b'_{\alpha}) = k(b)$. So, using (2.5), and the fact $\delta > 0$, for all $b \in \underline{B}$, we have

$$\begin{aligned} & \arg \max_{b'_{\alpha} \in B_{\alpha}} E_{\lambda_{\alpha}^*} [M_{\alpha}(s^*, b/b'_{\alpha})] \\ &= \arg \max_{b'_{\alpha} \in B_{\alpha}} E_{\lambda_{\alpha}^*} [u_{\alpha}(s_{k(b)}) + \delta u_{\alpha}(s_{b'}_{\alpha})] \\ &= \arg \max_{b'_{\alpha} \in B_{\alpha}} E_{\lambda_{\alpha}^*} [u_{\alpha}(s_{b'}_{\alpha})]. \end{aligned} \tag{3.17}$$

Hence, Definition 2.1(a) reads, for all $\alpha \in \Omega$,

$$b_{\alpha}^* \in \arg \max_{b_{\alpha} \in B_{\alpha}} E_{\lambda_{\alpha}^*} [u_{\alpha}(s_{b_{\alpha}})] \tag{3.18}$$

Using the same argument as in Lemma 1, it follows that if $\alpha \in I$, the result of Lemma 2 holds. Hence we need only show that it holds for $\alpha \in U$.

Since $\mu(\{a\}) = 0$ for all $a \in \Omega$, we have $F^{-a}(x) = F(x)$ and $v_k^{-a}(b) = v_k(b)$ for all $x \in \mathbb{R}$, $k \in K$, $b \in \underline{B}$. Hence (ii) and (iii) of Definition 2.1(c) read, for all $a \in U$,

$$v_e^* > F(y_a^*) \Rightarrow \lambda_a^*(q^*s > y_a^*) = 1 \quad (3.19)$$

$$v_e^* > G(y_a^*) \Rightarrow \lambda_a^*(q^*s < y_a^*) = 1$$

where $v_e^* = v_e(s^*)$, and $v_e^* = v_e(s^*)$. But then, together with (i) of Definition 2.1(c), it follows that if $v_e^* > F(y_a^*)$, there is a set of $s \in \underline{S}$, of λ_a^* measure 1, for which $q^*s > y_a^*$ and for which $s_e < s_-$. For any such s , it follows that $|s_e - y_a^*| < |s_- - y_a^*| \Rightarrow u_a(s_e) > u_a(s_-)$, so $\{e\} = \arg \max_{k \in K_0} u_a(s_k)$. It follows from (3.18), that $b_a^* = e$. Using a similar argument when $v_e^* > G(y_a^*)$, we get that equations (3.19) imply, for all $a \in U$,

$$v_e^* > F(y_a^*) \Rightarrow b_a^* = e \quad (3.20)$$

$$v_e^* > G(y_a^*) \Rightarrow b_a^* = \bar{e}$$

Now, define $x_L = F^{-1}(v_e^*)$, and $x_R = G^{-1}(v_e^*)$. Then $x_L \leq x_R$, since $x_L = F^{-1}(v_e^*) \leq F^{-1}(\mu(\Omega) - v_e^*) = G^{-1}(v_e^*) = x_R$. And, by monotonicity of F and G ,

$$y_a^* < x_L \Rightarrow F(y_a^*) < F(x_L) = v_e^* \Rightarrow b_a^* = e \quad (3.21)$$

$$y_a^* > x_R \Rightarrow G(y_a^*) < F(x_R) = v_e^* \Rightarrow b_a^* = \bar{e}$$

Putting this together with the informed voters (for whom the result of the lemma holds), and using the fact that for all $x \in \mathbb{R}$,

$$E_I(x) = E_U(x) = 0, \text{ we get} \quad v_e^* \geq F_I(q^*s^*) + F_U(x_L) \quad (3.22)$$

$$v_e^* \geq G_I(q^*s^*) + G_U(x_R)$$

Adding and subtracting $F_I(x_L)$ and $G_I(x_R)$ respectively, to each of these equations, as in the proof of Lemma 1, we get, in the first equation, $F_I(q^*s^*) - F_I(x_L) \leq v_e^* - F(x_L) = 0$, or

$$F_I(x_L) \geq F_I(q^*s^*). \quad (3.23)$$

And the second yields

$$G_I(x_R) \geq G_I(q^*s^*). \quad (3.24)$$

From the monotonicity and invertibility of F_I and G_I , it follows that $x_R \leq q^*s^* \leq x_L$. But we have already shown $x_L \leq x_R$. So $x_L = x_R = q^*s^*$. Together with (3.21), this yields the desired result.

Q.E.D.

Thus, the above lemmata show that equilibrium behavior by all voters implies that the aggregate voting behavior of the voters extracts all the relevant information about the candidate positions. As we see, for any choice of strategies by the candidates, in equilibrium, all voters vote as if they had perfect information, regardless of whether they are informed or uninformed. It should be noted that in equilibrium the uninformed voters still do not know, or even have any common probability distribution of, the positions of the candidates. However they each have enough probabilistic information

about the location of the midpoint between s_1 and s_2 to allow them to make correct voting decisions. So the equilibrium extracts correct voting decisions without disseminating fully the information on candidate positions.

We next investigate the characteristics of full equilibria to the game. Here, for simplicity, we look only at the infinite voter case, where F_I and F are invertible. In addition to requiring the true c.d.f.'s to be invertible, we also require the c.d.f.'s of the candidate beliefs to be invertible. This we do by setting $\underline{Y} = \underline{Y}' = \{y \in \underline{Y}^0 | F(x|y) \text{ is invertible}\}$. Similar theorems can be proven for finite Ω , but they are messier, because of the non uniqueness of admissible strategies for uninformed voters with ideal points near the candidate midpoint. We do not present results for finite voters here, since (via Lemma 1), in the limiting case, as n gets large, they become equivalent to the infinite voter results presented here.

Theorem 1 There exists a full equilibrium to the game defined by

(2.5) - (2.6). Further, if $\mu(I) > 0$, F_I and F are invertible, $\underline{Y} = \underline{Y}'$, and $\delta > 0$, any equilibrium $((s^*, \gamma^*), (b^*, \lambda^*)) \in \underline{S} \times \underline{\Gamma} \times \underline{B} \times \underline{\Delta}$ must satisfy $s_1^* = s_2^*$.

Proof To show existence, we set $s_1^* = s_2^* = F^{-1}(t)$ where $t = \mu(\Omega)/2$. Let $\lambda_\alpha^*(s^*) = 1$ for all $\alpha \in \Omega$, and $\gamma_k^*(\{y^*\}) = 1$ for all $k \in K$. It is easily verified that this is a full equilibrium for any choice of b^* .

(In particular, we could choose $b_\alpha^* = 0$ for all α).

Now assume $((s^*, \gamma^*), (b^*, \lambda^*))$ is a full equilibrium, and $s_1^* \neq s_2^*$. Pick $k \in K$ with $s_k^* < s_{-k}^*$. I.e., $e = e(s^*) = k$, and $\bar{e} = \bar{e}(s^*) = \bar{k}$. Then by Lemma 2, we have

$$y_\alpha^* < q \cdot s^* \Rightarrow b_\alpha^* = k \quad (3.25)$$

$$y_\alpha^* > q \cdot s^* \Rightarrow b_\alpha^* = \bar{k}$$

So, since $E(x) = 0$ for all x , we have $v_0^* = 0$, $v_k^* = F(q \cdot s^*)$, and $v_{-k}^* = G(q \cdot s^*)$. Hence $v_k^* + v_{-k}^* = \mu(\Omega)$. So, writing $t = \mu(\Omega)/2$, either $v_k^* \leq t$ or $v_{-k}^* \leq t$. Assume, w.l.o.g., that $v_k^* \leq t$.

Now, for arbitrating $s \in \underline{S}$, $y \in \underline{Y}$,

$$M_k(s, \hat{b}(s, y)) = \begin{cases} 1 & \text{if } v_k(\hat{b}(s, y)) > v_{-k}(\hat{b}(s, y)) \\ -1 & \text{if } v_k(\hat{b}(s, y)) < v_{-k}(\hat{b}(s, y)) \\ 0 & \text{otherwise} \end{cases} \quad (3.26)$$

But, using the definitions of $\hat{b}$ and v_k , and manipulating the above expressions, we can rewrite

$$M_k(s, \hat{b}(s, y)) = \begin{cases} 1 & \text{if } |s_k - m(y)| < |s_{-k} - m(y)| \\ -1 & \text{if } |s_k - m(y)| > |s_{-k} - m(y)| \\ 0 & \text{otherwise} \end{cases} \quad (3.27)$$

where $m(y) \in \mathbb{R}$ is chosen to satisfy $F(m(y)|y) = t$. I.e., $m(y)$ is the median of the y_α . (Since $F(x|y)$ is invertible, it exists and is unique). It follows that

$$E_{\gamma_k^*}[M_k(s, \hat{b}(s, y))] = \gamma_k^*(|s_k - m(y)| < |s_{-k} - m(y)|) \quad (3.28)$$

$$- \gamma_k^* (|s_k - m(y)| > |s_k - m(y)|)$$

Now, we define m_k^* to be a median of the $m(y)$'s with respect to γ_k^* . I.e., $\gamma_k^*(m(y) \leq m_k^*) \geq \frac{1}{2}$ and $\gamma_k^*(m(y) \geq m_k^*) \geq \frac{1}{2}$. Then, for $s_k = m_k^*$, we have $E_{\gamma_k^*}[M_k(s^*/s_k, \hat{b}(s^*/s_k, y))] \geq \frac{1}{2} - \frac{1}{2} = 0$. So it follows from

Definition 2.2(a) that we must have

$$E_{\gamma_k^*}[M_k(s^*, \hat{b}(s^*|y))] \geq 0 \quad (3.29)$$

But invertibility of $F(\cdot|y)$ implies that $E(q^*s^*|y) = 0$ for all y .

Using this together with $v_0^* = 0$, Definition 2.2(b) becomes

$\gamma_k^*(F(q^*s^*|y) = v_k^*) = 1$. Since $v_k^* \leq t$, and $F(\cdot|y)$ is monotonic for all y , we must have $\gamma_k^*(m(y) \geq q^*s^*) = 1$. Now if $\gamma_k^*(m(y) > q^*s^*) > 0$, we have from (3.28), $E_{\gamma_k^*}[M_k(s^*, \hat{b}(s^*, y))] < 0$, a contradiction to (3.29).

Hence $\gamma_k^*(m(y) = q^*s^*) = 1$. But then, for $s_k = q^*s^*$, we have, from (3.28), that $E_{\gamma_k^*}[M(s^*/s_k, \hat{b}(s^*/s_k, y))] = 1$, while

$E_{\gamma_k^*}[M(s^*, \hat{b}(s^*|y))] = 0$. But this is a contradiction to Definition

2.2(a). Hence, $s_1 \neq s_2$ leads to a contradiction, so we must have

$s_1 = s_2$, which proves the theorem.

Q.E.D.

Unfortunately it happens that the full equilibria of the game (2.5)-(2.6) cannot be narrowed down any further than the set of candidate strategies defined by Theorem 1. In fact, it happens that if voters are in equilibrium, then all candidate profiles, (s^*, γ^*)

where $s_1^* = s_2^*$ and $\gamma_1^*(F(q^*s^*|y)=t) = \gamma_2^*(F(q^*s^*|y)=t) = 1$ yield full equilibria under the definition we have given. However it seems apparent that if $s_1^* = s_2^* \neq x^*$, where x^* is the median of the true distribution of voter ideal points, then the equilibrium is somewhat unstable. Under our definition, this is formally an equilibrium by virtue of the fact that both candidates maintain the same incorrect beliefs about the median. Since they both agree on their incorrect beliefs, they both adopt the same position as their strategy. There is no endorsement, and voters vote randomly between them, yielding an outcome which is consistent with the candidates' incorrect beliefs. However, this equilibrium is unstable in the sense that if either candidate makes a slight error in choice strategy, then the beliefs of both candidates will be subjected to reality testing, and will be found to be inconsistent with the observed voting behavior. These considerations lead us to define a somewhat stronger notion of equilibrium. This stronger version requires that beliefs must be consistent not only with the information that is generated when candidates adopt their equilibrium strategies, but also with the information that is generated when they make small errors:

Definition 3.1 Let $((s^*, \gamma^*), (b^*, \lambda^*)) \in \underline{S} \times \underline{\Gamma} \times \underline{B} \times \underline{\Delta}$ be a full equilibrium to the game (2.5)-(2.6). Then it is said to be an informationally stable equilibrium iff there is a neighborhood $N(s^*)$ of s^* , such that whenever $s' \in N(s^*)$, and (b', λ') is a voter equilibrium conditional on s' , then for each $k \in K$, if $e(s') \in K$,

$$\gamma_k^*(F(q^*s'|y) \geq v_e') = 1 \text{ and}$$

$$\gamma_k^*(G(q^*s'|y) \geq v_e') = 1$$

where $e = e(s')$, $\bar{e} = \bar{e}(s')$, and $v_e' = v_e(s')$, $v_{\bar{e}}' = v_{\bar{e}}(s')$.

Theorem 2 There exists an informationally stable equilibrium to the game defined by (2.5)–(2.6). Further, if $\mu(I) > 0$, F_I and F are invertible, $\underline{y} = \underline{y}'$, and $\delta > 0$, any equilibrium $((s^*, \gamma^*), (b^*, \lambda^*)) \in \underline{S} \times \underline{I}' \times \underline{B} \times \underline{\Delta}$ must satisfy $s_1^* = s_2^* = F^{-1}(t)$, where $t = \mu(\Omega)/2$.

Proof: Existence follows from the same example as in Theorem 1.

Assume the second part of the theorem is false. From Theorem 1, we must have $s_1^* = s_2^*$. Assume, w.l.o.g. that $s_1^* = s_2^* < F^{-1}(t)$. Pick $N(s^*)$ as in Definition 3.1, and pick $s' \in N(s^*)$ such that $s_1' < s_2'$ and such that $q^*s' = q^*s^*$. From Lemma 2, it follows that $v_1' = F(q^*s') = F(q^*s^*) < t$, $v_2' = G(q^*s') = G(q^*s^*) > t$, and $v_0' = 0$. From Definition 3.1 the invertibility of $F(\cdot|y)$ and $v_0' = 0$, it follows that $\gamma_1^*(F(q^*s'|y) = v_1') = 1$. So $\gamma_1^*(m(y) > q^*s^*) = 1$, where $m(y)$ is defined, as in theorem 1, to be the median of the y_α . But now, by an argument similar to that in the previous theorem. This leads to a contradiction to (3.29). Hence, we must have $s_1^* = s_2^* = F^{-1}(t)$.

Q.E.D.

In summary, we have shown that the only informationally stable equilibria involve both candidates converging to the true median of the entire electorate. If the candidates have converged exactly to

the equilibrium, then, of course, there is no useful endorsement or poll information generated for the voters. They must vote arbitrarily and the outcome is also arbitrary. If either of the candidates deviates at all from the equilibrium strategy, then the endorsement and poll information will be useful, and, in light of Lemma 1, the equilibrium behavior of the voters will extract all information. The outcome will be the same as the full information outcome, and all voters--informed and uninformed--will end up voting correctly.

4. Additional Results: Dynamics, Speed of Convergence, Bandwagons, and Manipulability

The previous section proves the existence of an equilibrium which extracts all relevant information. However, there is no guarantee that this equilibrium will ever be located. Here we concentrate on voter equilibria, when Ω is infinite, and we present a dynamic process by which such equilibria might be attained. The process corresponds to a series of successive polls. Candidate positions are fixed, and at each stage, all voters act rationally on the basis of information generated by the previous poll. We show that regardless of the initial starting behavior of the uninformed voters, this process converges to the full information voter equilibrium. The convergence properties of this process resemble in some respects a "Bandwagon effect." I.e., the vote share for one candidate increases monotonically, at the expense of the other candidate. Further, we obtain some results bearing on the speed of convergence. While any

positive density of informed at the candidate midpoint guarantees eventual convergence, the speed of this convergence depends on the ratio of the density of informed and uninformed voters at that point. Finally, although we do not prove this formally, it appears that the above process, as well as the equilibrium associated with it is non manipulable. I.e., given our restrictions on preferences, no voter can gain by adopting strategies different from those prescribed in the above dynamic. Similarly, and more obviously, in equilibrium no one can gain by misrepresenting his preferences.

Definition 4.1 Fix $s^* \in \underline{S}$ and $(b^0, \lambda^0) \in \underline{B} \times \underline{\Delta}$. We then define, inductively on t , a sequence of profiles $(b^t, \lambda^t) \in \underline{B} \times \underline{\Delta}$ satisfying:

- (a) $\forall \alpha \in \Omega, b_\alpha^t \in \arg \max_{b_\alpha \in B_\alpha} E_{\lambda^{t-1}}[u_\alpha(s_{b_\alpha})]$
- (b) $\forall \alpha \in I, \lambda_\alpha^t((s^*)) = 1$
- (c) $\forall \alpha \in U,$
 - (i) $\lambda_\alpha^t(s_e < s_-) = 1$
 - (ii) $v_e^{t-1} > F(y_\alpha^*) \Rightarrow \lambda_\alpha^t(q^* s > y_\alpha^*) = 1$
 - (iii) $v_-^{t-1} > G(y_\alpha^*) \Rightarrow \lambda_\alpha^t(q^* s < y_\alpha^*) = 1$

where $e = e(s^*)$ and $\bar{e} = \bar{e}(s^*)$, $v_e^{t-1} = v_e(b^{t-1})$, $v_-^{t-1} = v_-(b^{t-1})$.

Thus, each profile satisfies the conditions required in Definition 2.1 for a voter equilibrium with respect to the data generated from the previous profile. I.e., b^t is the best ballot conditional on the beliefs λ^{t-1} , and λ^t are beliefs consistent with the data generated by

the ballot b^{t-1} . For the case of Ω infinite, with F invertible, the conditions in Definition 2.1 become those given above.

Using an argument similar to that in Lemma 2, it is easily shown that if F is invertible, the ballots b^t in the above process satisfy:

$$\text{For } \alpha \in I, \quad b_\alpha^t = \begin{cases} e & \text{if } y_\alpha^* < x^* \\ \bar{e} & \text{if } y_\alpha^* > x^* \end{cases} \quad (4.1)$$

$$\text{For } \alpha \in U, \quad b_\alpha^t = \begin{cases} e & \text{if } y_\alpha^* < x^{t-1} \\ \bar{e} & \text{if } y_\alpha^* > x^{t-1} \end{cases} \quad (4.2)$$

where

$$x^* = q^* s^* = \frac{s_1 + s_2}{2} \quad (4.3)$$

and, for $0 \leq t$

$$x^t = F^{-1}(v_e^t) \quad (4.4)$$

It follows that the only voters voting incorrectly for $t \geq 1$ are the uninformed voters with ideal points in the interval between x^t and x^* . The measure of these voters is precisely $|v_e^t - v_e^*|$, where $v_e^* = F(q^* s^*) = F(x^*)$. We can now prove:

Theorem 3 If Ω is infinite, with $\mu(I) > 0$, with F , F_I and F_U invertible, and $\delta > 0$, then the process defined by Definition (4.1) converges to a voter equilibrium in the sense that $v_e^t \rightarrow v_e^*$, $v_-^t \rightarrow v_-^*$

and $x^t \rightarrow x^*$. The asymptotic speed of convergence of v_e^t to v_e^* is $f_U(x^*)/f(x^*)$, where f_U and f are the p.d.f.'s of F_U and F , respectively.

Proof Since F is monotone increasing and invertible, it is continuous, so all three convergence results follow if we prove $v_e^t \rightarrow v_e^*$. We have from (4.1) and (4.2)

$$\begin{aligned} v_e^t &= F_I(x^*) + F_U(x^{t-1}) \\ &= F_I(x^*) - F_I(x^{t-1}) + F_I(x^{t-1}) + F_U(x^{t-1}) \\ &= [F_I(x^*) - F_I(x^{t-1})] + F(x^{t-1}) \end{aligned} \quad (4.5)$$

But since $v_e^{t-1} = F(x^{t-1})$,

$$v_e^t - v_e^{t-1} = F_I(x^*) - F_I(x^{t-1}) \quad (4.6)$$

In a similar fashion, adding and subtracting $F_U(x^*)$ to equation (4.5), we get

$$v_e^* - v_e^t = F_U(x^*) - F_U(x^{t-1}) \quad (4.7)$$

Adding (4.6) and (4.7), we get the identity

$$v_e^* - v_e^{t-1} = F(x^*) - F(x^{t-1}) \quad (4.8)$$

Further, by monotonicity of F , F_I , and F_U , it follows that $(v_e^* - v_e^{t-1})$ has the same sign as $x^* - x^{t-1}$, which has the same sign as the right hand side of equations (4.6)-(4.8). But then, it follows that we can write v_e^t as the following convex combination of v_e^{t-1} and v_e^*

$$v_e^t = r_1 v_e^{t-1} + r_2 v_e^* \quad (4.9)$$

where

$$r_1 = \frac{F_U(x^{t-1}) - F_U(x^*)}{v_e^{t-1} - v_e^*}, \text{ and } r_2 = \frac{F_I(x^{t-1}) - F_I(x^*)}{v_e^{t-1} - v_e^*} \quad (4.10)$$

It is easily verified that $1 > r_1 > 0$ for $v_e^{t-1} \neq v_e^*$ and $r_1 + r_2 = 1$. Thus, it follows that the sequence $\{v_e^t\}_{t=0}^\infty$ is either a monotone increasing or monotone decreasing sequence, converging to v_e^* .

To address the speed of convergence, we note that if $\rho_t = |v_e^t - v_e^*|$, then, for large t ,

$$\frac{\rho_t}{\rho_{t-1}} = \frac{F_U(x^*) - F_U(x^{t-1})}{F(x^*) - F(x^{t-1})} = \frac{\frac{F_U(x^*) - F_U(x^{t-1})}{x^* - x^{t-1}}}{\frac{F(x^*) - F(x^{t-1})}{x^* - x^{t-1}}} \approx \frac{f_U(x^*)}{f(x^*)}. \quad (4.11)$$

Q.E.D.

To illustrate the above dynamic model, we give an example.

Figure 3 portrays the cumulative distribution of ideal points for the uninformed, informed, and all voters (denoted F_U , F_I and F respectively). The corresponding density functions for the uninformed, informed, and all voters are illustrated in the lower half of the figure and are denoted f_U , f_I and f . Now suppose that candidates 1 and 2 adopt $s^* = (s_1^*, s_2^*)$, as shown in Figure 3. An initial poll of voters might reveal a random response by uninformed voters (hence they split 50-50 between candidates 1 and 2) while the informed voters vote correctly, and split 30-70, an overall straw vote of .40 for candidate 1, .60 for candidate 2. We have assumed that uninformed voters know where they are on the issue relative to the entire electorate, and that each uninformed voter assumes he is the only uninformed voter. Hence, voters can infer where the midpoint between the candidates is and hence how they ought to vote.

candidate		candidate		candidate		candidate	
1	2	1	2	1	2	1	2
I .30	.70	I .30	.70	I .30	.70	I .30	.70
U .50	.50	U .72	.28	U .85	.15	U .94	.06
	.40 .60		.51 .49		.58 .43		.62 .38

Figure 3

Example of Successive Polls

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Specifically, using the poll data together with the endorsement information that candidate 1 is to the left of candidate 2, uninformed voters can infer that the candidate midpoint is at $x^0 = F^{-1}(v_1^0)$. So everyone to the left of the point x^0 ought to vote for candidate 1 and everyone to the right ought to vote for candidate 2. That is, in accordance with (4.2) and (4.4), uninformed voters who are below the 40th percentile on the overall distribution vote for candidate 1, the remaining vote for candidate 2. This produces a second poll result of 51% for candidate 1, 49% for candidate 2. Repeating this for a third poll yields a 57.5% - 42.5% division. Continuing in this fashion, the polls converge to a voter equilibrium, in which the vote is the correct vote of 63% for candidate 1, 37% for candidate 2. As was shown formally in Section 3, in equilibrium, all voters vote as if they had perfect information.

Finally, we say a few words about manipulability of the above dynamic process. Although we do not formally prove it here, it should be evident that given his state of information at time t , it will never be to any voter's advantage to vote differently than it is assumed he votes in the above dynamic process. The reason for this is because of the "multiplier effect" which drives the above process. Namely, uninformed voters cue off of the total vote, and the larger the vote for a given candidate, the greater is the number of uninformed voters who will infer it is their interest to vote for that candidate (since all uninformed voters to the left of $x^{t-1} = F^{-1}(v_e^{t-1})$ vote for e and those to the right vote for $\bar{e}$). Thus, given his state

of information at time t , in order to encourage other voters to vote for the candidate he believes he prefers, a voter should always vote his truthful preferences, as we assume he does.

5. Test of the Model

This section describes two experiments that are designed to test the model developed above. We wish to test both the hypothesis that uninformed voters use poll information to inform their vote and the hypothesis that candidates converge to positions that reflect the preferences of the uninformed as well as of the informed voters. Thus, it is necessary to design an experiment that allows candidates to adjust their policy positions, but at the same time keeps candidate positions stationary enough to allow voters to collect useful information on candidate positions through the poll results.

The experiments we conducted each had between forty and fifty subjects. Two of the subjects played the part of candidates, labeled candidate A and B for this experiment, while the rest of the subjects were voters. Each experiment consists of a sequence of periods, or elections. (See Figure 4 for a schematic diagram of the sequence of events.) In each period, the two candidates first adopt policy positions in a one dimensional policy space. Candidate positions are fixed for the duration of the period, after which the candidates are able to adopt new positions. Once the candidates have selected positions, a sequence of two polls is taken, followed by a final election. Each poll is like a Gallup poll, in that voters are asked

Figure 4

Sequence of Events for Experiments 1 and 2

Note: All polls and votes are by secret ballot.

how they would vote if the election were held now. There are two classes of voters: informed and uninformed, who are selectively provided with information about the candidate positions. The informed voters are told the positions of both candidates at the beginning of each period. The uninformed voters, on the other hand, are never told the position of the candidates. The uninformed voters are only told which candidate position is furthest to the left. All voters, however, are informed of the poll results, and hence, if they wish, can attempt to infer candidate positions on the basis of these results.

Voters are paid for their participation in each period on the basis of the position of the winning candidate and their individual payoff functions. A sample payoff function for a typical voter is given in Figure 5. The payoff function determines the amount the voter will be paid if the winning candidate adopts a given position. For example, in this sample, if the winning candidate adopted the position 70, the voter would earn \$1.28 in that period. All voters have single peaked, symmetric payoff functions, but the location of the individual ideal points will differ for different voters. Although voters do not know the distribution of voter ideal points, they do know where their own ideal point is in relation to those of the rest of the electorate. As seen in the sample of Figure 5, each voter is informed about how many voters have ideal points to the left and to the right of his.

Although the experiment consists of a number of periods, voter

Figure 5
Sample Payoff Functions

preferences remain fixed across periods, as does the partition of informed and uninformed voters. Voters know only their own ideal points, not those of any other voters, and candidates do not have any information about voter ideal points. Further, the uninformed voters never learn anything about the policy position adopted by either candidate in a given period until the termination of the entire experiment. Thus, there is no possibility for uninformed voters to make inferences about candidate positions from the historical record of candidate positions in previous periods, as they could in our previous experiments (McKelvey and Ordeshook [1983a]). For a complete listing of the instructions, see Appendix A.

The distribution of ideal points of the informed and uninformed voters for our two experiments is given in Figure 6.

Notice that in each experiment there are approximately equal numbers of informed and uninformed voters, with the distribution of the uninformed voters being stochastically dominated by that of the informed voters. The median informed voter is at 75, the median uninformed voter is at 45-48, and the overall median is at 60.

Our model makes predictions about the candidate behavior as well as about voter choice. First, with regard to voters, we would expect, in each period, the poll results to converge to the perfect information poll result. In the first poll, the informed voters should vote correctly, with the uninformed being indifferent or voting arbitrarily. In the second poll and in the final vote, uninformed voters should sort themselves into the appropriate category, by using

Figure 6

information available from the previous polls. If no voters err, then as proven in the previous section, this process should converge to the situation where uninformed voters vote as if they had complete information.

Table 1 summarizes the results of the final vote in each experiment, showing the number of voters making errors. An "error" is simply a difference from the behavior the subject would exhibit if he had full information. We see that the informed voters virtually always vote correctly. In Experiment 1 there is only one error in the entire experiment (for the final votes), while for Experiment 2, there are 8 errors, or an average of one per period. Of all the votes cast by the informed voters in the final period, over 97 percent are cast correctly. The error rate for the uninformed voters is substantially higher than that for the informed voters, but still, across both experiments, approximately 80 percent of the votes cast by the uninformed voters are cast correctly.

It is important to note that the above computation of the error rate is actually an over estimate of the individual level errors. Since uninformed voters can only make inferences about candidate positions on the basis of poll data, it follows that errors made by one voter can affect the decisions made by other voters. Under the above computation, a voter may be making a completely rational vote based on the information he observes, but if this information is itself incorrect, he will not necessarily vote as if he had complete information. We wish, therefore, to determine the

TABLE 1
SUMMARY OF ERRORS IN FINAL VOTE
(Errors Based on Assumption of Full Information)

<u>Informed Voters</u>			
	<u>Expt 1</u>	<u>Expt 2</u>	<u>All</u>
Correct Choice	160 (99.4)	149 (94.9)	309 (97.2)
Error	1 (0.6)	8 (5.1)	9 (2.8)
Total	161	157	318

<u>Uninformed Voters</u>			
	<u>Expt 1</u>	<u>Expt 2</u>	<u>All</u>
Correct Choice	121 (75.2)	131 (83.4)	252 (79.2)
Error	40 (24.8)	26 (16.6)	66 (20.8)
Total	161	157	318

TABLE 2
SUMMARY OF INDIVIDUAL LEVEL ERRORS
(Errors Based on Failure of Equation (2.1B))

<u>Second Poll</u>			
	<u>Expt 1</u>	<u>Expt 2</u>	<u>All</u>
Correct Choice	88 (71.5)	88 (68.8)	176 (70.1)
Abstain	14 (11.4)	29 (22.7)	43 (17.1)
Errors	21 (17.1)	11 (8.6)	32 (12.8)
Total	123	128	251

<u>Final Vote</u>			
	<u>Expt 1</u>	<u>Expt 2</u>	<u>All</u>
Correct Choice	119 (83.2)	118 (86.8)	237 (84.9)
Error	24 (16.8)	18 (13.2)	42 (15.1)
Total	143	136	279

proportion of voters who make correct voting decisions based on the information available to them. We assume, then, that uninformed voters use the decision rule in equation (3.6) modified for a dynamic setting—as described in section 4. Thus, we assume that for $a \in U$, $t = 2, 3$.

$$\begin{aligned} y_a^* \leq F^{-1}(v_e^{t-1}) &\Rightarrow b_a^t = e \\ y_a^* \geq G^{-1}(v_e^{t-1}) &\Rightarrow b_a^t = \bar{e} \end{aligned} \quad (5.1)$$

where $v_k^t = v_k(b^t)$ is the vote for candidate k in poll t . We look only at those voters who have a unique choice given the information available to them. Table 2 compiles this data for the second poll and final vote in all periods of Experiments 1 and 2. We see that the error rate for uninformed voters averages around fifteen percent across both experiments for both the final vote and the second poll.

Finally, a glance at Figures 7 and 8 illustrates that most of the errors which do occur can be attributed to two or three voters in each experiment.

Our second hypothesis concerns candidate behavior. Figure 9 shows the sequence of candidate positions in each experiment. We see that in both experiments, the candidates converge quickly to a point between 63 and 65. The point to which they converge lies between the median of the informed voters and that of the total electorate, but they are closer to the total median. This is consistent with what we should anticipate given the individual voting behavior of the uninformed voters. The fact that some proportion (about 1/3) of the

Figure 7

Key: The vote of each voter is listed above his ideal point.
Circled votes denote errors.

Y is the true candidate midpoint.

The two vertical lines represent $F^{-1}(v_e^2)$ and $G^{-1}(v_e^2)$

The numbers next to these lines are

v_e^2 and v_e^2 -bar : the votes for the left and right candidate in round 2.

Figure 8

Figure 9

Key
 — candidate A
 --- candidate B
 ○ winning candidate

uninformed voters are not utilizing the poll information causes the effective equilibrium for the candidates to slide up by several voters from the total median.

Overall, this experiment provides qualified support for model II. We do not have full support for either hypothesis. Rather about 2/3 of the uninformed voters appear to end up voting as if they had perfect information, and the positions to which the candidates converge is correspondingly to a point about 2/3 of the distance from the informed median to the total median.

APPENDIX A

INSTRUCTIONS

1-ISSUE RATIONAL EXPECTATIONS WITH POLLS

This experiment is a study of voting in two candidate elections. As subjects in the experiment, you will each be paid for your participation in the experiment on the basis of the decisions you make. If you are careful, and make good decisions, you can make a substantial amount of money.

In this experiment, there are two candidates, labeled A and B, and the rest of you are voters. The purpose of this experiment is to test certain ideas about how voters and candidates make decisions in elections and, in particular, how they make decisions when their information is imperfect or incomplete. The experiment will consist of a number of elections. In each election the candidates will adopt positions in a one dimensional policy space. Two successive polls will then be taken in which voters can indicate which candidate they prefer, and the outcome of each poll will be announced. After the second poll, voters will vote for the candidate of their choice, and the outcome of this vote will determine the winning candidate for that election. Candidates will then be permitted to adopt new positions, and the process will repeat itself. Voters are paid for their participation on the basis of their payoff function--to be described in more detail shortly, and candidates are paid for their participation on the basis of how many elections they win.

Before describing the experiment in detail, let me describe the policy space and the payoff function of the voters.

At the beginning of the experiment, voters will be given a payoff chart similar to the sample chart in front of you. This chart depicts the policy space and a sample payoff function for a voter. Candidates will be given a similar chart. However, the candidate chart will only contain the policy space, and will not have any voter payoff functions. The policy space is simply the set of all numbers between 0 and 100, and is represented on the horizontal axis in the diagram. During the experiment, in each period, candidates will select positions in the policy space. At the end of each period, each voter will be paid for his or her participation in that period on the basis of his payoff function and the position of the winning candidate. Thus, with the sample payoff chart, if the winning candidate were to adopt the position 48, then the voter would earn 90 cents for that period.

In the actual experiment, the payoff charts for each of the voters will be different from the sample chart. Further, the payoff functions for different voters may also be different. Each voter will have a payoff function which has a peak, or ideal point at some point in the policy space, and decreases symmetrically as we move away in either direction, as in the example in the sample chart. However, different voters' ideal points, or peaks, may be at different points in the space, and their payoff functions may decrease at different rates. One important rule in the experiment is that the information

on your payoff chart is private information. None of the other voters or candidates should know the information on your chart. At no time should you show, talk about, or in any other way reveal any information about your payoff chart to the other subjects. Further, at no time during the experiment are you to have any communications with any of the other subjects except those explicitly provided for in the rules.

Are there any questions about the payoff chart? If not, I will proceed to a description of the experiment itself.

The experiment itself is divided into a number of periods or elections. Each period will consist of a sequence of two polls followed by an election. At the beginning of each period, the two candidates, A and B, will each adopt policy positions. These positions will hold throughout that period, and the candidates will not be permitted to modify or change these positions until the next period. The positions adopted by the two candidates will not be made public. Rather, you, as voters will selectively be provided with information about the candidate positions. Before each poll, some of the voters will be informed as to the actual positions of the candidates. The remaining voters will only be given limited information about the candidate positions. They will only be told which candidate is farthest to the left and which is furthest to the right.

After the candidates have adopted their positions, and the voters have been given their information, we will then take a poll of

all voters. You may think of this as a Gallup Poll. Voters will be asked to indicate the candidate they would vote for if the election were held now. All voters will fill in their ballot cards and hand them in. This will be done as a secret ballot. The vote will be tallied and announced. At this point, additional information will be provided, and a second poll will be taken. After two such polls, the final information will be provided, and we will proceed to the final vote. These votes will also be cast in secret, and the experimenter will then tally the vote. We will then announce the winning candidate and proceed to the next period.

In order to selectively give information only to some voters, the following procedure will be used. Before each poll, the experimenter will write, on the blackboard, coded information about the position of each of the candidates. You will note, on the record sheet, (which is the second sheet in the sample packet you have been given), that for each period, and each poll, there is an entry for a code for each candidate. If this entry is filled in, you are an informed voter in that period and that poll. If it is not, you are uninformed. If you are an informed voter, you may obtain the correct position for the candidate from the coded information on the board. To do so, you just add the code to the coded information on the blackboard. Thus, in the example, if the coded positions on the board were $A = 106$, $B = 157$, then the correct position of the candidates in the first period are $A = 48$, $B = 34$. Thus, if you are an informed voter, you'll be able to obtain the exact positions adopted by the

candidates. If you are uninformed, the only information you will be given is information on which candidate is furthest to the left. This will be posted on the blackboard at the beginning of each period.

It should be emphasized that all voters will get the correct information if they are informed at all. Further, if you are informed for two successive polls in the same period, then using your code, you will get the same information in each period. So it is really only necessary for you to compute the candidate positions once. It is important to emphasize that there is no attempt in this experiment to mislead voters as to the position of the candidates. To emphasize this point, we invite any interested voter, after the experiment, to compare his decoded position with the actual position of the candidates. If there is any discrepancy, you will be awarded a \$10 bonus. Note that there are three possibilities in terms of the information you might receive. You may be an informed voter throughout an entire election (i.e. for both polls and the final vote), you may be uninformed for the entire election, or you may become informed part way through. On the sample record sheet, these possibilities are represented in periods 1, 2, and 3 respectively.

At this point the task of the uninformed voters might seem an impossible one. However, you will be given one additional piece of information to assist you in your decisions. Specifically, while neither you nor the candidates will know the exact distribution of voter ideal points, each of you, as indicated on the sample, will be told the total number of the voters who have ideal points to the left

and to the right of your most preferred position. You can use this information, in conjunction with the information you do have, as to which candidate is to the left and which candidate is on the right, and the fact that it is in the interest of the informed voters to vote sincerely, to formulate your guess about the locations of the candidates and hence to determine your preference for one candidate over the other.

To recapitulate, then, the sequence of events will be as follows. Candidates adopt positions, the experimenter will write the coded positions on the board for the first poll, and the first poll will be taken and announced. The experimenter will write the coded for the second poll, and the second poll will be taken and announced, then the coded for the final election will be written, and the final election will take place. We then proceed to the next period.

After a predetermined number of periods, the experiment will end. At this point, the position of the winning candidate will be announced, and voters will be paid the sum of their payoffs for the position of the winning candidate in each election. (Note that in each election all voters are paid for the position of the winning candidate, regardless of whether or not they voted for that candidate.) The candidate payoffs are as follows: A candidate will receive \$2 for each election won and nothing for each election lost.

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