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UNCERTAIN INNOVATION AND THE PERSISTENCE OF MONOPOLY

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ABSTRACT

In a recent paper published in this Review, Gilbert and Newbery (1982) show that, because an incumbent firm enjoys greater marginal incentives to engage in R and D (under their assumption of deterministic invention), the incumbent firm will engage in preemptive patenting. Thus the industry will tend to remain monopolized, and by the same firm. They then argue heuristically that this result extends to the case in which innovation is uncertain. One form of this conjecture is that the incumbent patents the innovation more often than not. We briefly review the Gilbert and Newbery argument as well as those in related papers (Gilbert, 1981 and Craswell, 1981). We then present a model which incorporates uncertainty and concludes the contrary; that is, in a Nash equilibrium the incumbent firm invests less on the innovation than a challenger. Consequently, the incumbent firm will patent the innovation <u>less</u> often than not. This result indicates that one need worry far less about persistent monopoly than would be suggested by the Gilbert and Newbery analysis.

UNCERTAIN INNOVATION AND THE PERSISTENCE OF MONOPOLY

Jennifer F. Reinganum[1]

I. Introduction

In a recent paper published in this Review, Gilbert and Newbery (1982) show that, because an incumbent firm enjoys greater marginal incentives to engage in R and D (under their assumption of deterministic invention), the incumbent firm will engage in preemptive patenting. Thus the industry will tend to remain monopolized, and by the same firm. They then argue heuristically that this result extends to the case in which innovation is uncertain. That is, they argue that the incumbent firm will patent the innovation first because it can act so as to ''guarantee'' that a rival firm will make negative profits (that it should <u>wish</u> to do so is taken for granted). However, we will also consider the weaker claim that the result is stochastically true; that is, that the incumbent firm will patent the innovation more often than not. The paper cited above and a related paper by Gilbert (1981) suggest two alternative ways to formalize this conjecture. The first is: an incumbent firm will invest more than a challenger on a particular project. The second is: an incumbent firm will run more parallel projects than a challenger. In Section II we briefly review the Gilbert and Newbery argument as well as those in related papers (Gilbert, 1981 and Craswell, 1981). In Section III we present a model which incorporates uncertainty and at least partially controverts the first statement; that is, in a Nash equilibrium the incumbent firm invests less on a given project than a challenger. Consequently, the incumbent firm will patent the innovation <u>less</u> often than not. Section IV presents a simple model which examines the second form of the conjecture in a rigorous framework. We conclude that for a nontrivial set of innovations, in a Nash equilibrium an incumbent firm will choose to conduct <u>fewer</u> parallel projects than a challenger. Hence again the incumbent will patent the innovation less often than not. These results indicate that one need worry far less about persistent monopoly than would be suggested by the Gilbert and Newbery analysis.

The intuition for these results is relatively straightforward, at least in the case where the first successful innovator captures the entire post-innovation market. When the inventive process is stochastic, the incumbent firm continues to receive flow profits during the time preceding innovation. This period is of random length but is stochastically shorter the greater the firms' investments in R and D. Since a successful incumbent merely ''replaces himself'' (albeit with a more profitable product), the incumbent firm has a <u>lower</u> marginal incentive to invest in R and D than does the challenger. The substance of our argument is that this intuition holds true even for innovations which have less dramatic consequences for the post-innovation market.

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II. Review of the Certainty Model

For simplicity, consider a case of cost-reducing innovation in an industry with constant returns to scale. Let $\bar{c}$ denote the incumbent firm's initial unit cost, and let c be the unit cost associated with the new technology. The case in which preemptive patenting is relevant is $c < \bar{c}$. Let the relevant profit rates be

R -- the current revenue flow to the incumbent firm

$\Pi(c)$ — the present value of monopoly profits using the new technology; also the present value of profits to the current incumbent if the incumbent receives a patent on the new technology

$\pi_I(c)$ — the present value of Nash-Cournot profits to the current incumbent firm if the challenger receives a patent on the new technology

$\pi_C(c)$ -- the present value of Nash-Cournot profits to the challenger if the challenger receives a patent on the new technology

Note that since $c < \bar{c}$, the present value of monopoly profits after innovation $\Pi(c)$ always exceeds the present value of monopoly profits before innovation R/r.

<u>Assumption 1.</u> The functions $\Pi(c)$, $\pi_I(c)$ and $\pi_C(c)$ are continuous, and piecewise continuously differentiable. Moreover, $\Pi(c)$ and $\pi_C(c)$ are nonincreasing in c while $\pi_I(c)$ is nondecreasing in c.

That is, if the incumbent patents the new technology, its profits will be lower the higher is the unit cost associated with the new technology. On the other hand, if the challenger patents the new technology (and the incumbent continues to use the old one), then the challenger's profits will be lower and the incumbent's higher the higher is the unit cost associated with the new technology.

<u>Definition 1.</u> The innovation will be termed <u>drastic</u> if $c \leq c_o$, where c_o is assumed to exist and to be uniquely defined as the maximum value of c such that $\pi_I(c) = 0$.

The important thing about the constant returns to scale assumption is that if profits are zero, so is output. Thus if $c \leq c_o$ then the current incumbent's output will be zero after the challenger patents the innovation. In this event, the challenger is a monopolist and $\pi_C(c) = \Pi(c)$. Note that $\Pi(c) \geq \pi_I(c) + \pi_C(c)$ with strict inequality whenever the innovation is not drastic.

The following example illustrates the preceding discussion and Assumption 1. If the demand curve is linear, $P = a - bQ$, then the functions above are $\Pi(c) = (a - c)^2/4b^2$, $\pi_I(c) = (a - 2\bar{c} + c)^2/9b^2$ and $\pi_C(c) = (a - 2c + \bar{c})^2/9b^2$, whenever the expressions in parentheses are nonnegative; otherwise the relevant value for the function is zero. Each of these functions is continuously differentiable except at the point at which the expression in parentheses becomes zero, and continuity is preserved at that point. The innovation is drastic whenever $c \leq c_o$, where $a - 2\bar{c} + c_o = 0$. From this equality, it is easy to see that $\Pi(c_o) = \pi_C(c_o)$ and $\pi_I(c_o) = 0$.

The Gilbert-Newbery argument proceeds as follows. If the inventive process is deterministic, then whoever is willing to bid most for the new technology receives the patent first with probability 1. The challenger will be willing to bid up to $\pi_C(c)$, while the incumbent will be willing to bid up to $\Pi(c) - \pi_I(c)$. Since $\Pi(c) \geq \pi_I(c) + \pi_C(c)$, with strict inequality for $c > c_o$, the incumbent preemptively patents the new technology. Only if the innovation is drastic will the incumbent and the challenger invest an equal amount. Consequently, preemption is the Nash equilibrium outcome in the bidding game. Thus Gilbert and Newbery conclude that the industry will remain monopolized and in the hands of the current incumbent.

This is clearly true when there is no uncertainty in the innovation process. However, Gilbert (1981, p. 229) subsequently argues: ''Uncertainty the invention process does not greatly change the deterministic analysis of preemption, provided R and D expenditures are sensitive to the expected returns and the established firm is no more averse to risk than rivals.''

In a laudatory comment on the Gilbert paper, Craswell (1981, p. 272) continues: ''Assuming any form of direct relationship between the amount spent on R and D and the likelihood of making the invention first, the incumbent will end up with the patent more often than not, and his monopoly will be maintained. In fact, the incumbent will

usually end up with the patent even if he is less efficient at R and D than are his rivals, so long as his inefficiency does not completely negate the advantage due to his larger expenditure on R and D.''

To summarize, Gilbert and Craswell evidently believe that the result that the incumbent invests more than the challenger extends straightforwardly to the case of uncertainty. In the next section, a simple model is presented which incorporates uncertainty. We come to quite the opposite conclusion regarding the persistence of monopoly: for an open set of technologies, the incumbent firm will, in a Nash equilibrium, invest less than a challenger. More precisely, for drastic innovations the incumbent always invests less than the challenger, so that the incumbency changes hands more often than not, contrary to Craswell's assertion. Due to the continuity of the equilibrium investment rates in the unit cost associated with the new technology, there will be an open neighborhood of c_o, representing innovations which are not drastic, for which the incumbent still invests less than the challenger.

A somewhat different argument is offered by Gilbert and Newbery (1982, p. 521) in support of the same basic claim. ''Similar results are obtained when the assumption of a deterministic patent date is replaced by a more general stochastic function which describes the probability of invention at date T conditional on a particular R and D plan. ... The argument is the same as before, except that the monopolist has to set up the correct number of rival research teams.''

Thus the implied claim is that, if allowed to select the number of parallel projects to be undertaken, an incumbent firm would choose a larger number than would a challenger. Section IV presents a simple model with parallel projects. Again there is a nontrivial set of innovations which the the incumbent is less likely to patent than is the challenger.

III. A Model Incorporating Uncertainty

The model developed in this section is a generalization of that of Lee and Wilde (1980) (which is itself based upon a model by Loury (1979)), and a specialization of that of Reinganum (1982). Again an incumbent and a challenger are simultaneously attempting to perfect a particular cost-reducing technology. Technological uncertainty takes the form of a stochastic relationship between the rate of investment and the eventual date of successful completion of the new technology. If x_I represents the rate of investment of the incumbent and $\tau_I(x_I)$ the random success date of the incumbent, then $\Pr\{\tau_I(x_I) \leq t\} = 1 - e^{-h(x_I)t}$, for $t \in [0,\infty)$. Similarly, if x_C and $\tau_C(x_C)$ represent the investment rate and the random success date for the challenger, then $\Pr\{\tau_C(x_C) \leq t\} = 1 - e^{-h(x_C)t}$. The expected success date for firm i (i = I,C) is $1/h(x_i)$, where the function $h(\cdot)$ is the hazard function used in much of the recent literature on patent races. In particular, following Loury (1979), Lee and Wilde (1980) and Dasgupta and Stiglitz (1980), we assume:

Assumption 2. The hazard function $h(\cdot)$ is nonnegative and increasing.

(i) $h(0) = 0 = \lim_{x \to \infty} h'(x)$,

(ii) $h''(x) \geq (\leq)\ 0$ as $x \leq (\geq)\ \bar{x} < \infty$ and

(iii) $h(x)/x \geq (\leq)\ h'(x)$ as $x \geq (\leq)\ \tilde{x} < \infty$.

Thus there may be initial increasing returns, but eventually the technology exhibits decreasing returns to scale. A typical hazard function is illustrated in Figure 1.

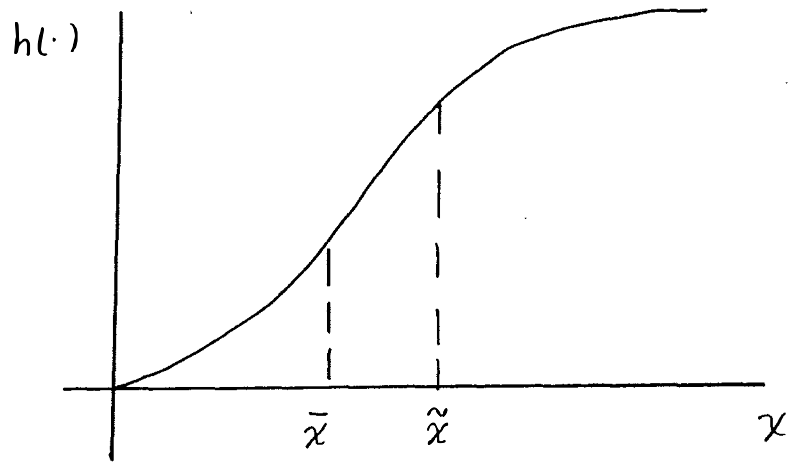

Figure 1

Suppose that the new technology is patentable so that the race ends with the first success. The expected profit to the incumbent for any pair of investment rates (x_I, x_C) is

$$V^I(x_I,x_C) = \int_0^\infty e^{-rt} e^{-(h(x_I) + h(x_C))t} [h(x_I)\Pi(c) + h(x_C)\pi_I(c) + R - x_I]dt$$

$$= [h(x_I)\Pi(c) + h(x_C)\pi_I(c) + R - x_I]/[r + h(x_I) + h(x_C)].$$

That is, the incumbent receives $\Pi(c)$ at t if the challenger has not yet succeeded and the incumbent succeeds at t; this event has probability density $h(x_I)e^{-(h(x_I) + h(x_C))t}$. The incumbent receives $\pi_I(c)$ at t if the incumbent has not yet succeeded and the challenger succeeds at t; this event has probability density $h(x_C)e^{-(h(x_I) + h(x_C))t}$. Finally, the incumbent receives flow profits of R and pays flow costs of x_I so long as no firm has succeeded; this event has probability $e^{-(h(x_I) + h(x_C))t}$.

The challenger's payoff is analogous.

$$V^C(x_I,x_C) = \int_0^\infty e^{-rt}e^{-(h(x_I) + h(x_C))t}[h(x_C)\pi_C(c) - x_C]dt$$
$$= [h(x_C)\pi_C(c) - x_C]/[r + h(x_I) + h(x_C)].$$

The differences between these payoffs arise from the incumbent's current profit flow and the fact that it shares the market in the event of successful innovation by the challenger.

<u>Definition 2.</u> A <u>strategy</u> for the incumbent (challenger) is an investment rate x_I (x_C). The <u>payoff</u> to the incumbent (challenger) is $V^I(x_I,x_C)$ $(V^C(x_I,x_C))$.

<u>Definition 3.</u> A <u>best response function for the incumbent</u> is a function $\phi_I:[0,\infty) \to [0,\infty)$ such that, for each x_C, $V^I(\phi_I(x_C),x_C) \geq V^I(x_I,x_C)$ for all $x_I \in [0,\infty)$. Similarly, a <u>best response function for the challenger</u> is a function $\phi_C:[0,\infty) \to [0,\infty)$

such that, for each x_I, $V^C(x_I,\phi_C(x_I)) \geq V^C(x_I,x_C)$ for all $x_C \in [0,\infty)$. The best response functions will also depend upon the parameters (c,R).

<u>Definition 4.</u> A strategy pair (x_I^*,x_C^*) is a <u>Nash equilibrium</u> if $x_I^* = \phi_I(x_C^*)$ and $x_C^* = \phi_C(x_I^*)$. That is, each firm's investment rate is a best response to the other's.

The proof of the following Proposition can be found in the Appendix.

<u>Proposition 1.</u> If $h'(0) \geq \max\{1/[\Pi(c) - R/r], 1/\pi_C(c)\}$, then there exists a best response function for the incumbent $\phi_I(x_C;c,R)$ which satisfies the first-order condition $\partial V^I(\phi_I,x_C)/\partial x_I = 0$ and the second-order condition $\partial^2 V^I(\phi_I,x_C)/\partial x_I^2 < 0$. The function ϕ_I is continuously differentiable in its argument x_C and continuous in the parameters c,R. Similarly, there exists a best response function for the challenger $\phi_C(x_I;c)$ which satisfies the analogous first- and second-order conditions, and is continuously differentiable in its argument x_I and continuous in the parameter c. Moreover, there exists a pair of Nash equilibrium strategies $x_I^*(c,R)$ and $x_C^*(c,R)$; each is continuous in the parameters c,R.

The first-order conditions which implicitly define the best response functions are

$$\partial V^I(\phi_I,x_C)/\partial x_I = [r + h(\phi_I) + h(x_C)][h'(\phi_I)\Pi(c) - 1]$$

$$- [h(\phi_I)\Pi(c) + h(x_C)\pi_I(c) + R - \phi_I]h'(\phi_I) = 0 \quad (1)$$

and

$$\partial V^C(x_I,\phi_C)/\partial x_C = [r + h(x_I) + h(\phi_C)][h'(\phi_C)\pi_c(c) - 1]$$
$$- [h(\phi_C)\pi_C(c) - \phi_C]h'(\phi_C) = 0. \quad (2)$$

Rearranging terms and noting the definitions of $V^I(\phi_I,x_C)$ and $V^C(x_I,\phi_C)$ yields

$$V^I(\phi_I,x_C) = [h'(\phi_I)\Pi(c) - 1]/h'(\phi_I) \quad (3)$$

and

$$V^C(x_I,\phi_C) = [h'(\phi_C)\pi_C(c) - 1]/h'(\phi_C). \quad (4)$$

<u>Remark 1.</u> Since the individual firm payoffs must be nonnegative, particularly when the firms play best responses, it follows that $h'(\phi_I)\Pi(c) - 1 \geq 0$ and $h'(\phi_C)\pi_C(c) - 1 \geq 0$.

<u>Proposition 2.</u> $\partial\phi_C(x_I;c)/\partial x_I \geq 0$.

<u>Proof.</u> By the implicit function theorem,

$$\partial\phi_C/\partial x_I = -[\partial^2 V^C(x_I,\phi_C)/\partial x_I \partial x_C]/[\partial^2 V^C(x_I,\phi_C)/\partial x_C^2].$$

The denominator is negative by the second-order condition, while the numerator is $-h'(x_I)[h'(\phi_C)\pi_C(c) - 1]$, which is nonpositive by Remark 1.

Q.E.D.

<u>Proposition 3.</u> If the innovation is drastic and $R > 0$, then

$\phi_I(x;c,R) < \phi_C(x;c)$ for all x,c.

<u>Proof.</u> Recall that if the innovation is drastic, $\pi_C(c) = \Pi(c)$ and $\pi_I(c) = 0$. Then the only difference between equations (1) and (2), which implicitly define the best response functions ϕ_I and ϕ_C, is the term R, representing current profit flows to the incumbent firm. If $R = 0$, and the innovation is drastic, then $\phi_I(x;c,0) = \phi_C(x;c)$ for all x,c. Again using the implicit function theorem, we see that

$$\partial\phi_I/\partial R = -[\partial^2 V^I(\phi_I,x)/\partial R\, \partial x_I]/\partial^2 V^I(\phi_I,x)/\partial x_I^2.$$

Since the denominator is negative and the numerator is $Rh'(\phi_I)$ which is positive, we have $\phi_I(x;c,R) < \phi_I(x;c,0) = \phi_C(x;c)$ for all $R > 0$, and all x,c.

Q.E.D.

<u>Proposition 4.</u> If the innovation is drastic and $R > 0$, then in a Nash equilibrium the incumbent invests less than the challenger; that is, $x_I^*(c,R) < x_C^*(c,R)$.

<u>Proof.</u> Suppose, contrary to the theorem, that $x_I^*(c,R) \geq x_C^*(c,R)$. Then Propositions 2 and 3 and the definition of a Nash equilibrium imply that $x_C^*(c,R) = \phi_C(x_I^*(c,R);c) \geq \phi_C(x_C^*(c,R);c) > \phi_I^*(x_C^*(c,R);c,R) = x_I^*(c,R)$.

But this is a contradiction. Thus $x_I^*(c,R) < x_C^*(c,R)$.

Q.E.D.

Proposition 5. If $R > 0$, then there exists an open neighborhood of c_0 (which may depend on R), denoted $N(c_0;R)$, such that if the technology is not drastic, but $c \in N(c_0;R)$, then $x_I^*(c,R) < x_C^*(c,R)$.

Proof. This follows immediately from Proposition 4 and the continuity of the Nash equilibrium investment rates $x_I^*(c,R)$ and $x_C^*(c,R)$ in the parameter c.

Q.E.D.

Thus we have concluded that in a nontrivial set of circumstances — that is, for technologies in the set $N(c_0;R)$ — it is precisely the assumption of certainty versus uncertainty which is responsible for the discrepancy between our results and those of Gilbert and Newbery. Obviously, their conjecture regarding the extent of applicability of their result needs to be tempered somewhat. To see the economics of the issue, consider what happens in our model with drastic innovation if the incumbent were to consider investing a tiny bit less. It would suffer a slightly increased probability of losing the patent to the challenger and a slightly decreased chance of collecting the patent itself, but would spend a bit less and would receive the flow revenue R stochastically longer. The challenger, by investing a bit less, suffers a slightly increased probability of losing the patent to the incumbent and a slightly decreased probability of collecting the patent for itself; on the other hand, it spends a bit less. Since it does not collect any additional current revenue, its marginal benefits due to investing a bit less are lower

than those of the incumbent, and hence in equilibrium the challenger invests more than the incumbent. Consider the same question under the assumption of certainty. What happens in the certainty model if the incumbent were to consider investing a tiny bit less? If the incumbent still invests more than the challenger, then the incumbent collects revenues R with probability 1 and suffers no threat of losing the patent to the challenger. If the incumbent was investing less than the challenger, then further reductions have no impact on their profits. The only important case is when the incumbent considers reducing its investment from above that of the challenger to below that of the challenger. This results in the incumbent receiving R for an infinitesimally short additional time, and losing the noninfinitesimal difference between the present values of monopoly profits and Nash-Cournot profits when the challenger patents the new technology. Consequently, the incumbent is always willing to invest more than the challenger when the innovation process is deterministic.

As long as a Nash equilibrium exists and possesses the requisite continuity properties, the key step in our argument will be Proposition 3. Therefore it is important to ask whether there exist values of c which will reverse the result of Proposition 3. That is, what happens to this result for less nearly drastic innovations?

Proposition 6. The best response functions $\phi_I(x;c,R)$ and $\phi_C(x;c)$ are nonincreasing in c.

Proof. These functions are continuous in c; where $\Pi(c)$, $\pi_C(c)$ and

$\pi_I(c)$ are continuously differentiable, $\phi_I(x;c,R)$ and $\phi_C(x;c)$ are continuously differentiable in c. Then the implicit function theorem implies that

$$\partial\phi_I(x;c,R)/\partial c = -[\partial^2 V^I(\phi_I,x)/\partial c\ \partial x_I]/\partial^2 V^I(\phi_I,x)/\partial x_I^2.$$

The denominator is negative by the second-order condition, while the numerator is $-h'(\phi_I)[(r + h(x))\Pi'(c) - h(x)\pi_I'(c)] > 0$ by Assumption 1. Similarly,

$$\partial\phi_C(x;c)/\partial c = -[\partial^2 V^C(x,\phi_C)/\partial c\ \partial x_C]/\partial^2 V^C(x,\phi_C)/\partial x_C^2.$$

The denominator is again negative; the numerator is $-h'(\phi_C)[r + h(x)]\pi_C'(c) > 0$ by Assumption 1. Continuity implies that $\phi_I(x;c,R)$ and $\phi_C(x;c)$ cannot increase at values of c at which they are not differentiable.

Q.E.D.

This Proposition suggests that both best response functions are shifted downward by less drastic innovations; the net effect, which is what we are interested in, remains ambiguous. If $\phi_I(x;c,R) - \phi_C(x;c)$ is nonincreasing in c, then the incumbent invests less than the challenger even for less drastic innovations.

On the other hand, if this expression is increasing, then it may be that there are innovations for which Proposition 3 does not hold, and consequently, for which the incumbent will invest more than the challenger.

IV. A Model With Parallel Projects

In this section, we present a model which allows the incumbent and challenger to select the number of parallel projects to undertake, denoted n_I and n_C for incumbent and challenger, respectively. The conjecture is that the incumbent is more likely than the challenger to patent the innovation; that is, the incumbent will choose a larger number of projects. For simplicity, suppose that the scale of each project is fixed at x, where $h(x)\pi_C(c) - x > 0$, so that the challenger has at least an opportunity of positive profit. In addition, there is a fixed cost of F per project.

<u>Definition 5.</u> A <u>strategy</u> for the incumbent (challenger) is a number of parallel projects, n_I (n_C). The <u>payoff</u> to the incumbent (challenger) is $V^I(n_I,n_C)$ ($V^C(n_I,n_C)$) as defined below. For simplicity, we treat the number of projects as real rather than integral.

$$V^I(n_I,n_C) = [n_I h(x)\Pi(c) + n_C h(x)\pi_I(c) + R - n_I x]/[r + n_I h(x) + n_C h(x)] - n_I F.$$

$$V^I(n_I,n_C) = [n_C h(x)\pi_C(c) - n_C x]/[r + n_I h(x) + n_C h(x)] - n_C F.$$

<u>Definition 6.</u> A <u>best response function for the incumbent</u> is a function $\eta_I:[0,\infty) \to [0,\infty)$ such that, for each n_C, $V^I(\eta_I(n_C),n_C) \geq V^I(n_I,n_C)$ for all $n_I \in [0,\infty)$. Similarly, a <u>best response function for the challenger</u> is a function $\eta_C:[0,\infty) \to [0,\infty)$ such that, for each n_I, $V^C(n_I,\eta_C(n_I)) \geq V^C(n_I,n_C)$ for all $n_C \in [0,\infty)$.

Again these functions will also depend upon the parameters (c,R).

Definition 7. A strategy pair (n_I^*, n_C^*) is a Nash equilibrium if $n_I^* = \eta_I(n_C^*)$ and $n_C^* = \eta_C(n_I^*)$. That is, each firm's number of projects constitutes a best response to that of the other firm.

First note that preemption (even stochastic preemption) can only be an issue if the challenger would choose $n_C > 0$ if the incumbent ignored the challenger and acted as if it were unopposed; that is, if $(\eta_I(0),0)$ is not a Nash equilibrium. (This is because $\eta_I(0)$ is the optimal number of projects for an unopposed incumbent). Otherwise, the incumbent would simply be behaving as an unopposed monopolist, and would not be taking any account of the challenger at all, and hence cannot be accused of preemptive behavior. A proof of the existence of a Nash equilibrium for this model can be constructed using the same methodology as the proof for the model of Section III. Since the existence proof is not of intrinsic interest, let us simply assume that a best response function exists for each agent; if it does, then it will be (at least) piecewise continuously differentiable in the rival's strategy and continuous in the parameters (c,R). Let us further assume that a Nash equilibrium exists, and that at the equilibrium point the best response functions are continuously differentiable and do not have inverse slopes of each other; in this case, the equilibrium strategies $n_I^*(c,R)$ and $n_C^*(c,R)$ will be continuous in the parameters (c,R) (see Hestenes 1980, p. 22, Theorem 7.1).

We have already determined that the challenger will choose a positive number of projects, by arguing above that $(\eta_I(0),0)$ cannot be an equilibrium in any case of interest. Similarly, let us assume that the incumbent also chooses a positive number of projects, so that the equilibrium is interior. The following first-order conditions must hold simultaneously at a Nash equilibrium:

$$\begin{aligned}\partial V^I(n_I^*, n_C^*)/\partial n_I = {} & [r + n_I^* h(x) + n_C^* h(x)][h(x)\Pi(c) - x] \\ & - [n_I^* h(x)\Pi(c) + n_C^* h(x)\pi_C(c) + R - n_I^* x]h(x) \\ & - F[r + n_I^* h(x) + n_C^* h(x)]^2 = 0. \qquad (5)\end{aligned}$$

and

$$\begin{aligned}\partial V^C(n_I^*, n_C^*)/\partial n_C = {} & [r + n_I^* h(x) + n_C^* h(x)][h(x)\pi_C(c) - x] \\ & - [n_C^* h(x)\pi_C(c) - n_C^* x]h(x) \\ & - F[r + n_I^* h(x) + n_C^* h(x)]^2 = 0. \qquad (6)\end{aligned}$$

Under the assumption that x is fixed, each firm varies its ''scale'' by choosing the number of (statistically independent) projects. Given this, we need only determine whether the incumbent or the challenger will invest in more parallel projects.

Proposition 7. If the innovation is drastic and R > 0, then $n_I^*(c,R) < n_C^*(c,R)$.

Proof. If the innovation is drastic, then $\pi_C(c) = \Pi(c)$ and $\pi_I(c) = 0$. Then equations (5) and (6) imply

$$n_I^* h(x)\Pi(c) + R - n_I^* x = n_C^* h(x)\Pi(c) - n_C^* x$$

or

$$(n_C^* - n_I^*)[h(x)\Pi(c) - x] = R. \tag{7}$$

Since $h(x)\Pi(c) - x > 0$, equation (7) requires that $n_C^*(c,R) > n_I^*(c,R)$.

Q.E.D.

<u>Proposition 8.</u> If $R > 0$, then there exists an open neighborhood of c_o (which may depend on R), denoted $\tilde{N}(c_o;R)$, such that if the technology is not drastic, but $c \in \tilde{N}(c_o;R)$, then $n_I^*(c,R) < n_C^*(c,R)$.

<u>Proof.</u> This result follows immediately from Proposition 7 and the continuity of $n_I^*(c,R)$ and $n_C^*(c,R)$ in the parameter c.

Q.E.D.

Thus the alternative form of the conjecture is also false for an open set of innovations. It fails for essentially the same reason as before; the incumbent has a lower marginal incentive to hasten the date of innovation, since it continues to receive the flow profit R until innovation, while the challenger does not.

V. Conclusion

It seems clear that the assumption of certainty in the

inventive process is not an innocuous one, particularly when one compares the policy implications of these two models. The Gilbert and Newbery model suggests that one ought to be very worried about the development of entrenched monopolies via preemptive patenting. This study suggests that one can reasonably worry far less on this score. The addition of incumbent advantages of various sorts can restore the result that an incumbent invests more than a challenger, but the initial, inertial effect of incumbency is to <u>reduce</u> the likelihood of persistent monopoly.

Of course, the models discussed in this paper also rely heavily upon simplifying assumptions. The assumption of constant returns to scale in the output production function is particularly useful, since it is allows us to use simple parametric expressions for the post-innovation profit functions. Taken together, this paper and that of Gilbert and Newbery (1982) indicate that the influence of current monopoly power on the persistence of monopoly is considerably more complicated than either paper taken alone might suggest.

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APPENDIX

Proof of Proposition 1.

$$\partial V^I/\partial x_I = [h'(x_I)(\Pi(c)r - R + h(x_C)\pi_I(c) + x_I) - B]/B^2,$$

where $B = r + h(x_I) + h(x_C)$. Let

$$f_I(x_I,x_C) = h'(x_I)[\Pi(c)r - R + h(x_C)\pi_I(c) + x_I] - B.$$

Note that sgn $\partial V^I/\partial x_I$ = sgn f_I. Under the assumption that $h'(0) \geq \max\{ 1/[\Pi(c) - R/r], 1/\pi_C(c) \}$, we see that $f_I(0,x_C) \geq 0$ for all x_C. Moreover, $f_I(\hat{x}_I(c,R),x_C) \leq 0$ for all x_C, where

$$\hat{x}_I(c,R) = \min\{x \geq \tilde{x} \mid h'(x) \leq \min(1/\pi_I(c), 1/[\Pi(c) - R/r])\}.$$

This value exists and is finite since $\lim_{x \to \infty} h'(x) = 0$. Since

$$\partial f_I/\partial x_I = h''(x_I)[\Pi(c)r - R + h(x_C)\pi_I(c) + x_I]$$

is first positive, is zero at $\bar{x}$ and then is negative, the function V^I is first increasing, eventually peaks and subsequently declines. Consequently, V^I is single-peaked and reaches its peak at or before $\hat{x}_I(c,R)$ for all x_C.

The value of x_I which provides the peak is $\phi_I(x_C;c,R)$, the unique best response for the incumbent to x_C. Moreover, since V^I is twice differentiable in (x_I,x_C) and continuous in (c,R), and since $\partial^2 V^I(\phi_I,x_C)/\partial x_I^2 < 0$, ϕ_I is implicitly defined as a continuously differentiable function of x_C (and a continuous function of (c,R)) by the first-order condition $\partial V^I/\partial x_I = 0$. A similar argument

establishes the analogous result for ϕ_C. Figure 2 illustrates this argument.

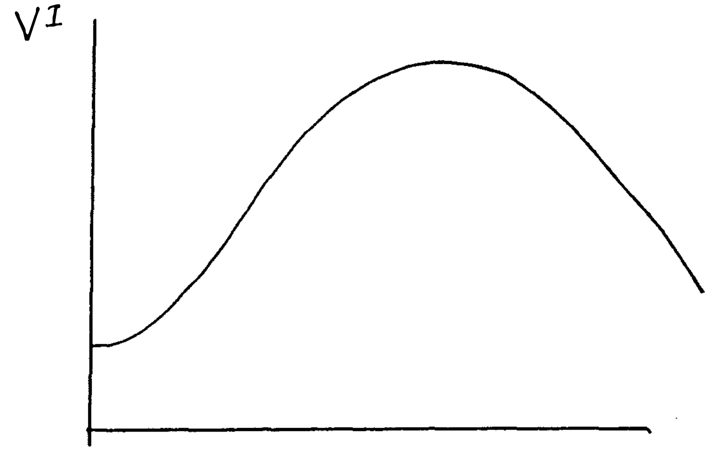

Figure 2 x_I

Define the composite function $\omega = \phi_I \circ \phi_C : [0,x_I(c,R)] \to [0,x_I(c,R)]$ (holding c and R fixed). The function $\omega(x;c,R)$ is continuously differentiable in x on a compact, convex and nonempty domain. Hence it has a fixed point $x_I^*(c,R)$ by Brouwer's theorem. That is, there is a point $x_I^*(c,R)$ such that $\omega(x_I^*(c,R);c,R) - x_I^*(c,R) = 0$. Under the assumption that $x_I^*(c,R)$ is not a critical point of $\omega(x;c,R) - x$ (that is, $\partial\omega(x_I^*(c,R);c,R)/\partial x \neq 1$), there exists a neighborhood of c in which the implicit function $x_I^*(\cdot,R)$ is continuous (see Hestenes 1980, p. 22, Theorem 7.1). Let $x_C^*(c,R) = \phi_C(x_I^*(c,R);c)$. The strategies $x_I^*(c,R)$ and $x_C^*(c,R)$ constitute a Nash equilibrium, and they are continuous in the parameter c.

Q.E.D.