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TECHNOLOGY ADOPTION UNDER IMPERFECT AND INCOMPLETE INFORMATION

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ABSTRACT

This paper presents a static game theoretic model of a firm's decision to adopt a technological innovation of uncertain profitabilty which will reduce the production cost associated with the firm's output. Given the levels of adoption costs, discount rates and expectations regarding the profitabilty of the innovation, we determine the (Nash equilibrium) range of initial production costs for which each firm prefers to adopt the innovation. In addition, we ask whether a high-cost or a low-cost firm will be more likely to innovate, and whether a firm will be more likely to innovate if its rival is a high-cost or a low-cost firm.

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I. INTRODUCTION

This paper presents a static game theoretic model of a firm's decision to adopt a technological innovation which is expected to reduce the production cost associated with the firm's output. The firm is assumed to share the market for the homogeneous output with one rival firm which faces the same decision problem regarding the innovation. There may be firm-specific or innovation-specific uncertainty regarding the profitability of the innovation (i.e., the extent of cost reduction), and uncertainty regarding the cost of adoption for the rival firm. Moreover, the decision problem is modeled as a simultaneous-move game so each firm must act in ignorance of its rival's intentions.

Basic notation is developed in Section II. Section III discusses the following questions in the context of a Nash equilibrium. Given the levels of adoption costs, discount rates and expectations regarding the profitability of the innovation, for what range of initial production costs will each firm prefer to adopt the innovation rather than forego adoption? Given the rival's initial production cost, will a high-cost firm or a low-cost firm be more likely to adopt the innovation? Given its own production cost, will a firm be more likely to innovate if its rival is a high-cost or a low-cost firm? What are the comparative static effects of a change in the level of adoption costs, discount rates or expectations about the profitability of the innovation? Section IV examines Bayesian equilibrium in a game of incomplete information where the rival's adoption costs are unknown. The analysis of this model follows that of Section III. Section V discusses the possibility of extending the static models to a dynamic one involving a sequence of innovations and places this model in the context of related literature.

II. BASIC NOTATION

Suppose that two firms currently produce a homogeneous good. They compete in a market characterized by Cournot-Nash quantity-setting behavior. Firm i produces at a constant unit cost of m_i. This generates profits for i at the rate $r_i(m) = r_i(m_1,m_2)$. If β_i is firm i's rate of discount, then $\pi_i(m) = r_i(m)/(1-\beta_i)$ represents the present value of the firm's profits using its current technology. Suppose that an alternative technology becomes available to firm i at a cost of k_i. Firm i is uncertain about the extent of the cost reduction the innovation will provide, but believes that the random variable c_i, representing the unit cost using the new technology, is drawn from an interval $M_i = [\underline{c}_i,\bar{c}_i]$ according to the distribution $F_i(\cdot)$. If $M_i = M_j$ and $F_i(\cdot) = F_j(\cdot)$, we will say that the uncertainty is <u>innovation-specific</u>. If $M_i \neq M_j$ and $F_i(\cdot) \neq F_j(\cdot)$, or if $M_i = M_j$ but $F_i(\cdot) \neq F_j(\cdot)$, then the uncertainty will be termed <u>firm-specific</u>.

Adoption costs may differ across firms due to firm-specific characteristics such as implementation and adjustment costs. These characteristics may also give rise to firm-specific uncertainty. However, it is assumed that both firms share the same beliefs regarding M_i and F_i, $i = 1,2$.

Assumption 1. Suppose that F_i is strictly increasing on M_i.

Assumption 2. $r_i(m_1,m_2)$ (and hence $\pi_i(m_1,m_2)$) is bounded, nonnegative and twice continuously differentiable with

(a) $\partial r_i/\partial m_i < 0$;

(b) $\partial r_i/\partial m_j > 0$; and

(c) $\partial^2 r_i/\partial m_i \partial m_j < 0$

for all $(m_1,m_2) \varepsilon M_1 \times M_2$. The Appendix discusses circumstances under which Assumption 2 can be expected to hold.

Since the extent of cost reduction is uncertain, the problem is characterized by imperfect information. If, in addition, firm i is uncertain about the value of firm j's adoption costs k_j, we will say that the problem is characterized by imperfect and incomplete information (Harsanyi, 1967-8). Although this is a static model, it is useful to describe the following timing conventions so as to clarify the informational assumptions. For the game with complete but imperfect information, both firms know $m = (m_1,m_2)$, β_1, β_2, F_1, F_2, k_1 and k_2 at the beginning of the period.

In the middle of the period both firms simultaneously decide whether or not to adopt the new technology. At the end of the period if any firm has chosen to adopt the new technology, its new unit cost -- the random variable c_i -- is realized and the payoffs are collected.[1] In the game with incomplete information, the same conventions apply except that only k_i is known initially by firm i; k_j is not known by, and is never revealed to, firm i. In what follows, the current costs m will be treated as state variables; dependence of the firms' decisions upon the remaining parameters will be suppressed except where it is useful in clarifying the underlying informational assumptions or when performing comparative static analysis.

III. THE GAME WITH COMPLETE INFORMATION

In this section we present a formal model of the economic problem outlined in Section II.

Definition 1. A strategy for firm i in the game with complete information is a function $d_i: M_1 \times M_2 \rightarrow [0,1]$. The expression $d_i(m)$ specifies the probability that i adopts the innovation when current costs are $m = (m_1,m_2)$.

The payoff to firm 1 when the firms play strategies $d(m) = (d_1(m),d_2(m))$ is denoted $V^1(m,d(m))$. For given $d(m)$,

1. It should be pointed out that the ''adoption'' decision is reversible; if the firm discovers that the new technology is actually inferior, it need not implement the new technology. Thus the term ''adoption'' is to be understood in this limited sense; the firm may stop short of full implementation.

$$V^1(m,d(m)) = d_1(m)d_2(m)V^1(m,1,1)$$
$$+(1 - d_1(m))d_2(m)V^1(m,0,1) + d_1(m)(1 - d_2(m)V^1(m,1,0)$$
$$+(1 - d_1(m))(1 - d_2(m))V^1(m,0,0). \quad (1)$$

Thus the payoff for any strategy pair $(d_1(m),d_2(m))$ is a weighted average of the payoffs obtainable from playing the degenerate strategies $d_j(m) = 0$ (i.e., don't adopt) and $d_j(m) = 1$ (i.e., adopt with certainty). These degenerate strategy payoffs are

$$V^1(m,0,0) = r_1(m) + \beta_1\pi_1(m) \quad (2)$$

$$V^1(m,1,0) = r_1(m) + \beta_1 \int_{\underline{c}_1}^{m_1} \pi_1(c_1,m_2)dF_1(c_1)$$
$$+ \beta_1(1-F_1(m_1))\pi_1(m) - k_1 \quad (3)$$

$$V^1(m,0,1) = r_1(m) + \beta_1 \int_{\underline{c}_2}^{m_2} \pi_1(m_1,c_2)dF_2(c_2)$$
$$+ \beta_1(1-F_2(m_2))\pi_1(m) \quad (4)$$

$$V^1(m,1,1) = r_1(m) + \beta_1 \int_{\underline{c}_1}^{m_1} \int_{\underline{c}_2}^{m_2} \pi_1(c_1,c_2)dF_1(c_1)dF_2(c_2)$$
$$+ \beta_1(1-F_1(m_1)) \int_{\underline{c}_2}^{m_2} \pi_1(m_1,c_2)dF_2(c_2)$$
$$+ \beta_1(1-F_2(m_2)) \int_{\underline{c}_1}^{m_1} \pi_1(c_1,m_2)dF_1(c_1)$$
$$+ \beta_1(1-F_1(m_1))(1-F_2(m_2))\pi_1(m) - k_1. \quad (5)$$

These are easily interpreted. Equation (2) says that the payoff to firm 1 if neither firm adopts the new technology is the flow profit $r_1(m)$ plus the present value of future profits under the current technology $\beta_1\pi_1(m)$. Equation (3) gives firm 1's payoff net of adoption costs given that firm 2 retains the current technology and firm 1 adopts the innovation. The term $\beta_1(1-F_1(m_1))\pi_1(m)$ allows for the possibility that the new production process is more costly than the current one. In this event, firm 1 reverts to the less costly technology. Note that this event may have probability zero. Equation (4) gives firm 1's payoff when firm 2 adopts the new technology and firm 1 retains its current production process. Finally, if both adopt the new technology firm 1's expected profits net of adoption costs are given by equation (5). Firm 2's payoffs are defined in the obvious way.

Definition 2. Given k_1, k_2, β_1, β_2, F_1 and F_2, a strategy pair $(d_1^*(\cdot),d_2^*(\cdot))$ is a <u>Nash equilibrium</u> if for all $m \in M_1 \times M_2$,

(a) $V^1(m,d_1^*(m),d_2^*(m)) \geq V^1(m,d_1(m),d_2^*(m))$

for all strategies $d_1(\cdot)$; and

(b) $V^2(m,d_1^*(m),d_2^*(m)) \geq V^2(m,d_1^*(m),d_2(m))$

for all strategies $d_2(\cdot)$.

A standard approach at this point is to attempt to determine firm 1's best response to an arbitrary strategy for firm 2. Unfortunately, with no information regarding the form of the opponent's strategy, this direct approach is not particularly useful. Instead we will go about characterizing the Nash equilibrium in a rather roundabout way.

Define

$$\eta_1(m) - k_1 = V^1(m,1,0) - V^1(m,0,0)$$
$$= \beta_1 \int_{\underline{c}_1}^{m_1} [\pi_1(c_1,m_2) - \pi_1(m_1,m_2)] dF_1(c_1) - k_1. \qquad (6)$$

The expression $\eta_1(m)$ represents the gain to firm 1 due to adoption of the innovation if firm 2 were to use the decision rule $d_2(m) = 0$; i.e., don't adopt. Note that $\eta_1(m) > 0$ for all $m_1 > \underline{c}_1$ and $\eta_1(\underline{c}_1,m_2) = 0$ for all m_2. In addition

$$\partial\eta_1/\partial m_1 = -\beta_1 F_1(m_1)\partial\pi_1(m)/\partial m_1 > 0 \qquad (7)$$

and

$$\partial\eta_1/\partial m_2 = \beta_1 \int_{\underline{c}_1}^{m_1} [\partial\pi_1(c_1,m_2)/\partial m_2 - \partial\pi_1(m_1,m_2)/\partial m_2] dF_1(c_1) > 0 \qquad (8)$$

by assumptions 2(a) and 2(c), respectively. That is, if firm 2 were to eschew adoption, then the net value of adoption to firm 1 is greater the greater are initial costs (m_1,m_2).

Define

$$\phi_1(m) - k_1 = V^1(m,1,1) - V^1(m,0,1)$$

$$= \beta_1 \int_{\underline{c}_1}^{m_1} [\pi_1(c_1,m_2) - \pi_1(m_1,m_2)] dF_1(c_1)$$

$$+\beta_1 \int_{\underline{c}_1}^{m_1} \int_{\underline{c}_2}^{m_2} [\pi_1(c_1,c_2) - \pi_1(c_1,m_2) - \pi_1(m_1,c_2)$$
$$+ \pi_1(m_1,m_2)] dF_1 dF_2 - k_1. \qquad (9)$$

The expression $\phi_1(m)$ represents the gain to firm 1 from adopting the innovation if firm 2 where to choose the strategy $d_2(m) = 1$; that is, adopt with certainty. Note that

$$\phi_1(m) = \eta_1(m) + \beta_1 \int_{\underline{c}_1}^{m_1} \int_{\underline{c}_2}^{m_2} [\pi_1(c_1,c_2) - \pi_1(c_1,m_2)$$
$$- \pi_1(m_1,c_2) + \pi_1(m_1,m_2)] dF_1 dF_2. \qquad (10)$$

Assumption 2(c) implies that the second term of (10) is negative for all $(m_1,m_2) > (\underline{c}_1,\underline{c}_2)$. Thus $\eta_1(m) > \phi_1(m)$ for all $(m_1,m_2) > (\underline{c}_1,\underline{c}_2)$. Moreover, $\phi_1(m_1,\underline{c}_2) = \eta_1(m_1,\underline{c}_2)$ for all m_1 and $\phi_1(\underline{c}_1,m_2) = \eta_1(\underline{c}_1,m_2) = 0$ for all m_2. This implies that if firm 1 chooses to adopt, its gain is greater if firm 2 chooses to forego adoption than if firm 2 chooses to adopt as well. Thus firm 1 always prefers that firm 2 forego adoption of the new technology. For $(m_1,m_2) > (\underline{c}_1,\underline{c}_2)$,

$$\eta_1(m) > \int_{\underline{c}_2}^{m_2} \eta_1(m_1,c_2) dF_2 = \beta_1 \int_{\underline{c}_2}^{m_1} \int_{\underline{c}_1}^{m_2} [\pi_1(c_1,m_2) - \pi_1(m_1,m_2)] dF_1 dF_2.$$

Thus for $(m_1,m_2) > (\underline{c}_1,\underline{c}_2)$

$$\phi_1(m) > \int_{\underline{c}_1}^{m_1}\int_{\underline{c}_2}^{m_2} [\pi_1(c_1,c_2) - \pi_1(m_1,c_2)]dF_1dF_2 > 0$$

by assumption 2(a). The dependence of $\phi_1(m)$ on m_1,m_2 is summarized below.

$$\partial\phi_1/\partial m_1 = -\beta_1F_1(m_1)(1-F_2(m_2))\partial\pi_1(m)/\partial m_1$$
$$-\beta_1F_1(m_1)\int_{\underline{c}_2}^{m_2}[\partial\pi_1(m_1,c_2)/\partial m_1]dF_2 > 0 \qquad (11)$$

by assumption 2(a). In addition,

$$\partial\phi_1/\partial m_2 = \beta_1(1-F_2(m_2))\int_{\underline{c}_1}^{m_1}[\partial\pi_1(c_1,m_2)/\partial m_2$$
$$-\partial\pi_1(m_1,m_2)/\partial m_2]dF_1 > 0 \qquad (12)$$

by assumption 2(c). If firm 2 chooses the strategy ''certainly adopt,'' then firm 1's gain due to adoption is greater the greater are initial costs (m_1,m_2).

The value of adoption to firm 1 (net of adoption costs) given an arbitrary strategy $d_2(m)$ for firm 2 is

$$V^1(m,1,d_2(m)) - V^1(m,0,d_2(m))$$
$$= d_2(m)\phi_1(m) + (1-d_2(m))\eta_1(m) - k_1$$
$$= \eta_1(m) + d_2(m)[\phi_1(m) - \eta_1(m)] - k_1. \qquad (13)$$

Define $\mu_1(m,d_2) = \eta_1(m) + d_2[\phi_1(m) - \eta_1(m)]$. Clearly $\phi_1(m) \leq \mu_1(m,d_2) \leq \eta_1(m)$ for all (m,d_2).

Given an arbitrary strategy for firm 2, $d_2(\cdot)$, a best response for firm 1 is to adopt the innovation if $\mu_1(m,d_2(m)) > k_1$, to forego adoption if $\mu_1(m,d_2(m)) < k_1$ and to do either (or, alternatively, to randomize) if $\mu_1(m,d_2(m)) = k_1$. Rather than attempting to characterize this best response directly, we consider the set $S_1^1 = \{m \in M \mid \phi_1(m) > k_1\}$. Since $\phi_1(m) \leq \mu_1(m,d_2)$ for all (m,d_2), if $m \in S_1^1$, then $\mu_1(m,d_2(m)) > k_1$ so regardless of d_2, firm 1 should play the strategy $d_1^*(m) = 1$. Thus for $m \in S_1^1$, firm 1 has the dominant strategy $d_1^* = 1$, implying that firm 1 should definitely adopt the innovation if $m \in S_1^1$. We can characterize the boundary of S_1^1, $\partial S_1^1 = \{m \in M \mid \phi_1(m) = k_1\}$, as follows.

Assumption 3. $\phi_1(\bar{c}_1,\underline{c}_2) - k_1 > 0$.

Under Assumption 3, $\phi_1(\bar{c}_1,m_2) > k_1$ for all $m_2 \in M_2$. Since $\phi_1(\underline{c}_1,m_2) = 0 < k_1$ for all $m_2 \in M_2$ and $\partial\phi_1/\partial m_1 > 0$, for each m_2 there exists $\tilde{m}_1(m_2) \in (\underline{c}_1,\bar{c}_1)$ such that $\phi_1(\tilde{m}_1(m_2),m_2) = k_1$. Moreover since $\partial\phi_1/\partial m_2 > 0$,

$$\frac{d\tilde{m}_1}{dm_2} = \frac{dm_1}{dm_2}\bigg|_{\phi_1(m)\,\equiv\,k_1} = \frac{-\partial\phi_1/\partial m_2}{\partial\phi_1/\partial m_1} < 0. \qquad (14)$$

Similarly, define $S_0^1 = \{m \in M \mid \eta_1(m) < k_1\}$. Since $\mu_1(m,d_2) \leq \eta_1(m)$ for all (m,d_2), $m \in S_0^1$ implies $\mu_1(m,d_2) < k_1$ so that regardless of d_2, firm 1 should play $d_1^* = 0$. Thus for $m \in S_0^1$, firm 1 has the dominant strategy $d_1^* = 0$. For $m \in S_0^1$, firm 1 doesn't want to adopt even if it knows that firm 2 will not adopt. The boundary of S_0^1, $\partial S_0^1 = \{m \in M \mid \eta_1(m) = k_1\}$, can be characterized as follows.

Since $\eta_1(\bar{c}_1,\underline{c}_2) = \phi_1(\bar{c}_1,\underline{c}_2)$, it follows from Assumption 3 that $\eta_1(\bar{c}_1,\underline{c}_2) > k_1$ which implies $\eta_1(\bar{c}_1,m_2) > k_1$ for all m_2. Since $\eta_1(\underline{c}_1,m_2) = 0 < k_1$ for all m_2 and $\partial\eta_1/\partial m_1 > 0$, for each m_2 there exists $\bar{m}_1(m_2) \in (\underline{c}_1,\bar{c}_1)$ such that $\eta_1(\bar{m}_1(m_2),m_2) = k_1$, with

$$\frac{d\bar{m}_1}{dm_2} = \frac{dm_1}{dm_2} \Big| \eta_1(m) \equiv k_1 = \frac{-\partial\eta_1/\partial m_2}{\partial\eta_1/\partial m_1} < 0. \tag{15}$$

Moreover, the locus $\eta_1(m) \equiv k_1$ lies everywhere to the left of the locus $\phi_1(m) \equiv k_1$ except at $m_2 = \underline{c}_2$, where the loci coincide. This can be summarized graphically as in Figure 1.

The inner strip remains to be characterized, but we can note that for $m \notin \bar{S}_0^1 \cup \bar{S}_1^1$ (where $\bar{S}$ denotes the closure of the set S), a best response for firm 1 to $d_2 = 1$ is $d_1 = 0$, and a best response for firm 1 to $d_2 = 0$ is $d_1 = 1$. More information can be garnered by performing the analogous analysis for firm 2. Graphed in (m_1,m_2) space, this analysis is summarized in Figure 2.

In the inner strip (that is, for $m \notin \bar{S}_0^2 \cup \bar{S}_1^2$), we have $\eta_2(m) > k_2$ and $\phi_2(m) < k_2$. That is, firm 2 prefers to adopt the innovation if firm 1 foregoes adoption and to forego adoption if firm 1 adopts.

Superimposing Figures 1 and 2 indicates that there are potentially three types of Nash equilibria: type (1) wherein both firms have dominant strategies; type (2) wherein a single firm has a dominant strategy and the other plays a best response; and type (3) wherein no firm has a dominant strategy. This is illustrated in Figure 3 with an underline indicating that the strategy is dominant.

The central region possesses two pure strategy equilibria. In addition, it possesses a mixed strategy equilibrium. For each m in the central region, $\eta_i(m) \geq k_i$ while $\phi_i(m) \leq k_i$, and $\eta_i(m) > \phi_i(m)$, i = 1,2. If d_2^* is firm 2's strategy, then firm 1 will be willing to randomize if and only if $V^1(m,1,d_2^*) = V^1(m,0,d_2^*)$. That is, if and only if

$$\eta_1(m) + d_2^*(m)[\phi_1(m) - \eta_1(m)] - k_1 = 0 \tag{16}$$

or equivalently

$$d_2^* = [\eta_1(m) - k_1]/[\eta_1(m) - \phi_1(m)]. \tag{17}$$

Notice that $0 \leq d_2^*(m) \leq 1$ so long as $\eta_1(m) - k_1 \geq 0$ and $\phi_1(m) \leq k_1$; but these are true for all m in the central region. Analogous analysis of firm 2's willingness to randomize yields

$$d_1^*(m) = [\eta_2(m) - k_2]/[\eta_2(m) - \phi_2(m)]. \tag{18}$$

It is easy to show that, in the central region, firm 1 prefers the pure strategy equilibrium (1,0) to the randomized strategy equilibrium, which is preferred by firm 1 to the pure strategy equilibrium (0,1). Of course, firm 2's preferences over these three equilibria are precisely opposite to those of firm 1.

The boundaries ∂S_0^1 and ∂S_1^1, which are of measure zero in $M_1 \times M_2$, may be arbitrarily assigned to the adjacent regions. Thus the following proposition has been established.

Proposition 1. The following policy is a Nash equilibrium strategy for firm 1.

$$d_1^*(m) = \begin{cases} 1 & m \in \overline{R}_1^1 \\ [\eta_2(m) - k_2]/[\eta_2(m) - \phi_2(m)] & m \in R_{10}^1 \\ 0 & m \in \overline{R}_0^1 \setminus \overline{R}_1^1 \cap \overline{R}_0^1 \end{cases} \quad (19)$$

where the regions R_1^1, R_{10}^1 and R_0^1 are as shown in Figure 4.

The other policy in the Nash equilibrium is

$$d_2^*(m) = \begin{cases} 1 & m \in \overline{R}_1^2 \\ [\eta_1(m) - k_1]/[\eta_1(m) - \phi_1(m)] & m \in R_{10}^2 \\ 0 & m \in \overline{R}_0^2 \setminus \overline{R}_1^2 \cap \overline{R}_0^2 \end{cases} \quad (20)$$

It is clear that $R_{10}^1 = R_{10}^2$. The map of the Nash equilibrium pair (d_1^*, d_2^*) is shown in Figure 5.

Consider the effect of a marginal increase in current costs m upon $d_i^*(m)$. For $m \in R_0^i$ or $m \in R_1^i$ it is clear that a marginal increase in either m_1 or m_2 leaves $d_i^*(m)$ unchanged. For $m \in R_{10}^i$,

$$\partial d_i^*/\partial m_i = [(\eta_i - \phi_i)(\partial\eta_i/\partial m_i) - (\eta_i - k_i)\partial(\eta_i - \phi_i)/\partial m_i](\eta_i - \phi_i)^2$$

$$= [(k_i - \phi_i)\partial\eta_i/\partial m_i + (\eta_i - k_i)\partial\phi_i/\partial m_i)]/(\eta_i - \phi_i)^2$$

Since $k_i > \phi_i$ and $\eta_i > k_i$ for $m \in R_{10}^i$, $\partial d_i^*/\partial m_i > 0$. Similarly, $\partial d_i^*/\partial m_j > 0$ for $m \in R_{10}^i$. Thus for $m \in R_1^i \cup R_0^i \cup R_{10}^i$, $d_i^*(m)$ is locally increasing in m_1 and m_2. However, it is apparent from Figure 4 that there are portions of the boundary of R_{10}^1 where a marginal increase in either m_1 or m_2 causes $d_1^*(m)$ to decrease. These portions consist of the curves from a to b and c to d in Figure 4. Thus $d_i^*(m)$ possesses no global monotonicity properties. Consequently, the questions posed at the beginning of this paper (Given the rival's initial production cost, will a high-cost firm or a low-cost firm be more likely to adopt the innovation? Given its own production cost, will a firm be more likely to innovate if its rival is a high-cost or a low-cost firm?) cannot be answered unambigously. However, the following qualified statements can be made as corollaries to Proposition 1.

Corollary 1. If initial costs (m_1, m_2) are sufficiently high (low) then both (neither) of the firms will adopt the new technology.

Corollary 2. If one firm's initial costs are sufficiently high and the other firm's costs are sufficiently low, then the high-cost firm will adopt the new technology and the low-cost firm will not.

Corollary 2 is the most interesting from the point of view of industrial organization. It says that if costs are too disparate, the high-cost firm will be the one to adopt the new technology. Thus in this framework there is a tendency toward more equal-sized firms rather than a tendency toward monopolization of the industry. This is despite the fact that the low-cost firm has a greater market share on which to gain by cost-reduction. Essentially, if the low-cost firm (say firm j) has costs which are sufficiently close to $\underline{c}_j$, then the new technology is much less likely to provide a (sufficiently) better technology, one which justifies the outlay of k_j.

Although the pattern of the Nash equilibrium strategies seems complex enough as it is, an implicit simplifying assumption has been made in Figures 1-5. Since the loci $\eta_1(m) \equiv k_1$ and $\phi_1(m) \equiv k_1$ each intersect the loci $\eta_2(m) \equiv k_1$ and $\phi_2(m) \equiv k_2$ <u>at least once,</u> they divide $M_1 \times M_2$ into at least 9 regions as in Figure 3. However, these loci may have multiple intersections. In this case, the adoption and nonadoption sets need not be connected. Nevertheless, the foregoing analysis is sufficient to characterize the Nash equilibrium pattern. This is illustrated in Figure 6 where the shaded region indicates randomization.

In order to examine comparative static effects, let us explicitly denote the dependence of the sets $\overline{R}_0^i$, $\overline{R}_1^i$ and R_{10}^i upon k and

β by $\overline{R}_0^i(k_1,k_2,\beta_1,\beta_2)$, $\overline{R}_1^i(k_1,k_2,\beta_1,\beta_2)$ and $R_{10}^i(k_1,k_2,\beta_1,\beta_2)$, i = 1,2. We will rely upon the following trivial lemmas to characterize comparative static effects.

Lemma A. $m \in \overline{R}_0^i(k_1,k_2,\beta_1,\beta_2)$ if and only if either (A1) $\eta_i(m) \leq k_i$ (that is, 0 is a dominant strategy for i); or (A2) $\phi_j(m) \geq k_j$ and $\phi_i(m) \leq k_i$ (that is, 1 is a dominant strategy for j and 0 is a best response for i).

Lemma B. $m \in \overline{R}_1^i(k_1,k_2,\beta_1,\beta_2)$ if and only if either (B1) $\phi_i(m) \geq k_i$ (that is, 1 is a dominant strategy for i), or (B2) $\eta_j(m) \leq k_j$ and $\eta_i(m) \geq k_i$ (that is, 0 is a dominant strategy for j and 1 is a best response for i).

Lemma C. $m \in R_{10}^i(k_1,k_2,\beta_1,\beta_2)$ if and only if both (C1) $\eta_i(m) > k_i > \phi_i(m)$ and (C2) $\eta_j(m) > k_j > \phi_j(m)$.

Proposition 2. $d_i^*(m)$ either decreases or remains the same in response to an increase in k_i (or, equivalently, a decrease in β_i).

Consider the impact of an increase in k_1 upon $d_1^*(m)$. The claim that $d_1^*(m)$ is nonincreasing in k_1 is equivalent to the following two claims.

Claim 1. $m \in \overline{R}_0^1(k_1,k_2,\beta_1,\beta_2)$ implies $m \in \overline{R}_0^1(k_1^o,k_2,\beta_1,\beta_2)$ for all $k_1^o \geq k_1$.

Claim 2. $m \in R_{10}^1(k_1,k_2,\beta_1,\beta_2)$ implies that there does not exist

$k_1^o \geq k_1$ such that $m \varepsilon \bar{R}_1^1(k_1^o, k_2, \beta_1, \beta_2)$. Moreover, if $m \varepsilon R_{10}^1(k_1^o, k_2, \beta_1, \beta_2)$, then $d_1^*(m)$ is unaffected.

Proof of Claim 1. $m \varepsilon \bar{R}_0^1(k_1, k_2, \beta_1, \beta_2)$ implies either (a) $\eta_1(m) \leq k_1$ or (b) $\phi_2(m) \geq k_2$ and $\phi_1(m) \leq k_1$ by Lemma A. If (a) holds at k_1, then (a) holds for all $k_1^o \geq k_1$. If (b) holds at k_1, again (b) holds a fortiori for all $k_1^o \geq k_1$. Lemma A then implies that $m \varepsilon \bar{R}_0^1(k_1^o, k_2, \beta_1, \beta_2)$ for all $k_1^o \geq k_1$.

Q.E.D.

Proof of Claim 2. $m \varepsilon R_{10}^1(k_1, k_2, \beta_1, \beta_2)$ implies both (a) $\eta_2(m) > k_2$ and (b) $\phi_1(m) < k_1$. Suppose, contrary to Claim 2, that there exists $k_1^o \geq k_1$ such that $m \varepsilon \bar{R}_1^1(k_1^o, k_2, \beta_1, \beta_2)$. Then Lemma B states that either (c) $\eta_2(m) \leq k_2$ and $\eta_1 \geq k_1^o$ or (d) $\phi_1(m) \geq k_1^o$. Since (c) contradicts (a) and (d) contradicts (b) whenever $k_1^o \geq k_1$, we are forced to conclude that there does not exist $k_1^o \geq k_1$ such that $m \varepsilon \bar{R}_1^1(k_1^o, k_2, \beta_1, \beta_2)$. The last statement of Claim 2 follows trivially from equation (19).

Q.E.D.

Proposition 3.

(a) If $m \not\varepsilon R_{10}^i(k_1, k_2, \beta_1, \beta_2)$, then $d_i^*(m)$ either increases or remains the same in response to an increase in k_j (or, equivalently, a decrease in β_j).

(b) If $m \varepsilon R_{10}^i(k_1, k_2, \beta_1, \beta_2)$, then $d_i^*(m)$ may either increase or decrease in response to an increase in k_j (or, equivalently, a decrease in β_j).

Proposition 3(a) is equivalent to Claim 3 below, for i = 1. Claim 4 below is a more precise statement of Proposition 3(b), for i = 1.

Claim 3. $m \varepsilon \bar{R}_1^1(k_1, k_2, \beta_1, \beta_2)$ implies that $m \varepsilon \bar{R}_1^1(k_1, k_2^o, \beta_1, \beta_2)$ for all $k_2^o \geq k_2$.

Claim 4. $m \varepsilon R_{10}^1(k_1, k_2, \beta_1, \beta_2)$ implies that there does not exist $k_2^o \geq k_2$ such that $m \varepsilon \bar{R}_0^1(k_1, k_2^o, \beta_1, \beta_2)$. However, if $m \varepsilon R_{10}^1(k_1, k_2^o, \beta_1, \beta_2)$, then $d_1^*(m)$ decreases with an increase in k_2.

Proof of Claim 3. $m \varepsilon \bar{R}_1^1(k_1, k_2, \beta_1, \beta_2)$ implies that either (a) $\phi_1(m) \geq k_1$ or (b) $\eta_2(m) \leq k_2$ and $\eta_1 \geq k_1$ by Lemma A. An increase in k_2 has no effect upon (a) while (b) holds a fortiori for all $k_2^o \geq k_2$. Thus $m \varepsilon \bar{R}_1^1(k_1, k_2^o, \beta_1, \beta_2)$.

Q.E.D.

Proof of Claim 4. $m \varepsilon R_{10}^1(k_1, k_2, \beta_1, \beta_2)$ implies both (a) $\eta_1(m) > k_1$ and (b) $\phi_2(m) < k_2$. Suppose, contrary to Claim 4, that there exists $k_2^o \geq k_2$ such that $m \varepsilon \bar{R}_0^1(k_1, k_2^o, \beta_1, \beta_2)$. Then either (c) $\eta_1(m) \geq k_1$ or (d) $\phi_2(m) \geq k_2^o$ and $\phi_1(m) \leq k_1$. But (c) contradicts (a) and (d) contradicts (b) for all $k_2^o \geq k_2$. Thus there does not exist $k_2^o \geq k_2$

such that $m \in \bar{R}_0^1(k_1,k_2^o,\beta_1,\beta_2)$. The final statement of Claim 4 follows from equation (19).

$$\partial d_1^*(m)/\partial k_2 = -1/(\eta_2(m) - \phi_2(m)) < 0.$$

Q.E.D.

We are now equipped to describe the impact of an increase in firm 1's adoption costs from k_1 to k_1^o upon the Nash equilibrium strategy pair $(d_1^*(m),d_2^*(m))$. If $m \notin R_{10}^1(k_1,k_2,\beta_1,\beta_2)$ or $m \in R_{10}^1(k_1,k_2,\beta_1,\beta_2)$ but $m \notin R_{10}^1(k_1^o,k_2,\beta_1,\beta_2)$, then firm 1 is no more likely to adopt and firm 2 is no less likely to adopt. However, if $m \in R_{10}^1(k_1,k_2,\beta_1,\beta_2)$ and $m \in R_{10}^1(k_1^o,k_2,\beta_1,\beta_2)$, then although firm 1 randomizes using the same probability as before, firm 1 now faces higher costs of adoption and is <u>willing to randomize</u> only if firm 2 lowers its probability of adoption. Thus in this case firm 1 is precisely as likely to adopt, while firm 2 is less likely to adopt.

IV. THE GAME WITH INCOMPLETE INFORMATION

In this section we will make an alternative assumption regarding the information the firms possess when making their decisions. Suppose that firm i knows its own adoption costs k_i but not k_j, firm j's adoption costs. This results in a game of incomplete information (Harsanyi, 1967-8). Using Harsanyi's reinterpretation of incomplete information as complete but imperfect information, we assume that the firms are Bayesian players with the same prior beliefs regarding the distributions of k_1 and k_2, now regarded as random

variables. That is, both firms believe that the adoption cost parameters k_1, k_2 are drawn (independently, for simplicity) from the intervals $K_1 = [\underline{k}_1,\bar{k}_1]$ and $K_2 = [\underline{k}_2,\bar{k}_2]$ according to the distributions $G_1(\cdot)$ and $G_2(\cdot)$, respectively. Firm i observes the random variable k_i before it selects its strategy; firm i does not observe k_j. Therefore a strategy for i may be contingent upon k_i, but not k_j. This informational assumption is emphasized by the definition of a strategy given below.

Definition 3. A <u>strategy</u> for firm i in the game with incomplete information is a function δ_i: $M_1 \times M_2 \times K_i \to [0,1]$. The expression $\delta_i(m,k_i)$ represents the probability that firm i will adopt the innovation when current costs are $m = (m_1,m_2)$ and firm i's adoption costs are k_i.

Since firm 1 cannot observe k_2 and knows that firm 2 cannot observe k_1, firm 1 can only make conjectures about what action firm 2 will take. These conjectures cannot depend upon k_2 since firm 1 can't observe k_2; nor can they depend upon k_1 since firm 2's <u>actual</u> behavior cannot depend upon k_1 and so 1's conjectures about 2's behavior should not depend upon information which firm 2 itself could not possibly possess in making its own decision. Thus firm 1's conjectures about firm 2's behavior can depend only upon current costs m. Define

$$\rho_2(m) = Pr_1\{\text{firm 2 adopts given m}\}$$

to be firm 1's <u>conjecture</u> regarding firm 2's probability of adoption.

Similarly, define $\rho_1(m)$ to be firm 2's conjectures about 1's probability of adoption when current costs are m. Then the expected value of adoption by firm 1 (gross of adoption costs), given firm 1's conjectures, is

$$\mu_1(m,\rho_2) = \rho_2(m)\phi_1(m) + (1 - \rho_2(m))\eta_1(m)$$
$$= \eta_1(m) + \rho_2(m)[\phi_1(m) - \eta_1(m)] > 0 \quad (21)$$

for $m_1 > \underline{c}_1$. For given k_1, firm 1 prefers to adopt if $\mu_1(m,\rho_2) < k_1$ and is indifferent if $\mu_1(m,\rho_2) = k_1$. Thus an optimal decision rule for firm 1 <u>contingent upon its conjectures</u> $\rho_2(m)$ is

$$f_1(m,k_1,\rho_2(m)) = \begin{cases} 1 & \mu_1(m,\rho_2) > k_1 \\ [0,1] & \mu_1(m,\rho_2) = k_1 \\ 0 & \mu_1(m,\rho_2) < k_1 \end{cases} \quad (22)$$

Before k_1 is revealed to firm 1, this contingent decision rule for firm 1 would generate an adoption probability for firm 1 of

$$\Pr\{\mu_1(m,\rho_2) \geq k_1\} = G_1(\mu_1(m,\rho_2(m))).$$

Definition 4. The conjectures $(\rho_1^*(\cdot),\rho_2^*(\cdot))$ are <u>consistent</u> if, for all $m \in M_1 \times M_2$,

$$\rho_1^*(m) = G_1(\mu_1(m,\rho_2^*(m)))$$

and

$$\rho_2^*(m) = G_2(\mu_2(m,\rho_1^*(m))).$$

That is, conjectures are consistent if they might arise from rational expectations on the parts of the firms.

Definition 5. A strategy pair (δ_1^*,δ_2^*) is a <u>Bayesian equilibrium</u> if for all $m \in M_1 \times M_2$ and for all $k_1 \in K_1$, $k_2 \in K_2$,

$$\delta_1^*(m,k_1) = f_1(m,k_1,\rho_2^*(m))$$

and

$$\delta_2^*(m,k_2) = f_2(m,k_2,\rho_1^*(m)).$$

That is, a Bayesian equilibrium is a pair of optimal decision rules which are based on consistent conjectures.

In general, finding a pair of consistent conjectures is equivalent to finding a fixed point of the mapping $T\rho_1(m)$, where T is defined by $T\rho_1(m) = G_1(\mu_1(m,G_2(\mu_2(m,\rho_1(m)))))$, in a function space. This is quite difficult, especially with no information about the distribution functions G_1,G_2. However, the following result is easily established.

Proposition 4. Suppose that $G_i(\cdot)$ is uniform on K_i and suppose that

$$\min_m \phi_i(m) \geq \underline{k}_i$$

and

$$\max_m \eta_i(m) \leq \overline{k}_i,$$

$i = 1,2$. That is, if $k_i = \underline{k}_i$, then firm i will adopt the innovation for all m even if firm j is certain to do so as well; and if $k_i = \overline{k}_i$

then firm i will never adopt the innovation, even if firm j foregoes it also. Then there exists a unique pair of consistent conjectures $(\rho_1^*(\cdot),\rho_2^*(\cdot))$.

Proof. $G_i(k_i) = (k_i - \underline{k}_i)/\Delta K_i$, where $\Delta K_i = \bar{k}_i - \underline{k}_i$. Applying the definition of consistent conjectures yields the equations

$$\rho_1^* = [\eta_1 - \underline{k}_1 + \rho_2^*(\phi_1 - \eta_1)]/\Delta K_1 \tag{23}$$

and

$$\rho_2^* = [\eta_2 - \underline{k}_2 + \rho_1^*(\phi_2 - \eta_2)]/\Delta K_2 \tag{24}$$

which can be uniquely solved for (ρ_1^*,ρ_2^*):

$$\rho_1^* = \frac{\Delta K_2(\eta_1 - \underline{k}_1) - (\eta_1 - \phi_1)(\eta_2 - \underline{k}_2)}{\Delta K_1\Delta K_1 - (\eta_1 - \phi_1)(\eta_2 - \phi_2)} \tag{25}$$

and

$$\rho_2^* = \frac{\Delta K_1(\eta_2 - \underline{k}_2) - (\eta_2 - \phi_2)(\eta_1 - \underline{k}_1)}{\Delta K_1\Delta K_2 - (\eta_1 - \phi_1)(\eta_2 - \phi_2)} \tag{26}$$

where the dependence of η_i, ϕ_i and ρ_i on m is implicit.

To see that $0 \leq \rho_1^*(m) \leq 1$, note that the numerator

$$\Delta K_2(\eta_1 - \underline{k}_1) - (\eta_1 - \phi_1)(\eta_2 - \underline{k}_2) \geq 0$$

because $\Delta K_2 \geq \eta_2 - \underline{k}_2$ and $\eta_1 - \underline{k}_1 \geq \eta_1 - \phi_1$ for all m. In addition, the denominator

$$\Delta K_1\Delta K_2 - (\eta_1 - \phi_1)(\eta_2 - \phi_2) > 0$$

since $\Delta K_1 > \eta_1 - \phi_1$ and $\Delta K_2 > \eta_2 - \phi_2$ for all m; finally, the numerator never exceeds the denominator.

$$\begin{aligned}\Delta K_1\Delta K_2 - (\eta_1 - \phi_1)(\eta_2 - \phi_2) - \Delta K_2(\eta_1 - \underline{k}_1) + (\eta_1 - \phi_1)(\eta_2 - \underline{k}_2) \\ = \Delta K_2(\bar{k}_1 - \eta_1) + (\eta_1 - \phi_1)(\phi_2 - \underline{k}_2) \geq 0\end{aligned}$$

under the hypotheses of Proposition 4.

Q.E.D.

We can conclude that the Nash equilibrium value function for firm 1 is

$$V^{1*}(m,k_1) = \max \begin{cases} \rho_2^*(m)V^1(m,1,1) + (1-\rho_2^*(m))V^1(m,1,0), \\ \rho_2^*(m)V^1(m,0,1) + (1 - \rho_2^*(m))V^1(m,0,0). \end{cases} \tag{27}$$

Unfortunately, the dependence of ρ_1^* and V^{1*} upon m_1 and m_2 is complicated.

$$\begin{aligned}\partial\rho_1^*/\partial m_i = (1/D)^2\{D[(\partial\eta_1/\partial m_i)(\bar{k}_2 - \eta_2) + (\partial\phi_1/\partial m_i)(\eta_2 - \underline{k}_2) \\ + (\phi_1 - \eta_1)\partial\eta_2/\partial m_i] - N_i[dD]\},\end{aligned}$$

where N_i is the numerator of ρ_i^* and D is the denominator. All terms in this expression are positive with the exception of $(\phi_1 - \eta_1)\partial\eta_2/\partial m_i < 0$. Thus although it seems likely that $\partial\rho_1^*/\partial m_i > 0$, $i = 1,2$, this cannot be directly established.

If we could establish that $\partial\rho_2^*/\partial m_1 > 0$, it follows that V^{1*} is decreasing in m_1.

$$\partial V^{1*}/\partial m_i = (\partial \rho_2^*/\partial m_i)(\ {}^1(m,1,1) - V^1(m,1,0)) + \rho_2^* \partial V^1(m,1,1)/\partial m_i + (1 - \rho_2^*)\partial V^1(m,1,0)/\partial m_i \quad (28)$$

for $\mu_1(m,\rho_2^*(m)) > k_1$, and

$$\partial V^{1*}/\partial m_i = (\partial \rho_2^*/\partial m_i)(V^1(m,0,1) - V^1(m,0,0)) + \rho_2^* \partial V^1(m,0,1)/\partial m_i + (1 - \rho_2^*)\partial V^1(m,0,0)/\partial m_i \quad (29)$$

for $\mu_1(m,\rho_2^*(m)) < k_1$. Each term of these expressions is negative for $i = 1$ and the continuity of V^{1*} implies that V^{1*} is decreasing in m_1.

A similar analysis for $i = 2$ does not permit us to conclude that V^{1*} is increasing in m_2, since the term $V^1(m,d_1,1) - V^1(m,d_1,0) < 0$ for $d_1 = 0, 1$ while $\partial\ {}^1(m,d_1,d_2)/\partial m_2 > 0$ for $d_1 = 0,1$ and $d_2 = 0,1$.

V. EXTENSIONS AND RELATED LITERATURE

A deterministic model of the adoption of an innovation (the telephone) which is assumed to generate positive externalities in consumption is developed by Rholfs (1974). Dybvig and Spatt (1980) extend and generalize the analysis of innovations which produce positive externalities. They discuss the means by which government can promote adoption of such innovations. They also address these questions in a model with negative externalities. The case of cost-reducing innovation is such a model if we consider only the

preferences of the two firms and not those of consumers. However, the Dybvig-Spatt results are inapplicable to our case of cost-reducing innovation since their assumptions (b) and (c) (p. 22) both fail to hold, even when the profitablity of the innovation is known. If we consider the preferences of consumers and the two firms, then adoption by one firm provides a positive externality to consumers and a uegative externality to the rival firm. A model of innovation adoption which allows one agent's actions to simultaneously generate negative externalities for some agents and positive externalities for other agents is that of Allen (1980). Using an alternative equilibrium concept, and under the hypothesis that each agent strictly randomizes, Allen proves the existence of a unique equilibrium distribution function describing the likelihood that each possible user set will occur.

A deterministic model involving a sequence of innovations is developed in Dasgupta and Stiglitz (1980). In this model the population grows at an exogenous rate, increasing the demand for the product and providing a growing incentive for cost-reducing innovation. Technical advance is assumed to take place continuously, but innovation -- the adoption of a new technology -- occurs only at discrete intervals due to the fixed cost associated with implementation. Since Dasgupta and Stiglitz allow their firms to innovate only once, there is no externality generated by the concurrent adoption decision of a rival firm. Instead, each firm simply selects a date at which it innovates and enters the industry, taking the time paths of demand, (feasible) unit production costs and adoption costs, as well as the innovation dates of the other firms, as given. Dasgupta and Stiglitz suggest that such a framework admits a steady state such that innovation occurs at regular intervals and involves a constant rate of technical progress, the magnitudes of which are generated by the equilibrium play of the innovating firms. A stochastic model involving a sequence of innovations can be found in Balcer and Lippman (1980). The innovations are generated in a manner which is exogenous to the industry, which consists of a single firm. The firm must decide whether (and when) to adopt the current best technology, the profitability of which is known, or to wait and adopt a later innovation. The firm is uncertain about both the timing and magnitude of future innovations.

The present paper is most closely related to that of Jensen (1980). In Jensen's model, two firms decide, for each of a finite number of decision periods, whether or not to adopt an innovation of uncertain profitability. Each is uncertain about the innovation's value (''good'' or ''bad'') and about the opponent's prior assessment that the innovation is ''good.'' Each period an external source provides a signal regarding the value of the innovation and priors are updated upon the basis of the information received that period. At first glance, this model appears to be dynamic. However, certain assumptions regarding the extreme myopia of the firms effectively reduces this to finitely many strategically independent static games linked together by a Bayesian updating rule.

In Jensen's model, the innovation is either ''good'' or ''bad.'' That is, if the innovation were known to be ''good'' then a firm would adopt it irrespective of its rival's decision. Alternatively, if the innovation were known to be ''bad'' then a firm would forego adoption regardless of its rival's decision. In this paper, in addition to these extreme possiblities, we admit the case wherein, if the profitability of the innovation were known, the value of adopting it still depends upon the rival's action. Even if an innovation is worth adopting if the rival foregoes adoption, it may not be worth adopting if the rival also adopts it, since this reduces the extent of the adoption benefits. This is because there is a continuum of possible costs associated with the new technology. In Jensen, the adoption decision is irreversible; it is reversible in this paper. That is, the firm needn't use the new technology if it discovers that it is unprofitable. In Jensen, the imperfect information is innovation-specific; this model allows firm-specific imperfect information as well. Both models contain elements of incomplete information; in Jensen the rival's prior assessment of the probability that the innovation is ''good'' is unknown, while in this paper the rival's cost of adoption may be unknown.

Finally, by considering initial costs as state variables, we are able to address questions of the sort discussed in the Introduction and in Section III, which is not done in Jensen (1980). Moreover, this formulation is the natural one to use in an attempt to extend the static model to a dynamic model involving a sequence of innovations. This would yield a dynamic, stochastic game theoretic model of the evolution of market structure as a result of technical advance. If we had been able to conclude from the model of Section IV that the Nash equilibrium value functions had inherited the properties of the profit functions described in Assumption 2, then we would be essentially finished. We could then extend the model recursively to an arbitrary finite number of innovations in the manner of dynamic programming. However, this goal appears to be out of reach at the moment since we cannot guarantee that these properties are inherited. One goal of future research in this area should be to determine reasonable sufficient conditions which would enable us to make the aforementioned extensions.

APPENDIX

SUFFICIENT CONDITIONS FOR ASSUMPTIONS 2(a) – (b).

Claim: Recall that $r_i(m) = (p(\bar{q}_1 + \bar{q}_2) - m_i)\bar{q}_i(m)$, where $(\bar{q}_1(m), \bar{q}_2(m))$ is a Cournot-Nash equilibrium. Then sufficient conditions for Assumptions 2(a)-(b) to hold is that $p' < 0$ and $p''\bar{q}_i + p' < 0$.

Proof. The existence of Cournot-Nash equilibrium is treated extensively elsewhere and will not be dealt with here. A Cournot-Nash equilibrium $(\bar{q}_1(m), \bar{q}_2(m))$ must satisfy

$$x_i = p'\bar{q}_i + p - m_i = 0, \quad i = 1,2,$$

and

$$x_{ii} = p''\bar{q}_i + 2p' < 0, \quad i = 1,2.$$

Thus $\partial r_i/\partial m_i = (p'\partial\bar{q}_j/\partial m_i - 1)\bar{q}_i$ and $\partial r_i/\partial m_j = p'\bar{q}_i(\partial\bar{q}_j/\partial m_j)$. Assuming that $p' < 0$, a sufficient condition for $\partial r_i/\partial m_i < 0$ is $\partial\bar{q}_j/\partial m_i > 0$; and a sufficient condition for $\partial r_i/\partial m_j > 0$ is $\partial\bar{q}_j/\partial m_j < 0$.

Differentiating x_1, x_2 totally and solving for the desired partial derivatives implies that

$$\partial\bar{q}_i/\partial m_i = x_{jj}/(x_{11}x_{22} - x_{12}x_{21})$$

and

$$\partial\bar{q}_i/\partial m_j = -x_{ij}/(x_{11}x_{22} - x_{12}x_{21}),$$

where $x_{ij} = p''\bar{q}_i + p'$. Notice that if $x_{ij} < 0$ and $p' < 0$, then $|x_{ii}| > |x_{ij}|$. Thus sufficient conditions for $\partial\bar{q}_i/\partial m_i < 0$ and $\partial\bar{q}_i/\partial m_j > 0$, $i = 1,2$ $(i \neq j)$ are that $x_{ij} < 0$, $i = 1,2$ $(j \neq i)$.

Q.E.D.

Assumption 2(c) states that $\partial^2 r_i/\partial m_i \partial m_j < 0$. Since

$$\begin{aligned}\partial^2 r_i/\partial m_i \partial m_j = {} & x_{ij}(\partial\bar{q}_i/\partial m_i)(\partial\bar{q}_j/\partial m_j) \\ & + p''\bar{q}_i(\partial\bar{q}_j/\partial m_j)(\partial\bar{q}_j/\partial m_i) \\ & + p'\bar{q}_i(\partial^2\bar{q}_j/\partial m_i \partial m_j),\end{aligned}$$

it is clear that sufficient conditions for $\partial^2 r_i/\partial m_i \partial m_j < 0$ are more complex. The first term is negative if $x_{ij} < 0$; the second term is negative if $p'' > 0$. The third term is of unknown sign. However, Assumptions 2(a) – (c) are easily shown to hold for the demand functions $P = a - b\ln(q_1 + q_2)$ (for $\bar{c}_i \leq b$), $P = a - b(q_1 + q_2)$ (for $\bar{c}_i \leq a/2$) and $P = a + b/(q_1 + q_2)$ (for $\underline{c}_i \geq a$), where the parameter restrictions are required for non-negativity of profits.

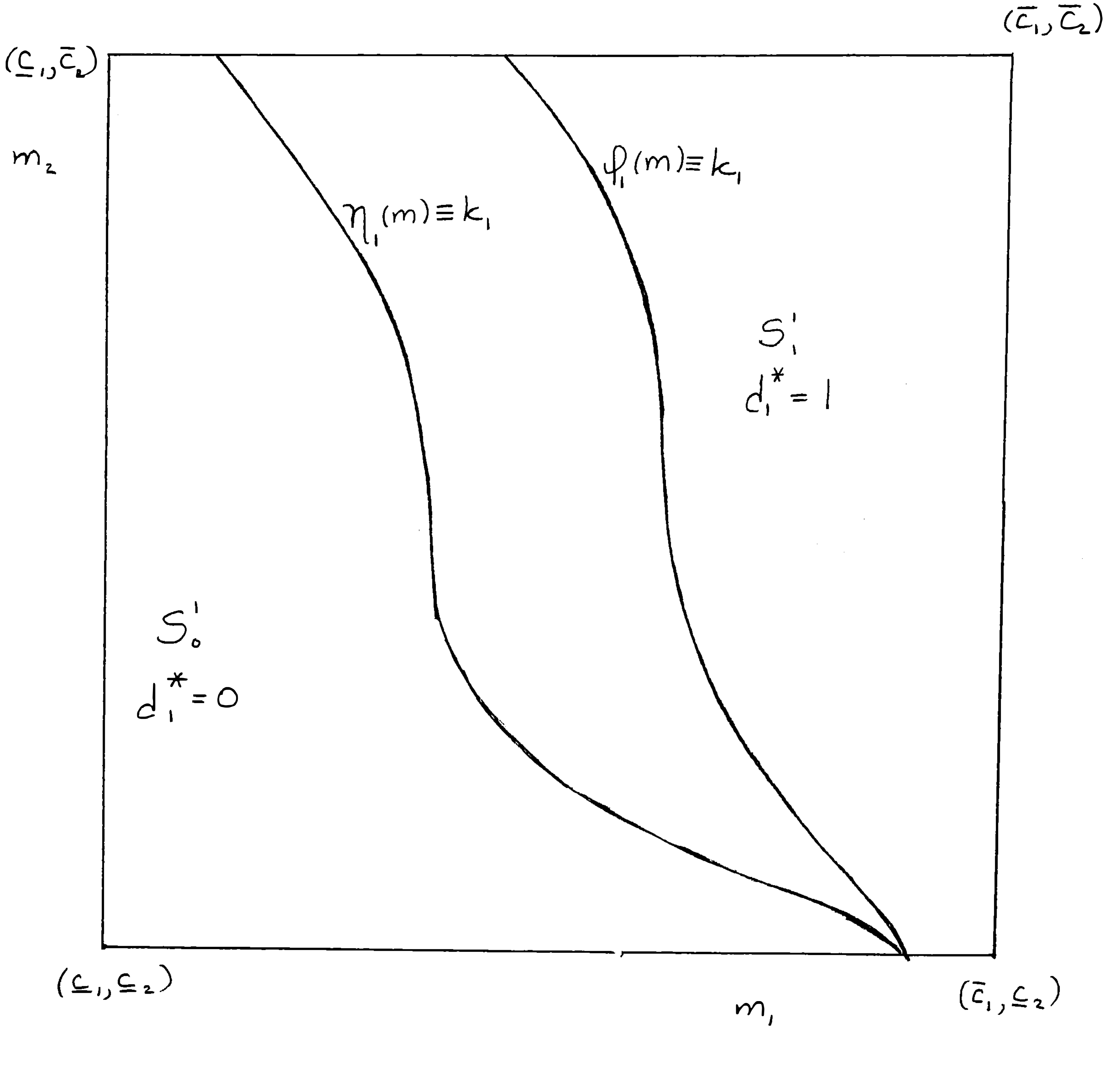
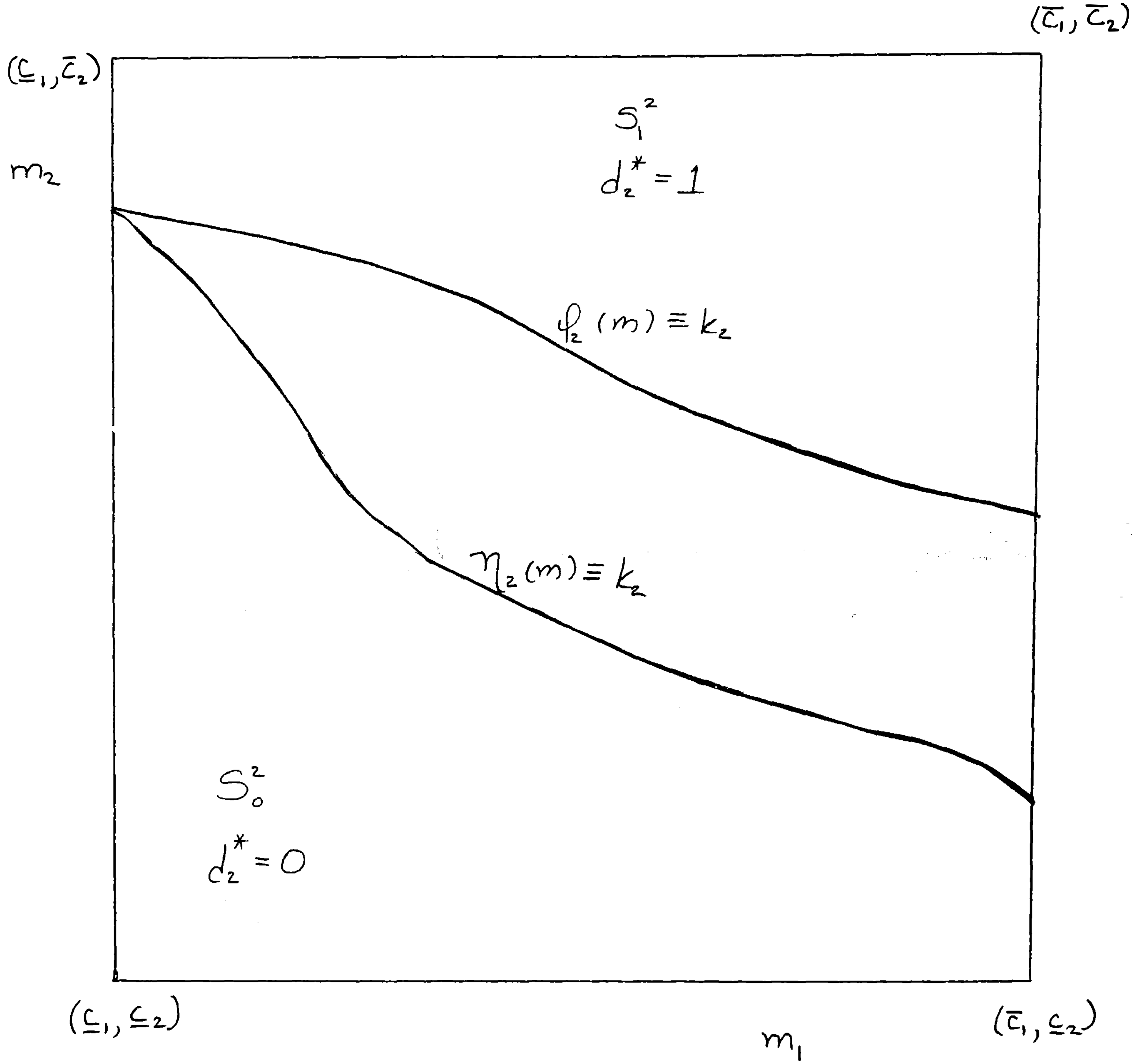

$(\underline{c}_1, \bar{c}_2)$
$(\bar{c}_1, \bar{c}_2)$
m_2
$\varphi_1(m) \equiv k_1$
$\eta_1(m) \equiv k_1$
S_1^1
$d_1^* = 1$
S_0^1
$d_1^* = 0$
$(\underline{c}_1, \underline{c}_2)$
m_1
$(\bar{c}_1, \underline{c}_2)$

Figure 1



$(\underline{c}_1, \bar{c}_2)$
$(\bar{c}_1, \bar{c}_2)$
m_2
S_1^2
$d_2^* = 1$
$\varphi_2(m) \equiv k_2$
$\eta_2(m) \equiv k_2$
S_0^2
$d_2^* = 0$
$(\underline{c}_1, \underline{c}_2)$
m_1
$(\bar{c}_1, \underline{c}_2)$

Figure 2

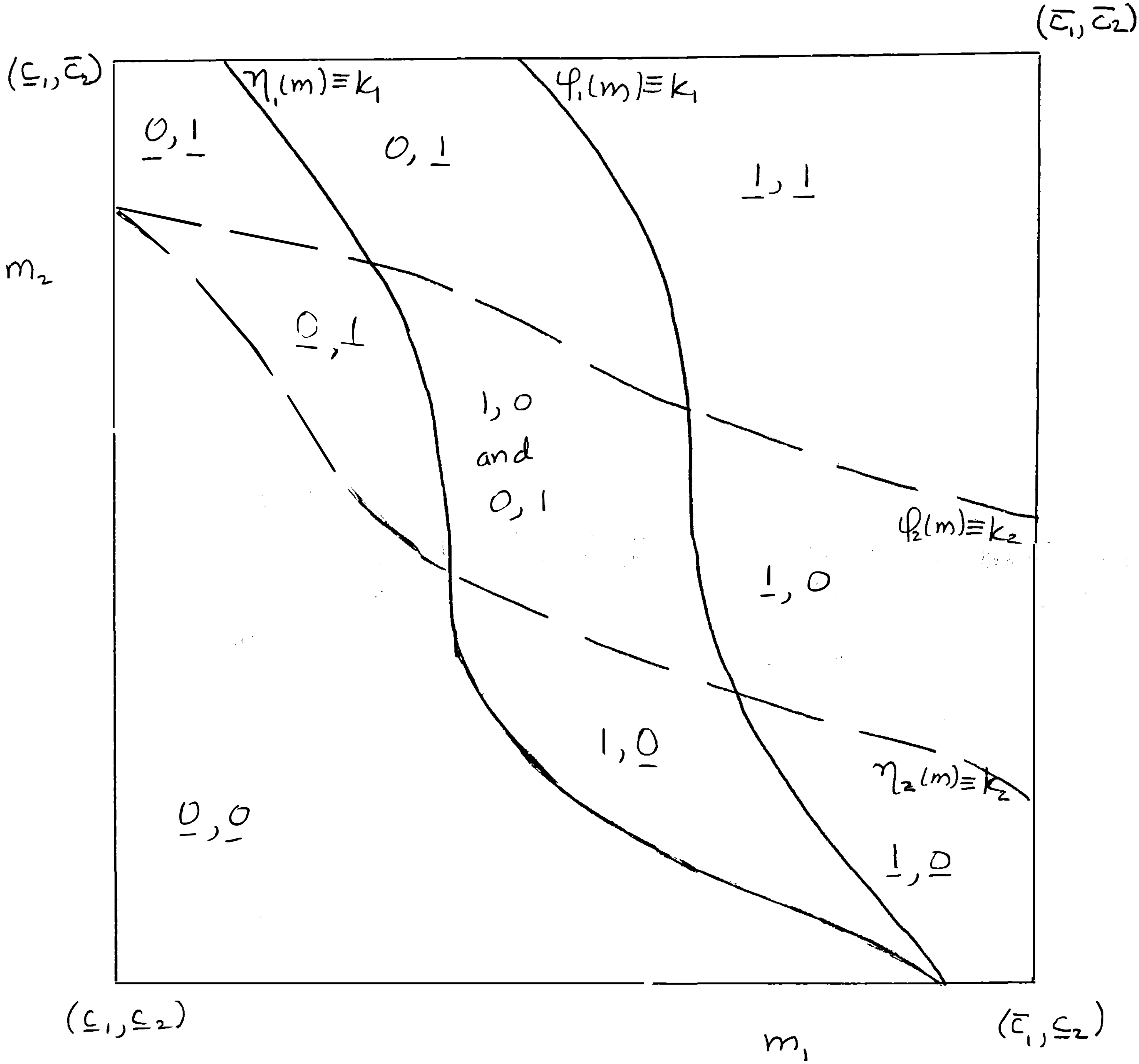
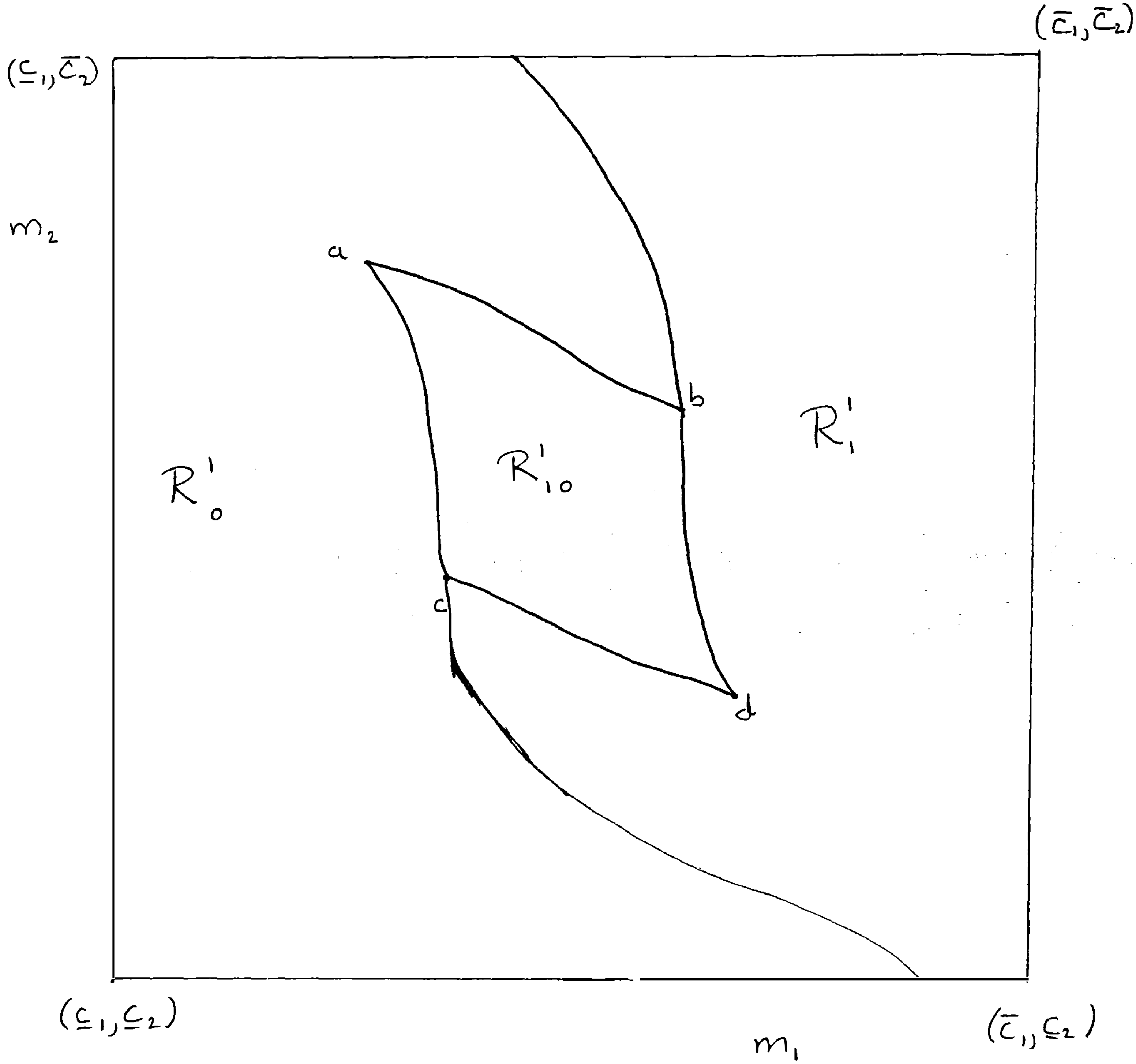
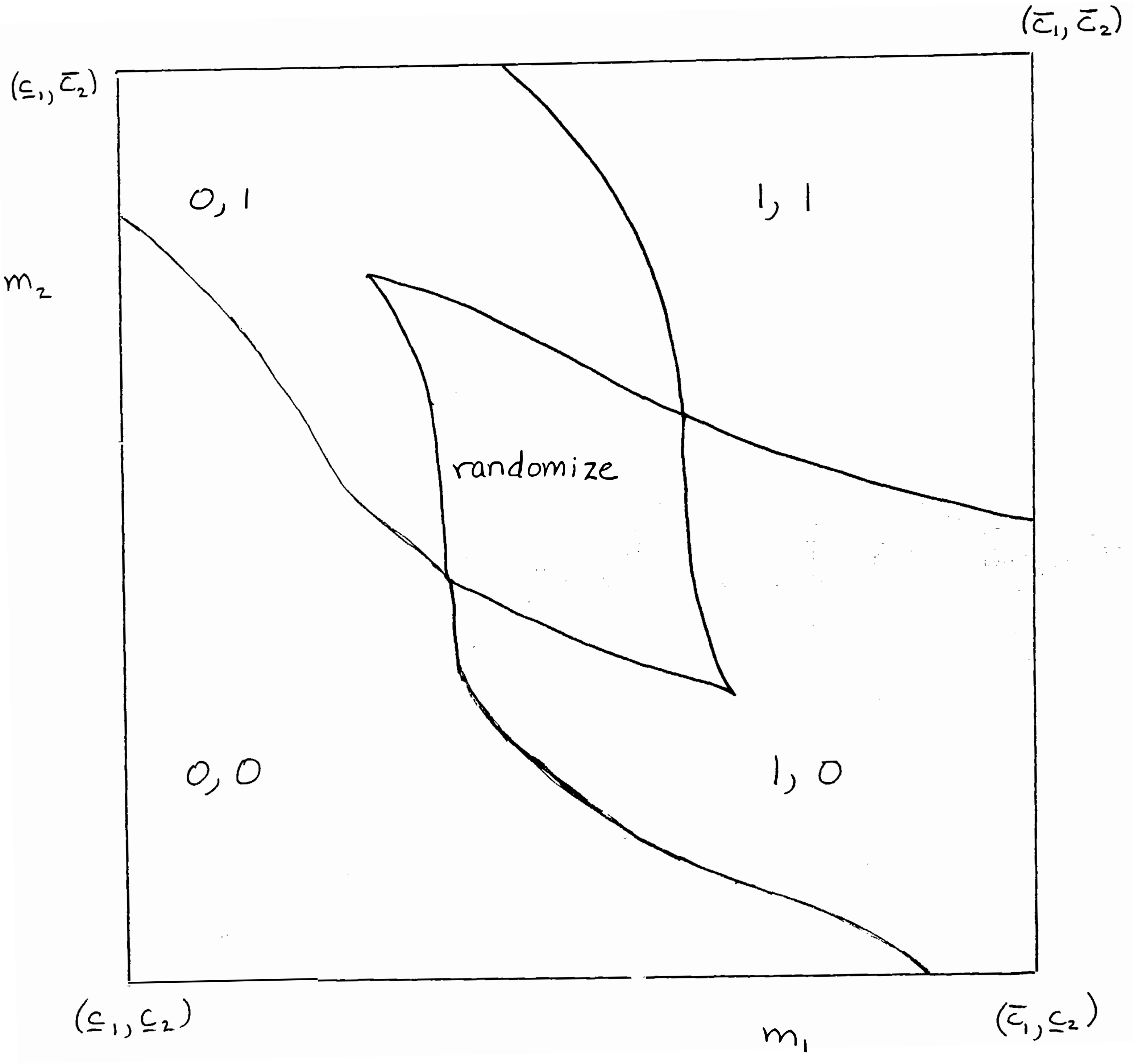

$(\underline{c}_1, \bar{c}_2)$
$(\bar{c}_1, \bar{c}_2)$
$\eta_1(m) \equiv k_1$
$\varphi_1(m) \equiv k_1$
$\underline{0}, \underline{1}$
$0, \underline{1}$
$\underline{1}, \underline{1}$
m_2
$\underline{0}, \underline{1}$
1, 0
and
0, 1
$\varphi_2(m) \equiv k_2$
$\underline{1}, 0$
$1, \underline{0}$
$\eta_2(m) \equiv k_2$
$\underline{0}, \underline{0}$
$\underline{1}, \underline{0}$
$(\underline{c}_1, \underline{c}_2)$
m_1
$(\bar{c}_1, \underline{c}_2)$

Figure 3



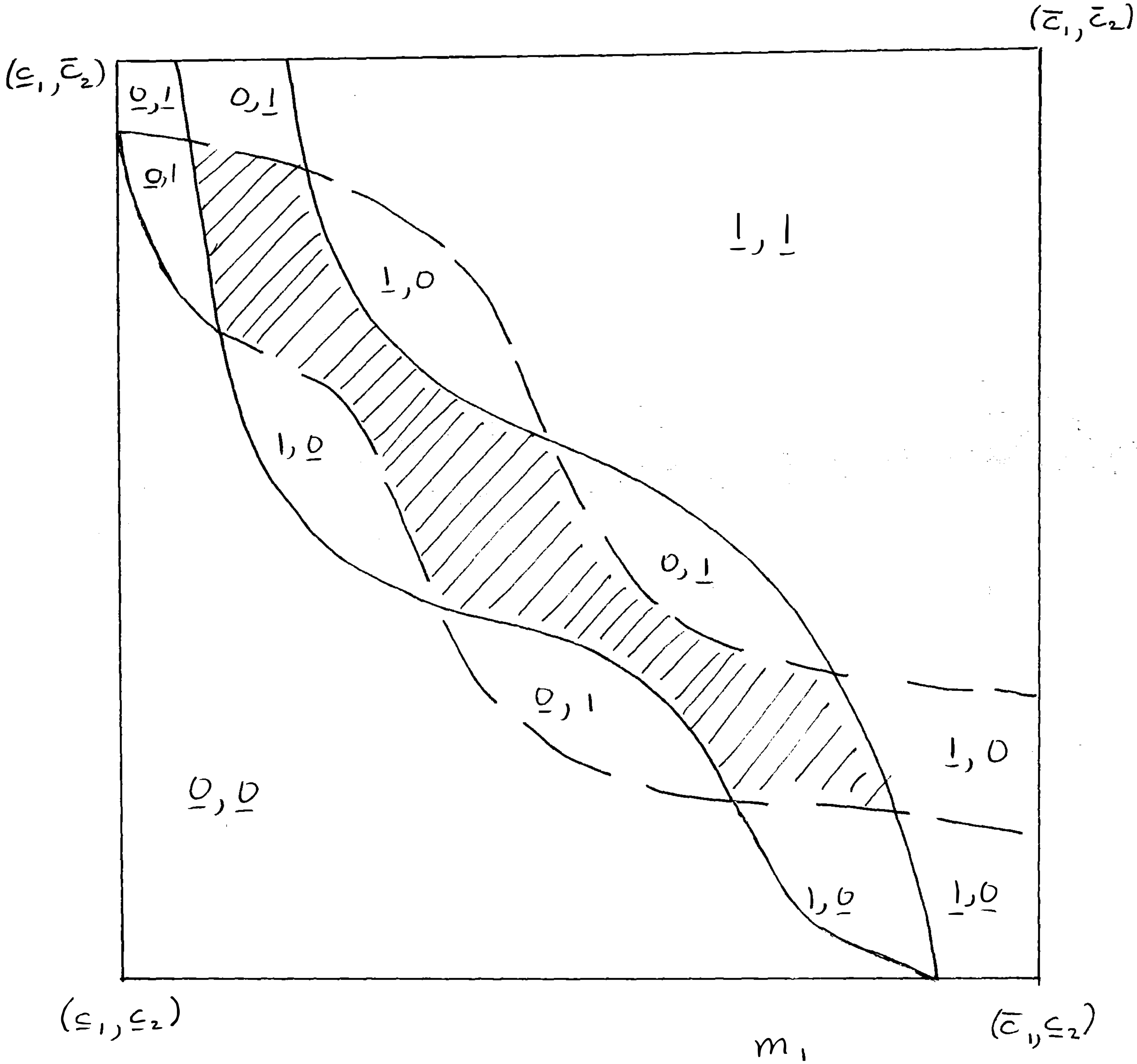
$(\bar{c}_1, \bar{c}_2)$
$(\underline{c}_1, \bar{c}_2)$
m_2
a
b
R'_1
R'_{10}
R'_0
c
d
$(\underline{c}_1, \underline{c}_2)$
$(\bar{c}_1, \underline{c}_2)$
m_1

Figure 4



$(\underline{c}_1, \overline{c}_2)$
$(\overline{c}_1, \overline{c}_2)$
0, 1
1, 1
m_2
randomize
0, 0
1, 0
$(\underline{c}_1, \underline{c}_2)$
m_1
$(\overline{c}_1, \underline{c}_2)$

Figure 5



$(\bar{c}_1, \bar{c}_2)$
$(\underline{c}_1, \bar{c}_2)$
$\underline{0}, \underline{1}$
$0, \underline{1}$
$\underline{0}, 1$
$\underline{1}, \underline{1}$
$\underline{1}, 0$
$1, \underline{0}$
$0, \underline{1}$
$\underline{0}, 1$
$\underline{1}, 0$
$\underline{0}, \underline{0}$
$1, \underline{0}$
$\underline{1}, \underline{0}$
$(\underline{c}_1, \underline{c}_2)$
$(\bar{c}_1, \underline{c}_2)$
m_1

Figure 6

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