Search for $R$-parity violating supersymmetry with displaced vertices in proton-proton collisions at $\sqrt{s} = 8$ TeV

V. Khachatryan et al.*
(CMS Collaboration)
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Results are reported from a search for $R$-parity violating supersymmetry in proton-proton collision events collected by the CMS experiment at a center-of-mass energy of $\sqrt{s} = 8$ TeV. The data sample corresponds to an integrated luminosity of 17.6 fb$^{-1}$. This search assumes a minimal flavor violating model in which the lightest supersymmetric particle is a long-lived neutralino or gluino, leading to a signal with jets emanating from displaced vertices. In a sample of events with two displaced vertices, no excess yield above the expectation from standard model processes is observed, and limits are placed on the pair production cross section as a function of mass and lifetime of the neutralino or gluino. At 95% confidence level, the analysis excludes cross sections above approximately 1 fb for neutralinos or gluinos with mass between 400 and 1500 GeV and mean proper decay length between 1 and 30 mm. Gluino masses are excluded below 1 and 1.3 TeV for mean proper decay lengths of 300 $\mu$m and 1 mm, respectively, and below 1.4 TeV for the range 2–30 mm. The results are also applicable to other models in which long-lived particles decay into multijet final states.

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I. INTRODUCTION

In spite of extensive efforts by the ATLAS and CMS Collaborations at the CERN LHC, the superpartners of standard model (SM) particles predicted by supersymmetry (SUSY) [1,2] have not yet been observed. If superpartners are produced and $R$ parity [3] is conserved, the lightest supersymmetric particle (LSP) passes through the detector unobserved, except for a potentially large amount of missing transverse energy. The assumption of $R$-parity conservation is motivated by experimental observations such as limits on the proton lifetime [4]. This assumption is not strictly required as long as either lepton or baryon number is conserved, or the associated $R$-parity violating (RPV) [5] terms in the Lagrangian are extremely small. Searches for a variety of signatures have not yet found any evidence for RPV SUSY [6–10].

In minimal flavor violating (MFV) models of RPV SUSY [11,12], the Yukawa couplings between superpartners and SM particles are the sole source of flavor symmetry violation, and the amplitudes for lepton- and baryon-number changing interactions are correspondingly small. At the LHC, the LSP typically decays within the detector volume, so there is no large missing transverse energy. The production processes of the superpartners are similar to those in the minimal supersymmetric standard model in that superpartners are produced in pairs, but the phenomenology depends on the identity of the LSP.

This analysis uses a benchmark signal model described in Ref. [12], in which the LSP is assumed to be either a neutralino or a gluino that is sufficiently heavy to decay into a top antiquark and a virtual top squark. The virtual top squark then decays via a baryon-number violating process to strange and bottom antiquarks, as shown in Fig. 1. Although this decay is heavily suppressed by the Yukawa coupling, it still dominates the top squark rate, with other partial widths being suppressed by a factor of 100 or more. As a consequence, the LSP is long-lived, with a lifetime that depends on the model parameters. For large parts of the parameter space, pair-produced LSPs lead to interesting signatures. Observable effects include increased top quark production rates, events with many jets, especially $b$-quark jets, and events with displaced vertices.

The decay of the LSP results in multiple jets emerging from a displaced vertex, often with wide opening angles. To identify the displaced vertices, we use a custom vertex

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*Full author list given at the end of the article.
reconstruction algorithm optimized for these distinctive features. This algorithm differs from standard methods used to identify $b$-quark jets [13], which assume a single jet whose momentum is aligned with the vertex displacement from the primary vertex. Our signature consists of two vertices, well separated in space. Studies based on event samples from Monte Carlo (MC) simulation show that SM background events rarely contain even one such reconstructed displaced vertex. In the even rarer events with two displaced vertices, the vertices are usually not well separated from each other.

The CMS Collaboration has also searched for pairs of displaced jets from a single vertex [14], while this analysis searches for a pair of displaced vertices, each of which is associated with a jet. The study reported here is sensitive to mean proper decay lengths between 300 μm and 30 mm, which are shorter than those probed by a similar analysis performed by the ATLAS Collaboration [15], and longer than those probed by a CMS analysis that looked for prompt LSP decays based on the jet and $b$-tagged jet multiplicity distributions [10].

This analysis applies not only to the MFV model described here, but more generally to models for physics beyond the SM with long-lived particles decaying to multiple jets. In addition to the results of the search with a neutralino or gluino LSP, we present a method for reinterpretation of the analysis.

II. THE CMS DETECTOR

The central feature of the CMS detector is a superconducting solenoid providing a magnetic field of 3.8 T aligned with the proton beam direction. Contained within the field volume of the solenoid are a silicon pixel and strip tracker, a lead tungstate electromagnetic calorimeter (ECAL), and a brass and scintillator hadronic calorimeter (HCAL). Outside the solenoid is the steel magnetic return yoke interspersed with muon tracking chambers. A more detailed description of the CMS detector, together with a definition of the coordinate system used and the relevant kinematic variables, can be found in Ref. [16].

The silicon tracker, which is particularly relevant to this analysis, measures the tracks of charged particles in the range of pseudorapidity, $|\eta| < 2.5$. For nonisolated particles with transverse momentum, $p_T$, of 1 to 10 GeV and $|\eta| < 1.4$, the track resolutions are typically 1.5% in $p_T$, 25–90 μm in the impact parameter in the transverse plane, and 45–150 μm in the impact parameter in the longitudinal direction [17]. When combining information from the entire detector, the jet energy resolution amounts typically to 15% at 10 GeV, 8% at 100 GeV, and 4% at 1 TeV, to be compared to about 40%, 12%, and 5% obtained when the ECAL and HCAL calorimeters alone are used [18].

The first level (L1) of the CMS trigger system, which is composed of custom hardware processors, uses information from the calorimeters and muon detectors to select the most interesting events in a fixed time interval of less than 4 μs. The high-level trigger (HLT) processor farm further decreases the event rate from around 100 kHz to less than 1 kHz, before data storage.

III. EVENT SAMPLES

The data sample used in this analysis corresponds to an integrated luminosity of 17.6 fb$^{-1}$, collected in proton-proton ($pp$) collisions at a center-of-mass energy of $\sqrt{s} = 8$ TeV in 2012. Events are selected using a trigger requiring the presence of at least four jets reconstructed from energy deposits in the calorimeters. At the L1 trigger, the jets are required to have $p_T > 40$ GeV, while in the HLT the threshold is $p_T > 50$ GeV. The latter threshold is afforded by a special data-taking strategy called “data parking” [19], in which the triggered events were saved but not promptly reconstructed, allowing a higher event rate. The data included in this analysis represent the fraction of the 2012 LHC operation for which this strategy was implemented.

Simulated events are used to model both the signal and background processes. Using PYTHIA 8.165 [20], signal samples with varying neutralino masses $M$ (200 $\leq M \leq 1500$ GeV) and lifetimes $\tau$ (0.1 $\leq \tau_r \leq 30$ mm) are produced. In these samples, neutralinos are produced in pairs; each neutralino is forced to undergo a three-body decay into top, bottom, and strange (anti-)quarks. Backgrounds arising from SM processes are dominated by multijet and top quark pair ($t\bar{t}$) events. The multijet processes include $b$-quark pair events. Smaller contributions come from single top quark production (single $t$), vector boson production in association with additional jets ($V+jets$), diboson production ($VV$), and top quark pairs with a radiated vector boson ($t\bar{t}+V$). Processes with a single vector boson include virtual photons, $W$ bosons, or $Z$ bosons, while the diboson processes include $WW$, $WZ$, and $ZZ$. Single top events are simulated with POWHEG 1.0 [21–25]; diboson events are simulated with PYTHIA 6.426 [26]; all other background samples are simulated using MADGRAPH 5.1 [27]. For all samples, hadronization and showering are done using PYTHIA 6.426 with tune Z2*. The Z2* tune is derived from the Z1 tune [28], which uses the CTEQ5L parton distribution set, whereas Z2* adopts CTEQ6L [29]. The detector response for all simulated samples is modeled using a GEANT4-based simulation [30] of the CMS detector. The effects of additional $pp$ interactions per bunch crossing (“pileup”) are included by overlaying additional simulated minimum-bias events, such that the resulting distribution of the number of interactions matches that observed in the experiment.

IV. EVENT PRESELECTION

To ensure that the four-jet trigger efficiency is high and well understood, more stringent criteria are applied off-line,
requiring at least four jets in the calorimeter with \( p_T > 60 \text{ GeV} \). These jets are reconstructed from calorimeter energy deposits, which are clustered by the anti-\( k_T \) algorithm \cite{31,32} with a distance parameter of 0.5. The trigger efficiency determined using events satisfying a single-muon trigger is \( (96.2 \pm 0.2)\% \) for events with four off-line jets with \( p_T > 60 \text{ GeV} \). The simulation overestimates this efficiency by a factor of \( 1.022 \pm 0.002 \), so, where used, its normalization is corrected by this amount.

Jets considered in the rest of the analysis are those obtained in the full event reconstruction performed using a particle-flow (PF) algorithm \cite{33,34}. The PF algorithm reconstructs and identifies photons, electrons, muons, and charged and neutral hadrons with an optimized combination of information from the various elements of the CMS detector. Before clustering the PF candidates into jets, charged PF candidates are excluded if they originate from a \( pp \) interaction vertex other than the primary vertex, which is the one with the largest scalar \( \Sigma |p_T|^2 \). The resulting particles are clustered into jets, again by the anti-\( k_T \) algorithm with a distance parameter of 0.5. Jets used in the analysis must satisfy \( p_T > 20 \text{ GeV} \) and \( |\eta| < 2.5 \).

For an event to be selected for further analysis, the scalar sum of the \( p_T \) of jets in the event \( H_T \) is required to be at least 500 GeV. This requirement has little impact on signal events but is useful for suppressing SM background.

### V. VERTEX RECONSTRUCTION, VARIABLES, AND SELECTION

#### A. Vertex reconstruction

Displaced vertices are reconstructed from tracks in the CMS silicon tracker. These tracks are required to have \( p_T > 1 \text{ GeV} \), at least eight measurements in the tracker including one in the pixel detector, and a transverse impact parameter with respect to the beam axis of at least 100 \( \mu \text{m} \). The impact parameter requirement favors vertices that are displaced from the primary vertex. The vertex reconstruction algorithm starts by forming seed vertices from all pairs of tracks that satisfy these requirements. Each vertex is fitted with the Kalman filter approach \cite{35}, and a fit is considered successful if it has a \( \chi^2 \) per degree of freedom \( \chi^2/d.o.f. \) that is less than 5. The vertices are then merged iteratively until no pair of vertices shares tracks. Specifically, for each pair of vertices that shares one or more tracks, if the three-dimensional (3D) distance between the vertices is less than 4 times the uncertainty in that distance, a vertex is fit to the tracks from both, and they are replaced by the merged vertex if the fit has \( \chi^2/d.o.f. < 5 \). Otherwise, each track is assigned to one vertex or the other depending on its 3D impact parameter significance with respect to each of the vertices, as follows:

(i) if the track is consistent with both vertices (both impact parameters less than 1.5 standard deviations), assign it to the vertex that has more tracks already;

(ii) if the track’s impact parameter is greater than 5 standard deviations from either vertex, drop it from that vertex;

(iii) otherwise, assign the track to the vertex to which it has a smaller impact parameter significance.

Each remaining vertex is then refit, and if the fit satisfies the requirement of \( \chi^2/d.o.f. < 5 \), the old vertex is replaced with the new one; otherwise it is dropped entirely.

This algorithm is similar in many regards to those used to identify (“tag”) \( b \)-quark jets \cite{13}. Typical \( b \) tagging algorithms, however, are optimized for identifying the decay in flight of a particle into a single jet and consequently make requirements that degrade sensitivity to the multijet final states sought here. For example, \( b \) tagging algorithms generally require that the tracks assigned to a vertex are approximately aligned with the flight direction from the primary vertex to the decay point, which is inefficient when there are multiple jets in the final state, including some that may be directed at large angles with respect to the flight path. The \( b \) tagging algorithms also discard tracks with impact parameters beyond those typical for \( b \)-quark daughters (>2 mm), thereby significantly reducing the efficiency for finding vertices with large displacements.

#### B. Vertex variables and selection

The vertexing procedure produces multiple vertices per event, only some of which are consistent with the signal. In order to select quality vertices, we impose additional requirements on the vertex and its associated tracks and jets. The requirements for each vertex are

(i) at least five tracks;

(ii) at least three tracks with \( p_T > 3 \text{ GeV} \);

(iii) at least one pair of tracks with separation \( \Delta R < 0.4 \), where \( \Delta R = \sqrt{ (\Delta \eta)^2 + (\Delta \phi)^2 } \), to favor vertices that include multiple tracks from a single jet;

(iv) at least one pair of tracks with \( \Delta R > 1.2 \) to favor vertices involving multiple jets;

(v) \( \Delta R < 4 \) for all pairs of tracks, to suppress wide-angle track coincidences;

(vi) at least one jet that shares one or more tracks with the vertex;

(vii) displacement in \( x \)-\( y \) of the vertex from the detector origin of less than 25 mm, to suppress vertices from interactions in the beam pipe or detector material;

(viii) uncertainty in the \( x \)-\( y \) distance of the vertex from the beam axis of less than 25 \( \mu \text{m} \).

In the data, 181 076 events have one vertex satisfying the above requirements, 251 have two of them, and no events have more than two. The candidate sample is composed of two-vertex events.

#### C. Signal discrimination in two-vertex events

The signal is extracted from the two-vertex events using the spatial separation between the vertices. In signal events,
the two LSPs are emitted approximately back to back, leading to large separations. We define the distance between the two vertices in the $x$-$y$ plane as $d_{VV}$, and fit this distribution to extract the signal. The fit to the observed $d_{VV}$ distribution is described in Sec. VIII.

The signal $d_{VV}$ templates are taken directly from simulation, with a distinct template for each LSP mass $M$ and lifetime $\tau$. In signal simulation, fewer than 10% of events in the candidate sample have more than two selected vertices. For these events, the two vertices with the highest number of tracks are selected for the $d_{VV}$ calculation, and in the case where two vertices have the same number of tracks, the vertex with decay products that have the higher invariant mass is chosen. The mass is reconstructed using the momenta of the associated tracks, assuming that the particles associated with the tracks have the charged pion mass.

Figure 2 shows the $d_{VV}$ distribution of an example simulated signal with $c\tau = 1$ mm, $M = 400$ GeV, and production cross section 1 fb, overlaid on simulated background. The bins in $d_{VV}$ are chosen to be sensitive to the peaking nature of the background at low $d_{VV}$; five 200 $\mu$m bins are used from 0 to 1 mm, then one bin from 1 to 50 mm where the contribution from the long-lived signal dominates.

Figure 3 shows the signal efficiency as a function of LSP mass and lifetime in the region $d_{VV} > 600$ $\mu$m, where the background is low. The signal efficiency generally increases as lifetime increases, until the lifetime is so long that decays more often occur beyond our fiducial limit at the beam pipe. The efficiency also generally increases as mass increases, up to approximately 800 GeV where it begins to decrease because of the event selection criteria, particularly the limit on the opening angle between track pairs in a vertex.

VI. BACKGROUND TEMPLATE

Background vertices arise from poorly measured tracks. These tracks can arise from the same jet, or from several jets in multijet events. Because it is an effect of misreconstruction, two-vertex background events are the coincidence of single background vertices.

Multijet events and $t\bar{t}$ production contribute 85% and 15% of the background in the two-vertex sample, respectively. Other sources of background, such as $V+$ jets and single $t$ events, are negligible. Approximately half of the background events include one or more $b$-quark jets, whose displaced decay daughters combine with misreconstructed tracks to form vertices.

Instead of relying on simulation to reproduce the background, we construct a background template, denoted by $d_{VV}^B$, from data. Taking advantage of the fact that two-vertex background events can be modeled using the one-vertex events, we define a control sample that consists of the 181 076 events with exactly one vertex. Each value entering the $d_{VV}^B$ template is the distance in the $x$-$y$ plane between two toy vertices, each determined by a value of the $x$-$y$ distance from the beam axis to the vertex, denoted by $d_{BV}$, and a value of the azimuthal angle of the vertex, denoted by $\phi_{BV}$.
The two values of $d_{BV}$ are sampled from the distribution of $d_{BV}$ for the one-vertex sample, which is shown in Fig. 4. The observed distribution is in good agreement with the sum of the background contributions from simulation.

The two values of $ϕ_{BV}$ are chosen using information about the jet directions in a one-vertex event. Since background vertices come from misreconstructed tracks, they tend to be located perpendicular to jet momenta. Therefore, we select a jet at random, preferring those with larger $p_T$ because of their higher track multiplicity, and sample a value of $ϕ_{BV}$ from a Gaussian distribution with width 0.4 rad, centered on a direction perpendicular to the jet in the transverse plane. To obtain the second value of $ϕ_{BV}$, we repeat this procedure using the same one-vertex event, allowing the same jet to be chosen twice.

The vertex reconstruction algorithm merges neighboring vertices. To emulate this behavior in our background template construction procedure, we discard pairs of vertices that are not sufficiently separated. We keep pairs of vertices with a probability parametrized by a Gaussian error function with mean $μ_{clear}$ and width $σ_{clear}$. The values of $μ_{clear}$ and $σ_{clear}$, which are related to the position uncertainties of the tracks, are varied in the fit to the observed $d_{VV}$ distribution. The values found in the fit are $μ_{clear} = 320 \mu$m and $σ_{clear} = 110 \mu$m.

Figure 5 compares the $d_{VV}$ distributions in simulated events, and shows the variation in $d_{VV}$ for values of $μ_{clear}$ and $σ_{clear}$ that are within one standard deviation of the fit values. The agreement is well within the statistical uncertainty. When normalized to the observed number of two-vertex events, the difference in their yields in the region $d_{VV} > 600 \mu$m is $0.6 \pm 2.6$ events.

**VII. SYSTEMATIC UNCERTAINITIES**

The signal is extracted from a fit of a weighted sum of the signal and background templates to the observed $d_{VV}$ distribution. For the signal, the simulation provides both the $d_{VV}$ distribution and its normalization, and systematic uncertainties arise from sources such as vertex reconstruction efficiency, track reconstruction, track multiplicity, pileup conditions, the detector alignment, and the jet energies. For the background, for which the template is derived from a control sample, the systematic uncertainties come from effects that could cause a discrepancy between the constructed $d_{VV}$ distribution and the nominal $d_{VV}$ distribution.

**A. Systematic uncertainties related to signal distribution and efficiency**

The dominant systematic uncertainty in the signal normalization arises from the difference between the
vertexing efficiencies in the simulation and data. This effect is evaluated in an independent study in which artificial signal-like vertices are produced in background events by displacing tracks associated with jets by a known displacement vector, and then applying the vertex reconstruction algorithm. The magnitude of the displacement vector is sampled from an exponential distribution with scale parameter 1 mm, restricted to values between 0.3 and 25 mm, similar to the expected distribution of signal vertices. The direction is calculated from the momentum of the jets in the event, but is smeared to emulate the difference between the flight and momentum directions in simulated signal events due to track inefficiency and unaccounted neutral particles. Events are required to satisfy the preselection requirements described in Sec. IV, and the displaced jets satisfy \( p_T > 50 \text{ GeV} \) and \( \Delta R < 4 \) for all pairs. To estimate the vertexing efficiency, we evaluate the fraction of events in which a vertex satisfying the requirements described in Sec. V B is reconstructed within 50 \( \mu \text{m} \) of the artificial vertex.

This fraction is evaluated for different numbers of displaced light parton or \( b \)-quark jets, with the ratio of efficiencies between data and simulation approaching unity for larger numbers of jets, independent of the size of the displacement. The largest disagreement between data and simulation occurs for the case where tracks from two light parton jets are displaced, where the fraction is 70\% in simulation and 64\% in data, with negligible statistical uncertainty. The ratio of efficiencies between data and simulation gives an 8.6\% uncertainty per vertex. For two-vertex events, the uncertainty is 17\%.

Additional studies explore the sensitivity of other effects that could alter the signal template. The vertex clustering depends on the number of charged particles in the event, which can vary based on the model of the underlying event used in PYTHIA [36]. The signal templates resulting from the choice of the underlying event model differ by no more than 1\% in any bin and the overall efficiency changes by no more than 3\%. This 3\% is taken as a systematic uncertainty.

To test the sensitivity to a possible misalignment, the signal samples have been reconstructed using several tracker misalignment scenarios corresponding to various “weak modes”: coherent distortions of the tracker geometry left over by the alignment procedure that lead to a systematic bias in the track parameters for no penalty in \( \chi^2 \) of the overall alignment fit [37]. These misalignments change the overall efficiency by no more than 2\%, which is taken as a systematic uncertainty.

To study sensitivity to the pileup distribution, we vary the inelastic \( pp \) cross section used in the pileup weighting by \( \pm 5\% \) [38]. This variation is found to have an effect of less than 1\% on the signal efficiency.

The uncertainty in the jet energy scale affects the total energy measured, and could change whether an event passes the jet \( p_T \) or \( H_T \) selections. This effect is studied by varying the jet energy scale and resolution [18], and is found to change the signal efficiency by less than 1\%. A 2.6\% uncertainty [39] is associated with the integrated luminosity for the 2012 data set and the derived signal cross section. The uncertainty in the trigger efficiency is less than 1\%.

Table I summarizes the systematic uncertainties in the signal efficiency. We assume there are no correlations among them, so we add them in quadrature to obtain the overall uncertainty.

<table>
<thead>
<tr>
<th>Systematic effect</th>
<th>Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex reconstruction</td>
<td>17</td>
</tr>
<tr>
<td>Underlying event</td>
<td>3</td>
</tr>
<tr>
<td>Tracker misalignment</td>
<td>2</td>
</tr>
<tr>
<td>Pileup</td>
<td>1</td>
</tr>
<tr>
<td>Jet energy scale/resolution</td>
<td>1</td>
</tr>
<tr>
<td>Integrated luminosity</td>
<td>3</td>
</tr>
<tr>
<td>Trigger efficiency</td>
<td>1</td>
</tr>
<tr>
<td>Overall</td>
<td>18</td>
</tr>
</tbody>
</table>

B. Systematic uncertainties related to background estimate

The \( \delta_{\mu}^{C_{VV}} \) background template is constructed from a large sample of events with a single vertex. Systematic uncertainties in the \( \delta_{\mu}^{C_{VV}} \) template are estimated by varying the \( \delta_{\mu}^{C_{VV}} \) construction method and taking the difference between the \( \delta_{\mu}^{C_{VV}} \) distributions using the default and alternate methods. The method for constructing \( \delta_{\mu}^{C_{VV}} \) involves drawing two values of \( \delta_{BV} \) and two values of \( \phi_{BV} \), with an angle between vertices \( \Delta \phi_{VV} \), so the main uncertainties come from effects related to the \( \delta_{BV} \) and \( \Delta \phi_{VV} \) distributions.

The production of \( b \) quarks in pairs introduces a correlation between the vertex distances in two-vertex events that is not accounted for when single vertices are paired at random. In simulation, events without \( b \) quarks have a mean \( \delta_{BV} \) of \( \sim 160 \mu \text{m} \), while events with \( b \) quarks, which account for 15\% of one-vertex events, have a mean \( \delta_{BV} \) of \( \sim 190 \mu \text{m} \), without significant dependence on \( b \)-quark momentum. We quantify this effect by sorting the simulated background events into those with and without \( b \) quarks, constructing the \( \delta_{C_{VV}} \) distributions for each, and then combining them in the proportions 45:55, which is the ratio of \( b \)-quark to non-\( b \)-quark events in two-vertex background events determined from simulation. The systematic uncertainty is taken to be the difference between the simulated yields obtained with this procedure and the standard one, scaled to the observed two-vertex yield.

The \( \delta_{C_{VV}} \) construction method discards pairs of vertices that would overlap, consistently leading to a two-vertex angular distribution that peaks at \( \pm \pi \) radians. To assess the systematic uncertainty related to assumptions about the
angular distribution between vertices, we draw $\Delta \phi_{VV}$ from the angular distribution between vertices in simulated two-vertex background events. This leads to a $d_{VV}$ distribution with a more strongly peaked $\Delta \phi_{VV}$ distribution, and provides a conservative estimate of the uncertainty.

The statistical uncertainty from the limited number of one-vertex events that are used to construct the two-vertex distribution is studied using a resampling method. Using one-vertex events that are used to construct the two-vertex parent, we randomly sample ten new $d_{BV}$ pseudodata distributions, and use each to construct a $d_{VV}$ distribution. The root-mean-square variation in bin-by-bin yields in the set of distributions gives the statistical uncertainty.

There is a small contribution to the uncertainty in the prediction of $d_{VV}$ in the first two scenarios, but observe no change in $d_{VV}$ for the majority of the templates. For the background templates is that due to the finite MC statistics; the uncertainty in the backgrounds is as high as 20% for the lowest lifetime and mass samples, but is generally no more than 1% in any bin for the majority of the templates. For the background templates, a Gaussian uncertainty is assumed in the value of the template in each bin, truncated at zero.

\begin{equation}
\log \mathcal{L}(n_i|s, b, \nu) = \sum_i [n_i \log a_i(s, b, \nu) - a_i(s, b, \nu)].
\end{equation}

Here $n_i$ is the number of observed events in bin $i$, $s$ and $b$ are the normalizations of the signal and background templates corresponding to the yields, $\nu$ denotes the shape parameters $\mu_{\text{clear}}$ and $\sigma_{\text{clear}}$, and $a_i(s, b, \nu)$ is the weighted sum of the signal and background frequencies $a_i(s)$ and $a_i(b)$ in bin $i$.

The only assumed shape uncertainty in the signal templates is that due to the finite MC statistics; the uncertainty is as high as 20% for the lowest lifetime and mass samples, but is generally no more than 1% in any bin for the majority of the templates. For the background templates, a Gaussian uncertainty is assumed in the value of the template in each bin, truncated at zero. To incorporate these uncertainties in the signal and background templates, a procedure similar to that of Barlow and Beeston [40] is followed, modified to allow a bin-by-bin Gaussian uncertainty in the background shape [41]. The final log-likelihood function is then given by
log \mathcal{L}(n_i|s, b, \nu, A^{(s)}_i, A^{(b)}_i) \\
= \sum_i n_i \log A_i - A_i + \sum_i M A^{(s)}_i \log MA^{(s)}_i - MA^{(s)}_i \\
+ \sum_i -\frac{1}{2} \left( \frac{a^{(b)}_i - A^{(b)}_i}{\sigma^{(b)}_i} \right)^2 ,
\tag{3}

with \( A_i = s A^{(s)}_i + b A^{(b)}_i \). The \( A^{(s)}_i \) and \( A^{(b)}_i \) replace the \( a^{(s)}_i \)
and \( a^{(b)}_i \) from above in the shape fit to the data, and are
allowed to vary as either Poisson (\( A^{(s)}_i \)) or Gaussian (\( A^{(b)}_i \))
distributed parameters. The quantity \( M \) is the number of events from the MC signal sample that produced the \( a^{(s)}_i \)
estimates, and \( a^{(b)}_i \) are the widths of the Gaussian distributions
taken to be the relative sizes of the uncertainties listed in Table II. The modified Barlow-Beeston procedure
finds the \( A^{(s)}_i \) and \( A^{(b)}_i \) that maximize \( \log \mathcal{L} \) given \( (s, b, \nu) \);
the difference here is that the \( A^{(b)}_i \) are Gaussian distributed parameters.

The likelihood function is only weakly dependent on the
background shape parameters \( \nu \), and when signal is
injected, the best fit values \( \hat{\nu} \) agree well with the back-
ground-only values. The fit is well behaved: for most signal
templates, in pseudoexperiments where the true signal and
background strengths are known, the distribution of the
fitted yields for \( s \) and \( b \) have means consistent with those
input, and the widths of the distributions as measured by
their root-mean-square are consistent with the uncertainties
in the fits. For the signal templates with low lifetimes,
however, the signal yield is biased downward when an
injected signal is present. This is due to the background
shape being allowed to vary upward at high \( d_{VV} \) within the
uncertainties assigned. When no injected signal is present,
there is a bias toward obtaining \( s > 0 \) when fitting using
templates with \( c \tau < 300 \ \mu \text{m} \). Therefore, we only consider
signals with \( c \tau \geq 300 \ \mu \text{m} \) in the fit and the search.

B. Statistical analysis

The test statistic \( q \) used to quantify any excess of signal
events over the expected background is given by a profile
likelihood ratio \cite{42}:
\[ q = \log \frac{\max_{s \geq 0,b \geq 0} \mathcal{L}(n_i|s, b, \hat{\nu}, \hat{A}^{(s)}_i, \hat{A}^{(b)}_i)}{\max_{s \geq 0} \mathcal{L}(n_i|s = 0, b, \hat{\nu}, \hat{A}^{(s)}_i, \hat{A}^{(b)}_i)}, \tag{4} \]

where for each value of \( s \) and \( b \) the nuisance parameters
\( \hat{A}^{(s)}_i \), \( \hat{A}^{(b)}_i \), and \( \hat{\nu} \) are found that maximize the relevant
likelihood. The probability under the background-only
hypothesis, \( p_0 \), to obtain a value of the test statistic
at least as large as that observed, \( q_{\text{obs}} \), is estimated as the
fraction of 10 000 pseudoexperiments with \( q \geq q_{\text{obs}} \). This
is referred to as the \( p \) value for a particular signal
hypothesis. The pseudoexperiments are generated using
the background \( \delta_{VV} \) distribution corresponding to the
background-only \( \hat{\nu} \), and background count \( b \) drawn from a
Poisson distribution with mean equal to \( n \), the number of
events in the data. The nuisance parameters \( \nu, A^{(s)}_i \), and \( A^{(b)}_i \)
are drawn from their corresponding Poisson or Gaussian
distributions in each pseudoeperiment.

We obtain limits on the signal yield, which can be
converted into limits on the product of the cross section for
neutralino or gluino pair production and the square of the
branching fraction for decay via the channel under study,
denoted by \( \sigma \mathcal{B}^2 \). To obtain limits on \( \sigma \mathcal{B}^2 \), for a given
number of signal events \( s_0 \), we calculate the probability
for the null hypothesis of \( s = s_0 \) versus the alternative that
\( s < s_0 \) denoted by \( p_{s_0} \). We do this in practice by generating
10 000 pseudoexperiments with \( s \) drawn from a Poisson
distribution with mean \( s_0 \), and \( b \) drawn from a Poisson
distribution with mean \( n - s_0 \). The background shape \( \delta_{VV} \)
is taken from the \( \nu \) from the original fit and signal shape
corresponding to the signal hypothesis in question, with
\( A^{(b)}_i \) from their Gaussian distributions. The null hypothesis
probability \( p_{s_0} \) is then the fraction of pseudoexperiments
where \( q \geq q(s_0) \). We protect against downward fluctuations
in the data by using the \( \text{CL}_s \) criterion \cite{43,44}, defining
the statistic as
\[ \text{CL}_s = \frac{p_{s_0}}{1 - p_0} . \tag{5} \]

The 95% confidence level (C.L.) upper limit on \( s \) is then the
biggest \( s_0 \) for which \( \text{CL}_s \) is still greater than 0.05.

The limit on the signal yield is converted to a limit on
\( \sigma \mathcal{B}^2 \) using the efficiencies calculated from simulation and
the integrated luminosity of the data sample, 17.6 fb\(^{-1}\). We
include the effect of the estimated 18% signal efficiency
uncertainty by varying the cross section in each pseudoe-
periment by the value sampled from a log-normal density
with location parameter 1 and scale parameter 0.18.

C. Results of the fit

The result of the fit to data is shown in Fig. 6, for the LSP
cross section of 1 mm, \( M = 400 \ \text{GeV} \) signal template. The observed
counts in each bin, along with the predictions from the
background-only fit and the related uncertainties, are listed
in Table III. There is a small excess of events with
\( 0.6 < d_{VV} < 50 \ \text{mm} \): seven in the data, while the back-
ground-only fit predicts \( 4.1 \pm 1.4 \), where the uncertainty is
the overall systematic uncertainty discussed in Sec. VII. In
the signal + background fits, a typical value for the signal
yield is \( 1.7 \pm 1.9 \), obtained with the \( c \tau = 1 \ \text{mm}, M = 400 \ \text{GeV} \) signal hypothesis. The associated \( p \) value
obtained from pseudoexperiments is in the range 0.05–
0.14 for signals with \( 0.3 \leq c \tau \leq 30 \ \text{mm} \), with the larger \( p \)
values coming from those with longer lifetimes.
D. Upper limits on signal cross section

Figure 7 shows the observed 95% C.L. upper limits on \( \sigma B^2 \). As an example, for a neutralino with mass of 400 GeV and \( c\tau \) of 10 mm, the observed 95% C.L. upper limit on \( \sigma B^2 \) is 0.6 fb.

Exclusion curves are overlaid, assuming the gluino pair production cross section \([45–49]\). In the context of the MFV model that we are studying, either a neutralino or a gluino LSP can decay into the final state targeted in the search.

The scan in \( c\tau \) is in steps of 100 \( \mu \)m from 300 \( \mu \)m to 1 mm, then in 1 mm steps up to 10 mm, and in 2 mm steps to 30 mm; the mass points are spaced by 100 GeV. The exclusion curves are produced by linear interpolation of the limit scan, which identifies the set of points for which the interpolated upper limit is less than the gluino pair production cross section (the neutralino pair production cross section is expected to be much smaller).

<table>
<thead>
<tr>
<th>Bin</th>
<th>( d_{VV} ) range</th>
<th>Observed ( n_i )</th>
<th>Expected event yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0–0.2 mm</td>
<td>6</td>
<td>6.2 ± 1.0</td>
</tr>
<tr>
<td>2</td>
<td>0.2–0.4 mm</td>
<td>193</td>
<td>192.6 ± 3.9</td>
</tr>
<tr>
<td>3</td>
<td>0.4–0.6 mm</td>
<td>45</td>
<td>48.1 ± 3.8</td>
</tr>
<tr>
<td>4</td>
<td>0.6–0.8 mm</td>
<td>5</td>
<td>3.5 ± 1.4</td>
</tr>
<tr>
<td>5</td>
<td>0.8–1.0 mm</td>
<td>1</td>
<td>0.3 ± 0.1</td>
</tr>
<tr>
<td>6</td>
<td>1.0–50 mm</td>
<td>1</td>
<td>0.3 ± 0.1</td>
</tr>
</tbody>
</table>

IX. EXTENDING THE SEARCH TO OTHER SIGNAL MODELS

The search for displaced vertices applies to other types of long-lived particles decaying to multiple jets. Here we present a generator-level selection that can be used to reinterpret the results of our analysis. For signal models in
which there are two well-separated displaced vertices, this generator-level selection approximately replicates the reconstruction-level efficiency. The selection is based on the displacements of the long-lived particles, and the momenta and angular distributions of their daughter particles, which are taken to be $u$, $d$, $s$, $c$, and $b$ quarks, electrons, and muons. The daughter particles are said to be "accepted" if they satisfy $p_T > 20$ GeV and $|\eta| < 2.5$, and "displaced" if their transverse impact parameter with respect to the origin is at least 100 µm. The criteria of the generator-level selection are

(a) at least four accepted quarks with $p_T > 60$ GeV;
(b) $H_T$ of accepted quarks > 500 GeV;
(c) for each vertex:
   (i) $x$-$y$ distance from beam axis < 25 mm;
   (ii) at least one pair of accepted displaced daughter particles with $\Delta R > 1.2$;
   (iii) $\Delta R < 4$ for all pairs of accepted displaced daughter particles;
   (iv) at least one accepted displaced daughter quark;
   (v) $\sum p_T$ of accepted displaced daughter particles > 200 GeV;
(d) $x$-$y$ distance between vertices > 600 µm.

In the region with $d_{xy} > 600$ µm, the background level is well determined and is insensitive to fit parameters. Use of this generator-level selection replicates the reconstruction-level efficiency with an accuracy of 20% or better for a selection of models for which the signal efficiency is high (>10%). The selection may underestimate the trigger efficiency because it does not take into account effects such as initial- and final-state radiation, and may overestimate the efficiency for reconstructing vertices with $b$-quark secondaries, since the $b$-quark lifetime can impede the association of their decay products with the reconstructed vertices.

X. SUMMARY

A search for $R$-parity violating SUSY in which long-lived neutralinos or gluinos decay into multijet final states was performed using proton-proton collision events collected with the CMS detector at $\sqrt{s} = 8$ TeV in 2012. The data sample corresponded to an integrated luminosity of 17.6 fb$^{-1}$, and was collected requiring the presence of at least four jets. No excess above the prediction from standard model processes was observed, and at 95% confidence level, the data excluded cross section times branching fraction squared above approximately 1 fb for neutralinos or gluinos with mass between 400 and 1500 GeV and $c\tau$ between 1 and 30 mm. Assuming gluino pair production cross sections, gluino masses below 1 and 1.3 TeV were excluded for mean proper decay lengths of 300 µm and 1 mm, respectively, and below 1.4 TeV for the range 2–30 mm. While the search specifically addressed $R$-parity violating SUSY, the results were relevant to other massive particles that decay to two or more jets. These are the most restrictive bounds to date on the production and decay of pairs of such massive particles with intermediate lifetimes.

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Università di Napoli ‘Federico II’, Napoli, Italy
Università della Basilicata, Potenza, Italy
Università G. Marconi, Roma, Italy
INFN Sezione di Padova, Padova, Italy
Universitá di Padova, Padova, Italy
Universitá di Trento, Trento, Italy
INFN Sezione di Pavia, Pavia, Italy
Universitá di Pavia, Pavia, Italy
INFN Sezione di Perugia, Perugia, Italy
Universitá di Perugia, Perugia, Italy
INFN Sezione di Pisa, Pisa, Italy
Universitá di Pisa, Pisa, Italy
Scuola Normale Superiore di Pisa, Pisa, Italy
INFN Sezione di Roma
Universitá di Roma
INFN Sezione di Torino, Torino, Italy
Universitá di Torino, Torino, Italy
Universitá del Piemonte Orientale, Novara, Italy
INFN Sezione di Trieste, Trieste, Italy
Universitá di Trieste, Trieste, Italy
Kyungpook National University, Daegu, Korea
Chonbuk National University, Jeonju, Korea
Hanyang University, Seoul, Korea
Korea University, Seoul, Korea
Seoul National University, Seoul, Korea
University of Seoul, Seoul, Korea
Sungkyunkwan University, Suwon, Korea
Vilnius University, Vilnius, Lithuania
National Centre for Particle Physics, Universiti Malaya, Kuala Lumpur, Malaysia
Centro de Investigacion y de Estudios Avanzados del IPN, Mexico City, Mexico
Universidad Iberoamericana, Mexico City, Mexico
Benemerita Universidad Autonoma de Puebla, Puebla, Mexico
Universidad Autonoma de San Luis Potosi, San Luis Potosi, Mexico
University of Auckland, Auckland, New Zealand
University of Canterbury, Christchurch, New Zealand
National Centre for Physics, Quaid-I-Azam University, Islamabad, Pakistan
Institute of Experimental Physics, Faculty of Physics, University of Warsaw, Warsaw, Poland
Laboratório de Instrumentação e Física Experimental de Partículas, Lisboa, Portugal
Joint Institute for Nuclear Research, Dubna, Russia
Petersburg Nuclear Physics Institute, Gatchina (St. Petersburg), Russia
Institute for Nuclear Research, Moscow, Russia
Institute for Theoretical and Experimental Physics, Moscow, Russia
Moscow Institute of Physics and Technology
National Research Nuclear University 'Moscow Engineering Physics Institute' (MEPhI), Moscow, Russia
P.N. Lebedev Physical Institute, Moscow, Russia
Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia
State Research Center of Russian Federation, Institute for High Energy Physics, Protvino, Russia
University of Belgrade, Faculty of Physics and Vinca Institute of Nuclear Sciences, Belgrade, Serbia
Centro de Investigaciones Energéticas Medioambientales y Tecnológicas (CIEMAT), Madrid, Spain
Universidad Autonóma de Madrid, Madrid, Spain
Universidad de Oviedo, Oviedo, Spain
Instituto de Física de Cantabria (IFCA), CSIC-Universidad de Cantabria, Santander, Spain
CERN, European Organization for Nuclear Research, Geneva, Switzerland
Paul Scherrer Institut, Villigen, Switzerland
Institute for Particle Physics, ETH Zurich, Zurich, Switzerland
Universität Zürich, Zurich, Switzerland
National Central University, Chung-Li, Taiwan
National Taiwan University (NTU), Taipei, Taiwan
Deceased.

Also at Vienna University of Technology, Vienna, Austria.

Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing, China.

Also at Institut Pluridisciplinaire Hubert Curien, Université de Strasbourg, Université de Haute Alsace Mulhouse, CNRS/IN2P3, Strasbourg, France.

Also at Universidade Estadual de Campinas, Campinas, Brazil.

Also at Université Libre de Bruxelles, Bruxelles, Belgium.

Also at Deutsches Elektronen-Synchrotron, Hamburg, Germany.

Also at Joint Institute for Nuclear Research, Dubna, Russia.

Also at Cairo University, Cairo, Egypt.

Also at Fayoum University, El-Fayoum, Egypt.

Also at British University in Egypt, Cairo, Egypt.

Also at Ain Shams University, Cairo, Egypt.

Also at Université de Haute Alsace, Mulhouse, France.

Also at CERN, European Organization for Nuclear Research, Geneva, Switzerland.

Also at Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia.

Also at Tbilisi State University, Tbilisi, Georgia.

Also at RWTH Aachen University, III. Physikalisches Institut A, Aachen, Germany.

Also at University of Hamburg, Hamburg, Germany.

Also at Brandenburg University of Technology, Cottbus, Germany.

Also at Institute of Nuclear Research ATOMKI, Debrecen, Hungary.

Also at MTA-ELTE Lendület CMS Particle and Nuclear Physics Group, Eötvös Loránd University, Budapest, Hungary.

Also at University of Debrecen, Debrecen, Hungary.

Also at Indian Institute of Science Education and Research, Bhopal, India.

Also at Institute of Physics, Bhubaneswar, India.

Also at University of Visva-Bharati, Santiniketan, India.

Also at University of Ruhuna, Matara, Sri Lanka.

Also at Isfahan University of Technology, Isfahan, Iran.

Also at University of Tehran, Department of Engineering Science, Tehran, Iran.

Also at Plasma Physics Research Center, Science and Research Branch, Islamic Azad University, Tehran, Iran.

Also at Università degli Studi di Siena, Siena, Italy.

Also at Purdue University, West Lafayette, USA.

Also at International Islamic University of Malaysia, Kuala Lumpur, Malaysia.

Also at Malaysian Nuclear Agency, MOSTI, Kajang, Malaysia.

Also at Consejo Nacional de Ciencia y Tecnología, Mexico City, Mexico.

Also at Warsaw University of Technology, Institute of Electronic Systems, Warsaw, Poland.

Also at Institute for Nuclear Research, Moscow, Russia.

Also at National Research Nuclear University ‘Moscow Engineering Physics Institute’ (MEPhI), Moscow, Russia.

Also at St. Petersburg State Polytechnical University, St. Petersburg, Russia.

Also at University of Florida, Gainesville, USA.

Also at P.N. Lebedev Physical Institute, Moscow, Russia.

Also at California Institute of Technology, Pasadena, USA.

Also at Faculty of Physics, University of Belgrade, Belgrade, Serbia.

Also at INFN Sezione di Roma, Università di Roma, Roma, Italy.

Also at Scuola Normale e Sezione dell’INFN, Pisa, Italy.

Also at National and Kapodistrian University of Athens, Athens, Greece.

Also at Riga Technical University, Riga, Latvia.

Also at Institute for Theoretical and Experimental Physics, Moscow, Russia.

Also at Albert Einstein Center for Fundamental Physics, Bern, Switzerland.

Also at Adiyaman University, Adiyaman, Turkey.

Also at Mersin University, Mersin, Turkey.

Also at Cag University, Mersin, Turkey.

Also at Piri Reis University, Istanbul, Turkey.

Also at Gaziosmanpasa University, Tokat, Turkey.

Also at Ozyegin University, Istanbul, Turkey.

Also at Izmir Institute of Technology, Izmir, Turkey.

Also at Marmara University, Istanbul, Turkey.

Also at Kafkas University, Kars, Turkey.

Also at Istanbul Bilgi University, Istanbul, Turkey.
Also at Yildiz Technical University, Istanbul, Turkey.
Also at Hacettepe University, Ankara, Turkey.
Also at Rutherford Appleton Laboratory, Didcot, United Kingdom.
Also at School of Physics and Astronomy, University of Southampton, Southampton, United Kingdom.
Also at Instituto de Astrofísica de Canarias, La Laguna, Spain.
Also at Utah Valley University, Orem, USA.
Also at University of Belgrade, Faculty of Physics and Vinca Institute of Nuclear Sciences, Belgrade, Serbia.
Also at Facoltà Ingegneria, Università di Roma, Roma, Italy.
Also at Argonne National Laboratory, Argonne, USA.
Also at Erzincan University, Erzincan, Turkey.
Also at Mimar Sinan University, Istanbul, Istanbul, Turkey.
Also at Texas A&M University at Qatar, Doha, Qatar.
Also at Kyungpook National University, Daegu, Korea.