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COMPETITIVE EQUILIBRIA IN MARKETS FOR HETEROGENEOUS GOODS UNDER IMPERFECT INFORMATION: A THEORETICAL ANALYSIS WITH POLICY IMPLICATIONS

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ABSTRACT

This paper characterizes necessary and sufficient conditions for heterogeneous search goods to trade at their competitive prices, and derives policy implications from these conditions. The model differs from earlier search equilibrium models in that it assumes the existence of product heterogeniety.

Our principle conclusions are that markets for heterogeneous search goods tend rather easily to segment into homogeneous subsets; when they do not, heterogeniety can work against the existence of competitive equilibria because it dilutes the effectiveness of search. Nevertheless, the likelihood of competitive equilibria obtaining in heterogeneous search goods markets can often be increased by reducing the costs to consumers of directly comparing purchase alternatives.

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Alan Schwartz* and Louis L. Wilde**

1. Introduction

For over a decade, the Federal Government has responded aggressively to apparent information imperfections in consumer markets. Examples of such responses include the Truth in Lending Law, the Magnusson-Moss Warranty-Federal Trade Commission Improvement Act, the Consumer Leasing Act and the Real Estate Settlement Procedures Act. Congress has passed almost all of this regulation, however, with no clear idea of what purposes it wanted to achieve, or could in fact achieve, or of the relation between various intervention strategies and the possible goals of government action. In previous papers,[1] we have argued that the state should be principally concerned with competitive equilibria; that is, an intervention based on *information grounds* is justified if and only if the costs to consumers of becoming informed about purchase choices are so high that noncompetitive equilibria exist, and interventions consequently should be directed to increasing the likelihood that competitive equilibria will obtain.

The formal model supporting these preliminary conclusions [Wilde and Schwartz, 1979] made fairly strong assumptions. In particular, it modeled markets for homogeneous search goods--identical products all of whose features consumers could observe before purchase. Thus it had direct applicability only to markets such as those for money or wheat. The model's relative simplicity nevertheless enabled us to derive intuitively plausible criteria for recognizing when a market is likely to be behaving badly because of information problems,[2] and to develop strategies that decisionmakers could use to move markets toward competitive equilibria. These strategies followed from the view that the likelihood of a competitive equilibrium obtaining varied directly with the extent of comparison shopping present in a market; they thus consisted mainly of methods of reducing the costs to consumers of directly comparing purchase alternatives. This paper begins to explore whether the conclusions that followed from our original model are applicable in more complex environments.

Consumers in the model developed below shop pursuant to a fixed sample size strategy in a market that potentially supplies a search good at two qualities, ''low'' and ''high''.[3] Although these consumers have preferences for low or high quality goods, they have imprecise information, when they begin to search, as to where either good can be found or what prices they are likely to face. Thus consumers shop randomly across quality levels. The firms in this model pursue relatively passive strategies, in that they do not advertise but instead only charge particular prices and change these prices when changes would increase expected profits. We derive necessary and sufficient conditions for an equilibrium to obtain in which both the low and the high quality goods trade at their competitive prices.

Surprisingly, it turns out that our original homogeneous search goods model has considerable generality. This is because it takes fairly strong assumptions to prevent the markets for high and low quality goods from ''segmenting''. For example, the two markets will segment nnless consumers who prefer low quality goods will buy high quality goods that trade at their competitive price if the consumers' search reveals only high quality goods; if these consumers would not purchase high quality goods in this circumstance, they actually are shoppers only for low quality. Similarly, the two markets will segment unless consumers who prefer high quality goods will buy low quality goods that trade at their competitive price if the consumers search reveals only low quality goods; if these consumers would not purchase low quality goods in this circumstance, they actually are shoppers only for high quality. If these two conditions fail, low and high quality goods will trade in distinct markets. To see more vividly the restrictiveness of these conditions, consider the market for compact cars. Unless the consumers who shop in it and who prefer BMWs or Mercedeses are willing to purchase Toyotas or Datsuns that trade at their competitive prices if the consumers' search fails to reveal a high quality dealer, the compact car market will segment. It will also segment unless the consumers who shop in it and who prefer Toyotas or Datsuns are willing to purchase BMWs or Mercedeses trading at competitive prices if the consumers' search fails to reveal a low quality dealer. The strength of these conditions, in many markets, implies that segmentation is

often likely for search goods. Thus, a model that presupposes product homogeneity has applicability to a fairly wide range of cases.

On the other hand, when the markets for low and high quality goods do interact, product heterogeneity can work against the existence of competitive equilibria, sometimes doing so in nonintuitive ways. This is because heterogeneity dilutes the effectiveness of search. As our earlier work has shown, prices are driven down to competitive levels in search equilibrium models as a result of competition among firms for shoppers. Suppose, however, that some consumers who shop twice visit one high and one low quality firm. These consumers effectively are nonshoppers in both markets. In consequence, if as much shopping occurs in a heterogeneous as in a homogeneous goods market, firms in the former market may face fewer actual comparison shoppers and thus be more likely to find it profitable to deviate from the competitive price. Further, because of the possibility that shoppers who prefer one quality will ''spill over'' into the market for the other quality--i.e. visit one or more firms that sell the less preferred quality--increasing the number of shoppers in a given market could sometimes cause that market to behave less competitively than before the increase.

Section 2 sets out the formal model and derives conditions for when the markets for low and high quality goods will interact. Sections 3, 4 and 5 derive necessary and sufficient conditions for when competitive equilibria will obtain in both markets and explain the intuition underlying these conditions. Section 6 then briefly discusses the policy implications and limitations of the analysis. In particular, we argue that while the opportunities of firms to make profitable deviations from the competitive price often can be unambiguously reduced because of the high probability of segmentation, when the markets for different qualities do in fact interact knowing when a market is behaving badly and whether a particular intervention will help become difficult questions.

2. A Search Equilibrium Model with Heterogeneous Goods

In this section, we develop a model for a heterogeneous search good--one that is described by price and ''quality'' but all of whose features are observable before purchase. This good is supplied at two distinct quality levels, ''low'' and ''high''. The adjectives low and high actually are conventions; the formal model requires only that the two goods be differentiated members of the same (narrowly defined) product class.

The technology associated with producing the low quality good is described by a fixed cost, F_L, a constant marginal cost, c_L, and a capacity constraint on firm size, s_L. Similarly, the technology associated with producing the high quality good is described by a fixed cost, F_H, a constant marginal cost, c_H, and a capacity constraint on firm size, s_H. The capacity constraint is an analytically convenient substitute for the usual assumption of U shaped average cost curves. In this model, $p_L^* = c_L + (F_L/s_L)$ and $p_H^* = c_H + (F_H/s_H)$ thus become the ''competitive'' prices associated with

the two quality levels. The only assumption we make regarding the relationship between the two technologies is that the competitive prices differ; to preserve consistency with our earlier terminology, we suppose that $p_L^* < p_H^*$. Firms offer either the low quality good or the high quality good but not both. The total number of firms is N, with N_L firms selling low quality goods and N_H firms selling high quality goods. We define $n_L = N_L/N$ and $n_H = N_H/N$. Firms also do not advertise. Instead, they charge a price, wait to see who buys and alter prices when this would increase expected profits.

Each consumer in the market lives for one period, demands one unit of the low quality good or one unit of the high quality good (but not both), and either purchases or gets a raincheck at the end of the period if he or she finds a firm whose price is acceptable. Consumers are partitioned in two distinct ways, according to those who shop and those who do not shop and according to those who ''prefer'' low quality and those who ''prefer'' high quality. The nature of this preference for quality will be made clear below. For now, let A_1 be the number of nonshoppers and A_2 be the number of shoppers; the total number of consumers is then $A = A_1 + A_2$, where $A_1 > 0$ and $A_2 > 0$. Also, let A_1^L be the number of nonshoppers who prefer low quality and A_2^L be the number of shoppers who prefer low quality. Using a similar notation for high quality, we have $A^L = A_1^L + A_2^L$ and $A^H = A_1^H + A_2^H$.

All consumers actually search pursuant to a fixed sample size strategy, in accordance with which each consumer creates and exhausts a preset sample of firms before he or she begins to shop.[4] For some consumers--the ''nonshoppers''--the sample size is one; for the others--the ''shoppers''--the sample size is n. For the remainder of this paper we restrict n to two for expositional convenience. The shoppers thus sample precisely two firms at random across both quality levels before purchasing. This shopping pattern is one of a set of sufficient conditions that allows the markets for the two quality types to interact. An alternative shopping pattern that would also allow the markets for the two types of goods to interact would arise if consumers were aware of the quality each firm offered but not its price; would buy either type of good if offered the opportunity to do so at its competitive price; and chose deliberately to shop across quality levels to compare price-quality tradeoffs. We rule out a ''cross-quality'' shopping pattern in this paper for two reasons. First, it is to some extent inconsistent with the model's formal assumption that consumers learn about prices and qualities only by direct sampling of firms. Second, the random shopping strategy has fairly broad application because low and high quality refer, as said above, only to differentiated members of the same (narrowly defined) product class.

A crucial aspect of modeling consumer behavior in a world of heterogeneous goods, as this discussion suggests, is to characterize consumer preferences across quality classes. In this paper, we consider only competitive equilibria, and thus we need not specify consumer behavior over all possible combinations of prices for the two types of goods; rather we can hold one or the other price fixed at its

competitive level and define preferences in terms of it.

To begin, suppose that if offered the opportunity to buy the goods at their competitive prices, denoted (p_L^*, p_H^*), members of A^L would buy the low quality good and members of A^H would buy the high quality good. Next let the price of the low quality good remain fixed at p_L^* but the price of the high quality good rise. Members of A^L would still want the low quality good, but members of A^H at some point would switch from buying high to buying low quality. Let $\bar{p}_H$ be the price for the high quality good at which these latter consumers are just indifferent to switching; in other words, suppose there to exist a price $\bar{p}_H > p_H^*$ such that, if offered the opportunity to buy the goods at prices (p_L^*, p_H), members of A^H will buy high quality if $p_H \leq \bar{p}_H$ and will buy low quality if $p_H > \bar{p}_H$. Similarly, suppose there to exist a price $\bar{p}_L > p_L^*$ such that, if offered the opportunity to buy the goods at prices (p_L, p_H^*), members of A^L will buy low quality if $p_L \leq \bar{p}_L$ and will buy high quality if $p_L > \bar{p}_L$.

Consumers also have ''limit prices'' for both quality levels. If no high quality goods were available, l_L is the maximum price that a consumer who prefers low quality would pay for the low quality good and h_L is the maximum price that a consumer who prefers high quality would pay for the low quality good. Similarly, if no low quality goods were available, l_H is the maximum price that a consumer who prefers low quality would pay for the high quulity good and h_H is the maximum price that a consumer who prefers high quality would pay for the high quality good. Three comments need to be made regarding these limit prices. First, although each consumer in reality would have an individual limit price, we suppose all members of A^L and all members of A^H to have common respective limit prices. This assumption is analytically convenient and does not affect the model's qualitative conclusions. Second, limit prices for the low quality good are independent of conditions in the market for the high quality good, and limit prices for the high quality good are independent of conditions in the market for the low quality good. These assumptions reflect the premise that consumers begin shopping with very imprecise information regarding the two markets.

The third important point respecting these limit prices is that since consumers are assumed to buy only one unit of the good, the difference between the limit price for a quality type and the purchase price of that quality type measures consumer surplus. This enables our assumptions concerning consumers' tastes for quality to be translated into constraints on the relationship between limit prices and competitive prices. We have already assumed that $\bar{p}_L > p_L^*$ and $\bar{p}_H > p_H^*$. It is also realistic to require that $\bar{p}_L \leq l_L$ and $\bar{p}_H \leq h_H$. That is, the price at which consumers who prefer low quality will switch from low quality to high quality (given that high quality can be purchased at its competitive price) is less than or equal to the maximum price consumers would pay for low quality rather than go without the good altogether; and the price at which consumers who prefer high quality will switch from high quality to low quality (given that low quality can be purchased at its competitive price) is

less than or equal to the maximum price they would pay for high quality rather than go without the good altogether. These assumptions yield two constraints on $\bar{p}_L$ and $\bar{p}_H$:

$$p_L^* < \bar{p}_L \leq l_L, \tag{1}$$

$$p_H^* < \bar{p}_H \leq h_H. \tag{2}$$

The ''switch price'' for consumers who prefer low quality has been defined as that price at which the consumer is indifferent between buying the low quality good and switching to the high quality good, assuming it can be purchased at its competitive price. In terms of consumer surplus, this means that $l_L - \bar{p}_L = l_H - p_H^*$. Rearranging this expression gives an analytical definition of $\bar{p}_L$:

$$\bar{p}_L = l_L - l_H + p_H^*. \tag{3}$$

An analogous argument provides an analytical definition of $\bar{p}_H$:

$$\bar{p}_H = h_H - h_L + p_L^*. \tag{4}$$

Substituting (3) and (4) into (1) and (2) yields:

$$p_L^* < l_L - l_H + p_H^* \leq l_L, \tag{5}$$

$$p_H^* < h_H - h_L + p_L^* \leq h_H. \tag{6}$$

The right hand inequalities in (5) and (6) reduce to $p_H^* \leq l_H$ and $p_L^* \leq h_L$. If these inequalities are not satisfied, the markets will necessarily segment--consumers who prefer low quality would never buy high quality and consumers who prefer high quality would never buy low quality.[5]

The left hand inequalities in (5) and (6) can be summarized as

$$l_H - l_L < p_H^* - p_L^* < h_H - h_L. \tag{7}$$

The terms $l_H - l_L$ and $h_H - h_L$ are interpreted as the marginal willingness to pay for high quality by consumers who prefer low quality and consumers who prefer high quality, respectively. This premium must be less than the marginal cost of high quality in competitive equilibrium for consumers who prefer low quality and greater than the marginal cost of high quality in competitive equilibrium for consumers who prefer high quality, in order for the two markets to interact. Because there is no compelling reason for equation (7) to be satisfied in most cases, this constraint also suggests that segmentation is likely to occur.

Finally, in this model, equilibrium is defined by a total consumer firm ratio, A/N, a distribution of firms across the two quality levels, (n_L, n_H), and a distribution of prices for each quality level such that (a) all consumers pursue specified shopping strategies, (b) given the equilibrium distribution of firms across the two quality levels all firms earn zero expected profits, and (c) no firm can earn positive profits by changing its price offer or its quality level.

3. Necessary and Sufficient Conditions for a Competitive Equilibrium I: Balancing Constraints

Under full information, the classical competitive equilibrium would satisfy the equilibrium conditions just specified. There would be A^L/s_L firms producing the low quality good, each charging p_L^*, and A^H/s_H firms producing the high quality good, each charging p_H^*. The assumptions of this model, however, preclude such an outcome. Because of ''spillovers'' between the markets, our model at best permits a ''pseudo-competitive'' equilibrium in which all firms earn zero profits and those which sell the low quality good do so at p_L^* and those which sell the high quality good do so at p_H^*. The difference between this pseudo-competitive equilibrium and the classical competitive equilibrium is that in the former N_L need not equal A^L/s_L and N_H need not equal A^H/s_H. This paper thus uses the phrase ''competitive equilibrium'' actually to describe a pseudo-competitive equilibrium.[6]

In our earlier homogeneous search goods model, the number of firms extant in competitive equilibrium was given simply by the ratio of total consumers to firm capacity. Hence the necessary and sufficient condition for a competitive equilibrium in that model was derived by asking whether deviating from the competitive price would be an unprofitable strategy for any firm. In the present heterogenous search goods model, an additional set of constraints is needed in order to maintain equality between expected demand and capacity in each market. We first derive these new ''balancing constraints'' in

Section 3. The ''breaking constraints,'' which imply that deviating from the competitive price would be an unprofitable strategy for either low or high quality firms, are derived in Section 4.

When $A_2^L + A_2^H > 0$, the only possible degenerate equilibrium is at (p_L^*, p_H^*).[7] To derive necessary and sufficient conditions for this pair to be an equilibrium, we initially take N_L and N_H as given, and assume that all firms selling the low quality good charge p_L^* and all firms selling the high quality good charge p_H^*. We then calculate expected demand for each type of firm, which we set equal to capacity in each market to allow us to solve directly for N_L and N_H.

To begin, all firms get an equal share of the nonshoppers, A_1/N, but do not get an equal share of the shoppers even though there is no price dispersion within quality levels. Members of A_2^L who sample two firms offering the high quality good will buy from one of them at random. Those who sample one firm offering the low quality good and one firm offering the high quality good will buy the low quality good. Those who sample two firms offering the low quality good will buy from one of them at random. Members of A_2^H behave similarly.

To calculate expected demand for firms offering the low quality good, first suppose that a member of A_2^L samples a firm offering the low quality good. The probability that this consumer buys from the firm is equal to the probability that his or her other observation is from a firm offering the low quality good times one-half plus the probability that his or her other observation is from a

firm offering the high quality good; i.e. $[(N_L/N)(1/2) + (N_H/N)]$.[8] The probability that a member of A_2^L samples any given firm is 2/N because shoppers sample precisely two firms. Hence the total expected demand from members of A_2^L is

$$A_2^L(2/N)[(N_L/N)(1/2) + (N_H/N)] = (A_2^L/N)[(N_L/N) + (2N_H/N)].$$

Next suppose that a member of A_2^H samples a firm offering the low quality good. The probability that this consumer buys from the firm is equal to the probability that his or her other observation is also from a firm offering the low quality good times one-half. The probability that a member of A_2^H samples any given firm is again 2/N. Hence the total expected demand from members of A_2^H is

$$A_2^H(2/N)(N_L/N)(1/2) = (A_2^H/N)(N_L/N).$$

In consequence, expected demand at firms offering the low quality good is

$$D_L = (A_2^L/N)[(N_L/N) + (2N_H/N)] + (A_2^H/N)(N_L/N) + (A_1/N).$$

Similarly,

$$D_H = (A_2^H/N)[(N_H/N) + (2N_H/N)] + (A_2^L/N)(N_H/N) + (A_1/N).$$

The next step in deriving necessary and sufficient conditions for a competitive distribution of firms over (p_L^*, p_H^*) to be an equilibrium is to note that $D_L = s_L$, and $D_H = s_H$ are implied by the zero profit constraint. This is because demand persistently greater than capacity implies that firms could profitably enter while demand persistently less than capacity implies that firms are earning negative profits or charging noncompetitive prices. From the definitions of D_L and D_H and the zero profit constraint we thus get the following necessary condition for a competitive equilibrium:

$$(A_1/N) + (A_2^L/N_L)[1-(N_H/N)^2] + (A_2^H/N_L)(N_L/N)^2 = s_L$$

$$(A_1/N) + (A_2^H/N_H)[1-(N_L/N)^2] + (A_2^L/N_H)(N_H/N)^2 = s_H$$

These will determine N_L and N_H. The details of this derivation are presented in the appendix. It turns out that

$$N_L = [A(s_L-s_H) - s_H(A_2^L-A_2^H)]/(s_L-s_H)^2 \quad (8)$$

$$N_H = [s_L(A_2^L-A_2^H) - A(s_L-s_H)]/(s_L-s_H)^2 \quad (9)$$

Equations (8) and (9) imply that

$$N(s_L-s_H) = (A_2^L- A_2^H) \quad (10)$$

We also require $N_L \geq 0$ and $N_H \geq 0$. From (8) and (9), these constraints are equivalent to

$$s_L(A_2^L- A_2^H) \geq A_1(s_L- s_H) \geq s_H(A_2^L- A_2^H). \quad (11)$$

Condition (11) is not sufficient to guarantee that the competitive equilibrium will occur. It does establish constraints (necessary conditions), however, on the mix of shoppers and

nonshoppers that must be associated with a proper balance of firms offering the low quality good and firms offering the high quality good for a competitive distribution of firms to be an equilibrium.

These balancing constraints nevertheless are strong and nonintuitive. For example, equation (10) shows that for a competitive equilibrium in both markets to exist, the difference between the number of shoppers who prefer the low quality good and the number of shoppers who prefer the high quality good must have the same sign as the difference between the capacity constraint for firms offering the low quality good and the capacity constraint for firms offering the high quality good. The explanation of this necessary condition is that firms with large capacity need to attract more consumers in competitive equilibrium than firms with small capacity. If capacity constraints are roughly similar--s_L is not much greater than s_H--but A_2^L is considerably larger than A_2^H, a competitive equilibrium could still occur through adjustments in the proportions of firms: n_L will increase and n_H will decline. These adjustments, however, could never overcome an absolute advantage in the opposite direction of the capacity constraints because each firm has an inherent advantage in attracting those shoppers who prefer its own quality level. Thus, if $s_H > s_L$ while $A_2^L > A_2^H$, firms which offer high quality goods would experience persistent excess capacity.

4. Necessary and Sufficient Conditions for a Competitive Equilibrium II: Breaking Constraints

Suppose that the constraint in equation (11) is satisfied; N_L firms offer the low quality good at p_L^*; and N_H firms offer the high quality good at p_H^* (where N_L and N_H are given by equations (8) and (9)). We wish to characterize conditions under which this situation is stable with respect to individual firms raising their prices. A firm that deviates from the competitive equilibrium has several pricing options, with the profitability of each depending on the relationships between switch prices and limit prices in the relevant market. We first consider firms offering the low quality good.

4(a): Deviations by Low Quality Firms

Suppose that all high quality firms charge p_H^* and all low quality firms except one charge p_L^*. The deviant firm has three pricing options, to charge $\bar{p}_L$, l_L or h_L. The relative profitability of each option depends on the relationships between the three prices.

Case L1: $p_L^* < h_L < \bar{p}_L < l_L$.

If all other firms charge the competitive price, any firm selling the low quality good that raises its price to h_L will lose only those shoppers who have sampled both it and another firm offering the low quality good, regardless of whether they prefer low quality or high quality. The firm retains all nonshoppers and those shoppers who prefer low quality and sample one firm offering the low quality good and one firm offering the high quality good. Expected profits become

$$\pi_1^L(h_L) = [(A_1/N) + 2(A_2^L)/N)(N_H/N)](h_L - c_L) - F_L.$$

The competitive distribution at (p_L^*, p_H^*) is an equilibrium if expected profits from deviating (in this case charging h_L) are nonpositive; that is, if

$$(A_1/N) + 2(A_2^L/N)(N_H/N) < F_L/(h_L - c_L).$$

Recall that $n_L = N_L/N$ and $n_H = N_H/N$. Let $\bar{s} = n_L s_L + n_H s_H$ be "average" capacity under a competitive distribution. Then the constraint requisite for a competitive equilibrium is

$$a_1 + 2a_2^L n_H \leq F_L/\bar{s}(h_L - c_L) \qquad (12)$$

where $a_1 = A_1/A$ and $a_2^L = A_2^L/A$.

Now suppose the deviant firm raises its price to $\bar{p}_L$. In this case it loses, in addition to shoppers who have sampled another low quality firm, those nonshoppers who prefer high quality. This is because $\bar{p}_L > h_L$. Expected profits become

$$\pi_1^L(\bar{p}_L) = [(A_1^L/N) + 2(A_2^L/N)(N_H/N)](\bar{p}_L - c_L) - F_L.$$

The associated constraint requisite for a competitive equilibrium is

$$a_1^L + 2a_2^L n_H \leq F_L/\bar{s}(\bar{p}_L - c_L) \qquad (13)$$

where $a_1^L = A_1^L/A$.

Finally, suppose that the deviant firm raises its price to l_L. In this case, it loses all shoppers and those nonshoppers who prefer high quality. This is because $l_L > h_L$. Expected profits become

$$\pi_1^L(l_L) = (A_1^L/N)(l_L - c_L) - F_L.$$

The associated constraint requisite for a competitive equilibrium is

$$a_1^L \leq F_L/\bar{s}(l_L - c_L). \quad (14)$$

A low quality firm would have no incentive to depart from the competitive price in case L1 only if equations (12), (13) and (14) __all__ hold; in equivalent summary notation, $\max[\pi_1^L(h_L), \pi_1^L(\bar{p}_L), \pi_1^L(l_L)] \leq 0$.

__Case L2__: $p_L^* < \bar{p}_L < h_L < l_L$

The analysis for this case proceeds as in case L1. If a deviant firm charges $\bar{p}_L$, it loses only those shoppers who have sampled two low quality firms. Hence profits are

$$\pi_2^L(\bar{p}_L) = [(A_1/N) + 2(A_2^L/N)(N_H/N)](\bar{p}_L - c_L) - F_L,$$

and the analogue to (13) is

$$a_1 + 2a_2^L n_H \leq F_L/\bar{s}(\bar{p}_L - c_L). \quad (15)$$

If the firm charges h_L, it loses all shoppers since $h_L > \bar{p}_L$. Hence profits are

$$\pi_2^L(h_L) = (A_1/N)(h_L - c_L) - F_L,$$

and the analogue to (12) is

$$a_1 \leq F_L/\bar{s}(h_L - c_L). \quad (16)$$

Finally, if it charges l_L it again loses all shoppers and those nonshoppers who prefer high quality. Hence profits are

$$\pi_2^L(l_L) = (A_1^L/N)(l_L - c_L) - F_L,$$

and (14) remains unchanged:

$$a_1^L \leq F_L/\bar{s}(l_L - c_L). \quad (17)$$

The breaking constraints for the low quality firm in case L2 consist of equations (15, (16) and (17) __all__ holding, or in equivalent summary notation, $\max[\pi_2^L(h_L), \pi_2^L(\bar{p}_L), \pi_2^L(l_L)] \leq 0$.

__Case L3__: $p_L^* < \bar{p}_L < l_L < h_L$

The logic here works precisely as in the first two cases. Profits are

$$\pi_3^L(h_L) = (A_1^H/N)(h_L - c_L) - F_L,$$

$$\pi_3^L(\bar{p}_L) = [(A_1/N) + 2(A_2^L/N)(N_H/N)](\bar{p}_L - c_L) - F_L,$$

$$\pi_3^L(l_L) = (A_1/N)(l_L - c_L) - F_L;$$

and the analogues to (12), (13), and (14) are, respectively,

$$a_1^H \le F_L/\bar{s}(h_L - c_L), \tag{18}$$

$$a_1 + 2a_2^L n_H \le F_L/\bar{s}(\bar{p}_L - c_L), \tag{19}$$

$$a_1 \le F_L/\bar{s}(l_L - c_L). \tag{20}$$

The breaking constraints for the low quality firm in case L3 consist of equations (18), (19) and (20) **all** holding, or in equivalent summary notation, $\max(\pi_3^L(h_L), \pi_3^L(\bar{p}_L), \pi_3^L(l_L)) \le 0$.

4(b): Deviations by High Quality Firms

Suppose that all low quality firms charge p_L^* and all high quality firms except one charge p_H^*. The deviant firm has three pricing options, analogous to the low quality deviant analyzed in section 4(a); it can charge $\bar{p}_H$, l_H or h_H. Again, three cases of interest arise. These cases and the associated breaking constraints follow.

<u>Case H1</u>: $p_H^* < l_H < \bar{p}_H < h_H$.

$$\pi_1^H(l_H) = [(A_1/N) + 2(A_2^H/N)(N_L/N)](l_H - c_H) - F_H,$$

$$\pi_1^H(\bar{p}_H) = [(A_1^H/N) + 2(A_2^H/N)(N_L/N)](\bar{p}_H - c_H) - F_H,$$

$$\pi_1^H(h_H) = (A_1^H/N)(h_H - c_H) - F_H.$$

The breaking constraints are thus

$$a_1 + 2a_2^H n_L \le F_H/\bar{s}(l_H - c_H), \tag{21}$$

$$a_1^H + 2a_2^H n_L \le F_H/\bar{s}(\bar{p}_H - c_H), \tag{22}$$

$$a_1^H \le F_H/\bar{s}(h_H - c_H). \tag{23}$$

Equation (21), (22) and (23) must **all** hold, or equivalently, $\max(\pi_2^H(l_H), \pi_1^H(\bar{p}_H), \pi_1^H(h_H)) \le 0$ for the competitive distribution at (p_L^*, p_H^*) to be an equilibrium.

<u>Case H2</u>: $p_H^* < \bar{p}_H < l_H < h_H$.

$$\pi_2^H(l_H) = (A_1/N)(l_H - c_H) - F_H,$$

$$\pi_2^H(\bar{p}_H) = [(A_1/N) + 2(A_2^H/N)(N_L/N)](\bar{p}_H - c_H) - F_H,$$

$$\pi_2^H(h_H) = (A_1^H/N)(h_H - c_H) - F_H.$$

The breaking constraints are thus

$$a_1 \le F_H/\bar{s}(l_H - c_H), \tag{24}$$

$$a_1 + 2a_2^H n_L \le F_H/\bar{s}(\bar{p}_H - c_H), \tag{25}$$

$$a_1^H \le F_H/\bar{s}(h_H - c_H). \tag{26}$$

Equations (24), (25) and (26) must **all** hold, or equivalently,

$\max\{\pi_2^H(l_H), \pi_2^H(\bar{p}_H), \pi_2^H(h_H)\} \leq 0$ for the competitive distribution at (p_L^*, p_H^*) to be an equilibrium.

__Case H3__: $p_H^* < \bar{p}_H < h_H < l_H$.

$$\pi_3^H(l_H) = (A_1^L/N)(l_H - c_H) - F_H,$$

$$\pi_3^H(\bar{p}_H) = [(A_1/N) + 2(A_2^H/N)(N_L/N)](\bar{p}_H - c_H) - F_H,$$

$$\pi_3^H(h_H) = (A_1/N)(h_H - c_H) - F_H.$$

The breaking constraints are

$$a_1^L \leq F_H/\bar{s}(l_H - c_H), \tag{27}$$

$$a_1 + 2a_2^H n_L \leq F_H/\bar{s}(\bar{p}_H - c_H), \tag{28}$$

$$a_1 \leq F_H/\bar{s}(h_H - c_H). \tag{29}$$

Equations (27), (28) and (29) must __all__ hold, or equivalently, $\max\{\pi_3^H(l_H), \pi_3^H(\bar{p}_H), \pi_3^H(h_H)\} \leq 0$ for the competitive distribution at (p_L^*, p_H^*) to be an equilibrium.

The constraints reflected in equations (15) through (29) are roughly analogous to the single constraint derived in Wilde and Schwartz [1979] for the homogeneous search goods case. Together with equation (11), these constraints provide a set of necessary and sufficient conditions for the competitive distribution of firms at

(p_L^*, p_H^*), defined by (8) and (9), to be an equilibrium; that is, they provide a set of necessary and sufficient conditions for a competitive outcome in both markets.

5. An Analysis of the Breaking Constraints: The Relevance of Product Heterogeneity

To understand the relevance of the constraints derived above, it is important to recognize that the markets for low and high quality goods interact in two distinct ways. First, a member of A_2^L can spill over ''completely'' into the market for high quality goods if both of his or her observations are taken at firms which offer only the high quality good. Such a member of A_2^L is effectively a comparison shopper in the market for high quality goods. A member of A_2^H similarly can spill over completely into the market for low quality goods. This complete spillover has less effect on the equilibrium that will obtain than ''partial'' spillover: a member of A_2^L can spillover ''partially'' into the market for high quality goods if precisely one of his or her observations is taken at a firm which offers the high quality good. Such a member of A_2^L is effectively a nonshopper in both markets. A member of A_2^H similarly can spill over partially into the market for low quality goods if precisely one of his or her observations is taken at a firm which offers the low quality good. Partial spillover is another term for the dilution in the effectiveness of search that product heterogeneity creates. (see page 4 above) As an example, let $A_1 = 0$ so that all consumers are shoppers. Even in this case, a competitive equilibrium may not obtain for either good because, although everyone shops, some nonshoppers will inevitably exist in both markets.

Partial spillover or ''search dilution'' also helps explain

the nature of the breaking constraints. Equations (15) through (29) reveal two kinds of breaking constraints, those which only include terms associated with nonshoppers (a_1, a_1^L, or a_1^H) on the left-hand side, and those which include additional terms associated with shoppers (a_2^L or a_L^H) on the left-hand side. The effects of changes in consumer shopping patterns on the likelihood that a competitive equilibrium will obtain is sensitive to which type of constraint is actually binding. For example, in case L3, if $\max\{\pi_3^L(h_L), \pi_3^L(\bar{p}_L), \pi_3^L(l_L)\} = \pi_3^L(\bar{p}_L)$, then equation (19)—$a_1 + 2a_2^L n_H \leq F_L/\bar{s}(\bar{p}_L - c_L)$—is the operative constraint, but if $\max\{\pi_3^L(h_L), \pi_3^L(\overline{p_L}), \pi_3^L(l_L)\} = \pi_3^L(l_L)$, then equation (20)—$a_1 \leq F_L/\bar{s}(l_L - c_L)$—is the operative constraint. Whether a low quality firm is more or less likely to deviate from p_L^* when consumer shopping patterns change in particular ways depends crucially on which type of constraint is operative.

Understanding more clearly a firm's incentives to deviate from the competitive price requires a two step approach. First, we increase the proportion of shoppers in such a way as to keep N_L and N_H constant. This is called a ''balanced'' change in shoppers. Second, we hold the total proportion of shoppers constant but shift consumers between A_2^L and A_2^H. This is called an ''unbalanced'' change in shoppers.

5(a) Balanced Changes in Shoppers

Define $K_1 = a_2^L - a_2^H$. Suppose that a_2^L increases subject to two conditions: K_1 remains constant and neither a_1^L nor a_1^H rises (*i.e.* we allow no absolute redistribution between A_1^L and A_1^H). Then from (8) and (9) it is immediate that $n_L = [1/(a_2^L - a_2^H)] - [s_H/(s_L - s_H)]$ and $n_H = [s_L/(s_L - s_H)] - [1/a_2^L - a_2^H)]$, so that

$$(\partial n_L/\partial_2^L)|_{K_1} = 0 = (\partial n_H/\partial a_2^L)|^{K_1}$$

Hence $\bar{s} = n_L s_L + n_H s_H$ is constant with respect to balanced changes in shoppers. Now consider the effects of such changes on the likelihood that a competitive equilibrium will obtain when the operative constraints on both kinds of firms do not include terms associated with shoppers. A balanced increase in shoppers will never increase the left hand side of these constraints and will usually lower it since the increase in a_2^L and a_2^H comes at the expense of a_1^L and a_1^H. This implies that a decrease in the number of nonshoppers makes a competitive equilibrium more likely to occur in both markets. The intuition behind this result is clear: the operative constraints fail to include a_2^L and a_2^H only when the price that maximizes profits for a deviant firm necessarily eliminates all shoppers from consideration. In such a case, that product heterogeneity can dilute the effectiveness of search is irrelevant; the deviant firm sells only to nonshoppers, and when their number is reduced it can become unprofitable for the firm to deviate.

As an example, consider case L_3, in which $p_L^* < \bar{p}_L < l_L < h_L$, and suppose a deviant low quality firm to consider charging $p_L = l_L$. At this price, the firm would lose all shoppers whose other firm is

low quality because by assumption that firm charges p_L^* and $p_L^* < l_L$. It also would lose any shoppers whose other firm is high quality: members of A_2^H prefer high quality when it trades at its competitive price, and members of A_2^L switch to high quality in this circumstance since $l_L > \bar{p}_L$. Thus a low quality firm contemplating a deviation from p_L^* to l_L will ask only whether the total percentage of nonshoppers is high enough to make the deviation profitable; this firm would never sell to a shopper. Since a balanced increase in the number of shoppers leaves the distribution of firms unchanged (and thus the equilibrium consumer firm ratio), a decrease in the percentage of nonshoppers will always make a deviation from the competitive equilibrium less likely in this case. This discussion is summarized in the following proposition:

<u>Proposition 1</u>: Let the operative constraints on both types of firm be independent of a_2^L and a_2^H. Then a balanced increase (decrease) in shoppers which does not increase (decrease) either a_1^L or a_1^H increases (decreases) the likelihood that both markets are competitive.

Next suppose the operative constraints on both kinds of firm to include a_2^L or a_2^H and initially consider low quality firms. Most constraints which include a_2^L or a_2^H take the form $a_1 + 2a_2^L n_H$ on the left-hand side (the exception is (13) in case L1). The term $2a_2^L n_H$ is associated with partial spillovers; it only arises when the price associated with the operative constraint is less than or equal to the switchprice for consumers who prefer low quality ($\bar{p}_L$). When a deviant low quality firm charges a price above the competitive price but below the switchprice, it retains those members of A_2^L who have partially spilled over into the market for high quality. As long as the price it charges is less than or equal to $\min\{l_L, h_L\}$, it also retains all members of A^1. The increase in a_2^L makes it more likely that the firm will wish to deviate from p_L^*, because of the effect of partial spillovers, but the decrease in a_1 makes such a motivation less likely. In this case, the latter effect dominates because a_2^L is weighted by n_H and balanced increases do not affect n_L or n_H. When the price that the deviant low quality firm charges is greater than $\min\{l_L, h_L\}$, (i.e. case L2, $p_L^* < \bar{p}_L < h_L < l_L$), the net effect is ambiguous, depending on the extent to which the decrease in a_1 comes at the expense of a_1^L or a_1^H.

A similar discussion applies to possible deviations by high quality firms. If the price charged by a deviant firm (i.e. the price associated with the operative constraint) is less than or equal to $\min\{l_H, h_H\}$, then a balanced increase in shoppers makes it more likely that a competitive equilibrium will obtain. If the price is greater than $\min\{l_H, h_H\}$ (i.e. case H1, $p_H = \bar{p}_H$), then the net effect is again ambiguous. We thus have the following proposition.

Proposition 2: Assume that the operative constraints on both types of firm depend on a_2^L or a_2^H. Let p_L^D and p_H^D be the prices which maximize profits for deviant firms. Then (1) $p_L^D \leq \min\{l_L, h_L\}$ and $p_H^D \leq \min\{l_H, h_H\}$ imply that a balanced increase (decrease) in shoppers will make it more (less) likely that both markets are competitive, and (2) $p_L^D > \min\{l_L, h_L\}$ and $p_H^D > \min\{l_H, h_H\}$ imply that a balanced increase

in shoppers has ambiguous effects on the likelihood of a competitive equilibrium occurring.

The implication of Propositions 1 and 2 is that balanced increases in shoppers generally tend to make it more likely that both markets are competitive. The second step in understanding how changes in the mix of consumers affect the likelihood of a competitive equilibrium in both markets is to consider unbalanced changes in a_2^L and a_2^H--changes which do not keep $a_2^L - a_2^H$ constant.

5(b) Unbalanced Changes in Shoppers

Unbalanced changes in a_2^L and a_2^H induce changes in the mix of low and high quality firms (and thus in $\bar{s}$). To begin to understand the effect of these changes, suppose that $s_L > s_H$ and consider an increase in a_2^L that comes entirely at the expense of a_2^H; that is, a_1^L and a_1^H are held constant while some shoppers shift from the group that prefers high quality to the group that prefers low quality. Define $K_2 = a_2^L + a_2^H$. It is shown in the appendix that

$$(\partial n_L/\partial a_2^L)|_{K_2} < 0 \quad \text{and} \quad (\partial n_H/\partial a_2^L)|_{K_2} > 0.$$

These derivatives imply that $\bar{s}$ must decrease since we have assumed that $s_L > s_H$.

The next step in the analysis is again to divide the breaking constraints into those that depend on a_2^L or a_2^H and those that do not. The constraints that do not depend on a_2^L or a_2^H are more likely to be satisfied—i.e. deviations from the competitive price are less likely--when the increase in a_2^L is unbalanced. To see why this is so, recall that such a shift both increases the proportion of shoppers who are predisposed to buy low quality and decreases the number of low quality firms. Unless more of the shoppers who prefer low quality spill over totally into the market for high quality, excess demand will occur in the low quality market. To avoid this disequilibrium phenomenon - i.e. to facilitate total spillovers - the number of high quality firms must increase. When high quality firms are assumed to have lower capacity, the equilibrium consumer firm ratio thus must also decline. Such a decline makes deviations from competitive prices less profitable for firms that would depend only on the business of nonshoppers after the price rise. This is because, with a lowered consumer firm ratio, each firm has fewer expected customers, including fewer nonshoppers, and so is less equipped to withstand loss of patronage. This leads to Proposition 3:

Proposition 3: Suppose that $s_L > s_H$ and that the operative constraints on both types of firm are independent of a_2^L and a_2^H. Then an unbalanced increase in a_2^L (a_2^H) will increase (decrease) the likelihood that both markets are competitive.

The final step in our formal analysis is to consider unbalanced changes in shoppers when the operative constraints on both types of firm do depend on a_2^L or a_2^H. It is shown in the appendix that:

$$(\partial \bar{s}(a_1 + 2a_2^L n_H)/\partial a_2^L)|_{K_2} = (\partial \bar{s}(a_1^L + 2a_2^L n_H)/\partial a_2^L|_{K_2} > 0$$

and

$$(\partial \bar{s}(a_1 + 2a_2^H n_L)/\partial a_2^L)|_{K_2} = (\partial \bar{s}(a_1^H + 2a_2^H n_L)/\partial a_2^L)|_{K_2} < 0.$$

These derivatives imply that an unbalanced increase in a_2^L will make it less likely that the market for low quality goods is competitive and more likely that the market for high quality goods is competitive. This is a somewhat startling result: An increase in the number of shoppers who prefer the low quality good which comes entirely at the expense of shoppers who prefer the high quality good makes it less likely that firms offering the low quality good will find it unprofitable to increase prices above the competitive price but makes it more likely that firms offering the high quality good will find it unprofitable to increase prices above the competitive price.

To see why this result obtains, observe that the operative constraint for low quality deviants depends on a_2^L if and only if the price that maximizes profits for the deviant firm is less than or equal to the switchprice $\bar{p}_L$. Similarly, the operative constraint for high quality deviants depends on a_2^H if and only if the price that maximizes profits for the deviant firm is less than or equal to the switchprice $\bar{p}_H$. In these cases, partial spillover matters. Furthermore, while a shift from a_2^H to a_2^L increases the number of comparison shoppers who prefer the low quality good, it also increases the number of these consumers who will take only one observation in the market for low quality goods because some members of A_2^L will partially spillover into the market for high quality goods. A firm offering the low quality good that wishes to deviate from p_L^* knows that it will lose all comparison shoppers but get all nonshoppers. Because of partial spillover, the number of nonshoppers in the low quality market actually increases when a_2^L increases entirely at the expense of a_2^H. Thus, a firm offering the low quality good is more likely to deviate from p_L^* . In the market for high quality goods, as a_2^H decreases, the number of shoppers who will take only one observation in the market for high quality goods declines. So does the number of shoppers in this market who prefer high quality goods and take both observations in it. But from the point of view of a firm offering the high quality good, fewer nonshoppers exist as a result of the decline in a_2^H. Thus, this firm will find it less profitable to deviate from p_H^* and will be less likely to do so. The result is that when a_2^L increases entirely at the expense of a_2^H, a competitive equilibrium in the market for low quality goods is less likely to occur while a competitive equilibrium in the market for high quality goods is more likely to occur.

As with the case in which the operative constraints do not depend on a_2^L or a_2^H, there are again indirect effects associated with unbalanced changes in a_2^L and a_2^H. These indirect effects are associated with ''rebalancing'' the market when $a_2^L-a_2^H$ changes. When $s_L > s_H$ and $a_2^L + a_2^H$ is constant, it turns out that

$$(\partial n_L/\partial a_2^L)|_{K_2} < 0 \text{ and } (\partial n_H/\partial a_2^L)|_{K_2} > 0.$$

As before, the signs of these derivatives are explained by the need to

keep spillovers between the markets balanced with the capacity constraints, and work to reinforce the direct effects of a shift from a_2^H to a_2^L. An increase in n_H and a decrease in n_L makes it more likely that a member of A_2^L will partially spillover into the market for the high quality good because more high quality firms then exist. Also, this shift makes it less likely that a member of either A_2^L or A_2^H will act as a comparison shopper in the market for the low quality good. Similarly, an increase in n_H and a decrease in n_L makes it less likely that a member of A_2^H will partially spillover into the market for the low quality good, and more likely that a member of either A_2^L or A_2^H will act as a comparison shopper in the market for the high quality good.

These direct and indirect affects lead to a final proposition.

<u>Proposition 4</u>: Suppose that $s_L > s_H$ and that the operative constraints on both types of firm depend on a_2^L or a_2^H. Then an unbalanced increase (decrease) in a_2^L will decrease (increase) the likelihood that the market for low quality goods is competitive and will increase (decrease) the likelihood that the market for high quality goods is competitive.

6. Policy Implications and Limitations of the Analysis

To best perceive the policy implications of the analysis made above, consider first a market for a homogeneous search good. We have previously shown[9] that a competitive equilibrium will obtain in such a market if and only if $A_2/(A_1 + A_2) \geq 1-[F/s(p_L-\hat{p})]$, where F is a fixed cost, p_L is the common limit price of all consumers and $\hat{p}$ is the constant marginal cost. This model is helpful to decisionmakers in two ways. Fist, it suggests methods of recognizing whether markets are in competitive equilibrium or not, the principal criteria having to do with the extent of comparison shopping and the degree to which prices cluster. Second, it implies that the best way to move badly behaving markets toward competitive equilibria is to increase comparison shopping (increase $A_2/(A_1 + A_2)$) by reducing the costs of comparing purchase alternatives. How are these suggestions affected by the existence of product heterogeneity?

Initially, because heterogeneous goods markets seems likely to segment into homogeneous subsets, decisionmakers can very much simplify the task of deciding when an intervention in a market on information grounds is necessary by assuming that apparently distinct products actually do trade in separate markets. As an example, a decisionmaker deciding whether information problems make a market for stereo equipment noncompetitive could suppose, as a working assumption, that the markets for ''high end'' and ''low end'' equipment do not interact; he or she could thus treat each market as dealing with roughly homogeneous goods, and evaluate the markets' competitive states with the aid of our earlier model. This is not to say that interactions between markets will never occur or that goods in markets such as that for low end stereo equipment are truly homogeneous. It is only to say that some formal support exists to believe that a decisionmaker would seldom go seriously wrong if he or

she adopted a presumption of segmentation when segmentation seemed intuitively plausible. The savings in administrative costs from so acting perhaps could outweigh the errors that this presumption would cause.[10]

If markets for goods of different qualities do interact and are badly behaved for information reasons, the principal policy prescription of our earlier model--increasing the number of comparison shoppers--also remains plausible.[11] Initially, suppose that one or both of the markets is in equilibrium at the monopoly price (generally h_H or l_L). In this case, firms that lower their prices would get all shoppers who visit them; increasing the number of shoppers thus makes monopoly equilibria less likely. Also, driving prices below the monopoly price probably would increase welfare. This is because firms in the model described here earn zero expected profits in all equilibria and so would be indifferent to the equilibrium that actually obtained, while consumers would benefit ex ante from the lower prices. Suppose instead that a market is in equilibrium at a price between the competitive and monopoly prices. Because an increase in the number of shoppers can be decomposed into ''balanced'' and ''unbalanced'' increases and because some unbalanced increases can decrease the likelihood of competitive equilibria obtaining in one of the markets (see Proposition 4), an intervention to reduce the costs of comparison shopping in this ''intermediate'' case could have undesirable effects. Further, it would be difficult for decisionmakers to predict these effects in advance.

Even in this intermediate case, however, a strategy of increasing the amount of comparison shopping is probably wise. This is because increasing the amount of comparison shopping is likely also to increase the intensity of search, and the ability of product heterogeneity to dilute the effectiveness of search varies inversely with the amount of search that occurs. As an example, in the formal model above shoppers visited precisely two firms; it was thus relatively easy for them to visit one firm in each market and so effectively to be nonshoppers. If these shoppers' ssmple size were to increase, they would be less likely to get ''stuck'', and the undesirable effects of incressing the number of shoppers in the intermediate case might not occur. Another way to put this is that segmentation is more likely when search intensity increases because, for example, shoppers who prefer low quality may take enough observations in the low quality market to be comparison shoppers in it despite any high quality observations they may also have made. Given that the undesirable effects of an intervention apparently would obtain in a minority of intermediate cases even with the smallest shopping ssmple size possible (see Propositions 1-3), and that reducing the costs of comparison shopping actually could increase sample sizes, the model developed here suggests that a decisionmaker should always seriously consider reducing the costs of comparison shopping when a market is badly behaved for information reasons.

Recent evidence also is consistent with the view that this strategy can reduce prices in heterogeneous goods markets. Devine and

Marion provided consumers with some comparative price information and with a weighted index of prices on sixty-five common food items for supermarkets in a Canadian city for a five week period. A weighted index of supermarket prices is analogous to a single price for a heterogeneous good. Prices in the ssmple market declined substsntially and price dispersion decreased during the experimental period, while prices and dispersion were largely unaffected in the control market.[12]. Also, search intensity apparently incressed. In addition, the Devine and Marion study tentatively suggested that the gain in consumer surplus from these price declines exceeded the sum of the program's administrative costs and the decline in producers surplus. Thus reducing the costs of comparison shopping in badly behaved consumer markets is a potentially useful policy option.

A serious difficulty for a decisionmaker when markets for different quality goods do in fact interact, however, is how to know when these markets are badly behaved. The earlier homogeneous search goods model was sufficiently tractable for us to be able to characterize all possible equilibria from competitive to monopoly. We thus could develop criteria that would aid in identifying the kind of equilibrium that actually obtained in a market. Because this paper only characterizes competitive equilibria, we have little to say to a decisionmaker who wants to know, when markets do interact, whether problems exist or not. Our future work will attempt formally to characterize noncompetitive equilibria in heterogeneous goods markets.

The snalysis made above is limited as s source of advice to decisionmakers for two other reasons. Initially, we assume that consumers are aware of quality differences before they begin to search. Shopping, however, sometimes performs an educative function, in which persons learn about market options as they go along. Markets of this kind might be less well behaved than the markets described above since search is more likely to involve wasted effort, but in the absence of formal analysis it is difficult to know. Further, we dealt with a heterogeneous search good, but many goods, including some used in our intuitive explanations, have important experience aspects. Whether our conclusions are applicable in these cases again is a very open question, which we shall address elsewhere.

7. Conclusion

Markets that are said to behave badly because consumers are imperfectly informed have been significant objects of public concern for over a decade. Our earlier papers dealing with homogeneous search goods suggested that this concern may be exaggerated. This is because the principal factor causing markets to behave competitively is the extent of comparison shopping, and it apparently takes less comparison shopping to generate competitive outcomes than had previously been supposed. The present paper, dealing with markets for heterogeneous search goods, reaches conclusions largely consistent with our earlier work, for it shows that markets for heterogeneous search goods tend rather easily to segment into roughly homogeneous subsets. When heterogeneous goods markets do interact, however, product heterogeneity can work against the existence of competitive equilibria

equilibria because it dilutes the effectiveness of search. Increasing the extent of comparison shopping in these markets nevertheless remains a sensible remedy; they too are more likely to reach competitive equilibria if more direct comparison shopping occurs in them. Recognizing when markets that interact actually are behaving badly, however, is a difficult problem, respecting which the model described above has little to say. Also, any recommendations we do make to decisionmakers must be importantly qualified by the limitations of this model, in particular its assumptions that consumers are fully aware of quality differences before they begin to search and shop only for search goods. Further work will attempt to generalize the research described here.

APPENDIX

Claim 1 (p. 15):

$$N_L = [A(s_L - s_H) - s_H(A_2^L - A_2^H)/(s_L - s_H)^2$$
$$N_H = [s_L(A_2^L - A_2^H) - A(s_L - s_H)]/(s_L - s_H)^2$$

Proof: Expected demand equal to capacity for firms of each quality type yields (see p. 15)

$$(A_1/N) + (A_2^L/N_L)[1 - (N_H/N)^2] + (A_2^H/N_L)(N_L/N)^2 = s_L$$

and

$$(A_1/N) + (A_2^H/N_H)[1 - (N_L/N)^2] + (A_2^L/N_H)(N_H/N)^2 = s_H$$

Rearranging each of these and eliminating (A_1/N) yields

$$s_L - (A_2^L/N_L)[1 - (N_H/N)^2] - (A_2^H/N_L)(N_L/N)^2$$
$$= s_H - (A_2^H/N_H)[1 - (N_L/N)^2] - (A_2^L/N_H)(N_H/N)^2.$$

A number of tedious operations reduces this equation substantially:

$$s_LN_LN_H - A_2^LN_H[1 - (N_H/N)^2] - A_2^HN_H(N_L/N)^2$$
$$= s_HN_LN_H - A_2^HN_L[1 - (N_L/N)^2] - A_2^LN_L(N_H/N)^2$$

$$s_LN_LN_H - A_2^L[N_H - N_H(N_H/N)^2 - N_L(N_H/N)^2]$$
$$= s_HN_LN_H - A_2^H[N_L - N_L(N_L/N)^2 - N_H(N_L/N)^2]$$

$$s_LN_LH_H - A_2^L[N_H - N(N_H/N)^2] = s_HN_LN_H - A_2^H[N_L - N(N_L/N)^2]$$

$$s_LN_LN_H - A_2^LN_H[1 - (N_H/N)] = s_HN_LN_H - A_2^HN_L[1 - (N_2/N)]$$

$$s_LN_LN_H - A_2^LN_H(N_L/N) = s_HN_LN_H - A_2^HN_L(N_H/N)$$

$$Ns_L - A_2^L = Ns_H - A_2^H$$

$$N(s_L - s_H) = A_2^L - A_2^H$$

or, finally,

$$N = (A_2^L - A_2^H)/(s_L - s_H). \quad (A1)$$

Equation (A1) can be used to find N_L and N_L. To do this note that

$$N_Ls_L + N_Hs_H = A_1 + (A_2^L + A_2^H). \quad (A2)$$

Solving (A1) and (A2) for N_L and setting the results equal to each other yields:

$$[(A_2^L - A_2^H) - N_H(s_L - s_H)]/(s_L - s_H) = (A - N_Hs_H)/s_L.$$

Hence

$$(A_2^L - A_2^H)s_L - N_H(s_L - s_H)s_L = (A - N_Hs_H)(s_L - s_H)$$
$$N_H[s_H(s_L - s_H) - s_L(s_L - s_H)] = A(s_L - s_H) - s_L(A_2^L - A_2^H)$$

and, finally,

$$N_H = [s_L(A_2^L - A_2^H) - A(s_L - s_H)]/(s_L - s_H)^2.$$

Thus

$$N_L = N - N_H$$
$$= [(A_2^L - A_2^H)(s_L - s_H) - s_L(A_2^L - A_2^H) + A(s_L - s_H)]/(s_L - s_H)^2$$
$$= [A(s_L - s_H) - s_H(A_2^L - A_2^H)]/(s_L - s_H)^2$$

Q.E.D.

__Claim 2__ (p. 29):

$$(\partial n_L/\partial a_2^L)|_{K_2} > 0 \text{ and } (\partial n_H/\partial a_2^L)|_{K_2} > 0.$$

__Proof:__ By definition, $K_2 = a_2^L + a_2^H$, and

$$n_L = N_L/N$$
$$= [A(s_L - s_H) - s_H(A_2^L - A_2^H)]/(s_L - s_H)^2 \quad (A_2^L - A_2^H)/(s_L - s_H)$$
$$= [A(s_L - s_H) - s_H(A_2^L - A_2^H)]/(s_L - s_H)(A_2^L - A_2^H)$$

$$= [A/(A_2^L - A_2^H)] - [s_H/(s_L - s_H)] = [1/(a_2^L - a_2^H)] - [s_H/(s_L - s_H)].$$

Thus

$$(\partial n_L/\partial a_2^L)|_{K_2} = [-(\partial a_2^L/\partial a_2^L)|_{K_2} + (\partial a_2^H/\partial a_2^L)|_{k_2}]/(a_2^L - a_2^H)^2$$
$$= (-1 - 1)/(a_2^L - a_2^H)^2 = -2/(a_2^L - a_2^H)^2 < 0.$$

A similar calculation shows $(\partial n_H/\partial a_2^L)|_{K_2} > 0$.

Q.E.D.

__Claim 3__ (p. 30):

$$(\partial \bar{s}(a_1 + 2a_2^L n_H)/\partial a_2^L)|_{K_2} = (\partial \bar{s}(a_1^H + 2a_2^L n_H)/\partial a_2^L)|_{K_2} > 0$$

and

$$(\partial \bar{s}(a_1 + 2a_2^H n_L)/\partial a_2^L)|_{K_2} = (\partial \bar{s}(a_2^L + 2a_2^H n_L)/\partial a_2^L)|_{K_2} > 0.$$

__Proof:__

$$(\partial \bar{s}(a_1 + 2a_2^L n_H)/\partial a_2^L)|_{K_2} = (\partial \bar{s}/\partial a_2^L)|_{K_2} \quad (a_1 + 2a_2^L n_H)$$
$$+ \bar{s} \quad 2[a_2^L(\partial n_H/\partial a_2^L)|_{K_2} + n_H(\partial a_2^L/\partial a_2^L)|_{K_2}].$$

Now $\bar{s} = n_L s_L + n_H s_H = (s_L - s_H)/(a_2^L - a_2^H)$. Thus

$$(\partial \bar{s}/\partial a_2^L)|_{K_2} = -(s_L - s_H)\,[(\partial a_2^L/\partial a_2^L)|_{K_2} - (\partial a_2^H/\partial a_2^L)|_{K_2}]/(a_2^L - a_2^H)^2$$
$$= -2(s_L - s_H)/(a_2^L - a_2^H)^2 = -2\bar{s}/(a_2^L - a_2^H).$$

From Claim 2 we have

$$(\partial n_H/\partial a_2^L)|_{K_2} = 2/(a_2^L - a_2^H)^2$$

Thus

$$+ 2a_2^L n_H)/\partial a_2^L)|_{K_2} = -[2\bar{s}(a_1 + 2a_2^L n_H)/(a_2^L - a_2^H)] + 2\bar{s}[a_2^L[2/(a_2^L - a_2^H)^2] + n_{}$$

$$= [2\bar{s}/(a_2^L - a_2^H)^2]\ [- a_1 + 2a_2^L n_H)\ (a_2^L - a_2^H) + 2a_2^L + n_H(a_2^L - a_2^H)^2]$$

$$= [2\bar{s}/(a_2^L - a_2^H)^2]\ [- a_1(a_2^L - a_2^H) - 2a_2^L(1 - n_L)(a_2^L - a_2^H) + 2a_2^L + (1 - n_L)(a_2^L - a_2^H)^2]$$

$$= [2\bar{s}/(a_2^L - a_2^H)^2]\ [- a_1(a_2^L - a_2^H) - 2a_2^L(1 - n_L)(a_2^L - a_2^H) + 2a_2^L + (1 - n_L)a_2^L(a_2^L - a_2^H) - (1 - n_L)a_2^H(a_2^L - a_2^H)]$$

$$= [2\bar{s}/(a_2^L - a_2^H)^2]\ [-a_1(a_2^L - a_2^H) - a_2^L(1 - n_L)(a_2^L - a_2^H) + 2a_2^L - (1 - n_L)a_2^H(a_2^L - a_2^H)]$$

$$= [2\bar{s}/(a_2^L - a_2^H)^2]\ [-(a_1 + a_2^L + a_2^H)\ (a_2^L - a_2^H) + n_L a_2^L(a_2^L - a_2^H) + 2a_2^L + n_L a_2^H(a_2^L - a_2^H)]$$

$$= [2\bar{s}/(a_2^L - a_2^H)^2]\ [-(a_2^L - a_2^H) + 2a_2^L + n_L(a_2^L + a_2^H)(a_2^L - a_2^H)]$$

$$= [2\bar{s}/(a_2^L - a_2^H)^2]\ [(a_2^L + a_2^H) + (a_2^L + a_2^H)(a_2^L - a_2^H)n_L]$$

$$= [2\bar{s}/(a_2^L - a_2^H)^2]\ [(a_2^L + a_2^H)\ [1 + (a_2^L - a_2^H)n_L] > 0$$

Similar tedious calculations establish the remainder of the claim.

Q.E.D.

FOOTNOTES

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1. See Wilde and Schwartz (1979); Schwartz and Wilde (1979).

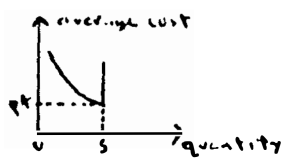
2. These criteria included the extent to which prices cluster in a market, the extent of comparison shopping taking place there, the relative ease with which consumers can compare purchase alternatives directly and the relative difficulty that firms would have in discriminating among consumers on the bases of knowledge or sophistication. See Schwartz and Wilde (1979).

3. Recent search equilibrium models have sought in interesting fashion to characterize equilibria in environments where search costs are positive for some fraction of consumers. See Salop and Stiglitz (1977), Varian (1980), Bagnoli (1980). The model described below differs from these papers in two important ways; it assumes that search costs are positive for all consumers and it deals with heterogeneous goods.

4. We have explained elsewhere why consumers might use fixed sample size strategies. See Wilde and Schwartz (1979); Schwartz and Wilde (1979). In brief, a fixed sample size strategy may be optimal when consumers have very imprecise information respecting the prices and qualities they are likely to see, because consumers seem to make fewer mistakes--searching neither too much nor too little--when using this strategy than when using sequential search strategies.

5. The Introduction argued that these conditions are quite strong but we do not want to overstate the point. If consumer preferences for quality are heterogeneous, then some consumers might exist at the tail of the distribution of those who prefer low quality who would purchase high quality if they saw only it, and similarly for the distribution of those who prefer high quality. The importance of such ''spillovers'' is an empirical question.

6. In a world of perfect information, our formal assumptions would preclude the existence of a competitive equilibrium. To see why, observe how the capacity constraint (s) actually looks:



Suppose that all firms charged the competitive price and one firm then raised its price. Because all firms in the market are at capacity, no firm could expand output to supply the customers of the deviant firm. Also, since there is a positive cost to entry, a range of price rises exists for this firm such that no entry would occur. Thus the competitive price could not be an equilibrium. This objection is not compelling in the framework of our model for two reasons. First, if the right-hand derivative of the average cost curve at s were finite, existing firms could expand output and make the deviation unprofitable. The use of a capacity constraint in our model is no more than a convenient way to avoid the complexities of using U-shaped average cost curves, but it is realistic to suppose that firms' average cost curves will have a positive slope at the profit maximizing level of output. Thus a competitive equilibrium could exist in real world cases. Second, consumers in our model shop stochastically and take rain checks from firms whose price is acceptable but who are stocked out in the shopping period. Thus consumers would not buy from the deviant firm in the short run, and this eliminates the incentive of the firm to deviate from the competitive price.

7. The proof of this claim follows from arguments which are now standard. See Lemma 2 of Wilde and Schwartz [1979].

8. The text assumes sampling with replacement whereas it is more realistic to assume sampling without replacement. This is because a member of A_2^L, for example, who visits one low quality firm will choose a second firm to visit from a distribution that

does not include the firm already visited. When the number of firms in a market is large, however, the difference between sampling with and without replacement vanishes. We suppose the number of firms to be large and assume sampling with replacement because this simplifies calculation. This is the standard approach used in the search literature.

9. See Wilde and Schwartz (1979).

10. The propensity of heterogeneous goods markets to segment is relevant to a variety of legal issues. As an example, suppose that a decisionmaker wants to know, for purposes of evaluating the legality of a horizontal merger, whether two physically similar but not identical goods that trade at different prices are in the same market. The markets for these goods may interact in such fashion that, for example, both goods are trading at their competitive prices, the price distinction being attributable to different production functions and consumer preferences. Alternatively, no interaction between the two markets is occurring. The analysis above suggests that where price distinctions between roughly similar goods persist, the latter situation is the more likely.

11. We suggest that the state should increase the number of comparison shoppers by reducing the costs to consumers of directly comparing purchase alternatives. For example, comparative price information could be made widely available.

Such a policy prescription is to be distinguished from proposals that would provide consumers with ''institutional knowledge'' of the sort: ''Only firms A, B and C sell high quality''. Providing consumers with institutional knowledge alone would be unwise because some consumers could get trapped in the wrong market. To see how, suppose that a consumer with such knowledge perceives himself to prefer high quality before he begins to shop and thus plans to sample only high quality firms. This consumer could end up paying his limit price for a high quality item whereas, if he shopped randomly, he might have seen a low quality good selling at its competitive price, which in many cases would be preferred to buying a high quality good at the limit price. Providing consumers with institutional knowledge as well as reducing the costs to them of directly comparing purchase alternatives might be wise, but providing institutional knowledge alone in a world of heterogeneous goods would not be.

12. See Devine and Marion (1979). Other empirical studies also report price declines following induced reductions in the cost to consumers of directly comparing purchase alternatives. See McNeil; Nevin, Trubeck and Miller (1979) (used cars); Russo, Kreiser and Miyashita (1975) (dishwashing liquid, canned dog food, facial tissues).

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