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COLLECTIVE DECISION MECHANISMS AND EFFICIENT STOCK MARKET
ALLOCATIONS: EXISTENCE OF A PARTICIPATION EQUILIBRIUM

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ABSTRACT

The recent literature on economies with an incomplete set of markets has been devoted to the study of the efficiency properties of collective stockholder decision mechanisms for guiding the behavior of firms when the restrictive Ekern-Wilson spanning condition is not satisfied. The results have been essentially negative; a majority voting rule and controlling interest rules will not yield efficient equilibrium allocations in general. However, in a recent paper, Helpman and Razin (1978) suggested a decision rule that assures constrained Pareto optimality of equilibrium allocations. Their rule is patterned on the recent contributions to the theory of incentive compatibility. In this paper, we show that an equilibrium relative to the Helpman-Razin Mechanism rarely exists, making their optimality result essentially vacuous. We then demonstrate that an equilibrium does exist in general relative to the Shared Cost Mechanism developed by Hurwicz (1976), and that all equilibrium allocations in the Helpman-Razin model are constrained Pareto optima. Finally, we suggest that the optimality of equilibrium allocations is as much a consequence of how technology is modeled as of the incentives induced by the decision mechanism. Existence, on the other hand, is very sensitive in general to the decision mechanism adopted.

COLLECTIVE DECISION MECHANISMS AND EFFICIENT STOCK MARKET ALLOCATIONS: EXISTENCE OF A PARTICIPATION EQUILIBRIUM

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1. INTRODUCTION

Much of the recent work in the theory of general equilibrium under uncertainty has focused on the choice of objective functions for guiding the behavior of firms when the set of markets in the economy is not complete. The literature generally is devoted to establishing conditions under which firms can choose production plans with the unanimous approval of their stockholders although some attention has been given to other decision mechanisms that include voting schemes such as majority rule or controlling interest. Until recently, the results of these investigations have been in the form of impossibility theorems; except under the very restrictive conditions consistent with unanimity, the proposed objectives do not achieve constrained Pareto optimal equilibrium stock ownership plans in general. One purpose of this paper is to correct this deficiency. We exhibit a collective decision mechanism relative to which an equilibrium stock ownership program does exist for a wide class of economies, which subsumes the unanimity model as a special case. We show that all equilibrium stock ownership programs are constrained Pareto optimal for the class of economies we consider. Our model requires that information be transferred between agents and formal mechanisms for achieving this are considered. In each of these mechanisms stockholders must at

least be given a naive incentive to communicate truthfully if efficient equilibria are to be attained.

The general problem concerns firms' choices of production plans and investors' choices of a portfolio of firms' revenue shares in the absence of a complete set of contingent securities markets. There are two periods, present and future, and there are several possible future states of the world. There is a single commodity in the present which must be consumed now or invested in firms to yield consumption in the future. The income of each consumer (or stockholder) is a fixed share of random profits plus some fixed nonrandom income. Any objective function a firm may use must reflect the tastes of its stockholders if efficiency is to be achieved. Dreze (1974) has shown that production efficiency will be attained if each firm maximizes its value to the stockholders. Necessarily, the firm's value to stockholders is a function of each stockholder's marginal rate of substitution between second period random consumption and first period certain consumption, which is the individual's implicit contingent claim price, weighted by his or her relative shareholding. Thus, for firms to make efficient decisions, they must know or be able to learn final stockholders' true contingent claim prices. To assume that firms know stockholders' contingent claim prices is not satisfactory, and to assume that consumers are pure competitors in the stockmarket, taking firms' prevailing revenue share prices as given, does not eliminate this information problem for those firms concerned with making efficient decisions.

One approach to this problem is to let the final stockholders of each firm become participants in the firm's decision process, making a collective production decision. Conceptually, the firm is then like a large cooperative or partnership, and the principal difficulty becomes the design of a voting or collective decision procedure that yields efficient equilibria which exist for an interesting class of economies.¹ Gevers (1974) and Jordan (1979) have observed, respectively, that a majority voting rule and a controlling interest voting rule are logically inconsistent with constrained Pareto optimality in general.² This problem vanishes, however, if stockholders are unanimous in their preferences.

Unanimity prevails if and only if the Ekern-Wilson (1974) spanning condition is satisfied. Loosely stated, the spanning condition requires that any small adjustment in stockholders' returns achievable by altering firms' production plans must also be achievable by portfolio changes. In short, the set of available state-distributions of returns cannot be affected by firms' decisions. That is, the value of any change in the production plans must equal the cost of making the corresponding portfolio change. Since the latter cost is calculated from observable market values, it is the same for all stockholders. Therefore, each firm's manager can use his own preferences when selecting a production plan, and an efficient allocation will be obtained. But, the spanning condition is highly restrictive since it is not satisfied in many nonpathological economies.

An alternative approach is that suggested by Helpman and Razin (1978). If consumers are pure competitors in the stock market, then firms' revenue shares can be allocated with a price mechanism; a collective decision procedure is to be used to determine the distribution of firms' production costs among stockholders as well as firms' input-output levels. The decision rule, based on a mixture of an I-player noncooperative game and ordinary competition, plays a role that is conceptually similar to — but different in application from — collective choice mechanisms for achieving Pareto efficient public good allocations in private ownership economies with complete markets and no uncertainty. What is extraordinary about the Helpman-Razin decision mechanism is their rule for distributing firms' costs among stockholders.

An equilibrium in the Helpman-Razin model — called a participation equilibrium — is a Nash noncooperative equilibrium relative to their decision mechanism. Firms' state-dependent technologies are given, and each stockholder is asked to communicate the increment (or decrement) of each firm's input he or she would like to add to (or subtract from) the amount requested by the other stockholders. The input level selected by each firm is the sum of the increments (or decrements) communicated by all stockholders. Clearly, the resulting equilibrium input levels must be those most desired by stockholders. The Helpman-Razin cost sharing rule assesses each consumer in the economy a preassigned and fixed equal share of the economy's aggregate production costs plus a personalized lump sum

transfer. They then show that an equilibrium allocation is a constrained Pareto optimum.

Since Helpman and Razin do not rely on the Ekern-Wilson spanning condition to prove optimality of equilibrium allocations, their theorem has considerable appeal. However, it is significant that a participation equilibrium relative to the Helpman-Razin decision mechanism usually will not exist — even if the spanning condition is satisfied — making their optimality result essentially vacuous. The general nonexistence of a participation equilibrium relative to their mechanism is a consequence of their cost sharing rule. Indeed, as long as the cost shares are preassigned and fixed, an equilibrium will not exist in general.

If the cost sharing rules are changed, then so is the incentive structure of the decision mechanism, and the question to be asked is whether there is a collective decision mechanism that maintains the desired incentive structure and relative to which a participation equilibrium exists for a general class of economies. A natural mechanism to consider is the Shareholding Mechanism which equates stockholders' cost and revenue shares. Leland (1973) has shown that a necessary and sufficient condition for an equilibrium with the Shareholding Mechanism is the Ekern-Wilson spanning condition,³ so it should not be surprising that this mechanism can only work for a highly restricted class of economies.

Another candidate that does work for a general class of economies is the Shared Cost Mechanism developed by Hurwicz (1976).

The Shared Cost Mechanism is in the spirit of both the Helpman-Razin and Shareholding Mechanisms. However, the stockholders' cost shares are not exogenous and fixed nor are they necessarily equal to the stockholders' revenue shares; rather the cost shares are choice variables for each stockholder. Relative to the Shared Cost Mechanism, a participation equilibrium exists for general stockholders' ownership economies and equilibrium stock ownership programs are constrained Pareto optima.

In the next section, we present the general model. We also consider examples for which there are no participation equilibria relative to the Helpman-Razin Mechanism or the Shareholding Mechanism, but for which equilibria relative to the Shared Cost Mechanism can be demonstrated. In section 3, we prove that a participation equilibrium relative to the Shared Cost Mechanism exists for a general class of economies, and we show that the equilibrium allocations are efficient. To prove existence, we define the notion of a full stockholders equilibrium, and show that such equilibria exist. We then demonstrate that the set of full stockholders equilibria is the set of participation equilibria relative to the Shared Cost Mechanism. A point of interest is that a full stockholders equilibrium in stock ownership economies is structurally the same as a Lindahl equilibrium in private ownership economies with public goods.

Within the context of our model, the set of full stockholders equilibria is also the set of stockholders equilibria as defined by Dreze (1974). It follows that Dreze stockholders' equilibrium

allocations are efficient. In section 4, we reconcile this conclusion with the apparently contrary result proven by Dreze (1974) for similarly specified economies by showing that the two models have a very important difference in how technologies are specified across states of nature. It is this difference rather than the incentive structure induced by the decision mechanism that drives the optimality result, both here and in the original Helpman-Razin formulation. However, the use of a decision mechanism like the Shared Cost Mechanism sidesteps the implicit contingent claim price revelation questions that arise otherwise. Thus, permitting an individual's revenue share to differ from his cost share allows us to characterize a wide class of economies in which stockholders equilibria are constrained Pareto optimal. Our conclusions are discussed more fully in section 5.

2. THE MODEL

The model we consider is a stock ownership economy consisting of I consumers, indexed $i = 1, \dots, I$, and J firms, indexed $j = 1, \dots, J$. There are two periods of time labeled 0 and 1, respectively. There are S possible mutually exclusive states of the world that can occur in period 1, indexed $s = 1, \dots, S$. Which of these occurs is not known in period 0.

A consumption plan for consumer i is a nonnegative vector $c_i = (c_{i0}, c_{i1}, \dots, c_{iS}) \in \mathbb{R}_+^{S+1}$, where c_{i0} denotes consumption of i in period 0 and c_{is} is the consumption of i in period 1 if state s

obtains. The consumption set of i is the nonnegative orthant, R_+^{S+1} . Further, we assume that the preferences of consumer i are representable by a utility function $U^i(c_i)$ which is defined over the consumption set.

Each consumer i is endowed with w_i units of a good in period 0 which may be used for current consumption or invested in firms to yield future consumption. Consumer i is also endowed with a revenue share $\bar{\theta}_{ij}$ of each firm j , where $0 \leq \bar{\theta}_{ij} \leq 1$ and $\sum_i \bar{\theta}_{ij} = 1$ all $j \leq J$. Let $\bar{\theta}_i = (\bar{\theta}_{ij}, j \leq J)$.

A production plan for firm j is a vector $y_j = (x_j, y_{j1}, \dots, y_{jS}) \in R_+^{S+1}$ where x_j denotes the input of firm j in period 0 and y_{js} denotes the output of firm j in period 1 if state s occurs. We further assume that the choice of input completely determines the level of each firm's output in each state. That is, we assume that each firm's technology is representable by a set of state specific production functions $R_{js}(x_j)$, all s, j .

A stock ownership economy will be denoted by $E = \{(U^i(\cdot), w_i, \bar{\theta}_i, i \leq I), (R_{js}(\cdot), s \leq S, j \leq J)\}$. We assume,

A1: for each i , $U^i(\cdot)$ is a twice continuously differentiable semi-strictly quasi-concave function⁴ such that

$$\frac{\partial U^i}{\partial c_{i0}} > 0, \frac{\partial U^i}{\partial c_{is}} > 0 \text{ for all } s \text{ and for all } c_i \in R_+^{S+1};$$

A2: for each j and each s , $R_{js}(x_j)$ is a twice continuously differentiable concave function such that $\frac{\partial R_{js}}{\partial x_j} > 0$ for all s and all $x_j \in R_+$.

To include a framework in the model for explicitly analyzing alternative managerial decision procedures that utilize information collected from stockholders, we formulate the investment and production decisions for each firm j as a non-cooperative I -person game among stockholders. The firms use information received from stockholders to compute input levels and individual stockholder cost shares. These computations are made according to an allocation rule and stockholder cost sharing rules. Messages which stockholders may communicate to each firm are restricted to a particular message space. The message space, allocation rule and stockholder cost sharing rules for each firm together comprise a stylized managerial decision mechanism for the firm. Collectively, the firms' decision mechanisms define a managerial decision mechanism for the stock ownership economy. Formally,

Definition 2.1: A managerial decision mechanism G in an economy E is defined by: (i) a message space M ; (ii) an allocation rule $f(\cdot) = (f_1(\cdot), \dots, f_J(\cdot)): M^I \rightarrow R^J$; (iii) individual cost sharing rules $C_i: M^I \rightarrow R^J$ for each $i \leq I$.

To provide for consumption in period 1, individual i chooses a portfolio of revenue shares θ_i to hold in period 1, entitling him or her to the corresponding fractions of firms' outputs in period 1. Individual i then pays (or receives) $p(\bar{\theta}_i - \theta_i)$ where $p = (p_j, j \leq J)$; p_j denotes the market value of firm j .

Each individual desires a portfolio θ_i and input levels

$x_i = (x_{i1}, \dots, x_{iJ})$ that are feasible which maximize his or her utility. Thus, given a managerial decision mechanism G , the decision problem of individual i is

$$\text{to maximize } U^i(c) \\ (c_i, \theta_i, m_i)$$

subject to

- (i) $c_{i0} + \sum_j p_j \theta_{ij} + \sum_j c_{ij}(m) \leq w_i + \sum_j p_j \bar{\theta}_{ij},$
- (ii) $c_{is} \leq \sum_j R_{js}(f_j(m)) \theta_{ij}, \quad s \leq S,$
- (iii) $m^i \in M,$
- (iv) $\theta_{ij} \geq 0, \quad x_j \geq 0, \quad j \leq J,$
- (v) $c_{i0} \geq 0, \quad c_{is} \geq 0, \quad s \leq S.$

We assume that each stockholder is competitive in the stock market and with respect to messages communicated by other stockholders. That is, we assume,

A3: Each stockholder i considers prices $p \in R_+^J$ and the messages of other stockholders $m_{-i} = (m_1, \dots, m_{i-1}, m_{i+1}, \dots, m_I) \in M^{I-1}$ as given.

In this formulation, a competitive equilibrium is a Nash noncooperative equilibrium relative to the decision mechanism G , given the prevailing price system. Such an equilibrium is called a participation equilibrium.

Definition 2.2: A participation equilibrium in an economy E relative to a decision mechanism G is a price system $p \in R_+^J$, a consumption allocation $c_i = (c_i, i \leq I)$, a portfolio allocation $\theta = (\theta_i, i \leq I)$ and a vector of messages $m = (m_i, i \leq I) \in M^I$ such that

- (i) for each $i \leq I$, (c_i, θ_i, m_i) is a solution to stockholder i 's decision problem given prices p , the mechanism G , and the messages of other stockholders m_{-i} .
 - (ii) $\sum_i c_{i0} + \sum_j f_j(m) = \sum_i w_i,$
 - (iii) $f_j(m) = x_j, \quad j \leq J,$
 - (iv) $\sum_i \sum_j c_{ij}(m) = \sum_j x_j, \quad j \leq J$
- and
- (v) $\sum_j \theta_{ij} = 1, \quad j \leq J.$ ⁵

In this formulation of decision making in stock ownership economies, managers are just functionaries that implement the decisions of stockholders given the managerial decision mechanism G . Choosing a firm production plan is a collective decision for stockholders, and any production plan implemented must be acceptable to each stockholder given the feasible alternatives under the decision mechanism. Thus, a firm's production plan is an equilibrium plan only if it is unanimously supported by all stockholders given the corresponding cost sharing arrangement. This unanimity property is, in general, dependent on transfers between stockholders implicit from

the equilibrium revenue share holdings and cost sharing arrangements. It follows that our model is a generalization of the simple unanimity model examined by Leland (1973) and Ekern-Wilson (1974) which requires cost shares and revenue shares to be identical for all stockholders and thus does not permit transfers between them. Since uncertainty may be — but is not restricted to being — multiplicative, the Diamond (1967) model is subsumed as a special case as well.

Perhaps the most natural mechanism to consider is the Shareholding Mechanism which assesses each stockholder cost shares equal to his revenue shares. Under this mechanism, each stockholder is asked to communicate an increment (or decrement) that he or she would like added to (or subtracted from) the input level of each firm desired by other stockholders as reflected by their messages given the allocation rule. Thus, in equilibrium, the resulting input levels are those most desired by all stockholders given the prevailing price system and the cost sharing rules. Formally, we define the mechanism as follows:

Definition 2.3: The Shareholding Mechanism is given by: (i) the message space $M = R^J$; (ii) the allocation rule $x_j = \sum_i m_{ij}$, for each $j \leq J$; and (iii) the cost functions $C(m) = (C_{ij}(m), j \leq J)$ where $C_{ij}(m) = \theta_{ij} \sum_h m_{hj}$.

Unfortunately, the existence of an equilibrium under the Shareholding Mechanism cannot be assured, as the following example illustrates.

Example 1 Consider an economy with two consumers, one firm, and two states of the world. Let $R_1(x) = x^{1/2}$ and $R_2(x) = x$, and define for each i ,

$$U^i(c_{i0}, c_{i1}, c_{i2}) = c_{i0}^{1/2} (\beta_1^i c_{i1}^{1/2} + \beta_2^i c_{i2}^{1/2})$$

It is easy to see that we must have an interior solution in equilibrium: if either $x = 0$ or $\theta_i = 0$, then $c_{is} = 0$, $s = 1, 2$, and since $\frac{\partial U^i(c_{is})}{\partial c_{is}} \rightarrow \infty$ as $c_{is} \rightarrow 0$ for $i = 1, 2$, $s = 1, 2$, we may immediately rule out corner solutions. The necessary first order conditions with respect to θ_i and m_i require

$$p + x = \frac{c_{i0}}{\beta_1^i c_{i1}^{1/2} + \beta_2^i c_{i2}^{1/2}} \left[\frac{\beta_1^i x^{1/2}}{x^{1/4} \theta_i^{1/2}} + \frac{\beta_2^i x}{x^{1/2} \theta_i^{1/2}} \right] = \frac{c_{i0}}{\theta_i} \quad \text{for all } i$$

and

$$\begin{aligned} 1 &= \frac{c_{i0}}{\beta_1^i c_{i1}^{1/2} + \beta_2^i c_{i2}^{1/2}} \left[\frac{\beta_1^i}{2(c_{i1}x)^{1/2}} + \frac{\beta_2^i}{c_{i2}^{1/2}} \right] \\ &= \frac{c_{i0}}{\theta_i} \left[\frac{(\beta_1^i + 2\beta_2^i x^{1/4})}{2x(\beta_1^i + \beta_1^i x^{1/4})} \right] \quad \text{for all } i \quad (2.2) \end{aligned}$$

Substituting for $\frac{c_{i0}}{\theta_i}$ in (2.2) from (2.1) gives

$$1 = (p + x) \left[\frac{\beta_1^i + 2\beta_2^1 x^{1/4}}{2x(\beta_1^i + \beta_2^i x^{1/4})} \right] \text{ for each } i. \quad (2.3)$$

Solving (2.3) for p gives

$$p = \frac{x\beta_1^i}{(\beta_1^i + 2\beta_2^i x^{1/4})}, \quad i = 1, 2 \quad (2.4)$$

and, since both stockholders must agree on an input level satisfying (2.4),

$$\frac{x\beta_1^1}{(\beta_1^1 + 2\beta_2^1 x^{1/4})} = \frac{x\beta_1^2}{(\beta_1^2 + 2\beta_2^2 x^{1/4})}$$

or

$$(\beta_2^2 \beta_1^1 - \beta_2^1 \beta_1^2) x^{5/4} = 0 \quad (2.5)$$

It is now apparent that, for arbitrary $\beta_s^1 \neq \beta_s^2$, $s = 1, 2$, (2.5) can only be satisfied if $x = 0$. However, as demonstrated above, this input level cannot be an equilibrium. Thus, no equilibrium relative to the Shareholding Mechanism exists for this economy. ||

The Shareholding Mechanism is essentially a formalization of the decision procedure consistent with the simple unanimity model within the context of our more general model. Therefore, Example 1 should not be surprising since there are economies in the class permitted by A1 and A2 that do not satisfy the Ekern-Wilson spanning

condition. However, Example 1 does make clear that the difficulty with the Shareholding Mechanism stems from the fact that the portfolio holdings of each individual must perform two distinct purposes. They determine each individual's pattern of consumption across states of the world, and also the share of cost each consumer must incur when making input decisions.

Helpman and Razin suggest an alternative mechanism which is a step towards separating these two functions. In particular, they assign exogenous cost shares to each consumer which will not in general coincide with portfolio holdings. Their mechanism is given by:

Definition 2.4: The Helpman-Razin Mechanism is defined by: (i) the message space $M = R^J$; (ii) the allocation rule $x_j = \sum_i m_{ij}$, for each $j \leq J$; and (iii) the individual cost functions $C_i(m) = (C_{ij}(m))$, $j \leq J$ $C_{ij}(m_j) = \frac{1}{I} \sum_h m_{hj}$ for each i and j .⁶

The difficulty that arises here is that an equilibrium rarely exists under this mechanism for a wide class of regular economies. This is due to the fact that the cost shares have been preassigned, $a_{ij} = \frac{1}{I}$, for each i and j . This is demonstrated in the following two examples.

Example 2 Consider the class of economies with two consumers, two firms and two states of the world where

$$R_{11}(x_1) = x_1, \quad R_{12}(x_1) = 2x_1, \quad R_{21}(x_2) = x_2, \quad R_{22}(x_2) = 2x_2, \text{ and}$$

$$U^i(c_{i0}, c_{i1}, c_{i2}) = c_{i0} + \beta_1^i \log c_{i1} + \beta_2^i \log c_{i2},$$

$$\beta_1^i + \beta_2^i = 1, \beta_s^i > 0 \text{ for } s = 1, 2 \text{ and } i = 1, 2.$$

Each consumer maximizes his utility subject to

$$c_{i0} + \sum_j p_j \theta_{ij} + \frac{1}{I} \sum_h \sum_j m_{hj} \leq w_i + \sum_j p_j \bar{\theta}_{ij}$$

$$c_{is} \leq \sum_j R_{js} \left(\sum_h m_{hj} \right) \theta_{ij}.$$

The necessary first order conditions with respect to input proposals, m_{ij} , are

$$\frac{1}{2} = \left[\frac{2\beta_1^i}{c_{i1}} + \frac{\beta_2^i}{c_{i2}} \right] \theta_{i1}, \text{ for } i = 1, 2 \quad (2.6a)$$

$$\frac{1}{2} = \left[\frac{\beta_1^i}{c_{i1}} + \frac{2\beta_2^i}{c_{i2}} \right] \theta_{i2}, \text{ for } i = 1, 2. \quad (2.6b)$$

Similarly, the necessary first order conditions with respect to θ_{ij} are

$$p_1 = \left[\frac{2\beta_1^i}{c_{i1}} + \frac{\beta_2^i}{c_{i2}} \right] x_1, \text{ for } i = 1, 2 \quad (2.7a)$$

$$p_2 = \left[\frac{\beta_1^i}{c_{i1}} + \frac{2\beta_2^i}{c_{i2}} \right] x_2, \text{ for } i = 1, 2 \quad (2.7b)$$

Combining (2.6a) with (2.7a) and (2.6b) with (2.7b) it is easily seen that

$$\frac{p_i}{x_j} = \frac{1}{2\theta_{ij}}, \text{ for } i = 1, 2 \text{ and } j = 1, 2$$

and multiplying this by θ_{ij} and summing over both individuals gives $p_j = x_j$ which requires that $\theta_{ij} = 1/2$. Further, multiplying (2.6a) by x_1 and (2.6b) by x_2 and summing gives $x_1 + x_2 = 2$. Thus, making use of these substitutions in (2.6a) and (2.6b) gives

$$\frac{1}{2} = \frac{2\beta_1^i}{2x_1 + x_2} + \frac{1 - \beta_1^i}{x_1 + 2x_2} \quad (2.8a)$$

$$\frac{1}{2} = \frac{\beta_1^i}{2x_1 + x_2} + \frac{2(1 - \beta_1^i)}{x_1 + 2x_2} \quad (2.8b)$$

Multiplying (2.8b) by two and subtracting from (2.8a) gives

$$\frac{1}{2} = \frac{3(1 - \beta_1^i)}{x_1 + 2x_2}$$

and using this, together with the fact that $x_1 + x_2 = 2$ gives

$$\begin{aligned} x_1 &= 2(3\beta_1^i - 1) \\ \text{and} \\ x_2 &= 2(2 - 3\beta_1^i) \end{aligned}$$

This can only be satisfied in the event that both individuals have identical utility functions, i.e., $\beta_j^1 = \beta_j^2$ for $j = 1, 2$. In this case all individuals will be unanimous in their preferences over input

decisions and this choice becomes trivial. In all other cases, a competitive equilibrium under a Helpman-Razin Mechanism will not exist. Since the set of parameters for which $\beta_s^1 = \beta_s^2$, $s = 1, 2$ has measure zero in the class of allowable parameters, non-existence of an equilibrium relative to the Helpman-Razin Mechanism is generic for the class of regular economies defined by this example.⁷

As one additional remark, it should be noted that there is a complete set of contingent claim markets in Example 2 yet no equilibrium exists.||

Example 1a Consider the economy specified in Example 1. Let $w_1 = 4/3$, $w_2 = 1$, $\bar{\theta}_1 = \bar{\theta}_2 = 1/2$, and $\beta_1^i = \beta_2^i = 1/2$, $i = 1, 2$. The necessary first order conditions under the Helpman-Razin Mechanism with respect to θ_i and m_i are

$$p = \frac{c_{i0}}{\theta_i} \quad \text{for all } i \quad (2.9)$$

and

$$1/2 = c_{i0} \left[\frac{1 + 2x^{1/4}}{2x(1 + x^{1/4})} \right] \quad \text{for all } i \quad (2.10)$$

Summing (2.10) over both individuals and making use of the fact that

$$\sum_i c_{i0} = \sum_i w_i - x = 7/3 - x \text{ gives}$$

$$1 = (7/3 - x) \left[\frac{1 + 2x^{1/4}}{2x(1 + x^{1/4})} \right]$$

and it may be verified that the only solution to this equation which is feasible, i.e., $x \in [0, \sum_i w_i]$, is given by $x = 1$. Substituting this into equation (2.10) gives $c_{10} = c_{20} = 2/3$ and, from (2.9), $p = 4/3$ and $\theta_1 = \theta_2 = 1/2$. Recall that individual 2's budget constraint in period 0 is

$$c_{20} + p\theta_2 + x/2 \leq w_2 + p\bar{\theta}_2 = 1 + 2/3 = 5/3.$$

However, the allocation requires this individual to spend

$c_{20} + p\theta_2 + x/2 = 2/3 + 2/3 + 1/2 = 11/6$. Since this allocation is not affordable, no equilibrium will exist for the economy in this example.||

Note that although individuals have identical preferences in this example, there may not exist an equilibrium under the Helpman-Razin Mechanism if individuals have different endowments.

Another "natural" mechanism would be to have initial shareholders bear the input costs in proportion to their endowed holdings, $\bar{\theta}_{ij}$. This is nothing more than a special case of the Helpman-Razin Mechanism where $a_{ij} = \bar{\theta}_{ij}$ is set exogenously. From Example 1a it may be seen that an equilibrium will still fail to exist in general.

In principle, the Helpman-Razin Mechanism is focused in the proper direction since it separates ownership shares from cost shares. However, both the Shareholding Mechanism and the Helpman-Razin Mechanism employ linear cost sharing functions which, in general, do

not permit sufficient flexibility in the assignment of costs corresponding to individual consumption levels, given here by the revenue share holdings and production plans, to ensure existence of an equilibrium. This difficulty derives from the public good nature of firms' production decisions, and is basically the same problem that was identified by Groves-Ledyard (1977) for allocation mechanisms in Arrow-Debreu private ownership economies with public goods. Groves-Ledyard (1977, 1980) circumvented this difficulty by introducing a public good allocation mechanism with non-linear (quadratic) cost sharing functions where the selection of cost shares is endogenous in the model.⁸ The efficient public good allocation mechanisms suggested by Hurwicz (1976) (the Shared Cost Mechanism) and Walker (1981) (the Paired Difference Mechanism)⁹ also use non-linear cost sharing functions and the selection of cost shares are again endogenous in the mechanism.

This suggests that such mechanisms should solve the stock ownership allocation problem as well, an observation that is indeed correct. Any of the three mechanisms is easily applied to stock ownership economies using the general model presented above, and major properties of the mechanisms in public good economies carry over to stock ownership economies. With any of these mechanisms, an efficient equilibrium exists even if stockholders behave strategically. In the remainder of the paper, we focus on the Shared Cost Mechanism. However, in our concluding remarks, we note those similarities and differences that arise from application of the other mechanisms.

Definition 2.5: The Shared Cost Mechanism is defined by: (i) a message space $M = R^{2J}$ where $m_i = (x_{ij}, a_{ij}, j \leq J) \in M$ for all $i \leq I$; (ii) an allocation rule $x_j = \sum_i x_{ij}$, for $j \leq J$; and (iii) a set of cost functions $C_i(m) = (C_{ij}(m), j \leq J)$ where

$$C_{ij}(m) = (1 - \sum_{k \neq i} a_{kj})x_j + (1 - \sum_k a_{kj})^2, \text{ for each } i \text{ and } j.$$

The interpretation given to this mechanism is that the component a_{ij} of each message m_{ij} is the share of cost which individual i is willing to transfer to others. In equilibrium, however, $\sum_k a_{jk} = 1$, for each j , since a_{ij} affects an individual i only through the term $(1 - \sum_k a_{kj})^2$ and thus the cost which he incurs can be decreased without changing x_j merely by changing a_{ij} if $\sum_k a_{kj} \neq 1$. Thus, in equilibrium a_{ij} becomes the share of the input cost of firm j which is transferred to individual i .

To interpret this mechanism in terms of the general stockholders problem, we imagine stockholders and firm managers coming together in a stockholders' meeting where production plans are to be selected and stockholders' cost shares assigned. Each firm's manager announces that these decisions will be made according to the Shared Cost Mechanism which is designed to function in the stockholders' interests. Then management polls stockholders by secret ballot, asking that each stockholder communicate desired incremental changes in the previously announced tentative production plans and the share of production costs each stockholder would like transferred to others. Stockholders are permitted to trade shares on the stock market at

prevailing prices even as new tentative production plans are being announced; a firm's stock prices are revised upward (downward) by a market auctioneer if there is an excess demand (supply) for the firm's shares. The stockholders' meetings are adjourned and the stock market closed when an equilibrium decision is signaled by no change in any stockholder's ballot or proposal to trade on the stock market.

In the next section, we consider the general equilibrium properties of the Shared Cost Mechanism in stock ownership economies. In particular, we provide a proof of the existence of a participation equilibrium relative to the Shared Cost Mechanism. Prior to doing this, however, we show that the previously considered examples have equilibria relative to this mechanism.

Example 1b Consider the economy specified in Example 1. Further let $w_1 = 21/10$ and $w_2 = 2/10$ and let $\bar{\theta}_1 = \bar{\theta}_2 = 1/2$. We also wish to specify the parameters in the utility functions by $\beta_1^1 = \beta_2^1 = 1/2$ and $\beta_1^2 = 1/3$, $\beta_2^2 = 2/3$. The necessary first order conditions under the Shared Cost Mechanism with respect to θ_i , a_i , and x_i are

$$p = \frac{c_{i0}}{\theta_i},$$

$$a_i = 1 - \sum_{k \neq i} a_k,$$

and

$$a_i = 1 - \sum_{k \neq i} a_k = c_{i0} \left[\frac{\beta_1^i + 2\beta_2^i x^{1/4}}{2x(\beta_1^i + \beta_2^i x^{1/4})} \right].$$

It may be verified that the following allocation is an equilibrium under the Shared Cost Mechanism:

$$c_1 = (1, 10/13, 10/13), \quad c_2 = (3/10, 3/13, 3/13)$$

$$\theta_1 = 10/13, \quad \theta_2 = 3/13, \quad x = 1, \quad p = 13/10$$

$$a_1 = 3/4, \quad a_2 = 1/4.$$

This example points up the need of allowing cost shares to vary endogenously independently of ownership shares since, as can be seen, $\theta_1 \neq a_1$ and $\theta_2 \neq a_2$. ||

Example 2a: Consider the economy specified in Example 2. Let

$$w_1 = 2, \quad w_2 = 3, \quad \bar{\theta}_{1j} = \bar{\theta}_{2j} = 1/2, \quad j = 1, 2 \text{ and}$$

$\beta_1^1 = \beta_2^1 = 1/2$, $\beta_1^2 = 1/3$, $\beta_2^2 = 2/3$. The necessary first order conditions under the Shared Cost Mechanism with respect to θ_{ij} are given in (2.6) and the ones with respect to θ_{ij} and x_{ij} are

$$a_{ij} = 1 - \sum_{k \neq i} a_{kj}, \quad j = 1, 2$$

$$a_{ij} = (1 - \sum_{k \neq i} a_{kj}) = (2\beta_1^i/c_{i1} + \beta_2^i/c_{i2})\theta_{ij}, \quad i = 1, 2$$

$$a_{ij} = 1 - \sum_{k \neq i} a_{kj} = \left[\frac{\beta_1^i}{c_{i1}} + \frac{2\beta_2^i}{c_{i2}} \right] \theta_{i2}, \quad i = 1, 2.$$

It may be verified that, for this example, an equilibrium is given by

$$c_1 = (1, 3/2, 3/2), \quad c_2 = (2, 1, 2)$$

$$\theta_{11} = a_{11} = 1, \theta_{21} = a_{21} = 1/3, p_1 = x_1 = 1/2.$$

$$\theta_{12} = a_{12} = 0, \theta_{22} = a_{22} = 2/3, p_2 = x_2 = 3/2.$$

This example demonstrates that the Shared Cost Mechanism continues to perform satisfactorily when there are complete markets. The reader may verify that the Shareholding Mechanism also chooses the same allocation whereas for the Helpman-Razin Mechanism, an equilibrium does not exist, as was previously demonstrated.||

3. EQUILIBRIUM AND OPTIMALITY

In this section, we prove that a participation equilibrium relative to the Shared Cost Mechanism exists for the class of economies given by assumptions A1, A2 and A3. We show too that an equilibrium allocation is constrained Pareto optimal, and that each constrained Pareto optimal allocation can be implemented as a participation equilibrium (the two Fundamental Welfare Theorems). To prove existence, we appeal to an existence theorem established by Dreze (1974). We proceed by first reviewing his definition of a stockholders equilibrium, then we define a full stockholders equilibrium and show that every stockholders equilibrium can be supported as a full stockholders equilibrium. Finally we show that a full stockholders equilibrium can be implemented as a participation equilibrium using the Shared Cost Mechanism. A full stockholders equilibrium is the analog for stock ownership economies of a Lindahl

equilibrium in private ownership economies with public goods, and their similarity establishes a definitive analytical link between stock ownership economies and public good economies. We conclude the section with the Fundamental Welfare Theorems.

3.1. Existence

A stock ownership program is a vector $z = (x, c, \theta) \in R_+^{J+I(S+1+J)}$ consisting of firms' input levels $x = (x_j, j \leq J)$, stockholders consumptions $c = (c_{i0}, c_{is}, s \leq S, i \leq I)$ and revenue shares $\theta = (\theta_i, i \leq I) = (\theta_{ij}, j \leq J, i \leq I)$. Let $Z \subset R_+^{J+I(S+1+J)}$ denote the set of attainable stock ownership programs, and let $w = \sum_i w_i$ and $x = (x_j, j \leq J)$. Following Dreze (1974), define for each $z \in Z$,

$$F^j(\hat{z}) = \{z \in Z | x_k = \hat{x}_k \text{ for } k \neq j \text{ and } \theta = \hat{\theta}\} \quad (3.1)$$

$$= \{z \in Z | \sum_i c_{i0} + x_j \leq w - \sum_{k \neq j} \hat{x}_k;$$

$$c_{is} - \hat{\theta}_{ij} R_{js}(x_j) \leq \sum_{k \neq j} \hat{\theta}_{ik} R_{ks}(\hat{x}_k), i \leq I, s \leq S\} \text{ for each } j \leq J,$$

$$E(\hat{z}) = \{z \in Z | x = \hat{x}\} \quad (3.2)$$

$$= \{z \in Z | \sum_i c_{i0} \leq w - \sum_j \hat{x}_j; c_{is} - \sum_j \theta_{ij} R_{js}(\hat{x}_j) \leq 0, \\ i \leq I, s \leq S; \sum_i \theta_{ij} = 1, j \leq J; \theta \geq 0\}.$$

$F^j(\hat{z})$ is the set of stock ownership programs attainable from a given starting point $\hat{z}$ through decisions of the j^{th} firm and adjustments in current consumptions, the input plans of other firms being given.

$E(\hat{z})$ is the set of stock ownership programs attainable from the starting point $\hat{z}$ through exchanges of revenue shares accompanied by adjustments in current consumptions, the input levels of all firms being given. Notice that $E(\hat{z})$ is also the set of feasible allocations for a pure exchange economy $\xi(\hat{z}) = \{(W^i(\cdot), i \leq I), w\}$ where the commodities are revenue shares θ_i and current consumptions c_{i0} . $\hat{w} \equiv w - \sum_j \hat{x}_j$ and $W^i(c_{i0}, \theta_i) \equiv U^i(c_{i0}; \sum_j R_{js}(\hat{x}_j) \theta_{ij}, s \leq S)$ for each $i \leq I$.

Definition 3.1. A pseudo equilibrium for firm j relative to z is a stock ownership program $z^* \in F^j(z)$ and a set of I vectors $\pi_i^* \in R_+^{S+1}$, $\pi_{i0}^* = 1$ such that

(i) $U^i(c_i) > U^i(c_i^*)$ implies $\pi_i^* c_i > \pi_i^* c_i^*$ for each $i \leq I$;

(ii) x_j^* maximizes $\sum_s R_{js}(x_j) \sum_i \theta_{ij} \pi_{is}^* - x_j$.

Definition 3.2. A price equilibrium for the economy $\xi(z)$ is a stock ownership program $z^* \in E(z)$ and a price vector $(p_0^*, p^*) \in R_+^{1+J}$, $p_0^* = 1$ such that $W^i(c_{i0}, \theta_i) > W^i(c_{i0}^*, \theta_i^*)$ implies $c_{i0} + p^* \theta_i > c_{i0}^* + p^* \theta_i^*$.

Definition 3.3. A stock ownership program $z^* \in Z$ is a stockholders equilibrium if and only if

- (i) there exist I vectors π_i^* such that $(z^*; \pi_i^*, i \leq I)$ is a pseudo equilibrium for firm j relative to z^* , each $j \leq J$;
- (ii) there exists a price vector (p_0^*, p^*) such that $(z^*; (p_0^*, p^*))$ is a price equilibrium for $\xi(z^*)$.

Dreze (1974, p. 144) proves that

Theorem 3.1. (Dreze) Under assumptions A1 and A2, if $w > 0$, then there exists a stockholders equilibrium.

Dreze also shows that, under differentiability,

$$p_j^* \geq \sum_s \pi_{is}^* R_{js}(x_j^*) \quad \text{all } i, j, \quad (3.3)$$

$$p_j^* \theta_{ij}^* = \sum_s \pi_{is}^* R_{js}(x_j^*) \theta_{ij}^* \quad \text{all } i, j, \quad (3.4)$$

so that $p_j^* > \sum_s \pi_{is}^* R_{js}(x_j^*)$ in equilibrium only if $\theta_{ij}^* = 0$. Further,

$$\pi_{is}^* = \frac{\partial U^i(c_i^*)}{\partial c_{is}} / \frac{\partial U^i(c_i^*)}{\partial c_{i0}} \quad \text{each } i, s. \quad (3.5)$$

The Dreze framework, given by (3.1), (3.2) and Definitions 3.1 - 3.3, does not provide for a distribution of production costs among the I stockholders. That is, Dreze does not consider directly the question of production financing, a question we must consider to prove existence of a participation equilibrium since the distribution of

production costs and its impact on the incentive structure are integral parts of the problem. Therefore, to invoke Dreze's existence theorem, we must extend the concept of a stockholder equilibrium to include the distribution of production costs among stockholders, incorporating the input levels of firms that are willingly financed by stockholders into the definition as stockholder decision variables.

Let $V^i(c_{i0}, \theta_i, x) \equiv U^i(c_{i0}; \sum_j R_{js}(x_j) \theta_{ij}, s \leq S)$.

Definition 3.4. A stock ownership program $z^* \in Z$, a vector of prices

$(p_0, p^*) \in R_+^{1+J}$, $p_0^* = 1$ and a vector of stockholders' cost shares

$\alpha^* = (\alpha_i^*, i \leq I) = (\alpha_{ij}^*, j \leq J, i \leq I)$ is a full stockholders equilibrium

if and only if there exist I vectors $\pi_i^* \in R_+^{1+S}$, $\pi_{i0}^* = 1$ such that

- (i) $U^i(c_i) > U^i(c_i^*)$ implies $\pi_i^* c_i > \pi_i^* c_i^*$;
- (ii) $V^i(c_{i0}, \theta_i, x) > V^i(c_{i0}^*, \theta_i^*, x^*)$ implies $c_{i0} + p^* \theta_i + \alpha_i^* x > c_{i0}^* + p^* \theta_i^* + \alpha_i^* x^*$;
- (iii) for each $j \leq J$, x_j^* maximizes $\sum_s \pi_{is}^* R_{js}(x_j) \theta_{ij}^* - \alpha_{ij}^* x_j$ each $i \leq I$;
- (iv) $c_{i0}^* + p^* \theta_i^* + \alpha_i^* x^* = w_i + p^* \bar{\theta}_i$ each $i \leq I$;
- (v) $\sum_i \alpha_{ij}^* = 1$ each $j \leq J$;
- (vi) $\sum_i \theta_{ij}^* = 1$ each $j \leq J$;

Definition 3.4 is a straightforward generalization of

Definition 3.3. Condition (i) says that individually preferred points must have a greater consumption value to the stockholder, condition

(ii) requires that any individually preferred action entail an

increase in individual costs, and condition (iii) requires that equilibrium production costs be distributed among stockholders in such a way as to make the j^{th} firm's production decision maximal for each stockholder given his revenue share θ_{ij}^* , cost share α_{ij}^* and implicit contingent claim prices π_i^* . Conditions (iv), (v) and (vi) are just feasibility and market clearing conditions. Now assume

A4: Each stockholder i is competitive with respect to his cost shares α_i and revenue share prices (p_0, p) .

Theorem 3.2. If $w > 0$, then there exists a full stockholders equilibrium under assumptions A1, A2, and A4.

Proof: To prove the theorem, we must show there is a nonnegative vector $(z^*, (p_0^*, p^*), (\alpha_i^*, i \leq I), (\pi_i^*, i \leq I))$ that satisfies conditions (i)-(vi) of definition 3.4.

Let $z^* \in Z$ be a stockholders equilibrium, and let $(p_0^*, p^*), (\pi_i^*, i \leq I)$ be the vectors corresponding to z^* in accordance with definitions 3.1 - 3.3. The existence of $z^*, (p_0^*), (\pi_i^*, i \leq I)$ is guaranteed by Theorem 3.1.

Define for each $i \leq I$, $j \leq J$,

$$\bar{\theta}_i = \theta_i^*. \quad (3.6)$$

$$\alpha_{ij}^* = \sum_s \pi_{is}^* \frac{dR_{js}(x_j^*)}{dx} \theta_{ij}^*, \quad (3.7)$$

$$w_i = c_{i0}^* + a_i^* x^*. \quad (3.8)$$

(i) and (vi) follow immediately from Theorem 3.1. To establish (iii) and (v), we note that

$$\sum_i a_{ij}^* = \sum_i \sum_s \pi_{is}^* \frac{dR_{is}(x_j^*)}{dx_j} \theta_{ij}^* = 1 \quad \text{for each } j$$

by (3.7), Theorem 3.1 and Definition 3.1 (ii). Since

$\sum_s \pi_{is}^* \frac{dR_{is}(x_j^*)}{dx_j} \theta_{ij}^*$ is nonnegative and monotone non-increasing as x_j increases for all i, j , there exist unique $a_{ij}^*, i \leq I$ for each $j \leq I$

such that $a_{ij}^* = \sum_s \pi_{is}^* \frac{dR_{is}(x_j^*)}{dx_j} \theta_{ij}^*$.

(iv) follows from (3.6) and (3.8). To establish (ii), we note that,

$$\begin{aligned} dV^i(c_{i0}, \theta_i, x) &= \frac{\partial U^i(c_i)}{\partial c_{i0}} dc_{i0} \\ &+ \sum_s \frac{\partial U^i(c_i)}{\partial c_{is}} \left[\sum_j \frac{dR_{is}(x_j)}{dx_j} \theta_{ij} dx_j \right. \\ &\left. + \sum_j R_{js}(x_j) d\theta_{ij} \right] > 0 \end{aligned}$$

implies by hypothesis and (3.5),

$$(c_{i0} - c_{i0}^*) > \sum_j \sum_s \pi_{is}^* \frac{dR_{is}(x_j^*)}{dx_j} \theta_{ij}^* (x_j^* - x_j)$$

$$+ \sum_j \sum_s \pi_{is}^* R_{js}(x_j^*) (\theta_{ij}^* - \theta_{ij}).$$

Hence, by (3.3), (3.4) and (3.7),

$$\begin{aligned} c_{i0} + p^* \theta_i &> c_{i0}^* + \sum_j \sum_s \pi_{is}^* \frac{dR_{is}(x_j^*)}{dx_j} \theta_{ij}^* (x_j^* - x_j) \\ &+ \sum_j \sum_s \pi_{is}^* R_{js}(x_j^*) (\theta_{ij}^* - \theta_{ij}) + p^* \theta_i \\ &\geq c_{i0}^* + a_i^* (x^* - x) + p^* \theta_i^*, \end{aligned}$$

and therefore,

$$c_{i0} + p^* \theta_i + a_i^* x > c_{i0}^* + p^* \theta_i^* + a_i^* x^* ||$$

The next theorem establishes that the set of full stockholders equilibrium allocations is also the set of participation equilibrium allocations relative to the Shared Cost Mechanism.

Theorem 3.3. $(z^*, (p_0^*, p^*), a^*)$ is a full stockholders equilibrium under assumptions A1, A2, A4 if and only if there exist messages $m_i = (x_i, a_i) \in R^{2J}$, $i \leq I$, current consumption-revenue share bundles $(c_{i0}, \theta_i) \geq 0$, $i \leq I$ and prices $(q_0, q) \in R_+^{1+J}$ such that $((m_i, c_{i0}, \theta_i), i \leq I; (q_0, q))$ is a participation equilibrium relative to the Shared Cost Mechanism under assumptions A1, A2, A3, where $\sum_i x_{ij} = x_j^*$, $c_{i0} = c_{i0}^*$, $\theta_i = \theta_i^*$, $a_{ij}^* = 1 - \sum_{h \neq i} a_{hj}^*$, for all i, j , and $(q_0, q) = (p_0^*, p^*)$.

Proof: Given $x = (x_{ij}, i \leq I, j \leq J)$, let $x^* = \sum_i x_i$, and given x^* , let $x \in R^{IJ}$ be any vector such that $x_{ij} = x_j^* - \sum_{h \neq j} x_{ih}$ each $j \leq J$. Observe that, in equilibrium, $a_{ij} = 1 - \sum_{h \neq i} a_{hj}$ for each $j \leq J$ since the message component a_{ij} affects stockholder i only through the term $(1 - \sum_h a_{hj})^2$ and thus i can decrease his cost share, thereby increasing c_{i0} simply by changing a_{ij} if $\sum_h a_{hj} \neq 1$, any j . Hence, we may let $a_{ij} = a_{ij}^*$ all i, j . The theorem follows by letting $c_{i0} = c_{i0}^*$, $\theta_i = \theta_i^*$ and $(q_0, q) = (p_0^*, p^*)$, since the alternative competitive assumptions, A3 and A4, imply the same incentives exist in both systems with respect to prices and cost shares.||

Theorems 3.2 and 3.3 prove that,

Theorem 3.4. (Existence) Under assumptions A1-A3, if $w > 0$, then there exists a participation equilibrium relative to the Shared Cost Mechanism.

3.2. Optimality and Unbiasedness

An appropriate concept of efficiency for economies with incomplete markets is constrained Pareto optimality. Pareto optimality cannot be achieved in general because stockholders cannot contract for revenue shares in the first period contingent on the state of the world that occurs in the second period. Rather, a stockholder must contract for a single revenue share for each firm that will remain the same under each state of the world.

Definition 3.5. A stock ownership program $z \in Z$ is constrained Pareto optimal if there does not exist a stock ownership program $z \in Z$, such that $U^i(c_i) \geq U^i(c_i)$ for all $i \leq I$ and $U^h(c_h) > U^h(c_h)$ for some $h \leq I$.

Theorem 3.5: (Optimality) If $(m^*, c^*, \theta^*, (p_0^*, p^*))$,

$p_0^* = 1$, $m_i^* = (x_i^*, a_i^*) \in R^{2J}, i \leq I$, is a participation equilibrium relative to the Shared Cost Mechanism, then the equilibrium stock ownership program $z^* = (x^*, c^*, \theta^*)$ is constrained Pareto optimal.

Proof: The proof is by contradiction. Suppose there is an attainable Pareto superior stock ownership program $z \in Z$.

Since $(z^*, (p_0^*, p^*), (a_i^*, i \leq I))$ where $a_i^* = a_i^*$ is a full stockholders equilibrium by Theorem 3.3, it follows by hypothesis from Definition 3.4 (ii) that, $c_{i0} + p^* \theta_i + a_i^* x \geq c_{i0}^* + p^* \theta_i^* + a_i^* x^*$ all i and $c_{h0} + p^* \theta_h + a_h^* x > c_{h0}^* + p^* \theta_h^* + a_h^* x^*$ some h . By Definition 3.4 (iv)-(vi), summing over stockholders gives,

$$\sum_i c_{i0} + \sum_j x_j > \sum_i c_{i0}^* + \sum_j x_j^*$$

and $\sum_i c_{i0}^* + \sum_j x_j^* = \sum_i w_i$, which imply $\sum_i c_{i0} + \sum_j x_j > w$, a contradiction since w is the total of resources available for current consumption and input into production.||

To prove that the Shared Cost Mechanism is unbiased, we use an unbiasedness theorem proven by Dreze (1974, p.144) and the equivalence of the sets of stockholders equilibria and full stockholders equilibria.

Theorem 3.6: (Dreze) If $z^* \in Z$ is a constrained Pareto optimal stock ownership program, such that $c_{i0} > 0$ for all $i \in I$ then there exist I vectors π_i^* , $i \in I$ and prices (p_0^*, p^*) , $p_0^* = 1$ that support z^* as a stockholders equilibrium.

Theorem 3.7: $z^* \in Z$ is a stockholders equilibrium if and only if $z^* \in Z$ can be supported as a full stockholders equilibrium stock ownership program.

The proof of Theorem 3.7 follows immediately from the proof of Theorem 3.2. Using Theorems 3.6, 3.7, and 3.3, one can show,

Theorem 3.8: If $z^* \in Z$ is a constrained Pareto optimal stock ownership program, such that $c_{i0} > 0$ for all $i \in I$, then there exist messages $m_i = (x_i, a_i)$, $i \in I$, and prices (p_0^*, p^*) , $p_0^* = 1$ that support z^* as a participation equilibrium relative to the Shared Cost Mechanism.

4. INFORMATION, TECHNOLOGIES AND OPTIMALITY

The implication of Theorems 3.5, 3.7 and 3.3 that every stockholders equilibrium in our model is constrained Pareto optimal is of particular interest. Using a more general model, Dreze [1974] exhibits three examples of stockholders equilibria that are not constrained Pareto optima, each illustrating a different feature of his model that can lead to inefficient equilibrium stock ownership programs. In this section, we show that none of these features can cause difficulties in our model, thus reconciling the apparently contrary optimality results. Our model is a special case of the

Dreze model, so the Dreze existence theorem remains valid.

One of the Dreze examples [1974, p. 151, ex. 4.4] exploits the non-differentiability of utility functions and can be dismissed immediately in view of assumption A1. The remaining examples arise because of informational imperfections that may prevent adjustment away from certain types of inefficient stock ownership programs once they have been attained. These adjustment problems can exist only when each firm is able to employ resources in more than one production activity, and this is not possible in our model.

The only decision open to a firm in our model is the choice of a scale of activity; each firm is identified with a single production activity, and, for each input level, the firm's output level is fully determined by the state of the world. Dreze, on the other hand, permits firms to choose production activities as well as input levels, and a firm may, in fact, use more than one activity although inputs used in one activity cannot also be used in another activity. Thus, in the Dreze framework, if a stockholder possesses only local information about a firm's production possibilities, the consequences can be more serious than in our model since the stockholder would be ignorant of possible tradeoffs between activities.

To illustrate this point, consider the following example given by Dreze (1974, p. 146, ex.4.1). There are two states (t and s), two stockholders (i and h) and two firms (j and k). Preferences for the stockholders are given for fixed current consumption c_{i0} and c_{h0} , and thus are represented by indifference curves in the space of future

consumption for the two states of the world. The input quantity available to each firm is fixed: $x_j = 1 = x_k$, so that $c_{i0} + c_{h0} + 2 = w$. Stockholders' preferences are represented by curves ii and hh , and firms' production possibilities by the triangles $0t_j s_j$ and $0t_k s_k$. Each firm has available to it two distinct activities $R(\cdot)$ and $P(\cdot)$, each of which yield output in only one state of the world:

$$R_{jt}(x_j) = P_{js}(x_j) = R_{kt}(x_k) = P_{ks}(x_k) = 0, \text{ all } x_j, x_k,$$

$$R_{js}(1) = s_j, P_{jt}(1) = t_j, R_{ks}(1) = s_k, P_{kt}(1) = t_k.$$

Thus each firm's production possibilities frontier $F(\cdot)$, given the fixed input available, represents the tradeoff between output in state s and output in state t and each firm's input must be divided between its activities:

$$F_j(1) = \{(R_{js}(x_{jR}), P_{jt}(x_{jP})) \in R^2 | x_{jR} + x_{jP} = 1\},$$

$$F_k(1) = \{(R_{ks}(x_{kR}), P_{kt}(x_{kP})) \in R^2 | x_{kR} + x_{kP} = 1\}.$$

Let $\theta_{ij} = 1 = \theta_{hk}$, in which case $\theta_{ik} = 0 = \theta_{hj}$. Then, in Figure 1, an inefficient equilibrium stock ownership program occurs when $x_{jR} = 1 = x_{kP}$. The inefficiency is obvious. Both stockholders can be made better off by exchanging revenue shares so that $\theta_{ik} = 1 = \theta_{hj}$, in which case the firms would value maximize at

Figure 1

$x_{jP} = 1 = x_{kR}$ given the input constraints $x_j = 1 = x_k$. Notice that small moves are not profitable.

The difficulty is one of information: each firm produces optimally given the preferences of its owner, and each stockholder carries an optimal portfolio given the production plans of the firms. Clearly, if the stockholders know the production sets of both firms, this situation cannot arise. If firms consider the line t_{ksj} to be an opportunity line that provides information about the direction of desirable changes in production plans, then they may reasonably alter their production plans with the expectation that optimal portfolio changes will follow. Although it is reasonable to believe that firms can observe the production plans of other firms, Dreze [1974] considers two variations of the above example, one where the opportunity line t_{ksj} is missing so that firms lack the information which justifies changing their production plans in the previous example, and one where the opportunity line provides misleading information so that the adjusting firm can actually force the economy away from a constrained Pareto optimal allocation.

In our model, these difficulties cannot arise because firms cannot effect a tradeoff between activities (since each firm has only one productive activity available to it). We observe, however, that the inefficiency shown in Figure 1 can be generated by unbundling the activities and associating a distinct firm with each possible activity. In this instance, there would be four firms, each of which could produce in only one of the two states of the world. But, this

would violate Assumption A2 of our model since each firm would have a constant zero marginal productivity for one state of the world, namely the state in which the firm does not produce.

5. CONCLUDING REMARKS

In this paper, we have developed a general decision making framework that formalizes the notion of communication between agents in the conventional stock market model. This enables us to rigorously examine the equilibrium properties of alternative behavioral objectives for guiding the behavior of firms within a single model of stock market economies. This model also requires that investors be given at least a naive incentive to communicate the information required for an efficient decision. Finally we demonstrated the general existence and optimality for participation equilibria which includes the equilibria in the Leland, Ekern-Wilson, Diamond, and Helpman-Razin models whenever they exist.

The general model without activity choice has not really been distinguished in the literature from the model with activity choice. As a consequence, all existing optimality theorems for equilibrium stock market allocations are attributed to highly restrictive conditions such as the spanning condition in Leland and Ekern-Wilson, multiplicative uncertainty in Diamond, and a very special decision mechanism in Helpman-Razin. Our results show that, in fact, the optimality of the equilibrium allocations in each of these models is due to the lack of activity choice.

The significance of this observation is further enhanced by

the work of Grossman-Hart (1979) who introduced a new concept of efficiency for stock market models called production social Nash optimality. One of the motivations for this concept was to exhibit a reasonable notion of efficiency for which the sets of efficient allocations and equilibrium allocations in stock market economies would coincide. Yet, the technological structure assumed by Grossman-Hart in their introductory model¹⁰ is the same as that which we assume in this paper so that the set of constrained Pareto optimal allocations itself coincides with the set of equilibrium allocations, provided sufficient freedom is allowed in the assignment of cost shares to obtain existence in general.¹¹

Another interesting comparison of our development to Grossman-Hart (1979) is possible if we view the notion of a full stockholders equilibrium as a shareholding equilibrium (i.e. cost shares = revenue shares) with sidepayments. Grossman-Hart justify a value maximization criteria for firms on the basis of the existence of sidepayments (in terms of period 0 income transfers) from those shareholders who favor a change in a firm's production plan to those who do not, such that all shareholders are made better off. They do not, however, permit these sidepayments to be made but rather assume that the manager of the firm has knowledge of the set of contingent claim prices of each shareholder and may perform this computation directly.

As the examples in section 2 illustrate, an equilibrium in our framework will generally require stockholders to receive revenue

shares θ_{ij} that do not coincide with their cost share a_{ij} . If we define $\sigma_{ij} = a_{ij} - \theta_{ij}$ and note that

$$p_j \theta_{ij} + a_{ij} x_j = (p_j + x_j) \theta_{ij} + \sigma_{ij} x_j$$

where $\sum_i \sigma_{ij} = 0$ for all j , then we may interpret our equilibrium as a shareholding equilibrium with sidepayments $\sigma_{ij} x_j$. The Shared Cost Mechanism then represents a specific decision procedure that implements these equilibria, and may be viewed as a modified Shareholding Mechanism where, in order to ensure shareholder unanimity with respect to firm j 's input decision, $\sigma_{ij} x_j$ is the period 0 income transfer to shareholder i .¹²

Finally, the Shared Cost Mechanism, the Paired Difference Mechanism and the Quadratic Mechanism exhibit the same equilibrium properties when applied to stock market economies with an incomplete set of contingent markets as they do when applied to private ownership economies with public goods. In particular, the Shared Cost Mechanism is efficient, unbiased and individually rational. It is also not individually feasible away from equilibrium. This and the fact that the concept of a full stockholders equilibrium is a clear analog of a Lindahl equilibrium for stock ownership economies reinforces the analytical similarity between public good economies and stockownership economies first noted by Dreze (1974).

FOOTNOTES

1. Dreze (1978) and others have noted the public good nature of equilibria in stockholders' ownership economies: in equilibrium, no stockholder desires a (local) change of a firm's input-output decision. The participation approach to these economies implicit in virtually all the voting or collective decision schemes considered in the literature enhances the public aspect of the problem.
2. Hart (1977) has suggested that the Condorcet paradox which arises under majority rule is not a definitive objection to the rule because it does not account for informational difficulties that can prevent the formation of majority coalitions. However, as Jordan (1979) points out, if the majority coalition is a single stockholder, then Hart's caveat is inappropriate. Jordan's (1979) concept of controlling interest is a refinement of the concept of majority control that builds on the notion of single stockholder control.
3. There do exist examples without spanning which have an equilibrium relative to the Shareholding Mechanism. These examples, however, all use a restricted class of preference profiles. Leland's necessity result relies upon using a general class of profiles.
4. A function $f(x)$ is said to be semi-strictly quasi-concave if

$$f(x^1) \geq f(x^2) \text{ implies } f(\lambda x^1 + (1 - \lambda)x^2) \geq f(x^2), 0 \leq \lambda \leq 1.$$

and

$$f(x^1) > f(x^2) \text{ implies } f(\lambda x^1 + (1 - \lambda)x^2) > f(x^2), 0 < \lambda < 1.$$

5. The equilibrium condition that the second period consumption in each state of this world equals total output in that state, i.e., $\sum_i c_s^i = \sum_j R_s^j(x^j)$, for all s , is unnecessary since it is assured by (iii), (iv), and (v).
6. In the Helpman-Razin formulation each individual pays the same fraction $\alpha_j^i = 1/I$ of the cost of his own input proposal, m_{ij} . Requiring each individual to pay a fraction of each firm's total cost eliminates the need for lump sum transfers to achieve a balanced budget in their model.
7. Generic non-existence relative to the Helpman-Razin Mechanism can be similarly demonstrated for the class of economies that satisfy the Ekern-Wilson spanning condition.
8. Definition. The Quadratic Mechanism is defined by,

$$(i) \quad M = R^J, \quad (ii) \quad x_j = \sum_i m_{ij} \text{ for each } j, \text{ and } (iii) \\ C_{ij}(m) = \alpha_{ij} \sum_i m_{ij} + \frac{\gamma}{2} \left[\frac{I-1}{I} (m_{ij} - \frac{1}{I-1} \sum_{h \neq i} m_{hj})^2 \right. \\ \left. - \frac{1}{2(I-1)(I-2)} \sum_{h \neq i} \sum_{h' \neq i} (m_{hj} - m_{h'j})^2 \right] \text{ for all } i \text{ and each } j \text{ where} \\ \gamma > 0 \text{ and } \sum_i \alpha_{ij} = 1 \text{ each } j.$$
9. Definition. The Paired Difference Mechanism is defined by,

- (i) $M = R^T$, (ii) $x_j = \sum_i m_{ij}$ for each j , and (iii)
 $C_{ij}(m) = \left[\frac{1}{I} + m_{i+2,j} - m_{i+1,j} \right] \sum_i m_{ij}$ where $I + 1 = 1$ and
 $I + 2 = 2$ for each j and all i .

10. See sections 2 and 3 of Grossman-Hart (1978).
11. It should be noted that Grossman-Hart (1979) use an objective function for firms which differs from that which we use here. They have firms maximizing net value relative to initial shareholders rather than final shareholders.
12. Hart [1977] has shown that the use of sidepayments makes the existence of a majority rule equilibrium less likely than without them, where his definition of majority excludes unanimity.

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