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EX ANTE OPTIMALITY AND SPOT MARKET ECONOMIES

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ABSTRACT

Arrow's result linking ex ante Pareto optimality for a pure
trade world to competitive equilibrium positions under a complete set
of contingent claim markets is summarized, as is his reinterpretation
of ex ante optimality for the case of an economy with active spot
markets. Possible difficulties arising from this reinterpretation are
noted. The final section of the paper examines conditions under which
an economy with active spot markets will achieve an ex ante optimum in
the original sense of this term and summarizes the behavior of such an
economy.

EX ANTE OPTIMALITY AND SPOT MARKET ECONOMIES

Stuart Burness, Ronald Cummings and James Quirk*

1. INTRODUCTION

The most important result in the literature dealing with the
optimality properties of the competitive equilibrium under uncertainty
is Arrow's proof (1964) that in a pure trade world of risk averse
consumers, and with a complete set of markets in contingent claims,
any ex ante Pareto optimum can be achieved as a competitive
equilibrium, given essentially the same caveats as in the certainty
case. In Arrow's original paper on this topic, this result was
established for a world in which all market transactions take place
before the state of the world is revealed; no spot markets are
permitted to function in future periods. In the same paper, however,
Arrow also considers an alternative institutional arrangement, one in
which there are ''security'' markets that operate before the state of
the world is revealed, a security of type s paying $1 if state of the
world s occurs, and nothing otherwise. There is a security market for
every possible state of the world. When the state of the world is
revealed, spot markets open and consumers can make purchases of goods
in such markets, using the proceeds from the securities that pay off

in that state of the world. Arrow shows that if there is agreement among consumers as to the market clearing price vectors in these spot markets, then again any _ex ante_ Pareto optimum can be achieved as a competitive equilibrium.

Arrow's approach was criticized by Radner (1968, 1970), who noted that the assumption that spot markets do not open after clearing of the markets for contingent claims is highly restrictive, as is the assumption in the security market model that there is unanimous agreement among consumers as to the market clearing price vectors in the spot markets. Radner observed that these assumptions effectively rule out consideration of a number of interesting issues in market theory including uncertainty about future prices, speculation, liquidity, hedging, and so forth. In particular, Radner introduced a distinction of critical importance in models of the competitive economy under uncertainty, namely the distinction between uncertainty about the environment—events over which market participants have no control—and uncertainty about events where actions by market participants _do_ affect or determine the outcome, such as market clearing prices in future spot markets. Radner's interpretation of the Arrow model was that it incorporated only uncertainty about the environment, but not uncertainty about the equilibrium prices in future spot markets.

This interpretation of the Arrow model underlies the exchange between Nagatani (1975) and Arrow (1975) on Nagatani's view that ''... there is actually no mechanism (in the Arrow model) that generates

knowledge of the (spot prices).'' Consequently, Nagatani argues that consumers in an Arrow economy with active spot markets will be uncertain as to which price vector will prevail in the spot markets at the time they make their decisions as to the purchase and/or sale of securities. This uncertainty leads in turn to a two-stage dynamic programming decision rule for consumers, which results in market outcomes that are generally inconsistent with _ex ante_ Pareto optimality.

Arrow's reply to Nagatani introduced a new element into the picture: ''As Nagatani notes, my construction requires each economic agent to be aware of what commodity prices will prevail for each possible state of nature. ... An alternative interpretation (of the Arrow model) is that the definition of a state of the world includes a statement of the prices that will prevail. The money claims, then, are payable conditioned on the occurrence of specified possible price vectors. ... [This] interpretation obviously eliminates a good many difficulties; there can be no uncertainty about the prices that will prevail in a given state if those prices are made part of the very definition of the state. But it must be admitted that there are some difficulties with this interpretation. Implicitly, at least, the uncertainties in the model are exogenous to the economic system; but prices are endogenous to it, and this might complicate our understanding of the model.''

We would like to examine the nature of the ''difficulties'' that arise in this reinterpretation of the Arrow model. Our argument

is to the effect that Radner and Nagatani have correctly identified a fundamental problem with the Arrow model so far as optimality is concerned, and that the device of incorporating the market clearing spot market price vector into the definition of a state of the world resolves this difficulty only by creating other more fundamental problems for the concept of optimality under uncertainty. It is convenient to begin by restating the basic Arrow model.

2. THE ARROW MODEL

We consider a world in which at time 0, securities paying off at the occurrence of various states of the world are bought and sold. After this market closes, the state of the world is revealed and at time 1, spot markets open and trading in spot markets for commodities occurs, following which consumers consume those goods they have acquired. This is a pure trade world, with I consumers, C goods and S states.

Let:

x_{isc}^{o} = number of units of good c in the endowment of consumer i in state s;

x_{isc} = number of units of good c consumed by consumer i in state s;

y_{is} = number of ''type s'' securities purchased by consumer i, each security promising payment of \$1 if state s occurs. If y_{is} is negative, this

represents the sale of securities rather than purchases;

p_s = price for a security of type s;

q_{sc} = spot market price in state s for 1 unit of good c;

π_{is} = consumer i's subjective probability as to the occurrence of state s;[*]

$u^i(x_{is})$ = the (measurable) utility function of consumer i over consumption in state s, where

$$x_{is} = (x_{is1}, \ldots, x_{isC}).$$

In terms of this notation, an ex ante Pareto optimum allocation $x^* = (x_{11}^*, \ldots, x_{I1}^*; \ldots, x_{1S}^*, \ldots, x_{IS}^*)$ is an allocation satisfying:

(i) feasibility, that is,

$$\sum_{i=1}^{I} x_{isc}^* = \sum_{i=1}^{I} x_{isc}^{o}, \text{for all s, c.}$$

(ii) non-dominance, that is, if x is a feasible allocation with

$$\sum_{s=1}^{S} \pi_{is} u^i(x_{is}) > \sum_{s=1}^{S} \pi_{is} u^i(x_{is}^*) \text{ for some i, then}$$

$$\sum_{s=1}^{S} \pi_{js} u^i(x_{js}) < \sum_{s=1}^{S} \pi_{js} u^j(x_{js}^*) \text{ for some j.}$$

Assuming a classical environment with differentiable utility functions and interior maximum positions, an ex ante Pareto optimum

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* For simplicity, we assume $\pi_{is} > 0$ for all i, s.

can be characterized in terms of the marginal conditions derived from solving the problem

$$\max_{\{x_i\}} \sum_{i=1}^{I} \alpha_i \sum_{s=1}^{S} \pi_{is} u^i(x_{is})$$

subject to $\sum_{i=1}^{I} x_{isc} = \sum_{i=1}^{I} x_{isc}^o$ for all s, c, where α_i is the weight assigned to consumer i in the welfare function.

Hence at an _ex ante_ Pareto optimum we have

$$\left(\frac{\pi_{is}}{\pi_{it}}\right)\left(\frac{\partial u^i}{\partial x_{isc}}\bigg/\frac{\partial u^i}{\partial x_{itd}}\right) = \left(\frac{\pi_{is}}{\pi_{jt}}\right)\left(\frac{\partial u^j}{\partial x_{jsc}}\bigg/\frac{\partial u^j}{\partial x_{jtd}}\right) \qquad (1)$$

for all i, j, s, t, c, d, together with

$$\sum_{i=1}^{I} x_{isc} = \sum_{i=1}^{I} x_{isc}^o = \text{ for all s, c.} \qquad (2)$$

Consider next a model of a competitive economy with security markets open at time 0, with active spot markets at time 1, and with consumers in unanimous agreement at time 0 as to the market clearing spot market price vector q_s associated with any state s. This is the version of a security market economy first proposed by Arrow.

Then consumer i acts to solve the problem (at time 0):

$$\max_{\{y_i, x_i\}} \sum_{s=1}^{S} \pi_{is} u^i(x_{is})$$

subject to $\sum_{s=1}^{S} p_s y_{is} = 0$ and $\sum_{c=1}^{C} q_{sc} x_{isc} = \sum_{c=1}^{C} q_{sc} x_{isc}^o + y_{is}$ for all

s.*

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* The appropriate constraint to impose on time 0 dealings in the

First order conditions for this problem at an interior maximum involve the constraints together with

$$\left(\frac{\pi_{is}}{\pi_{it}}\right)\left(\frac{\partial u^i}{\partial x_{isc}}\bigg/\frac{\partial u^i}{\partial x_{itd}}\right) = \frac{p_s q_{sc}}{p_t q_{td}} \quad \text{for all s,t,c,d.} \qquad (3)$$

Since (3) holds for any consumer i and every consumer faces the same prices in the security and spot markes, this implies that the marginal conditions (1) for an _ex ante_ Pareto optimum are satisfied. Moreover, market clearing conditions in the securities markets are given by

$$\sum_{i=1}^{I} y_{is} = 0 \quad \text{for all s;} \qquad (4)$$

while market clearing conditions in the spot markets are that

$$\sum_{i=1}^{I} x_{isc} = \sum_{i=1}^{I} x_{isc}^o \quad \text{for all s,c.} \qquad (5)$$

It immediately follows that the competitive equilibrium is _ex ante_ Pareto optimal, and, under an appropriate distribution of endowments among consumers, any _ex ante_ Pareto optimum can be achieved as a competitive equilibrium. This is precisely Arrow's second theorem. But this result relies, of course, on the assumption that there is unanimous agreement among consumers as to the spot market

security markets is problematical. In the formulation presented here, a barter type process is assumed in which securities are traded for securities. We do not attempt to solve the default problem, which is a basic difficulty in futures market–spot market models. See Green (1974) and Stigum (1974) for a discussion of the issues involved in the default risk controversy.

price vector in any state of the world. This gives rise to Nagatani's objection that there is no mechanism in the Arrow model that leads to this restrictive condition; and in turn this leads to Arrow's reinterpretation of his model.

3. THE REINTERPRETATION OF THE ARROW MODEL

The reinterpretation proposed by Arrow in his reply to Nagatani fits nicely into the framework outlined above. The only change is that up to the present, the symbol π_{is} has referred to consumer i's subjective probability as to the occurrence of state of the environment s, with q_s, the spot market price vector in state s, taken to be known with certainty (and with unanimous agreement) by all consumers. In Arrow's reinterpretation, π_{is} is consumer i's subjective probability with respect to state s, where the specification of state s includes not only the state of the environment but also the value taken on by the market clearing spot market price vector. There is no longer any need to assume that there is unanimous agreement among consumers as to the spot market price vector in any state of the world, since the spot market price vector is now part of the specification of a state of the world. With this reinterpretation, the number of states has been expanded, of course, but otherwise the argument given above applies in its entirety so that again any ex ante Pareto optimum can be achieved as a competitive equilibrium. There is, however, one essential difference between this result and Arrow's original result, namely that the probabilities used

in defining an ex ante Pareto optimum now refer not to exogenous states of the environment but rather to states of the world specified both in terms of exogenous states of the environment and endogenous market clearing spot market price vectors.

It is here that the ''difficulties'' arise that Arrow was referring to in his reply to Nagatani. By incorporating the probability beliefs of consumers as to spot market price vectors into the definition of an ex ante optimum, a number of paradoxical complications arise. To begin with, the definition of an ex ante Pareto optimum has now been tied directly into one specific institutional device for allocating resources, namely a system of futures (security) and spot markets. Consider for example the same set of consumers placed in the setting of a centrally planned economy. We suppose that in this centrally planned economy, once the state of the environment is known, the central planning board announces a spot market price vector (not necessarily market clearing) to govern transactions in that state of the environment, together with some rationing rules. At time 0, before the state of the environment is known, a market opens in contracts that pay off on the basis of the state of the environment and the spot market price vector to be announced by the central planning committee. Presumably consumers have probability beliefs as to the planning committee's price announcements, and will use such beliefs in their dealings in contracts; but does this mean that such beliefs should be a part of the definition of an ex ante Pareto optimum? In terms of Arrow's

reinterpretation of the concept, the answer must be yes; in general, the definition of an ex ante optimum now varies with the institutions used to allocated resources, incorporating the probability beliefs of consumers as to any random events that can take place in that institutional setting. And with different probability beliefs in different institutional settings, this means that allocations under different institutions will be Pareto non-comparable. This seems to imply that it is only in the exceptional case that any ranking at all of allocative mechanisms will be possible, because generally probability beliefs even as to the same events will differ simply because different resource allocation institutions are being employed. Moreover, as in the case of a futures market-spot market economy, the institutions themselves create uncertainties that are now incorporated into the definition of an optimum. Among other things, this certainly lessens the policy significance of the fact that a competitive equilibrium is an ex ante Pareto optimum.

Paradoxes relating to both the original and the reinterpreted notion of ex ante optimality include the following. Hirshleifer (1971) has argued that in a pure trade world of identical consumers, knowledge at time 0 of spot market prices in time 1 markets or even of the state of the environment at time 1 has no social value; on the other hand, consumers are willing to expend sizeable amounts of resources to acquire such information because of the private (distributional) gains that are possible through speculation. Hirshleifer argues that this introduces an inefficiency into the

system. But under either the original or the reinterpreted version of ex ante optimality, the situations before and after the expenditure of resources to acquire information are both optimal, since the probability beliefs of consumers differ in the two situations, and in both cases a complete set of contingent claim and spot markets exist. On the other hand, given the expenditure of resources to acquire information, it is possible through lump sum transfers to make every individual better off (ex post) in every state of the environment, if resources are not expended in information gathering. An even more striking instance is the case in which there is no uncertainty about states of the environment on the part of consumers, so that in the absence of futures markets, Pareto optimality is achieved by simply equating marginal rates of substitution between goods among consumers in spot markets. But establishing a futures market now leads to a new set of ex ante optimality conditions because of the possibility of speculating in terms of future spot market prices, and another (Pareto non-comparable) set of optimality conditions covering the case in which resources are expended for information gathering as well.

Finally, there is the problem of endogenous probability beliefs, as emphasized by Radner (1970). Beliefs as to future spot market prices presumably are influenced to some degree by market clearing prices on futures markets. In the extreme case of an individual who believes that markets are efficient in the strict sense of the term, the individual's probability distribution over spot prices is determined by futures prices. This again raises problems in

terms of using the notion of ex ante optimality to rank or compare
allocative mechanisms. In fact, given that perturbations of
institutions lead to effects on market prices (including futures
market prices), it begins to appear that ex ante optimality in the
reinterpreted sense might lead essentially to no welfare comparisons
at all. A related set of problems arises from the fact that
probability beliefs about future spot prices involve beliefs
concerning other individuals' subjective beliefs as to other
individuals' beliefs, and so on—the so-called ''Keynes problem''
(Keynes (1936, p. 156). This is a further dimension of Nagatini's
concern as to the mechanism which generates knowledge of future spot
market prices, and it raises questions as to whether one can expect
beliefs of consumers with respect to spot market prices to be
representable in the subjective probability framework.

It is also of interest that by converting optimality into a
concept whose specification varies with institutions, the unbiasedness
property of the competitive equilibrium generally fails. That is, it
is no longer true that any ex ante Pareto optimum can be achieved by a
system of competitive markets, since there is no mechanism in the
competitive setting that induces the beliefs that characterize
optimality in non-competitive settings. Thus in effect the
reinterpretation of ex ante optimality preserves efficiency of the
competitive mechanism at the sacrifice of unbiasedness.

As Starr (1973), Harris (1978), Radner (1970), and others have
pointed out, there are difficult problems in arriving at an acceptable

notion of optimality for a world of uncertainty; the arguments pro and
con ex ante and ex post optimality reveal some of those difficulties.
We have no solution to suggest to the problem, but we do want to
emphasize that both the original and the reinterpreted version of ex
ante optimality do indeed pose difficulties of interpretation, as
Arrow has noted. And the difficulties are especially pronounced in
the reinterpreted version since uncertainties introduced into the
problem solely by the institution generally lead in the direction of
optimality more or less by default, and destroy the unbiasedness
property of the competitive mechanism. Given these difficulties, one
might ask under what conditions does a futures market–spot market
economy satisfy ex ante optimality in Arrow's original sense of the
term? In the next section we examine the performance of an economy
with a complete set of contingent claim markets and with active spot
markets, the contingent claims paying off on the occurrence of states
of the environment only; we then consider a similar structure for a
security market–active spot market economy.

With contingent claims paying off on the occurrence of states
of the environment and with active spot markets, equilibrium positions
of this economy are ex ante Pareto optimal (in the original sense of
this term) if and only if all consumers agree with certainty as to the
spot market price vector that will occur in any state of the
environment. The same result holds for the security market model.
Moreover, it is easy to see that this result also extends to the case
in which contingent claims or securities are indexed by spot market

price vectors as well as by states of the environment.

However, even under this highly restrictive condition (unanimous agreement with certainty as to spot market prices), in the contingent claim economy there is no assurance that planned purchases in spot markets will agree with actual purchases after spot markets open, and speculative gains and losses will typically occur even though they are not planned. This paradoxical feature of future market-spot market economies was first pointed out by Svensson (1976). It is of interest that this does not occur in a security economy as in the one proposed in Arrow's original paper.

4. EX ANTE OPTIMALITY AND ACTIVE SPOT MARKETS

We can identify two distinct kinds of risk that are present when spot markets are active in future periods. There are environmental risks, associated with uncertainty as to the state of the environment that will occur, and there are market risks, associated with uncertainty as to the spot market price vectors that will occur in future spot markets. Environmental risks are present regardless of the institution adopted to allocate resources, but market risks arise only with active spot markets in future periods. Market risks are not present in the original Arrow models; either no spot markets function or consumers have no uncertainty about spot market prices. In the reinterpretation of the Arrow model, market risks are present but are ''insured against'' through the purchase and sale of securities paying off on the occurrence of specific spot

market price vectors.

We next consider a model of an economy in which, at time 0, markets open in contingent claims to commodities, each such claim promising delivery of one unit of the specified commodity on the occurrence of a specific state of the environment. No securities or contingent claims paying off on the occurrence of spot market price vectors exist. At time 1, spot markets in commodities open after the state of the environment is revealed. This means that at time 0, each consumer buys and sells of contingent claim contracts in part on the basis of expectations of capital gains at time 1, rather than simply on the basis of the consumer's desire for consuming a certain mix of goods in that state of the environment. Market risk and speculative opportunities arise because of uncertainty about future spot prices.

We use the same notation employed earlier. In addition, let z_{isc} denote the number of contingent claim contracts purchased (or, if z_{isc} is negative, the number of contingent claim contracts sold) by consumer i, each such contract promising the delivery of 1 unit of commodity c in state of the environment s. In a world with contingent claim markets and spot markets, the consumer first enters the contingent claim markets at time 0 and then, at time 1, after the state of the environment is revealed, he enters the spot markets. We use the index t to refer to spot market price vectors, with q_{stc} denoting the spot price of commodity c in state of the environment s and price vector t, and x_{istc} denoting the number of units of commodity c consumed by consumer i in state of the environment s with

price vector t. Then, after state s has been revealed, with price vector t clearing spot markets, the consumer solves the problem

$$\max_{\{x_{ist}\}} u^i(x_{ist})$$

subject to $\sum_{c=1}^{C} q_{stc}x_{istc} = \sum_{c=1}^{C} q_{stc}(z_{isc} + x_{isc}^o)$ with the maximizing bundle satisfying

$$\left(\frac{\partial u^i}{\partial x_{istc}} \Big/ \frac{\partial u^i}{\partial x_{istd}}\right) = q_{stc}/q_{std} \quad \text{for all } c,d, \tag{8}$$

and

$$\sum_{c=1}^{C} q_{stc}x_{istc} = \sum_{c=1}^{C} q_{stc}(z_{isc} + x_{isc}^o) \tag{9}$$

Conditions (8) and (9) determine the demand functions for consumer in the joint state (s,t), that is

$$x_{istc} = x_{istc}(q_{st}, M_{ist})$$

where $M_{ist} = \sum_{c=1}^{C} q_{stc}(z_{isc} + x_{isc}^o)$.

Market clearing in the spot markets occurs when

$$\sum_{i=1}^{I} x_{istc} = \sum_{i=1}^{I} x_{istc}^o \quad \text{for all } s,c. \tag{10}$$

We might note that the volume of contingent claim contracts does not enter directly into this market clearing condition, since purchase of such a contract by consumer i involves an offsetting sale

by some consumer j.*

Having characterized the solution to the time 1 problem, we now move backwards in time to examine the time 0 problem. At time 0, the consumer decides on the number of contingent claim contracts to buy and sell, under uncertainty both as to the state of the environment that will occur and as to the spot market price vector that will clear markets in that state of the environment. We assume that there are T possible spot market price vectors as well as S possible states of the environment, with π_{ist} denoting consumer i's subjective probability as to the state of the environment s and spot market price vector t. Then, at time 0, consumer i solves the problem

$$\max_{\{z_i\}} \sum_{s=1}^{S} \sum_{t=1}^{T} \pi_{ist} u^i(x_{ist}(q_{st}, M_{ist}))$$

subject to $\sum_{s=1}^{S} \sum_{c=1}^{C} P_{sc}z_{isc} = 0$,

where P_{sc} is the price of a contingent claim promising delivery of 1 unit of commodity c in state of the environment s.

Because contingent claim markets exist only for claims paying off on the occurrence of states of the environment, z and p are indexed only by s and not by t. Assuming an interior maximum, the first order conditions for the time 0 problem are given by

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* Of course they enter indirectly, however, since x_{istc} is a function of M_{ist}, which depends on the contracts owned by the consumer.

$$\left(\sum_{t=1}^{T} \pi_{ist}\left(\frac{\partial u^i}{\partial x_{istc}}\right)\left(\frac{\partial x_{istc}}{\partial M_{is}}\right)q_{stc}\right) \bigg/ \left(\sum_{t=1}^{T} \pi_{irt}\left(\frac{\partial u^i}{\partial x_{irtd}}\right)\left(\frac{\partial x_{irtd}}{\partial M_{ir}}\right)q_{rtd}\right)$$

$$= \frac{P_{sc}}{P_{rd}} \quad \text{for all } s,r,c,d, \quad (11)$$

and

$$\sum_{s=1}^{S}\sum_{c=1}^{C} P_{sc}z_{isc} = 0. \quad (12)$$

With (11) and (12) holding for each consumer i, market clearing in the contingent commodity markets occurs when

$$\sum_{i=1}^{I} z_{isc} = 0 \quad \text{for all } s,c, \text{ and} \quad (13)$$

$$\frac{I_{isc}}{I_{ird}} = \frac{I_{jsc}}{I_{jrd}} \quad \text{for all } i,j,s,r,c,d \quad (14)$$

where $I_{isc} = \left(\sum_{t=1}^{T} \pi_{ist}\left(\frac{\partial u^i}{\partial x_{istc}}\right)\left(\frac{\partial x_{istc}}{\partial M_{ist}}\right)q_{stc}\right)$.

Given an _ex ante_ Pareto optimum as characterized by conditions (1), assume that the utility function u^i is strictly quasi-concave and twice differentiable so that the marginal rate of substitution between any two commodities for any consumer is 1-to-1 with relative price ratios. Then it is clear that conditions (8) and (14) are not consistent with an _ex ante_ Pareto optimum. To put it another way, a necessary condition for an _ex ante_ Pareto optimum in the original version of this term, given the economy as described, is that each consumer knows with certainty the market clearing spot market price vector that will occur with any state of the environment. Let t(i,s) denote the spot market price vector that consumer i knows with certainty will occur in state of the environment s. Then the decision problem at time 0 for consumer i becomes

$$\max_{\{z_i, X_i\}} \sum_{s=1}^{S} \pi_{ist(i,s)} u^i(x_{ist(i,s)})$$

subject to $\displaystyle\sum_{s=1}^{S}\sum_{c=1}^{C} P_{sc}z_{isc} = 0$, and

$$\sum_{c=1}^{C} q_{st(i,s)c}x_{ist(i,s)c} = \sum_{c=1}^{C} q_{st(i,s)c}(z_{isc} + x_{isc}^o) \quad \text{for all } s.$$

At an interior maximum we have

$$\left(\frac{\pi_{ist(i,s)}}{\pi_{irt(i,r)}}\right)\left(\frac{\partial u^i}{\partial x_{ist(i,s)c}} \bigg/ \frac{\partial u^i}{\partial x_{irt(i,r)d}}\right) = \frac{q_{st(i,s)c}}{q_{rt(i,r)d}} \quad (15)$$

for all r,s,c,d.

Moreover, we can characterize the link between the time 0 price of a contingent claim to a commodity and the time 1 spot market price of the commodity, which is known with certainty, by using the envelope theorem, so that

$$\left(\frac{\partial En^i}{\partial W_i}\right) P_{sc} = \pi_{ist(i,s)} \frac{\partial u^i}{\partial M_{ist(i,s)}} q_{st(i,s)c} \quad \text{for all } s,c, \quad (16)$$

where $W_i = \displaystyle\sum_{s=1}^{S}\sum_{c=1}^{C} P_{sc}z_{isc}$, and $\dfrac{\partial En^i}{\partial W_i} = \displaystyle\sum_{s=1}^{S} \pi_{ist(i,s)}\frac{\partial u^i}{\partial W_i}$.

Condition (16) may be interpreted as follows. At a maximum of expected utility, the gain in expected utility from selling one more

contingent claim contract at time 0 on c deliverable in state s must be equal to the loss in utility that will be experienced when this contract is covered in state s times the probability that state s occurs.

Returning to the conditions (15), recall that in the characterization of an _ex ante_ Pareto optimum from (1), the probabilities that appear are π_{is}, where $\pi_{is} = \sum_{t=1}^{T} \pi_{ist}$ for all i, s. Given that each consumer is certain as to the spot market price vector that will occur in any state of the environment s, it thus turns out that $\pi_{ist(i,s)} = \pi_{is}$ for all i, s. But, using (15), this means that at _ex ante_ Pareto optimum (in the original sense of this term), every consumer expects with certainty the same spot market price vector for any given state of the environment. In turns this means that the conditions (1) are satisfied.

Since by definition there is market clearing at any equilibrium, it follows that a necessary and sufficient condition for an equilibrium of this economy to be an _ex ante_ Pareto optimum is that there is agreement among consumers as to the spot market price vector that will occur with certainty in any state of the environment.

Write $q_{st(i,s)c} = q_{sc}$, since there is agreement among consumers as to spot market prices in any state s. Then the conditions (16) imply that

$$P_{sc}/q_{sc} = P_{sd}/q_{sd} \quad \text{for all c,d and for all s.} \tag{17}$$

The idea behind the conditions (17) is this. With spot

markets open at time 1, purchases and sales in the contingent claim markets at time 0 can be for speculative purposes (buying a claim on c to be delivered in state s, in order to resell it at a profit if state s occurs) or for the purpose of rearranging the distribution of the consumer's income among states. Suppose that (17) did not hold; in particular, suppose that $P_{sc}/q_{sc} < P_{sd}/q_{sd}$ for some c, d. Then any consumer buying state s claims on c or d would purchase only claims on c, because for any given outlay at time 0, this maximizes the amount of income received in state s. And any consumer selling claims would of course sell claims on d only, since this minimizes the amount of income sacrificed in state s in order to buy claims for other states. Hence arbitrage at time 0 will insure that there are no speculative profits to be made and that (17) holds. Further, when (17) holds, consumers are perfectly indifferent among portfolios of state s claims having the same market value at time 0, because any such portfolio will have the same value at time 1 (a value larger, of course, than the time 0 value of the portfolio).

With indifference as to portfolios on the part of consumers, then for any state of the environment, the volume of claims on particular goods and the distribution of these claims among consumers are both unpredictable, as Svensson (1976) has noted. On the other hand, the spot market prices that will actually occur on spot markets at time 1 in state s depend, of course, in part on the volume of claims and the distribution of these claims. But this means that the market clearing spot market price vector for any state is then

unpredictable as well. Thus we have arrived at Svensson's paradoxical conclusion that in a futures market-spot market economy in which every consumer believes with certainty that a certain spot market price vector will occur in a given state of the environment, this induces behavior and market clearing conditions in futures markets that essentially guarantee that the expected spot market price vector will _not_ occur.

This means that while there is no planned speculation in this economy at an _ex ante_ Pareto optimum, nonetheless generally there will be speculative gains and losses for all consumers. Furthermore, there is another aspect of this interesting situation that raises added difficulties for the reinterpretation of the optimality criterion. The problem is that consumers will presumably realize what we have seen—that if all of them are in unanimous agreement with certainty as to the occurrence of a spot market price vector, this in and of itself acts to essentially guarantee that this outcome will not occur. This will certainly have feedback effects on the beliefs of consumers; that is, as Radner (1970) has emphasized, probability beliefs of consumers as to the occurrence of endogenous events such as future spot market prices tend themselves to be endogenous and in particular to be influenced by observed prices in futures markets. An extreme instance of this would be a naive efficient market believer, who would take the time 0 prices in the contingent claims markets (discounted by probabilities of occurrence of states of the environment) as the best estimators of time 1 spot prices.

An alternative modeling of the competitive economy is suggested by Nagatani (1975), who envisages a security market open at time 0, securities paying off on the basis of occurrence of states of the environment, and spot markets for commodities active at time 1. First order conditions characterizing such an economy include

$$\frac{K_{is}}{K_{ir}} = \frac{K_{js}}{K_{jr}} = \frac{p_s}{p_r} \quad \text{for all } i,j,r,s, \tag{18}$$

where

$$K_{is} = \sum_{t=1}^{T} \pi_{ist} \sum_{c=1}^{C} \left(\frac{\partial u_i}{\partial x_{istc}}\right)\left(\frac{\partial x_{istc}}{\partial M_{ist}}\right); \text{ as before,}$$

from the dynamic programming approach

$$x_{istc} = x_{istc}(q_{st}, M_{ist}) \text{ where } M_{ist} = \sum_{c=1}^{C} q_{stc} x^{o_{isc}} + y_{is}.$$

Market clearing requires that

$$\sum_{i=1}^{I} x_{istc} = \sum_{i=1}^{I} x^{o}_{istc} \quad \text{for all } s,t,c; \tag{19}$$

and

$$\sum_{i=1}^{I} y_{is} = \quad \text{for all } s. \tag{20}$$

where y_{is} is the number of securities owned by i that pay \$1 if state s occurs.

As in the contingent claim economy, given a twice differentiable strictly quasi-concave utility function and an interior maximum, at an _ex ante_ Pareto optimum, all consumers must be certain

as to the spot market price vector that will occur in any state of the environment. Given certainty as to spot market price vectors, the conditions that hold at an equilibrium become

$$\left(\frac{\pi_{ist(i,s)}}{\pi_{irt(i,r)}}\right)\left(\frac{\partial u^i}{\partial x_{ist(i,s)c}} \middle/ \frac{\partial u^i}{\partial x_{irt(i,r)d}}\right) = \frac{q_{st(i,s)c}}{q_{rt(i,r)d}} \qquad (21)$$

for all i,r,s,c,d; and

$$\left(\frac{\partial Eu^i}{\partial V_i}\right) p_s = \pi_{ist(i,s)} \frac{\partial u^i}{\partial M_{is}} \quad \text{for all i,s,} \qquad (22)$$

where $V_i = \displaystyle\sum_{s=1}^{S} p_s y_{is}$.

Equation (22) has an interpretation similar to that for (16). At a maximum of expected utility, the gain in expected utility from selling a security paying \$1 if state s occurs must be equal to the loss in utility from paying \$1 in state s, times the probability that state s will occur.

As in the contingent claim economy, from (21) we can derive the conclusion that t(i,s) is independent of i, so that every consumer expects with certainty the same spot market price vector in any state s. It immediately follows that equilibrium positions of this economy are _ex ante_ Pareto optimal if and only if there is unanimous agreement among consumers as to the spot market price vector that will occur with certainty in any state of the environment.

As in the contingent claim economy, no individual has anticipations of speculative gains. In contrast, however, in the

security market economy there are no realized speculative gains as well. Thus for the security market economy, certainty beliefs as to spot market prices are self-fulfilling whereas in the contingent claim economy they tend to be self-negating. This is a crucial distinction between the two institutions, in terms of allocative outcomes.

Finally, we simply note that in an economy in which contingent claims or securities are indexed by spot market price vectors as well as by states of the environment, it clearly is still the case that _ex ante_ optimality (in the original sense) occurs if and only if there is unanimous agreement (with certainty) as to spot market prices in any state of the environment. The reason is that the original version of _ex ante_ optimality requires that the commodity bundles chosen by consumers not vary within a state of the environment, hence indexing by spot market price vectors offers no advantages over contracts indexed by states of the environment only, so far as attainment of an optimum is concerned.

5. SUMMARY

Formally, the optimality properties of a contingent claim economy or a security market economy can be preserved with active spot markets by introducing claims that pay off on the joint occurrence of states of the environment and spot market price vectors. However this formal equivalence, as in the reinterpretation of the Arrow model, involves redefining an _ex ante_ Pareto optimum in terms of consumers' subjective probabilities over states of the environment and spot

market price vectors. We have argued that this is a major revision of
the original notion of ex ante optimality by making the concept of
optimality a variable depend indirectly on the institutions for
allocating resources. It also raises problems related to basing
probability beliefs on observed future market prices, and it
ressurects the ''Keynes problem'' and the applicability of the
subjective probability framework to characterize beliefs. Beyond
this, the institution specific character of the revised notion of ex
ante optimality preserves efficiency of the competitive system at the
expense of destroying unbiasedness.

Returning to the original definition of an ex ante optimum, a
condition necessary and sufficient for an economy with active spot
markets to attain such an optimum has been derived. This condition is
highly restrictive, involving unanimous agreement among consumers as
to the spot market price vector that will occur with certainty in any
state of the environment. Moreover, even with this restrictive
condition satisfied, in a world with a complete set of contingent
claims the resulting spot market equilibrium will generally not
generate those spot market prices forecasted with certainty by all
consumers, and will generally result in (unplanned) speculative gains
and losses for all consumers.

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