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DEMAND-REVEALING MECHANISMS FOR PRIVATE GOOD AUCTIONS

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ABSTRACT

The set of demand revealing mechanisms for allocating private goods is characterized and examples are given. Auctions in which multiple units of either homogenous or nonhomogeneous commodities are studied and, in particular, it is demonstrated that there will generally not exist a demand revealing mechanism with which each bidder will pay the same price for each unit purchased. The experimental literature on these bidding mechanisms is discussed and several additional inquiries are suggested.

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Robert Forsythe and R. Mark Isaac

1. INTRODUCTION

Nearly two decades have passed since the publication of Vickrey's seminal article [1961] in which he proposed a mechanism for achieving an optimal allocation in sealed bid auctions. His mechanism had the demand revealing property that each participant had a dominant bidding strategy to truthfully reveal his demand schedule. Since that time, an extensive body of literature has been developed which examines methods for the auctioning of private goods (see Stark and Rothkopf [1977]). Over the same period, there has been considerable interest in demand revealing properties of mechanisms for the provision of public goods. Loeb [1977] demonstrated an isomorphism between the private good auction problem and the public good institutional design problem but he focused his attention on one particular mechanism. None of the literature since his work has provided any more complete characterization of demand revealing mechanisms for private good auctions. In this paper we attempt to fill this void. Making use of the isomorphism demonstrated by Loeb we translate the relevant theorems from the literature and then demonstrate particular properties of auctions which are demand revealing.

In practice, most auctions are held for the sale of either multiple units of a single homogeneous commodity (as in the case of the Treasury security auction) or single units of several nonhomogeneous commodities (as, for example, the offshore oil lease auction). Due to this, we then examine demand revealing mechanisms for conducting these auctions. Further, we show that in either of these cases there will not generally exist a demand revealing mechanism with which each bidder will pay the same price for each unit purchased. Even though a "competitive" pricing policy is not possible, the existence of demand revealing mechanisms insure the optimal allocation of the goods from these auctions.

In conclusion we discuss the growing body of literature which has analyzed several demand revealing mechanisms in experimental markets and suggest several experimental inquiries into more general market situations.

2. A CHARACTERIZATION OF DEMAND REVEALING AUCTIONS

Let $\mathcal{N} = \{1,\ldots,N\}$ be the set of N goods, possibly nonhomogeneous, to be auctioned and let $\mathcal{I} = \{1,\ldots,I\}$ be the set of I agents in the auction. Further, let χ be the set of possible distributions of the N goods to the I agents. A typical element of χ, $X = [x_j^i] \in \chi$ is a $N \times I$ allocation matrix where x_j^i is the amount of good j that that is awarded to bidder i. The set χ may be any compact set in a topological space. Some examples include: (a) the allocation of a single indivisible good to one bidder, i.e., X is a $1 \times I$ and χ contains all I possible unit vectors; (b) the allocation of a fixed supply of a single, perfectly divisible good, $\bar{x}$, i.e.,

each $X \in \chi$ is a $1 \times I$ vector such that $\sum_{i\in\mathcal{I}} x^i \leqq \bar{x}$; (c) the allocation of both supply and demand allocations of a single, perfectly divisible good, i.e., X is a $1 \times I$ vector such that $\sum_{i\in\mathcal{B}} x^i \leqq \sum_{i\in\mathcal{S}} x^i$; where $\mathcal{B}$ is the set of buyers, $\mathcal{S}$ is the set of sellers, and $\mathcal{B} \cup \mathcal{S} = \mathcal{I}$; (d) the allocation of N nonhomogeneous goods to each agent as a straightforward generalization of either (a), (b) or (c). We will postpone until Section 5 any discussion of nonhomogenous goods and until then we will focus on the homogeneous good case with an exogenous inelastic supply (i.e., either (a) or (b) aobve). The interested reader will note that all the propositions stated below directly extend to the N good case with a suitable redefinition of the notation. Further the introduction of sellers into the auction is also straightforward and for ease of exposition we will indicate any adjustments necessary to incorporate them into the analysis through footnotes.

Each bidder $i \in \mathcal{I}$ posses a utility function $u^i(x^i,y^i)$ defined over the allocation $x^i \in \mathbb{R}$ and money $y^i \in \mathbb{R}$ which is additively separable in its arguments, that is

$$u^i(x^i,y^i) = v^i(x^i) + y^i, \quad i \in \mathcal{I} \tag{1}$$

The function $v^i(\cdot)$ will be referred to as bidder i's valuation function for possible allocations from the auction.[1] We will assume that agent i's valuation function is restricted to some prescribed set of valuation functions V^i. We assume further that valuation function is normalized such that $v^i(0) = 0$.

From (1), an optimal allocation of objects to bidders, $X^*(v)$, must satisfy[2]

$$X^*(v) \in \underset{X \in \chi}{\operatorname{argmax}} \sum_{i \in I} v^i(x^i), \tag{2}$$

where $v = (v^1, \ldots, v^I) \in V = \underset{i \in I}{X} V^i$ are the true valuation functions of the bidders. This simply states that the items are awarded to the bidders who value them the most. Since only bidder i knows his own valuation function, all decisions must be made using the information he reveals about this function. We will assume that each bidder is asked to report his true valuation function and that an allocation is made according to a decision rule satisfying (2) with respect to the reported valuations. In order to induce each agent to tell the truth, we will use a set of payment schedules $\{p^i(w)\}_{i \in I}$ where $p^i(w)$ is the amount of money which bidder i must pay when reported valuations of all agents are $w = (w^1, \ldots, w^I) \in V$. Schedules which induce each bidder to report his true valuation function as a dominant strategy are referred to as being demand revealing. They will result if and only if

$$v^i \in \underset{w^i \in V^i}{\operatorname{argmax}}\ v^i(x^{i*}(w^i, w^{)i(})) - p^i(w^i, w^{)i(}),\ \forall\ w^{)i(} \in V^{)i(},\ \forall i \in I, \tag{3}$$

where

$$w^{)i(} = (w^1, \ldots, w^{i-1}, w^{i+1}, \ldots, w^I), (w^i, w^{)i(}) = w, \text{ and } V^{)i(} = \underset{\ell \in I \setminus \{i\}}{X} V^\ell .$$

In the literature on public goods, payment schedules which have this property are known as the family of Groves' mechanisms and are of the form

$$p^i(w) = -\sum_{\ell \in I \setminus \{i\}} w^\ell(x^{\ell *}(w)) + h^i(w^{)i(}),\ \forall\ w \in V,\ \forall\ i \in I, \tag{4}$$

where $h^i(\cdot)$ is an arbitrary function which is independent of w^i.[3] This requires that each bidder pays a lump sum charge, $h^i(w^{)i(})$, which is independent of his reported valuation, less the amount of the surplus of all other bidders who are awarded some positive amount of the good (since $w^\ell(0) = 0$). To see that this family of mechanisms are demand revealing suppose that given the reported valuations of all other bidders except i, $w^k \in V^k$, $k \neq i$, X^* is the allocation matrix when bidder i reports his true valuation function, $w^i = v^i$, and let $\hat{X}$ be the allocation matrix for any other valuation function, w^i, he may submit. Then

$$\begin{aligned}
u^i(x^{i*}, y^{i*}) - u^i(\hat{x}^i, \hat{y}^i) &= v^i(x^{i*}) - p^i(v^i, w^{)i(}) - v^i(\hat{x}^i) + p^i(w^i, w^{)i(}) \\
&= \Big(v^i(x^{i*}) + \sum_{\ell \in I \setminus \{i\}} w^\ell(x^{\ell *})\Big) - \Big(v^i(\hat{x}^i) \\
&\qquad + \sum_{\ell \in I \setminus \{i\}} w^\ell(\hat{x}^\ell)\Big) \\
&= \max_{x \in \chi} \Big\{\Big(v^i(x^i) + \sum_{\ell \in I \setminus \{i\}} w^\ell(x^\ell)\Big)\Big\} - \Big(v^i(\hat{x}^i) \\
&\qquad + \sum_{\ell \in I \setminus \{i\}} w^\ell(\hat{x}^\ell)\Big) \\
&\geqq 0
\end{aligned}$$

When dealing with the allocation of public goods, Green and Laffont [1977] have established the payment scheme given in (4) is also unique in solving the incentive problem we have posed. (See also Holmstrom [1979] for the most general result over restricted domains for the admissible valuation functions.) It may be verified that their proofs do not depend upon the "publicness" of the goods, and so the Groves' family of mechanisms remain unique for solving the private good auction problem. Thus we may state the following proposition:

Proposition 1: All demand revealing mechanisms are completely characterized by the Groves' family of mechanisms.

The one shortcoming of these mechanisms that should be noted at this point is that no decisive mechanism exists which is efficient, since aggregate net payments can not be assured to sum to zero. This has been shown by Hurwicz [1975] and Green, Kohlberg and Laffont [1976]. In particular, when sellers are included in the analysis this means that no mechanism exists such that aggregate net payments by all buyers will necessarily equal aggregate net receipts to all sellers.

3. PROPERTIES OF DEMAND REVEALING AUCTION MECHANISMS

The most familiar demand revealing mechanism is Vickrey's second price mechanism. It can be seen that the second price auction is a Groves' mechanism by specifying $h^i(\cdot)$ as

$$h^i(w^{\setminus i}) = \max_{X\varepsilon\chi} \sum_{\ell\varepsilon I\setminus\{i\}} w^\ell(x^\ell) \qquad (5)$$

This gives the amount that all bidders, excluding the i^{th}, would be willing to pay for the allocation which arises when bidder i is excluded from the auction. This is the maximum reported surplus others would realize if bidder i dropped out of the market. To see that the payment schedule which results from using the function given in (5) is the appropriate generalization of the Vickrey second price auction, consider the case where a single indivisible good is being auctioned. Without loss of generality, assume that all bidders are numbered in descending order of their bids. In this instance, the winning bidder (bidder 1) must pay

$$p^1(w) = h^1(w^{\setminus 1}) \qquad \text{since } w^\ell(0) = 0,\ \ell \neq 1$$

$$= \max_{x\varepsilon\chi} \sum_{\ell\varepsilon I\setminus\{i\}} w^\ell(x^\ell)$$

$$= w^2(1)$$

where bidder 2 would receive the good if bidder 1 was excluded from the auction. Any losing bidder i will pay

$$p^i(w) = -\sum_{\ell\varepsilon I\setminus\{i\}} w^\ell(x^{\ell *}) + h^i(w^{\setminus i})$$

$$= -w^1(1) + \max_{x\varepsilon\chi} \sum_{\ell\varepsilon I\setminus\{i\}} w^\ell(x^\ell)$$

$$= -w^1(1) + w^1(1)$$

$$= 0$$

since bidder 1 will continue to receive the good if any other bidder i is excluded from the auction.

Alternative mechanisms may be constructed by specifying other functional forms for $h(\cdot)$. For example, an n^{th} price auction is given by $h^i(w^{)i(}) = \sum_{\ell \varepsilon I \setminus \{i\}} w^\ell(\hat{x}^\ell)$, where $\hat{X} \varepsilon \chi$ is the allocation matrix which yields the $(n - 1)^{st}$ highest value of $\sum_{\ell \varepsilon I \setminus \{i\}} w^\ell(x^\ell)$. Returning to the single indivisible good auction it can be seen that, for the winning bidder,

$$p^1(w) = w^n(x^n),$$

where $w^n(x^n)$ is the bid of the n^{th} highest bidder; and for all other bidders

$$p^i(w) = -w^1(x^1) + w^n(x^n) < 0 \text{ for } 1 < i < n$$

$$= -w^1(x^{*1}) + w^{n-1}(x^{n-1}) < 0 \text{ for } i \geq n.$$

The illustration of the n^{th} price auction indicates a difficulty which arises with all dominant strategy mechanisms except for Vickrey's. For an n^{th} price auction all losing bidders must be compensated and, in general, for all other auctions except the second price one, all losing bidders must pay or receive some nonzero amount. The following proposition formalizes the result.

<u>Proposition 2</u> If a demand revealing auction mechanism possesses the property that $p^i(w) = 0$ whenever $x^{i*} = 0$, then it must be a second-price auction.

<u>Proof</u>: Let $x^{i*} = 0$ then, since $v^i(0) = 0$,

$$0 = p^i(w) = -\sum_{\ell \varepsilon I \setminus \{i\}} w^\ell(x^{\ell*}) + h^i(w^{)i(})$$

or that

$$h^i(w^{)i(}) = \sum_{\ell \varepsilon I \setminus \{i\}} w^\ell(x^{\ell*})$$

$$= \max_{x \varepsilon \chi} \sum_{\ell \varepsilon I \setminus \{i\}} w^\ell(x^\ell)$$

since the allocation, X^*, will be the same when bidder i is excluded from the auction.

In the event we wish to guarantee that the auction does not lose revenue we could alternatively require that $p^i(w) \geq 0 \ \forall \ i$. In this case, the following lemma is easily shown by a straightforward modification of the previous proof.

<u>Lemma 1</u>: If a demand revealing auction mechanism possesses the property that $p^i(w) \geq 0 \ \forall \ i$, then $h^i(w^i) \geq \max_{x \varepsilon \chi} \sum_{\ell \varepsilon I \setminus \{i\}} w^\ell(x^\ell)$.

Thus the second price auction provides a lower bound on the set of demand-revealing auctions which insure nonnegative revenue generations.

A further difficulty with this class of auctions has been studied by Green and Laffont [1978] for public goods. If, in designing an auction, we concern ourselves with the fact that some bidders may be required to pay an amount greater than their budget, then we must further restrict the set of admissible auction mechanisms. For the analogous public goods case to the auctioning of one unit of an indivisible private good they require that the bid of each agent be bounded by

$$w^i(1) \leq M^i, \qquad (6)$$

where M^i is the budget of agent i. With this bidding constraint they show that the set of admissible functions $h^i(\cdot)$ is bounded below by

$$h^i(w)^i() \leq \max_{x \in \chi} \sum_{\ell \in I \setminus \{i\}} w^\ell(x^\ell).$$

Combining this result with Lemma 1, it is clear that the only auction which restricts bankruptcy and insures a non-negative revenue generation is the second price auction. This, however, no longer insures that the item is awarded to the bidder who values it the most. As they show, each bidder has a dominant strategy to report $w^i(1) = \min[v^i(1), M^i]$ and as the following example indicates, the allocation specified by (2) need no longer be attainable.

Let there be two bidders with the valuation functions $v^1(1) = 10$ and $v^2(1) = 5$. Suppose that the budgets of the bidders are $M^1 = 4$ and $M^2 = 6$ and that they enter a second price auction for the object, with no bidding constraint. Then bidder 1 will be awarded the object and be required to pay 5 units of the private good which exceeds his budget. Alternatively, if the bidding constraint (6) is imposed, bidder 2 will be awarded the object and will be required to pay 4 units of the private good.

If there are multiple units of a commodity to be awarded, the second price auction loses its dominant strategy property. Consider an

example where there are two items to be awarded using a second price auction and there are two bidders. If the first bidder wins both items then he must pay $w^2(2)$ and if he wins only one item he must pay $w^2(2) - w^2(1)$. If we assume that both bidders have non-negative marginal utility for this commodity, i.e., $w^i(2) \geq w^i(1) \geq 0$, then for the first bidder to remain solvent we must be assured that $w^2(2) \leq M^1$ if he wins both items. Such will be the case if we constrain his bidding in an analogous manner to the single unit case above by requiring[4]

$$w^1(1) \leq w^1(2) \leq M^1.$$

In this instance, if bidder one wins two units the amount he will pay, $w^2(2)$, must be less than M^1 or else bidder two would have received at least one unit.

In order to see that there are no dominant strategies for bidder one under this constraint, suppose $v^1 = (v^1(0), v^1(1), v^1(2)) = (0, 6, 10)$ and let his budget be given by $M^1 = 7$. Further suppose for the following situations bidder two's bidding constraint is never binding so that he always will truthfully reveal his valuations. We may consider strategies for the first bidder of the form $w^1 = (w^1(0), w^1(1), w^1(2)) = (0, 2, 7)$. We consider the following three cases:

Case I: If $v^2 = (0, 7, 13 - \varepsilon)$ and $\varepsilon > 0$, then for ε arbitrarily small $w \geq 6$ and bidder one will receive one unit of the commodity and pay $6 - \varepsilon$.

Case II: If $v^2 = (0, 7, 13 + \varepsilon)$ and $\varepsilon > 0$, then for ε arbitrarily small $w \leq 6$ and bidder one will not receive any of the commodity and will pay nothing.

Case III: From the first two cases, we need only to concern ourselves with considering $w = 6$ as a dominant strategy. Let $v^2 = (0, 3, 5)$, then $w^1 = (0, 6, 7)$ will reward bidder one with one unit of the commodity and he must pay $v^2(2) - v^2(1) = 2$ units of the private good. His net payoff is 4 in this instance. Alternatively let $w^1 = (0, 3\ 1/2, 7)$. Here bidder one will receive both units of the commodity and will pay $v^2(2) = 5$. With this outcome his net payoff is $10 - 5 = 5$. Thus $w = 6$ is not a dominant strategy.

Thus the lack of a dominant strategy in these environment provides a caution in instances where auctions are being designed where the solvency of agents is potentially a problem. For the remainder of this paper, however, we shall overlook these considerations.

4. THE GENERAL NONEXISTENCE OF ONE-PRICE DEMAND REVEALING AUCTIONS

In an one-price auction each successful buyer pays the same price for each unit he is awarded. Unlike discriminative auctions in which successful buyers pay what they have bid, one-price auctions are analogous to competitive market mechanisms. Smith [1967], Belovicz

[1979], Miller and Plott [1980], and Grether, Isaac and Plott [1979] have examined one-price auctions within the context of experimental markets. In fact, Miller and Plott study a market in which buyers may purchase multiple units in which they all pay the highest rejected bid for each unit. For their parameters, the one-price auction is demand revealing and they find that after replication, bidders report their true valuation.

Indeed, Engelbrecht-Wiggans [1980] states "If several identical objects are being auctioned, and players may bid on how much they would pay for the first, second, etc., objects awarded to them, then an appropriate extension of the [Vickrey] model is to set the price of all objects equal to the highest unsuccessful rejected bid." Unfortunately, even though this institution is seemingly analogous to Vickrey's second-price auction the demand revealing properties of that mechanism do not extend. We will show that one-price auctions are not in general demand revealing in the multiple unit case. In addition, we will demonstrate the correct extension of the Vickrey mechanism in the case of multiple units.

Prior to presenting these results, one observation is in order. Namely, if successful bidders are to pay the same price there must be at least one excluded bidder (i.e., a bidder who does not receive any units). From (4), bidders must face a payment schedule which is unrelated to their own bids. Thus, all rejected bids which are made by buyers who also are awarded units of the commodity must be excluded from consideration. Even if we were assured that one excluded bidder were to exist (for example, in an auction in which there were more bidders than objects to be awarded), demand revealing one-price auctions still do not exist as the following example illustrates.

Suppose there are three indivisible, identical objects to be awarded and there are four bidders. Consider an auction in which all winning bidders must pay the highest rejected bid for each item they receive. The bidders' marginal valuation functions are given in the following table:

bidder \ x^i	1	2	3
1	100	70	50
2	75	40	20
3	60	30	20
4	30	20	10

TABLE 1

If each bidder were to bid his true valuation it is easy to see that bidder 1 would receive two of the objects and bidder 2 would receive one. They each would pay the highest bid of bidder 3, namely 60 for each item purchased. In this case bidder 1's net change in utility form the auction would be

$$100 + 70 - 2(60) = 50.$$

Alternatively suppose bidder 1 reported the valuation function

$$w^1(1) = 100,\ w^1(2) = 150,\ w^1(3) = 200,$$

so that his marginal valuations would be 100, 50, and 50, respectively. In this case one unit would be awarded to bidders 1, 2, and 3 and they each would pay 30 (the highest rejected bid of bidder 4). In this case bidder 1's net change in utility from the auction would be 100 - 30 = 70, and thus he is better off misreporting his true preferences.

The difficulty with one price auctions may be seen by examining the payment schedule in (4). If each buyer is to pay the same price, say q, for each unit purchased then the arbitrary function $h^i(w^{)i(})$ must be given by

$$h^i(w^{)i(}) = \sum_{\ell \in I \setminus \{i\}} w^\ell(x^{\ell *}(w)) + qx^{i*}. \qquad (7)$$

Since $h^i(\cdot)$ must be independent of w^i, the price q must be independent of the reported valuations of all successful bidders. The difficulty arises since X* in general depends on w^i. As a direct application of Proposition 2, we know that for (7) to hold for all possible values of $x^{i*} \geq 0$ then, when $x^{i*} = 0$, the function is specified by Vickrey's mechanism, where

$$h^i(w^{)i(}) = \sum_{\ell \in I \setminus \{i\}} w^\ell(x^{\ell *}(w)) = \max_{x \in \chi} \sum_{\ell \in I \setminus \{i\}} w^\ell(x^\ell(w)), \qquad (8)$$

Unfortunately, when extended to the case of multiple units, Vickrey's mechanism does not yield a one-price auction. This has been previously demonstrated in Forsythe, Isaac, and Walker [1979].

If there are N units to be sold, then using (8) it can be seen that the Vickrey mechanism specifies that a bidder who is awarded J units must pay the J highest losing bids excluding his own. Using the valuations given in Table 1, bidder 1 would receive two units and must pay the sum of the two highest nonwinning bids excluding his own, i.e.,

$$p^1(w) = 60 + 40 = 100.$$

Bidder 2 would receive one unit and would pay the highest nonwinning bid excluding his own, i.e.,

$$p^2(w) = 60.$$

Although this mechanism is demand revealing it may be readily verified that it cannot be a one-price auction since bidder 1's payment is not twice as much as bidder two. If there are multiple units to be awarded there will be a single price auction only in the event where there is a sufficient number of identical bids at the highest rejected bid (as in the design of Miller and Plott).

Alternatively, in the case where x^{i*} can take on only two values, say 0 and N, a single-price auction specifies that a bidder will pay

$$p^i(w) = \begin{cases} 0 & \text{if } x^{i*} = 0 \\ w^k(N) & \text{if } x^{i*} = N \end{cases}$$

where bidder k is the first excluded bidder in the auction. Here it is always true that there is a sufficient number of identical bids at the highest rejected bid since only one such bid is required.

A second example of a one-price auction which is found in the literature is due to Marschak.[5] In this mechanism, the seller of a single object writes a price on a card and places it face down on a table. There is one potential buyer who is then permitted to make a bid with the understanding that he will be awarded the object at the seller's price if and only if his bid is greater than the price recorded by the seller. It is easily seen that the buyer has a dominant strategy to truthfully reveal his valuation for the object since if he overstates his valuation he will be forced to buy at some unfavorable prices and if he understates it he will lose the opportunity to buy the object at some desirable prices.

If there are two or more potential buyers and the seller has only a single object for sale then each buyer no longer has a dominant strategy to truthfully reveal his valuation. This will occur when the buyer with the second highest valuation is also willing to pay more than the seller requires. This buyer has an incentive to overstate his valuation since if he wins the item he will only pay the seller's price. Thus the incentive properties of the mechanism will be restored in this instance only if the seller is willing to sell as many objects as there are potential buyers.

In order to extend this mechanism to include many potential buyers it must be assumed that the seller is willing to report a perfectly elastic supply curve over the relevant range. Only then will all buyers necessarily face a one-price auction. This is best seen by examining the payment schedule for this mechanism. Using (4), this is given by

$$p^i(w) = -\sum_{\ell \varepsilon B\setminus\{i\}} w^\ell(x^{\ell*}) + cx^* + \max_{x \varepsilon \chi}\left[\sum_{\ell \varepsilon B\setminus\{i\}} w^\ell(x^\ell) - cx\right], \quad \forall\, i \,\varepsilon\, B \tag{8}$$

where c is the seller's reported constant marginal cost. From footnote 1, recall that the seller is assumed to reported the negative of his total cost function. Examining the form of (8) it should be obvious that the seller's price auction is the same as the Vickrey second-price auction. With a perfectly elastic supply curve, the buyers who receive items when bidder i is included in the auction will coincide exactly with the allocation which results when bidder i is excluded. The only difference is that the seller will now sell $\hat{x} = \sum_{\ell \varepsilon B\setminus\{i\}} x^{\ell*} \leq x^* = \sum_{\ell \varepsilon B} x^{\ell*}$ units of his product. Thus the payment schedule in (8) becomes

$$\begin{aligned} p^i(w) &= -\sum_{\ell \varepsilon B\setminus\{i\}} w^\ell(x^{\ell*}) + cx^* + \sum_{\ell \varepsilon B\setminus\{i\}} w^\ell(x^{\ell*}) - c\hat{x} \\ &= c(x^* - \hat{x}) \\ &= cx^{i*} \end{aligned}$$

thus all buyers will pay the same price for each unit they receive.

It is the assumption of a perfectly elastic supply curve that renders the seller's price mechanism useless for the case of Treasury security auctions, for example. In order to maintain both one-price and demand revealing properties of this mechanism, the Treasury would have to be willing to set a price and fill the orders of all buyers who report a willingness to pay in excess of this price.

Unless an accurate prediction of demand schedules can be made, the Treasury would effectively be unable to limit the total supply of securities sold in a given auction.

Another difficulty with the seller's price auction stems from the fact that the seller is not included in the mechanism and that it will not be a dominant strategy for him to reveal his true reservation price. By overstating his true prices, he will increase his revenues in many cases. If the seller does indeed have a perfectly elastic supply curve, it would be possible to include him in the auction in order to have his true marginal cost reported as a dominant strategy.

By exactly the analogous argument used to derive Proposition 2, if a seller is to receive no payments and pay no charges when he makes no sales then he must also face a second-price auction. Thus,

$$p^i(w) = -\sum_{\ell \varepsilon B} w^\ell(x^{\ell*}) - \sum_{\ell \varepsilon S\setminus\{i\}} w^\ell(x^{\ell*}) + \max_{x \varepsilon \chi}\left[\sum_{\ell \varepsilon B} w^\ell(x^\ell) + \sum_{\ell \varepsilon S\setminus\{i\}} w^\ell(x^\ell)\right], \quad \forall\, i \,\varepsilon\, S \tag{9}$$

In the case described above with a single seller, (9) becomes

$$p^i(w) = -\sum_{\ell \varepsilon B} w^\ell(x^{\ell*})$$

since there is no supply to be sold when the seller is excluded from the auction and $w^\ell(0) = 0$, $\forall\, \ell \,\varepsilon\, B$. This implies that the seller must receive the total surplus of all buyers in the auction or, in other words, be rewarded as a perfectly discriminating monopolist. It should be noted that this result does not depend upon a perfectly elastic supply curve although it most certainly holds in that case also. This is precisely what drives the result of Loeb and Magat [1979] who find the monopolistic utility can be regulated in a decentralized fashion since it will truthfully reveal its demand function if it is promised the total consumers' surplus as a reward.

5. DEMAND REVEALING MECHANISMS AND MULTIPLE GOOD AUCTIONS

When there are multiple nonidentical goods to be auctioned, the design of an institution becomes trickier. For the case of m objects to be allocated among n bidders, there are $2^m - 1$ valuations which each bidder may possess. If either the objects are auctioned off sequentially or, as in the case of offshore oil lease sales, they are auctioned off similtaneously but only m bids may be submitted by each bidder, the resulting allocations may be far from optimal. Further, it is not at all clear what constitutes demand revelation in either of these two institutions since, for example, a bidder may be unable to even express that his valuation on an object depends upon whether or not he receives one or more other objects.

Any member of the family of Groves' mechanisms may be used to overcome these difficulties. If the number of objects, m, is large, however, the advantages of demand revealing mechanism may be outweighed by the large computational cost which arises from each bidder having to submit $2^m - 1$ bids. For a small number of objects the auction remains relatively simple as the following example illustrates for the second price auction.

Consider an auctioneer wishing to sell the last two remaining specimens of a rare stamp (designated for convenience the left (L) and right (R) stamps). These stamps could conceivably be sold as a pair, P, or as individual items, L and R. With the appropriate extension of the Vickrey mechanism, the "bundling" decision is endogenous to the auction.

For ease of exposition, let us assume that there are three bidders, and that each bidder values the pair more than his sum of valuations on the individual stamps. (Neither of these assumptions is necessary for the mechanism to work). Specifically, suppose that the true valuations of the three bidders are as follows:

Bidder	L	R	P
A	$ 5	$10	$16
B	$12	$ 6	$21
C	$ 2	$ 3	$17

For the optimal allocation decision (2) to hold, the auctioneer will "package" all possible combinations of left and right stamps, and award the stamps to the package which maximizes reported willingness to pay. With truthful revelation, the nine packages from this example would be

L = B,	R = A	$22
P = B		21
P = C		17
P = A		16
L = B,	R = C	15
L = C,	R = A	12
L = A,	R = B	11
L = C,	R = B	8
L = A,	R = C	8

The left stamp would be awarded to person B, the right stamp to person A. The payments for each would be figured as in Table 2.

Note that not only is this extension of the Vickrey auction not a one-price auction, it is not even a second price auction in that the sum of the payments does not equal the second highest package value either including or excluding persons A and B.

6. MULTIPLE GOOD AUCTIONS AND EXPERIMENTAL INQUIRY

Interest in the properties of auctions and bidding behavior has always been grounded in naturally occurring allocation problems. The use of laboratory experimental methods has proven to be especially fruitful in testing hypotheses about auction behavior.

For Person A

$$-\sum_{\ell\in I\setminus\{A\}} (w^{\ell}(x^*(w)) = \$ -12$$

$$\max_{x\in\chi} \sum_{\ell\in I\setminus\{A\}} w^{\ell}(x^{\ell}) = \$ \ 21$$

$$\$ \ 9 \quad \text{for A}$$

For Person B

$$-\sum_{\ell\in I\setminus\{B\}} (w^{\ell}(x^*(w)) = \$ -10$$

$$\max_{x\in\chi} \sum_{\ell\in I\setminus\{B\}} w^{\ell}(x^{\ell}) = \$ \ 17$$

$$\$ \ 7 \quad \text{for B}$$

For Person C

$$-\sum_{\ell\in I\setminus\{C\}} (w^{\ell}(x^*(w)) = \$ -22$$

$$\max_{x\in\chi} \sum_{\ell\in I\setminus\{C\}} w^{\ell}(x^{\ell}) = \$ \ 22$$

$$\$ \ 0 \quad \text{for C}$$

TABLE 2

Some versions of demand revealing mechanisms have already been the subject of laboratory experimental testing. In particular, the Vickrey mechanism for the single unit case (i.e., the second-price auction) has been examined in the laboratory. Smith [1978] and Coppinger, Smith and Titus [1980] report the results of experiments in which the second-price auction was compared to a first price auction and to the seller's price auction discussed in Section 4. It was found that many subjects "learned" their dominant strategy fairly rapidly, but that violations of single period dominant strategy behavior were common, especially in the "early" trials of an experimental session.

More often than not, however, the naturally occurring allocation problems concern multiple good auctions: either the seller is disposing of N > 1 units of a homogenous commodity (Treasury security auctions, for example) or of several different, nonhomogenous commodities (the auction of offshore oil leases). Not surprisingly, there has been a tremendous amount of interest in laboratory experiments involving multiple good, sealed-bid auctions (some examples are Smith [1967]; Belovicz [1979]; Smith, Williams, Bratton, and Vannoni [1979]; Grether, Isaac and Plott [1979]; Miller and Plott [1980]; Cox, Roberson, and Smith [1980]; Cox, Smith, and Walker [1980], and Palfrey [1980].) Much of the work has focused on multi-unit discriminative auctions or some form of multiple unit "competitive" auctions. To our knowledge, there have been no systematic tests of the family of demand revealing mechanisms whihc we have described (although in Cox, Smith, and Walker [1980], the restriction that each person may bid for at most one unit allows the one-price competitive auction to have truth as a dominant strategy). There are several potentially interesting areas of inquiry about the extension of Vickrey mechanism to multiple good auctions:

1) The demand revealing mechanisms we have described tend to be comparatively more complicated than when only a single unit of a good is at auction. Would subjects do as well at "learning" their dominant strategy as they did in the Smith [1978] and Coppinger, Smith, and Titus [1980] experiments?

2) The simple competitive one-price auction for multiple units is not, in general, incentive compatible. Yet, it may be that, for certain parameters, truthful relevation is a dominant or perhaps a Nash equilibrium strategy. Nontruthful behavior could be very complicated to discover or risky to attempt. Smith [1967], Grether, Isaac, and Plott [1979], and Miller and Plott [1980] have found that the "single-price" multi-unit auctions were relatively successful in providing demand revealing behavior. Smith, Williams, Bratton, and Vannoni [1979] found that a two-sided, uniform price sealed bid auction performed poorly when bidders could submit only one bid for all units; but when participants could submit complete inverse demand and supply schedules, the amount of demand revealing behavior increased. How would the performance of these uniform price processes compare with the theoretically demand revealing, but operationally more complicated, extended Vickrey mechanisms. The comparisons could include demand revealing behavior, stability, revenue generating properties, etc.

3) With the exception of recent work by Palfrey [1980], there have been few experimental examinations of multiple-commodity private good auctions. As discussed in Section 5 the extended Vickrey mechanism for multi-commodity auctions has very strong informational requirements. It would be interesting to test this mechanism against simultaneous and/or sequential auctions which have milder informational requirements but which also has poorer efficiency properties.

4) Given the interest in one-price auctions, it would be interesting to examine a seller's price auction as outlined in Section 4. In particular assume that a seller does not have a perfectly elastic supply curve but nonetheless is asked to report some price at which

he will be willing to fulfill the demand of those buyers who report a willingness to pay no less than that amount. Let buyers face the demand revealing mechanism described by (8) and let the seller receive the revenue from the auction. With this one-price mechanism the size of the inefficiency generated by the reported price of the seller could be examined and the revenue generating properties of this auction could be compared to previous work.

FOOTNOTES

1. For sellers, $u^i(x^i,y^i)$ may be thought of as a profit function with $v^i(x^i)$ as the negative of producer i's cost function.

2. If supply decisions are also endogenous then (2) becomes

$$X^*(v) \in \underset{X\in\chi}{\operatorname{argmax}}\left(\sum_{i\in B} v^i(x^i) + \sum_{i\in S} v^i(x^i)\right).$$

where, recall $v^i(x^i) \leq 0$, $\forall\ i \in S$ since it is the negative of producer i's cost function. As in the usual competitive model, X^* is the equilibrium quantity.

3. As noted in the introduction, Loeb [1977] has previously recognized that this family of mechanisms, originally described for public goods, also satisfies the private good problem. However, he focused his attention on one particular member of this family.

4. For a formal derivation of constraining the admissible strategy space in this manner, see Green and Laffont [1978].

5. As reported in Smith [1979], Jacob Marschak described this procedure in the early 1950s.

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