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A FORMAL MODEL OF GOVERNMENT SPONSORED RESEARCH (WITH APPLICATIONS TO SOLAR POWER SYSTEMS)

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SOCIAL SCIENCE WORKING PAPER 339

August 1980

ABSTRACT

This paper analyzes the behavior of a single firm engaged in R and D for a ''sponsor.'' We are interested in two particular aspects of the interaction between the two agents: (1) the revelation to the sponsor of new information generated by the firm's research, and (2) the firm's choice of research strategy. We show that contractual forms which provide good incentives in a static environment may introduce incentive problems in a dynamic setting. More specifically, we show that a firm engaged in a sequence of R and D contracts is more likely to do research (1) the lower are the costs of R and D, (2) the better is the state of sponsor knowledge, and (3) the longer is the sequence of contracts (given an appropriately high discount factor). We also show that the firm reveals a larger share of its results (1) the better is the state of sponsor knowledge, (2) the better is the state of private knowledge possessed by the firm, and (3) the shorter is the sequence of contracts. Finally, somewhat surprisingly, we find that the amount of information a firm reveals is independent of the costs of R and D.

A DYNAMIC MODEL OF RESEARCH CONTRACTING

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1. INTRODUCTION

A considerable body of economic theory has been developed over the last fifteen years to model (i) R and D decisionmaking by private firms, (ii) general equilibrium aspects of industrial innovation, and (iii) principal-agent relations. Of particular interest to those concerned with public policy are papers by Scherer [1969], Bernholdt [1967] and Cummins [1973], who have analyzed the behavior of defense contractors as reflected in cost over-runs or under-runs. These authors consider the rational choice of risk-assumption provisions in the uncertain environment typically characterizing a bilateral procurement contract. Related papers are by Williamson [1967], who has attempted to ascertain what factors are responsible for performance results obtained in military contracts, and Agapos and Dunlop [1970], who have developed a price-determination model for bilateral procurement. In spite of this work, our understanding of the behavior of firms engaged in sponsored research is very limited. In this paper we are particularly interested in two aspects of the interaction between private firms and a research sponsor (who may or may not be a government bureaucrat): (1) the revelation to the sponsor of technical change embodied in the firm's physical and human capital and (2) the firm's choice of research strategy.

In the next section of this paper we introduce a discrete-time dynamic programming model which captures the essence, we believe, of the problem from the firm's perspective. It turns out that the form of the payoff function is especially important in our model. Section 3 analyzes the model for the most ''natural'' form of the payoff function. We show there that a firm engaged in a sequence of R and D contracts is more likely to do research (1) the lower are the costs of R and D, (2) the better is the state of public (or sponsor) knowledge, and (3) the longer is the sequence of contracts (given an appropriately high discount factor). We also show that the firm reveals a larger share of its results (1) the better is the state of public (or sponsor) knowledge, (2) the better is the state of (private) knowledge possessed by the firm, and (3) the shorter is the sequence of contracts. We also show, somewhat surprisingly, that the amount of information a firm reveals is independent of the costs of R and D.

Section 4 discusses alternate forms of the payoff function as well as a number of extensions of the basic model. A final section offers some concluding remarks.

We want to emphasize that the purpose of this exercise is not to characterize ''optimal'' incentive contracts. Indeed, we never even go so far as to specify an objective function for the research sponsor. Rather, we are interested in examining the performance of contract forms which currently exist, or are likely to emerge in the future (especially in response to increased levels of government

sponsored R and D). One reason we take this approach is that the conditions which characterize optimal incentive contracts may often not be operational. For example, if the sponsor is only interested in getting the firm to reveal the results of its research, then introducing nonconvexity into the payoff function will suffice. But this isn't an operational solution since the sponsor is unlikely to know how much nonconvexity is ''enough.'' Furthermore, other informational and/or institutional constraints often limit the set of feasible instruments.

2. THE MODEL: PRELIMINARIES

The focus of this paper is on the interaction between a research sponsor and a particular firm. In this section a stylized model is developed which makes several strong assumptions about the nature of the research process and the reward to the firm. While we only analyze one version of the model in detail, the purpose of the exercise is in part to develop a methodology which can be modified in a variety of ways to address different problems.

Suppose that a research sponsor is interested in reducing the cost of some technology. It contracts with a firm to undertake research designed to accomplish this goal. We make two crucial assumptions regarding the nature of contract incentives. First, we assume that the reward earned by the firm in each period is a function of the current state of sponsor knowledge -- a level of unit costs R -- and the new state of sponsor knowledge created by the firm's reported research — a level of unit costs r. We of course require that r not exceed R since knowledge of the technology required to produce at unit cost R is presumed to be known to the sponsor. We denote the payoff function by $W(r,R)$.[1]

If the research sponsor could costlessly monitor the firm, then the firm would always report fully its research output; that is, in each period it would reveal to the research sponsor the true minimum cost yielded by its research. But monitoring is not costless, and the firm may wish to sign a new contract with the research sponsor when the current contract expires. Hence it has an incentive to withhold information temporarily. We assume that the firm never lies (in other words, it must document each contract's final report) but it need not reveal everything it knows.[2] If σ is the true minimum unit cost which the firm ''holds in inventory'' then its reported unit costs cannot be lower than σ; i.e. we require that $\sigma \leq r \leq R$. In the analysis which follows, we are particularly interested in the relationship between the form of the payoff function $W(r,R)$ and the firm's incentives to reveal its research output.

To finish specifying the firm's problem we need only to describe the research process. In this paper we use a very simple search-theoretic approach. Assume that the length of a contract is fixed at one period. In each period the firm can either engage in research or not. The cost of research is fixed at c, $c \geq 0$. The output of research is uncertain, though. We assume that by paying c the firm gets a random draw from a distribution of potential unit

costs, F. For analytical convenience, we assume that F is differentiable with $f(x) = F'(x)$ strictly positive on some interval $[a,b]$.[3]

The firm's objective is to maximize its discounted expected profit from engaging in a series of contracts with the sponsor. Thus the problem can be formalized using simple discrete-time dynamic programming techniques.

Let $V_t(\sigma,R)$ be the discounted expected profit when there are t periods remaining in the planning horizon given that the state of sponsor knowledge is a level of unit costs R and the state of the firm's private knowledge is a level of unit costs σ. In this model t can be thought of as the maximum remaining number of contracts in which the firm expects to participate. By the Principle of Optimality,

$$V_t(\sigma,R) = \max \begin{cases} -c+E \max\limits_{R\geq r\geq \min\{\sigma,X\}} [W(r,R)+\beta V_{t-1}(\min\{\sigma,X\},r)] \\ \max\limits_{R\geq r\geq\sigma} [W(r,R)+\beta V_{t-1}(\sigma,r)] \end{cases} \qquad (1)$$

The first term on the right-hand-side of (1) represents discounted expected profit when the firm conducts research (X is the random variable associated with research output). The second term on the right-hand-side of (1) represents discounted expected profit when no research is conducted (even here some ''new'' information might still be revealed by the firm if R is strictly greater than σ). In either case, the relevant discount rate is β, $0 < \beta < 1$.[4]

Equation (1) holds for all $t \geq 1$. For $t = 0$, define

$V_0(\sigma,R) \equiv 0$ for all σ and R. If $R > \sigma$ there might be some profit to the firm from selling its residual information stock to other private parties, but we assume penalties for such action are so severe as to eliminate the possibility.[5]

This model is very simple, yet it is surprisingly rich. The only technical assumptions needed in addition to those already introduced concern the payoff function $W(r,R)$. It is quite natural to assume that the payoff to the firm increases as the difference between the level of unit costs previously known by the sponsor and the level of unit costs reported by the firm increases. To this end we assume $W_r(r,R) < 0$ and $W_R(r,R) > 0$ for all $r \leq R$.[6] We further assume that $W_{rr}(r,R) < 0$ $W_{RR}(r,R) < 0$ for all $r \leq R$. The latter simply reflect diminishing returns.

Many of the results derived below depend on the sign of $W_{rR}(r,R)$. We initially analyze the model under the assumption that $W_{rR}(r,R) > 0$ for all $r \leq R$. This is perhaps the most realistic case. It implies that a decrease in the level of unit costs previously known to the sponsor increases the <u>marginal</u> return to further reductions in it. As Figure 1 illustrates, this is equivalent to saying that as the level of sponsor-known unit costs decreases, the payoff to reporting low levels of unit costs remains relatively high. Thus, in this case, the research sponsor is interested in achieving a low level of unit costs, not just in being able to attain a ''breakthrough.'' An example of such a payoff function is $W(r,R) = U(R-r)$ where $U' > 0$ and $U'' < 0$.

[FIGURE 1 HERE]

Section 3 of this paper will analyze the model under the assumption that $W_{rR} > 0$. Section 4 will consider how the results are effected by assuming either $W_{rR} = 0$ or $W_{rR} < 0$. It will also discuss a number of other extensions of the basic model.

To ease notation throughout the remainder of the paper, let

$$g_t(r,\sigma,R) = W(r,R) + \beta V_{t-1}(\sigma,r) \quad (2)$$

and

$$G_t(\sigma,R) = \max_{R \geq r \geq \sigma} g_t(r,\sigma,R). \quad (3)$$

Then (1) becomes

$$V_t(\sigma,R) = \max\{-c+EG_t(\min\{\sigma,X\},R), G_t(\sigma,R)\}. \quad (4)$$

3. THE MODEL: $W_{rR} > 0$

To characterize the optimal policy implicit in the solution to the functional equation (1), we start with the single-period problem and then use induction to generalize the results to $t \geq 1$.

Given $W_r < 0$ and $V_0 \equiv 0$, it will always pay the firm to report everything it knows at the end of the project horizon. Hence $G_1(\sigma,R) = g_1(\sigma,\sigma,R) = W(\sigma,R)$, and

$$\begin{aligned} EG_1(\min\{\sigma,X\},R) &= Eg_1(\min\{\sigma,X\}, \min\{\sigma,X\},R) \\ &= EW(\min\{\sigma,X\},R) \\ &= \int_a^\sigma W(x,R)f(x)dx + \int_\sigma^b W(\sigma,R)f(x)dx. \end{aligned} \quad (5)$$

Given (5), the first question we ask of the firm's optimal policy is whether or not it engages in research. Research is preferred to no research if $-c + EG_1(\min\{\sigma,X\},R) > G_1(\sigma,R)$. Thus we seek to characterize, in some useful way, the set of all (σ,R) such that $-c + EG_1(\min\{\sigma,X\},R) > G_1(\sigma,R)$. Using (5) and the fact that $G_1(\sigma,R) = W(\sigma,R)$ we have

$$-c + EG_1(\min\{\sigma,X\},R) > G_1(\sigma,R)$$

if and only if

$$-c + \int_a^\sigma W(x,R)f(x)dx + \int_\sigma^b W(\sigma,R)f(x)dx > W(\sigma,R)$$

which, in turn, holds if and only if

$$c < \int_a^\sigma [W(x,R) - W(\sigma,R)]f(x)dx \equiv H^1(\sigma,R). \tag{6}$$

Using $H^1(\sigma,R)$ as defined in (6) we define a ''reservation level of privately known unit costs,'' $\sigma_1^*(R)$, by $H^1(\sigma_1^*,R) = c$. The following property of H^1 guarantees σ_1^* is unique:

$$H^1_\sigma(\sigma,R) = -W_r(\sigma,R)F(\sigma) > 0. \tag{7}$$

Furthermore,

$$\{(\sigma,R)|-c + EG_1(\min\{\sigma,x\},R) > G_1(\sigma,R)\} = \{(\sigma,R)|\sigma > \sigma_1^*(c,R)\},$$

so that σ_1^* is indeed a ''reservation'' quantity. Figure 2 illustrates σ_1^*.[7]

[FIGURE 2 HERE]

It is obvious from inspection of Figure 2 that $d\sigma_1^*/dc > 0$. The formal proof of this result is given in Appendix 1. It is also shown there that $d\sigma_1^*/dR > 0$. Hence we have the following proposition.[8]

<u>Proposition 1</u>: (a) $d\sigma_1^*/dc > 0$,

(b) $W_{rR} > 0$ implies $d\sigma_1^*/dR > 0$.

The lower is σ_1^*, the more likely is the firm to engage in research. Thus, Proposition 1 shows that either lowering the cost of research or lowering the level of sponsor-known unit costs is likely to encourage research. The latter result depends crucially on the assumption that $W_{rR} \geq 0$. It is easy to show, for example, that $W_{rR} < 0$ implies $d\sigma_1^*/dR < 0$.

Establishing results for $t \geq 2$ requires knowledge of the properties of $V_1(\sigma,R)$. Using σ_1^* we can write $V_1(\sigma,R)$ in a form which makes analysis of it much easier. That is,

$$V_1(\sigma,R) = \begin{cases} -c + \int_a^\sigma W(x,R)f(x)dx + \int_\sigma^b W(\sigma,R)f(x)dx & \text{if } \sigma > \sigma_1^* \\ (W\sigma,R) & \text{if } \sigma \leq \sigma_1^* \end{cases} \tag{8}$$

[9]

The following lemmas follow directly upon differentiating expression (8). The proofs are therefore omitted.

<u>Lemma 1</u>: (a) $dV_1(\sigma,R)/dR > 0$,

(b) $d^2V_1(\sigma,R)/dR^2 < 0$.

<u>Lemma 2</u>: $dV_1(\sigma,R)/d\sigma < 0$.

Lemma 3: (a) $W_{rR} > 0$ implies $d^2V_1(\sigma,R)/dRd\sigma > 0$,

(b) $d^2V_1(\sigma,R)/dRdc = 0$.

We are now ready to consider the two-period problem. This is inherently more interesting than the one-period problem since the potential to withhold information now exists. Recall from (1) that

$$V_2(\sigma,R) = \max \begin{cases} -c + E \max_{R \geq r \geq \min\{\sigma,X\}} [W(r,R) + \beta V_1(\min\{\sigma,X\},r)] \\ \max_{R \geq r \geq \sigma} [W(r,R) + \beta V_1(\sigma,r)] \end{cases} \tag{9}$$

Also, from (2) and (3), we have

$$g_2(r,\sigma,R) = W(r,R) + \beta V_1(\sigma,r)$$

and

$$G_2(\sigma,R) = \max_{R \geq r \geq \sigma} g_2(r,\sigma,R)$$

where $\arg\max_{R \geq r \geq \sigma} g_2(r,\sigma,R) = r_2^*(\sigma,R)$ is the optimal amount of private information to reveal. Equation (9) can thus be rewritten as in (4):

$$V_2(\sigma,R) = \max\{-c + EG_2(\min\{\sigma,X\},R),\ G_2(\sigma,R)\}. \tag{10}$$

As before, we wish to characterize the set of all (σ,R) such that $-c + EG_2(\min\{\sigma,X\},R) > G_2(\sigma,R)$. To begin, consider the properties of $r_2^*(\sigma,R)$.

Proposition 2: $r_2^*(\sigma,R)$ is a well-defined maximum.[10] Furthermore,

(a) $W_{rR} > 0$ implies $dr_2^*/dR > 0$,

(b) $W_{rR} > 0$ implies $dr_2^*/d\sigma > 0$,

(c) $dr_2^*/dc = 0$.

Proposition 2 suggests that the firm will tend to report a lower unit cost the lower is the level of sponsor-known unit costs or the lower is the level of unit costs privately known by the firm. The remarkable result is that $r_2^*(\sigma,R)$ is independent of c -- the optimal amount of private information to reveal is independent of the cost of research. In other words, how much of its knowledge a firm reveals in any given period is independent of how much it will cost to acquire more information later.

Consider next $EG_2(\min\{\sigma,X\},R)$. By definition $G_2(\sigma,R) = g_2(r_2^*(\sigma,R),\sigma,R)$. Hence

$$EG_2(\min\{\sigma,x\},R) = \int_a^{\sigma} g_2(r_2^*(x,R),x,R)f(x)dx + \int_{\sigma}^{b} g_2(r_2^*(\sigma,R),\sigma,R)f(x)dx.$$

Thus

$$-c + EG_2(\min\{\sigma,X\},R) > G_2(\sigma,R)$$

if and only if

$$-c + \int_a^{\sigma} g_2(r_2^*(x,R),x,R)f(x)dx + \int_{\sigma}^{b} g_2(r_2^*(\sigma,R),\sigma,R)f(x)dx > g_2(r_2^*(\sigma,R),\sigma,R)$$

which, in turn, holds if and only if

$$c < \int_a^{\sigma} [g_2(r_2^*(x,R),x,R) - g_2(r_2^*(\sigma,R),\sigma,R)]f(x)dx \equiv H^2(\sigma,R) \tag{11}$$

Using $H^2(\sigma,R)$ as defined in (11) we define a reservation level of privately known unit costs for the two-period problem, $\sigma_2^*(R)$, by

$H^2(\sigma_2^*,R) = c$. Consider the properties of $H^2(\sigma,R)$.

$$H_\sigma^2(\sigma,R) = -\frac{dg_2(r_2^*(\sigma,R),\sigma,R)}{d\sigma} F(\sigma)$$

$$= -\left[\frac{\partial g_2(r_2^*,\sigma,R)}{\partial r}\frac{dr_2^*}{d\sigma} + \frac{\partial g_2(r_2^*,\sigma,R)}{\partial \sigma}\right] F(\sigma).$$

But $\partial g_2(r_2^*,\sigma,R)/\partial r = 0$ by the first-order-condition defining r_2^*. Furthermore, $\partial g_2(r_2^*,\sigma,R)/\partial\sigma = \beta\partial V_2(\sigma,r)/\partial\sigma < 0$. Hence

$$H_\sigma^2(\sigma,R) = -\beta \frac{\partial V_1(\sigma,r_2^*)}{\partial\sigma} F(\sigma) > 0.$$

Thus

$$\{(\sigma,R)|-c + EG_2(\min\{\sigma,X\},r) > G_2(\sigma,R)\} = \{(\sigma,R)|\sigma > \sigma_2^*(c,R)\}.$$

We also have the following analogue to Proposition 1.

<u>Proposition 3</u>: (a) $d\sigma_2^*/dc > 0$,

(b) $W_{rR} > 0$ implies $d\sigma_2^*/dR > 0$.

Proposition 3 shows that σ_2^* behaves in a similar fashion as σ_1^* to changes in c and R. The next result shows that as the project horizon increases, the firm may or may not be more likely to do research depending on the size of the discount rate.

<u>Proposition 4</u>: $W_{rR} > 0$ implies that there exists $\hat{\beta}_2\varepsilon(0,1)$ such that $\sigma_2^*(R) \leq \sigma_1^*(R)$ if $\beta \geq \hat{\beta}_2$.

This is an important result because it highlights the interaction between the firm's incentives to do research and its ability to withhold information. Normally we would expect that an increase in the project horizon would increase the incentives to do research. But in this model an increase in the project horizon causes the firm to reveal less information (at least when $W_{rR} > 0$).[11] Hence the benefits of doing research (which can be received only when the information is revealed) are postponed into the future while the costs of research are paid in the present. If the firm discounts the future too much, then an increase in the project horizon will reduce the incentives to do research in the present.

Finally, we need to derive the properties of $V_2(\sigma,R)$. Not surprisingly they turn out to be almost identical to those of $V_1(\sigma,R)$ — almost, but not quite. The only difference is that $d^2V_2(\sigma,R)/dR^2$ cannot be signed under the assumptions made so far. Unfortunately, this turns out to be crucial to the induction argument. However, for a wide class of payoff functions, $d^2V_2(\sigma,R)/dR^2$ will be negative so an <u>assumption</u> that V_2 is concave will not be overly strong.

<u>Lemma 4</u>: $dV_2(\sigma,R)/dR > 0$,

<u>Lemma 5</u>: $dV_2(\sigma,R)/d\sigma > 0$.

<u>Lemma 6</u>: (a) $W_{Rr} > 0$ implies $d^2V_2(\sigma,R)/dRd\sigma > 0$,

(b) $d^2V_2(\sigma,R)/dRdc = 0$.

As pointed out above, Lemma 4 is not as strong as we would like. It is necessary for the induction argument which generalizes the above results to all $t \geq 1$ that $d^2V_t(\sigma,R)/dR^2 < 0$. Since for the most general class of payoff functions meeting our assumptions $d^2V_t(\sigma,R)/dR^2$ is of ambiguous sign, we will simply assume that it is negative and state our results under that condition. However, it should be noted that when $W(r,R) = U(R - r)$ where $U' > 0$ and $U'' < 0$, our prototypic payoff function for $W_{rR} > 0$, it can be shown (see Appendix II) that $d^2V_2(\sigma,R)/dR^2$ _is_ negative. Thus, for the class of payoff functions given by $\{U(R - r)|U' > 0 \text{ and } U'' < 0\}$ all the results of this section generalize to $t \geq 1$. We summarize them in the following Proposition:

Proposition 5: Suppose $W_{rR}(\sigma,R) > 0$ for all σ and R. If $d^2V_t(\sigma,R)/dR^2 < 0$ for all $t \geq 1$, then

(a) There exists a unique value of σ, σ_t^*, such that

$$\{(\sigma,R)|-c + EG_t(\min\{\sigma,X\},R) > G_t(\sigma,R)\} = \{(\sigma,R)|\sigma > \sigma_t^*(c,R)\}.$$

Furthermore,

(i) $d\sigma_t^*/dc > 0$,

(ii) $d\sigma_t^*/dR > 0$,

(iii) there exists $\hat{\beta}_t \varepsilon (0,1)$ such that

$\sigma_{t+1}^*(R) \leq \sigma_t^*(R)$ for $\beta \geq \hat{\beta}_t$.

(b) Define $\underset{R \geq r \geq \sigma}{\arg\max}\, g_t(r,\sigma,R) \equiv r_t^*(\sigma,R)$. Then r_t^* is a well-defined maximum. Furthermore,

(i) $dr_t^*/dR > 0$,

(ii) $dr_t^*/d\sigma > 0$,

(iii) $dr_t^*/dc = 0$,

(iv) $r_{t+1}^*(\sigma,R) \geq r_t^*(\sigma,R)$.

The new result in Proposition 5, that $r_{t+1}^*(\sigma,R) \geq r_t^*(\sigma,R)$ for all $t \geq 1$, is very important. It shows that when $W_{rR} > 0$, the longer the sequence of projects the firm envisions as possible, the less incentive it has to reveal the results of its research. However, this effect is offset to some extent by the fact that when $\beta \geq \hat{\beta}_t$, $\sigma_{t+1}^*(R) \leq \sigma_t^*(R)$ so that the longer the sequence of projects the firm envisions as possible, the more incentive it has to do research in the first place.

It is standard at this point to show that as t goes to infinity, the sequence of finite horizon value functions $\{V_t\}$ converge to a function V which is the unique solution to the infinite horizon analogue to equation (1). Such arguments are straightforward for this kind of problem and won't be detailed in this paper. The properties of V are analogous to members of $\{V_t\}$. Furthermore, the sequences $\{\sigma_t^*\}$ and $\{r_t^*\}$ converge to functions σ^* and r^* which have properties analogous to members of $\{\sigma_t^*\}$ and $\{r_t^*\}$ respectively.

4. VARIATIONS AND EXTENSIONS

a. $W_{rR} = 0$: There are a number of variations of our basic model which are of interest both from a theoretical point of view and from a practical point of view. The most obvious concern alternate forms of the payoff function. Consider first the case in which

$W_{rR}(r,R) = 0$ for all $r \leq R$. There are two problems with this form of payoff function, one is technical and the other has to do with the realism of the model.

Consider first the technical problem. If $W_{rR}(r,R) = 0$ for all $r \leq R$, then W can be decomposed into $W(r,R) = K + m(r) + n(R)$ where K is a constant term and m and n are functions with no constant terms. In general we would like to assume that if r = R then the firm earns no ''bonus'' over and above the fixed reward K. This, however, requires that $m(x) = -n(x)$ for all $x \geq 0$, in which case

$$W(r,R) = K - n(r) + n(R)$$

Hence $W_{rr} = -n''$ and $W_{RR} = n''$. But we require $W_{rr} < 0$ and $W_{RR} < 0$. With $W(r,R) = K - n(r) + n(R)$ these cannot both be satisfied.

In spite of the technical problem with the assumption that $W_{rR} = 0$, some might be reluctant to abandon this case because it includes payoff functions which are linear in r and R.[12] However, it is a common observation that many firms, especially those engaged in government sponsored R and D, tend to be very risk averse. To see the implications of risk aversion, let a monetary payoff function be denoted by $\pi(r,R)$ and let a utility of money function for the firm be denoted by $U(\pi)$. Assume $U' > 0$ and $U'' < 0$. Then $W(r,R) = U(\pi(r,R))$, so that

$$W_{rr} = U'\pi_{rr} + U''\pi_r^2,$$

$$W_{rR} = U'\pi_{rR} + U''\pi_r\pi_R,$$

and

$$W_{RR} = U'\pi_{RR} + U''\pi_R^2.$$

Now $\pi_{rr} \leq 0$ and $\pi_{RR} \leq 0$ imply $W_{rr} < 0$ and $W_{RR} < 0$, respectively. But even if $\pi_{rR} = 0$, $W_{rR} > 0$. In fact, it might even be that $\pi_{rR} < 0$ and still $W_{rR} < 0$ if the firm is sufficiently risk averse.

b. $W_{rR} < 0$: The second argument presented against the assumption that $W_{rR} = 0$ also applies to some extent to this case. However, in the above example, if $\pi_{rR} < 0$ then it might be the case that $W_{rR} < 0$ as well. This would imply that a decrease in the level of sponsor-known unit costs decreases the marginal return to further reductions in it. As Figure 3 illustrates, this is equivalent to saying that as the level of sponsor-known unit costs decreases, the payoff to reporting low levels of unit costs becomes relatively low -- the research sponsor rewards initial ''breakthroughs'' from high unit costs disproportionately well.

[FIGURE 3 HERE]

It does this in a strange way though. A more ''natural'' way to accomplish the same goal would be to let $W_{rr} < 0$ and $W_{RR} > 0$.[13] Nevertheless, one can easily derive results analogous to those presented in Section 3 of this paper for the case of $W_{rR}(r,R) < 0$. We will simply state the analogue to Proposition 5 as the techniques of proof are the same.

<u>Proposition 6</u>: Suppose $W_{rR}(r,R) < 0$ for all $r \leq R$. If $d^2V_{t(\sigma,R)}/dR^2 < 0$ then for all $t \geq 1$:

(a) There exists a unique value of σ, σ_t^*, such that

$\{(\sigma,R)|-c + EG_t(\min\{\sigma,R\},R) > G_t(\sigma,R)\} = \{(\sigma,R)|\sigma > \sigma_t^*(R)\}$.

Furthermore,

(i) $d\sigma_t^*/dc > 0$,

(ii) $d\sigma_t^*/dR < 0$,

(iii) $\sigma_{t+1}^*(R) \geq \sigma_t^*(R)$.

(b) Define $\underset{R > r > \sigma}{\operatorname{argmax}} \; g_t(r,\sigma,R) \equiv r_t^*(\sigma,R)$. Then r_t^* is a well-defined maximum. Furthermore

(i) $dr_t^*/dR < 0$,

(ii) $dr_t^*/d\sigma < 0$,

(iii) $dr_t^*/dc = 0$,

(iv) $r_{t+1}^*(\sigma,R) \gtreqless r_t^*(\sigma,R)$.

On the surface, $W_{rR} < 0$ seems to yield desirable behavior on the part of the firm. The higher are sponsor-known unit costs, the more likely the firm is to engage in research and the more likely it is to report the results of that research (for any given σ). However, as σ falls, $r_t^*(\sigma,R)$ rises -- a decrease in privately known unit costs <u>increases</u> the optimal level of unit costs to report. Even more troublesome effects concern increases in the planning horizon. First, $\sigma_{t+1}^* \geq \sigma_t^*$; the longer the sequence of projects the firm envisions as possible, the less likely it is to do research. Second,

it is quite possible that $r_{t+1}^* < r_t^*$; it might be that an increase in the planning horizon induces the firm to reveal more of its privately known information. This possibility suggests that more is going on here than meets the eye.

Suppose $r_{t+1}^*(\sigma,R) < r_t^*(\sigma,R)$ for some t. Furthermore, suppose $\sigma < r_{t+1}^*(\sigma,R)$ and $\sigma < \sigma_t^*$. Now $\sigma_t^* \leq \sigma_{t+1}^*$ so that $\sigma < \sigma_{t+1}^*$ as well. Under these circumstances, when t + 1 periods remain in the planning horizon then the firm reveals $r_{t+1}^*(\sigma,R)$ and does no research. In the next period the state of the firm's private knowledge is still σ but the state of sponsor knowledge is now $r_{t+1}^*(\sigma,R)$. But $r_{t+1}^*(\sigma,R) < r_t^*(\sigma,R)$ so when t periods remain in the planning horizon, the firm does nothing. It neither reveals any new information (although it could) nor does research. But it is difficult to see how this can be a profit maximizing strategy. In any given period the firm should either reveal some new information or do research unless $r_t^*(\sigma,R) = \sigma$ and $\sigma \leq \sigma_1^*$; i.e. unless it never intends to do more research and has already revealed all of its knowledge. The problem here is that the withholding of information might create incentives not to do research, just as when $W_{rR} > 0$. But when $W_{rR} < 0$, $r_{t+1}^*(\sigma,R) < r_t^*(\sigma,R)$ is possible. It is our conjecture that when $W_{rR} < 0$, additional restrictions (such as those imposed when $W_{rR} = 0$) will imply that $\sigma \leq \sigma_{t+1}^*(R)$ and $r_{t+1}^*(\sigma,R) \leq r_t^*(\sigma,R)$ are incompatible, but we have not yet been able to get formal results along these lines.

c. Targets: The model developed in this paper has much in common with the recent planning literature.[14] One of the major differences is the lack of ''targets'' in the payoff function. An example of the latter is a contract in which the firm specifies in advance what its output will be in a future period. It receives no reward if the pre-set target is not met. Similarly, it receives no ''bonus'' if a level of performance higher than the target is reported.[15] The methodology used in sections 2 and 3 of this paper can easily be extended to deal with such a case. The relevant functional equation is:

for $\sigma > S$,

$$V_t(\sigma,R,S) = \max \begin{cases} -c+[W(S,R)+\beta E \max\limits_{S\geq r\geq 0} V_{t-1}(X,S,r)]F(S) \\ 0 \end{cases}$$

and for $\sigma \leq S$,

$$V_t(\sigma,R.S) = \max \begin{cases} W(S,R)-c+\beta E \max\limits_{S\geq r\geq 0} V_{t-1}(\min\{\sigma,x\},S,r) \\ W(S,R)+\beta \max\limits_{S\geq r\geq 0} V_{t-1}(\sigma,S,r) \end{cases} \quad (12)$$

where the expectation in the first case is over $x \leq S$. In (12) σ is the firm's privately known level of unit costs, R is the current state of sponsor-known unit costs and S is the target level of unit costs the firm elected (when t+1 periods remained in the planning horizon) to reveal when t periods remain in the planning horizon ($\sigma \leq S \leq R$). In this case r becomes the ''target'' for the next period (as opposed to the amount revealed in the current period). It is easy to see that this model can also be extended to incorporate

bonuses and penalties of the sort discussed by Bonin [1976] and Weitzman [1976].

d. Uncertain Renewal: The model analyzed in this paper focuses on the firm's incentives to under-report its research output. Of course situations exist in which the problem is precisely the opposite. That is, in many cases firms have an incentive to claim results which they do not have. This problem is especially acute when contract renewal is uncertain.[16]

Contract renewal can be uncertain for two basic reasons. First, the level of unit costs which is satisfactory to the research sponsor may be vague. In other words, the length of the project horizon may be sensitive to the level of reported unit costs and the precise nature of the sensitivity may be uncertain. However, whether this effect causes the firm to report a higher or lower level of unit costs is unclear. The firm may believe that in order to get a contract renewal, interim reports should be exaggerated with the hope that later research output can justify them. On the other hand, if it believes the research sponsor has an underlying target in mind which, once reached, terminates the project, then it may further under-report its research output.

The second reason why contract renewal may be uncertain is that there may be competition from other firms for the contracts awarded by the research sponsor. Several game-theoretic models of R and D which focus on the relationship between innovation and market structure have recently appeared in the literature (Loury [1979], Lee and Wilde [1980], and Reinganum [1980]). It appears that the model developed in this paper can be extended to a similar game-theoretic setting in which firms compete for the right to do R and D.

5. CONCLUSION

Section 4 has already suggested a number of extensions of the model developed in this paper which might be of interest to pursue. In this conclusion we would simply like to make a few comments on the literature. Most of the recent literature on dynamic incentive systems in planned economies has considered production of typical economic commodities [Murrell, 1979; Weitzman, 1980]. While costs might be assumed to vary from period to period, there is no systematic exploitation of finite resources in these models. On the other hand, our model is set in an environment in which the primary output, knowledge, is depletable. Many of the unique aspects of our model derive from this feature.

There are other differences between our model and those of Murrell [1979] and Weitzman [1980]. For example, those authors are interested in the incentive effects of various ''bonus'' schemes in which the reward is sensitive to a current target (selected by the firm or the planner), while we use a ''smooth'' payoff function of a more neoclassical type. In spite of this, the fact that in our model output in one period is an argument in the next period's payoff function creates a link with the extant literature.

FOOTNOTES

*This research was sponsored by the Jet Propulsion Laboratory under Department of Energy funding from the Solar Thermal Parabolic Dish Program. The views expressed reflect those of the authors only. We would like to thank Steve Lippman and Bill Rogerson for helpful comments on an earlier draft.

1. This specification of contract incentives is fairly general -- it is qualitatively equivalent to a number of contractual relationships now in use or being considered by both government and private research sponsors. See <u>United States Code, Annotated</u>, Title 41, Public Contracts (Subpart 1-3.4, Types of Contract, 1-3.407-2).

2. This is not a trivial assumption. One of the major problems with many existing government projects (e.g. the DOE Solar R and D program) is that firms tend to overstate the results of their research. See section 4 for more discussion of this issue.

3. This approach is similar to that used in much of the recent R and D literature -- see Lee [1980] and the references sited therein for examples. One could assume that F is sensitive to σ with $\partial F/\partial \sigma < 0$ in order to capture the notion that a good draw from F increases the likelihood of even better draws in the future. It does not appear that such a modification would effect our qualitative conclusions.

4. It is implicit in the form of (1) that the research sponsor cannot monitor the firm's research activity at all. To relax this assumption would integrate the present model with the well-known principal-agent literature.

5. Again, this assumption precludes analysis of a class of issues which is clearly of interest from a practical point of view. However, as section 4 illustrates, the methodology developed in this paper can easily be extended to the more general case which allows this assumption to be relaxed.

6. Throughout this paper partial derivatives will often be indicated by subscripts. Hence $W_r(r,R) = \partial W(r,R)/\partial r$, $W_{rr}(r,R) = \partial^2 W(r,R)/\partial r^2$, etc.

7. Figure 2 shows $H^1(\sigma,R)$ as concave. This is true for t=1 but fails to generalize to t>1. Nevertheless, $\sigma_t^*(R)$ will be well-defined since $H_\sigma^t(\sigma,R)$ will be positive.

8. The proofs of all the results in this paper follow standard dynamic programming techniques. Unless otherwise stated, the proofs are included in Appendix I.

9. We adopt the convention that if $\sigma = \sigma_1^*$ then no research is conducted.

10. It is implicitly assumed in Proposition 2 that an interior maximum is achieved. Using (A1) and the fact that $dV_1(\sigma,r_2^*)/dR = W_R(\sigma,R)$ for $\sigma \leq \sigma_1^*$, we have

(i) $r_2^*(\sigma,R) < R \iff W_r(R,R) + \beta W_R(\sigma,R) < 0$

(ii) $r_2^*(\sigma,R) > \sigma \iff W_r(\sigma,R) + \beta W_R(\sigma,\sigma) > 0$

Both (i) and (ii) hold given appropriate assumptions regarding the slope of $W(\sigma,R)$ when $\sigma = R$, e.g. $W_r(x,x) = -\infty$ and $W_R(x,x) = \infty$ for all x. These conditions also imply $R > r_2^*(\sigma,R) > \sigma$ for $\sigma > \sigma_1^*$. Throughout the rest of this paper they are presumed to hold.

11. See Lemma 6 below.

12. Furthermore, $W_{rR} = 0$ does not mean that a solution to the functional equation (1) fails to exist. In fact, in many cases it leads to an ''incentive compatible'' solution in the sense that $r_t^*(\sigma,R) = \sigma$ for all $\sigma \leq R$ and $t \geq 1$ (i.e. when $W(r,R)$ is convex in r). While this may seem to be a more satisfactory outcome than an interior solution (see footnote 10) the text following suggests it has significant problems of its own.

13. Of course the two assumptions are not technically equivalent. For example, if $W(r,R) = U(R - r)$ where $U' > 0$ and $U'' > 0$, a given absolute reduction in unit costs yields the same payoff independent of R. This is generally not the case when $W_{rR} > 0$.

14. For example, see Murrell [1979] and the references sited therein.

15. This case also applies to the type of grant typically awarded academics by organizations such as the NSF.

16. From a modeling point-of-view, over-reporting of research output is a tricky issue because the nature of the associated penalties is unclear. It appears that the only sanction typically available to a research sponsor is cancellation of future contracts, although even this is rarely observed.

APPENDIX I

<u>Proof of Proposition 1</u>: Taking the total derivative of $H^1(\sigma_1^*,R) = c$, we have $H^1_\sigma(\sigma_1^*,R)d\sigma_1^* = dc$ or, rearranging,

$$\frac{d\sigma_1^*}{dc} = 1/H^1_\sigma(\sigma_1^*,R) > 0.$$

Similarly,

$$\frac{d\sigma_1^*}{dR} = -H^1_R(\sigma_1^*,R)/H^1_\sigma(\sigma_1^*,R).$$

But, from (6),

$$H^1_R(\sigma,R) = \int_a^\sigma [W_R(x,R) - W_R(\sigma,R)]f(x)dx.$$

Thus the assumption that $W_{rR}(\sigma,R) > 0$ implies $H^1_R(\sigma,R) < 0$. This, in turn, implies $d\sigma_1^*/dR > 0$.

Q.E.D.

<u>Proof of Proposition 2</u>: By definition $r_2^*(\sigma,R)$ is given by the first order condition $\partial g_2(r_2^*,\sigma,R)/\partial r = 0$, or

$$W_r(r_2^*,R) + \beta\frac{dV_1(\sigma,r_2^*)}{dR} = 0 \qquad \text{(A1)}$$

The second-order condition, $W_{rr}(r,R) + \beta d^2V_1(\sigma,r)/dR^2 < 0$, is implied by Lemma 1(b). Hence $r_2^*(\sigma,R)$ is a well-defined maximum. Taking the total derivative of (A1) with respect to R, σ and c yields the following:

$$\frac{dr_2^*}{dR} = \frac{-W_{rR}(r_2^*,R)}{W_{rr}(r_2^*,R) + \beta d^2V_1(\sigma,r_2^*)/dR^2}$$

$$\frac{dr_2^*}{d\sigma} = \frac{-\beta d^2V_1(\sigma,r_2^*)/dRd\sigma}{W_{rr}(r_2^*,R) + \beta d^2V_1(\sigma,r_2^*)/dR^2}$$

$$\frac{dr_2^*}{dc} = \frac{-\beta d^2V_1(\sigma,r_2^*)/dRdc}{W_{rr}(r_2^*,R) + \beta d^2V_1(\sigma,r_2^*)/dR^2}$$

The Proposition now follows from the assumption that $W_{rR} > 0$ and Lemma 3.

Q.E.D.

<u>Proof of Proposition 3</u>: Taking the total derivative of $H^2(\sigma_2^*,R) = c$ we have $d\sigma_2^*/dc = 1/H_\sigma^2(\sigma_2^*,R) > 0$. Also,

$$\frac{d\sigma_2^*}{dR} = \frac{-H_R^2(\sigma_2^*,R)}{H_\sigma^2(\sigma_2^*,R)} .$$

But

$$H_R^2(\sigma_2^*,R) = \int_a^\sigma \left[\frac{\partial g_2(r_2^*(x,R),x,R)}{\partial R} - \frac{\partial g_2(r_2^*(\sigma,R),\sigma,R)}{\partial R}\right] f(x)dx$$

$$= \int_a^\sigma \left[W_R(r_2^*(x,R),R) - W_R(r_2^*(\sigma,R),R)\right] f(x)dx .$$

But $r_2^*(x,R) < r_2^*(\sigma,R)$ since $x < \sigma$. Thus $H_R^2(\sigma_2^*,R) < 0$ and $d\sigma_2^*/dR > 0$.

Q.E.D.

<u>Proof of Proposition 4</u>: First, note that $H^1(\sigma,R) = 0 = H^2(\sigma,R)$ for all $\sigma \le a$. Furthermore, $H^1(\sigma,R) > 0$ and $H^2(\sigma,R) > 0$ for all $R \ge \sigma > a$. Consider $\sigma \le \sigma_1^*$. For such σ,

$$H_\sigma^1(\sigma,R) = -W_r(\sigma,R)F(\sigma)$$

and

$$H_\sigma^2(\sigma,R) = -\beta\frac{dV_1(\sigma,r_2^*(\sigma,R))}{d\sigma} F(\sigma)$$

$$= -\beta\, W_r(\sigma,r_2^*(\sigma,R))F(\sigma) .$$

Hence,

$$H_\sigma^2(\sigma,R) > H_\sigma^1(\sigma,R) \Leftrightarrow \beta W_r(\sigma,r_2^*(\sigma,R)) > -W_r(\sigma,R)$$
$$\Leftrightarrow \beta W_r(\sigma,r_2^*(\sigma,R)) < W_r(\sigma,R) .$$

Now $W_{rR} > 0$ and $r_2^*(\sigma,R) \le R$ so that $W_r(\sigma,r_2^*(\sigma,R)) < W_r(\sigma,R)$. But $0 < \beta < 1$ and $W_r < 0$. Thus $W_r(\sigma,r_2^*(\sigma,R)) < W_r(\sigma,R)$ only if β is ''close to'' one. Figure 4 illustrates the relationship between H^1 and H^2.

Q.E.D.

[FIGURE 4 HERE]

<u>Proof of Lemma 4</u>: Using $\sigma_2^*(R)$, $V_2(\sigma,R)$ can be written as

$$V_2(\sigma,R) = \begin{cases} \int_a^\sigma g_2(r_2^*(x,R),x,R)f(x)dx + \int_\sigma^b g_2(r_2^*(\sigma,R),\sigma,R)f(x)dx & \text{if } \sigma > \sigma_2^* \\ g_2(r_2^*(\sigma,R),\sigma,R) & \text{if } \sigma \le \sigma_2^* \end{cases} \quad (A2)$$

Taking the derivative of (A2) with respect to R, and noting that $\partial g_2(r_2^*,\sigma,R)/\partial r = 0$ for all σ and R, we have

$$\frac{dV_2(\sigma,R)}{dR} = \begin{cases} \int_a^\sigma W_R(r_2^*(x,R),R)f(x)dx + \int_\sigma^b W_R(r_2^*(\sigma,R),R)f(x)dx & \text{if } \sigma > \sigma_2^* \\ W_R(r_2^*(\sigma,R),R) & \text{if } \sigma \le \sigma_2^* \end{cases}$$

since $\partial g_2(r_2^*,\sigma,R)/\partial R = W_R(\sigma,R)$ for all σ and R. Hence, if $\sigma > \sigma_2^*$,

$$\frac{d^2V_2(\sigma,R)}{dR^2} = \int_a^\sigma [W_{rR}(r_2^*(x,R),R)\ \frac{dr_2^*(x,R)}{dR} + W_{RR}(r_2^*(x,R),R)]f(x)dx$$

$$+ \int_\sigma^b [W_{rR}(r_2^*(\sigma,R),R)\frac{dr_2^*(\sigma,R)}{dR} + W_{RR}(r_2^*(\sigma,R),R)]f(x)dx,$$

and if $\sigma \le \sigma_2^*$,

$$\frac{d^2V_2(\sigma,R)}{dR^2} = W_{Rr}(r_2^*(\sigma,R),R)\ \frac{dr_2^*(\sigma,R)}{dR} + W_{RR}(r_2^*(\sigma,R),R).$$

Now $W_R > 0$ so $dV_2(\sigma,R)/dR > 0$. But $W_{rR} > 0$, $dr_2^*/dR > 0$ and $W_{RR} < 0$. Hence $d^2V_2(\sigma,R)/dR^2$ is of ambiguous sign.

Q.E.D.

Proof of Lemma 5: Differentiating (A2) with respect to σ and using the facts that $\partial g_2(r_2^*,\sigma,R)/\partial r = 0$ and $\partial g_2(r_2^*,\sigma,R)/\partial\sigma = \beta\ \partial V_1(\sigma,r_2^*)/\partial\sigma$, we have

$$\frac{dV_2(\sigma,R)}{d\sigma} = \begin{cases} \int_\sigma^b \beta\ \frac{dV_1}{d\sigma}(\sigma,r_2^*(\sigma,R))f(x)dx & \text{if } \sigma > \sigma_2^* \\ \beta\ \frac{dV_1}{d\sigma}(\sigma,r_2^*(\sigma,R)) & \text{if } \sigma \le \sigma_2^* \end{cases}$$

From Lemma 2, $dV_1(\sigma,R)/d\sigma < 0$ for all σ and R.

Q.E.D.

Proof of Lemma 6: Again, from (A2),

$$\frac{d^2V_2(\sigma,R)}{d\sigma dR} = \begin{cases} \int_\sigma^b W_{Rr}(r_2^*(\sigma,R),R)f(x)dx & \text{if } \sigma > \sigma_2^* \\ W_{Rr}(r_2^*(\sigma,R),R) & \text{if } \sigma \le \sigma_2^* \end{cases}$$

Thus $W_{Rr} > 0$ implies $d^2V_2(\sigma,R)/d\sigma dR > 0$. Part (b) is obvious.

Q.E.D.

Proof of Proposition 5: All parts of the induction are immediate from our earlier analysis except part e(iv). For all $t \ge 2$, $r_t^*(\sigma,R)$ solves $\partial g_t(r_t^*(\sigma,R),\sigma,R)/\partial r = 0$. Since g_t is concave (either by assumption or from $W(r,R) = U(R - r)$ where $U' > 0$ and $U'' < 0$), $r_{t+1}^*(\sigma,R) \ge r_t^*(\sigma,R)$ is implied by $\partial g_t(r_{t+1}^*(\sigma,R),\sigma,R)/\partial r \le 0$. Now $\partial g_t(r,\sigma,R)/\partial r = W_r(r,R) + \beta dV_{t-1}(\sigma,r)/dR$. Hence

$$\partial g_t(r_{t+1}^*(\sigma,R),\sigma,R)/\partial r = W_r(r_{t+1}^*(\sigma,R),R) + \beta dV_{t-1}(\sigma,r_{t+1}^*(\sigma,R))/dR.$$

But $\partial g_{t+1}(r_{t+1}^*(\sigma,R),\sigma,R)/\partial r = 0$ implies

$$W_r(r_{t+1}^*(\sigma,R),R) + \beta dV_t(\sigma,r_{t+1}^*(\sigma,R))/dR = 0.$$

Hence

$$\partial g_t(r_{t+1}^*(\sigma,R),\sigma,R)/\partial r = \beta\Big[\frac{dV_{t-1}}{dR}(\sigma,r_{t+1}^*(\sigma,R)) - \frac{dV_t}{dR}(\sigma,r_{t+1}^*(\sigma,R))\Big]. \qquad \text{(A3)}$$

Consider $\sigma > \sigma_t^* > \sigma_{t+1}^*$. Over this range

$$\frac{dV_\tau(\sigma,\rho)}{dR} = \int_a^\sigma W_R(r_\tau^*(x,\rho),\rho)f(x)dx + \int_\sigma^b W_R(r_\tau^*(\sigma,\rho),\rho)f(x)dx$$

for either $\tau = t$ or $\tau = t-1$, where $\rho = r_{t+1}^*(\sigma,R)$. Thus, $\partial g_t(r_{t+1}^*(\sigma,R),\sigma,R)/\partial r \leq 0$ if

$$\int_a^\sigma W_R[r_t^*(x,\rho),\rho]f(x)dx + \int_\sigma^b W_R[r_t^*(\sigma,\rho),\rho]f(x)dx$$

$$\geq \int_a^\sigma W_R[r_{t-1}^*(x,\rho),\rho]f(x)dx + \int_\sigma^b W_R[r_{t-1}^*(\sigma,\rho),\rho]f(x)dx. \qquad (A4)$$

But by the induction hypothesis, $r_t^*(\sigma,R) \geq r_{t-1}^*(\sigma,R)$ for all σ and R. Hence, because $W_{rR} > 0$, it must be that $W_R[r_t^*(x,\rho),\rho] \geq W_R[r_{t-1}^*(x,\rho),\rho]$ for all $x < \sigma$ and, in particular, for $x = \sigma$. Hence the inequality in (A4) holds, implying (A3) is negative, implying $r_{t+1}^*(\sigma,R) \geq r_t^*(\sigma,R)$ for $\sigma > \sigma_t^*$. Similar arguments show the result holds for $\sigma \leq \sigma_t^*$ as well.

Q.E.D.

APPENDIX II

__Claim__: $W(r,R) = U(R-r)$ with $U' > 0$ and $U'' < 0$ implies $d^2V_2(\sigma,R)/dR^2 < 0$.

__Proof of claim__: In this case, we have $W_{RR} = U''(R-r) = W_{rr}$ and $W_{rR} = -U''(R-r)$. Hence

$$\frac{dr_2^*}{dR} = \frac{U''(R-r_2^*)}{U''(R-r_2^*) + \beta d^2V_1(\sigma,r_2^*)/dR^2}$$

and thus

$$\int_a^\sigma \Big[W_{rR}(r_2^*(x,R),R)\frac{dr_2^*(x,R)}{dR} + W_{RR}(r_2^*(x,R),R)\Big]f(x)dx$$

$$= \int_a^\sigma \Big[\frac{-U''(R-r_2^*(x,R))^2}{U''(R-r_2^*(x,R)) + \beta d^2V_1(x,r_2^*(x,R))/dR^2} + U''(R-r_2^*(x,R))\Big] f(x)dx$$

$$= \int_a^\sigma \Big[-U''(R-r_2^*(x,R))\frac{U''(R-r_2^*(x,R))}{U''(R-r_2^*(x,R)) + \beta d^2V_1(x,r_2^*(x,R))/dR^2} - 1\Big] f(x)dx.$$

Now

$$\frac{U''(R-r_2^*(x,R))}{U''(R-r_2^*(x,R)) + \beta d^2V_1(x,r_2^*(x,R))/dR^2} - 1 < 0$$

since $U'' < 0$ and $d^2V_1(x,r_2^*(x,R))/dR^2 < 0$. Thus $W(r,R) = U(R-r)$ where $U' > 0$ and $U'' < 0$ implies

$$\int_a^\sigma \left[W_{rR}(r_2^*(x,R),R)\frac{dr_2^*(x,R)}{dR} + W_{RR}(r_2^*(x,R),R)\right] f(x)dx < 0 .$$

Similarly, it implies

$$\int_\sigma^b \left[W_{rR}(r_2^*(\sigma,R),R)\frac{dr_2^*(\sigma,R)}{dR} + W_{RR}(r_2^*(\sigma,R),R)\right] f(x)dx < 0.$$

Hence it implies $d^2V_{2(\sigma,R)}/dR^2 < 0$ for $\sigma > \sigma_2^*$. For $\sigma \leq \sigma_2^*$, an analogous argument also implies $d^2V_{2(\sigma,R)}/dR^2 < 0$.

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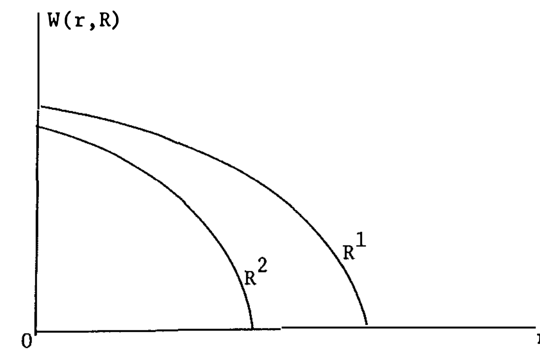


Figure 1. Form of W(r,R) when $W_{rR} > 0$: $R^1 > R^2$.

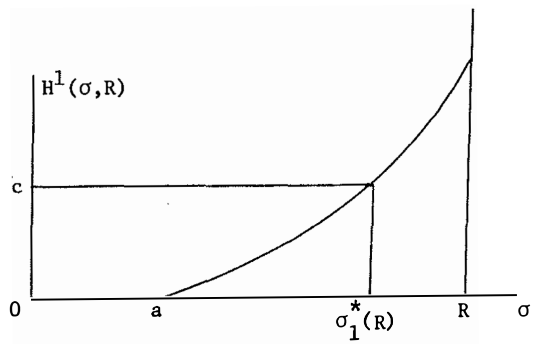


Figure 2: Definition of $\sigma_1^*(R)$

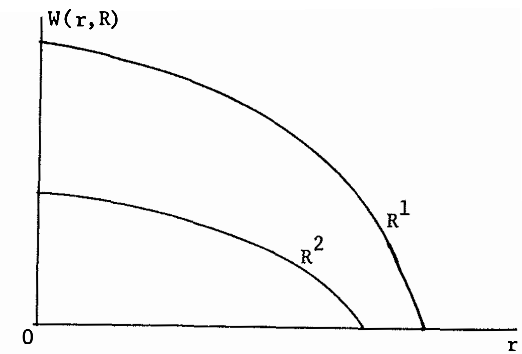
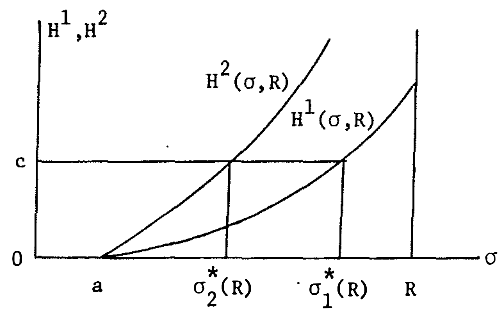


Figure 3: Form of W(r,R) when $W_{rR} < 0$: $R^1 > R^2$



<u>Figure 4</u>: $W_{rR} > 0$ implies $\sigma_2^*(R) \leq \sigma_1^*(R)$ if $\beta \geq \hat{\beta}_2$.