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EQUILIBRIUM AND EFFICIENCY UNDER PURE ENTITLEMENT SYSTEMS

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ABSTRACT

An entitlement system is a political institution which accords various citizens rights to veto specific changes of social state. This paper concerns the performance of such institutions, in the absence of other centralized decision-making institutions. Specifically, questions relating to unbiasedness (the potential of an institution to support any Pareto-efficient social state as an equilibrium) in situations with externalities are discussed. Necessary conditions for an entitlement system to be unbiased regardless of a society's technology are found, and these conditions are shown to be sufficient when technology and preferences satisfy some geometric conditions (e.g., convexity). However, counter-examples to unbiasedness are provided when these conditions fail. It is argued that, even when an entitlement institution is unbiased, extensive information is required to verify this fact. The difficulty of systematically designing an unbiased system qualifies Hayek's assertion that such a system would be informationally efficient in operation.

I. INTRODUCTION

Two types of answer have been given to the question of how government should contribute to the happiness of its citizens. One is that it should take a direct (and perhaps coercive)role in the pursuit of optimality or efficiency. The other is that it should indirectly promote voluntary cooperation among its citizens by providing procedural justice (e.g., enforcement of property rights) to them.[1]

This paper addresses the question of how cooperative equilibria of a "minimal government" (one which guarantees rights but which does not make direct allocative decisions) would compare with social states[2] which are Pareto efficient for its citizens. Specifically it examines whether or not such a government would be unbiased (i.e., whether every Pareto-efficient social state would be possible to support as a cooperative equilibrium[3]) in an environment where its citizens might impose externalities on one another. Unbiasedness of equilibrium could be interpreted to mean that under ideal circumstances (i.e., that cooperative outcomes would in fact be achieved) a minimal government would be procedurally fair. However, unbiasedness would also imply that such a government could not guarantee an egalitarian distribution of wealth, or any other feature except Pareto efficiency, in an equilibrium social state.

The specific form of a "minimal government" to be studied here is suggested by the political theories of Fredrick Hayek [4], John Rawls[4] [8], and Robert Nozick [6]. Following Nozick, this sort of government will be called an entitlement system. An entitlement is a right accorded to some person to veto a particular change of social state. An entitlement system will be called pure if it is the only institution of government (as Hayek and Nozick seem to envision) rather than being a branch of a more extensive government (as Rawls recommends).

A formal theory of entitlement systems is presented, and a cooperative equilibrium concept for pure entitlement systems is defined, in section two.[5] Some examples of unbiased entitlement systems, including markets in private-goods economies, are presented in section three. Then, in section four, necessary conditions are stated for an entitlement system to be unbiased regardless of the technology available to society, and informal examples of how bias may occur are proposed. Section five presents a restatement of section four within the formal entitlement theory. The informational requirements for the conditions to be satisfied are examined in section six. Sufficient conditions for an efficient state to be an entitlement equilibrium (and hence, if they hold for every state, for a system to be unbiased) are established in section seven, and their indispensability is shown in section eight. Section nine, in which the implications of this paper for political theory are discussed, concludes the paper.

There are three appendices. Appendix A explains why unbiasedness may be of interest even to libertarians who reject utilitarianism as a foundation for political philosophy. Appendix B describes the relationship between this paper and the work of Ronald Coase [2] on property-rights systems. Appendix C contains the proof of a technical result used in the paper.

Material has been organized so that sections 2,3,4,6, and 9 constitute a fairly complete informal exposition, and so that sections 2,5,7 and 8 provide a logically self-contained formal treatment.

II. ENTITLEMENT SYSTEMS AND THEIR EQUILIBRIA

Consider a society which is in some historically given situation or social state. An entitlement system is a government which does not perturb the social state directly, but which allows private individuals to make changes. These changes are proposed unilaterally, but are subject to veto. The role of the government is to assign entitlements, or veto rights against particular changes of social state, to various individuals. A social state is an equilibrium of the government if no one can change it to his advantage without harming someone else who is entitled to exercise a veto. Such an equilibrium state will be called final.[6]

Entitlement systems are now described formally. The veto rights of an individual (or agent) are specified indirectly in terms of a relation, his ascribed welfare, which will determine the set of changes of social state which he is not entitled to veto. This indirect specification of veto rights corresponds to the notion of "legitimate expectation" in political theory. Technically, it will simplify the statement of conditions on entitlements to be introduced later.

Let N be a finite set of agents (i,j range over N), and let X be a set of social states (x,y,z range over X). It is assumed that

(1) X is a subset of E^k (k-dimensional Euclidean space) with nonempty interior.

A subset $P_i \subseteq X^2$ describes i's preference. Interpret xP_iy to mean that i considers x to be strictly better than y. Define $R_i \subseteq X^2$

by xR_iy if and only if not yP_ix. It is assumed that

(2) R_i is a total preordering, and is locally nonsatiated.

Define the Pareto ordering $P \subseteq X^2$ by

(3) $xPy \equiv [\exists i \; xP_iy \text{ and } \forall j \; xR_jy]$.

An entitlement system is an assignment of relations $W_i \subseteq X^2$ to agents. The relation W_i will be called agent i's ascribed welfare. Define xV_iy to mean that x belongs to the topological closure of $\{z|zW_iy\}$. V_i is the set of changes of social state which agent i does not have the right to veto. I.e., $\{x|xV_iy\}$ is i's legitimate expectation when state y is the status quo.

The basic idea of an entitlement system is that an agent's legitimate expectation must not be violated without his consent. A change of social state from y to x will be made only if (a) some agent would desire this change, and (b) there is no agent for whom it would violate both his wishes and his legitimate expectation. Call a change which meets these criteria a libertarian transition. Formally, define the libertarian-transition relation $L \subseteq X^2$ by

(4) $xLy \equiv [\exists i \; xP_iy \text{ and } \forall j \; (xR_jy \text{ or } xV_jy)]$.

Let $A \subseteq X$ be the set of attainable social states. A state $x \in A$ is Pareto efficient in A (or simply efficient in A, for short) if it has no P-successor in A, and it is libertarian final in A (or simply final in A, for short) if it has no L-successor in A. Being a final state is precisely the equilibrium concept for pure entitlement systems which has been informally described earlier. Thus, a pure entitlement system is unbiased in an environment (specified by agents' preferences and by a set of attainable states) if every efficient state is final.

Two features of finality as an equilibrium concept deserve notice. First, since every feasible Pareto improvement is a libertarian transition, all final states are efficient. This fact provides the answer to the converse of the question about unbiasedness which is studied here. Second, the set of final states can be interpreted as the core of a game without side payments in characteristic-function form. Say that x improves y for a coalition $C \subseteq N$ if (a) xP_iy for some $i \in C$, (b) xR_iy for all $i \in C$, and (c) xV_iy for all $i \notin C$. The right-hand side of (4) holds if and only if x improves y for some coalition, so the core (i.e., the set of states which cannot feasibly be improved) consists exactly of the final states. This characterization makes it clear that finality is a cooperative equilibrium concept, and suggests that questions of "transaction costs" and of strategic manipulability would have to be considered before one could assert that an entitlement institution would achieve final outcomes in practice.

III. SOME PARETO-UNBIASED ENTITLEMENT SYSTEMS

There is at least one entitlement system which is unbiased in all environments: that which accords to every individual the right to veto any proposed change of social state. In a Pareto-efficient state no one can then make himself better off without there being some aggrieved person who would be entitled to veto the change. Thus every efficient state will be final.

However, even among those who strongly emphasize the need to protect individuals' rights, there are many who doubt the necessity or desirability of requiring unanimous consent for every action or change. For instance, according to Coase, "The problem which we face in dealing with actions which have harmful effects is not simply one of restraining those responsible for them. What has to be decided is whether the gain from preventing the harm is greater than the loss which would be suffered

elsewhere as a result of stopping the action which produces the harm." [2, p. 27] Similarly, Hayek has stated that "in the course of this process [of legal evolution] it will be found not only that not all expectations can be protected by general rules, but even that the chance of as many expectations as possible being fulfilled will be most enhanced if some expectations are systematically disappointed." [4, v.1, p.102]

Another entitlement system which is unbiased in an important class of environments is a market: A private -goods economy is an environment in which social states are allocations and in which each agent's preference between social states depends only on a comparison of the consumption which they yield to him. A market is an entitlement system which allows an agent to veto a change of state if it affects his own consumption bundle. (Formally, each agent has a consumption set X_i, $X = \prod_{i \in N} X_i$, and $xW_iy \equiv xV_iy \equiv x_i = y_i$.) Since in a private-goods environment an agent's consumption bundle is affected whenever he is harmed, all libertarian transitions must be Pareto improvements. Thus, as with the unanimous-consent system, all efficient states are final. Note that the final states in this example are precisely the 'contract curve'. In the absence of geometric restrictions (e.g., convexity of A), final allocations are not assured to have supporting prices.

In the preceding examples, unbiasedness was proved by showing that the Pareto ordering and the libertarian-transition relation are identical. These relations do not have to coincide in order to assure unbiasedness. In fact, the libertarian-transition relation of an unbiased entitlement system may even have cycles. An example of this phenomenon is shown in figure 1. Here ascribed-welfare relations are constructed by linear local approximation to agents' preference relations. It is assumed that, for every agent i and every state x, the upper-contour set of P_i at x (i.e., $\{y|yP_ix\}$) is open and has a unique supporting hyperplane at x. Then yW_ix will be defined to mean that y lies on the same side of this hyperplane as the upper-contour set does, and yV_ix if y belongs to the corresponding closed half space. This is shown in figure 1(a).

Figure 1(b) is an Edgeworth box. Two indifference curves are shown for each agent. State x (resp. y) lies on the lower indifference curve of agent 1 (resp. 2) and on the higher indifference curve of agent 2 (resp. 1). However, because the curves are not linear, xV_1y and yV_2x. This the changes from x to y and back again are both libertarian transitions. The libertarian-transition relation contains a cycle.

Figure 1(c) shows a representative efficient state in the Edgeworth box. A hyperplane through z (i.e., the budget line at the market-clearing price) supports the upper-contour sets of both agents, and separates them. Thus, either agent would want to veto and would be entitled to veto any change which would benefit the other. The efficient state is final. The supporting-price argument which establishes unbiasedness in this example presents the intuition underlying the sufficiency theorem for unbiasedness to proved later.

That the libertarian-transition relation of an unbiased entitlement system may have cycles indicates a limitation of the significance of unbiasedness. Unbiasedness means that an efficient state will be "stable" once it is reached, but it does not guarantee that one will ever be reached. Since a libertarian-transition path can cycle forever between inefficient states, unbiasedness does not imply that an entitlement system would necessarily converge from an arbitrary initial state to an efficient state. Even in the absence of "transaction costs" and of strategic manipulation, unbiasedness does not rule out the possibility of long-run inefficiency.

IV. NECESSARY CONDITIONS FOR UNBIASEDNESS

One way to characterize unbiased entitlement systems is to say that they guarantee "enough" veto rights so that any change from a Pareto-efficient state will be prevented. In the example just given, this was accomplished by making agents' ascribed welfare relations closely resemble their actual preferences. The question arises, how far can ascribed welfare diverge from true preference in an unbiased system? In this section, two conditions describing the relation between ascribed welfare and preference will be introduced. These conditions must be satisfied by an entitlement system in order for it to be unbiased for every set of attainable states. (That is, these are necessary conditions for an entitlement system operating in a society of agents with fixed preferences to be unbiased regardless of the society's technological capability. Such an entitlement system will be called uniformly unbiased.) Two further conditions will be introduced, in the absence of which one of the necessary conditions might hold vacuously. Informal examples will be presented to illustrate the four conditions. The conditions and examples will be formalized in the next section.

The first necessary condition for an entitlement system to be uniformly unbiased is that, if there is one agent whose preference is violated by a change of state and if every other agent would at least weakly prefer that change, then the change must violate the aggrieved agent's welfare. The aggrieved agent will be called a lone dissenter, and the condition will be called the lone-dissenter requirement.

An externality problem shows the force of this requirement. In a two-agent situation, the requirement is violated whenever one agent is able to benefit by imposing a negative externality on the other agent without his permission. If the initial state and the state in which the externality is imposed are the only attainable states, then the initial state is efficient but not final. This violates unbiasedness.

If there are only two agents, then the lone-dissenter requirement rules out all changes which are not Pareto improvements. When there are more than two agents, though, a change may be advantageous to one of them and disadvantageous to several others. Such a change would not be a Pareto improvement, but it would not violate the lone-dissenter requirement because more than one agent is aggrieved. Again, if the two states in question were the only attainable states, an entitlement system which permitted the change would be biased. A necessary condition to rule out such changes is that, whenever a change of social state violates an agent's preference, there must be some agent (possibly, but not necessarily, the aggrieved one) who is entitled to veto the change. The agent ascribed a welfare loss will be called a surrogate for the agent whose preference is violated, and the condition

that every aggrieved agent must have a surrogate will be called the surrogates requirement. This is the second necessary condition for an entitlement system to be uniformly unbiased.

Although two-agent externality situations violate the lone-dissenter requirement, they will satisfy the surrogates requirement because the perpetrator of the externality is a surrogate for the victim. When there are more than two agents, it is possible for the surrogates requirement to fail even in cases where the lone-dissenter requirement is satisfied. (I.e., each condition is independent of the other.) For instance, consider the hypothetical case of three automobile manufacturers. Volkswagen makes a small diesel car, Honda makes a small gasoline-powered car, and Mercedes-Benz makes large cars of both types. Suppose that a social state is a pair of market prices, one for gasoline and one for diesel fuel. Both Volkswagen and Honda perceive Mercedes-Benz to be a threat, and wish the price of both fuels to remain high so that small cars remain desirable. However, each small-car manufacturer benefits most from an increase in the price of the fuel which its own cars do not use. Mercedes-Benz wants the prices of both fuels to remain low. (To make this example precise, firms' profits would be expressed as functions of weighted price averages. This is done in example 3 of the next section.)

In this situation, every social state is Pareto efficient relative to the three firms. To see this, first suppose that prices change in a way favorable to Volkswagen. Then the price of gasoline must rise by more than the price of diesel fuel drops, so the average of the two prices is higher and Mercedes-Benz is hurt. In the same way, Mercedes-Benz is hurt by a price change which favors Honda. Finally, suppose that a price change helps Mercedes Benz. Then the average price of fuel goes down, and the price of at least one of the fuels must drop. The small-car manufacturer whose cars use the other fuel must then be hurt.

Consider an entitlement system according to which each firm may veto a change when the price of the fuel that its cars use is raised. If both prices drop, both Volkswagen and Honda are hurt so there is no lone dissenter. However, every firm is viewed as being better off, so Volkswagen and Honda lack surrogates. Since Mercedes-Benz actually does benefit from the drop in both prices, the change is a libertarian transition. The efficient initial price vector is not final, so the entitlement system is biased.

It might be hoped that the lone-dissenter and surrogates requirements together would be sufficient, as well as necessary, for uniform unbiasedness. It will be shown that they are jointly sufficient for unbiasedness in a class of environments satisfying some geometric restrictions, if two kinds of situation which allow the lone-dissenters requirement to be satisfied vacuously (because there are "too many" dissenters) are ruled out. One kind of situation occurs when two agents have the same preference ordering. Then it can happen that a change of social state violates only one preference <u>ordering</u>, so that some agent with that ordering ought to be able to veto the change, but that the lone-dissenter requirement does not insure this because several <u>agents</u> are harmed. The other kind of situation is one in which preferences are distributed densely in the population. If one agent is harmed by a change, there is someone else sufficiently like him so that he is also harmed, so that neither is a lone dissenter.

Identity of different agents' preferences is presumably an artificial situation, but denseness of preferences might be approximated in a large and diverse society. For example, consider a town where a hospital is to be built. Everyone would like the hospital to be fairly close to his home, but not in his immediate neighborhood. Let a social state be specified by a location of the hospital within the city (which is densely populated). Every state is efficient, because moving the hospital inevitably puts it closer to someone at its new location.

Suppose that agents are allowed to veto changes when the hospital is moved away from them. The surrogates requirement is satisfied because moving the hospital puts it farther from someone, who is a surrogate for those to whom it is now next door. There are no lone dissenters, because the hospital cannot be put next door to someone without becoming extremely close to his immediate neighbors also. No two agents have exactly identical preferences, although neighbors' preferences are very similar to one another. Yet, although every location for the hospital is efficient, none is final. It can always be moved away from someone, and those who suffer from this move (i.e., those in whose immediate neighborhood it is placed) are exactly those who are ascribed an improvement in welfare. Every change of location is a libertarian transition.

In order that efficiency should coincide with finality, preferences must not be completely disparate. In the presence of the surrogates requirement, this can be guaranteed by a restriction on ascribed welfare. A welfare ascription will be called <u>cohesive</u> if there is some potential change of social state (which need not be feasible) which would be considered to increase the welfare of every agent.

(According to the surrogates requirement, this change cannot violate the preference of any agent.) It will be required that a welfare ascription should be cohesive.

V. FORMALIZATION OF THE REQUIREMENTS

In this section, the requirements which have just been discussed will be represented in the formal theory defined in section two and will be shown to be independent. That is, geometric models of the theory will be constructed in which exactly one of the requirements fails, and in which some efficient state is not final. The geometric implications of the requirements will be used further in proving the sufficiency theorem for unbiasedness. The class of preference and welfare relations for which the implications hold will first be defined.

Let $Q \subseteq X^2$. Define Q to be regular[7] at $x \in X$ if there is a $q \in E^k$ such that the following are equivalent for all $y \in X$:

(5) $q \cdot (y-x) > 0$

(6) $\exists r > 0 \quad (ry + (1-r)x)Qx$

(7) $\exists t > 0 \; \forall r$ [if $0<r<t$, then $(ry + (1-r)x)Qx$].

For example, if Q is a strict preference represented by a differentiable, quasiconcave utility function, then Q is regular at points where this function has a nonzero gradient. (q is the gradient vector.) Regularity is a strong condition which combines elements of local nonsatiation, irreflexivity, differentiability and convexity. In particular, if Q is regular at a point x in the interior of X, then q is unique up to a positive scalar multiple and (x,x) is a boundary point of $\{y|yQx\}$. Note that $(x,x) \notin Q$ by the equivalence of

(5) and (6). Furthermore, if a total preordering R has an asymmetric part which is regular at every point of X, then every upper-contour set of R is convex.

Henceforth it will be assumed that relations P_i and W_i are regular at point x being considered. The normal vector q in the definition of regularity will be denoted by p^i and w^i for P_i and W_i, respectively. Preference relations $\{P_i\}_{i \in N}$ will be called first-order distinct at x if p^i and p^j are not positive scalar multiples of one another for $i \neq j$. This condition makes precise the third requirement discussed in the last section, that agents' preferences be distinct.

The remaining requirements are now stated: The surrogates requirement holds at x if

(8) $\exists i \; xP_iy$ implies $\exists j$ not yV_jx.

The lone dissenter requirement holds at x if, for all i

(9) xP_iy implies [not yV_ix or $\exists j \neq i \; xP_jy$].

The relations W_i are cohesive at x if, for some $y \in X$,

(10) $\forall i \; yW_ix$.

These three requirements do not refer to regularity in their statements. However, under the hypothesis of regularity, the requirements have implications in terms of the vectors p^i and w^i. Specifically, these implications concern convex cones generated by vectors. Recall that, if $q^1,\dots,q^n$ are vectors in E^k, the convex cone generated by them is $\{\sum_{i=1}^{n} r_iq^i | r_1,\dots,r_n$ are nonnegative scalars$\}$.

This set will be denoted by $K[q^1,\ldots,q^n]$ or by $K[\{q^i\}_{i \le n}]$. A fundamental result about cones is

<u>Farkas' lemma</u> [5, p. 3] Let $q, q^1,\ldots,q^n$ be vectors in E^k. The following are equivalent:

(11) $q \in K[q^1,\ldots,q^n]$, and

(12) $\forall z \in E^k[(\forall i \le n \; q^i \cdot z \ge 0)$ implies $q \cdot z \ge 0]$.

Geometric implications of the requirements are stated in the following three lemmas.

<u>Lemma 1</u>: If the relations W_i are cohesive at a point x in the interior of X, then there is a vector $z \in E^k$ such that $\forall i \in N \;\; w^i \cdot z > 0$. (Geometrically this means that some open half-space of E^k contains all of the w^i.)

<u>Proof</u>: Since the W_i are cohesive, there exists $y \in X$ with yW_ix for all i. Let $z = y - x$. By the equivalence of (5) and (6) for each W_i, $w^i \cdot z > 0$. (Note that yW_ix implies (6).) q.e.d.

<u>Lemma 2</u>: If the relations W_i are cohesive at an interior point x of X, and if the surrogates requirement is met at x, then for all i, $p^i \in K[\{w^j\}_{j \in N}]$.

<u>Proof</u>: Suppose that $p^i \notin K[\{w^j\}_{j \in N}]$. By Farkas' lemma, there is a $z \in E^k$ such that $w^j \cdot z \ge 0$ for all j but $p^i \cdot z < 0$. By lemma 1, there is a $z' \in E^k$ such that $w^j \cdot z' > 0$ for all j. A positive scalar t can

be chosen sufficiently small that, if $u = z + tz'$, then $w^j \cdot u > 0$ for all j but $p^i \cdot u < 0$. Because x is in the interior of X, $y_r = x + ru \in X$ for sufficiently small positive r. By the equivalence of (5) and (7), r can be chosen so that y_rW_jx for all j. Since $W_j \subseteq V_j$, y_rV_jx for all j. But $p^i \cdot (y_r - x) < 0$, so not y_rR_ix by the equivalence of (5) and (6). (N.B. Since R is locally nonsatiated by (2), (5) and (6) imply that $p^i \cdot (y_r - x) \ge 0$ if y_rR_ix.) Thus the surrogates condition fails. q.e.d.

<u>Lemma 3</u>: If the relations W_i are cohesive at an interior point x of X, and if both the surrogates requirement and the lone-dissenter requirement are met at x, then for all i, $p^i \in K[w^i, \{p^j\}_{j \ne i}]$.

<u>Proof</u>: By lemma 2, $p^j \cdot z' > 0$ for all j, where z' is the vector described in lemma 1 with $w^j \cdot z' > 0$ for all j. Thus, as in lemma 2, a state $y_r \in X$ can be found with y_rV_ix, y_rP_jx for $j \ne i$, but xP_iy_r. This violates the lone-dissenter requirement. q.e.d.

Call a relation $Q \subseteq X^2$ <u>linear</u> if there is a nonzero vector $q \in E^k$, independent of x, such that $yQx \equiv q \cdot (y - x) > 0$. If the relations P_i and W_i are linear, then $\forall i \; p^i \in K[\{w^j\}_{j \in N}]$ and $\forall i \; p^i \in K[w^i, \{p^j\}_{j \ne i}]$ are sufficient for the surrogates requirement and the lone-dissenter requirement, respectively, to hold. These facts make it easy to construct and examine some geometric models of preference aggregation.

Four such models will now be presented. In each, $X = E^2$. N, A, $\{p^i\}_{i \in N}$ and $\{w^i\}_{i \in N}$ will be specified, and P_i and W_i will be linear for each i. In each example, it will be shown that one of the four requirements for uniform unbiasedness will be violated and that some efficient state will not be final. The remaining three requirements will all be satisfied. (This is left for the reader to verify, using the equivalences just described.) These examples demonstrate that no proper subset of the requirements is sufficient to assure unbiasedness.

Example 1 (Figure 2(a)) -- Violation of the lone-dissenter requirement: $N = \{1,2\}$, $A = \{(a,b) | a + b = 0\}$, $p^1 = w^2 = (1,0)$, $p^2 = w^1 = (0,1)$. Let $x = (0,0)$, $y = (-1,1)$. State x is efficient (as is every state in A). xP_1y, but yP_2x and yV_1x, so yLx. Thus x is not final.

Example 2 (Figure 2(a)) -- Violation of the first-order distinctness of preferences: This example is identical to the last, except that there are two new agents. Agents 3 and 1, and 4 and 2, are identical respectively. Again, x is efficient but yLx.

Example 3 (Figure 2(b)) -- Violation of the surrogates requirement: This formalizes the automobile-manufacturer example. $N = \{1,2,3\}$, $A = E^2$, $p^1 = (-2,-1)$, $p^2 = (-1,-2)$, $p^3 = w^3 = (1,1)$, $w^1 = (1,0)$, $w^2 = (0,1)$, let $x = (0,0)$, $y = (1,1)$. Every state is efficient because $K[p^1, p^2, p^3] = E^2$. However yV_1x, yV_2x and yP_3x, so yLx.

Example 4 (Figure 2(c)) -- Violation of the cohesiveness of ascribed welfare: This example formalizes the hospital example, if every agent is imagined to live at the endpoint of his welfare vector. $N = \{1,2,3,4,5\}$, $A = E^2$ p^i is the unit vector at angle $2i\pi/5$ radians. w^i is the unit vector diametrically opposed to p^i. Every state is efficient, since $K[\{p^i\}_{i \leq 5}] = E^2$. However, if $x = (0,0)$ and y is the endpoint of p^2, then yP_ix for $i \leq 3$ and yV_jx for $j > 3$, so yLx.

VI. INFORMATION NEEDED TO ASSURE UNBIASEDNESS

The question arises of how much information about an environment would be needed in order to decide whether a particular entitlement system would be unbiased there. Specifically, what information about preferences is needed in order to verify that the surrogates and lone-dissenter requirements are met? The purpose of this section is to point out that such verification is possible only if the entitlement system accords veto rights very liberally or if a substantial amount is known about agents' preferences.

The justification for this claim is that, in order to verify that the surrogates and lone-dissenter requirements are met, it must be possible to guarantee that entitlements are granted whenever states are related in certain ways determined by preference.

Specifically, in order to verify the surrogates requirement, one needs to be sure that no one strictly prefers any state x to any other state y such that xV_iy for all i. That is, it must be known that no agent is hurt by any change of state which no one has a right to veto. If the entitlement system grants veto rights sparingly, then there will be many such changes which must be known to be weak Pareto improvements (i.e., to hurt no one).

In addition, the lone-dissenter requirement stipulates that in certain situations an agent's ascribed welfare must reflect his actual preference. In particular this is true whenever an agent has a strict preference which is not shared by anyone else.[8] If the entitlement system does not accord some agent the right to veto a change from x to y, then it must be known either that this agent does not strictly prefer x to y or else that some other agent shares the strict preference for x over y. Again, whenever an entitlement is not given, some knowledge about agents' preferences is presupposed in order to verify that a requirement for uniform unbiasedness is met.

Hayek [4, v. 2, pp. 115-120] has stressed that, compared to an activist government, an entitlement system makes very modest demands for central acquisition and processing of information in its operation. The argument just given suggests that the informational requirements to make an *ex ante* choice among alternative entitlement systems (e.g., in a constitutional convention) on the basis of a comparison of their final states would be far more extensive. That is, in order to have either an unbiased government or a government that is biased in a particular way (e.g., towards egalitarianism), the amount of information about citizens' preferences that would be needed to choose an appropriate entitlement system is comparable to the amount of information that a social planner would need to choose an appropriate social welfare function. While Hayek's comparison of the informational requirements of an entitlement system versus an activist government (i.e., one which directly maximizes a social welfare function) *after adoption* of a constitution may be correct, the amount of information that must be gathered *at some stage* in order for government to be deliberate (i.e., in order for the relation between political equilibrium and Pareto efficiency to be foreseen) seems to be approximately equal between the two forms of government.

VII. SUFFICIENT CONDITIONS FOR UNBIASEDNESS

The notion of a system of rights induced by an ascription of welfare to agents in society is intended to serve the role which Coase

[2] has given to a system of 'property rights'. Coase emphasizes that an 'incomplete' assignment of rights would be ineffective; the surrogates requirement and the lone-dissenter requirement specify explicitly in the present theory how many 'veto rights' have to be provided in order for an assignment to be complete. It is further specified that these criteria may not be sufficient if preferences are not first-order distinct or if the ascription of welfare relations to agents is incohesive.

Coase argues that a society with a complete assignment of rights would perform just like a private-goods economy with voluntary exchange: the efficient social states would be the stable outcomes of the process of social change constrained by respect for agents' rights. He supposes that, like the argument given in section three concerning voluntary exchange, the argument for this general proposition would involve only consideration of the relation between rights and preferences, and not of the specific attributes of individuals' preferences or of society's technology. In this respect, the present theory does not conform to Coase's assumption. It will be shown in this section that 'complete' entitlement systems are unbiased in environments which satisfy some geometrical conditions, but examples to be given in the next section will establish that these conditions are required to assure unbiasedness.

The theorem on the finality of efficient states requires two hypotheses which have not been used yet. One is the convexity of the set A of attainable states. The other is a generalization of the minimum-wealth condition of equilibrium theory. This general condition will now be stated. A nonempty subset of N is called a *coalition*. A coalition C is a *proponent coalition* at x if, for some $y \in A$, $\forall i \in N \; i \in C \equiv y P_i x$. A state $z \in A$ is called a *concession* from coalition C at x if $\forall i \notin C \; zP_i x$. It will be required that every proponent coalition can make a concession. A theorem about the efficiency of final states (and hence about sufficient conditions for unbiasedness) can now be stated.

<u>Theorem</u>: Consider a finite society N of agents, $i \in N$ having a preference relation R_i which is a locally-nonsatiated total preordering of a set $X \subseteq E^k$ of social states. For each i, let P_i be the asymmetric part of R_i. Let A be the set of attainable social states for this society, and let $x \in A$ be a Pareto-efficient state in A. Suppose that:

(a) x is an interior point of X

(b) Every P_i is regular at x (i.e. (5), (6) and (7) are equivalent with $Q = P_i$ and $q = p^i$)

(c) $\{P_i\}_{i \in N}$ is first-order distinct at x (i.e. $i \neq j$ implies that $p^i \neq p^j$)

(d) Every proponent coalition at x has a concession at x

(e) A is convex.

Consider an ascription of welfare relations $W_i \subseteq X^2$ to agents in this society. Suppose that:

(f) Every W_i is regular at x (i.e. (5), (6) and (7) are equivalent with $Q = W_i$ and $q = w^i$)

(g) $\{W_i\}_{i \in N}$ is cohesive at x (i.e. $\exists y \in X \; \forall i \in N \; yW_i x$)

(h) The surrogates requirement holds at x (i.e. for all $y \in X$, all $i \in N$, $xP_i y$ implies $\exists j$ not $yV_j x$)

(i) The lone-dissenter requirement holds at x (i.e. for all $y \in X$, all $i \in N$, $xP_i y$ implies [not $yV_i x$ or $\exists j \neq i \; xP_j y$]).

Under assumptions (a) - (i), x is final in A, i.e. not $\exists y \in A \; \exists i \in N \; [yP_i x$ and $\forall j \in N[yR_j x$ or $yV_j x]]$.

The proof of the theorem depends on two lemmas:

<u>Lemma 4</u>: Let $x \in A$, $y \in A$, $yP_i x$ for some $i \in N$, and $\forall j \in N$ $p^j \cdot (y-x) \geq 0$. Then x is not Pareto-efficient in A, if (b), (d) and (e) hold.

<u>Proof</u>: Since $yP_i x$, there is a (non-empty) proponent coalition C for y at x. By regularity, $p^j \cdot (y-x) > 0$ for all $j \in C$. Let $z \in A$ be a concession by C at x. Then $p^j \cdot (z-x) > 0$ for all $j \notin C$ by regularity. Define $u_r = y + rz$. For sufficiently small positive r, $p^j \cdot (u_r - x) > 0$ for all j. For such an r, define $v_t = x + tu_r$. By regularity, $v_t P_j x$ for all j, for sufficiently small positive t. u_r and v_t are in A, by convexity, so x is not efficient in A. q.e.d.

Lemma 5: Let J be a finite set of vectors in E^k and suppose that, for some $z \in E^k$, $\forall q \in J\ q \cdot z > 0$. Let $I \subseteq J$ and suppose that, for some q, $q \in J\ \ K[I]$. Then for some q', $q' \in J \backslash I$ and $q' \notin K[J \backslash \{q'\}]$ or else q' is a positive scalar multiple of another vector $q'' \in J \backslash I$.

This lemma is proved in Appendix C.

Proof of the theorem: Suppose that, contrary to the theorem, x is not final in A. It will be shown that, contrary to hypothesis, either x is not efficient in A or the lone-dissenter requirement is not satisfied at x.

If x is not final in A, let $y \in A$ and yLx (i.e. $\exists i[yP_i x$ and $\forall j[yR_j x$ or $yV_j x]]$). Either $\forall i\ p^i \cdot (y-x) \geq 0$ or $\exists i\ p^i \cdot (y-x) < 0$.

Case 1: $\forall i\ p^i \cdot (y-x) \geq 0$. Then the hypotheses of lemma 4 are satisfied, so x is not efficient, contrary to hypothesis.

Case 2: $p^i \cdot (y-x) < 0$ for some $i \in N$. Let C be the proponent coalition for y at x. Define $D = \{p^j | p^j \cdot (y-x) \geq 0\}$. $C \subseteq D$ by regularity, and $i \notin D$. Define $J = \{p^j\}_{j \in N} \cup \{w^j\}_{j \notin D}$, and define $I = \{p^j\}_{j \in D} \cup \{w^j\}_{j \notin D}$. By lemmas 1 and 2, $\exists z \in X\ \forall q \in J\ q \cdot z > 0$.

$p^j(y-x) \geq 0$ for $j \in D$, and by regularity $w^j \cdot (y-x) \geq 0$ for $j \notin D$. Thus $\forall q \in I\ q \cdot (y-x) \geq 0$. By Farkas' lemma, then, $p^i \notin K[I]$. Thus lemma 5 is applicable. $J \backslash I \subseteq \{p^j\}_{j \notin D}$, so by lemma 5, $p^j \notin K[J \backslash \{p^j\}]$ for some $j \notin D$. By first-order distinctness of preferences at x, $\{p^h\}_{h \neq j} \subseteq J \backslash \{p^j\}$. Since $p^j \notin D$, $xP_j y$ by regularity and local nonsatiation, so $yV_j x$ and thus $w^j \cdot (y-x) \geq 0$, so $p^j \neq w^j$. Therefore $w^j \in J \backslash \{p^j\}$, so $K[w^j, \{p^h\}_{h \neq j}] \subseteq K[J \backslash \{p^j\}]$. Then $p^j \notin K[w^j, \{p^h\}_{h \neq j}]$, which contradicts the lone-dissenter requirement by lemma 3. q.e.d.

VIII. ROLE OF THE GEOMETRIC CONDITIONS

In private-goods environments, the unbiasedness of a market entitlement system does not depend on the geometric conditions (i.e., convexity of preferences and technology, and the minimum-wealth constraint) which are required to assure the existence of supporting prices for efficient allocations. In the theorem of the last section, though, geometric conditions were used as hypotheses. These conditions were satisfied in all of the examples of section five, so whether they are indispensable for the theorem does not affect how the examples are interpreted. However it is of interest to know whether, even when the requirements relating ascribed welfare to preferences are satisfied, there may be efficient states of some environments which are not final. It will now be shown that this is the case. An efficient state can fail to be final if some proponent coalition lacks a concession (example 5), if the set of attainable states is not convex (example 6), or if preference and welfare relations are not regular (example 7).

Local-approximation welfare ascriptions are used in examples 5 and 6, so the surrogates requirement and the lone-dissenter will be satisfied because $P_i \subseteq W_i$ for every i. Example 7 is isomorphic to example 6, so the requirements will be satisfied there also. It will easily be seen that preferences are first-order distinct and welfare ascriptions are cohesive, as well. In each of the following three examples, $N = \{1,2\}$ and $X = E^2$. A state of X will sometimes be denoted by (a,b), where a and b are the horizontal and vertical coordinates, respectively.

Example 5: A proponent coalition lacks a concession (figure 3 (a)): Let $p^1 = w^1 = (1,0)$, and $P_1 = W_1 = \{(x,y) \mid p^1 (x-y) > 0\}$ Let $p^2 = w^2 = (1,0)$. Define (a,b) P_2 (c,d) if $(b - a^2) > (d - c^2)$ and $(a,b) W_2 (c,d)$ if $(-2c,1) \cdot (a-c, b-d) > 0$ [N.B. This is the local approximation to P_2 at (c,d).] Let $A = \{(a,b) \mid b \leq 0\}$. Then (0,0) and (1,0) are in A, $(1,0)P_1(0,0)$ and $(1,0)V_2(0,0)$, so $(1,0)L(0,0)$ and (0,0) is not final in A. However, (0,0) is efficient in A because it is agent 2's most-preferred feasible state. (Specifically it is the only feasible state (a,b) for which $b - a^2 \geq 0$.) This fact also entails that {1}, the proponent coalition of (1,0)at (0,0), has no concession at (0,0).

Example 6: A is not convex (figure 3(b)): Let preference and welfare relations be as in the last example. Define $A = \{(a,b) \mid [a \leq 0 \text{ and } b \leq 4a^2] \text{ or } [a > 0 \text{ and } b \leq 0]\}$. Now (-1/2,1) is a concession of 1 at (0,0). However (0,0) is still efficient because if $(a,b) \in A$ and $(a,b)\ P_2\ (0,0)$, then $b > 0$ so $a < 0$, which implies that $(0,0)P_1(a,b)$. $(1,0)\ L\ (0,0)$ as before, so the efficient state (0,0) is not final.

Example 7: P_1 and W_1 are not regular: Define $f: E^2 \to E^2$ by $f(a,b) = (a, b + 4a^2)$ if $a \leq 0$ and $f(a,b) = (a,b)$ if $a > 0$. The function f is a differentiable 1-1 mapping of the plane onto itself. Define (x,y) to be in a relation P_1 or W_1 in this example if (f(x), f(y)) was in the corresponding relation in example 6, and define $x \in A$ in this example if $f(x) \in A$ in example 6. $f(0,0) = (0,0)$ and $f(1,0) = (1,0)$, so (0,0) and (1,0) are in A still and $(1,0)\ L\ (0,0)$, and (0,0) is still efficient since the image of the Pareto ordering in this example is the Pareto ordering of example 6. A is now the lower half plane,

IX. CONCLUSION

Entitlement theory describes a government which guarantees procedural justice to its citizens, but which does not intervene directly to move society toward particular goals. Some philosophers have argued that ethical restrictions on the institutional form of a government make an entitlement system the only justifiable government. Nozick, who holds this position, calls it a "side-constraint" view. However, most proponents of entitlement systems, even Nozick and others whose advocacy is based on considerations which are not primarily economic, have suggested informally that these governments might be optimal in terms of allocative efficiency.

A particularly interesting claim about the efficiency of an entitlement system is that its equilibria coincide precisely with the Pareto-efficient social states. Depending on one's philosophical convictions, such a government might be favored because its institutional safeguards of rights succeed in preventing any group in the population from benefiting systematically at the expense of others, or else it might be rejected because it fails to assure distributional equity or other substantive conditions which not all efficient states satisfy.

This paper has been devoted to studying the relationship between Pareto efficiency and entitlement equilibrium as a question in economic theory. Throughout, at attempt has been made to isolate the technical issues raised by this relationship from philosophical questions. Now, in conclusion, it is appropriate to consider the import of the results presented here for a libertarian defense of entitlement theory. Specifically, would these results make an entitlement system of government seem more attractive or less, if the coincidence of efficient social states and final states were held to be an important and desirable property? The implications are mixed.

On the positive side, a handful of intuitively natural requirements on the relation between rights and preferences have been shown, under some geometric restrictions on preferences and technology, to be sufficient for an efficient state to be final. In particular, the theorem shows that the presence of externalities in an environment does not *per se* cause entitlement systems to be biased. Moreover, it is evident that in the environments considered in the theorem there are unbiased systems which provide much greater latitude for redistributive social change than to the unanimous-consent systems which previously were known to be unbiased.

However, due to limitations of the analysis offered here, the sufficiency theorem must be interpreted cautiously. These limitations are summarized by the statement that finality is a static cooperative equilibrium concept. It is plausible that a society beginning in a historically determined social state and evolving in a way consistent with the libertarian-successor relation would somehow converge to a final state, but the forces that would lead it to do so (in a direct or "purposeful" way, one would hope), if the relation has cycles, have not been specified. Of more pressing concern is the problem that limitations on the ability of coalitions to forms under laissez-faire, and opportunities for them to engage in strategic manipulation if they do form, may make finality an inappropriate equilibrium concept for a

positive theory of how entitlement systems would perform.

The possibility of strategic manipulation should particularly concern libertarians, who argue that the incentive of those in authority to advance their own welfare rather than 'social welfare' are not adequately taken into account by advocates of centralization. This argument, which boils down to a claim that an institution designed to implement a social welfare function would be subject to strategic manipulation, is not compelling unless libertarians can show that their own institutions are strategy proof.

If it is granted that final states would be reached and supported as equilibria, there remain two results which suggest that, in environments other than private-goods economies, entitlement institutions are likely to be biased. The first such result is the observation that, except for unanimous-consent institutions, entitlement systems cannot be ascertained to be uniformly unbiased unless substantial information about agents' preferences is available. Thus, even though an unbiased system for some environment might exist in principle, there is little reason to believe that it would be discovered through a process of constitutional choice or evolution. The stringent informational requirements for choice *ex ante* of an unbiased constitution have been ignored by libertarian theorists, who have concentrated their attention on the informational parsimony of entitlement systems *ex post*.

The second negative result is that none of the geometric restrictions placed on environments in the sufficiency theorem can be removed. These restrictions correspond closely to the hypotheses used by T. C. Bergstrom [1] to prove the existence of a Lindahl equilibrium. Thus, the same phenomena which would prevent an efficient state from being support by a Lindahl price vector may also cause the state not to be final. In particular, although externality does not _per se_ cause an entitlement system to be biased, the nonconvexity with which externality is often associated (cf. D. Starrett [9]) raises a serious problem. In light of this fact, the inference often drawn from the "Coase Theorem," that complete private-goods markets are representative of entitlement systems in general, should be rejected.

This author's opinion is that the negative results presented here are the most compelling ones. Unbiasedness of an entitlement system, which prior to investigation had seemed like a property that would be easy to guarantee, turns out to be fragile (outside of private-goods environments) except in the case of a particular system (i.e., the unanimous-consent institution) which is widely considered unattractive for other reasons. If unbiasedness is accepted as a criterion of procedural fairness, then whether a pure entitlement system would be more fair in practice than a government which engages in well-defined and carefully limited direct activity (as advocated, for instance, by Rawls) is open to question.

APPENDIX A: UNBIASEDNESS AND FAIRNESS

In welfare economics, Pareto unbiasedness of an institution is commonly accepted as an indication that the institution is procedurally fair. Some libertarians (notably Nozick [6, p. 165 n]) are willing to interpret it this way as well. Such libertarians may hold unbiasedness to be an important and desirable property of a government even if they reject utilitarian justifications of government.

Specifically these libertarians may consider unbiasedness of an entitlement system to refute the criticism implicit in the characterization by Anatole France of "the majestic equalitariansim of the law, which forbids rich and poor alike to sleep under bridges, to beg in the streets, and to steal bread." Such a description suggests that, although laws are written in universal terms which seems to promise equal treatment, in fact they work systematically to the advantage of some persons at the expense of others.

A possible answer to this charge is that, although any institution will affect different persons in different ways accidentally as a result of differences in their circumstances, the law does not discriminate systematically on the basis of persons' intrinsic characteristics. One way to suggest this is to find for each person the social state which he most prefers among all attainable states (typically this most favored state will be Pareto efficient), and to show that it can be supported as an equilibrium. Depending on the initial state of society, then, the law might work to the advantage of any citizen. This idea is what Pareto unbiasedness is intended to express.

The use of unbiasedness to represent fairness raises numerous questions. To begin with, if an entitlement system is adopted in some particular historical circumstance from which it will forseeably lead to a highly skewed distribution of welfare, why should its different performance in some other hypothetical initial situation be allowed as an excuse for this skewed outcome? It is important to recognize that the normative interpretation of the results presented in this paper depends on the answers to this and other such philosophical questions.

APPENDIX B: ON THE "COASE THEOREM"

Coase´s model of rights as market goods has sometimes been interpreted as a formalizatin of entitlement theory. Coase has argued that all equilibria would be efficient, a proposition which is the converse of the one studied here. He considers only market-entitlement systems such as are described in section three of this paper. The ascribed-welfare relations which define those systems are not regular, so the main theorem proved here does not apply directly to Coase´s model. Nevertheless, because a device identical to Coase´s model is employed in a standard existence proof for Lindahl equilibrim and because the conditions given here for finality to be unbiased are the same as those given by Bergstrom [1] for Lindahl equilbrium to be unbiased, the theorem proved here for regular entitlement systems (and also its geometric restrictions) might be expected to hold as well for complete property-rights systems.

There are two formal advantages of the approach taken here to studying entitlement systems over that of Coase. First, equilibrium is defined here explicitly in terms of a cooperative game in characteristic-function form. This enables problems to be squarely faced which Coase does not notice, especially the problem that equilibrium may not exist. Coase assumes that the potential rents from cooperation will in fact be appropriated, but he does not indicate how agreement will be reached concerning their distribution. The unbiasedness theorem proved here indicates that cooperative agreements can be reached in many environments, but Example 4 shows that agreement is not automatic.

Second, the present theory improves on Coase's by representing social states in a "natural" geometrical way. An example of an environment with congestion externalities will make clear what this means. Suppose that there are two agents, each of whom drives a car and each of whom prefers to be alone on the road. Suppose that it is not feasible for these agents to coordinate the times at which they drive, so that the probability that either driver faces congestion is simply the proportion of times that the other is on the road. Intuitively a social state can be represented in two dimensions by specifying the amount of driving that each agent does. However, it requires four dimensions to describe a complete property-rights allocation which specifies both an amount of driving and also a (redundant) probability of facing congestion for each driver. Because each agent in the property-rights market is entitled to veto changes of allocation in two dimensions (i.e., his driving time and his freedom from congestion), which is the full dimension of the space of social states, the property-rights system in this environment is in reality a unanimous-consent institution. It is a much more restrictive entitlement system when viewed in two dimensions (the "natural" space of social states) than it would seem when considered in four.

Moreover, there is a philosophical problem about taking an assignment of rights to be a feature of the social state (i.e., of the allocation of appropriable goods) rather than of a political institution. The exercise of a right needs to be distinguished from

the right itself. A person who owns neither a house nor an atomic bomb may have the right to own a house (i.e., he would be allowed to keep a house if he were to build it or acquire it in trade), but not to own an atomic bomb. The right to own a house is best represented as the right to veto its confiscation in any state (actual or not) where the holder of the right is in possession of a house.

Hayek [4, v. 1, pp. 106-109] has expressed other reservations about the adequacy of the material-property model to represent his version of libertarian theory.

APPENDIX C: PROOF OF LEMMA 5

Lemma 5: Let J be a finite set of vectors in E^k and suppose that, for some $z \in E^k$, $\forall q \in J \; q \cdot z > 0$. Let $I \subseteq J$ and suppose that, for some q, $q \in J\backslash K[I]$. Then for some q', either $q' \in J\backslash I$ and $q' \notin K[J\backslash\{q'\}]$ or else q' is a positive scalar multiple of another $q'' \in J\backslash I$.

Proof: Let $H^* = \{H \subseteq J \mid I \subseteq H, J \not\subseteq K[H]\}$. H^* is finite and nonempty (because $I \in H^*$), and it is partially ordered by $\subseteq$, so it has a maximal element H. $J\backslash K[H]$ is nonempty, and the lemma will be proved if it is shown either to have only one element or to have two elements which are positive scalar multiples of one another. Assume that q' and q" belong to $J\backslash K[H]$. It will be shown that q' is a positive scalar multiple of q".

Since H is maximal in H^*, $q' \in K[H \cup \{q''\}]$ and $q'' \in K[H \cup \{q'\}]$. Let $q' = r_1 q'' + q_1$ and $q'' = r_2 q' + q_2$, where $r_1 > 0$, $r_2 > 0$, $q_1 \in K[H]$ and $q_2 \in K[H]$. From these two equations it follows that $(1-r_1 r_2)q' = q_1 + r_1 q_2$. Since $q' \notin K[H]$, this last equation can only be satisfied if $r_1 r_2 = 1$ and $q_1 = q_2 = 0$. (Note that $q_1 + r_1 q_2 = 0$ only if $q_1 = q_2 = 0$, because otherwise $(q_1 + r_1 q_2) \cdot z > 0$.) Then $q' = r_1 q''$ and $q'' = r_2 q'$. q.e.d.

FOOTNOTES

1. **These are the principal positions in modern constitutional theory. Some markedly different answers have been given, notably by Plato and Marx. The positions are not exclusive. Activists admit that some limits should be put on the government's power of confiscatory redistribution, and proceduralists see the necessity of centrally imposed decisions in emergency situations like war or natural disasters.**

2. **The term 'state' will always refer in this paper to a social state (e.g., to an allocation in a private-goods economy), and will never be used here as a synonym for 'government.'**

3. **The term unbiasedness (or Pareto unbiasedness) is used throughout the paper in this technical sense. This is a well established usage in welfare economics. The relationship of the technical condition to the intuitive notion of unbiasedness is considered in Appendix A.**

4. **Rawls is often misinterpreted as advocating use of a maximin criterion to choose among social states. In fact, he advocates applying the criterion to equilibrium states of institutions in order to choose among the institutions. He puts greater stress on the role of the criterion in justifying rights of the sort modelled here (which enable individuals to veto social states which would be disastrous for them) than on its role in justifying redistributin of endowments. Thus, although he does**

advocate some carefully limited redistributive policies, his theory does not have the predisposition towards activism with which it is frequently charged.

5. It is often suggested that the work of Coase [2] provides an alternative formalization of entitlement theory. The relation between that formalization and the one offered here is discussed in Appendix B. Also Gibbard [3] has formalized a somewhat different libertarian theory.

6. there are several passages in libertarin writings which suggest that the role of government should be to guarantee a system of veto rights. Kayek's statement is that "Coercion can assist free men in the pursuit of their ends only by the enforcement of a framework of universal rules which do not direct them to particular ends, but, by enabling them to create for themselves a protected domain against unpredictable disturbance caused by other men — including agents of government — to pursue their own ends" [4, v. 3, p. 131]. A system of entitlements is envisioned also in Rawls's discussion of the rule of law: "A legal system is a coercive order of public rules addressed to rational persons for the purpose of regulating their conduct and providng the framework for social cooperation. When these rules are just they establish a basis for legitimate expectations. They constitute grounds upon which persons can rely on one another and rightly object when their expectations are not fulfilled" [i, p. 235].

7. This use of the term 'regular' is very closely related, although not precisely identical, to its use in differential topology (i.e., that a function is regular at those points where it has a nonzero gradient). The idea of a regular relation was introduced by Rader [7], who used the term 'directionally dense relation.'

8. If x is in the interior of X, preferences are regular at x, and the requirements of first-order distinctness and cohesiveness are met, then there is a lone dissenter to a change from x to some state y. This follows from Farkas' Lemma and Lemma 5 (to be stated in the next section). Thus a very substantial amount of information will be required to verifying that the lone-dissenter requirement holds everywhere.

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FIGURE 1

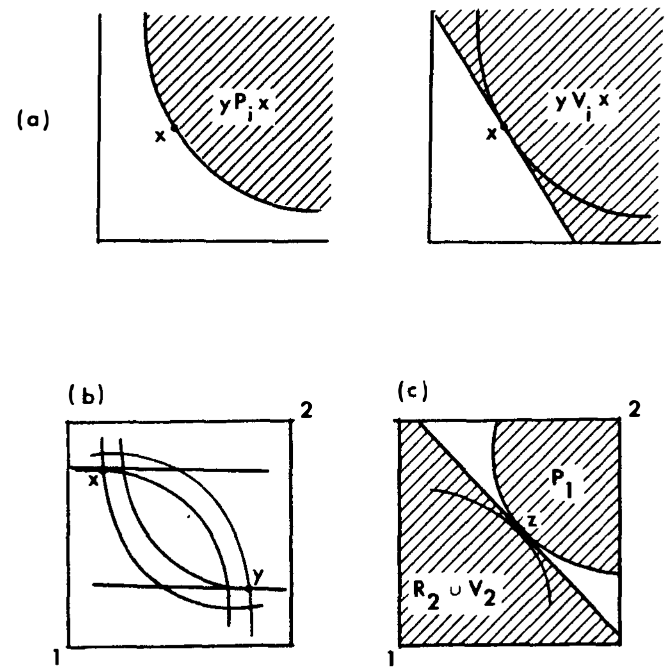

FIGURE 2

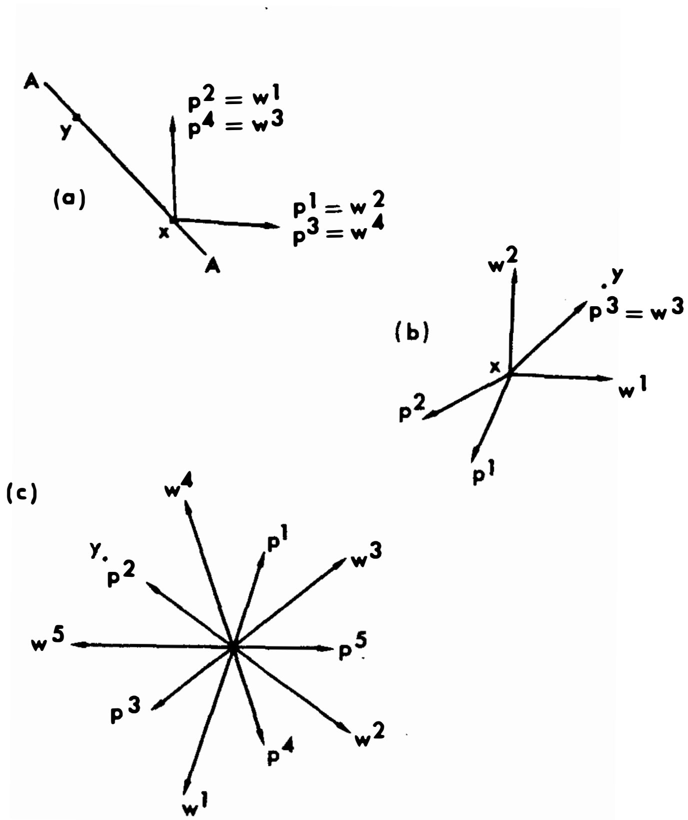

FIGURE 3