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A NOTE ON 'EXPERIMENTAL AUCTION MARKETS AND THE WALRASIAN HYPOTHESIS'*

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ABSTRACT

This note serves to correct an erroneous inference regarding price dynamics and to graphically illustrate the importance of model specification in the context of a very simple and fascinating structure. In an earlier JPE article, Vernon Smith concluded that excess supply by itself was an unreliable predictor of the speed of price adjustment. On the basis of regression procedures applied to experimental data he found that the potential rent to be captured exerts the dominant influence. Two alternative statistical procedures, a Tobit specification and a nonparametric test, dramatically deny this inference. Excess supply dominates excess rent as a predicter of the rate of adjustment, but in fact neither Hypothesis adequately captures the random behavior of price movement.

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Vernon Smith [1965] reports the results of a test of Walrasian versus expected rent hypotheses of price adjustment mechanisms in a double oral auction. On the basis of regression results, he rejects the former in favor of the latter. The purpose of this note is to point out certain statistical details which cast doubt on his result, suggest alternative statistical specifications which, when estimated, reverse the direction of the inference, and further to argue that neither result can be viewed as conclusive. Resolution awaits further detailed theory on price adjustment mechanisms and further emperical evidence.

1. The Data

The data used by Smith consisted of the series of contract prices P_t agreed to in a double oral auction. The market was experimentally controlled with a given number, N_B, of buyers seeking to obtain an artificial commodity for later redemption at $4.20 and a fixed number, N_s, of sellers seeking to sell contracts for the commodity which they "produce" at $3.10. Each agent was allowed to trade a single unit per trading period, yielding market demand and supply schedules as illustrated in figure 1. The Equilibrium price, P_o,

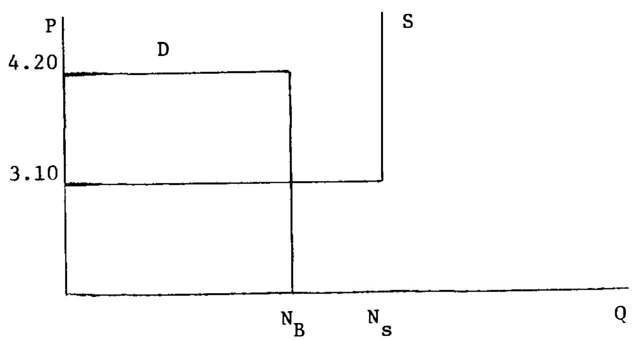
[See Figure 1]

was $3.10 in all trials while market excess supply, $e = N_s - N_B$, varied between trials by design. The designed homogeniety of sellers and buyers implies an excess demand which is constant and independent of the sequence of trades, making the study particularly suitable for the examination of price adjustment.

Each market session consisted of a series of trading periods with the design parameters held constant across periods. The session was terminated after apparent stabilization of contract prices. Smith analyzed the results from six market sessions. N_B was fixed at eleven (11) in all six while N_s varied between sessions with two markets conducted at each of three values, 13, 16, and 19. The analysis reported here includes a seventh market with $N_s = 16$ reported in Smith [1974]. In six of the seven experiments, prices had stabilized at $3.10 and the session was terminated after four trading periods. The seventh market, one with excess demand of two units, required an additional two periods.

FIGURE 1

MARKET DEMAND AND SUPPLY



2. Tests of alternative price adjustment hypotheses

The two alternative adjustment hypotheses considered by Smith were the Walrasian model, in which the rate of price change is proportional to excess supply, and an excess rent hypothesis, in which the rate of change is proportional to excess rent. Letting e_t be the excess supply and P_o be the equilibrium price, the Walrasian model may be written as

$$P_{t+1} = P_t + \beta e_t \quad \text{if right hand side} > P_o \qquad (1)$$
$$= P_o \quad \text{otherwise}$$

and the excess rent model is

$$P_{t+1} = P_t + \alpha e_t (P_t - P_o) \qquad (2)$$

Combining the two models into one equation and adding an additive disturbance, Smith obtained the general stochastic formulation

$$P_{t+1} = P_t + \alpha e_t (P_t - P_o) + \beta e_t + u_t \qquad (3)$$

If the Walrasian hypothesis holds, $\alpha=0$ while $\beta < 0$, and if the excess rent model holds, $\beta = 0$ while $\alpha < 0$.

Using 299[1] observations from the seven experiments, equation (3) was estimated by least squares. The results, with standard errors in parentheses, appear in equation (4) below

$$\hat{P}_{t+1} = P_t - \underset{(.0047)}{.0230}\, e_t (P_t - 3.10) + \underset{(.00108)}{.0011}\, e_t \qquad (4)$$

The coefficient on the excess rent term is significant and negative as expected while the excess supply coefficient has the wrong sign and is not significantly different from zero. Thus we are led to reject the Walrasian model in favor of the excess rent model.[2]

Several statistical details are worthy of note. The apparent decrease in price change dispersion as prices converge might invalidate use of standard tests but would not bias the estimates. A more serious criticism is that serial correlation and lagged prices lead to inconsistent estimates. But a priori the bias could go in either direction.

One problems which does lead to a bias in favor of the excess rent model is the exogenous price floor of \$3.10. Consider the phase diagrams for the two hypotheses in figure 2.

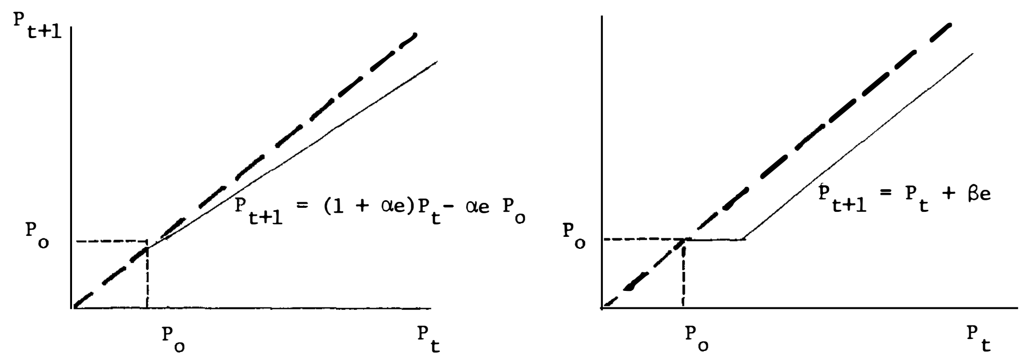
[See Figure 2]

Equation (3) suggests a linear model while the Walrasian hypothesis requires a kink at and flat segment below $P_t = 3.10 - \beta e$. Even in a deterministic environmnet, observations below the kink would tend to flatten the linear approximation to the Walrasian phase line, that is to bias the results in favor or the excess rent phase line. To control for this bias, Smith repeated the least squares estimation of equation (3) after discarding those observations for which P_{t+1} as predicted by equation (4) would lie at or below \$3.10. Equation (5) presents estimates obtained after deleting the 76 observations with P_t = \$3.10

$$\hat{P}_{t+1} = P_t - \underset{(.0067)}{.0201}\, e_t(P_t - 3.10) + \underset{(.0019)}{.00004}\, e_t \qquad (5)$$

FIGURE 2

PHASE DIAGRAMS FOR ALTERNATIVE HYPOTHESES



Excess Rent Hypothesis   Walrasian Hypothesis

Thus, the sample reduction diminishes the support of the excess rent hypothesis slightly but does not alter the test result.

Elimination of observations with small $\hat{P}_t$ does not, however, eliminate the bias. With the exogenous price floor, observations with positive disturbance terms u_t are more likely to remain in the sample than those with negative u_t. That is, the expectation of u_t is not zero and, worse, it varies inversely with P_t. Estimates in equation (5), therefore, are still biased. A formulation which appears to account for the censoring is the Tobit specification

$$\begin{aligned} P_{t+1} &= P_t + \alpha e_t(P_t - P_o) + \beta e_t + u_t && \text{if RHS} > P_o \qquad (6) \\ &= P_o && \text{otherwise} \end{aligned}$$

Assuming $u_t \sim IN(0,\sigma^2)$, α, β and σ may be estimated by maximum likelihood. Such results for the sample of 299 observations appear in equation (7).

$$\begin{aligned} \hat{P}_{t+1} &= P_t - \underset{(.0064)}{.00035}\, e_t(P_t - P_o) - \underset{(.0017)}{.0077}\, e_t && \text{if RHS} > P_o \qquad (7) \\ &= P_o && \text{otherwise} \end{aligned}$$

The inferences drawn from this Tobit specification are the reverse of those from the regression model! Both terms have the hypothesized sign but the Walrasian coefficient is significantly negative while the excess rent coefficient is not.

This last result must, however, be viewed with suspicion. Limited dependent variable models are not robust against misspecification. Unlike the least squares case, for example, even heteroscedasticity causes an asymptotic bias in parameter estimates (see Nelson [1979]). Any one of three likely specification errors (nonnormal disturbances, serial correlation, heteroscedasticity) may bias the results and invalidate the Walrasian versus excess rent comparison.

Application of the asymptotic specification test proposed in Nelson [1979][3] yields a test statistic of 8.064. If the specification of the Tobit model is correct, that statistic should follow an asymptotic chi square distribution with three degrees of freedom. Since it exceeds the 95% critical value of 7.81 we are led to reject the hypothesis of correct specification. The inference favoring the Walrasian model on the basis of the Tobit specification must therefore be regarded as suspect.

Inspection of the phase diagrams in figure 2 suggests an alternative nonparametric test. Since no contracts can be negotiated below \$3.10, downward price movements must clearly be hetergeneous, the potential adjustment depending on current price. There is less reason to reject homogeneous upward movements. (There is an explicit ceiling of \$4.20 but it was never binding. The largest observed contract was at \$3.80). If contract prices are randomly distributed about the phase line such that this distribution is independent of P_t, the likelihood of an upward movement in prices depends only on the distance between the phase line and the forty five degree line for any given P_t. According to the Walrasian model, that distance is constant with respect to P_t but increases with increasing e_t.

According to the excess rent model, the distance increases with both increasing e_t and increasing P_t. An examination of the frequency of price increases at each value of e_t and P_t may, therefore, distinguish between the two theories.[4]

Specifically, the two theories imply the following hypotheses. By the Walrasian model we have

$$H_1: \quad Pr(\Delta P_t > 0 | P_t, e_t) = Pr(\Delta P_t > 0 | P_s, e_s), \quad \text{for } P_t \neq P_s \text{ and } e_t = e_s, \tag{8}$$

while according to the excess rent model, we have

$$H_2: \quad Pr(\Delta P_t > 0 | P_t, e_t) < Pr(\Delta P_t > 0 | P_s, e_s), \quad \text{for } P_t > P_s \text{ and } e_t = e_s. \tag{9}$$

Under both models we have

$$H_3: \quad Pr(\Delta P_t > 0 | P_t, e_t) < Pr(\Delta P_t > 0 | P_s, e_s), \quad \text{for } e_t > e_s \text{ and } P_t = P_s \tag{10}$$

Table 1 below contains the frequency of observations by direction of price change, current price, and level of excess supply. Current price categories have been collapsed to achieve numbers in each cell sufficient to justify the relevant Chi-square tests.

[Table 1]

Contrasting H_1 and H_2 we see that a test of the Walrasian (H_1) versus the excess rent (H_2) models amounts to a test of independence between direction of price change and current price. The Chi-square test of independence can be computed at each level of e_t and aggregated over all e_t. These statistics appear in the last

TABLE 1

FREQUENCY OF OBSERVATIONS BY ΔP, P_t AND e_t

e_t	P_t	$\Delta P \leq o$	$\Delta P > o$	% $\Delta P > o$	χ^2(d.f.)
2	3.10-3.15	21	5	19.2	
	3.20-3.35	35	5	12.5	
	3.40-3.80	26	8	23.5	
	Total	82	18	18.0	1.6 (3)
5	3.10	30	5	14.3	
	3.15-3.20	26	3	10.3	
	3.25-3.45	17	10	37.0	
	3.50-3.80	22	6	21.4	
	Total	95	24	20.2	7.3 (4)
8	3.10	27	8	22.9	
	3.15-3.25	20	6	23.1	
	3.30-3.80	15	4	21.1	
	Total	62	18	22.5	.03 (3)
	Overall	239	60	20.1	8.9 (10)

column of Table 1. None are significantly large at a 90 percent confidence level. It would appear that the likelihood of a price increase is independent of the current price, contrary to the expected rent model.

The Walrasian model fares better, but even it is not wholly supported by the data. The proportion of observations with rising prices increases with excess demand rather than decreases, contrary to H_3. Note, though, that the χ^2 with 3 d.f. for independence of e_t and ΔP, ignoring P_t, is .56 so that the trend is not significant.

It would thus appear that either the assumptions required of this test are violated or that neither model adequately describes the data. Indeed the results so contradict those from the regression and Tobit formulations, which detected negative effects of e_t on price changes, as to cast doubt on their validity. Inspection of the data suggests that the range and variance of price changes is greatest for the e = 8 experiments and lowest for the e = 2 experiments. This may simultaneously explain the contradiction and question the assumptions of the nonparametric test.

3. Summary and Conclusions

This note applies three standard statistical procedures to a sample of experimental data for the purpose of distinguishing between two straightforward hypotheses. The results prove contradictory and inconclusive. In particular, it was argued that (a) Smith's inference favoring the excess rent model on the basis of regression results are suspect, (b) an alternative Tobit specification reverses the inference in favor of the Walrasian hypothesis, (c) the Tobit specification is also suspect, (d) a nonparametric test favors the Walrasian over the excess rent model but in fact rejects both, and (e) that there may be reason to question the results of this nonparametric test as well.

The difficulty lies with the assumption, in all three formulations, that the random component of price changes is independent and identically distributed. Any definitive test would require explicit allowance for the failure of both and, moreover, an explicit accounting of the nature of the dependence and the changing distribution. Consideration of the double oral auction process would suggest that early in a market session there is little information available regarding likely success of bids or offers. Thus the contract prices should exhibit erratic movement. As information about price agreements and their trend is accumulated, it must be incorporated in the process generating new bids and offers and, therefore, contract prices. These later prices, as they stabilize and ultimately converge, must surely be arising from a changing distribution with, specifically, less dispersion.

None of these factors are accounted for in the tests performed here. Note, for example, that the regression formulation of equation (3) and the Tobit model given in (6) do not even allow for convergence to equilibrium! With u_t distributed IID as assumed in both models and with $P_t = P_o$, there is a constant and substantial probability, namely $Pr(u_t > -\beta e_t)$, that the next price will lie above equilibrium. This is true even if prices have been maintained at P_o for many periods. Indeed the models predict that convergence, in the sense of sustained equilibrium prices, never occurs. A modification which dictates that $V(u_t)$ declines with successive trades might artificially correct the problem. But then questions of the rate and form of decline and whether its the same for all groups of traders arise. Indeed that decline in price dispersion ought to be regarded as an explicit and essential component of the price adjustment process itself.

In summary, then, it would appear that, as an approximation, the Walrasian model is a better predictor of the rate of price adjustment than is the excess rent model - the rent to be captured does not appear to affect the adjustment speed. It is also apparent that the Walrasian model is itself not wholly adequate. This deterministic hypothesis made stochastic by the addition of an additive error does not fully capture the behavior of prices. Any definitive analysis of the rate of price adjustment must involve a model in which the random component is an inherent part. The development of such a model is beyond the scope of this note.

FOOTNOTES

*Vernon Smith suggested the possible application of limited dependent variable techniques and Charles Plott provided helpful discussion and encouragement. Viewpoints expressed and responsibility for errors or omission or commission are those of the author.

The seven experiments included a total of thirty (3) trading periods. In all but one trading period, the maximum of eleven trades were observed, yielding 329 contract prices. Data is required on the triple (P_t, P_{t+1}, e_t) for each observation. One observation per period was dropped (the first on P_{t+1} and the last on P_t) to allow for interperiod changes, leaving 299 observations. e_t is of course constant across all observations and trading periods for a given market.

As noted above the sample size used here is larger than the one employed by Smith. He also included a constant term in all regression equations and scaled prices in cents. All regression results reported here and by Smith are qualitatively identical in spite of these two differences. His estimate of equation (3), for example, was

$$P_{t+1} = P_t - \underset{(1.108)}{.6134} - \underset{(.0051)}{.0226}\, ER_t + \underset{(.1952)}{.2198}\, e_t .$$

3. The test statistic is $m = N(\hat{\Theta}_1 - \hat{\Theta}_0)' \; [V(\hat{\Theta}_1) - V(\hat{\Theta}_0)]^{-1} \; (\hat{\Theta}_1 - \hat{\Theta}_0)$ where $\hat{\Theta}_1$ is the observed sum of cross products between the dependent and independent variables,

$$\hat{\Theta}_1 = \frac{1}{N} \sum_1^N P_{t+1} \; (P_t, ER_t, e_t)',$$

and $\hat{\Theta}_0$ is its theoretical expectation according to the specification of the model and evaluated at the maximum likelihood estimates. Under the assumption of correct specification, both $\hat{\Theta}_1$ and $\hat{\Theta}_0$ are consistent and asymptotically normal so that $\sqrt{N}\,(\hat{\Theta}_1 - \hat{\Theta}_0)$ is also normal and m follows an asymptotic chi square distribution. Any of a number of misspecifications will cause $\hat{\Theta}_1$ and $\hat{\Theta}_0$ to diverge in the limit so that under this alternative hypothesis, m will not follow a chi square. For convenient implementation of the test, the model was reestimated, allowing for a coefficient on P_t. These estimates were

$$\hat{P}_{t+1} = \underset{(.052)}{.969}\, P_t + \underset{(.012)}{.0055}\, ER_t - \underset{(.0017)}{.0077}\, e_t.$$

4. Specifically the test used will be a Chi-square test for independence. The test is less powerful than those based on regression or Tobit specifications above, but its assumptions are weaker and it may be more robust against violation of those assumptions. Let Y_t be a bernoulli variable which assumes the value 1 if $\Delta P_t = P_{t+1} - P_t > 0$ and zero otherwise. The assumptions of the Chi-square test are that, under the null hypothesis, Y_t is independently distributed with constant $\Pr(Y_t = 1)$.

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