DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

DISEQUILIBRIUM, SELF-SELECTION AND SWITCHING MODELS

G.S. Maddala
California Institute of Technology
and University of Florida

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ABSTRACT

The present paper outlines the similarities in the structure of self-selectivity models and disequilibrium models. Both these models fall under the category of switching models--with sample separation known and sample separation unknown. Curiously enough the econometric models with self-selectivity are all switching models with sample separation known, whereas the econometric models with disequilibrium are mostly formulated as switching models with unknown sample separation. The paper argues that the reasons for this are that not much attention is devoted to the reasons for the existence of disequilibrium and the models are all formulated as "rationing" models. It is suggested that many empirical applications of disequilibrium fall in the category of "trading" models and here the sample separation is known and the reasons for the existence of disequilibrium are also clear.

TABLE OF CONTENTS

DISEQUILIBRIUM, SELF-SELECTION AND SWITCHING MODELS *

G.S. Maddala

1. INTRODUCTION

The title of this chapter stems from the fact that there is an underlying similarity between econometric models involving disequilibrium and econometric models involving self-selection, the similarity being that both of them can be considered switching structural systems. We will first consider a switching regression model and show how the simplest models involving disequilibrium and self-selection fit in this framework. We will then discuss switching simultaneous equation models.

Suppose the observations on a dependent variable y can be classified into two regimes and are generated by different probability laws in the two regimes. Define

$$y_1 = X\beta_1 + u_1 \tag{1.1}$$

$$y_2 = X\beta_2 + u_2 \tag{1.2}$$

and

$$y = y_1 \text{ iff } Z\alpha - u > 0 \tag{1.3}$$

$$y = y_2 \text{ iff } Z\alpha - u \le 0 \tag{1.4}$$

X and Z are (possibly overlapping) sets of explanatory variables.

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β_1, β_2 and α are sets of parameters to be estimated. u_1, u_2 and u are residuals that are only contemporaneously correlated. We will assume that (u_1, u_2, u) are jointly normally distributed with mean vector 0, and covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{1u} \\ \sigma_{12} & \sigma_2^2 & \sigma_{2u} \\ \sigma_{1u} & \sigma_{2u} & 1 \end{pmatrix}$$

We have set var(u)=1 because, by the nature of the conditions (1.3) and (1.4) α is estimable only up to a scale factor.

The model given by equations (1.1) to (1.4) is called a switching regression model. If $\sigma_{1u} = \sigma_{2u} = 0$ then we have a model with exogenous switching. If σ_{1u} or σ_{2u} is non-zero, we have a model with endogenous switching. This distinction between switching regression models with exogenous and endogenous switching has been discussed at length in Maddala and Nelson (1975).

We will also distinguish between two types of switching regression models.

Model A: Sample separation known.

Model B: Sample separation unknown.

In the former class we know whether each observed y is generated by (1.1) or (1.2). In the latter class we do not have this information. Further, in the models with known sample separation we can consider two categories of models:

Model A-1: y observed in both regimes.

Model A-2: y observed in only one of the two regimes.

We will discuss the estimation of these types of models in the next section. But first, we will give some examples for the three different types of models.

Example 1: Disequilibrium Market Model:

Fair and Jaffe (1972) consider a model of the housing market. There is a demand function and a supply function but demand is not always equal to supply. (As to why this happens is an important question which we will discuss in a later section.) The specification of the model is:

Demand function: $D= X\beta_1 + u_1$

Supply function: $S= X\beta_2 + u_2$

The quantity transacted, Q, is given by

$Q= \mathrm{Min}\ (D,S)$

Thus $Q=X\beta_1 + u_1$ if $D < S$

$Q=X\beta_2 + u_2$ if $D > S$

The condition $D < S$ can be written as:

$$X \left(\frac{\beta_2-\beta_1}{\sigma} \right) - \left(\frac{u_1-u_2}{\sigma} \right) > 0$$

where $\sigma^2 = \mathrm{var}\ (u_1-u_2) = \sigma_1^2 + \sigma_2^2 - 2\sigma_{12}$

Thus the model is the same as the switching regression model in equations (1.1) to (1.4) with $Z = X$, $\alpha = \frac{\beta_2-\beta_1}{\sigma}$ and $u = \frac{u_1-u_2}{\sigma}$

If sample separation is somehow known, i.e. we know which observations correspond to excess demand and which correspond to excess supply, then we have Model A-1. If sample separation is not known, we have Model B.

Example 2: Model with self-selection.

Consider the labor supply model considered by Gronau (1974) and Lewis (1974). The wages offered W_o to an individual, and the reservation wages W_r (the wages at which the individual is willing to work) are given by the following equations:

$$W_o = X\beta_1 + u_1 \qquad W_r = X\beta_2 + u_2$$

The individual works and the observed wage $W = W_o$ if $W_o \geq W_r$. If $W_o < W_r$, the individual does not work and the observed wages are $W = 0$.

This is an example of Model A-2. The dependent variable is observed in only one of the two regimes. The observed distribution of wages is a truncated distribution - it is the distribution of wage offers truncated by the "Self-selection" of individuals - each individual choosing to be 'in the sample' of working individuals or not, by comparing his (or her) wage offer with his (or her) reservation wage.

Example 3: Demand for Durable Goods:

This example is similar to the labor-force participation model in Example 2. Let y_1 denote the expenditures the family can afford to make, and y_2 denote the value of the minimum acceptable car to the family (the threshhold value).

The actual expenditures y will be defined as $y = y_1$ iff $y_1 \geq y_2$

$= 0$ otherwise.

Example 4: Needs vs. Reluctance Hypothesis.

Banks are reluctant to frequent the discount window too often for fear of adverse sanctions from the Federal Reserve. One can define:

y_1 = Desired borrowings

y_2 = Threshhold level below which banks will not use the discount window.

The structure of this model is somewhat different from that given in examples 2 and 3, because we observe y_1 all the time. We do not observe y_2 but we know for each observation whether $y_1 \leq y_2$ (the bank borrows in the Federal funds market) or $y_1 > y_2$ (the bank borrows from the discount window).

Some other examples of the type of switching regression model considered here are the unions and wages model by Lee (1978), the housing demand model by Lee and Trost (1978), and the education and self-selection model of Willis and Rosen (1979).

2. ESTIMATION OF THE SWITCHING REGRESSION MODEL: SAMPLE SEPARATION KNOWN.

Returning to the model given by equations (1.1) to (1.4), we

note that the likelihood function is given by (dropping the t subscripts on u,X,Z,y and I)

$$L\,(\beta_1,\beta_2,\alpha,\sigma_1^2,\ \sigma_2^2,\ \sigma_{12},\ \sigma_{1u},\ \sigma_{2u})$$

$$= \prod_{t=1}^{T} \left[g_1(y-X\beta_1) \int_{-\infty}^{Z\alpha} f_1(u|\,y-X\beta_1)\,du \right]^{I} \left[g_2(y-X\beta_2) \int_{Z\alpha}^{\infty} f_2(u|\,y-X\beta_2)\,du \right]^{1-I} \quad (2\text{-}1)$$

where $I = 1$ iff $Z\alpha - u > 0$

$= 0$ otherwise.

and the bivariate normal density of (u_1,u) has been factored into the marginal density $g_1(u_1)$ and the conditional density $f_1(u|u_1)$, with a similar factorization of the bivariate normal density of (u_2,u). Note that σ_{12} does not occur at all in the likelihood function and thus is not estimable in this model. Only σ_{1u} and σ_{2u} are estimable. In the special case $u = \frac{u_1-u_2}{\sigma}$ where $\sigma^2 = \mathrm{Var}(u_1-u_2)$ as in the examples in the previous section, it can be easily verified that from the consistent estimates of σ_1^2, σ_2^2, σ_{1u} and σ_{2u} we can get a consistent estimate of σ_{12}.

The maximum likelihood estimates can be obtained by an iterative solution of the likelihood equations using the Newton-Raphson method or the Berndt et-al. (1974) method. The latter involves obtaining only the first derivatives of the likelihood function and has better convergence properties. In Lee and Trost (1978) it is shown that the log-likelihood function for this model is uniformly bounded from above. The maximum likelihood estimates of this model can be shown to be consistent and asymptotically efficient following the lines of proof that Amemiya (1973) gave for the Tobit model. To start the iterative solution of the likelihood equations, one can use preliminary consistent estimates of the parameters which can be obtained by using a two-stage estimation method which is described in Lee and Trost (1978)[1], and will not be reproduced here.

There are some variations of this switching regression model that are of considerable interest. The first is the case of the labor supply model where y is observed in only one of the two regimes (Model A-2). The model is given by the following relationships:

$$y = y_1 \quad \text{if} \quad y_1 \geq y_2$$
$$= 0 \text{ otherwise}$$

For the group $I = 1$, we know $y_1 = y$ and $y_2 \leq y$

For the group $I = 0$, all we know is $y_1 < y_2$

Hence the likelihood function for this model can be written as:

$$L(\beta_1, \beta_2, \sigma_1^2, \sigma_2^2, \sigma_{12})$$

$$= \prod_{t=1}^{T} \left[\int_{-\infty}^{g_{2t}} f(g_{1t}, u_{2t})\, du_{2t} \right]^{I_t} \left[1-\Phi_t \right]^{1-I_t}$$

where $g_{1t} = y_t - X_t\beta_1$

$$g_{2t} = y_t - X_t\beta_2$$

$$\Phi_t = \Phi\left[\frac{X_t(\beta_2-\beta_1)}{\sigma}\right]$$

$$\sigma^2 = \text{Var}(u_1-u_2) = \sigma_1^2 + \sigma_2^2 - 2\sigma_{12}$$

$\Phi(.)$ is the distribution function of the standard normal and f is the joint density of (u_{1t}, u_{2t}). Since y is observed only in one of the regimes, we need to impose some identifiability restrictions on the parameters of the model. These restrictions are:

(a) There should be at least one explanatory variable in (1.1) not included in (1.2)

or

(b) $\text{cov}(u_1,u_2) = 0$.

These conditions were first derived in Nelson (1975) and since then have been re-derived by others. The most straightforward way of deriving these conditions (see Maddala (1978) for this) is to note that, in the switching regression model given by equations (1.1) - (1.4).

(i) The probit estimation based on the dichotomous variable I gives us a consistent estimate of α.

(ii) The two-stage estimation based on observation on y in regime 1 gives us consistent estimates of β_1, σ_1^2 and σ_{1u}.

(iii) The two-stage estimation based on observations on y in regime 2 gives us consistent estimates of β_2, σ_2^2 and σ_{2u}.

In the present case, where $\alpha = \frac{\beta_1-\beta_2}{\sigma}$ and $u = \frac{u_2-u_1}{\sigma}$ and there are no observations on y in regime 2, this translates to the fact that we have consistent estimates of only β_1, σ_1^2, $\frac{\sigma_{12}-\sigma_1^2}{\sigma}$, and $\frac{\beta_1-\beta_2}{\sigma}$. If there is one explanatory variable in (1.1) not included in (1.2), then corresponding to this variable, since we have estimates of β_{1j} and $\frac{\beta_{1j}}{\sigma}$ we can get a consistent estimate of σ and hence consistent estimates of all the elements of β_2. From the estimates of σ^2, σ_1^2 and $\frac{\sigma_{12}-\sigma_1^2}{\sigma}$ we now get an estimate of σ_{12} and hence also of σ_2^2. Thus all parameters are estimable. Alternatively, if $\sigma_{12} = 0$, since we have estimates of σ_1^2 and $-\frac{\sigma_1^2}{\sigma}$ we get an estimate of σ. This enables us to get estimates of β_2 and also σ_2^2. Thus again all parameters are estimable.

Note that the identification conditions are the same in the case of example 4 in the previous section where y_1 is observed for all observations. The important fact is that there are no observations on y_2.

The second variation of the switching regression model that has found wide application is where the criterion function determining the switching also involves y_1 and y_2 i.e. equations (1.3) and (1.4) are replaced by

$$y = y_1 \quad \text{iff } I^* > o$$

$$y = y_2 \quad \text{iff } I^* \le o$$

Where
$$I^* = \gamma_1\, y_1 + \gamma_2\, y_2 + Z\alpha - u. \qquad (2.3)$$

Examples of this model are the unions and wages model by Lee (1978) and the education and self-selection model by Willis and Rosen (1979). In both cases, the choice function (2.3) determining the switching involves the income differential $(y_1 - y_2)$. Thus $\gamma_2 = -\gamma_1$. Interest

centers on the sign and significance of the coefficient of $(y_1 - y_2)$.

The estimation of this model proceeds as before. We first write the criterion function in its reduced form and estimate the parameters by the probit method. Note that, for normalization purposes, instead of imposing the condition Var(u) = 1, it is more convenient to impose the condition that the variance of the residual u^* in the reduced form for (2.3) is unity.

$$\text{i.e. } \mathrm{Var}(u^*) = \mathrm{Var}(\gamma_1\, u_1 + \gamma_2\, u_2 - u) = 1 \qquad (2.4)$$

This means that $\mathrm{Var}(u) = \sigma_u^2$ is a parameter to be estimated. But, in the switching regression model, the parameters that are estimable are: β_1, β_2, σ_1^2, σ_2^2, σ_{1u*}, and σ_{2u*} where $\sigma_{1u}^* = \mathrm{cov}(u_1, u^*)$ and $\sigma_{2u}^* = \mathrm{cov}(u_2, u^*)$.

The estimates of σ_{1u*} and σ_{2u*} together with the normalization equation (2.4) give us only 3 equations from which we still have to estimate four parameters σ_{12}, σ_{1u}, σ_{2u} and σ_u^2. Thus, in this model we have to impose the condition that one of the covariances σ_{12}, σ_{1u}, σ_{2u} is zero. The most natural assumption is $\sigma_{12} = 0$.

As for the estimation of the parameters in the choice function (2.3), again we have to impose some conditions on the explanatory variables in y_1 and y_2. After obtaining estimates of the parameters β_1 and β_2, we get the estimated values $\hat{y}_1$ and $\hat{y}_2$ of y_1 and y_2 respectively and estimate the parameters in (2.3) by the probit method using these estimated values for y_1 and y_2. The condition for the estimability of the parameters in (2.3) is clearly that there be no perfect multicollinearity between $\hat{y}_1$, $\hat{y}_2$ and z.

This procedure, called the "two-stage probit method" gives consistent estimates of the parameters of the choice function. Note that since $(y_1-\hat{y}_1)$ and $(y_2-\hat{y}_2)$ are heteroscedastic, the residuals in this two-stage probit method are heteroscedastic. But this heteroscedasticity exists only in small samples and the residuals are homoscedastic asymptotically, thus preserving the consistency properties of the two-stage probit estimates. For a proof of this proposition and the derivation of the asymptotic covariance matrix of the two-stage probit estimates, see Lee (1979).

3. ESTIMATION OF THE SWITCHING REGRESSION MODEL: SAMPLE SEPARATION UNKNOWN

In this case we do not know whether each observation belongs to regime 1 or regime 2. The labor supply model clearly does not fall in this category because the sample separation is known automatically. In the disequilibrium market model, where the assumption of unknown sample separation has been often made, what this implies is that given just the data on quantity transacted and the explanatory variables, we have to estimate the parameters of both the demand and supply functions. Once we estimate these parameters, we can estimate the probability that each observation belongs to the demand and the supply function.

Consider the simplest disequilibrium model with sample separation unknown:

$$D_t = X_{1t}\beta_1 + u_{1t} \quad \text{(Demand function)}$$
$$S_t = X_{2t}\beta_2 + u_{2t} \quad \text{(Supply function)}$$
$$Q_t = \text{Min}\ (D_t, S_t)$$

The probability that observation t belongs to the demand function is:

$$\lambda_t = \text{Prob}\ (D_t < S_t)$$
$$= \text{Prob}\ (u_{1t} - u_{2t} < X_{2t}\beta_2 - X_{1t}\beta_1) \tag{3.1}$$

Let $f(u_1,u_2)$ be the joint density of (u_1,u_2) and $g(D,S)$ the joint density of D and S derived from it.

If observation t is on the demand function, we know that $D_t = Q_t$ and $S_t > Q_t$. Hence,

$$h(Q_t|Q_t = D_t) = \int_{Q_t}^{\infty} g(Q_t,S_t)dS_t/\lambda_t \tag{3.2}$$

The denomination λ_t in (3.2) is the normalizing constant. It is equal to the numerator integrated over Q_t over its entire range. Similarly, if observation t is on the supply function, we know that $S_t = Q_t$ and $D_t > Q_t$. Hence,

$$h(Q_t|Q_t = S_t) = \int_{Q_t}^{\infty} g(D_t,Q_t)dD_t/(1 - \lambda_t) \tag{3.3}$$

Hence, the unconditional density of Q_t is:

$$h(Q_t) = \lambda_t h(Q_t|Q_t = D_t) + (1 - \lambda_t)h(Q_t|Q_t = S_t)$$
$$= \int_{Q_t}^{\infty} g(Q_t,S_t)dS_t + \int_{Q_t}^{\infty} g(D_t,Q_t)dD_t \tag{3.4}$$

The likelihood function is:

$$L = \prod_t h(Q_t) \tag{3.5}$$

As will be shown later, the likelihood function for this model is unbounded for certain parameter values.

Once the parameters in the model have been estimated, we can estimate the probability that each observation is on the demand function or the supply function. Maddala and Nelson (1974) suggest estimating the expressions λ_t in (3.1). These were the probabilities calculated in Sealy (1979) and Portes and Winter (1980). Kiefer (1980) has pointed out that (3.1) does not use all the sample information viz. The data on Q_t. He, therefore, suggests calculating:

$$P(D_t < S_t | Q_t) \tag{3.6}$$

Kiefer derives this probability in a model where u_1 and u_2 are uncorrelated. We will derive it for a general model.

Noting that $P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(B|A) \cdot P(A)}{P(B)}$, we can write (3.6) as:

$$\text{Prob } (D_t < S_t | Q_t) = \frac{\text{Prob } (Q_t | D_t < S_t) \cdot \text{Prob } (D_t < S_t)}{\text{Prob } (Q_t)} \tag{3.7}$$

But the numerator of (3.7), from (3.2), is just

$$\int_{Q_t}^{\infty} g(Q_t, S_t) dS_t.$$

Hence,

$$\text{Prob } (D_t < S_t | Q_t) = \int_{Q_t}^{\infty} g(Q_t, S_t) dS_t / h(Q_t) \tag{3.8}$$

where $h(Q_t)$ is defined in (3.4).

If u_1 and u_2 are independent so that $g(D_t, S_t)$ can be written as $g_1(D_t) \cdot g_2(S_t)$ and G_1 and G_2 are the distribution functions corresponding to the density functions g_1 and g_2 respectively, then (3.8) simplifies to:

$$\text{Prob } (D_t < S_t | Q_t) = \frac{g_1(Q_t)\ [1 - G_2(Q_t)]}{g_1(Q_t)\ [1 - G_2(Q_t)] + g_2(Q_t)\ [1 - G_1(Q_t)]} \tag{3.9}$$

The likelihood function (3.5) simplifies in this case to:

$$L = \prod_t \{g_1(Q_t)[1 - G_2(Q_t)] + g_2(Q_t)\ [1 - G_1(Q_t)]\} \tag{3.10}$$

Noting the relationship between (3.9) the likelihood function and (3.10) or between (3.8) and the corresponding likelihood function (3.5), it is clear that the calculation of the conditional probabilities (3.5) are computationally feasible. Even in a complicated model, these relationships hold good. Note that in a more complicated model (say with stochastic price adjustment equations) to calculate λ_t as in (3.1) we have to derive the marginal distributions of D_t and S_t. To compute (3.8) we need to derive the joint distribution of D_t and S_t. This is the main difference between the expressions (3.1) and (3.8).

There are two major problems with the models with unknown sample separation, one conceptual and the other statistical. The conceptual problem is that we are asking too much from the data when we do not know which observations are on the demand function and which are on the supply function. The results cannot normally be expected to be very good though the frequency with which 'good' results are reported with this method are indeed surprising. For instance, in

Sealey (1979) the standard errors for the disequilibrium model (with sample separation unknown) are in almost all cases lower than the corresponding standard errors for the equilibrium model! Goldfeld and and Quandt (1975) analyze the value of sample separation information by Monte-Carlo methods and Kiefer (1979) analyzes analytically the value of such information by comparing the variances of the parameter estimates in a switching regression model from a joint density of (y,D) and the marginal density of y (where y is a continuous variable and D is a discrete variable). These results show that there is considerable loss of information if sample separation is not known. In view of this, some of the empirical results being reported from the estimation of disequilibrium models with unknown sample separation are surprisingly good. Very often, if we look more closely into the reasons why disequilibrium exists, then we might be able to say something about the sample separation itself. This point will be discussed later in our discussion of disequilibrium models.

The statistical problem is that the likelihood functions for this class of models are usually unbounded unless some restrictions (usually unjustifiable) are imposed on the error variances. As an illustration, consider the model in equations (1.1) to 1.4):

Define $\text{Prob}\ (y=y_1) = \Pi$

$\text{Prob}\ (y=y_2) = 1-\Pi$

The conditional density of y given $y=y_1$ is:

$$f(y|y_1) = f_1(y-X\beta_1)/\Pi.$$

Similarly, $f(y|y_2) = f_2(y-X\beta_2/(1-\Pi)$

Hence, the unconditional density of y is:

$$f(y) = \left[f_1(y-X\beta_1) + f_2(y-X\beta_2)\right]$$

Where f_1 and f_2 are the density functions of u_1 and u_2 respectively. Thus, the distribution of y is the mixture of two normal distributions. Given n observations y_i, we can write the likelihood function as:

$$L = (A_1 + B_1)\ (A_2 + B_2)\text{----}(A_n + B_n) \tag{3.11}$$

where $A_i = \frac{1}{\sigma_1} \exp \left[-\frac{1}{2\sigma_1{}^2} (Y_i - X_i\beta_1)^2\right]$

and $B_i = \frac{1}{\sigma_2} \exp \left[-\frac{1}{2\sigma_2{}^2} (Y_i - X_i\beta_2)^2\right]$

Take $\sigma_2 \neq o$ and consider the behaviour of L as $\sigma_1 \to o$

If $X_1\hat{\beta}_1 = y_1$, then $A_1 \to \infty$ and $A_2, A_3, \text{-----} A_n$ all $\to o$.

But $B_1, B_2, \text{----} B_n$ are finite. Hence $L \to \infty$. Thus, as $\sigma_1 \to o$ the likelihood function tends to infinity if $X_i\hat{\beta}_1 = y_i$ for any value of i. Similarly, if $\sigma_1 \neq o$, then as $\sigma_2 \to o$ the likelihood function tends to infinity if $X_i\hat{\beta}_2 = y_i$ for any value of i.

The case of the disequilibrium model with unknown sample separation is similar. Consider the simplest formulation:

$$D_t = \beta_1 P_t + u_{1t}$$

$$S_t = \beta_2 P_t + u_{2t}$$

$$Q_t = \text{Min}\ (D_t, S_t)$$

with (u_{1t}, u_{2t}) both contemporaneously and serially uncorrelated. If Q is on the demand function, D = Q and S > Q. Similarly, if Q is on the supply function, S = Q and D > Q. Hence likelihood function for this model is of the form (3.11) with

$$A_i = f_1\,(Q_i)\left[1-F_2\,(Q_i)\right]$$
$$B_i = f_2\,(Q_i)\left[1-F_1(Q_i)\right]$$

Where f_1 and f_2 are the density functions of u_1 and u_2 respectively and F_1 and F_2 are the corresponding distribution functions.

Take $\hat{\beta}_1 = \text{Max}\ (\frac{Q_t}{P_t})$

Suppose this maximum is $\frac{Q_k}{P_k}$.

Suppose $\sigma_2 \neq 0$ and consider the behaviour of A_i and B_i as $\sigma_1 \to 0$.

Since $Q_k - \beta_1 P_k = 0$ and $Q_j - \beta_1 P_j < 0$ for $j \neq k$ we will have,

$f_1(Q_k) \to \infty$ and $f_1(Q_j) \to 0$ for $j \neq k$

$F_1(Q_k) \to 1/2$ and $F_1(Q_j) \to 0$ for $j \neq k$

$f_2(Q_j)$ and $F_2(Q_j)$ are finite for all j.

Thus, all B_i will be finite.

$A_k \to \infty$ and all the other A_i will be finite. Hence $L \to \infty$ as $\sigma_1 \to 0$.

In more complicated models, for instance the watermelon market model considered by Goldfeld and Quandt (1975) where unboundedness of the likelihood function is demonstrated, the proof is more complicated, but the structure of the proof is the same as in the simple model above.

This unboundedness does not occur in models with known sample separation, nor does it occur if we assume $\sigma_2^2 = c\sigma_1^2$ where c is known. In this case if one of the $A_i \to \infty$, we can show that some of the other factors $(A_j + B_j) \to 0$. The consequence of the unboundedness of the likelihood function is that in practical work, the maximization of the likelihood function might pose problems in that successive iterations might produce higher and higher values of the likelihood without ever converging. This is the reason why Goldfeld and Quandt imposed the condition $\sigma_2^2 = c\sigma_1^2$ with c known. One can see this problem occuring if the successive higher values of the likelihood correspond to lower and lower values of one of the residual variances. In such cases one might want to fix the values of these residual variances so as to prevent them from declining and then maximize with respect to the other parameters. The maximum we locate this way will of course be a local maximum rather than a global one. However, Amemiya and Sen [1977] show that the true parameter value in this model is a local maximum likelihood estimate even if the global maximum likelihood estimate diverges. Therefore, if we can choose the initial estimate in our iterations close enough to the true value, the local maximum likelihood estimate will converge to the true value.[2]

Goldfeld and Quandt (1980) examine in detail the simple disequilibrium model with unknown sample separation and with $\text{Cov}\,(u_1,u_2) = \zeta\sigma_1\sigma_2$. They derive the likelihood function (3.5) and simplify it by factoring the joint density $g(D_t, S_t)$ into the marginal density of one variable (D_t or S_t, whichever is equal to Q_t) and the conditional density of the other variable.

Define for compactness of notation

$$W_{1t} = \frac{Q_t - X_{1t}\beta_1}{\sigma_1}$$

$$W_{2t} = \frac{Q_t - X_{2t}\beta_2}{\sigma_2} \qquad (3.12)$$

$$Z_{1t} = \frac{1}{\sqrt{1 - \zeta^2}} [W_{2t} - \zeta W_{1t}]$$

$$Z_{2t} = \frac{1}{\sqrt{1 - \zeta^2}} [W_{1t} - \zeta W_{2t}]$$

Then $h(Q_t)$ in (3.4) can be written as:

$$h(Q_t) = \frac{1}{\sigma_1} \phi(W_{1t})[1 - \Phi(Z_{1t})] + \frac{1}{\sigma_2} \phi(W_{2t})[1 - \Phi(Z_{2t})] \qquad (3.13)$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ and the density function and distribution function of the standard normal. Goldfeld and quandt then show that if a set of values for β_1, β_2, σ_1 and σ_2 are chosen such that:

$$W_{1t} + W_{2t} < 0 \text{ for all } t \qquad (3.14)$$

then the likelihood function (3.13) can only increase as $\zeta \to -1$. Similiarly, if

$$W_{2t} > W_{1t} \text{ if } \frac{1}{\sigma_2} \phi(W_{2t}) > \frac{1}{\sigma_1} \phi(W_{1t}) \qquad (3.15)$$

$$W_{2t} < W_{1t} \text{ otherwise}$$

then the likelihood function (3.13) increases as $\zeta \to +1$. Goldfeld and Quandt claim that conditions (3.14) and (3.15) are often encountered in the empirical estimation of this model and, in fact, that this problem is more frequent than the problem of unboundedness of the likelihood function.

The disequilibrium model with unknown sample separation that we have been discussing is a switching regression model with endogenous switching. The case of a switching regression model with exogenous switching and unknown sample separation has been extensively discussed in Quandt and Ramsay (1978) and the discussion that followed their paper.

The model in this case is:

Regime 1: $y_i = X_{1i}'\beta_1 + \varepsilon_{1i}$ with probability λ

Regime 2: $y_i = X_{2i}\beta_2 + \varepsilon_{2i}$ with probability $(1 - \lambda)$

$\varepsilon_{1i} \sim IN(0, \sigma_1 2)$ $\varepsilon_{2i} \sim IN(0, \sigma_2 2)$

As noted earlier, the likelihood function for this model becomes unbounded for certain parameter values. However, the following results are known for this model:

(a) Kiefer (1978) has shown that a root of the likelihood equations corresponding to a local maximum is consistent, asymptotically normal and efficient.

(b) Hartley (1977) suggests an algorithm for the iterative solution of the likelihood equations which, he says, can be shown to be equivalent to the EM algorithm of Dempster, Laird and Rubin (1977) for this problem. He finds, in the limited Monte Carlo experiments he conducted, that convergence to a solution of the likelihood equations corresponding to a local maximum of the likelihood function always obtains and that point estimates are very close to the parameter values (for moderate sample sizes of 100 observations).

The EM method involves substitution of expected values for the missing variables (E-part) and then maximizing the likelihood function (M-part). To implement it in this case Hartley suggests defining an auxiliary variable $Z_i \sim IN(\mu,1)$.

If $Z_i < 0$ then y_i belongs to Regime 1.

If $Z_i > 0$ y_i belongs to Regime 2.

Thus $\lambda = \text{prob}\ (Z_i < 0) = \Phi(-\mu)$.

The case where Z_i are observed corresponds to known sample separation. Since Z_i are "missing", Hartley suggests evaluating $E(Z_i|y_i)$ and substituting these missing values for Z_i. The details of this method are described for a more general case (where λ varies with each observation) in Hartley (1977, 1978) and need not be reproduced here.

(c) Quandt and Ramsay (1978) suggest an MGF (moment generating function) estimator for this model. Note that the moment generating function of y is:

$$E(e^{\theta y}) = \lambda \exp\ [x_1'\beta_1\theta + \frac{\theta^2\sigma_1^2}{2}]$$

$$+ (1 - \lambda) \exp\ [x_2'\beta_2\theta + \frac{\theta^2\sigma_2^2}{2}] \qquad (3.16)$$

Select a set of θ_j (j = 1,2...k) and replace in equation (3.16).

$$E(e^{\theta_j y}) \text{ by } \frac{1}{n}\sum_{i=1}^{n} e^{\theta_j y_i}$$

$$\exp\ (\theta_j x_1'\beta_1) \text{ by } \frac{1}{n}\sum_{i=1}^{n} \exp\ (\theta_j x_{1i}'\beta_1)$$

and

$$\exp\ (\theta_j x_2'\beta_2) \text{ by } \frac{1}{n}\sum_{i=1}^{n} \exp\ (\theta_j x_{2i}'\beta_2)$$

Quandt and Ramsay's MGF method is to estimate the parameters $\gamma = (\lambda,\beta_1,\beta_2,\sigma_1^2,\sigma_2^2)$ by minimizing

$$\sum_{j=1}^{k} \left[\frac{1}{n} \sum_{i=1}^{n} Z_i(\theta_j) - \frac{1}{n} \sum_{i=1}^{n} G(\gamma, x_i, \theta_j)\right]^2 \tag{3.17}$$

where

$$Z_i(\theta_j) = \exp\ (\theta_j y_i)$$

and $G(\gamma, x_i, \theta_j)$ is the value of the expression on the right hand side of (3.16) for $\theta = \theta_j$ and the ith observation.

The normal equations obtained by minimizing (3.17) with respect to γ are the same as those obtained by minimizing

$$\sum_{j=1}^{k} \sum_{i=1}^{n} [Z_i(\theta_j) - G(\gamma, x_i, \theta_j)]^2 \tag{3.18}$$

The normal equations in both cases are

$$\sum_i \sum_j [Z_i(\theta_j) - G(\gamma, x_i, \theta_j)]\frac{\partial G}{\partial \gamma} = 0 \tag{3.19}$$

One major problem with the MGF method is the choice of the θ_j values. Quandt and Ramsay say that the θ_j should be chosen so as to ensure the non-singularity of the equation system (3.19). They derive the asymptotic distribution of the MGF estimates and present some Monte Carlo evidence to show that its performance is satisfactory. The discussants of the Quandt and Ramsay paper pointed out that the authors had perhaps exaggerated the problems with the ML method, that they should compare their method with the ML method, and perhaps use the MGF estimates as starting values for the iterative solution of the likelihood equations.

In summary, there are many problems with the estimation of switching models with unknown sample separation and much more work needs to be done before one can judge the empirical results in this area. The literature on self-selection deals with switching models with known sample separation but the literature on disequilibrium models contains several examples of switching models with unknown sample separation (see Sealey (1979), Rosen and Quandt (1979) and Portes and Winter (1980)). These studies are all based on the hypothesis of the minimum condition holding on the aggregate so that the aggregate quantity transacted switches between being on the demand curve and the supply curve. The validity of this assumption could be as much a problem in the interpretation of the empirical results as the estimation problems discussed above. The problems of aggregation are as important as the problems of estimation with unknown sample separation.

4. SWITCHING SIMULTANEOUS SYSTEMS

We now consider generalizations of the model (1.1) to (1.4) to a simultaneous equation system. Suppose the set of endogenous variables Y are generated by the following two probability laws:

$$B_1Y_1 + \Gamma_1 X = U_1 \tag{4.1}$$

$$B_2Y_2 + \Gamma_2 X = U_2 \tag{4.2}$$

and

$$Y = Y_1 \quad \text{iff} \quad Z\alpha - v > o \tag{4.3}$$

$$Y = Y_2 \quad \text{iff} \quad Z\alpha - v < o \tag{4.4}$$

If v is uncorrelated with U_1 and U_2, we have switching simultaneous systems with exogenous switching. Goldfeld and Quandt (1978) consider models of this kind. Davidson (1978) and Richard (1978) consider switching simultaneous systems where the number of endogenous variables could be different in the two regimes. The switching is still exogenous. An example of this type of model mentioned by Davidson is the estimation of a simultaneous equation model where exchange rates are fixed part of the time and floating the rest of the time. Thus the exchange rate is endogenous in one regime and exogenous in the other regime.

If the residual v is correlated with U_1 and U_2 we have endogenous switching. The analysis of such models proceeds the same way as section 2 and the details, which merely involve algebra, will not be pursued here. (See Lee [1979] for the details). Problems arise, however, when the criterion function in (4.3) and (4.4) involves

some of the endogenous variables in the structural system. In this case we have to write the criterion function in its reduced form and make sure that the two reduced form expressions amount to the same condition. This point can be illustrated by a couple of examples.

Consider the watermelon market model discussed by Goldfeld and Quandt.

Let q = crop of watermelons

p = price of watermelons

x = desired harvest ($x \leq q$)

y = actual harvest

Z_1, Z_2, Z_3 are sets of exogenous variables.

The model consists of the following equations:

$$q = b_1Z_1 + b_2 + u_1 \tag{4.5}$$

$$x = b_3p + b_4q + b_5Z_2 + b_6 + u_2 \text{ if } x < q \tag{4.6}$$

$$= q \text{ otherwise.}$$

$$p = b_7Z_3 + b_8y + b_9 + u_3 \tag{4.7}$$

$$y = \text{Min}\ (x,\ q) \tag{4.8}$$

We have rewritten equations (4.6) in a more illuminating way than in the Goldfeld-Quandt paper. The other equations are the same as in their paper. Goldfeld and Quandt consider the estimation of the model with q unobserved and q observed. Let us consider the latter case. We can divide the observations into two regimes:

$x < q$ in which case $y = x$

$x > q$ in which case $y = q$

The model is essentially a switching simultaneous system which can be written as follows:

Regime 1: $x < q$	Regime 2: $x > q$
$y = x$	$y = q$
$q = b_1Z_1 + b_2 + u_1$	$q = b_1Z_1 + b_2 + u_1$
$x = b_3p + b_4q + b_5Z_2 + b_6 + u_2$	$x = b_3p + b_4q + b_5Z_2 + b_6 + u_2$
$p = b_7Z_3 + b_8x + b_9 + u_3$	$p = b_7Z_3 + b_8q + b_9 + u_3$

The third structural equation is different in the two regimes.

Writing the reduced form for x and q in Regime 1, the condition $x < q$ implies:

$$\frac{1}{1 - b_3b_8}(b_4u_1 + u_2 + b_3u_3) - u_1 < b_1Z_1 + b_2 - \frac{1}{1 - b_3b_8}(b_4b_1Z_1 + b_5Z_2 + b_3b_7Z_3 + b_6 + b_3b_9 + b_4b_2)$$

If we assume $(1 - b_3b_8) > 0$ we can multiply throughout by this factor and get

$$(b_4 + b_3b_8 - 1)\ u_1 + u_2 + b_3u_3 < (1 - b_3b_8 - b_4)(b_1Z_1 + b_2)-(b_5Z_2 + b_6)-b_3(b_7Z_3 + b_9) \qquad (4.9)$$

Similarly, in Regime 2, we obtain the reduced forms for x and q and the condition $x > q$ gives

$$(b_4 + b_3b_8 - 1)u_1 + u_2 + b_3u_3 > (1 - b_3b_8 - b_4)(b_1Z_1 + b_2) - (b_5Z_2 + b_6) - b_3(b_7Z_3 + b_9) \qquad (4.10)$$

Conditions (4.9) and (4.10) are thus mutually exclusive and exhaustive. On the other hand, if $(1 - b_3b_8) < 0$ we would have both the conditions $x < q$ in Regime 2 producting the condition (4.10) thus leading to contradictions. Thus, the condition for logical consistency of this model is $(1 - b_3b_8) > 0$. Since b_3 is expected to be positive and b_8 is expected to be negative, this condition is automatically satisfied in this case.

Such conditions for logical consistency have been pointed out by Amemiya (1974), Maddala and Lee (1976) and Heckman (1978). They need to be imposed in switching simultaneous systems where the switch depends on some of the endogenous variables. Gourieroux et.al. (1978) have derived some general conditions which they call "coherency conditions" and illustrate them with a number of examples. These conditions are derived from a theoreum by Samelson et.al. (1958) which gives a necessary and sufficient condition for a linear space to be partitioned in cones. We will not go into these conditions in detail here. In the case of the switching simultaneous system considered here, the condition they derive is that the determinants of the matrices giving the mapping from (q,x,p) to (u_1,u_2,u_3) are of the same sign.

In regime 1 $(x < q)$ we have:

$$A_1\begin{bmatrix} q \\ x \\ p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -b_4 & 1 & -b_3 \\ 1 & -b_8 & 1 \end{bmatrix}\begin{bmatrix} q \\ x \\ p \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

In regime 2 (x >q) we have:

$$A_2 \begin{bmatrix} q \\ x \\ p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -b_4 & 1 & -b_3 \\ 1 & -b_8 & 1 \end{bmatrix} \begin{bmatrix} q \\ x \\ p \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Hence, $|A_1| = 1 - b_3 b_8$ and $|A_2| = 1$. The condition that both have the same sign gives the condition that $1 - b_3 b_8 > 0$ derived earlier. We will use the conditions derived by Gourieroux et.al. (1978) again in our discussion of multi-market disequilibrium models (section 8).

As yet another example. Consider the model:

$$Y_1 = \gamma_1 Y_2 + \beta_1' X_1 + u_1$$

$$Y_2 = \gamma_2 Y_1 + \beta_2' X_2 + u_2 \quad \text{if } Y_1 < c$$

$$= \gamma_2' Y_1 + \beta_2' X_2 + u_2 \quad \text{if } Y_1 \geq c$$

The two determinants under consideration are $(1-\gamma_1\gamma_2)$ and $(1-\gamma_1\gamma_2')$. The condition for logical consistency of the model is that they are of the same sign or $(1-\gamma_1\gamma_2)\ (1-\gamma_1\gamma_2') > 0$. A question arises about what to do with these conditions. One can impose them and then estimate the model. Alternatively, since the condition is algebraic, if it cannot be given an economic interpretation, it is important to check the basic structure of the model. As an illustration consider the dummy endogenous variable model by Heckman (1976). The model is defined as follows:

Let Y_i = Wages of blacks relative to whites in state i

X_i = vector of exogenous variables

S_i = Sentiment favoring fair employment legislation in state i

D_i = 1 if state i has a fair employment law

= 0 otherwise.

D_i = 1 if $S_i > 0$

= 0 otherwise. (4.11)

The structural equations are:

$$Y_i = X_{1i}\beta_1 + \delta_1 D_i + \gamma_1 S_i + u_{1i} \tag{4.12}$$

$$S_i + X_{2i}\beta_2 + \delta_2 D_i + \gamma_2 Y_i + u_{2i} \tag{4.13}$$

This is a switching simultaneous system which can be written as follows:

Regime 1: $S_i > 0$	Regime 2: $S_i \leq 0$
$Y_i = X_{1i}\beta_1 + \delta_1 + \gamma_1 S_i + u_{1i}$	$Y_i = X_{1i}\beta_1 + \gamma_1 S_i + u_{1i}$
$S_i = X_{2i}\beta_2 + \delta_2 + \gamma_2 Y_i + u_{2i}$	$S_i = X_{2i}\beta_2 + \gamma_2 Y_i + u_{2i}$

The reduced form for S_i in Regime 1 is:

$$S_i = \frac{1}{1-\gamma_1\gamma_2} \quad X_{2i}\beta_2 + \gamma_2 X_{1i}\beta_1 + u_{2i} + \gamma_2 u_{1i} + \delta_2 + \gamma_2\delta_1$$

The reduced form for S_i in Regime 2 is:

$$S_i = \frac{1}{1-\gamma_1\gamma_2} \quad X_{2i}\beta_2 + \gamma_2 X_{1i}\beta_1 + u_{2i} + \gamma_2 u_{1i}$$

Since both of these expressions have to be the same we require that $\delta_2 + \gamma_2\delta_1 = 0$. This is the condition for the logical consistency of the model. The question is whether it can be given a meaningful interpretation.

Consider the interpretation of equations (4.12) and (4.13). Equation (4.12) can be very easily justified since our objective is to study the effect of the passage of the fair employment laws per se after allowing for the sentiment in favor of fair employment laws. It is, however, the "sentiment" equation - equation (4.13) that is hard to justify. How can the passage of the law affect sentiment in the same period? If we set $\delta_2 = 0$, the logical consistency condition $\delta_2 + \gamma_2\delta_1 = 0$ implies either $\delta_1 = 0$ or $\gamma_2 = 0$. But, δ_1 is precisely the coefficient we are interested in. Hence we ought to have $\gamma_2 = 0$. But the reason why Heckman included the variable Y_i in equation (4.13) is to capture the fact that in "states with much market discrimination the demand for antidiscrimination on the part of blacks is high, and through logrolling, this lends to a greater incidence of fair employment legislation in these states" (Heckman [1976] p. 236). This is an important argument but what this says is that it is equation (4.11) that needs to be changed. Equation (4.13) should still be defined with $\delta_2 = \gamma_2 = 0$. But since the passage of the law depends on two factors:

(i) the positive sentiment in favor of blacks

and (ii) the pressure from blacks for passage of such laws because of a low value of y_i due to the presence of market discrimination.

it is not S_i that determines the dummy variable D_i (passage of the law). It is some combination of S_i and y_i^* where y_i^* is the value of y_i for $D_i = 0$ (the wages of blacks relative to whites that would have prevailed in the absence of the law) that determines the passage of the law.

Instead of (4.11) we now have

$$D_i = 1 \text{ if } S_i + \theta y_i^* > 0$$
$$= 0 \text{ otherwise.} \tag{4.11}$$

where $y_i^* = X_{1i}\beta_1 + \gamma_1 S_i + u_{1i}$

and θ measures the weight attached to the pressure from blacks for legislation of fair employment laws.

Define $S_i^* = S_i + \theta y_i^*$

S_i^* is pressure for legislation.

We have

$$S_i^* = (1 + \gamma_1\theta)S_i + X_{1i}(\beta_1\theta) + \theta u_{1i} \tag{4.14}$$

Note that in this formulation S_i is not observed even as a dichotomous variable. We have here a model with an unobserved latent variable S_i, which occurs as an explanatory variable in two equations:

Y_i as given by (4.12)

and S_i^* as given by (4.14)

This is similar to the MIMIC model discussed by Joreskog and Goldberger (1975) except that of the two indicators, one is continuous and the other dichotomous. Estimation of this model is discussed in Maddala

(1980). The important point to note is that a careful examination of the "logical consistency" condition can reveal problems with the original formulation of the model and lead to alternative formulations of the samp problem.

The simultaneous equations models with truncated dependent variables considered by Amemiya (1974) are also switching simultaneous equations models which require conditions for logical consistency. Again, one needs to examine whether these conditions need to be imposed exogenously or whether a more logical formulation of the problem leads to a model where these conditions are automatically satisfied. For instance, Waldman (1979) gives an example of tome allocation of young men to school and work where the model is formulated in terms of underlying behavioural relations and the conditions derived by Amemiya follow naturally from economic theory. On the other hand, these conditions have to be imposed exogenously (and are difficult to give an economic interpretation) if the model is formulated in a mechanical fashion where time allocated to work was modelled as a linear function of school time and exogenous variables and time allocated to school was modelled as a linear function of work time and exogenous variables.

The point of this lengthy discussion is that in switching variables, we often have to impose some conditions for the logical consistency of the model. If these conditions cannot be given a meaningful economic interpretation, it is worthwhile checking the original formulation of the model rather than imposing these conditions exogenously and estimating the parameters in the model subject to these conditions.

An interesting feature of the switching simultaneous systems is that is is possible to have underidentified systems in one of the regimes. As an illustration, consider the following model estimated by Avery (1979):

$$D = \beta_1' X_1 + \alpha_1 Y + u_1 \quad \text{Demand for Durables} \tag{4.15}$$

$$Y_1 = \beta_2' X_1 + \alpha_2 D + u_2 \quad \text{Demand for Debt} \tag{4.16}$$

$$Y_2 = \beta_3' X_3 + \alpha_3 D + u_3 \quad \text{Supply of Debt} \tag{4.17}$$

$$Y = \min\,(y_1, y_2) \quad \text{Actual quantity of Debt} \tag{4.18}$$

D, Y_1, Y_2 are the endogenous variables and X_1 and X_3 are sets of exogenous variables. Note that the exogenous variables in the demand for durables equation and the demand for debt equation are the same.

The model is a switching simulteneous equations model with endogenous switching. We can write the model as follows:

Regime 1: $Y_1 < Y_2$	Regime 2: $Y_2 < Y_1$
$D = \beta_1' X_1 + \alpha_1 Y + u_1$	$D = \beta_1' X_1 + \alpha_1 Y + u_1$
$Y = \beta_2' X_1 + \alpha_2 D + u_2$	$Y = \beta_3' X_3 + \alpha_3 D + u_3$

If we get the reduced forms for Y_1 and Y_2 in the two regimes and simplify the expression $Y_1 - Y_2$, we find that:

$$(Y_1 - Y_2) \text{ in Regime 2} = \frac{1 - \alpha_1\alpha_3}{1 - \alpha_1\alpha_2}\{(Y_1 - Y_2) \text{ in Regime 1}\} \qquad (4.19)$$

Thus, the condition for the logical consistency of this model is that $(1 - \alpha_1\alpha_2)$ and $(1 - \alpha_1\alpha_3)$ are of the same sign - a condition that can also be derived by using the theorems in Gourieroux et al (1978).

The interesting thing to note is that the simultaneous equation system in Regime 1 is under-identified. However, if the system of equations in Regime 2 is identified, the fact that we can get consistent estimates of the parameters in the demand equation for durables from regime 2, enables us to get consistent estimates of the parameters in the Y_1 equation. Thus the parameters in the simultaneous equations system in Regime 1 are identified. One can construct a formal and rigorous proof but this will not be attempted here. Avery (1979) found that he could not estimate the parameters of the structural equation for Y_1 but this is possibly due to the estimation methods used.

The likelihood function for this model is

$$L = \Pi\left[\int_Y^\infty f(D,Y,Y_2)\,dY_2 + \int_Y^\infty f(D,Y_1,Y)\,dY_1\right] \qquad (4.20)$$

where $f(D,Y_1,Y_2)$ is the joint density of D, Y_1, Y_2 derived from equations (4.15) - (4.17).

The ML estimation of this model is not much difficult than the two-stage estimation. The two-stage estimation involves the

following steps:

(i) Write equations (4.16) and (4.17) in their reduced form and estimate these with equation (4.18) using the ML method. However, note that the residuals in these reduced forms are correlated even if the residuals in the structural equations are not.

(ii) Next, obtain consistent estimates $\hat{Y}_1$ and $\hat{Y}_2$ of Y_1 and Y_2 from the estimates of these reduced form equations. Also, let $\hat{\sigma}_1^2$, $\hat{\sigma}_2^2$ and $\hat{\sigma}_{12}$ be the estimates of the variances and covariance between the residuals in these reduced forms. Then a consistent estimate of Y is

$$\hat{Y} = \lambda\hat{Y}_1 + (1 - \lambda)\hat{Y}_2 - \hat{\sigma}\,\phi\,\frac{(\hat{Y}_1 - \hat{Y}_2)}{\hat{\sigma}} \qquad (4.21)$$

where $\lambda = \Phi\left[\frac{(\hat{Y}_1 - \hat{Y}_2)}{\hat{\sigma}}\right]$

$\Phi(\cdot)$ and $\phi(\cdot)$ are respectively the distribution function and the density function of the standard normal, and

$$\hat{\sigma}^2 = \hat{\sigma}_1^2 + \hat{\sigma}_2^2 - 2\,\hat{\sigma}_{12} \qquad (4.22)$$

(iii) Substitute $\hat{Y}$ in (4.15) and estimate it by OLS. This gives the two-stage estimates of the parameters in (4.15).

(iv) Next, obtain a consistent estimate $\hat{D}$ of D and re-estimate the equations (4.16) to 4.18) by ML using the same procedure as in step (i).

This procedure gives consistent estimates of all the structural parameters. The major problem with this procedure is that the standard errors obtained in the second stage are not the correct standard errors. Further, the derivation of the correct asymptotic covariance matrix of the two-stage estimates is much more involved than the derivation in Lee, et al. (1980) which is for the switching simultaneous system with known sample separation. In view of all this, and the fact that the two-step procedure itself involves the use of the ML Procedure twice, it is preferable to estimate this model by the ML method. The paper by Goldfeld and Quandt (1975) demonstrates the feasibility of the ML estimation procedure in such models.

There are also some switching simultaneous equations models where a variable is endogenous in one regime and exogenous in another and, unlike the cases considered by Richard (1978) and Davidson (1978), the switching is endogenous. An example is the disequilibrium model in Maddala (1979) which will be discussed in a later section.

5. DISEQUILIBRIUM MODELS: DIFFERENT FORMULATIONS OF PRICE ADJUSTMENT

Econometric estimation of disequilibrium models has a long history. The partial adjustment models are all disequilibrium models and in fact this is the type of model that the authors had in mind when they talked of "disequilibrium model." Some illustrative examples of this are Rosen and Nadiri (1974), and Jonson and Taylor (1977).

The recent literature on disequilibrium econometrics considers a different class of models and has a different structure. These models are more properly called "rationing models." This literature started with the paper by Fair and Jaffee (1972). The basic equation in their models is

$$Q_t = \text{Min}\ (D_t, S_t) \tag{5.1}$$

where Q_t = quantity transacted

D_t = quantity demanded

S_t = quantity supplied.

Fair and Jaffee considered two classes of models

(i) Directional models: In these we infer whether Q_t is equal to D_t or S_t based on the direction of price movement, i.e.,

$D_t > S_t$ and hence $Q_t = S_t$ if $\Delta P_t > 0$

$D_t < S_t$ and hence $Q_t = D_t$ if $\Delta P_t < 0$

where $\Delta P_t = P_t - P_{t-1}$.

and (ii) Quantitative models: In these the price change is proportional to excess demand (or supply), i.e.,

$$P_t - P_{t-1} = \gamma(D_t - S_t). \tag{5.2}$$

The maximum likelihood estimation of the quantitative model is discussed in Amemiya (1974a). The maximum likelihood estimation of the directional model, and models with stochastic sample separation (i.e., where only (5.1) is used or (5.2) is stochastic) is discussed in Maddala and Nelson (1974).

The directional method is logically inconsistent since the condition that ΔP_t gives information on sample separation implies that P_t is endogenous, in which case there are not enough equations to determine the endogenous variables Q_t and P_t. We will, therefore, discuss only models with the price determination equation (5.2) included. One can then use two-stage least squares methods to estimate these models. Suppose the demand and supply functions are specified as:

$$D_t = \beta_1' X_{1t} + \alpha_1 P_t + u_{1t}$$

$$S_t = \beta_2' X_{2t} + \alpha_2 P_t + u_{2t}$$

Then, Fair and Jaffee (1972) note that we can derive the following equation:

For $\Delta P_t > 0$, we know from (5.2) that $D_t > S_t$. Hence $S_t = Q_t$ and we have:

$$Q_t = \beta_2' X_{2t} + \alpha_2 P_t + u_{1t} \tag{A}$$

Also equation (5.2) can be written as $Q_t = D_t - \frac{1}{\gamma}\Delta P_t$. Hence we have

$$Q_t = \beta_1' X_{1t} + \alpha_1 P_t + u_{1t} - \frac{1}{\gamma}\Delta P_t \tag{B}$$

Similarly, for $\Delta P_t < 0$ we have $D_t < S_t$ and hence $D_t = Q_t$. Thus,

$$Q_t = \beta_1' X_{1t} + \alpha_1 P_t + u_{1t} \tag{B'}$$

Also writing equation (5.2) as $Q_t = S_t - \frac{1}{\gamma}\Delta P_t$ we get

$$Q_t = \beta_2' X_{2t} + \alpha_2 P_t + u_{2t} - \frac{1}{\gamma}\Delta P_t \tag{A'}$$

Combining equations (B) and (B') we get

$$Q_t = \beta_1' X_{1t} + \alpha_1 P_t + \frac{1}{\gamma} Z_{1t} + u_{1t} \tag{C}$$

where
$$Z_{1t} = \begin{cases} -\Delta P_t & \text{if } \Delta P_t > 0 \\ 0 & \text{if } \Delta P_t < 0 \end{cases}$$

Similarly, combining (A) and (A') we get

$$Q_t = \beta_2' X_{2t} + \alpha_2 P_t + \frac{1}{\gamma} Z_{2t} + u_{2t} \qquad \text{(D)}$$

where $$Z_{2t} = \begin{cases} 0 & \text{if } \Delta P_t > 0 \\ -\Delta P_t & \text{if } \Delta P_t < 0. \end{cases}$$

Amemiya (1974a) suggests the following two-stage estimation method: Regress P_t, Z_{1t}, Z_{2t} on all the exogenous variables (using all the observations). Substitute the estimated P_t, Z_{1t}, Z_{2t} in equations (C) and (D) and estimate these by OLS. The resulting two stage estimates can be shown to be consistent.

We need not go through the algebraic details of the ML methods. What we need to discuss is the adequacy of the basic equations. First of all, the condition (5-1) is valid in only "rationing models" of disequilibrium i.e. if there is excess demand, the supply is "rationed out" to the demanders, and similarly, if there is excess supply, the available demand is "rationed out" to the suppliers. As to how this is accomplished, is not usually specified or discussed. The second point is that, there is no reason why the direction of price movement ΔP_t should give any information on excess demand or excess supply. One can visualize

shifts in the demand and supply functions (produced by changes in exogenous variables) such that $P_{t+1} > P_t$ and still there is excess supply in period (t+1). Thus, equation (5.2) is hard to justify, though we will describe one set of conditions under which it makes sense. As for the quantitative method, suppose we argue, as Fair and Jaffee did, that if there is excess demand, prices rise and if there is excess supply, prices fall. The question, however, is when? In this case it may be more meaningful to substitute $\Delta P_{t+1} = P_{t+1} - P_t$ for ΔP_t in equation (5.2) so that we have

$$\Delta P_{t+1} = \gamma(D_t - S_t) \qquad (5.2')$$

This is what Laffont and Garcia (1977) do. What this does to the estimation of the model is that P_t is exogenous and not endogenous at time t. Laffont and Garcia also allow for different speeds of price adjustment for periods of excess demand and excess supply in their quantitative methods. In their re-formulation equation (5.2') is written as

$$\begin{aligned} \Delta P_{t+1} &= \gamma_1 (D_t - S_t) \quad \text{if } D_t > S_t \\ &= \gamma_2 (D_t - S_t) \quad \text{if } D_t < S_t \end{aligned} \qquad (5.3)$$

The method of estimation does not change much with these re-formulations. The only difference is that there is one extra parameter. We can derive the likelihood function for this model following the procedure used by Amemiya (1974a) for the Fair and Jaffee model. The only thing to note is that P_t is exogenous and it is now P_{t+1} that is endogenous.

Let $f_1(Q_t, P_{t+1})$ and $f_2(Q_t, P_{t+1})$ be the joint densities of Q_t and P_{t+1} when $Q_t = D_t$ and $Q_t = S_t$ respectively. To derive f_1 note that

$$Q_t = D_t = \beta_1' X_{1t} + \alpha_1 P_t + u_{1t}$$

and $P_{t+1} - P_t = \gamma_2 (D_t - S_t) = \gamma_2 (Q_t - \beta_2' X_{2t} - \alpha_2 P_t - u_{2t})$

These equations can be written as

$$u_{1t} = Q_t - \beta_1' X_{1t} - \alpha_1 P_t$$

$$u_{2t} = Q_t - \beta_2' X_{2t} - (\alpha_2 - \frac{1}{\gamma_2}) P_t - \frac{1}{\gamma_2} P_t + 1 \qquad (5.4)$$

The Jacobian of the transformation from (u_{1t}, u_{2t}) to $Q_t, P_{t+1})$ is $\frac{1}{\gamma_2}$

Hence $f_1(Q_t, P_{t+1}) = \frac{1}{\gamma_2} g_1(u_1, u_2)$ and we substitute the expressions in (5.4) in the observed variables.

The derivation of f_2 is similar. We have

$$Q_t = S_t = \beta_2' X_{2t} + \alpha_2 P_t + u_{2t}$$

and

$$P_{t+1} - P_t = \gamma_1 (D_t - S_t) = \gamma_1 (\beta_1' X_{1t} + \alpha_1 P_t + u_{1t} - Q_t)$$

These equations can be written as:

$$u_{1t} = Q_t - \beta_1' X_{1t} - (\alpha_1 - \frac{1}{\gamma_1}) P_t - \frac{1}{\gamma_1} P_{t+1}$$

$$u_{2t} = Q_t - \beta_2' X_{2t} - \alpha_2 P_t$$

The Jacobian of the transformation from (u_{1t}, u_{2t}) to (Q_t, P_{t+1}) is $\frac{1}{\gamma_1}$

Hence $f_2(Q_t, P_{t+1}) = \frac{1}{\gamma_1} g_2(u_1, u_2)$

where $g_2(u_1, u_2)$ is the density of (u_1, u_2) with the expressions in (5.5) substituted for u_1 and u_2.

Finally, the likelihood function to be maximized is:

$$L = \prod_{\Delta P_{t+1} < 0} f_1(Q_t, P_{t+1}) \prod_{\Delta P_{t+1} > 0} f_2(Q_t, P_{t+1})$$

Thus, the likelihood function and estimation problems are not much altered by making the alternative assumptions in equations (5.3) instead of (5.2)[3].

Though the modifications of the price-adjustment equations suggested by Laffont and Garcia given by equations (5.3) make sense, there is an alternative interpretation of the price adjustment equation that one can think of under which equation (5.2) is more meaningful. This interpretation actually goes to the root of the question as to why disequilibrium exists at all.

Let P_t^* be the price that equilibrates demand and supply. If there are no costs of price adjustment, then $P_t = P_t^*$ and we have an equilibrium model. On the other hand if firms cannot adjust prices immediately (even though they know the market clearing price), we have a partial adjustment model:

$$P_t - P_{t-1} = \lambda\,(P_t^* - P_{t-1}) \qquad 0<\lambda<1 \qquad (5.6)$$

$$= \lambda(P_t^* - P_t + P_t - P_{t-1})$$

Hence $$P_t - P_{t-1} = \frac{\lambda}{1-\lambda}\,(P_t^* - P_t). \qquad (5.7)$$

If $P_t < P_t^*$ there will be excess demand and if $P_t > P_t^*$ there will be excess supply. Hence, if $\Delta P_t<0$ we have a situation of excess supply.

Note that in this case it is ΔP_t (not ΔP_{t+1} as in the Laffont-Garcia case) that gives the sample separation. But the interpretation is not that prices rise in response to excess demand (as implicitly argued by Fair and Jaffee) but that there is excess demand (or excess supply) because prices do not fully adjust to the equilibrating values.[4]

Equation (5.7) can also be written as

$$P_t - P_{t-1} = \gamma(D_t - S_t) \qquad (5.8)$$

if we assume that the excess demand $(D_t - S_t)$ is proportional to the difference $(P_t^* - P_t)$, i.e., the difference between the equilibrating price and the actual price. The interpretation of the coefficient γ in (5.8) is of course different from what Fair and Jaffee gave to the same equation.

Thus, there are two interpretations of the price adjustment equations that one can think of:

(i) Prices rise or fall in response to excess demand or supply. Here the formulation of Laffont and Garcia using ΔP_{t+1} makes more sense than the formulation of Fair and Jaffee using ΔP_t.

(ii) Excess demand or excess supply exist because prices do not adjust fully (due to costs of adjustment) to the equilibrium level. Here the formulation of Fair and Jaffee makes more sense than that of Laffont and Garcia.

Different speeds of upward and downward adjustment as in equation (5.4) can also be derived in the partial adjustment framework. Consider the following formulation:

$$P_t - P_{t-1} = \lambda_1\,(P_t^* - P_{t-1}) \text{ if } P_t^* > P_{t-1}$$

$$= \lambda_2\,(P_t^* - P_{t-1}) \text{ if } P_t^* < P_{t-1} \qquad (5.9)$$

These equations imply

$$P_t - P_{t-1} = \frac{\lambda_1}{1-\lambda_1}\,(P_t^* - P_t) \text{ if } P_t^* > P_t. \qquad (5.10)$$

$$= \frac{\lambda_2}{1-\lambda_2}\,(P_t^* - P_t) \text{ if } P_t^* < P_t.$$

Note first that the conditions $P_t^* > P_{t-1}$, $P_t > P_{t-1}$, $P_t^* > P_t$ and $D_t > S_t$ are all equivalent. Also assuming that excess demand is proportional to $P_t^* - P_t$ we can write equations (5.10) as

$$\Delta P_t = \gamma_1(D_t - S_t) \text{ if } D_t > S_t$$

$$= \gamma_2(D_t - S_t) \text{ if } D_t < S_t$$

Again note that we get ΔP_t and not ΔP_{t+1} in these equations.

Ito and Ueda (1979) use Bowden's formulation with different speeds of adjustment as given by (5.9) to estimate the rates of adjustment in interest rates for business loans in the U.S. and Japan. They prefer this formulation to that of Fair and Jaffee or Laffont and Garcia because in equation (5.9), λ_1 and λ_2 are pure numbers which can be compared across countries. The same cannot be said about the parameters γ_1 and γ_2 in equation (5.4).

There is still one disturbing feature about the partial adjustment equation (5.6) that Bowden adopts and under which we have given a justification for the Fair and Jaffee directional and quantitative methods. This is that ΔP_t unambiguously gives us an idea about whether there is excess demand or excess supply. As mentioned earlier this does not make intuitive sense. On closer examination one sees that the problem is with equation (5.6), in particular the assumption that λ lies between 0 and 1. This is indeed a very strong assumption and implies that prices are sluggish but never change to overshoot P_t^* the equilibrium prices. There is, however, no apriori reason why this should happen. Once we drop the assumption that λ should lie between 0 and 1, it is no longer true that we can use ΔP_t to classify observations as belonging to excess demand or excess supply. As noted earlier the assumption $0<\lambda<1$ implies that the conditions $P_t^* > P_{t-1}$, $P_t > P_{t-1}$, $P_t^* > P_t$ and $D_t > S_t$ are all equivalent. With $\lambda>1$, this no longer holds good.

To see the full implications of the partial adjustment equation (5.6), define Q_t^* as the quantity that would be transacted if the market were to be in equilibrium. Consider the situation in Figure 1 where the supply function SS is stable but the demand function shifts

to the left. Then $P_t^* < P_{t-1}^*$ and $Q_t^* < Q_{t-1}^*$. As for P_{t-1}, we do not really know whether at time (t - 1), the market was in equilibrium or not. But for any value of P_{t-1} between X and P_t^*, the value of Q_{t-1} would be greater than Q_t^* but less than Q_{t-1}^*. As for Q_t, a partial adjustment model for P_t given by (5.6) will trace out points along YZ and so Q_t would be less than Q_t^*. Thus the partial adjustment equation for prices given by (5.6) implies a quantity adjustment equation.

$$Q_t - Q_{t-1} = \mu(Q_t^* - Q_{t-1}) \text{ where } \mu > 1. \qquad (5.11)$$

Of course in this particular case if P_{t-1} were to be $< P_t^*$ then the value of μ in the quantity adjustment equation (5.11) also satisfies $0 < \mu < 1$. But we can show that anything is possible for the parameter μ in the quantity adjustment equation (5.11).

If Q_t and P_t are both simultaneously determined. as the model says they are, then it is inconsistent to have a price adjustment equation of the form (5.6) in isolation that leads to a quantity adjustment equation of the form (5.11). The solution out of this dilemma is either to drop the minimum condition (5.1) or drop the assumption that P_t is endogenous. If one has a market clearing model, of course one should not consider equation (5.1) in the first place. But if one is dealing with a rationing model, then one has to live with condition (5.1). The logical thing then is to drop equation (5.6). Obviously, equation (5.6) itself implies that prices are being "adjusted" or set by <u>someone</u>. If this is the case, P_t has to be exogenous and a price "adjustment" equation like (5.2') suggested by Laffont and Garcia is more reasonable. But the proper way to look at this equation is

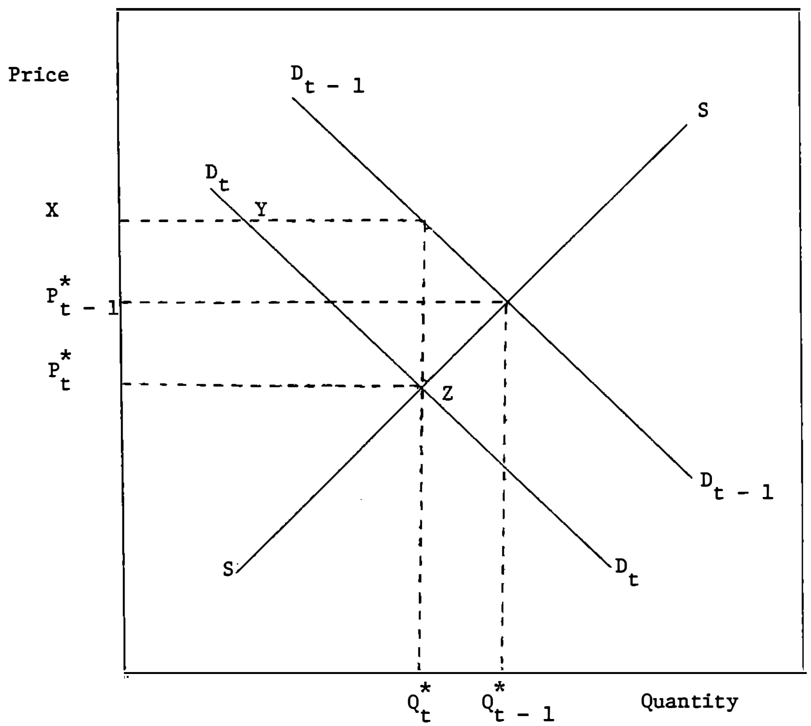

Figure 1

as a forecast equation for future prices, in which case equation (5.2') needs some further thinking. The source of disequilibrium now is not imperfect "adjustment" of prices but imperfect forecasts of market equilibrating prices.

6. DISEQUILIBRIUM MODELS: CONTROLLED PRICES AND "RATIONING" VS. "TRADING" MODELS.

The most important criticism one can level against the

econometric literature on disequilibrium is that there is usually no discussion of why disequilibrium exists in the first place. There are two major sources of disequilibrium:

(1) Imperfect adjustment of prices

(2) Controlled prices

In the previous section we discussed the case of imperfect adjustment to the market equilibrating price. In this section we will discuss the case of controlled prices.

The case of controlled prices is different from the case of fixed prices. The disequilibrium model considered earlier in example 1, section 1 is one with fixed prices. With fixed prices, the market is almost always in disequilibrium. With controlled prices, the market is sometimes in equilibrium and sometimes in disequilibrium.

Consider the following model:

$$D_t = X_{1t}\beta_1 + \alpha_1 P_t + u_{1t} \tag{6.1}$$

$$S_t = X_{2t}\beta_2 + \alpha_2 P_t + u_{2t} \tag{6.2}$$

D_t is quantity demanded.

S_t is quantity supplied.

P_t is price.

X_{1t} and X_{2t} are explanatory variables.

u_{1t} and u_{2t} are residuals which are only contemporaneously correlated.

Let P_t be controlled to lie between $\overline{P}_{1t}$ and $\overline{P}_{2t}$. i.e. $\overline{P}_{1t} < P_t < \overline{P}_{2t}$.

There are several examples of this. In the case of natural gas $\overline{P}_{2t}$ is the price-ceiling. There is no price-floor and hence $\overline{P}_{1t} = -\infty$. In the case of price supports for agricultural commodities, $\overline{P}_{1t}$ is the price-floor. There is no price-ceiling. Hence $\overline{P}_{2t} = \infty$. In the case of commodity futures markets, there are both lower and upper limits for the price variation.

In all these models, if the equilibrating price is within the specified limits, we have equilibrium and the quantity transacted, Q_t, is given by $D_t = S_t = Q_t$. In this case both P_t and Q_t are endogenous variables. If the price falls outside the specified limits, P_t is exogenous and we have disequilibrium. We thus have a switching simultaneous system where P_t is sometimes endogenous and sometimes exogenous. Earlier, Barten and Bronsard (1970) derived some two-stage least squares estimators for the case where a regressor may be exogenous or endogenous at different times. Richard (1978) studied some wider aspects of this problem and Davidson (1978) derived the exact maximum likelihood estimators for a fairly general class of models involving shifts between the endogenous and exogenous variables. But the switching between regimes considered in these papers is exogenous rather than endogenous as in the model we are considering. It is a consequence of some abrupt institutional changes or policy changes like shifts from fixed to floating echange rates. Shifts in Federal Reserve policy from manipulation of interest rates to control of money supply etc. By contrast, the switch in our model is produced by controls in the market equilibrating price.

We can classify the observations into three regimes:

Regime 1: $\overline{P}_{1t} < P_t < \overline{P}_{2t}$. Denote this set of points by ψ_1. These are the equilibrium points and Q_t and P_t are both endogenous.

Regime 2: $P_t \geq \overline{P}_{2t}$. Denote this set of points by ψ_2. As to what happens to Q_t depends on whether we are considering a "rationing model" or a "trading model." In the case of natural gas, $Q_t = S_t$ and $D_t \geq Q_t$. Also $\overline{P}_{2t}$ is an exogenous variable. In "trading models," since no trading takes place $Q_t = 0$.

Regime 3: $P_t \geq P_{1t}$. Denote this set by ψ_3. This set corresponds to excess supply. As to what happens to Q_t depends on the type of model we are considering. In "rationing models," we have $Q_t = D_t$, $S_t \geq Q_t$ and $\overline{P}_{1t}$ is exogenous. In the case of agricultural price supports, we observe both D_t and S_t in this regime since we know the market demand and the surplus purchased by the government. In the case of "trading models," since no trading takes place, we have $Q_t = 0$.

The appropriate likelihood functions for the different classes of models are as follows:

For a model with price-ceiling and "rationing" the likelihood function is:

$$L = \prod_{\psi_1} f(Q_t, P_t) \cdot \prod_{\psi_2} \int_{Q_t}^{\infty} g(D_t, Q_t)\, dD_t \tag{6.3}$$

where $f(Q_t,P_t)$ is the joint density of Q_t and P_t derived from the joint density of u_1,u_2) as in any simultaneous equations model, and $g(D_t,S_t)$ is the joint density of D_t and S_t derived from the joint density of (u_1, u_2) treating $P_t = \overline{P}_t$ and exogenous. Note that the jacobian of transformation for $f(Q_t,P_t)$ is $|\alpha_1 - \alpha_2|$ which is expected to be nonzero since α_1 and α_2 are of opposite signs and nonzero. The jacobian of transformation for $g(D_t,S_t)$ is, of course, unity.

For a model with price supports as in agricultural commodity programs (no rationing) the likelihood function is:

$$L = \prod_{\psi_1} f(Q_t,P_t) \prod_{\psi_3} g(D_t,S_t) \tag{6.4}$$

where $f(Q_t,P_t)$ is as defined in (6.3) and $g(D_t,S_t)$ is the joint density of D_t and S_t derived from the joint density of (u_1,u_2) treating $P_t = \overline{P}_{1t}$ as exogenous.

For a model with both price ceilings and price floor, and "rationing," the likelihood function is

$$L = \prod_{\psi_1} f(Q_t,P_t) \cdot \prod_{\psi_2} \int_Q^\infty g_2(D_t,Q_t)dD_t \cdot \prod_{\psi_3} \int_Q^\infty g_1(Q_t,S_t)dS_t \tag{6.5}$$

where g_1 and g_2 are the joint densities of D_t and S_t derived from (1) and (2) after substituting $P_t = \overline{P}_{2t}$ respectively.

For a "trading" model where no transactions take place if there is excess demand or excess supply, the likelihood function is:

$$L = \prod_{\psi_1} f(Q_t,P_t) \quad \prod_{\psi_2} \int_{\overline{P}_{2t}}^{\infty} g(P_t)dP_t \quad \prod_{\psi_3} \int_{-\infty}^{\overline{P}_{1t}} g(P_t)dP_t \tag{6.6}$$

where $g(P_t)$ is the distribution of the equilibrium price P_t, i.e., the distribution of P_t derived from the reduced form equation for P_t implied by the structural equations (6.1) and (6.2).

In practice, with commodity trading, there will be a series of trades that take place at different prices within the admissible range $\overline{P}_{1t} \leq P_t \leq \overline{P}_{2t}$. In this case all we observe is Q_t (the total volume of trading) and all we know was that price P_t was within the admissible range. In this case, the first term in the likelihood function (6.6) should be changed to:

$$\prod_{\psi} \int_{\overline{P}_{1t}}^{\overline{P}_{2t}} f(Q_t,P_t)dP_t$$

Note that here we have a simultaneous equations model with two endogenous variables P_t and Q_t. Q_t is observed only if P_t is within a specified range and P_t is observed only in a qualitative way - which of the three different sets it belongs to. The parameters of the demand and supply functions (6.1) and (6.2) can be estimated with these data.

Further details of estimation, two stage methods, and estimation of the market equilibrating price if controls are removed are discussed in Maddala (1979) and will not be repeated here.

7. TESTS FOR DISEQUILIBRIUM:

There have been many tests suggested for the "disequilibrium hypothesis" i.e., to test whether the data have been generated by an equilibrium model or a disequilibrium model. Quandt (1978) discusses several tests and says that there does not exist a uniformly best procedure for testing the hypothesis that a market is in equilibrium against the alternative that it is not.

A good starting point for "all" tests for disequilibrium is to ask the basic question of what the disequilibrium is due to. In the case of the partial adjustment model given by equation (5.7), the disequilibrium is clearly due to imperfect adjustment of prices. In this case the proper test for the equilibrium vs. disequilibrium hypothesis is to test whether $\lambda = 1$. As discussed in section 5, this leads to a test that $\frac{1}{\gamma} = 0$ in the Fair and Jaffee quantitative model, since γ is proportional to $\frac{1}{1-\lambda}$. This is the procedure Fair and Jaffee suggest. However, if the meaning of the price adjustment equation is that prices adjust in response to either excess demand or excess supply, then as argued in Section 5, the price adjustment equation should have ΔP_{t+1} not ΔP_t, and also it is not clear how one can test for the equilibrium hypothesis in this case. The intuitive reason is that now the price adjustment equation does not give any information about the source of the disequilibrium.

Quandt (1978) argues that there are two classes of disequilibrium models which are:

(a) Models where it is known for which observations $D_t < S_t$ and for which $D_t > S_t$ i.e., the sample separation is known, and

(b) Models in which such information is not available.

He says that in case (a) the question of testing for disequilibrium does not arise at all. It is only in case (b) that it makes sense.

The example of the partial adjustment model (5.7) is a case where we have sample separation given by ΔP_t. However, it still makes sense to test for the disequilibrium hypothesis which in this case merely translates to a hypothesis about the speed of adjustment of prices to levels that equilibrate demand and supply. Adding a stochastic term u_{3t} to the price adjustment equation does not change the test. When $\lambda=1$ this says $P_t = P_t^* + u_{3t}$.

There is considerable discussion in Quandt's paper on the question of nested vs. non-nested hypothesis. Quandt argues that very often the hypothesis of equilibrium vs. disequilibrium is non-nested i.e. the parameter set under the null hypothesis that the model is an equilibrium model is not a subset of the parameter set for the disequilibrium model. The problem in these cases may be that there is no adequate explanation of why disequilibrium exists in the first place.

Consider for instance, the disequilibrium model: with the demand and supply functions specified by equations (6.1) and (6.2).

Quandt argues that if one takes the limit of the likelihood function for this model with price adjustment equation as:

$$\Delta P_t = \gamma(D_t - S_t) + u_{3t} \quad (7.1)$$

and $\sigma_{23} = \mathrm{Cov}(u_2,u_3) = 0$

$\sigma_{13} = \mathrm{Cov}(u_1,u_3) = 0$

$\sigma_3^2 \neq 0$

and $\gamma \to \infty$

then we get the likelihood function for the equilibrium model ($Q_t = D_t = S_t$) and thus the hypothesis is "nested"; but that if $\sigma_3^2 = 0$, the likelihood function for the disequilibrium model does not tend to the likelihood function for the equilibrium model even if $\gamma \to \infty$ and thus the hypothesis is not nested. The latter conclusion, however, is counter-intuitive and if we consider the correct likelihood function for this model derived in Amemiya (1974) and if we take the limits as $\gamma \to \infty$, we get the likelihood function for the equilibrium model.

Quandt also shows that if the price adjustment equation is changed to

$$\Delta P_{t+1} = \gamma(D_t - S_t) + u_{3t} \quad (7.2)$$

then the limit of the likelihood function of the disequilibrium model

as $\gamma \to \infty$ is not the likelihood function for the equilibrium model. This makes intuitive sense and is also clear when we look at the likelihood functions derived in section 5. In this case the hypothesis is nonnested, but the problem is that as discussed earlier, this price adjustment equation does not tell us anything about what disequilibrium is due to. As shown in section 5, the price adjustment equation (7.1) follows from the partial adjustment equation (5.7) and thus throws light on what disequilibrium is due to, but the price adjustment equation (7.2) says nothing about the source of the disequilibrium. If we view the equation as a forecast equation, then the disequilibrium is due to imperfect forecasts of the market equilibrating price. In this case it is clear that as $\gamma \to \infty$, we do not get perfect forecasts. What we need to have a nested model is a forecasting equation which for some limiting values of some parameters yields perfect forecasts at the market equilibrating prices.

Consider now the case where we do not have a price adjustment equation and the model merely consists of a demand equation and a supply equation. Now, clearly the source of the disequilibrium is that P_t is exogenous. Hence the test boils down to testing whether P_t is exogenous or endogenous. The methods developed by Wu (1973) and Hausman (1978) would be of use here.

In summary, tests for disequilibrium should be based on a discussion of the source of disequilibrium. The test would then be a test of a nested hypothesis, and what the appropriate test is would be obvious from a statement of the problem.

8. MULTIMARKET DISEQUILIBRIUM MODELS

The analysis in the preceeding sections on single market disequilibrium models has been extended to multimarket disequilibrium models by Gourieroux et.al. (1980) and Ito (1980). Quandt (1976) first considered a two-market disequilibrium model of the following form: (the exogenous variables are omitted):

$$\begin{aligned} D_{1t} &= \alpha_1 Q_{2t} + U_{1t} \\ S_{1t} &= \beta_1 Q_{2t} + U_{2t} \\ D_{2t} &= \alpha_2 Q_{1t} + V_{1t} \\ S_{2t} &= \beta_2 Q_{1t} + V_{2t} \end{aligned} \qquad (8.1)$$

$$\begin{aligned} Q_{1t} &= \text{Min}\,(D_{1t}, S_{1t}) \\ Q_{2t} &= \text{Min}\,(D_{2t}, S_{2t}) \end{aligned} \qquad (8.2)$$

Quandt did not consider the logical consistency of the model. This is considered in Amemiya (1977) and Gourieroux et.al. (1978).

Consider the regimes:

$$\begin{aligned} R_1&: D_1 \geq S_1 \;\cdot\; D_2 \geq S_2 \\ R_2&: D_1 \geq S_1 \;\cdot\; D_2 < S_2 \\ R_3&: D_1 < S_1 \;\cdot\; D_2 < S_2 \\ R_4&: D_1 < S_2 \;\cdot\; D_2 \geq S_2 \end{aligned} \qquad (8.3)$$

In regime 1, we have $Q_1 = S_1$, $Q_2 = S_2$ and substituting these in (8.1) we have

$$A_1 \begin{bmatrix} D_1 \\ S_1 \\ D_2 \\ S_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\alpha_1 \\ 0 & 1 & 0 & -\beta_1 \\ 0 & -\alpha_2 & 1 & 0 \\ 0 & -\beta_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_1 \\ S_1 \\ D_2 \\ S_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

Similarly, we can define the corresponding matrices A_2, A_3, A_4 in regimes R_2, R_3, R_4 respectively that give the mapping from (D_1, S_1, D_2, S_2) to (u_1, u_2, u_3, u_4).

$$A_2 = \begin{bmatrix} 1 & 0 & -\alpha_1 & 0 \\ 0 & 1 & -\beta_1 & 0 \\ 0 & -\alpha_2 & 1 & 0 \\ 0 & -\beta_2 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & -\alpha_1 & 0 \\ 0 & 1 & -\beta_1 & 0 \\ -\alpha_2 & 0 & 1 & 0 \\ -\beta_2 & 0 & 0 & 1 \end{bmatrix}$$

and $$A_4 = \begin{bmatrix} 1 & 0 & 0 & -\alpha_1 \\ 0 & 1 & 0 & -\beta_1 \\ -\alpha_2 & 0 & 1 & 0 \\ -\beta_2 & 0 & 0 & 1 \end{bmatrix}$$

The logical consistency or 'coherency' conditions derived by Gourieroux et.al. are that the determinants of these four matrices i.e. $(1-\beta_1\beta_2)$, $(1-\alpha_2\beta_1)$, $(1-\alpha_1\alpha_2)$, $(1-\alpha_1\beta_2)$ must be the same sign.

The major problem that the multimarket disequilibrium models are supposed to throw light on (which the models in equations (8.1) and (8.2) does not) refers to the "spill-over effects" -- the effects of unsatisfied demand or supply in one market on the demand and supply in other markets. Much of this discussion on spill-over effects has been in the context of macro-models, the two markets considered are the commodity market and the labor market. The commodity is supplied by producers and consumed by households. Labor is supplied by households and used by producers. The quantities actually transacted are given by

$$C = \text{Min}\ (C^d, C^s) \qquad L = \text{Min}\ (L^d, L^s) \tag{8.4}$$

The demands and supplies actually presented in each market are called "effective" demands and supplies and these are determined by the exogenous variables and the endogenous quantity constraints (8.4). By contrast, the "notional" demands and supplies refer to the unconstrained values. Denote these by $\bar{C}^d$, $\bar{C}^s$, $\bar{L}^d$, $\bar{L}^s$. The different models of multi-market disequilibrium differ in the way 'effective' demands and "spill-over effects" are defined. Gourieroux et.al. (1980) define the effective demands and 'spill-over effects' as follows:

<u>Model I</u>:

$$\begin{aligned} C^d &= \bar{C}^d && \text{if} \quad L = L^s \le L^d \\ &= \bar{C}^d + \alpha_1(L - \bar{L}^s) && \text{if} \quad L = L^d < L^s \end{aligned} \tag{8.5}$$

$$\begin{aligned} C^s &= \bar{C}^s && \text{if} \quad L = L^d \le L^s \\ &= \bar{C}^s + \alpha_2(L - \bar{L}^d) && \text{if} \quad L = L^s < L^d \end{aligned} \tag{8.6}$$

$$\begin{aligned} L^d &= \bar{L}^d && \text{if} \quad C = C^s \le C^d \\ &= \bar{L}^d + \beta_1(C - \bar{C}^s) && \text{if} \quad C = C^d < C^s \end{aligned} \tag{8.7}$$

$$\begin{aligned} L^s &= \bar{L}^s && \text{if} \quad C = C^d \le C^s \\ &= \bar{L}^s + \beta_2(C - \bar{C}^d) && \text{if} \quad C = C^s < C^d \end{aligned} \tag{8.8}$$

This specification is based on Clower (1965) and Malinvand (1977) and assumes that agents on the short-side of the market present their notional demand as their effective demand in the other market. For instance equation (8.5) says that if households are able to sell all the labor they want to, then their effective demand for goods is the same as their 'notional' demand. On the other hand, if they cannot sell all the labor they want to, there is a "spill-over effect" but note that this is proportional to $L - \bar{L}^s$ not $L - L^s$. (i.e. it is proportional to the difference between actual labor sold and the 'notional' supply of labor).

The model considered by Ito (1980) is as follows:

<u>Model II</u>:

$$C^d = \bar{C}^d + \alpha_1(L - \bar{L}^s) \tag{8.5'}$$

$$C^s = \bar{C}^s + \alpha_2(L - \bar{L}^d) \tag{8.6'}$$

$$L^d = \bar{L}^d + \beta_1(C - \bar{C}^s) \tag{8.7'}$$

$$L^s = \bar{L}^s + \beta_2(C - \bar{C}^d) \tag{8.8'}$$

An alternative model suggested by Portes (1977) based on work by Benassy is the following:

<u>Model III</u>:

$$C^d = \overline{C}^d + \alpha_1(L - L^s) \quad (8.5'')$$

$$C^s = \overline{C}^s + \alpha_2(L - L^d) \quad (8.6'')$$

$$L^d = \overline{L}^d + \beta_1(C - C^s) \quad (8.7'')$$

$$L^s = \overline{L}^s + \beta_2(C - C^d) \quad (8.8'')$$

Portes compares the reduced forms for these three models and argues that econometrically, there is little to choose between the alternative definitions of effective demand.

The conditions for logical consistency (or coherency) are the same in all these models viz: $0<\alpha_i\beta_j<1$ for i,j = 1,2. Both Gourieroux et.al. (1980) and Ito (1980) derive these conditions, suggest price and wage adjustment equations similar to those considered in Section 5, and discuss the maximum likelihood estimation of their models. Ito also discusses two-stage estimation similar to that proposed by Amemiya for the Fair and Jaffee model, and derives sufficient conditions for the uniqueness of a quantity-constrained equilibrium in his model. We cannot go into the details of all these derivations here. The details involve more of algebra than any new conceptual problems in estimation. In particular, the problems mentioned in Section 5 about the different price adjustment equations apply here as well. There is as yet no empirical example illustrating the estimation of these multi-market disequilibrium models. There is, on the other hand, an enormous amount of theoretical literature in this area. One major problem, from the empirical point of view is that the discussion of the multi-market disequilibrium models has

been entirely in the context of a macro-model. Thus, when people think of an empirical application, they think only of a macro-model. One can consider spill-over effects in other models as well.

For instance, consider two commodities which are substitutes in consumption (say natural gas and coal) one of which has price controls. We can define the demand and supply functions in the two markets (omitting the exogenous variables) as follows:

$$\begin{aligned} D_1 &= \alpha_1 P_1 + \beta_1 P_2 + u_1 \\ S_1 &= \alpha_2 P_1 + u_2 \\ Q_1 &= \text{Min } (D_1, S_1) \\ P_1 &\le \overline{P} \\ D_2 &= \gamma_1 P_2 + \delta_1 P_1 + \lambda(D_1 - S_1) + V_1 \\ S_2 &= \gamma_2 P_2 + V_2 \\ Q_2 &= D_2 = S_2 \text{ i.e. the second market is always in equilibrium.} \end{aligned} \quad (8.9)$$

If $P_1 \le \overline{P}$, we have the usual simultaneous equations model with the two quantities and two prices as the endogenous variables. If $P_1 > \overline{P}$, then there is excess demand in the first market and a spill-over of this into the second market. This model is still in a "partial equilibrium" framework but would have interesting empirical applications. It is at least one step forward from the single-market disequilibrium model which does not say what happens to the unsatisfied demand or supply.

In actual practice the unsatisfied demand spills over to other markets. But it will also 'spill over' into future trading sessions of the same market. This implies that the demand function is of the form:

$$D_t = \beta_1' X_t + \alpha(D_{t-1} - S_{t-1}) + u_t.$$

Models that consider such "inter-temporal" spill-overs are, however, more difficult to estimate than those that consider contemporaneous spill-overs into other markets.

9. MODELS WITH SELF-SELECTIVITY

There are many problems where the data we have are generated by individuals making choices of belonging to one group or the other (i.e. by individuals' self-selection). An early discussion of this problem of self-selectivity is in Roy (1951) who discusses the problem of individuals choosing between two professions: hunting and fishing, based on their productivity in each. The observed distribution of incomes of hunters and fishermen is determined by these choices.

Suppose Y_{1i} is the output of the i-th individual in hunting and Y_{2i} the output in fishing. Individual i will choose to be a hunter if $Y_{1i} > Y_{2i}$.

Assume that $(Y_1 Y_2)$ have a joint normal distribution with means (μ_1, μ_2) and covariance matrix $\begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$

Define $u_1 = Y_1 - \mu_1$

$u_2 = Y_2 - \mu_2$ $\qquad \sigma^2 = \text{Var}(u_1 - u_2)$

$$Z = \frac{\mu_1 - \mu_2}{\sigma} \text{ and } u = \frac{u_2 - u_1}{\sigma}$$

The condition $Y_1 > Y_2$ implies $u < Z$.

The mean income of hunters is given by

$$E(Y_1 | u<Z) = \mu_1 - \sigma_{1u} \frac{\phi(Z)}{\Phi(Z)} \qquad (9.1)$$

Where $\sigma_{1u} = \text{Cov}(u_1, u)$ and $\phi(\cdot)$ and $\Phi(\cdot)$ are respectively the density function and the distribution function of the standard normal.[5] The mean income of fishermen is given by

$$E(Y_2 | u>Z) = \mu_2 + \sigma_{2u} \frac{\phi(Z)}{1-\Phi(Z)} \qquad (9.2)$$

where $\sigma_{2u} = \text{Cov}(u_2, u)$.

Since $\sigma_{1u} = \frac{\sigma_{12} - \sigma_1^2}{\sigma}$ and $\sigma_{2u} = \frac{\sigma_2^2 - \sigma_{12}}{\sigma}$ we have $\sigma_{2u} - \sigma_{1u} > 0$.

We can now consider different cases.

Case (i)

$\sigma_{1u} < 0$, $\sigma_{2u} > 0$. In this case the mean income of hunters is $> \mu_1$ and the mean income of fishermen is $> \mu_2$ i.e. those who have chosen hunting are better than average hunters and those who have chosen fishing are better than average fishermen.

Case (ii)

$\sigma_{1u} < 0$, $\sigma_{2u} < 0$. In this case the mean income of hunters is $> \mu_1$

and the mean income of fishermen is $< \mu_2$. In this case those who chose hunting are better than average in both hunting and fishing but they are better in hunting than fishing. Those who chose fishing are below average in both hunting and fishing but they are better in fishing than hunting.

Case (iii)

$\sigma_{1u} > 0$, $\sigma_{2u} > 0$. This is the reverse case of case (ii).

Case (iv)

$\sigma_{1u} > 0$, $\sigma_{2u} < 0$. This is not possible given the definitions of σ_{1u} and σ_{2u}.

Note that case (ii) typically occurs if σ_1 is very large compared to σ_2. Thus the better skilled individuals go into the profession with higher variance in earnings.

More detailed analysis of this model can be found in Roy (1951). The important thing to note here is the importance of the covariance terms σ_{1u} and σ_{2u} in the interpretation of the results. We will see later how they play an important role in discussions of selectivity bias.

The econometric discussion of the consequences of self-selectivity started with the papers by Gronau (1974), Lewis (1974) and Heckman (1974). In this case the problem is about women choosing to be in the labor force or not. The observed distribution of wages is a truncated distribution. It is the distribution of wage offers truncated by reservation wages. The Gronau-Lewis model consisted of two equations:

$$\begin{aligned} W_o &= \chi\beta_1 + u_1 \\ W_r &+ \chi\beta_2 + u_2 \end{aligned} \qquad (9.3)$$

We observe $W = W_o$ iff $W_o \geq W_r$. Otherwise $W = 0$. We discussed the estimation of this model in section 2 and we will not repeat it here. The term 'selectivity bias' refers to the fact that if we estimate equation (9.3) by OLS based on the observations for which we have wages W, we get inconsistent estimates of the parameters.

Note that $E(u_1 | W_o \geq W_r) = -\sigma_{1u} \frac{\phi(Z)}{\Phi(Z)}$

Where $Z = \frac{\chi\beta_1 - \chi\beta_2}{\sigma}$ and the other terms are as defined earlier. Hence we can write (9.3) as:

$$W = \chi\beta_1 - \sigma_{1u} \frac{\phi(Z)}{\Phi(Z)} + V \qquad (9.4)$$

Where $E(V)=0$.

A test for selectivity bias is a test for $\sigma_{1u}=0$. Heckman (1976) suggested a two-stage estimation method for such models. First get consistent estimates for the parameters in Z by the probit method applied to the dichotomous variable (in the labor force or not). Then estimate equation (9.4) by OLS using the estimated values $\hat{Z}$ for Z.

The self-selectivity problem has since been analyzed in different contexts by several people. Lee (1978) has applied it to the problem of unions and wages. Lee and Trost (1978) have applied it to the problem of housing demand with choices of owning and renting. Willis and Rosen (1979) have applied the model to the problem of education and self-selection. These are all switching regression models. Griliches et.al. (1979) and Kenny et.al. (1979) consider models with both selectivity and simultaneity. These models are switching simultaneous equations models. As for methods of estimation, both two-stage and maximum likelihood methods have been used. For two-stage methods, the paper by Lee et.al. (1980) gives the asymptotic covariance matrices when the selectivity criterion is of the probit and tobit types.

In the literature on self-selectivity a major concern has been with testing for selectivity bias. These are tests for $\sigma_{1u}=0$ and $\sigma_{2u}=0$ in equations of the form (9.1) and (9.2). However, a more important issue is the sign and magnitude of these covariances and often not much attention is devoted to this. In actual practice we ought to have $\sigma_{2u}-\sigma_{1u}>0$ but σ_{1u} and σ_{2u} can have any signs. It is also important to estimate the mean values of the dependent variables for the alternate choice. For instance, in the case of college education and income, we should estimate the mean income of college graduates had they chosen not to go to college, and the mean income of non-college graduates had they chosen to go to college. In the example of hunting and fishing we should compute the mean income of hunters had they chosen to be fishermen and the mean income of fishermen had they chosen to be hunters. Such computations throw light on the effects of self-selection and also reveal deficiencies in the model which simple tests for the existence of selectivity bias do not.

In the simplest two equation model given by equations (1.1) to (1.4), if we denote by C_1 and C_2 the choices of groups 1 and

2 respectively, we have:

$$E(u_j|C_1) = -\sigma_{ju}\frac{\phi(Z\alpha)}{\Phi(Z\alpha)} \qquad j = 1,2$$

and

$$E(u_j|C_2) = \sigma_{ju}\frac{\phi(Z\alpha)}{1-\Phi(Z\alpha)} \qquad j = 1,2$$

Once the parameters in the model have been estimated, one can use these expressions to compute the mean value of y for the two groups of individuals under the alternative choice. One should also interpret the error covariances, as was done in the simple example of hunting and fishing earlier.

In the literature on labor supply, there has been considerable discussion of "individual heterogeneity" i.e. the observed self-selection is due to individual characteristics not captured by the observed variables (some women want to work no matter what and some women want to sit at home no matter what). Obviously, these individual specific effects can only be analyzed if we have panel data. This problem has been analyzed by Heckman, but since these problems will be discussed in the chapters on labor supply models and analysis of cross-section and time-series data they will not be elaborated here.

10. MULTIPLE CRITERIA FOR SELECTIVITY

There are several practical instances where selectivity could be due to several sources rather than just one as considered in the examples in the previous section. Griliches et.al. (1979) cite several problems with the NLS young men data set that could lead to selectivity bias. Prominent among these are attrition and (other) missing data problems. In such cases we would need to formulate the model as switching regression or switching simultaneous equations models where the switch depends on more than one criterian function. One such example is that by Abowd and Farber (1979) who consider the union and wages example of Lee (1978). The model consists of a union wage equation (Y_1) and a non-union wage equation (Y_2). There are two decision functions: the decision of individuals to join a queue for union jobs (I_1^*) and the decision of employers to draw individuals from the queue (I_2^*). The specification of the model is:

$$Y_1 = \chi_1\beta_1 + u_1 \tag{10.1}$$

$$Y_2 = \chi_2\beta_2 + u_2 \tag{10.2}$$

$$I_1^* = Z_1\gamma_1 - \varepsilon_1 \tag{10.3}$$

$$I_2^* = Z_2\gamma_2 - \varepsilon_2 \tag{10.4}$$

If $I_1^* > 0$ the individual decides to join the queue for union jobs.

If $I_2^* > 0$ the individual is chosen from the queue for a union job. Here we observe Y_1 only if $I_1^* > 0$ and $I_2^* > 0$. In this example, the set $I_1^* < 0$ and $I_2^* > 0$ will be empty.

The analysis of the model in equations (10.1) to (10.4) will depend crucially on whether the two decisions are independent or correlated i.e. whether $\text{Cov}(\varepsilon_1,\varepsilon_2) = 0$ or not. In case $\text{Cov}(\varepsilon_1,\varepsilon_2) = 0$ we can easily extend the Heckman-Lee two-stage estimation methods to this model.

Define $\quad \lambda_{ij} = \text{Cov}(u_i,\varepsilon_j) \qquad i = 1,2$

$\qquad\qquad\qquad\qquad\qquad\qquad j = 1,2$

Then

$$E(u_1|I_1^* > 0,\ I_2^* > 0) = -\lambda_{11}\frac{(\phi Z_1\gamma_1)}{\Phi(Z_1\gamma_1)} - \lambda_{12}\frac{\phi(Z_2\gamma_2)}{\Phi(Z_2\gamma_2)}$$

Thus one gets preliminary consistent estimates of γ_1 and γ_2 by estimating equations (10.3) and (10.4) by the probit method. Next one regresses Y_1 on χ_1 and the constructed variables

$$\frac{\phi(Z_1\hat{\gamma}_1)}{\Phi(Z_1\hat{\gamma}_1)} \quad \text{and} \quad \frac{\phi(Z_2\hat{\gamma}_2)}{\Phi(Z_2\hat{\gamma}_2)}$$

In case ε_1 and ε_2 are correlated so that $\text{Cov}(\varepsilon_1,\varepsilon_2) = \sigma_{12}$ the expressions get very messy. In this case we have to use bivariate probit methods to estimate γ_1,γ_2 and σ_{12}. Further

$$E(u_1|I_1^* > 0,\ I_2^* > 0) = \lambda_{11}M_{12} + \lambda_{12}M_{21}$$

where

$$M_{ij} = (1 - \sigma_{12}^2)^{-1}\left[P_i - \sigma_{12}P_j\right]$$

$$\text{and } P_j = \frac{\int_{-\infty}^{Z_1\gamma_1}\int_{-\infty}^{Z_1\gamma_1} \varepsilon_j f(\varepsilon_1\varepsilon_2)d\varepsilon_2 d\varepsilon_1}{F(Z_1\gamma_1,Z_2\gamma_2)} \tag{10.5}$$

These expressions can still be evaulated numerically.

There are as yet not many empirical examples where the selectivity is based on multiple criterion functions. Abowd and Farber (1979) claim that their model with two decisions captures the effects of unions on wages better than a simple probit model for union status. However, their model with independent decision equations is not likely to be of general applicability. Fishe et.al. (1979) consider a model with two correlated decision criteria. The model is one that determines wages of young women -- some of whom have college education others not. The two decision equations (10.3) and (10.4) refer to the decision of whether to go to college or not and whether to join the labor force or not. Though the example is somewhat contrived, this is the only empirical illustration with two correlated selectivity criteria. Fishe et.al. estimate the parameters in equations (10.3) and (10.4) by the bivariate probit method and evaluate expressions of the form (10.5) by numerical methods. They then use the extension of the Heckman-Lee two-stage procedure.

11. CONCLUDING REMARKS AND SUGGESTIONS FOR FURTHER RESEARCH

We have examined the literature on disequilibrium and self-selection. Both classes of models fall in the category of switching models with endogenous switching. In some problems the model is a switching regression model; in others it is a switching simultaneous equations model. The estimation methods are by now well-known. Many investigators have used maximum likelihood methods and even when these are not computationally feasible, we have two-step methods available that are easily computable and that give consistent estimates for the parameters. Of greater importance at this time is the empirical application of these methods in problems of some practical consequence. The literature on self-selection contains interesting empirical applications in the areas of labor

supply, unions and wages and education and self-selfselection. However, the literature on disequilibrium models lacks any interesting empirical applications. Part of the problem here is that not much thought is often given to the substantive question of what the sources of disequilibrium are and also there are few micro data sets to which the methods have been applied. Almost all applications (Avery [1979] is perhaps an exception) are based on aggregate time-series data and there is not enough discussion of problems of aggregation[6]. The Fair and Jaffee example on the housing market as well as the different models of "credit rationing" are all based on aggregate data and there is much to be desired in the detailed specification of these models.

Perhaps the most interesting application of the disequilibrium models are in the areas of regulated industries. After all it is regulation that produces disequilibrium in these markets. Consider for instance the loan demand problem with interest rate ceilings. If one has data on individual loans, one can formulate a demand and supply model and examine the effects of interest rate ceilings. The appropriate model in this case is a switching simultaneous equations model which is similar to the disequilibrium model discussed in section 6. If the rate of interest that equilibrates demand and supply is at or below the rate ceiling, the loan is granted. Otherwise the loan is denied. We have:

$$\left.\begin{array}{ll}\text{Loan demand} & L_t = \alpha_1 R_t + \beta_1' X_{1t} + u_{1t} \\ \text{Loan supply} & L_t = \alpha_2 R_t + \beta_2' X_{2t} + u_{2t}\end{array}\right\} \text{ if } R_t \leq \bar{R}$$

$$L_t = 0 \text{ otherwise.}$$

Structurally, this model is similar to the labor supply model considered by Heckman (1974) where hours worked H adjust to equilibrate reservation wages and market wages and the individual works if $H > 0$.

Estimation of some disequilibrium models with micro-data sets for regulated industries and estimation of the effects of regulation would make the disequilibrium literature more intellectually appealing than it has been.

There are also some issues that need to be investigated regarding the appropriate formulation of the demand and supply functions under disequilibrium. The expectation of disequilibrium can itself be expected to change the demand and supply functions. Further, there will be a spill-over from past disequilibrium into the current demand and supply. Such spill-over effects have as yet not been analyzed in any of the disequilibrium models.

The literature on self-selection, by contrast to the disequilibrium literature, has several interesting empirical applications. However, even here a lot of work remains to be done. The case of selectivity being based on several criteria rather than one has been mentioned in the last section. Another problem is the extension of the methodology to other error distributions and the sensitivity of the results to the assumption of normality often

made (see Olsen [1979] for an initial attempt). Till some progress is made with alternative error distributions, one has to be careful in the choice of the functional form used. For instance where earnings functions are used, as often they are in these models, one can define the dependent variable in the log form, so that the assumption of normality is at least approximately valid.

FOOTNOTES

1. This procedure first used by Heckman (1976) for the labor supply model was extended to a wide class of models by Lee (1976).

2. Hartley and Mallela (1977) prove the strong consistency of the maximum likelihood estimator but on the assumption that σ_1 and σ_2 are bounded away from zero.

3. The two-stage least squares methods described earlier can be easily applied for these models as well. We replace ΔP_t by ΔP_{t+1} in the definitions of Z_{1t} and Z_{2t}, γ by γ_1 in equation (C) and γ by γ_2 in equation (D). Also, P_t is no longer endogenous. Only Z_{1t} and Z_{2t} are endogenous.

4. The formulation in terms of partial adjustment towards P* was suggested by Bowden (1978) though he does not use the interpretation of the Fair-Jaffee equation given here.

5. These are well-known formulae for the moments of truncated normal. See Johnson and Kotz (1972) pp. 112-113.

6. The papers by Batchelor (1977) and Muellbauer and Winter (1979) deal with aggregation problems in disequilibrium models.

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