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ABSTRACT

There is a surprising lack of congruity between the A-J literature and the peak load pricing literature. Much of the A-J literature assumes increasing returns to scale. This can be contrasted with the theory of peak-load pricing which focuses on the case of decreasing returns to scale. This paper extends the theory of peak load pricing by considering the case where the average variable cost curve initially exhibits increasing returns to scale. The principal result is that off-peak users should rarely shoulder the burden of capacity costs.

Peak Load Pricing: Who Should Pay?

A central issue in the peak load pricing debate is the allocation of capacity costs. While the conventional wisdom dictates that peak users should pay for marginal capacity costs, recent articles in this journal argue that capacity costs should be apportioned between peak and off-peak users (see Wenders, Spring 1976; Panzar, Autumn 1976). This comment examines peak load pricing under the assumption that the cost curve initially exhibits increasing returns to scale. The results support the view that off-peak users should not bear the burden of capacity costs. Furthermore, under the assumption of a classical U-shaped average cost curve, revenues from users in off-peak periods may fail to cover variable costs.

The possibility of a U-shaped average variable cost curve arises because it is sometimes technologically infeasible or economically inefficient to turn plants off during periods when they are not needed. While many old steam plants were built for cycling, often this is not feasible for newer models because of heating and cooling constraints. In the appendix, we outline a model which would yield a U-shaped curve in the case of discrete plant types. The body of the paper focuses on a more recent extension of the literature to the continuous case.

The model employed here is exactly analogous to the one which Panzar uses in his neoclassical approach. The results are driven by the assumptions regarding the shape of the production function. These remarks are focused on Panzar's second proposition, which states that "consumers in all periods make a positive contribution toward the cost of capital

inputs," assuming the neoclassical production function exhibits decreasing returns to scale for <u>all</u> positive levels of output.[1]

The proof will be reconstructed in a simpler notation. This will be followed by a discussion of the robustness of the results and an illustration of the model in the two factor case. We assume initially that the production function f is a continuously differentiable, quasi-concave function. Inputs consist of a vector of fixed factors K and a vector of variable factors L^t, where the superscript t denotes the fact that the vector L can differ over time. Output, Q^t, is given by the following equation:

$$f(K,L^t) - Q^t \geq 0; \; K = (K_1,\ldots,K_m); \; t = 1,\ldots,T; \; L^t = (L_1^t,\ldots,L_n^t) \qquad (1)$$

For positive output, the marginal product of all fixed factors is assumed to be strictly greater than zero; in addition, positive amounts of each factor are required for positive production. Analogous to Panzar, we initially assume the elasticity of scale, e_s, is less than one.

$$e_s \equiv \sum_{i=1}^{n} L_i^t \frac{\partial f}{\partial L_i^t} \Big/ f(K,L^t) < 1 \;\; \forall L^t, \; K \neq 0 \qquad (1a)$$

Prices for inputs are determined exogenously and will be denoted by the vectors W and B:[2]

$$W = (W_1,\ldots,W_n) \qquad B = (B_1,\ldots,B_m)$$

The price of the ith variable input is given by W_i, and similarly, the cost of hiring the ith capital input for all T periods is given by B_i.

Output prices are expressed as a function of demands which are assumed to be independent across periods. Denoting the price in period t by P^t, we have

$$P^t = P^t(Q^t) \geq 0, \; dP^t/dQ^t \leq 0 \quad t = 1,\ldots,T.$$

The maximization of producer's and consumer's surplus yields the following Lagrangian expression:

$$\max_{K,L^t,Q^t} \; L = \sum_{t=1}^{T} \int_0^{Q^t} P^t(Q)dQ - \sum_{t=1}^{T} W\cdot L^t - B\cdot K - \sum_{t=1}^{T} \lambda^t \left[Q^t - f(K,L^t)\right]$$

subject to $Q^t \geq 0$, $L^t \geq 0$, $K \geq 0$,

where the Lagrange multiplier, λ^t, represents the marginal value of an incremental increase in output in period t.

From the Kuhn-Tucker theorem, we derive the following abbreviated set of necessary conditions for an optimum:

$$P^t = \lambda^t \;\forall Q^t > 0 \quad 1 = 1,\ldots,T \tag{2}$$

$$W_i = \lambda^t \frac{\partial f}{\partial L_i^t} \;\forall L_i^t > 0 \quad i = 1,\ldots,n \quad t = 1,\ldots,T \tag{3}$$

$$B_i = \sum_{t=1}^{T} \lambda^t \frac{\partial f}{\partial K_i} \;\forall K_i > 0 \quad i = 1,\ldots,m \tag{4}$$

$$Q^t = f(K,L^t) \;\forall \lambda^t > 0 \quad t = 1,\ldots,T \tag{5}$$

If we assume output and prices are greater than zero in all periods, it is straight-forward to show that consumers in every period make a positive contribution to capital costs.

Proof: Define N^t as the excess of revenues over variable costs in period t. Then, we would like to show that N^t is positive in all periods.

Definition: $N^t \equiv P^tQ^t - W\cdot L^t$

Substitution from (2) and (5) yields

$$N^t = \lambda^t f(K,L^t) - W\cdot L^t$$

Summing (3) over all variable factors and multiplying by the respective quantities gives

$$W\cdot L^t = \sum_{i=1}^{n} \lambda^t \frac{\partial f}{\partial L^t} \cdot L_i^t$$

Finally, substitution into the expression for N^t gives

$$N^t = \lambda^t[f(K,L^t) - \sum_{i=1}^{n} \frac{\partial f}{\partial L^t} \cdot L_i^t] \tag{6}$$

which is the desired result, since the expression in parentheses is positive by assumption (1a), and $\lambda^t = P^t(Q^t) > 0$ by hypothesis.

To better understand the implications of this theorem, it is useful to consider the case where there is one variable input (L^t), and one fixed input (K). In this case, equation (6) can be rewritten in more familar terms by factoring out the variable input.

$$N^t = \lambda^t L^t[f(K,L^t)/L^t - \partial f/\partial L^t] \quad (6a)$$

Inspection of the terms inside the parentheses reveals that it is merely the difference between the average product of the variable factor and its marginal product. Since the prices of output are given, the average product and marginal product are inversely related to the average variable cost and marginal cost, respectively. For the two factor case, the general relationship between the cost curves is depicted in Figure 1:

Figure 1

Thus, the two factor case reveals that assumption (1a) leads to cost curves with the somewhat special attribute that marginal cost exceeds average variable cost for all positive levels of output.[3] Unfortunately, relaxation of this assumption calls Panzar's Proposition 2 into question. Let us consider the conventional case of the U-shaped average cost curve. It has been shown elsewhere that a necessary and sufficient condition for such a curve is that the elasticity of scale decreases through unity.[4] Using this result, we will show that Panzar's Proposition 2 no longer obtains.

Since Proposition 2 is concerned with the set of outputs greater than zero, equations (2)-(5) form a subset of the necessary conditions for an optimum,[5] and the expression given for N^t in equation (6) is still valid. Our assumption of a U-shaped average cost curves implies the elasticity of scale is greater than unity over some positive interval, which

is sufficient to guarantee equation (6) is negative for some positive levels of output, thus disproving the proposition. An example of a negative value for the contribution to capital in a particular period is given below for the two factor case. The marginal cost curve (MC) and the average variable cost curve (AVC) exhibit the conventional shapes. DD' represents a hypothetical single period demand curve.

Figure 2

Figure 2 is constructed so that the integral under the demand curve from zero to Q_o is greater than the total variable cost associated with producing Q_o. Since DD' intersects the marginal cost curve only once, (P_o,Q_o) is the equilibrium price-output combination in this period. From the diagram, we see that $AC(Q_o)$ exceeds $MC(Q_o)$, which implies the marginal product is greater than the average product of labor. In conjunction with equation (6a), this last relationship gives the desired result: $N^t < 0$.

Note that if output is positive in all periods, it must be the case that the excess of surplus over variable costs must be nonnegative, i.e.

$$\int_0^{Q^t} P^t(Q)\,dQ - W \cdot L^t \geq 0 \;\forall\; t \quad (7)$$

If, for any given period t, there were no positive level of output for which surplus was greater than or equal to variable costs, output should be set equal to zero. For example, consider the two factor case illustrated in Figure 3. The notation is the same as that of Figure 2; however, the position of the demand curve, DD', is different.

Figure 3

If, for the moment, we were to ignore the restriction imposed by (7), the price-quantity combination associated with equations (2)-(5) would correspond to (P_o,Q_o). However, since this point violates condition (7), the value of the Lagrangian would actually decrease if Q_o were produced. We avoid this decrement in the value of the objective function by not producing in this period.

Relaxing the assumption of strict short-run concavity also calls two other important results into question: the positive relationship between output and contribution toward capacity cost, and the strictly monotonic relationship between price and output. These results no longer obtain because it is possible to construct an example where production takes place at a point where MC is decreasing. An illustrative example for the two factor case is shown below:

Figure 4

Since the only restriction on demand is that it be downward sloping, it will always be possible to satisfy (7) by making DD' less elastic, and hence, ensuring an interior solution.

What remains to be shown within the context of the neoclassical paradigm is the extent to which off-peak users should be responsible for capital costs. The answer to this problem hinges on how capacity is defined. Panzar begs the question by defining capacity in such a way as

to make it uneconomical to operate there.[6] An alternative approach which seems intuitive is to define capacity as the point where the elasticity of scale equals unity.[7] In the two factor model presented earlier, this is equivalent to defining capacity as the minimum point on the average variable cost curve. Using this definition, one arrives at the somewhat curious result that off-peak[8] users not only fail to contribute to capital costs, but also do not cover the costs of variable inputs.[9] This conclusion is diametrically opposed to those of Panzar and Wenders and, interestingly enough, tends to agree with earlier approaches which assume a rigid production technology (e.g., see Steiner and Boiteux).

APPENDIX

This section derives a U-shaped average variable cost curve for the case of two plants, A and B, with constant operating costs, b_1 and b_2. The idea motivating the example is that during the off peak-period, there is at least one plant which must operate at some percent of capacity. For example, it might be the case that it is cheaper to operate a plant at 20% of capacity during off-peak than actually turning the plant off and then on again.

To obtain a U-shaped cost curve, it is sufficient to assume that any plant except the one with lowest operating costs must be run at a minimal level during the off-peak period. The illustrative example considers the case of two plants with capacities $\overline{Q}_A$ and $\overline{Q}_B$, respectively. For simplicity, we assume there are two time periods, peak and off-peak. Treating capital costs as fixed, the problem is to maximize consumer surplus less variable costs, subject to capacity constraints, technological constraints, and feasibility constraints. Mathematically, this reduces to the following problem:

$$\underset{(Q^1,Q^2,Q_A^1,Q_A^2,Q_B^1,Q_B^2)}{\text{MAXIMIZE}} \quad \sum_{t=1}^{2} \int_0^{Q^t} P^t(Q)dQ - \sum_{t=1}^{2} (b_1 Q_A^t - b_2 Q_B^t)$$

subject to

$$\text{(8a)} \quad Q_A^t \leq \overline{Q}_A \qquad t=1,2$$

$$\text{(8b)} \quad Q_B^t \leq \overline{Q}_B \qquad t=1,2$$

$$\text{(8c)} \quad Q_A^t \geq Q_A^* \qquad t=1,2$$

$$\text{(8d)} \quad Q_B^t \geq Q_B^* \qquad t=1,2$$

$$\text{(8e)} \quad Q^t \geq Q_A^* \qquad t = 1,2$$

$$\text{(8f)} \quad Q^t \leq Q_A^t + Q_B^t \quad t = 1,2$$

There are two plants, A and B. 'A' can be thought of as the baseload plant since we will assume b_1 is less than b_2. The amount of electricity supplied by plant A in period t is Q_A^t. Similarly, the electricity supplied by B is Q_B^t. Q_A^* and Q_B^* represent the minimum output level of plants A and B under the assumption that they will be used for both the peak and off-peak periods. Constraints (8a) and (8b) merely say that the supply from either plant cannot exceed capacity; (8c) and (8d) indicate supply must be greater than some minimum amount. (8e) assumes that demand in period t, denoted by Q^t, will always exceed the minimum amount supplied by the base plant. Period 1 is taken to be the off-peak period, (i.e., $Q^1 < Q^2$). The peak and off-peak problem is probably best couched in the time frame of a day as opposed to a seasonal variation. The last set of constraints, (8f), impose the restriction that demand cannot exceed supply.

Setting up the Lagrangian yields four cases of interest which are shown in Table 1.

<u>Table 1</u>

Case 1 assumes that off-peak demand is less than the minimum supply $(Q_A^* + Q_B^*)$. In this case, off-peak users should not be charged for the use of electricity. Note that peak users are always charged b_2. This results from the assumption that even peak users do not bump up against the capacity constraint.

Columns 3 and 4 give the average variable costs and the direction of change in AVC as Q^1 increases. Note that these costs decrease until Q^1 exceeds $\overline{Q}_A + Q_B^*$. Combining the results on marginal cost with those on average variable costs generates the following graph:

Figure 5

Thus, we see that the MC curve is given by a step function while the AVC curve exhibits the desired shape.

FOOTNOTES

1. Panzar, p. 526.

2. All factor prices are assumed to be positive.

3. Proof: In the problem, minimize variable cost (VC) subject to $Q^t - f(K,L^t) = 0$, let λ be the multiplier. Then $\lambda = \frac{\partial VC}{\partial Q^t}$ and $W_i = \lambda \frac{\partial f}{\partial L_i^t}$. $AVC = \frac{\sum W_i L_i^t}{Q^t} = \frac{\lambda \sum \frac{\partial f}{\partial L_i^t} L_i^t}{Q^t}$ by the first order conditions.

Thus, $MC > AVC \iff \lambda > \frac{\lambda \sum \frac{\partial f}{\partial L_i^t} L_i^t}{Q^t} \iff Q^t > \sum \frac{\partial f}{\partial L_i^t} L_i^t$ true by (1a).

4. More specifically, we would like to guarantee the production function, f, yields U-shaped average cost curves for all p>>0. A necessary and sufficient condition is that the elasticity of scale, e_s, decreases through 1 along any expansion path (Hanoch, p. 495).

5. The assumption of an interior solution implies necessary conditions for a local maximum will also be necessary for a global maximum. Since a form of the rank condition is satisfied, equations (2)-(5) are necessary for a local maximum (See Takayama, pp. 91, 94).

6. In a recent extension of Panzar's work, Marino imposes bounds on the use of the variable input, but retains the assumption that the production function exhibits strict short-run concavity. This formulation of the problem allows for the possibility that excess capacity will not exist in all periods. Using a mode of analysis similar to the one presented in this paper, it is easily shown that Marino's principal theorems which extend Panzar's results do not obtain when the assumption of strict short-run concavity is relaxed.

7. Of course, this definition is not feasible in Panzar's paper because he assumes the elasticity of scale is always less than unity.

8. Off-peak refers to levels of output which are less than those associated with capacity.

9. This follows immediately from the result that N^t is less than zero at output levels where the elasticity of scale is less than unity.

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Figure 1

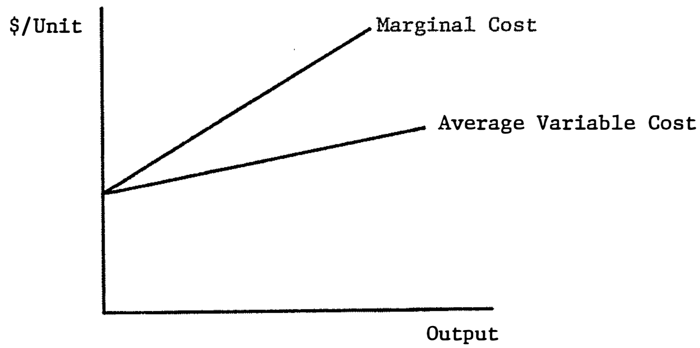

Figure 2

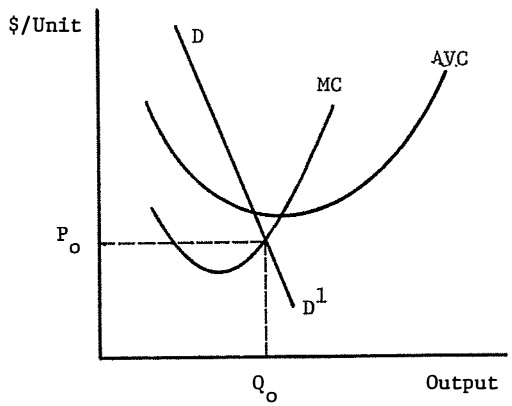

Figure 3

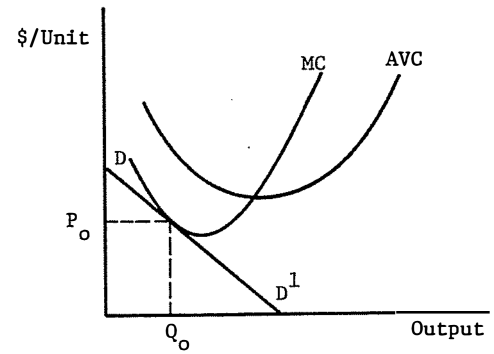

Figure 4

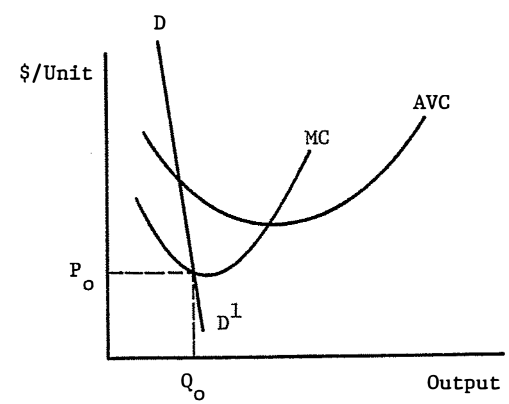

Figure 5: A U-shaped Average Variable Cost Curve: The Discrete Case

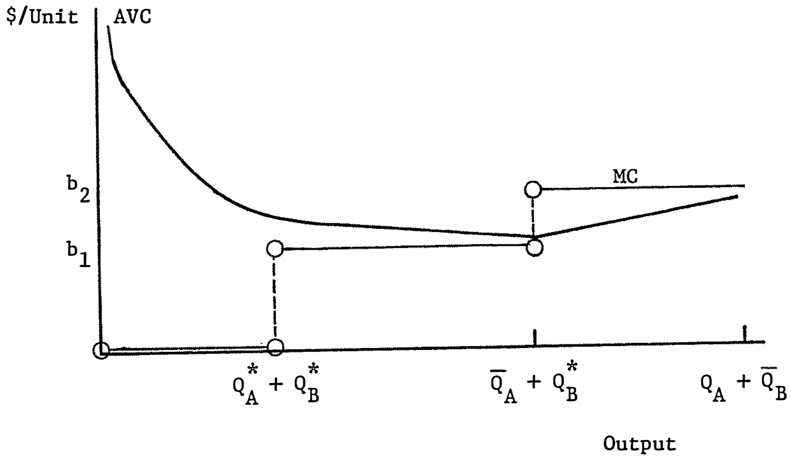

TABLE 1:

How Costs Change as a
Function of Off-Peak Output

	P^1	P^2	Average Variable Costs	$\frac{d(AVC)}{dQ^1}$
Case 1: $Q_A^* < Q^1 < Q_A^* + Q_B^*$	0	b_2	$(b_1 Q_A^* + b_2 Q_B^*)/Q^1$	<0
Case 2: $Q_A^* + Q_B^* < Q^1 < \overline{Q}_A$	b_1	b_2	$(b_1(Q^1-Q_B^*) + b_2 Q_B^*)/Q^1$	<0
Case 3: $\overline{Q}_A < Q^1 < \overline{Q}_A + Q_B^*$	$b_1 \le P^1 \le b_2$	b_2	(Same as Case 2)	<0
Case 4: $\overline{Q}_A + Q_B^* < Q^1 < Q^2$	b_2	b_2	$(b_1\overline{Q}_A + b_2(Q^1-\overline{Q}_A))/Q^1$	>0